What do we plan on modeling exactly:

A. Radiation produced by free electron in constant magnetic field
    a. Evolution of trajectory (single particle to start with) with different time steps
    b. Power radiated as a function of the solid angle (non-relativistic vs. relativistic)

B. Radiation produced by electron in undulator
    a. Total energy emitted as function of magnetic field strength in undulator
    b. Total energy emitted as function of spatial frequency of undulator magnets

Equations needed to effectively model electron emitting synchrotron radiation in undulator (part B):

$$\bar{P}_{\mathrm{cen}}\Big|_{e^-} \simeq \frac{\pi e^2 c \gamma^2}{\epsilon_0 \lambda_u^2 N} \frac{K^2}{(1+K^2/2)^2}$$

The above equation is the power radiated to the central cone (will define in a moment) by a single electron in an undulator. This equation can be derived from the dipole radiation equation. The angular width of the central cone is defined as:

$$\theta_{\mathrm{cen}} = \frac{1}{\gamma^* \sqrt{N}}$$

Where the gamma-star is the effective lorentz factor, and N is the ratio of the mean photon wavelength divided by the range of photon wavelengths produced.

Self Reflection:

Brian - Did research on Synchrotron radiation to familiarize himself on the topic. Found equations for the power given off by an electron in an undulator, as well as the power give off in a solid angle from an electron in a circular path.

Chris - Derived equations for theta(t) and R(t) to find the trajectory of a particle in a circular magnetic field. Found a relativistic correction for the Lamor formula. Gave my previous paper and research to Brian

https://people.eecs.berkeley.edu/~attwood/srms/2007/Lec10.pdf

www.astro.utu.fi/~cflynn/astroII/l4.html

https://en.wikipedia.org/wiki/Synchrotron_radiation

Equations needed to effectively model electron emitting synchrotron radiation in circular path (part A):

$$\frac{dP}{d\Omega} = \frac{q^2}{16\pi^2 \epsilon_0 c} \frac{|a|^2}{1-\beta\cos\theta}\left[1 - \frac{\sin^2\theta\cos^2\phi}{\gamma^2(1-\beta\cos\theta)^2}\right]$$

Integrating this equation with respect to the azimuthal angle phi will allow us to plot the power radiated as a function of the polar angle which should tell us about the “focus” of the synchrotron radiation beam

$$P_{lamor} = \frac{(q^2 a^2)}{(6\pi\varepsilon_0 c^3)}$$

This equation describes the power lost by an particle that is being accelerated. This version is for non relativistic speeds.

$$\theta(t) = \frac{qBt}{m}$$

Derived theta equation from particle in circular motion. Constant change in theta.

$$R(t) = \frac{(T - P_{lamor})(2m)}{(q^2 B^2)}$$

Equation for the radial distance from the center of circular motion for a particle. It decreases as energy is radiated by the Lamor equation. T is initial Kinetic energy.

Also could calculate trajectory for relativistic particle speeds and see differences between non-relativistic and relativistic

$$P_{rel} = \frac{2}{3}\frac{(e^2 c)}{(R^2)}\left(\frac{E}{mc^2}\right)^4$$

Relativistic Power radiated equation

Runge-Kutta Method:
https://ac.els-cdn.com/0307904X88900698/1-s2.0-0307904X88900698-main.pdf?_tid=f7e042fe-8380-4706-863f-5d4f72a12084&acdnat=1522436004_d77c737ea54bb29ff5498ca5f8fad32e\

This discrete method to solving initial value problems is the method we plan on using to model the time evolution of the trajectory of the particle.