

Options dans le cadre Black-Scholes

TP-2: Pricing Vanna-Volga

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The purpose of this problem set is to explore the Vanna-Volga pricing model. In this problem set, you will use the following functions:

GBSPrice: Price of a vanilla option:

$$P = f(\text{PutCall}, S, K, T, r, b, \sigma)$$

where:

PutCall 'c' for a call, 'p' for a put

b cost of carry: risk free rate r less dividend yield d

r risk-free rate

```
GBSPrice <- function(PutCall, S, K, T, r, b, sigma) {  
  d1 <- (log(S/K) + (b+sigma^2/2)*T)/(sigma*sqrt(T))  
  d2 <- d1 - sigma*sqrt(T)  
  
  if(PutCall == 'c')  
    px <- S*exp((b-r)*T)*pnorm(d1) - K*exp(-r*T)*pnorm(d2)  
  else  
    px <- K*exp(-r*T)*pnorm(-d2) - S*exp((b-r)*T)*pnorm(-d1)  
  
  px  
}
```

GBSVega: Vega ($\frac{\partial P}{\partial \sigma}$) of a Vanilla option:

```
GBSVega <- function(PutCall, S, K, T, r, b, sigma) {  
  d1 <- (log(S/K) + (b+sigma^2/2)*T)/(sigma*sqrt(T))  
  S*exp((b-r)*T) * dnorm(d1)  
}
```

Volatility Interpolation

Given the implied volatility at three strikes, we will use the Vanna-Volga pricing method to interpolate the volatility curve. Assume $r = 0, b = 0, T = 1, \text{Spot} = 100$.

```
# Benchmark data: (strike, volatility)
VolData <- list(c(80, .32), c(100, .30), c(120, .315))
```

Let's first define an array of pricing functions for the benchmark instruments:

```
C1 <- function(vol=sigma, spot=Spot) GBSPPrice(PutCall='c', S=spot, K=VolData[[1]][1], T=T, r=r, b=b, si=si)
C2 <- function(vol=sigma, spot=Spot) GBSPPrice(PutCall='c', S=spot, K=VolData[[2]][1], T=T, r=r, b=b, si=si)
C3 <- function(vol=sigma, spot=Spot) GBSPPrice(PutCall='c', S=spot, K=VolData[[3]][1], T=T, r=r, b=b, si=si)
C <- c(C1, C2, C3)
```

1. Write a utility functions to compute the risk indicators, all by finite difference:

```
Vega <- function(f, vol, spot=Spot) {
  # ???
}

Vanna <- function(f, vol, spot=Spot) {
  # ???
}

Volga <- function(f, vol) {
  # ???
}
```

Then, the calculation of vega for the three benchmark options may be performed by:

```
B.vega <- sapply(1:3, function(i) Vega(C[[i]], sigma))
```

2. Compute vectors of vega, vanna, volga for the three hedge instruments

```
«echo=T»= B.vega <- 0 B.vanna <- 0 B.volga <- 0 @
```

3. Choose a new strike for which we want to compute the implied volatility.
4. Compute the risk indicators for a call option struck at that strike.
5. Compute the Vanna-Volga price adjustment and the corresponding implied volatility.
6. Wrap the above logic in a function in order to interpolate/extrapolate the vol curve from $K = 70$ to $K = 130$

Pricing a digital call

Recall that a digital call with strike K pays one euro if $S_T \geq K$, and nothing otherwise.

Using the same logic as in the previous question, price a digital call, maturity $T = 1$, struck at $K = 105$.