Programmation Stochastique avec Recours

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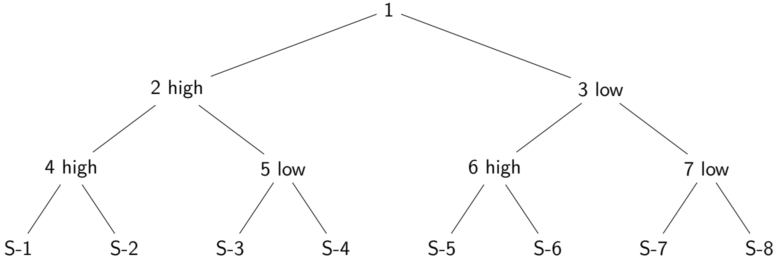
Arbre de décision



Solution

Table 1: Optimal asset allocation at each node in the 3-period tree

Node	Stock	Bond
1	41479	13521
2	65095	2168
3	36743	22368
4	83840	0
5	0	71429
6	0	71429
7	64000	0

Surplus or Shortfall by scenario

Table 2: Shortfall or excess by scenario

Scenario	Shortfall	Excess
1	0	24800
2	0	8870
3	0	1429
4	0	0
5	0	1429
6	0	0
7	0	0
8	12160	0

Stochastic vs Myopic Solution

Table 3: Expected Value vs. Recourse solutions of the asset allocation problem.

	EV solution		Recourse solution	
Scenario	Shortfall	Surplus	Shortfall	Surplus
1		27422		24800
2		11094		8870
3		11094		1429
4	2752			
5		11094		1429
6	2752			
7	2752			
8	14494		12160	

Augmented Lagrangian

Algorithm 1: Augmented Lagrangian algorithm

Input : ρ, tol, k_{max}

Output: x^*

$k \leftarrow 0; \quad \lambda_{\leftarrow} 0$

while $k < k_{max}$ *and* $t > tol$ **do**

$\quad x_{k+1} \leftarrow \underset{x}{\operatorname{argmin}} \phi_k(x)$

$\quad \lambda_{k+1} \leftarrow \lambda_k - \rho g(x_{k+1})$

$\quad k \leftarrow k + 1$

$\quad t_1 \leftarrow ||\lambda_{k+1} - \lambda_k||$

$\quad t_2 \leftarrow ||g(x_{k+1}||$

$\quad t \leftarrow \max(t_1, t_2)$

end

Algo Progressive Hedging

Algorithm 2: Progressive Hedging algorithm

Input : ρ, tol, i_{max}

Output: x^*

$i \leftarrow 0; \lambda_i \leftarrow 0;$ converged $\leftarrow$ *False*

while $k < k_{max}$ *and* !*converged* **do**

  Solve the K subproblems to obtain $x_k^{i+1}, k = 1, \ldots, K$

  Compute $\hat{x}^{i+1} = \sum_{k=1}^{K} \pi_k x_k^{i+1}$

  Update the multipliers: $\lambda_k^{i+1} = \lambda_k^i - \rho(x_k^{i+1} - \hat{x}^{i+1})$

  $i \leftarrow i + 1$

  $t_1 \leftarrow ||\lambda^{i+1} - \lambda^i|| < \text{tol}_1$

  $t_2 \leftarrow ||g^1(x^{i+1}|| < \text{tol} < \text{tol}_2$

  $t_3 \leftarrow ||g^2(x^{i+1}|| < \text{tol} < \text{tol}_3$

  converged $\leftarrow \max(t_1, t_2, t_3)$

end

First Example

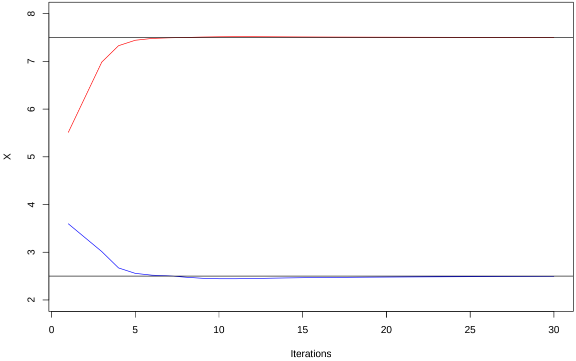
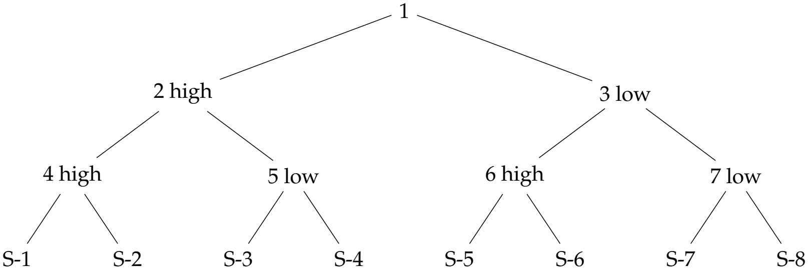


Figure 1: Progressive Hedging Iterations

Second example: A 3-stage optimization problem



Linear Convergence of the Progressive Hedging Algorithm