Options dans le cadre Black-Scholes

TP-1: Modèle de Shimko

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In this problem set, you will use the following functions:

GBSPrice: Price of a vanilla option:

$$P = f(\text{PutCall}, S, K, T, r, b, \sigma)$$

where:

PutCall 'c' for a call, 'p' for a put

b cost of carry: ridk free rate r less dividend yield d

r risk-free rate

```
GBSPrice <- function(PutCall, S, K, T, r, b, sigma) {
  d1 <- (log(S/K) + (b+sigma^2/2)*T)/(sigma*sqrt(T))
  d2 <- d1 - sigma*sqrt(T)

  if(PutCall == 'c')
    px <- S*exp((b-r)*T)*pnorm(d1) - K*exp(-r*T)*pnorm(d2)
  else
    px <- K*exp(-r*T)*pnorm(-d2) - S*exp((b-r)*T)*pnorm(-d1)

px
}
```

GBSVega: Vega ($\frac{\partial P}{\partial \sigma}$) of a Vanilla option:

```
GBSVega <- function(PutCall, S, K, T, r, b, sigma) {
  d1 <- (log(S/K) + (b+sigma^2/2)*T)/(sigma*sqrt(T))
  S*exp((b-r)*T) * dnorm(d1)
}
```

Implied dividend yield and risk-free rate

We observe a set of call and put prices, maturity 60 days.

The strikes of the options are:

```
K <- c(325, 345, 360, 365, 375, 385, 390, 395, 400, 405, 410, 425)
```

The corresponding call prices are:

```
C <- c(66.5, 46, 33, 27.75, 20.125, 13.5, 9.625, 7.25, 5.375, 3.375, 1.875, 0.25)
```

The corresponding put prices are:

```
P <- c(0.3125, 0.875, 2, 2.625, 4.25, 7.125, 8.75, 11, 13.75, 17, 19.75, 34)
```

The spot is $S_0 = 390.02$. Using the Call-Put parity, estimate by linear regression the implied risk-free rate (r) and dividend yield (d).

Implied Volatility calculation

1. Using the functions above, write a function that computes the implied volatility of a Vanilla option. Let:

$$g(\sigma) := P - f(\text{PutCall}, S, K, T, r, b, \sigma)$$

where P is the observed price of the option of interest.

We look for the volatility σ such that $g(\sigma) = 0$.

The function should have the following signature:

```
ImpliedVol <- function(p, TypeFlag, S, X, Time, r, b, sigma=NULL, maxiter=500, tol=1.e-5) {
}
```

where:

p price of the option

$sigma$ an optional initial value for the volatility

$maxiter$ an optional maximum number of iterations

tol an optional tolerance for the error $|g(\sigma)|$.

2. Test the accuracy of your procedure on options that are deep in the money and deep out of the money, and report the results of your tests.
3. Compute the implied volatility of the calls and puts in the data set.
4. Fit a quadratic function to the call and put implied volatilities (one function for the calls, one for the puts), and plot actual vs. fitted data. Interpret the results.

Breeden-Litzenberger formula

Compute the implied density of S_T using the Breeden-Litzenberger formula. Estimate

$$\frac{\partial^2 f}{\partial K^2}$$

by finite difference. Remember that now σ is a function of strike. Plot the implied distribution and compare to the distribution implicit in the standard Black-Scholes model. Interpret your observations.

Shimko's Model

Compute the implied distribution of S_T using Shimko's model and the quadratic smile function estimated above. Plot this distribution and compare with the result of the Breeden-Litzenberger formula. Interpret your observations.

Pricing a digital call

Recall that a digital call with strike K pays one euro if $S_T \geq K$, and nothing otherwise.

Using the implied density computed above, compute the price of a digital call by numerical integration.

Perform this calculation for strikes ranging from 350 to 420. Compare with the price obtained using a log-normal distribution for S_T. Interpret your observations.