MTH 627: Advanced PDEs notes

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Hello.

Definition 1 (Semi-norm)**.** Let V be a vector space and define $p : V \to \mathbb{R}$. Then p is a semi-norm if

(1) p has absolute homogeneity; $p(cx) = |c|p(x)$ for all $x \in V$ and $c \in \mathbb{R}$, and

(2) p has the triangle inequality; $p(x + y) \leq p(x) + p(y)$ for all $x, y \in V$.

Property 1 ((semi-norms))**.** If $p : V \to \mathbb{R}$ is a semi-norm, then

(1) p has the reverse triangle inequality; $|p(x) - p(y)| \leq p(x - y)$, for all $x, y \in V$.

Solution.
Let $x, y \in V$ and suppose p is a semi-norm on V. Using the triangle inequality we compute,

$$p(x) = p(x - y + y) \leq p(x - y) + p(y)$$
$$p(y) = p(y - x + x) \leq p(x - y) + p(x),$$

meaning that

$$p(x) - p(y) \leq p(x - y) \text{ and } p(y) - p(x) \leq p(x - y)$$

which results in

$$-p(x - y) \leq p(x) - p(y) \leq p(x - y)$$

by multiplying the second inequality by -1. Therefore, $|p(x) - p(y)| \leq p(x - y)$, for all $x, y \in V$.

(2) p is non-negative $p(x) \geq 0$ for all $x \in V$

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