Lab Sessions 3–5: Radar Detection

Modern Theory of Detection and Estimation

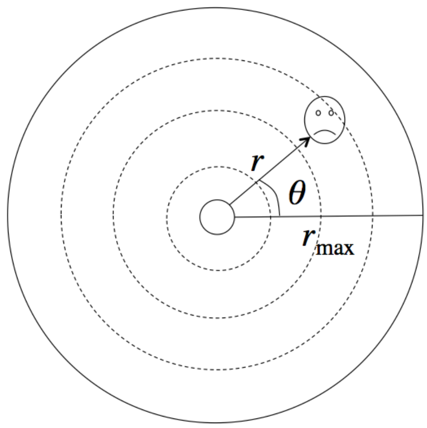
November 13, 2012

1 Introduction

During these lab sessions the students will work on a simplified radar detection scenario, a case for which a statistical model is readily available. Therefore, we will consider the design of detectors using an analytical approach. The students will analyze characteristic operation curves (ROC) of LRT detectors, and use them to implement the Neyman-Pearson detector. They will also study how such classifiers performance change as a function of the radar-to-target distance, the number of available observations, maximum admissible exploration time, etc.

The following definition is taken from the English version of the wikipedia: *"Radar (an acronym for RAdio Detection And Ranging) is an object detection system which uses radio waves to determine the range, altitude, direction, or speed of objects. It can be used to detect aircraft, ships, spacecraft, guided missiles, motor vehicles, weather formations, and terrain. The radar dish or antenna transmits pulses of radio waves or microwaves which bounce off any object in their path. The object returns a tiny part of the wave's energy to a dish or antenna which is usually located at the same site as the transmitter."*.

During these lab sessions we will analyze the detection phase of a radar system, i.e., the process of deciding the presence or absence of a target, as well as its position, based on the received signal. We will consider a simplified model which incorporates the most relevant elements for the detection phase.



In order to search for potential targets, the radar emits pulses sweeping a complete circumference. For each angle θ, it emits a pulse and then listens any received signals coming from the same direction. These received signals are the basis for target detection. Since electromagnetic waves propagate with the speed of light, we can align the elapsed time between the emitted and received signals with the distance at which an hypothetical target would lie:

$$r = \frac{c\,t}{2}, \qquad (1)$$

where r is the radar-to-target distance, c = 300.000 km/s, and t is the elapsed time. The total time the radar has to remain in listening mode after emitting a pulse will depend on the maximum distance to explore, which we will set to $r_{\max} = 15$ km. Consequently, for each angle the antenna has to remain in listening mode for 0.1 ms.

The assumed model takes also into consideration that the amplitude of any received echoes will decrease as the distance to target increases. Concretely, it can be shown that the received power decays with the fourth power of the distance; equivalently, the amplitude of the pulse will decay with r^2,

$$A_r = \alpha / r^2, \tag{2}$$

where A_r is the amplitude of the received signal, r is the distance expressed in meters, and α is a constant which depends on the amplitude of the emitted pulse, the directivity and aperture of antennas, etc. Along the practical, we will consider $\alpha = 5 \cdot 10^5$.

Finally, it is also necessary to take into account that any measurements in the reception antenna are subject to observation noise, that we will model as Gaussian, with zero mean and variance $v = 10^{-4}$. Thus, defining the absence of targets as the null hypothesis, the likelihoods of both hypotheses (target presence or absence) at a distance r will be

$$p_{X_r|H}(x_r|0) \sim G(0, v)$$
$$p_{X_r|H}(x_r|1) \sim G(\alpha/r^2, v)$$

x_r being the corresponding observation for the test.

2 Characterizing LRT deciders using ROC curves

Verify that the LRT for detecting a target at distance r can be expressed as a threshold test on the observation variable:

$$x_r \underset{D=0}{\overset{D=1}{\gtrless}} \eta \tag{3}$$

1. Express, as a function of the distance r, the value of the threshold η for the maximum likehood test. Plot, again as a function of r, the false alarm and detection probabilities (P_{FA} and P_{D}, respectively) of such ML test. If you find it convenient, consider using Matlab function `normcdf`.
2. In radar, it is customary to use the Neyman-Pearson criterion to design the detector. Consider a maximum admissible probablity of false alarm $P_{\mathrm{FA}} < 10^{-3}$. Using Matlab function `norminv`, obtain the thresholds corresponding to the Neyman-Pearson test and plot the evolution of P_{D} as a function of r. If the specification of the system requires that the probability P_{D} exceeds 0.9, find the maximum distance for which radar specifications would be satisfied.
3. Plot the ROCs characterizing the LRTs for the following radar-to-target distances: $r = 2000, 5000$ y 10000 m. Use Matlab function `norminv` to obtain a smooth curve taking equally-spaced (and sufficiently close) values for the probability of false alarm.

3 Improving performance through multiple measurements

In order to increase the range in which the radar system fulfills the P_{FA} and P_{D} specifications, we consider sending multiple pulses for each observation angle. Accordingly, if the emission and reception process is repeated l times, a set of observations $\{x_r^{(k)}\}, k = 1, \ldots, l$ is available for decision. If measurements are independent and identically distributed, it turns out that the decision depends only on the value of the sufficient statistic $t_r = \frac{1}{l} \sum_k x_r^{(k)}$, which is normally distributed under both hypotheses.

1. Compute the pdf of t_r under each hypothesis. Verify that maximum likelihood test can be expressed as a threshold test with respect to t_r.

2. For $r = r_{\max} = 15000$ m, plot the ROC curves corresponding to the LRT for t_r with the following number of observations $l = 1, 3, 10, 30, 100, 300$. Contrast those curves with the ones that would be obtained for a smaller distance.

3. For $r = 10000$, check how the P_{D} of the Neyman-Pearson decider with $P_{\mathrm{FA}} \leq 10^{-3}$ varies with l. State which is the minimum number of measurements l_0 that yields a detection probability above 0.9.

4. For the value of l_0 obtained in the previous step, plot the evolution of the P_{D} of the Neyman-Pearson decider with $P_{\mathrm{FA}} \leq 10^{-3}$ with respect to distance. Compare with the result obtained for $l = 1$.

5. Knowing that constant α is proportional to transmitted amplitude A_t, by which factor should we scale it in order to achieve the same performance at $r = 10000$ m using a single observation?

   As you can see, from an energy consumption perspective, the repeated emission of pulses is as efficient as the use of a single, larger-amplitude pulse. However, there are other practical elements that can limit the maximum usable power in the system. Also, since the radar system must work in real time, updating its estimates about presence or absence of target at each location, the l observations can be obtained at further explorations sweeps, i.e., the observations at each sweep can be used together with observations from previous sweeps. In this way, better performance can be achieved without extending the amount of time required for full exploration of the observation space.

4 Working with data

In this section, students will be able to work with synthetic data in order to determine the presence or absence of targets in a concrete case. Data file `data_P2.mat` provided along with this text includes the following variables:

- Variable `X`: It is 100×1000 matrix containing the collected observations, associated to the emission of 100 different pulses for a single angle θ. Therefore, each row of `X` contains measurements associated with each of the emitted pulses at 1000 consecutive instants after the emission.

- Variable `t`: It is a length 1000 vector containing time references. The i-th element of `t` is the delay associated to the i-th column of `X`.

1. Visualize the provided observations of matrix `X`. You may want to use Matlab functions `imagesc` and `mesh`. Try to locate larger values suggesting the presence of targets.

2. Build a matrix `T` of the same size as `X`, such that the i-th row of `T` contains the average of the first i rows of `X`. In this way, matrix `T` contains the sufficient statistics for detection considering a number of observations between 1 and 100. Visualize matrix `T`. If a single target exists, how far away from the radar is it?

3. Finally, compute and visualize the output of Neyman-Pearson decider with $P_{\mathrm{FA}} \leq 10^{-3}$. Note that the threshold to use over `T` depends on the number of available observations, but not on their delay (equivalently, on the distance to the potential target). Is it reasonable to think that there is a single target? Why do you think that there are other time instants in which the decider selects the non-null hypothesis?

5 Extra exercise

In this section, students should locate the position of some target associated to their NIA. In order to do this, a Matlab function `explora` is provided. It needs two arguments: the six last digits of the student's NIA and the exploration angle. The function then returns two variables: `t`, which is a vector containing the delays between the emission of the pulse and each of the observed measurements and said measurements, which are in the second output variable, `x`.

In order to locate the target, each student should explore all angles in the interval $[0, 2\pi]$, using at least 500 equispaced values. Simulate a situation in which, for each exploration angle, l_0 pulses are emitted; then use the results to compute each sufficient statistic for each [angle, distance] pair.

For each point of the obtained radar image, apply the corresponding Neyman-Pearson test. Visualize the output of such test using the provided function `polarimagesc`.