

Optional Lab: Cost Function for Logistic Regression

Goals

In this lab, you will:

- examine the implementation and utilize the cost function for logistic regression.

```
In [1]: import numpy as np
        %matplotlib widget
        import matplotlib.pyplot as plt
        from lab_utils_common import plot_data, sigmoid, dlc
        plt.style.use('./deeplearning.mplstyle')
```

Dataset

Let's start with the same dataset as was used in the decision boundary lab.

```
In [2]: X_train = np.array([[0.5, 1.5], [1,1], [1.5, 0.5], [3, 0.5], [2, 2], [1, 2.5]]) # (m,n)
        y_train = np.array([0, 0, 0, 1, 1, 1]) # (m,)
```

We will use a helper function to plot this data. The data points with label $y = 1$ are shown as red crosses, while the data points with label $y = 0$ are shown as blue circles.

```
In [3]: fig, ax = plt.subplots(1,1,figsize=(4,4))
        plot_data(X_train, y_train, ax)

        # Set both axes to be from 0-4
        ax.axis([0, 4, 0, 3.5])
        ax.set_ylabel('$x_1$', fontsize=12)
        ax.set_xlabel('$x_0$', fontsize=12)
        plt.show()
```

Cost function

In a previous lab, you developed the *logistic loss* function. Recall, loss is defined to apply to one example. Here you combine the losses to form the **cost**, which includes all the examples.

Recall that for logistic regression, the cost function is of the form

$$J(\mathbf{w}, b) = \frac{1}{m} \sum_{i=0}^{m-1} \left[\text{loss}(f_{\mathbf{w},b}(\mathbf{x}^{(i)}), y^{(i)}) \right] \quad (1)$$

where

- $\text{loss}(f_{\mathbf{w},b}(\mathbf{x}^{(i)}), y^{(i)})$ is the cost for a single data point, which is:

$$\text{loss}(f_{\mathbf{w},b}(\mathbf{x}^{(i)}), y^{(i)}) = -y^{(i)} \log(f_{\mathbf{w},b}(\mathbf{x}^{(i)})) - (1 - y^{(i)}) \log(1 - f_{\mathbf{w},b}(\mathbf{x}^{(i)})) \quad (2)$$

- where m is the number of training examples in the data set and:

$$f_{\mathbf{w},b}(\mathbf{x}^{(i)}) = g(z^{(i)}) \quad (3)$$

$$z^{(i)} = \mathbf{w} \cdot \mathbf{x}^{(i)} + b \quad (4)$$

$$g(z^{(i)}) = \frac{1}{1 + e^{-z^{(i)}}} \quad (5)$$

Code Description

The algorithm for `compute_cost_logistic` loops over all the examples calculating the loss for each example and accumulating the total.

Note that the variables \mathbf{X} and \mathbf{y} are not scalar values but matrices of shape (m, n) and $(m,)$ respectively, where n is the number of features and m is the number of training examples.

```
In [4]: def compute_cost_logistic(X, y, w, b):
        """
        Computes cost

        Args:
            X (ndarray (m,n)): Data, m examples with n features
            y (ndarray (m,)) : target values
            w (ndarray (n,)) : model parameters
            b (scalar)       : model parameter

        Returns:
            cost (scalar): cost
        """

        m = X.shape[0]
        cost = 0.0
        for i in range(m):
            z_i = np.dot(X[i],w) + b
            f_wb_i = sigmoid(z_i)
            cost += -y[i]*np.log(f_wb_i) - (1-y[i])*np.log(1-f_wb_i)

        cost = cost / m
        return cost
```

Check the implementation of the cost function using the cell below.

```
In [ ]: w_tmp = np.array([1,1])
        b_tmp = -3
        print(compute_cost_logistic(X_train, y_train, w_tmp, b_tmp))
```

Expected output: 0.3668667864055175

Example

Now, let's see what the cost function output is for a different value of w .

- In a previous lab, you plotted the decision boundary for $b = -3$, $w_0 = 1$, $w_1 = 1$. That is, you had $b = -3$, $w = \text{np.array}([1,1])$.
- Let's say you want to see if $b = -4$, $w_0 = 1$, $w_1 = 1$, or $b = -4$, $w = \text{np.array}([1,1])$ provides a better model.

Let's first plot the decision boundary for these two different b values to see which one fits the data better.

- For $b = -3$, $w_0 = 1$, $w_1 = 1$, we'll plot $-3 + x_0 + x_1 = 0$ (shown in blue)
- For $b = -4$, $w_0 = 1$, $w_1 = 1$, we'll plot $-4 + x_0 + x_1 = 0$ (shown in magenta)

```
In [ ]: import matplotlib.pyplot as plt

# Choose values between 0 and 6
x0 = np.arange(0,6)

# Plot the two decision boundaries
x1 = 3 - x0
x1_other = 4 - x0

fig,ax = plt.subplots(1, 1, figsize=(4,4))
# Plot the decision boundary
ax.plot(x0,x1, c=dlc["dlblue"], label="$b$=-3")
ax.plot(x0,x1_other, c=dlc["dlmagenta"], label="$b$=-4")
ax.axis([0, 4, 0, 4])

# Plot the original data
plot_data(X_train,y_train,ax)
ax.axis([0, 4, 0, 4])
ax.set_ylabel('$x_1$', fontsize=12)
ax.set_xlabel('$x_0$', fontsize=12)
plt.legend(loc="upper right")
plt.title("Decision Boundary")
plt.show()
```

You can see from this plot that $b = -4$, $w = \text{np.array}([1,1])$ is a worse model for the training data. Let's see if the cost function implementation reflects this.

```
In [ ]: w_array1 = np.array([1,1])
b_1 = -3
w_array2 = np.array([1,1])
b_2 = -4

print("Cost for b = -3 : ", compute_cost_logistic(X_train, y_train, w_array1,
b_1))
print("Cost for b = -4 : ", compute_cost_logistic(X_train, y_train, w_array2,
b_2))
```

Expected output

Cost for b = -3 : 0.3668667864055175

Cost for b = -4 : 0.5036808636748461

You can see the cost function behaves as expected and the cost for $b = -4$, $w = \text{np.array}([1,1])$ is indeed higher than the cost for $b = -3$, $w = \text{np.array}([1,1])$

Congratulations!

In this lab you examined and utilized the cost function for logistic regression.

In []: