Finance Quantitative

Risque de Taux

Patrick Hénaff

Version: 16 Dec 2024

Relative Price Change (MPP)

Calculate the percentage price change for 4 bonds with different annual coupon rates (5% and 10%) and different maturities (3 years and 10 years), starting with a common 7.5% YTM (with annual compounding frequency), and assuming successively a new yield of 5%, 7%, 7.49%, 7.51%, 8% and 10%.

Treasury Bond

Suppose that the Treasury department issues a new 2-year bond that settles today and matures in exactly 2 years. It has a yield of 6% and a coupon rate of 6%. Coupon frequency and compounding frequency are assumed to be semiannual. There are 182 days in the first coupon period. Answer the following questions.

1. What is the price of the bond?
2. What is the accrued interest?
3. What is the duration and modified duration?

Duration

1. Find the duration of a 10 year, 8% bond trading at par.
2. Analytical formula for duration.
   (a) Using the formula for bond price:

   $$P = \frac{c}{r}[1-(1+r)^{-n}] + (1+r)^{-n}$$

   compute $\frac{\partial \ln(P)}{\partial r}$

   (b) Use this result to derive a formula for duration

   (c) Show that the limiting value of duration as maturity is increased to infinity is:

   $$\lim_{n \to \infty} D = \frac{1+r}{r}$$

Cash flow matching

Suppose the price is \$212 for a 2-year coupon bond with face of \$200 and an annual coupon (first one is one year from now) of \$40. Suppose also that the price is \$150 for a 1-year coupon bond with face of \$150 and an annual coupon (one remaining, one year from now) of \$15.

Remaining pension benefits in a plan having two more years to go are \$95,000 one year from now and \$60,000 two years from now.

1. What replicating portfolio of the two coupon bonds covers the pension liabilities exactly?
2. What is the price of the replicating portfolio?

Bond dedication

In this problem, we construct a bond portfolio that generates a cash flow stream that matches a liability.

Assume that you must pay the amounts summarized in table~1. You can invest in a portfolio of 5 bonds described in table~2.

Year	Cash Flow
1	-100
2	-200
3	-150
4	-400
5	-300

Table 1: Liability cash flow stream

Bond	Maturity	Coupon	Yield
A	1	.05	.05
B	2	.07	.075
C	3	.06	.058
D	4	.05	.049
E	5	.08	.081

Table 2: Available bonds for dedication

At every period, we can re-invest excess cash flow at a rate of .02, but cannot borrow.

Let's define the following notation:

q_i quantity of bond i

$C(t)$ cash balance at time t

$F_i(t)$ cash flow from 1 unit of bond i at time t.

The purpose of the problem is to determine $q_i, i \in A, B, C, D$ and $C(t), t = 0, \dots, 4$. The cash balance at end of year 5, $C(5)$ should be 0.

1. Write the accounting identity defining the cash-flow balance at each period (i.e. the balance between the money received and the money paid out).
2. Write the constrains on the variables q_i and $C(t)$.
3. Your goal is to minimize the cost of this dedication strategy. Write the corresponding objective function.
4. Use the linprog package in R to solve the problem

Reinvestment Risk and Market Risk

Consider a 3-year standard bond with a 6% YTM and a 100 € face value, which delivers a 10% coupon rate. Coupon frequency and compounding frequency are assumed to be annual. Its price is 110.69 € and its duration is equal to 2.75. We assume that YTM changes instantaneously to become 5%, 5.5%, 6.5% or 7% and stays at this level during the life of the bond. Whatever the change in this YTM, show that the sum of the bond price and the reinvested coupons after 2.75 years is always the same.

Duration Hedging

An investor holds 100,000 units of bond A whose features are summarized in the following table. He wishes to be hedged against a rise in interest rates by selling some bond H.

Bond	Maturity	Coupon rate (%)	YTM (%)	Duration	Price
A	18	9.5	8	9.5055	114.181
H	20	10	8	9.87	119.792

Coupon frequency and compounding frequency are assumed to be semiannual. YTM stands for yield to maturity. The YTM curve is flat at an 8% level.

1. What is the quantity of the hedging instrument H that the investor has to sell?
2. We suppose that the YTM curve increases instantaneously by 0.1%.
   (a) What happens if the bond portfolio has not been hedged?
   (b) And if it has been hedged?
3. Same question as the previous one when the YTM curve increases instantaneously by 2%.
4. Conclude.

What is the quantity of the hedging instrument H that the investor has to sell?

Rich-Cheap Bond Strategy

A trader implements a duration-neutral strategy that consists in buying a cheap bond and selling a rich bond. Today, the rich and cheap bonds have the following characteristics:

Bond	Maturity	Coupon rate (%)	YTM (%)
Rich	10	5	7.50
Cheap	12	5.5	7.55

Coupon frequency and compounding frequency are assumed to be annual. Face value is 100 € for the two bonds.

Compute the PV01 of the two bonds and find the hedged position.