Technische Informatik: Abgabe 3

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Exercise 3.1 (MUX Wizardry)

a) We can construct a n-MUX from a 1-MUX and two $(n-1)$-MUX'es, clamping the two larger MUX'es to the inputs of the 1-MUX. A trivial induction immediately yields that we need $2^n - 1$ 1-MUX'es to construct a n-MUX in that way. For example, for a 4-MUX we need 15 1-MUX'es:

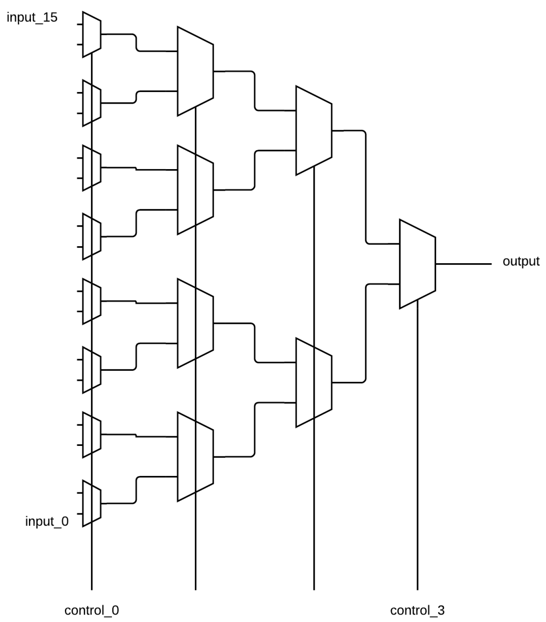


Abbildung 1: Solution to 1 (a)

b) We have $f = m_0 + m_3 + m_6 + m_{13}$. (Notice that we read the numbers from right to left, interpreting x_4 as least significant bit and x_1 as most significant bit).

All we have to do is to mark the corresponding inputs with ones (notice that the order of control signals x_1 to x_4 corresponds to reading the numbers from right to left, the topmost input corresponds to the least significant bit).

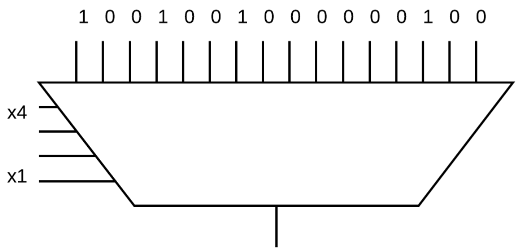


Abbildung 2: Solution to 1 (b)

c) Our goal is to implement the function from part b) with only two 2-MUX'es and one inverter. To derive the optimized circuit, we need some notation. We shall denote an N-MUX as follows:

$$MUX_{c_0,\ldots,c_{N-1}}\left(i_0,\ldots,i_{2^N-1}\right) := \bigvee_{k=0}^{2^N-1} i_k m_k\left(c_0,\ldots,c_{2^N-1}\right).$$

where m_k is the 0-indexed minterm. More concretely, for 2-MUX we use the notation:

$$MUX_{l,h}(a,b,c,d) := a\bar{h}\bar{l} + b\bar{h}l + ch\bar{l} + dhl.$$

Notice that from this definition and idempotence it immediately follows:

$$MUX_{l,h}(a,b,c,d) = MUX_{l,h}(a,b\bar{h},c,dh). \qquad (*)$$

Using this, we calculate:

$$\begin{aligned}
f(x_1,x_2,x_3,x_4) &= \bar{x}_1\bar{x}_2\bar{x}_3\bar{x}_4 + \bar{x}_1\bar{x}_2x_3x_4 + \bar{x}_1x_2x_3\bar{x}_4 + x_1x_2\bar{x}_3x_4 \\
&\overset{\text{def MUX}}{=} MUX_{x_2,x_1}\left(\bar{x}_3\bar{x}_4 + x_3x_4, x_3\bar{x}_4, 0, \bar{x}_3x_4\right) \\
&\overset{\text{def XOR}}{=} MUX_{x_2,x_1}\left(\neg(x_3 \not\leftrightarrow x_4), (x_3 \not\leftrightarrow x_4)x_3, 0, (x_3 \not\leftrightarrow x_4)x_4\right) \\
&\overset{(*),\text{ idempotence}}{=} MUX_{x_2,x_1}\left(\neg(x_3 \not\leftrightarrow x_4), (x_3 \not\leftrightarrow x_4)x_3\bar{x}_1\bar{x}_1, 0, (x_3 \not\leftrightarrow x_4)x_4x_1x_1\right) \\
&\overset{+0}{=} MUX_{x_2,x_1}\Big(\neg(x_3 \not\leftrightarrow x_4), MUX_{x_1,(x_3\not\leftrightarrow x_4)}\left(0,0,x3,x4\right)\bar{x}_1, 0, MUX_{x_1,(x_3\not\leftrightarrow x_4)}\left(0,0,x3,x4\right)x_1\Big) \\
&\overset{(*)}{=} MUX_{x_2,x_1}\Big(\neg(x_3 \not\leftrightarrow x_4), MUX_{x_1,(x_3\not\leftrightarrow x_4)}\left(0,0,x3,x4\right), 0, MUX_{x_1,(x_3\not\leftrightarrow x_4)}\left(0,0,x3,x4\right)\Big)
\end{aligned}$$

In the step marked as "+0"we used two transformations of the type

$$(x_3 \not\leftrightarrow x_4)x_3\bar{x}_1 = (0 + 0 + x_3(x_3 \not\leftrightarrow x_4)\bar{x}_1 + x_4(x_3 \not\leftrightarrow x_4)x_1)\bar{x}_1 = MUX_{x_1,(x_3\not\leftrightarrow x_4)}\left(0,0,x3,x4\right)\bar{x}_1$$

two times. Now we can replace the XOR expression by another MUX, and obtain the following circuit:

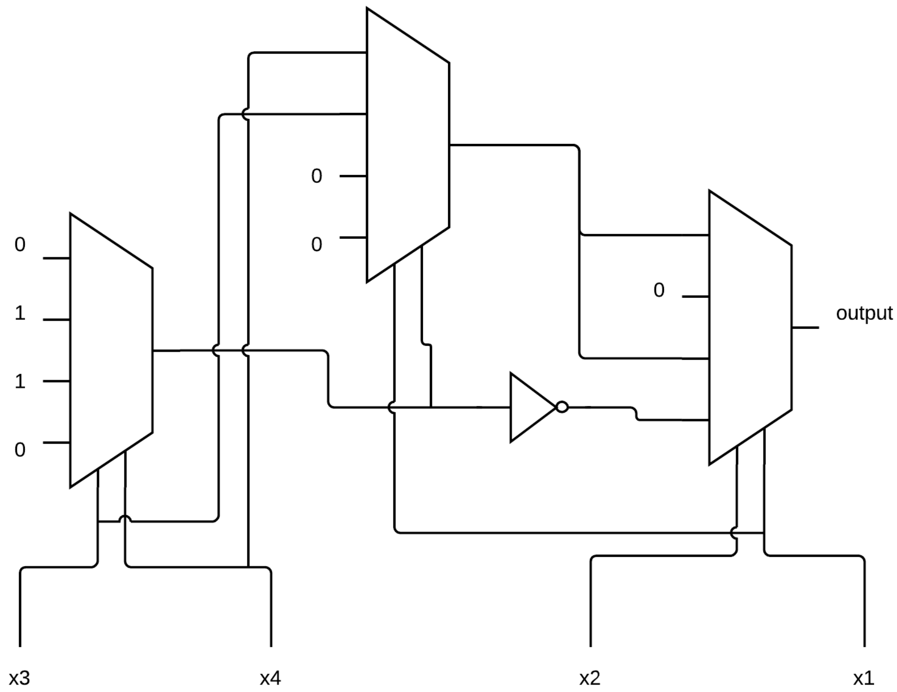


Abbildung 3: Solution to 3.1 (c)

Exercise 2 (Air conditioner)

We have an air contitioner, that reacts to three binary sensors (temperature t, wind w, humidity h). The rule of the automatic part a of the system consist of three parts:

- if $t \wedge \neg(w \wedge \neg h)$ then switch conditioner on
- if $h \wedge \neg(\neg t \wedge w)$ then switch conditioner on
- if $w \wedge \neg(t \wedge h)$ then switch conditioner off

All together, it yields:

$$\begin{aligned}
a &= ((t \wedge \neg(w \wedge \neg h)) \vee (h \wedge \neg(\neg t \wedge w))) \wedge \not(w \wedge \neg(t \wedge h)) \\
&\overset{DeMorgan}{=} ((t \wedge (\neg w \vee h)) \vee (h \wedge (t \vee \neg w))) \wedge (\neg w \vee (t \wedge h)) \\
&\overset{distr,idempot}{=} (t\bar{w} + th + ht + h\bar{w})(\bar{w} + th) \\
&\overset{distr,assoc}{=} t\bar{w} + th\bar{w} + h\bar{w} + th\bar{w} + th + th\bar{w} \\
&\overset{absorp}{=} t\bar{w} + th + h\bar{w}
\end{aligned}$$

This can be implemented as a circuit consisting of a single 2-MUX. Additionally, the user u shall be able to override the sensors and switch the conditioner ON manually. Alltogether, this yields following circuit:

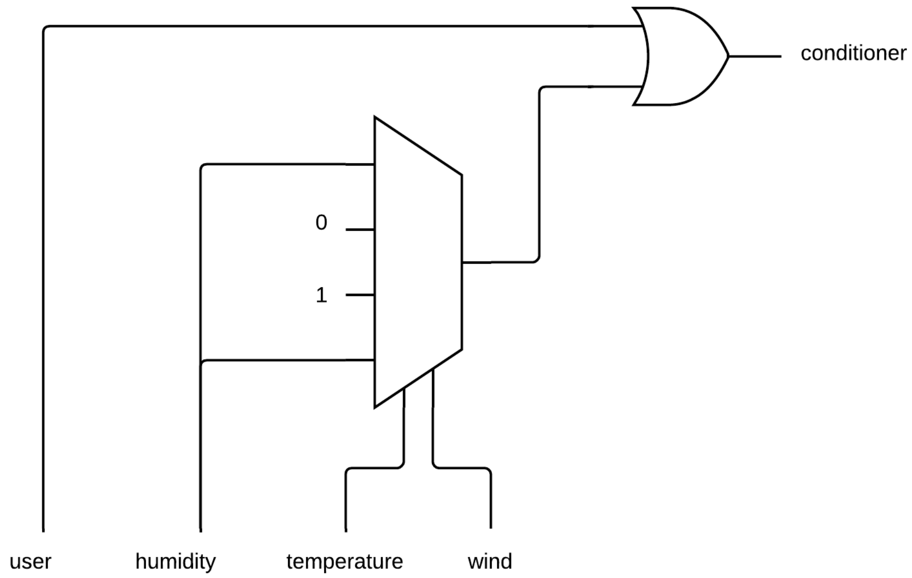
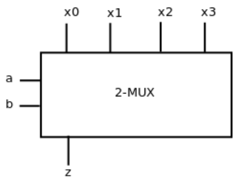


Abbildung 4: Solution to 3.2

Exercise 3: MUX is universal



1. AND: $x_0 = x_1 = x_2 = 0$ and $x_3 = 1$ yields output $z = a \wedge b$
2. OR: $x_0 = 0$ and $x_1 = x_2 = x_3 = 1$ yields output $z = a \vee b$
3. NOT: since NOT is an unary operand $b = 0$ and $x_0 = 1, x_1 = 0, (x_2 = x_3 = 0)$ yields output $z = \neg a$

Exercise 4 (4-DeMUX from smaller parts)

If we want to construct a 4-DeMUX, but have only 4 2-DeMUX'es and infinite number of simpler gatters at our disposal, we can construct an additional 2-DeMUX from scratch from simpler AND gatters and inverters, and then clamp the four 2-DeMUX'es to it's outputs:

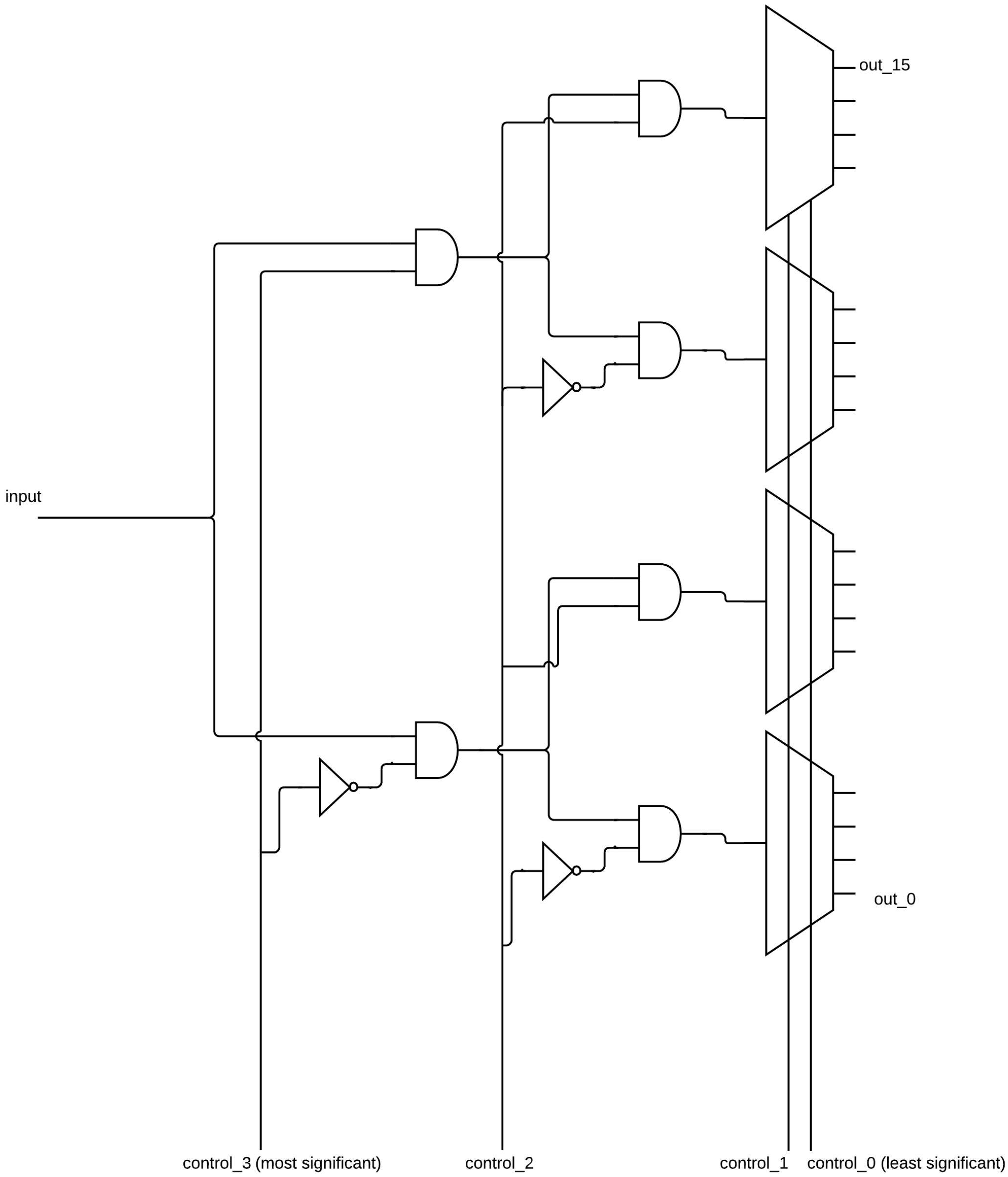


Abbildung 5: Solution to 3.4