ICONHIC2019
2nd International Conference on Natural Hazards & Infrastructure
23-26 June, 2019, Chania, Greece

The effects of stiffness anisotropy on elastic solutions of circular foundations under complex loading

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ABSTRACT

Current wind turbine (WT) foundation codes and design approaches rely on rather conservative geotechnical assumptions to aid the simplification of design processes. Moreover, the industry often treats WT foundations as an extension of the turbine itself, using catalog approaches for design; which can lead to over-design and cost increases. Existing design codes incorporate uncoupled analytical elastic solutions that can be used for preliminary designs under idealized soil conditions such as linear elasticity and isotropy. However, most natural soil deposits exhibit some degree of stiffness anisotropy and stiffness degradation with cyclic strains. This paper describes the results from a series of analyses performed to obtain coupled elastic solutions of shallow and embedded foundations founded on anisotropic soils with linear increase of stiffness with depth under complex loading. The effect of the domain dimensions was examined and a domain width of 50 times the circular foundation diameter was found to be sufficiently accurate for the finite element analysis. Correction factors were derived to account for the effects of soil vertical non-homogeneity and foundation embedment. The results show that the vertical stiffness rate of change affects the vertical, horizontal and moment stiffness, but does not affect the coupling between horizontal and moment responses. The effect of soil stiffness anisotropy on foundation responses was investigated by an application of the stiffness equations. The comparison shows that the stiffness anisotropy can considerably affect the vertical and horizontal translations of the foundation but has a limited influence on the rocking behavior.

Keywords: stiffness, wind turbine foundation, anisotropy, Gibson soil

INTRODUCTION

With increasing wind energy demand and wind turbines developing higher rated powers (with associated bigger towers and blades), the necessity for design standards that can optimize construction and operation for local conditions have become even more important. The most common codes and guidelines for wind turbine foundation design rely on relatively conservative geotechnical assumptions (e.g. linear elasticity, isotropy, uncoupled bearing capacity factors, circular foundations, etc.) to simplify the design process of wind turbine foundations. Such simple assumptions can have an important impact on the overall final design of wind turbine foundations. Soil stiffness is often approximated as spring stiffness and can be defined as the ratio of load or moment to the deformation in the direction of the action, respectively. The foundation design must ensure the deformations of the soil remain below threshold values defined by the serviceability limit state (SLS). Four types of uniaxial loading cases are considered: vertical (V), horizontal (H), moment (M), and torsion (T), resulting in 4 different types of elastic stiffness. For wind turbines, the rocking stiffness is considered to be a critical design parameter because it controls the location of the center of gravity with respect to the foundation of the WT system (Lang, 2012). Borowicka (1943) developed the classical rocking stiffness equation (see Equation 1) for circular foundations on isotropic elastic half-spaces:

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$$k_R = \frac{8GR^3}{3(1-\mu)} = \frac{M}{\theta} \tag{1}$$

where G is the shear modulus; μ is the Poisson's ratio; R is the radius of the foundation; θ is the angle of rotation; and M is the applied moment. G and μ will depend on the type of the soil underlying the foundation and the strain range associated with the applied loads.

Poulos & Davies (1974) summarized available analytical solutions to estimate the elastic response of surface rigid circular footings resting on homogeneous elastic half-spaces subjected to individual VHMT loads. Likewise, Whitman (1976) described a set of equivalent-linear "spring" solutions for circular foundations on elastic half-spaces subjected to loading frequencies approaching zero (static stiffness). Moreover, Kausel et al. (1978) illustrated the connection between direct finite element (FE) procedures for the evaluation of soil-structure interaction effects of embedded structures. From their finite element analysis, they proposed approximate solutions for embedded circular foundations. Bell (1991) performed a series of three-dimensional finite element analyses on surface and embedded rough rigid circular footings subjected to *combined* VHMT loads. Bell (1991) demonstrated the cross-coupling when horizontal or moment loads act simultaneously on a foundation; that is, in a compressible soil, when a foundation is subjected to pure horizontal load, it not only undergoes translation, but also rotation around a horizontal axis perpendicular to the direction of the load. Similarly, when a moment is applied it produces both rotation and translation (see Figure 1). This cross-coupling is important as is additive when moment arises from a horizontal load applied at some distance above the footing, which is a typical case for wind turbine foundations. Bell (1991) expressed the response of a complex loading case on a footing using a matrix approach as defined by Equation 2:

$$\begin{Bmatrix} V/GR^2 \\ H/GR^2 \\ M/GR^3 \end{Bmatrix} = \begin{bmatrix} K_V & 0 & 0 \\ 0 & K_{HH} & K_{MH} \\ 0 & K_{MH} & K_{MM} \end{bmatrix} \begin{Bmatrix} w/R \\ u/R \\ \theta_M \end{Bmatrix} \tag{2}$$

where, K_V, K_{HH}, K_{MM} and K_{MH} are dimensionless elastic stiffness coefficients; G is the shear modulus; V, H and M are the vertical, horizontal and moment loads; and w, u and θ_M are the vertical, horizontal and rocking deformations. The dimensionless elastic stiffness coefficients depend only on the ratio of the foundation embedment to the radius of the foundation and the Poisson's ratio. K_V, K_{HH} and K_{MM} correspond to vertical, horizontal and moment degrees of freedom respectively. K_{MH} represents the cross-coupling of horizontal and moment degrees of freedom. For the case of surface circular rough footings resting on fully saturated soils (i.e. μ=0.5), Bell (1991) found that K_{MH} becomes zero, thus horizontal and moment loading become independent of each other. However, Bell (1991) also demonstrated that K_{MH} >0 with foundation embedment.

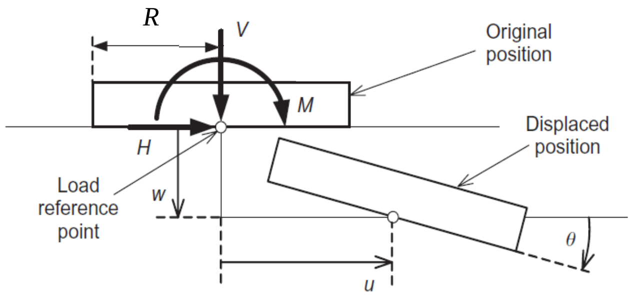


Figure 1. Nomenclature for foundation loading and geometry (adapted from Osman et al., 2007)

Non-homogeneity of soil stiffness is normally accounted for by increasing the elastic stiffness of soil with depth. A Gibson soil (Gibson, 1967) with a linear increase of the shear modulus with depth has been widely used (Randolph, 1981; Doherty & Deeks, 2003) due to its simplicity, although a power variation of the shear modulus with zero stiffness at the surface is more appropriate for normally consolidated soils (Hardin & Drnevich, 1972). Doherty & Deeks (2003) extended the approach presented by Bell (1991). They used a scaled boundary finite-element method to evaluate semi-analytical solutions for the elastic coefficients for different embedment cases considering the effect of soil non-homogeneity in their analyses. They assumed that the shear modulus (G) varied with depth following a power law and, for non-homogeneous cases, to have a value of zero at the surface. It should be noted that both Bell (1991) and Doherty & Deeks (2003), considered the soil to be isotropic. Thus, these methods fail to consider the potential effect of stiffness anisotropy of the soil material.

Bishop & Hight (1977) indicated that most natural soils will likely be anisotropic or at least transversely isotropic (cross-anisotropic) due to their deposition states and complex stress history. To describe an anisotropic elastic material, 21 independent elastic constants are required (Green, 1828). If cross-anisotropy is assumed, then only 5 parameters are necessary to define the horizontal plane of isotropy (Graham & Houlsby, 1983). These parameters are the Poisson's ratios (μ_{vh} or μ_{hv}, and μ_{hh}) and the stiffness parameters (G_{vh}, G_{hh}, E_h, E_v), where the subscripts h and v relate to horizontal or vertical directions in which the stiffness is measured. Graham & Houlsby (1983), proposed a simplified version of the cross-anisotropic model that consisted of only three independent parameters rather than seven as is ultilized in classical cross-anisotropic models. The three independent parameters are defined in Equations 3 to 5:

$$\text{Anisotropy factor: } \alpha^2 = E_h / E_v \tag{3}$$
$$\text{Modified Poisson's ratio: } \mu^* = \mu_{hh} \tag{4}$$
$$\text{Modified elastic modulus: } E^* = E_v \tag{5}$$

By assuming that the behavior in the horizontal plane only is isotropic and from the compliance matrix symmetry, the links between the elastic parameters are therefore defined as $\alpha^2 = E_h / E_v = \mu_{hv} / \mu_{vh}$, $\mu_{vh} = \mu_{hh} / \alpha$, and $G_{vh} = G_{hh} / \alpha = \alpha E_v / 2(1+\mu_{hh})$. Consequently, all of the cross-anisotropic elastic parameters can be calculated using the three-parameter model as follows:

$$E_v = E^*; E_h = \alpha^2 E^* \tag{6}$$
$$G_{vh} = \alpha E^*/2(1+\mu^*); G_{hh} = \alpha^2 E^*/2(1+\mu^*) \tag{7}$$
$$\mu_{hh} = \mu^*; \mu_{vh} = \mu^*/\alpha; \mu_{hv} = \alpha\mu^* \tag{8}$$

Due to the thermodynamic requirement that the strain energy function be non-negative in an elastic material (Love, 1927), the derived cross-anisotropic parameters can only take values within certain bounds. These bounds were summarized by Lings (2001) and are shown in Table 1.

Table 1. Bounds of the 3-parameter model (adapted from Lings, 2001)

$-1 < \mu_{hh} < 0.5$
$E_v/E_h\,(1-\mu_{hh}) - 2\mu_{vh}^2 \geq 0$
$G_{hv} \leq \dfrac{E_v}{2\mu_{vh}(1+\mu_{hh}) + 2\sqrt{E_v/E_h\,(1-\mu_{hh}^2)[1-E_h/E_v \cdot \mu_{vh}^2]}}$
$0 < \alpha^2 < 2$

The stiffness equations embedded in current codes (e.g. DNV, 2016) are uncoupled, and are applicable only for circular foundations on isotropic linear elastic homogeneous soils, and do not account for soil stiffness anisotropy or degradation. However, it is more likely that natural soil deposits may exhibit at least some degree of stiffness anisotropy. This paper shows results from numerical analyses performed to obtain coupled elastic solutions for shallow and embedded foundations founded on anisotropic soils with linear increase of stiffness with depth under coupled VHM loading. A thorough description of the numerical model and the undertaken theoretical assumptions for its development are discussed. The variations in the foundation response of a typical foundation with varying anisotropic conditions are also shown.

METHODOLOGY

Numerical model development and parametric study

A finite element model of a typical shallow foundation of a large wind turbine has been constructed with the software package ABAQUS (2016). The soil was represented by a mesh of around 180000 three-dimensional 8-noded continuum elements (C3D8R). To eliminate possible boundary effects, five model domain widths (5*D*, 10*D*, 20*D*, 50*D* and 100*D*) were examined. These dimensions represent the horizontal distance from the foundation edge to either side of the domain and the vertical depth of the soil below the foundation. Figure 2 shows the effect of the model dimensions on K_V for an isotropic soil and Bell's (1991) results with a 100*D* mesh for comparison. It can be seen that a domain size of 50*D* ~ 100*D* is sufficiently accurate, thus a model dimension of 50*D* was adopted throughout the remaining analyses. The horizontal displacement of the vertical sides was restricted, and the bottom of the model was kept fixed by restricting all six degrees of freedom. The base contact between the foundation and the soil was chosen to be a rough condition. Figure 3 shows the model geometry with a typical finite element mesh.

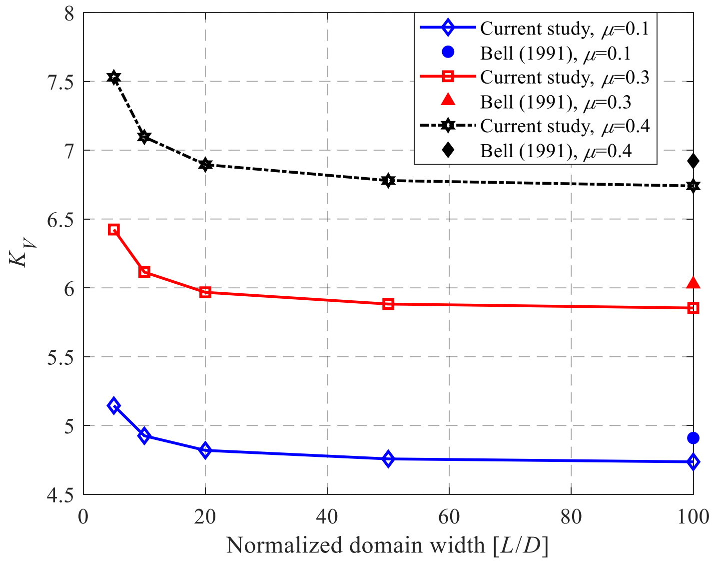


Figure 2. Effect of model dimensions

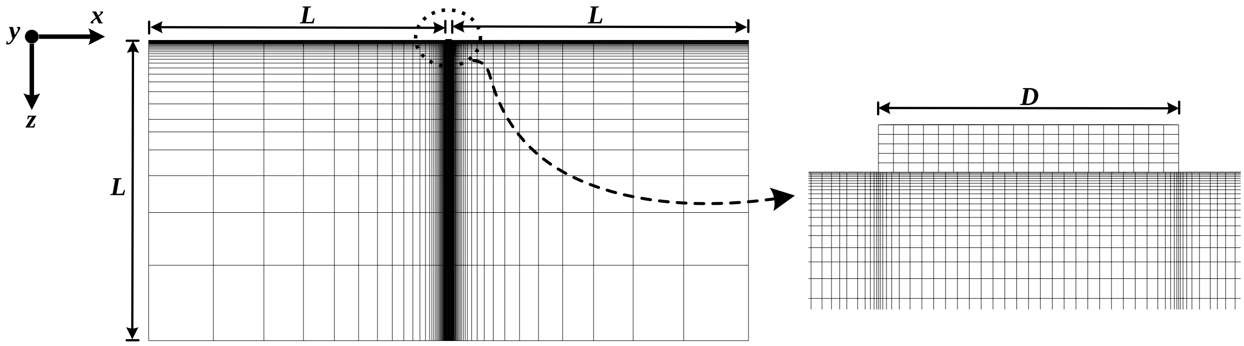


Figure 3. Mesh representation

The wind turbine shallow foundation was idealized as a circular rigid body and it was modelled with both surface and embedded conditions. The founding soil was modelled as an anisotropic linear elastic material represented by Equations (6) ~ (8), with isotropy as a special case (α^2=1). To assess the changes in the characteristics of the soil, this was conceived as a homogeneous layer with constant stiffness with depth, and as a non-homogeneous layer with linearly increasing stiffness with depth (Gibson, 1967). This last assumption better represents most natural deep soil deposits, as they often exhibit an increase of elastic stiffness with depth (Doherty & Deeks, 2003). Figure 4 shows the stiffness distribution with depth assumed for homogeneous and non-homogeneous soil profiles adopted herein.

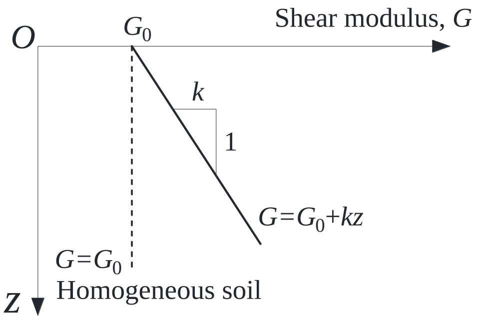


Figure 4. Stiffness distribution for homogenous and non-homogeneous soils

The effect of the foundation embedment was studied by changing the embedment ratios (d/D) from 0 to 0.158, and the effect of different slope (k) increments for the non-homogeneous soil stiffness increase with depth were considered with $1/\beta$ (where β is the normalized Gibson factor, defined as $\beta = \frac{E_{v0}}{kD}$) varying from 0 to 0.08. Table 2 shows a summary of the different cases studied with the finite element model, as well as the value of the parameters for the different analyses.

Table 2. Cases of analysis used in the finite element model

Embedment ratio d/D	**Homogeneous soil, $\frac{1}{\beta} = \frac{E_{v0}}{kD}$**	**Gibson soil $\frac{1}{\beta} = \frac{E_{v0}}{kD}$**				
	0	**0.2**	**0.4**	**0.6**	**0.8**	**1.0**
0	(1-1)	(1-2)	(1-3)	(1-4)	(1-5)	(1-6)
0.03	(2-1)	--	--	--	--	--
0.06	(3-1)	--	--	--	--	--
0.09	(4-1)	--	--	--	--	--
0.12	(5-1)	--	--	--	--	--
0.158	(6-1)	--	--	--	--	--

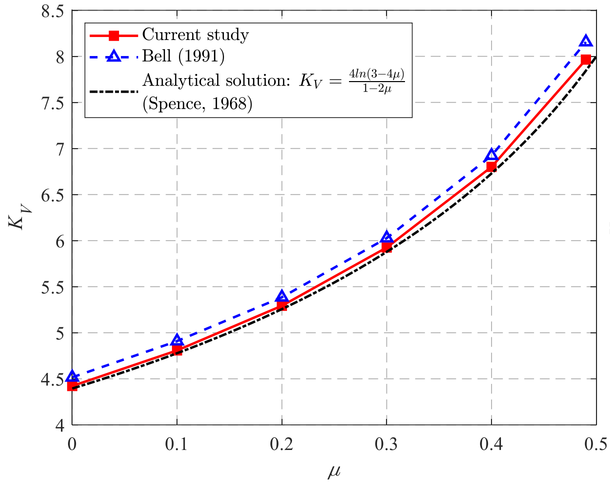
A parametric study was carried out to assess the variation of the elastic solutions due to changes in the anisotropic parameters α^2 and μ_{hh}. Six values of α^2 were adopted, varying from 0 to 2 (i.e. 0.2, 0.4, 0.6, 1.0, 1.5 and 2.0), and μ_{hh} was varied between 0 and 0.49 (i.e. 0, 0.10, 0.20, 0.30, 0.40 and 0.49). Therefore, each case shown in Table 2 includes 36 sub-cases (i.e. six values of α^2 × six values of μ_{hh}).

RESULTS

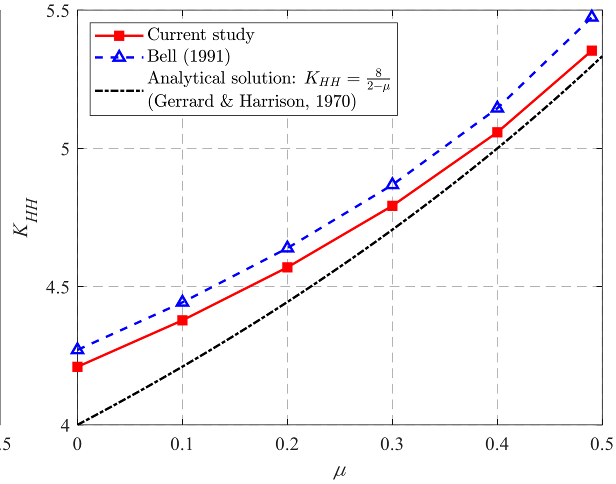
Model calibration

The results of the finite analysis were compared against the approximate solutions obtained by Bell (1991) and analytical solutions found in the literature for circular surface foundations resting on an isotropic homogeneous soil. Figure 5 shows a comparison between the vertical (K_V), horizontal (K_{HH}), moment (K_{MM}) static stiffness and the cross-coupling term (K_{MH}) obtained from the current study, Bell (1991) and the analytical solutions of Spence (1968), Gerrard and Harrison (1970), and Poulos and Davis (1974).

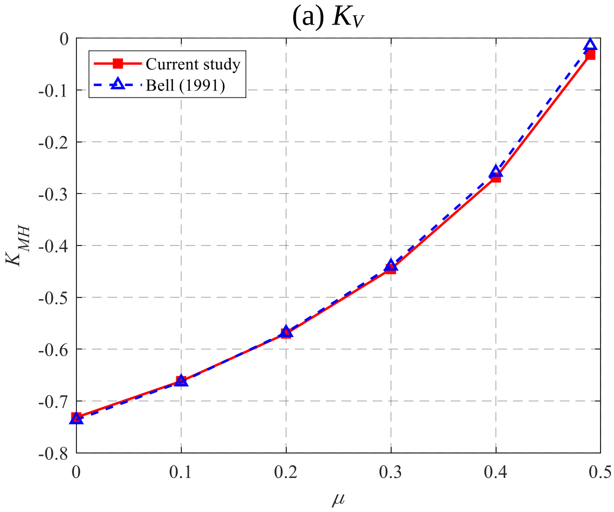
The figure shows that the current study compares well (difference less than 5%) with the approximate solutions of K_V, K_{HH} and K_{MM} from Bell (1991). Both the current study and Bell's (1991) results are larger than the analytical solutions, however, the current study lies closer to the analytical solutions due to the much finer mesh and improved analytical methods than that of Bell (1991). Likewise, the cross-coupling term in Figure 5c obtained from Bell (1991) shows good agreement with the obtained in this study. However, it should be noted that the differences of both K_{HH} and K_{MM} between the current study and the analytical solutions gradually decrease with μ. This is mainly because the analytical solutions of K_{HH} and K_{MM} are uncoupled. Figure 5c also shows a decreasing coupling between horizontal displacement and body rotation with μ and no coupling when the soil media is incompressible (i.e. undrained case: μ=0.5), which makes the uncoupled analytical solutions of K_{HH} and K_{MM} approach the coupled results obtained from the current study.



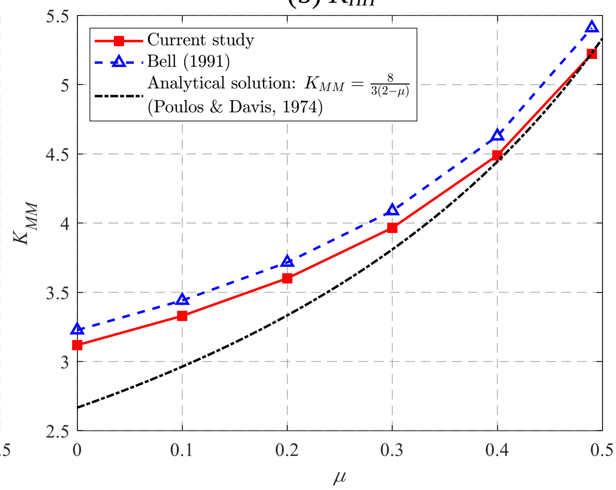
(a) K_V



(b) K_{HH}



(c) K_{MH}



(d) K_{MM}

Figure 5. Stiffness for surface circular foundations on isotropic homogeneous soils

Elastic solutions obtained from the parametric study

By analyzing surface foundations on homogeneous soils (i.e. Case (1-1) in Table 2), the basic stiffness equations can be obtained as a function of μ_{hh} and α^2, as shown below in Equation 9. Isotropy (i.e. $\alpha^2 = 1.0$) will reduce to the commonly-used isotropic stiffness values. Equations 9a and 9d show that K_V and K_{MM} decrease with α, while K_{HH} exhibits an opposite trend. This is because $\alpha^2 = E_h/E_v$, and K_V and K_{MM} rely more on the vertical stiffness, E_v, while K_{HH} is primarily dominated by the horizontal stiffness, E_h. It should also be noted that the coupling is not affected by α. In contrast, all of the stiffness coefficient values increase with larger μ_{hh}. Moreover, Equation 9c becomes zero when μ_{hh} is equal to 0.5, which indicates no coupling effect between horizontal and moment responses. This has also been confirmed by Bell (1991). The soil anisotropic ratio also shows no effect on the coupling term.

$$K_V = \frac{1}{(1.1 - \mu_{hh})(0.112\alpha + 0.0937)} \tag{9a}$$

$$K_{HH} = \frac{5.375\alpha + 4.58}{2.4 - \mu_{hh}} \tag{9b}$$

$$K_{MH} = K_{HM} = \frac{-1.5(0.5 - \mu_{hh})}{1 - \mu_{hh}} \tag{9c}$$

$$K_{MM} = \frac{1}{(1.2 - \mu_{hh})(0.145\alpha + 0.125)} \tag{9d}$$

Cases (1-1) ~ (1-6) in Table 2 can be used to obtain the effect of the Gibson case on the foundation stiffness. Similarly, the embedment effect on the foundation can be provided by analyzing Cases (1-1) ~ (6-1). The final stiffness equations including both the Gibson and embedment correction factors are shown below in Equation 10. It can be seen that the embedment ratios affect all four stiffness coefficients, while the Gibson modulus, β, affects everything except for the coupling behavior between horizontal and moment responses.

$$K_V = \left[\frac{1}{(1.1-\mu_{hh})(0.112\alpha+0.0937)}\right]\cdot\left[\frac{1.36}{\beta}+1\right]\cdot\left[\frac{1.03e}{0.90+4e}+1\right] \quad (10a)$$

$$K_{HH} = \left[\frac{5.375\alpha+4.58}{2.4-\mu_{hh}}\right]\cdot\left[\frac{0.63}{\beta}+1\right]\cdot\left[\frac{e}{0.32+e}+1\right] \quad (10b)$$

$$K_{MH} = K_{HM} = \left[\frac{-1.5(0.5-\mu_{hh})}{1-\mu_{hh}}\right]\cdot[-6.49e+1] \quad (10c)$$

$$K_{MM} = \left[\frac{1}{(1.2-\mu_{hh})(0.145\alpha+0.125)}\right]\cdot\left[\frac{0.30}{\beta}+1\right]\cdot\left[\frac{1.93e}{0.88+e}+1\right] \quad (10d)$$

where β is the normalized Gibson factor, defined as $\beta = \frac{E_{v0}}{kD}$; E_{v0} is the vertical elastic soil modulus at ground level; e is the embedment ratio, defined as $e = d/D$.

APPLICATION

In this section, the elastic behaviour of a typical wind turbine shallow foundation is studied based on the stiffness equations from the current study, DNV (2016) and Bell (1991). The dimension of the circular foundation and the properties of the anisotropic soil as estimated by the authors are summarized in Table 3. The design loads of this foundation were determined from the criterion IEC DLC6.1 (IEC 61400-1, 2005). The factored ultimate limit state loads from DLC6.1, for parked/idling gust load with 50-year return period, are shown in Table 4.

Table 3. Foundation and soil parameters

α^2	μ_{hh}	$G_{vh} = G_R$, [MPa]	k, [MPa/m]	R, [m]	e
1.30	0.24	82.0	2	9.5	0 & 0.16

α: anisotropic parameter; μ_{hh}: Poisson's ratio; G_{vh}: Small-strain shear modulus in the vertical direction; G_R: small-strain shear modulus at a depth equal to the foundation radius (Whitman, 1976); *k*: stiffness gradient; R: foundation radius; e: embedment ratio

Table 4. Design loads

Design loads	*V* [kN]	*H* [kN]	*M* [kN*m]
Ultimate loads (DCL 6.1)	21820	1100	76200

Table 5 presents a summary of the calculated elastic stiffnesses by the different approaches. Additional to the case outlined in Table 3 (presented as Case (a) in Table 5), two more cases were studied in light of showing the effect of the variation of the anisotropic parameter and the change of the stiffness gradient on the elastic stiffnesses. In Case (b), α^2 was varied from 0.5 to 2, following the limits reported by Lings (2001). In Case (c), the normalized Gibson factor (*β*), that depends on *k*, was varied between 0 to 5 following the ranged suggested by Rowe & Booker (1981), according to their study the proposed range of *β* encompass many practical situations. The deformations estimated using Equation 10 for Case (a) are generally higher when compared with the results given by the uncoupled isotropic solutions of the DNV (2016) and the isotropic coupled solutions of Bell (1991), however both DNV (2002) and Bell (1991) solutions are very closed to each other. From these results one could infer that the anisotropic parameter and the stiffness increase with depth of the soil profile are of high importance and can affect the estimation of the foundation deformations. This thought can be supported by further analyzing cases (b) and (c). As shown in Case (b), the variation of α^2 produces a narrow range for each elastic stiffness that get much broader when the Gibson factor is changed from 0 to 5.

Table 5. Stiffness comparison for surface foundations

Normalized stiffness		K_V	K_{HH}	K_{MH}/K_{HM}	K_{MM}
Current study	Case (a): $\alpha^2 = 1.3$ & $\frac{1}{\beta} = 0.21$ (i.e. k=2)	6.78	5.63	-0.51	3.82
	Case (b): $\alpha^2 = 0.5 \sim 2.0$ & $\frac{1}{\beta} = 0.21$	7.94~6.28	4.20~6.58	-0.51	4.76~3.41
	Case (c): $\alpha^2 = 1.3$ & $\frac{1}{\beta} = 0 \sim 5$	5.25~40.97	4.96~20.57	-0.51	3.59~8.97
DNV (2016)		5.26	4.55	--	3.51
Bell (1991)		5.62	4.73	-0.52	3.85

The responses of the foundation are compiled in Table 6 for both surface and embedded foundations. These responses were estimated using the recommendation of the DNV (2002) that suggests a working value of shear strains for wind turbine foundations of 10^{-3} to estimate the reduction in stiffness. This corresponds to approximately 30% reduction in the small-strain shear modulus (G_R) or G/G_{max} = 0.3 as estimated by the authors. It can be seen that the vertical translation, u_V, can be highly overestimated if the soil stiffness anisotropy is neglected. In contrast, the horizontal translation, u_H, of the surface foundation is considerably underestimated by DNV (2016) most likely for not considering the cross-coupling effect, while it is overestimated by Bell (1991) presumably due to the soil non-homogeneity. The rocking angle, θ, exhibits a similar trend to u_V, but the differences between the three methods are smaller.

Table 6. Responses of the foundation (G/G_{max}=0.3)

	u_V [mm]		u_H [mm]		θ [deg]	
	$d/D = 0$	$d/D = 0.158$	$d/D = 0$	$d/D = 0.158$	$d/D = 0$	$d/D = 0.158$
Current study – Case (a)	9.062	8.192	1.233	0.486	0.0486	0.0369
DNV (2016)	17.812	15.382	1.040	0.859	0.0592	0.0363
Bell (1991)	16.687	--	2.019	--	0.0556	--

SUMMARY & CONCLUSIONS

Coupled elastic stiffness of circular foundations founded on anisotropic soils under VHM loading has been studied using finite element analysis. A 3-parameter cross-anisotropic soil model was adopted. The effect of model dimensions was investigated and the results show that domain widths of 50 times the foundation diameter are sufficiently accurate for finite element analysis. The results for the stiffness coefficients of surface foundations resting on a homogeneous isotropic soil favorably compare with currently available values. The vertical and rocking stiffness decrease with α while the horizontal stiffness shows an increasing trend. On the other hand, the coupling stiffness showed no changes with varying values of α and seems to only depend on the values adopted by the Poisson's ratio. To account for the effects of soil non-homogeneity and foundation embedment, Gibson soils and embedded foundations were considered. The Gibson and embedment correction factors were derived accordingly. The analysis shows that a higher Gibson modulus can increase the vertical, horizontal and moment stiffness, while it does not affect the coupling between horizontal and moment responses. To study the effect of soil stiffness anisotropy on foundation responses and to show the application of the derived stiffness equations, a worked-out example was carried out. The results indicate that the vertical and horizontal translations of the foundation are considerably affected by stiffness anisotropy, making them prone to the adopted values of soil stiffness in the vertical and horizontal directions of the soil material, while its influence on the rocking behavior can be considered to be much smaller.

ACKNOWLEDGEMENTS

The writers would like to acknowledge the financial support of Natural Sciences and Engineering Research Council. The second author is grateful for the financial support of the China Scholarship Council (CSC).

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