Gestion de Portefeuille

Exo Risk Parity

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Risk parity portfolio

To set all risk contributions identical, minimize the squared difference between the contributions of pairs of assets.

$$\min_w \sum_i (CR_i - CR_{i-1})^2$$

```
sigma <- c(.1, .2, .3)
rho <- matrix(c(1,.8, .7,.8, 1, .6, .7, .6, 1), nrow = 3)
Sigma <- diag(sigma) %*% rho %*% diag(sigma)

obj <- function(w) {
  w <- matrix(w, ncol=1)
  Sw <- Sigma %*% w
  MRC <- w * Sw
  sum(diff(MRC)^2)*1000
}

Aeq <- matrix(rep(1,3), nrow=1)
Beq <- 1
lb <- rep(0,3)
ub <- rep(1,3)

w.0 <- rep(1/3,3)

res.optim <- solnl(X=w.0, objfun=obj, Aeq=Aeq, Beq=Beq, lb=lb, ub=ub)
```

w_0	0.5321702
w_1	0.2767711
w_3	0.1910586

Using Newton's method

Risk parity condition:

$$w_i \frac{\partial \sigma_P}{\partial w_i} = w_j \frac{\partial \sigma_P}{\partial w_j} = \lambda$$

Let $1/w = [1/w_1, \ldots, 1/w_n]$, the above expression in matrix form is, dropping the denominator $w^T \Sigma w$ which is common to both sides of the equalities:

$$\Sigma w = \lambda \times 1/w$$

Define the vector-valued function

$$F(w, \lambda) = \begin{bmatrix} \Sigma w - \lambda \times 1/w \\ 1^T w - 1 \end{bmatrix}$$

We look for w^*, λ^* such that $F(w^*, \lambda^*) = 0$.

```
f.obj <- function(x) {
  n <- length(x)
  w <- matrix(x[1:(n-1)], ncol=1)
  lambda <- x[n]
  res.1 <- Sigma %*% w - lambda * 1/w
  res.2 <- sum(w) - 1
  as.vector(rbind(res.1, res.2))
}

res.newton = nleqslv(c(.3, .3, .3, .1), f.obj)
```

w_0	0.5321836
w_1	0.2767601
w_3	0.1910563