Technische Informatik: Abgabe 5

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Exercise 5.1 (Quine McClusky and K-Maps)

a)

#(0)	decimal	$x_1x_2x_3x_4x_5$	Reduction-Links						
1	15	01111							
	30	11110	$\star^{1}$	$\star^{2}$					
	29	11101			$\star^{3}$	$\star^{4}$	$\star^{5}$		
2	28	11100	$\star^{1}$		$\star^{3}$			$\star^{6}$	
	25	11001			$\star^{8}$	$\star^{4}$			$\star^{7}$
	22	10110		$\star^{2}$					
	21	10101	$\star^{9}$				$\star^{5}$		
3	24	11000		$\star^{10}$				$\star^{6}$	$\star^{7}$
	17	10001	$\star^{9}$		$\star^{8}$				
	9	01001			$\star^{11}$				
4	8	01000	$\star^{12}$	$\star^{10}$	$\star^{11}$				
	2	00010		$\star^{13}$					
5	0	00000	$\star^{12}$	$\star^{13}$					

$\Downarrow$ $\Downarrow$

#(0)	decimal	$x_1x_2x_3x_4x_5$	Reduction-Links			
1	15	01111				
	28,30	111*0				
	22,30	1*110				
	28,29	1110*	$\star^{1}$			
	25,29	11*01		$\star^{2}$	$\star^{3}$	
	21,29	1*101				$\star^{4}$
2	24,28	11*00		$\star^{2}$		
	24,25	1100*	$\star^{1}$			
	17,25	1*001				$\star^{4}$
	17,21	10*01			$\star^{3}$	
3	8,24	*1000				
	8,9	0100*				
4	0,8	0*000				
	0,2	000*0				

$\Downarrow$ $\Downarrow$

#(0)	decimal	$x_1x_2x_3x_4x_5$
1	15	01111
	28,30	111*0
	22,30	1*110
	24,25,28,29	11*0*
	17,21,25,29	1**01
3	8,24	*1000
	8,9	0100*
4	0,2	000*0

$\Downarrow$ $\Downarrow$

Primimplicant	0	2	8	9	15	17	21	22	24	25	28	29	30	
01111					1									
111*0											1		1	reduced r 4
1*110								1					1	
11*0*									1	1	1	1		
1**01						1	1			1		1		
*1000			1						1					reduced r 4
0100*			1	1										
000*0	1	1												
reduced b/c			c9				c17			c17		c17	c22	

...here lives the big table for row and column cheating...

$\Longrightarrow f = \neg x_1 x_2 \neg x_3 \neg x_4 + \neg x_1 \neg x_2 \neg x_3 \neg x_5 + \neg x_1 x_2 x_3 x_4 x_5 + x_1 x_2 \neg x_4 + x_1 \neg x_4 x_5 + x_1 x_3 x_4 \neg x_5$

b) Use K-Maps to find a cost-minimal DNF.

$x_1 = 0$				
x_2x_3	x_4x_5: 00	01	11	10
00	1			1
01				
11			1	
10	1	1		

$x_1 = 1$				
x_2x_3	x_4x_5: 00	01	11	10
00		1		
01		1		1
11	1	1		1
10	1	1		

which yields: $f = \neg x_1 x_2 \neg x_3 \neg x_4 + \neg x_1 \neg x_2 \neg x_3 \neg x_5 + \neg x_1 x_2 x_3 x_4 x_5 + x_1 x_2 \neg x_4 + x_1 \neg x_4 x_5 + x_1 x_3 x_4 \neg x_5$
The same result as in part a).

Exercise 5.2 (Row and Column-Rules are no function)

Primimplic.	$C_1 = 0$	$C_2 = 1$	$C_3 = 2$
$R_1 = 01$		1	
$R_2 = 0*$	1	1	
$R_3 = *0$	1		1

We deleted Column C_1 with the Column-Rule, because $C_3 \leq C_1$ and overlaps it. Now we can either use $R_3 + R_2 = \neg x_2 + \neg x_1$ or $R_3 + R_1 = \neg x_2 + \neg x_1 x_2$ as our minimal solution.

Exercise 5.3 (?)

i	x_1	x_2	x_3	x_4	f_1	f_{2a}	f_{2b}
0	0	0	0	0			
1	0	0	0	1			
2	0	0	1	0	1 (B_2)		
3	0	0	1	1	1 (B_3)	1	1
4	0	1	0	0			
5	0	1	0	1	1 (B_3)	1	1
6	0	1	1	0			
7	0	1	1	1		1	1
8	1	0	0	0	1 (B_1)		
9	1	0	0	1	0 ($B_1 + B_3$)		
10	1	0	1	0	1 (B_1)		D
11	1	0	1	1	1 (B_1)		D
12	1	1	0	0			D
13	1	1	0	1			D
14	1	1	1	0			D
15	1	1	1	1		1	D

a1) $f_1 = 1 \Leftrightarrow B_1.\neg B_2.\neg B_3 + \neg B_1.B_2.\neg B_3 + \neg B_1.\neg B_2.B_3$
where

- $B_1 = 1 \Leftrightarrow x_1.\neg x_2 = 1$
- $B_2 = 1 \Leftrightarrow \neg x_1.\neg x_2.x_3.\neg x_4 = 1$
- $B_3 = 1 \Leftrightarrow x_4.\text{only one of}(x_1, x_2, x_3) = 1$

brings us to column f_1 in the table above and leeds to the Karnaugh table:

	x_3x_4			
x_1x_2	00	01	11	10
00		1	1	
01		1		
11				
10	1		1	1

which yields: $f_1 = x_1\neg x_2 x_3 + \neg x_1 \neg x_3 x_4 + \neg x_1 \neg x_2 x_4 + x_1 \neg x_2 \neg x_3 \neg x_4$

a2) $f_{2a} = 1 \Leftrightarrow (x_1x_2x_3x_4)_2 | 1101001 (\text{in decimal } 105 = 3 \times 5 \times 7)$
brings us to column f_{2a} in the table above and leeds to the Karnaugh table:

x_1x_2	x_3x_4 00	01	11	10
00			1	
01		1	1	
11			1	
10				

which yields $f_{2a} = \neg x_1 x_2 x_4 + \neg x_1 x_3 x_4 + x_2 x_3 x_4$

b)

$f_{2b} = 1 \Leftrightarrow 0000 \leq (x_1x_2x_3x_4)_2 \leq 1001|1101001$ gives column f_{2b} in the table above, Karnaugh table:

x_1x_2	x_3x_4 00	01	11	10
00			1	
01		1	1	
11	D	D	D	D
10			D	D

which yields $f_{2b} = x_2 x_4 + \neg x_1 x_3 x_4$