Technische Informatik: Abgabe 4

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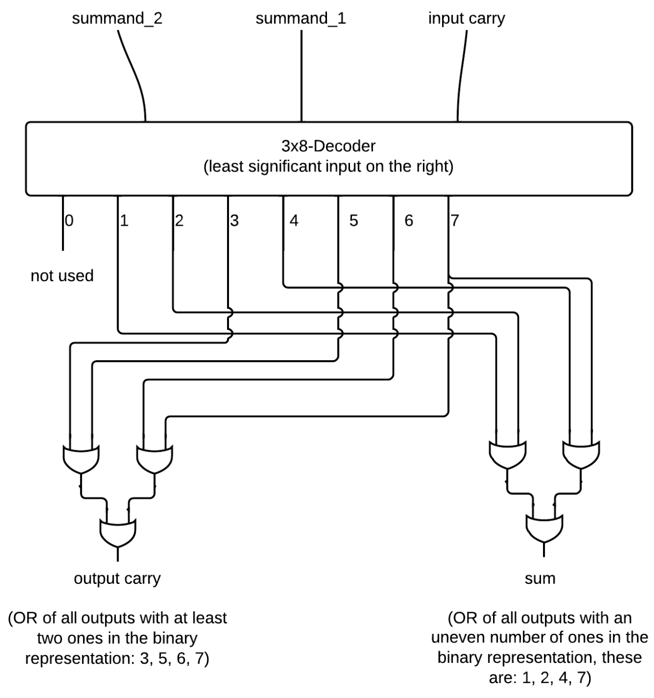
Exercise 4.1 (Full adder from decoder)



Exercise 4.2 (Subtractors)

a) Here are the tables for the two circuits we wish to implement (namely Half-Subtractor and Full-Subtractor):

minuend	subtrahend	underflow	difference
0	0	0	0
0	1	1	1
1	0	0	1
1	1	0	0

If we read ms as numbers with high order bit on the left:

$$u_{out} = m_1$$
$$d = m_1 + m_2.$$

minuend	subtrahend	underflow	underflow	difference
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	1	0
1	0	0	0	1
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Again, in SOP-notation with high-order bit on the left:

$$u_{out} = m_1 + m_2 + m_3 + m_7$$
$$d = m_1 + m_2 + m_4 + m_7.$$

b) More or less compact symbolic representations of these two circuits are as follows (first component is always the resulting underflow, second is the actual difference):

$$HalfSubtractor(m, s) = (\bar{m}s, m \not\leftrightarrow s)$$

$$FullSubtractor(m, s, u) = (\bar{m} \not\leftrightarrow su, m \not\leftrightarrow s \not\leftrightarrow u)$$

c) Now we want to simplify both components (difference and undeflow) of the full subtractor using Karnaugh diagrams. We begin with the difference:

		minuend / subtrahend			
		00	01	11	10
underflow	0		1		1
	1	1		1	

It seems that this diagram is not simplifiable at all: we have to cover every one by an own 1x1 block. The simpliest expression for difference is thus:

$$d = \bar{m}\bar{s}u + \bar{m}s\bar{u} + msu + m\bar{s}\bar{u}$$

The ones for the output-underflow can be covered by three 2x1 blocks, which all intersect at 011 (we use additive color combination, light gray is supposed to be combination of red, green and blue):

		minuend / subtrahend			
		00	01	11	10
underflow	0	0	1	0	0
	1	1	1	1	0

Thus, the simplified formula for the output-underflow is:

$$u_{out} = \bar{m}u + \bar{m}s + su.$$

d)

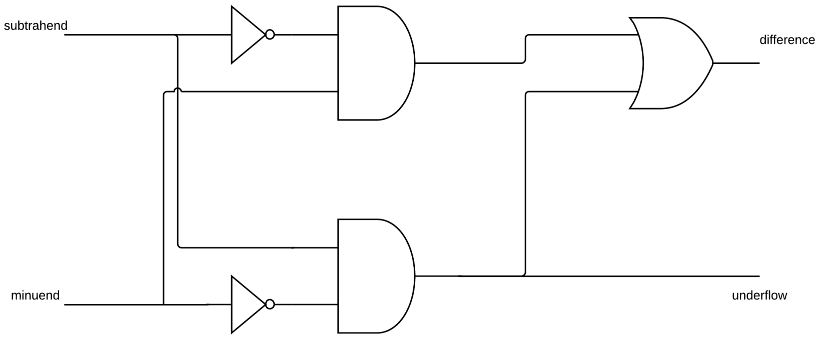


Abbildung 1: Half subtractor.

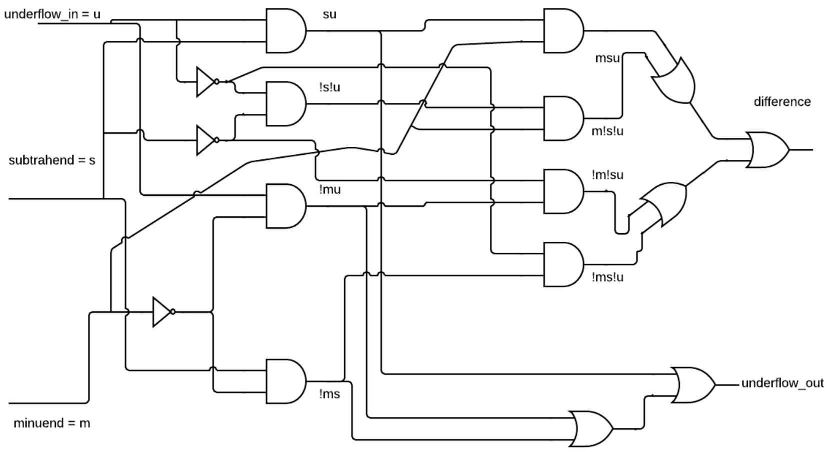


Abbildung 2: Full subtractor. We recycled as many gatters as we could for both sub-circuits. Reason: that's the maximal complexity allowed for free LucidChart accounts...

Exercise 4.3 (Betting and Racing)

i	x_1	x_2	x_3	x_4	f_1	f_{2a}	f_{2b}
0	0	0	0	0			
1	0	0	0	1			
2	0	0	1	0	1 (B_2)		
3	0	0	1	1	1 (B_3)	1	1
4	0	1	0	0			
5	0	1	0	1	1 (B_3)	1	1
6	0	1	1	0			
7	0	1	1	1		1	1
8	1	0	0	0	1 (B_1)		
9	1	0	0	1	0 ($B_1 + B_3$)		
10	1	0	1	0	1 (B_1)		D
11	1	0	1	1	1 (B_1)		D
12	1	1	0	0			D
13	1	1	0	1			D
14	1	1	1	0			D
15	1	1	1	1		1	D

a1) $f_1 = 1 \Leftrightarrow B_1.\neg B_2.\neg B_3 + \neg B_1.B_2.\neg B_3 + \neg B_1.\neg B_2.B_3$
where

- $B_1 = 1 \Leftrightarrow x_1.\neg x_2 = 1$
- $B_2 = 1 \Leftrightarrow \neg x_1.\neg x_2.x_3.\neg x_4 = 1$
- $B_3 = 1 \Leftrightarrow x_4$.only one of$(x_1, x_2, x_3) = 1$

brings us to column f_1 in the table above and leeds to the Karnaugh table:

x_1x_2 \ x_3x_4	00	01	11	10
00		1	1	
01		1		
11				
10	1		1	1

which yields: $f_1 = x_1\neg x_2x_3 + \neg x_1\neg x_3x_4 + \neg x_1\neg x_2x_4 + x_1\neg x_2\neg x_3\neg x_4$

a2) $f_{2a} = 1 \Leftrightarrow (x_1x_2x_3x_4)_2|1101001$(in decimal $105 = 3 \times 5 \times 7$)
brings us to column f_{2a} in the table above and leeds to the Karnaugh table:

x_1x_2 \ x_3x_4	00	01	11	10
00			1	
01		1	1	
11			1	
10				

which yields $f_{2a} = \neg x_1x_2x_4 + \neg x_1x_3x_4 + x_2x_3x_4$

b)

$f_{2b} = 1 \Leftrightarrow 0000 \leq (x_1x_2x_3x_4)_2 \leq 1001|1101001$ gives column f_{2b} in the table above, Karnaugh table:

x_1x_2 \ x_3x_4	00	01	11	10
00			1	
01		1	1	
11	D	D	D	D
10			D	D

which yields $f_{2b} = x_2x_4 + \neg x_1x_3x_4$