

QP Simple Example

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```
library(quadprog)
```

To illustrate the solution method of a quadratic programming problem with linear equality and inequality constraints, consider the following problem:

$$\min_x \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}^T x + \frac{1}{2} x^T \Sigma x$$

such that

$$\begin{aligned} \sum x_i &= 1 \\ x_1 &\leq 0.5 \\ x_2 + x_3 &\geq 0.8 \end{aligned}$$

The constraints are written in matrix notation, taking good care to separate the equality and inequality constraints:

$$\begin{aligned} A_e^T x &= b_e \\ A_n^T x &\geq b_n \end{aligned}$$

with

$$\begin{aligned} A_e &= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} & b_e &= 1 \\ A_n &= \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} & b_n &= \begin{bmatrix} -0.5 \\ 0.8 \end{bmatrix} \end{aligned}$$

Data

To stay in the domain of portfolio optimization, Σ is a covariance matrix, constructed from the correlation matrix and a column vector of standard deviation.

```
Cor <- matrix(c(1, 0.5, 0.2, 0.5, 1, 0.7, 0.2, 0.7, 1),3,3)
sigma <- matrix(c(1, 2, 1.5), nrow=3)
Sigma <- sigma %*% t(sigma) * Cor
```

$$\sigma = \begin{bmatrix} 1 \\ 2 \\ 1.5 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 1 & 1 & 0.3 \\ 1 & 4 & 2.1 \\ 0.3 & 2.1 & 2.25 \end{bmatrix}$$

QP

The QP solver is invoked as follows. Note that the A matrix must contain first the rows corresponding to the equality constraints, then the rows corresponding to the inequality constraints. Parameter `meq` indicates the number of equality constraints.

```
b <- rbind(b_e, b_n)
Amat <- cbind(A_e, A_n)
sol = solve.QP(Sigma, c(1,2,3), Amat, b, meq=1)
```

```
library(kableExtra)
names(sol$solution) <- seq(3)
kable(sol$solution, "latex", booktabs=T, row.names = TRUE) %>%
  kable_styling(full_width = F, position = "center")
```

	x
1	0.1316348
2	-0.4692144
3	1.3375796