Modèles de Black-Litterman

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Droite de Marché des Capitaux

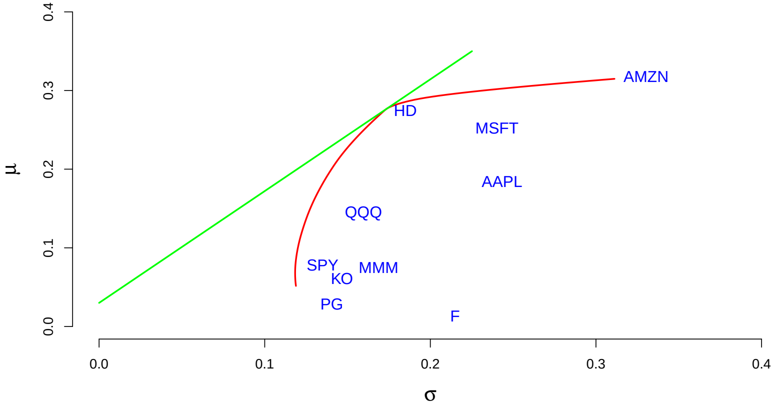

Figure 1: Droite de Marché des Capitaux

Black-Litterman (1)

- Par défaut: Accepter les espérances de rendement implicites dans le portefeuille de marché, et investir dans ce portefeuille.
- Exprimer des "vues" sur l'espérance de rendement de portefeuilles quelconques
- Utiliser ces "vues"pour modifier les espérances de rendement et la structure de covariance des actifs.

Information ex-ante

Distribution des rendements:

$$r \sim \mathcal{N}(\mu, \Sigma)$$

L'espérance de rendement μ est aussi aléatoire

$$\mu = \Pi + \epsilon^{(e)}$$

avec

$$\epsilon^{(e)} \sim \mathcal{N}(0, \tau\Sigma)$$

Optimisation inversée

On utilise le portefeuille de marché pour inférer l'espérance de rendement:

$$U(w) = w^T \Pi - \frac{\delta}{2} w^T \Sigma w$$

Solution "inversée" de Π en fonction de w:

$$\Pi = \delta \Sigma w_{eq}$$

Expression de prédictions à propos des rendements

Les prédictions sont exprimées par des portefeuilles dont on donne le rendement, avec une marge d'erreur.

$$P\mu = Q + \epsilon^{(v)}$$

avec

$$\epsilon^{(v)} \sim \mathcal{N}(0, \Omega)$$

Résumé

Deux équations pour μ

- Distribution ex-ante

$$\mu = \Pi + \epsilon^{(e)}$$

- Views

$$P\mu = Q + \epsilon^{(v)}$$

Conséquence

Distribution normale *ex-post* de l'espérance de rendement:

$$\mu \sim \mathcal{N}(\mu^*, M^{-1})$$

$$\mu^* = \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P^T\Omega^{-1}Q\right]$$

$$M^{-1} = \left[(\tau\Sigma)^{-1} + P^T\Omega^{-1}P\right]^{-1}$$

Distribution *ex-post* du rendement:

$$r \sim \mathcal{N}(\mu^*, \Sigma + M^{-1})$$

Exemple

```
##       IBM                   MS                  DELL
##  Min.   :-0.445480   Min.   :-0.53590   Min.   :-0.515656
##  1st Qu.:-0.060482   1st Qu.:-0.06699   1st Qu.:-0.086565
##  Median : 0.009032   Median : 0.02846   Median : 0.008809
##  Mean   : 0.006868   Mean   : 0.01264   Mean   : 0.002769
##  3rd Qu.: 0.070162   3rd Qu.: 0.10020   3rd Qu.: 0.079835
##  Max.   : 0.353799   Max.   : 0.50707   Max.   : 0.497706
##        C                   JPM                  BAC
##  Min.   :-0.3400743   Min.   :-0.444608   Min.   :-0.278997
##  1st Qu.:-0.0572979   1st Qu.:-0.076672   1st Qu.:-0.050389
##  Median : 0.0009806   Median : 0.013887   Median : 0.010103
##  Mean   : 0.0056924   Mean   :-0.003876   Mean   : 0.008242
##  3rd Qu.: 0.0539650   3rd Qu.: 0.082539   3rd Qu.: 0.065332
##  Max.   : 0.2533333   Max.   : 0.317181   Max.   : 0.173060
```

Correlation

	IBM	MS	DELL	C	JPM	BAC
IBM	1.0000000	0.3873395	0.4193389	0.4635322	0.4459814	0.3585381
MS	0.3873395	1.0000000	0.3981657	0.5929457	0.5226294	0.4646464
DELL	0.4193389	0.3981657	1.0000000	0.2701329	0.2671891	0.2321042
C	0.4635322	0.5929457	0.2701329	1.0000000	0.5477972	0.5070248
JPM	0.4459814	0.5226294	0.2671891	0.5477972	1.0000000	0.6832878
BAC	0.3585381	0.4646464	0.2321042	0.5070248	0.6832878	1.0000000

Exemple 1: IBM et Dell surperforme MS (sd $= 5\%$)

Rendement de (1/2 IBM - MSFT + 1/2 DELL) $= 6\% +$ terme d'erreur

```
sd <- .05
pickMatrix <- matrix(c(1/2, -1, 1/2, rep(0, 3)),
                     nrow = 1, ncol = 6 )
views <- BLViews(P = pickMatrix, q = 0.06,
                 confidences =  1/sd,
                 assetNames = colnames(monthlyReturns))
views
```

```
## 1 : 0.5*IBM+-1*MS+0.5*DELL=0.06  + eps. Confidence: 20
```

Traduction en distribution ex-post (voir note de cours)

```
## Prior means:
##  IBM   MS DELL    C  JPM  BAC
##    0    0    0    0    0    0
## Posterior means:
##           IBM            MS          DELL             C           JPM
##  0.0008752399 -0.0032537549  0.0034378545 -0.0006361924 -0.0019010603
##           BAC
## -0.0004723042
## Posterior covariance:
##              IBM          MS        DELL           C         JPM         BAC
## IBM  0.013336186 0.009807278 0.010943716 0.008423288 0.008347150 0.002232109
## MS   0.009807278 0.018635383 0.010119797 0.009082671 0.013620113 0.003707855
## DELL 0.010943716 0.010119797 0.027620885 0.006350919 0.008759737 0.002666050
## C    0.008423288 0.009082671 0.006350919 0.009073949 0.008062854 0.003977775
## JPM  0.008347150 0.013620113 0.008759737 0.008062854 0.016970264 0.006604006
## BAC  0.002232109 0.003707855 0.002666050 0.003977775 0.006604006 0.007002400
```

Exemple 2: Le rendement moyen du secteur financier sera de 15% (sd = .04)

Rendement de (C + JPM + BAC + MS)/4 = 15% + terme d'erreur

```
finViews <- matrix(ncol = 4, nrow = 1, dimnames = list(NULL, c("C","JPM","BAC","MS")))
finViews[,1:4] <- rep(1/4,4)
views <- addBLViews(finViews, q=0.15, confidences=1/sd, views)
views
```

```
## 1 : 0.5*IBM+-1*MS+0.5*DELL=0.06  + eps. Confidence: 20
## 2 : 0.25*MS+0.25*C+0.25*JPM+0.25*BAC=0.15  + eps. Confidence: 20
```

Traduction en distribution ex-post (voir note de cours)

```
marketPosterior <- BLPosterior(as.matrix(monthlyReturns), views,
                               tau = 1/2,
                               marketIndex = as.matrix(sp500Returns),
                               riskFree = as.matrix(US13wTB))
marketPosterior
```

```
## Prior means:
##         IBM          MS        DELL           C         JPM         BAC
## 0.020883598 0.059548398 0.017010062 0.014492325 0.027365230 0.002829908
## Posterior means:
##         IBM          MS        DELL           C         JPM         BAC
## 0.032542272 0.061826612 0.033876046 0.021064248 0.038104037 0.008792056
## Posterior covariance:
##              IBM          MS        DELL           C        JPM         BAC
## IBM  0.022102635 0.010855189 0.013551573 0.009009221 0.01138556 0.005716916
## MS   0.010855189 0.034015668 0.016902071 0.014078724 0.01630833 0.009054922
## DELL 0.013551573 0.016902071 0.048889764 0.007845313 0.01016152 0.005529305
## C    0.009009221 0.014078724 0.007845313 0.017113697 0.01213114 0.007032434
## JPM  0.011385556 0.016308332 0.010161517 0.012131145 0.02980206 0.012643858
## BAC  0.005716916 0.009054922 0.005529305 0.007032434 0.01264386 0.011729821
```

Optimisation MV classique

Portefeuille Tangent:

```
optPorts <- optimalPortfolios.fPort(marketPosterior,
            optimizer = "tangencyPortfolio")
```

Black-Litterman (7)

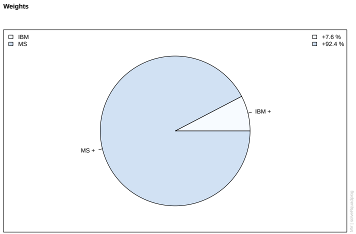

Figure 2: Prior Rdt/Risque

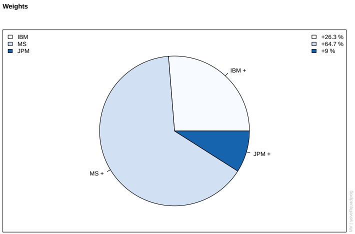

Figure 3: Posterior Rdt/Risque

Exercice

- Contraindre $w_i > 0$ en utilisant le code de la note de cours.
- BAC va surperformer Citibank (C)
- Dell aura un rendement de 0.5%