Monte-Carlo

Pricing by Simulation

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Use of Monte-Carlo Simulations

Simulations are used for:

- Risk measurement:
  - Scenario analysis, VaR
  - Stress testing (Basel II, Solvency II)
- Derivatives pricing
  - Easy to implement (just code the payoff formula)
  - Flexible: it dissociates the dynamics of the risk factors from the evaluation of the option.

Monte-Carlo Simulations: Historical vs. Risk-Neutral

- Simulations for scenario analysis and VaR are based on actual probabilities (historical VaR)
- Simulations for pricing are based on risk-neutral probabilities.
- Economic scenarii generation for Solvency II calculations use hybrid methods.
  - Market consistent
  - Use historical data when implied volatility from option market is not available

Simulating a Log-Normal Process

Given a stock with expected continuous compounded return μ:

$$\ln(S_t) - \ln(S_0) \approx \mathcal{N}\left((\mu - \frac{1}{2}\sigma^2)t, \sigma^2 t\right)$$

or

$$S_t = S_0 e^{x_t}$$

with $x_t \approx \mathcal{N}\left((\mu - \frac{1}{2}\sigma^2)t, \sigma^2 t\right)$.

$$x_t = (\mu - \frac{1}{2}\sigma^2)t + \sigma\sqrt{t}\epsilon$$

with $\epsilon \approx \mathcal{N}(0, 1)$.

How to simulate a log-normal process

To simulate a time series of stock prices following a log-normal process, observed at intervals Δt:

1. Start with S_0 at $t = 0$
2. for $t = \Delta t, 2\Delta t, \dots, T$:
   2.1 Simulate $\epsilon \approx \mathcal{N}(0, 1)$: ϵ_t
   2.2 Compute

$$S_t = S_{t-\Delta t} e^{\left(r - \frac{\sigma^2}{2}\right)\Delta t + \sigma\sqrt{\Delta t}\epsilon_t}$$