Gestion de Portefeuille

Ex-2: Modèle Moyenne-Variance (Solution)

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Version: 12 Feb 2025

1 Données

1.1 Séries de rendement quatidien pour 11 valeurs:

```
daily.ret.file <- file.path(get.data.folder(), "daily.ret.rda")
load(daily.ret.file)
kable(table.Stats(daily.ret), "latex", booktabs=T) %>% kable_styling(latex_options="scale_down")
```

1.2 Rendement annuel moyen:

```
kable(252*100*colMeans(daily.ret), "latex", booktabs=T, digits=1, col.names=c("Rendement (%)"),
      caption="Rendement annuel moyen")
```

1.3 Matrice de corrélation des rendements:

```
correl <- cor(daily.ret)
correl[lower.tri(correl)] <- NA
```

	AAPL	AMZN	MSFT	F	SPY	QQQ	XOM	MMM	HD	PG	KO
Observations	3308.0000	3308.0000	3308.0000	3308.0000	3308.0000	3308.0000	3308.0000	3308.0000	3308.0000	3308.0000	3308.0000
NAs	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Minimum	-0.1792	-0.1278	-0.1171	-0.2500	-0.0984	-0.0896	-0.1395	-0.1295	-0.0822	-0.0790	-0.0867
Quartile 1	-0.0077	-0.0094	-0.0073	-0.0103	-0.0038	-0.0047	-0.0068	-0.0055	-0.0067	-0.0046	-0.0047
Median	0.0010	0.0008	0.0005	0.0000	0.0006	0.0010	0.0001	0.0008	0.0006	0.0004	0.0007
Arithmetic Mean	0.0012	0.0015	0.0008	0.0005	0.0004	0.0006	0.0001	0.0004	0.0008	0.0004	0.0005
Geometric Mean	0.0010	0.0012	0.0006	0.0001	0.0003	0.0005	0.0000	0.0003	0.0006	0.0003	0.0004
Quartile 3	0.0112	0.0123	0.0088	0.0106	0.0056	0.0070	0.0073	0.0070	0.0082	0.0055	0.0059
Maximum	0.1390	0.2695	0.1860	0.2952	0.1452	0.1216	0.1719	0.0988	0.1407	0.1021	0.1388
SE Mean	0.0003	0.0004	0.0003	0.0005	0.0002	0.0002	0.0003	0.0002	0.0003	0.0002	0.0002
LCL Mean (0.95)	0.0005	0.0006	0.0002	-0.0005	0.0000	0.0002	-0.0004	-0.0001	0.0002	0.0000	0.0001
UCL Mean (0.95)	0.0019	0.0023	0.0013	0.0014	0.0008	0.0011	0.0006	0.0009	0.0013	0.0007	0.0009
Variance	0.0004	0.0006	0.0003	0.0007	0.0001	0.0002	0.0002	0.0002	0.0003	0.0001	0.0001
Stdev	0.0196	0.0243	0.0170	0.0266	0.0121	0.0130	0.0150	0.0140	0.0162	0.0109	0.0113
Skewness	-0.2151	1.4889	0.4319	0.7627	0.1379	-0.0084	0.4199	-0.3815	0.5114	0.0555	0.5004
Kurtosis	6.2706	16.8872	10.2176	20.9458	15.2824	7.3976	15.4203	7.3856	6.4641	8.1017	14.3236

Table 1: Rendement annuel moyen

	Rendement (%)
AAPL	30.2
AMZN	37.2
MSFT	19.0
F	11.4
SPY	9.9
QQQ	15.3
XOM	3.5
MMM	9.9
HD	19.2
PG	9.3
KO	12.5

Table 2: Corrélation des rendements quotidiens

	AAPL	AMZN	MSFT	F	SPY	QQQ	XOM	MMM	HD	PG	KO
AAPL	1	0.46	0.49	0.37	0.61	0.75	0.40	0.45	0.42	0.32	0.32
AMZN		1.00	0.50	0.33	0.56	0.66	0.39	0.41	0.44	0.27	0.30
MSFT			1.00	0.39	0.71	0.76	0.53	0.53	0.49	0.44	0.46
F				1.00	0.56	0.53	0.37	0.44	0.46	0.30	0.31
SPY					1.00	0.92	0.77	0.75	0.71	0.62	0.60
QQQ						1.00	0.64	0.69	0.66	0.52	0.52
XOM							1.00	0.60	0.47	0.52	0.49
MMM								1.00	0.55	0.50	0.47
HD									1.00	0.45	0.44
PG										1.00	0.57
KO											1.00

1.4 Modèle Moyenne Variance

Le portefeuille de variance minimale formé à l'aide des 11 titres ci-dessus:

```
cov.all <- cov(daily.ret)
n <- ncol(daily.ret)
dvec <- rep(0, n)
Amat <- matrix(1, nrow=n, ncol=1)
bvec <- 1
sol <- solve.QP(cov.all, dvec, Amat, bvec, meq=1)
```

Table 3: Solution Variance Minimale (%)

	weight
AAPL	-1.32
AMZN	-2.84
MSFT	-6.28
F	-2.66
SPY	-22.78
QQQ	38.75
XOM	3.33
MMM	10.34
HD	2.98
PG	42.89
KO	37.59

2 Questions

2.1 Modifier le programme ci-dessus pour imposer des poids positifs:

```
A.neq <- diag(n)
A.eq <- matrix(1, nrow=n, ncol=1)
Amat <- cbind(A.eq, A.neq)
bvec <- c(1, rep(0,n))
sol <- solve.QP(cov.all, dvec, Amat, bvec, meq=1)
```

Table 4: Solution Variance Minimale, poids positifs

	weight
AAPL	1.22
QQQ	11.84
MMM	8.18
HD	0.44
PG	42.24
KO	36.08

2.2 Calculer le portefeuille risqué qui procure un rendement espéré de 13% par an.

Solution numérique:

```
mu.star = .13
A.eq.1 <- matrix(1, nrow=n, ncol=1)
A.eq.2 <- matrix(252*colMeans(daily.ret), nrow=n, ncol=1)
Amat <- cbind(A.eq.1, A.eq.2)
bvec <- c(1, mu.star)
sol <- solve.QP(cov.all, dvec, Amat, bvec, meq=1)
```

Table 5: Solution Numérique Variance Minimale, espérance de Rdt=13%

	weight
AAPL	2.74
AMZN	-0.13
MSFT	-3.83
F	-2.83
SPY	-28.28
QQQ	32.75
XOM	1.37
MMM	10.47
HD	5.83
PG	42.83
KO	39.09

Solution analytique, en utilisant la notation de la note de cours:

```
Sigma <- cov.all
A.1 <- cbind(Sigma, -Amat)
A.2 <- cbind(t(Amat), diag(0,2))
A <- rbind(A.1, A.2)
b <- c(rep(0, n), 1, mu.star)
sol <- solve(A, b)
```

Table 6: Solution Analytique Variance Minimale, espérance de Rdt=13%

	weight
AAPL	2.74
AMZN	-0.13
MSFT	-3.83
F	-2.83
SPY	-28.28
QQQ	32.75
XOM	1.37
MMM	10.47
HD	5.83
PG	42.83
KO	39.09

2.3 Si le taux sans risque est de 4%, quel est le portefeuille tangent correspondant?

```
r.f <- .04
mu <- matrix(252*colMeans(daily.ret), nrow=n, ncol=1)
w.nom <- solve(Sigma, mu-r.f)
w.den <- sum(w.nom)
```

```r
w.t <- w.nom/sum(w.nom)
r.bar = sum(w.t * mu)
```

Table 7: Portefeuille tangent Rf=4%.

	weight
AAPL	116.55
AMZN	75.68
MSFT	64.95
F	-7.50
SPY	-182.50
QQQ	-135.67
XOM	-53.70
MMM	14.17
HD	85.71
PG	41.09
KO	81.22

On peut vérifier que le portefeuille tangent est sur la frontière efficiente en résolvant le modèle MV pour l'espérance de rendement calculé.

```r
mu.star = r.bar
A.eq.1 <- matrix(1, nrow=n, ncol=1)
A.eq.2 <- matrix(252*colMeans(daily.ret), nrow=n, ncol=1)
Amat <- cbind(A.eq.1, A.eq.2)
bvec <- c(1, mu.star)
sol <- solve.QP(cov.all, dvec, Amat, bvec, meq=1)
```

Table 8: Solution Numérique Variance Minimale, espérance de Rdt du portefeuille tangent

	weight
AAPL	116.55
AMZN	75.68
MSFT	64.95
F	-7.50
SPY	-182.50
QQQ	-135.67
XOM	-53.70
MMM	14.17
HD	85.71
PG	41.09
KO	81.22

2.4 Test with rank deficient matrix

We cancel the last 3 eigenvalues of the covariance matrix

```
s <- svd(cov.all)
ev <- s$d
ev[9:11] = 0
cov.low.rank <- s$u %*% diag(ev) %*% t(s$v)

cov.shrink <- cov.shrink(cov.low.rank)
```

```
## Estimating optimal shrinkage intensity lambda.var (variance vector): 0.7884
##
## Estimating optimal shrinkage intensity lambda (correlation matrix): 0.3733
```

```
n <- ncol(daily.ret)
dvec <- rep(0, n)
Amat <- matrix(1, nrow=n, ncol=1)
bvec <- 1
sol <- solve.QP(cov.shrink, dvec, Amat, bvec, meq=1)
```

Table 9: Solution Variance Minimale (%)

	weight
AAPL	12.92
AMZN	1.21
MSFT	3.99
F	-5.76
SPY	2.24
QQQ	7.55
XOM	7.50
MMM	15.04
HD	8.84
PG	25.34
KO	21.12