

# Modèles de Black-Litterman

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# Droite de Marché des Capitaux

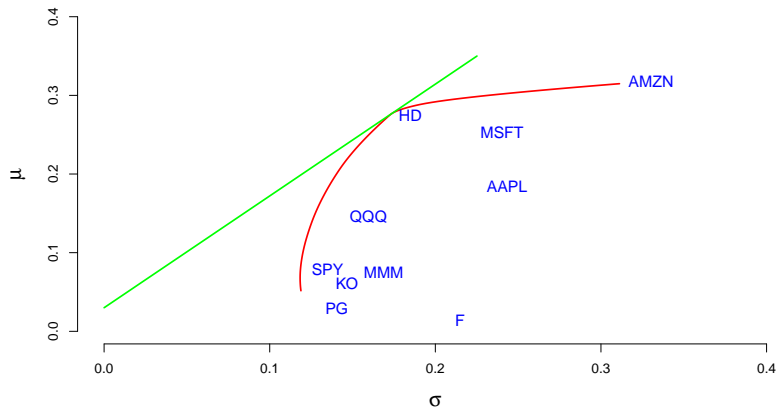


Figure 1: Droite de Marché des Capitaux

## Black-Litterman (1)

- ▶ Par défaut: Accepter les espérances de rendement implicites dans le portefeuille de marché, et investir dans ce portefeuille.
- ▶ Exprimer des “vues” sur l’espérance de rendement de portefeuilles quelconques
- ▶ Utiliser ces “vues” pour modifier les espérances de rendement et la structure de covariance des actifs.

## Information ex-ante

Distribution des rendements:

$$r \sim \mathcal{N}(\mu, \Sigma)$$

L'espérance de rendement  $\mu$  est aussi aléatoire

$$\mu = \Pi + \epsilon^{(e)}$$

avec

$$\epsilon^{(e)} \sim \mathcal{N}(0, \tau \Sigma)$$

## Optimisation inversée

On utilise le portefeuille de marché pour inférer l'espérance de rendement:

$$U(w) = w^T \Pi - \frac{\delta}{2} w^T \Sigma w$$

Solution “inversée” de  $\Pi$  en fonction de  $w$ :

$$\Pi = \delta \Sigma w_{eq}$$

## Expression de prédictions à propos des rendements

Les prédictions sont exprimées par des portefeuilles dont on donne le rendement, avec une marge d'erreur.

$$P\mu = Q + \epsilon^{(\nu)}$$

avec

$$\epsilon^{(\nu)} \sim \mathcal{N}(0, \Omega)$$

## Résumé

Deux équations pour  $\mu$

- Distribution ex-ante

$$\mu = \Pi + \epsilon^{(e)}$$

- Views

$$P\mu = Q + \epsilon^{(v)}$$

## Conséquence

Distribution normale *ex-post* de l'espérance de rendement:

$$\mu \sim \mathcal{N}(\mu^*, M^{-1})$$

$$\mu^* = \left[ (\tau \Sigma)^{-1} + P^T \Omega^{-1} P \right]^{-1} \left[ (\tau \Sigma)^{-1} \Pi + P^T \Omega^{-1} Q \right]$$

$$M^{-1} = \left[ (\tau \Sigma)^{-1} + P^T \Omega^{-1} P \right]^{-1}$$



Distribution *ex-post* du rendement:

$$r \sim \mathcal{N}(\mu^*, \Sigma + M^{-1})$$

# Exemple

##	IBM	MS	DELL
##	Min. : -0.445480	Min. : -0.53590	Min. : -0.515656
##	1st Qu.: -0.060482	1st Qu.: -0.06699	1st Qu.: -0.086565
##	Median : 0.009032	Median : 0.02846	Median : 0.008809
##	Mean : 0.006868	Mean : 0.01264	Mean : 0.002769
##	3rd Qu.: 0.070162	3rd Qu.: 0.10020	3rd Qu.: 0.079835
##	Max. : 0.353799	Max. : 0.50707	Max. : 0.497706
##	C	JPM	BAC
##	Min. : -0.3400743	Min. : -0.444608	Min. : -0.278997
##	1st Qu.: -0.0572979	1st Qu.: -0.076672	1st Qu.: -0.050389
##	Median : 0.0009806	Median : 0.013887	Median : 0.010103
##	Mean : 0.0056924	Mean : -0.003876	Mean : 0.008242
##	3rd Qu.: 0.0539650	3rd Qu.: 0.082539	3rd Qu.: 0.065332
##	Max. : 0.2533333	Max. : 0.317181	Max. : 0.173060

# Correlation

	IBM	MS	DELL	C	JPM	BAC
IBM	1.0000000	0.3873395	0.4193389	0.4635322	0.4459814	0.3585381
MS	0.3873395	1.0000000	0.3981657	0.5929457	0.5226294	0.4646464
DELL	0.4193389	0.3981657	1.0000000	0.2701329	0.2671891	0.2321042
C	0.4635322	0.5929457	0.2701329	1.0000000	0.5477972	0.5070248
JPM	0.4459814	0.5226294	0.2671891	0.5477972	1.0000000	0.6832878
BAC	0.3585381	0.4646464	0.2321042	0.5070248	0.6832878	1.0000000

## Exemple 1: IBM et Dell surperforme MS ( $sd = 5\%$ )

Rendement de  $(1/2 \text{ IBM} - \text{MSFT} + 1/2 \text{ DELL}) = 6\% + \text{terme d'erreur}$

```
sd <- .05
pickMatrix <- matrix(c(1/2, -1, 1/2, rep(0, 3)),
                      nrow = 1, ncol = 6)
views <- BLViews(P = pickMatrix, q = 0.06,
                 confidences = 1/sd,
                 assetNames = colnames(monthlyReturns))
views
```

```
## 1 : 0.5*IBM+-1*MS+0.5*DELL=0.06 + eps. Confidence: 20
```

# Traduction en distribution ex-post (voir note de cours)

```
## Prior means:
## IBM MS DELL C JPM BAC
## 0 0 0 0 0 0
## Posterior means:
## IBM MS DELL C JPM
## 0.0017305650 -0.0022402693 0.0077981956 -0.0004238026 -0.0003714824
## BAC
## 0.0001085247
## Posterior covariance:
## IBM MS DELL C JPM BAC
## IBM 0.018118357 0.011946685 0.015190295 0.010716615 0.010700802 0.004900854
## MS 0.011946685 0.016933332 0.009731609 0.009621199 0.011731245 0.003802819
## DELL 0.015190295 0.009731609 0.046699650 0.006220050 0.010740608 0.003295221
## C 0.010716615 0.009621199 0.006220050 0.010179856 0.008430558 0.004818358
## JPM 0.010700802 0.011731245 0.010740608 0.008430558 0.015815804 0.006248510
## BAC 0.004900854 0.003802819 0.003295221 0.004818358 0.006248510 0.007465990
```

Exemple 2: Le rendement moyen du secteur financier sera de 15% (sd = .04)

Rendement de  $(C + JPM + BAC + MS)/4 = 15\% + \text{terme d'erreur}$

```
finViews <- matrix(ncol = 4, nrow = 1, dimnames = list(NULL, c("C", "JPM", "BAC", "MS")))
finViews[,1:4] <- rep(1/4, 4)
views <- addBLViews(finViews, q=0.15, confidences=1/sd, views)
views
```

```
## 1 : 0.5*IBM+-1*MS+0.5*DELL=0.06 + eps. Confidence: 20
## 2 : 0.25*MS+0.25*C+0.25*JPM+0.25*BAC=0.15 + eps. Confidence: 20
```

# Traduction en distribution ex-post (voir note de cours)

```
marketPosterior <- BLPosterior(as.matrix(monthlyReturns), views,
                                tau = 1/2,
                                marketIndex = as.matrix(sp500Returns),
                                riskFree = as.matrix(US13wTB))
marketPosterior
```

```
## Prior means:
##      IBM      MS      DELL      C      JPM      BAC
## 0.020883598 0.059548398 0.017010062 0.014492325 0.027365230 0.002829908
## Posterior means:
##      IBM      MS      DELL      C      JPM      BAC
## 0.032542272 0.061826612 0.033876046 0.021064248 0.038104037 0.008792056
## Posterior covariance:
##      IBM      MS      DELL      C      JPM      BAC
## IBM  0.022102635 0.010855189 0.013551573 0.009009221 0.01138556 0.005716916
## MS   0.010855189 0.034015668 0.016902071 0.014078724 0.01630833 0.009054922
## DELL 0.013551573 0.016902071 0.048889764 0.007845313 0.01016152 0.005529305
## C    0.009009221 0.014078724 0.007845313 0.017113697 0.01213114 0.007032434
## JPM  0.01138556 0.016308332 0.010161517 0.012131145 0.02980206 0.012643858
## BAC  0.005716916 0.009054922 0.005529305 0.007032434 0.01264386 0.011729821
```

# Optimisation MV classique

Portefeuille Tangent:

```
optPorts <- optimalPortfolios.fPort(marketPosterior,  
  optimizer = "tangencyPortfolio")
```



# Black-Litterman (7)

Weights

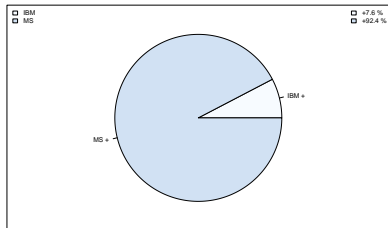


Figure 2: Prior Rdt/Risque

Weights

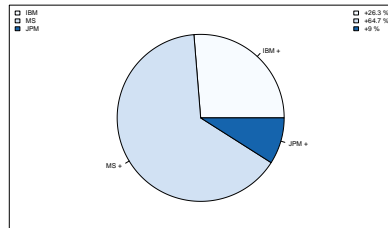


Figure 3: Posterior Rdt/Risque

## Exercice

- ▶ Contraindre  $w_i > 0$  en utilisant le code de la note de cours.
- ▶ BAC va surperformer Citibank (C)
- ▶ Dell aura un rendement de 0.5%

