Financial Time Series

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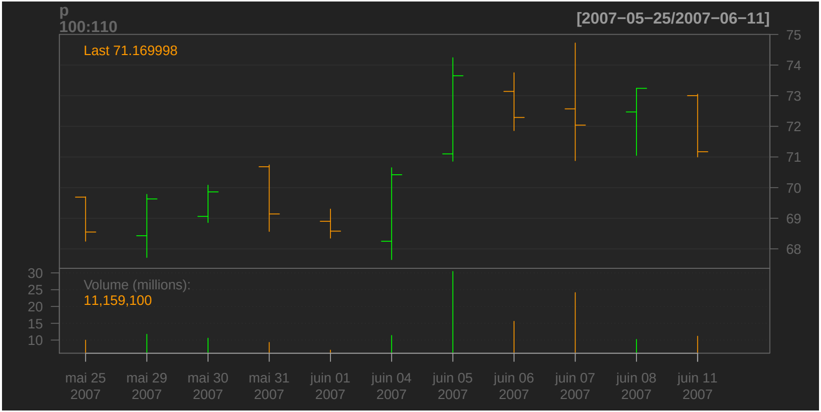
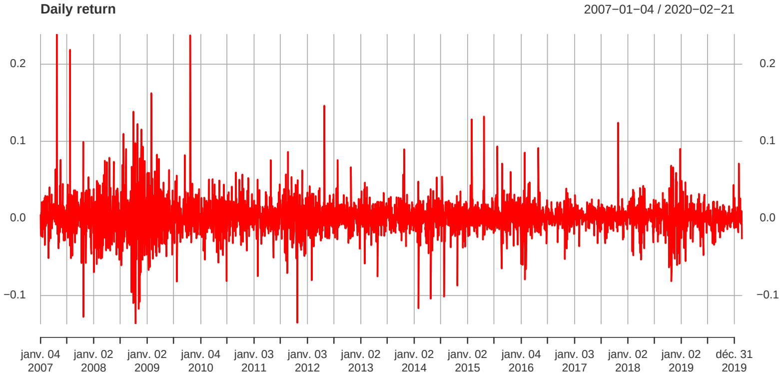
Financial Time Series (daily OHLC)

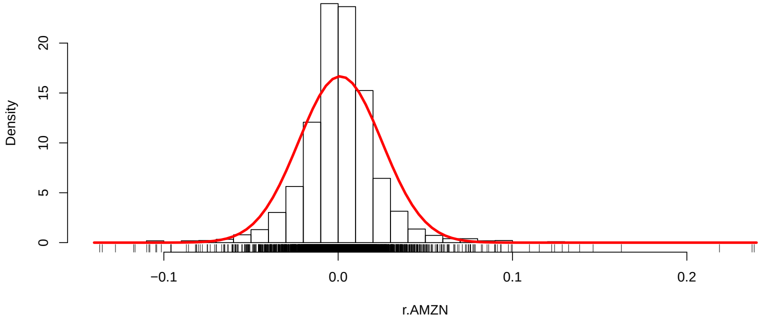


Daily Return - AMZN

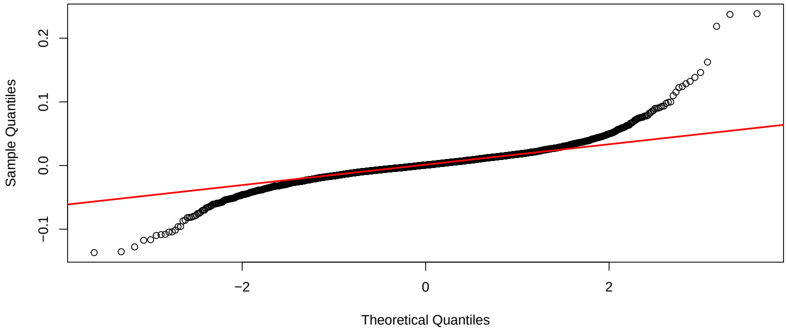
$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right)$$



Histogram of daily return - AMZN



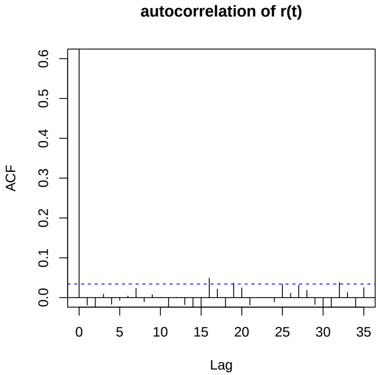
Analysis of return distribution - AMZN

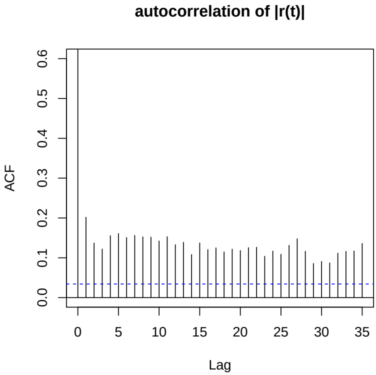


Moments of daily returns

	mean	std dev	skewness	kurtosis
AMZN	0.0012075	0.0239215	0.9526121	12.849686
GOOG	0.0005604	0.0178051	0.5008926	11.495964
AAPL	0.0010301	0.0196726	-0.4678985	7.176276
QQQ	0.0005412	0.0130159	-0.1803480	6.985970
DIA	0.0003516	0.0114022	0.2150189	15.397181
SPY	0.0003397	0.0121101	-0.1557741	14.109873
PG	0.0003238	0.0109203	-0.1052228	7.846785
KO	0.0004433	0.0112445	0.2436121	12.923579

Autocorrelation of Returns (AMZN)





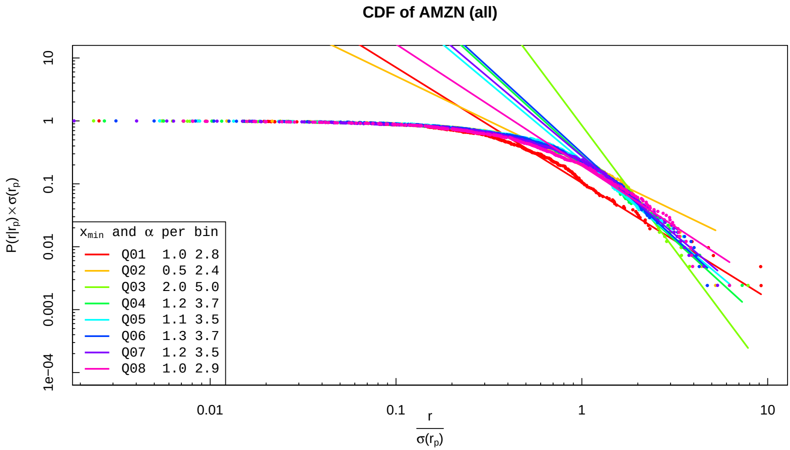
Rescaling daily return by $\sigma(r_{t-1})$ (Chen, Jayaprakash, and Yuan 2008)

$$z_t = \frac{r_t}{\sigma(r_{t-1})}$$

$$\left.\begin{aligned} p(z_t) &= \frac{\alpha - 1}{z_{min}} \left(\frac{z_t}{z_{min}}\right)^{-\alpha} \\ Pr(z_t > x) &= \left(\frac{z_t}{z_{min}}\right)^{-\alpha+1} \end{aligned}\right\} z_t > z_{min}$$

The density of z_t can be approximated by a power law. See paper for details of calculation.

Rescaling of daily return by $\sigma(r_{t-1})$



Unconditional distribution of return

The Johnson family of distributions is formed by various transformations of the normal density. Let X be the observed data, an define Z by:

$$Z = \gamma + \delta \ln \left(g \left(\frac{X - \xi}{\lambda} \right) \right)$$

where:

$$g(u) = \begin{cases} u & SL \\ u + \sqrt{1 + u^2} & SU \\ \frac{u}{1-u} & SB \\ e^u & SN \end{cases}$$

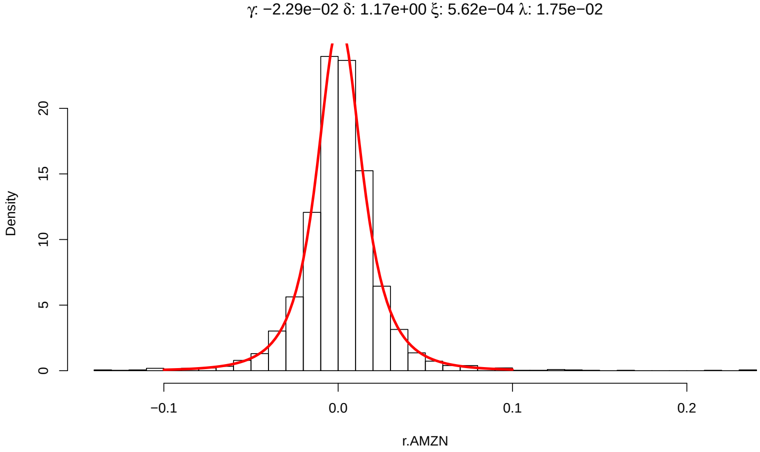
X follows a Johnson distribution if Z is normal.

Fitted Johnson SU distribution - AMZN (1)

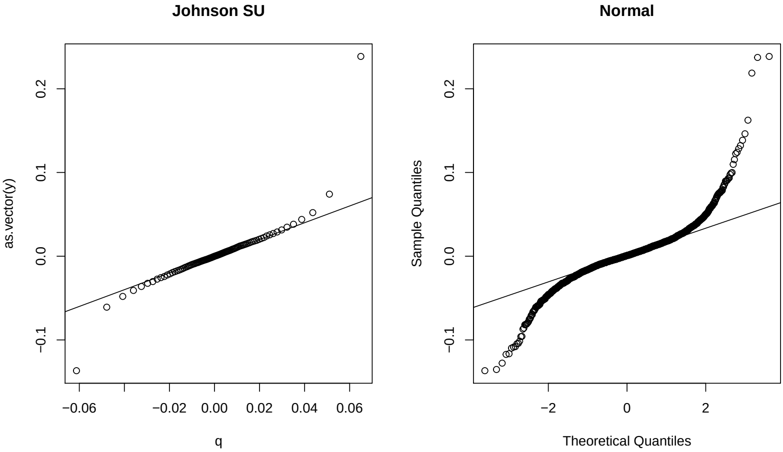
gamma	delta	xi	lambda	type
-0.0228945	1.16685	0.0005621	0.0174527	SU

	sample	johnson
mean	0.0012075	0.0010565
sigma	0.0239179	0.0225752
skew	0.9530444	-0.1098671
kurt	12.8592791	12.3022551

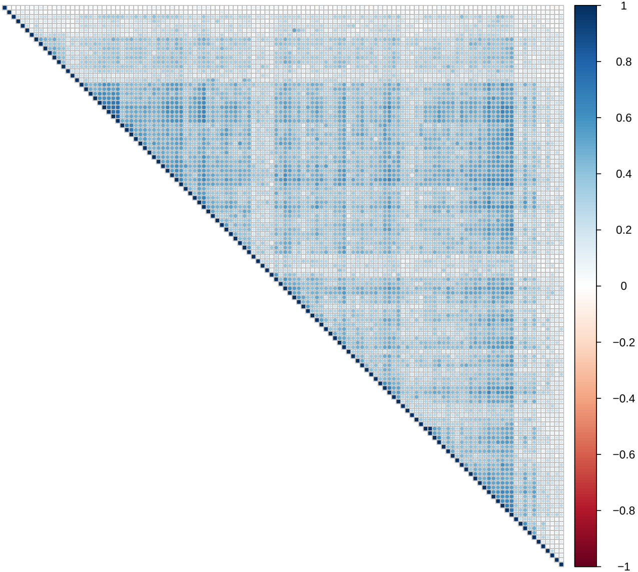
Fitted Johnson SU distribution - AMZN (2)



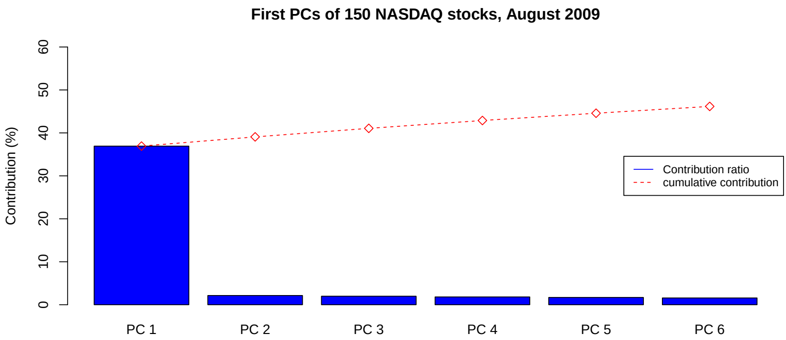
Fitted Johnson SU distribution - AMZN (3)



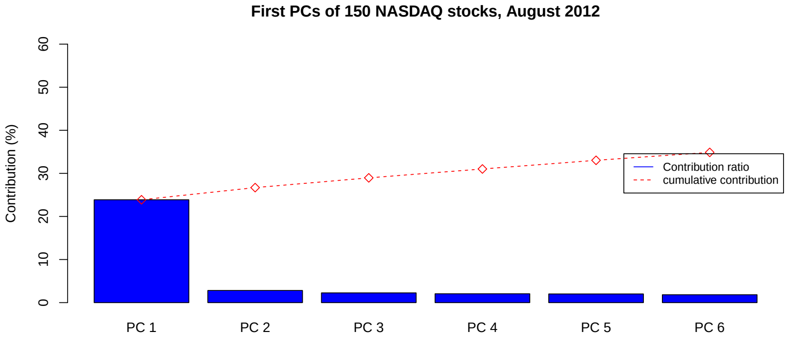
Correlation between assets (NASDAQ)



Correlation between assets

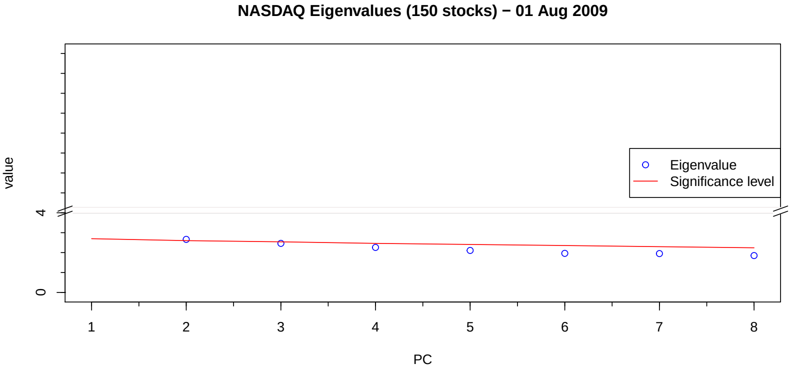


Correlation between assets

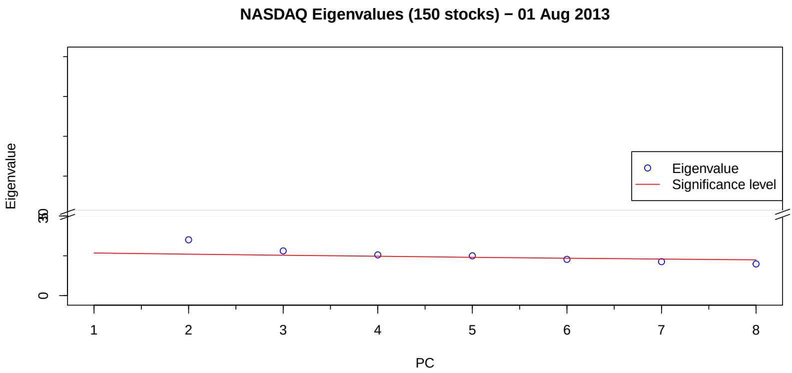


How many dimensions in a market?

Significance level (95%) for eigenvalues (252 observations, 127 variables):



How many dimensions in a market?



Summary

To summarize, empirical observations show that the distribution of returns exhibit features that strongly repart from the classical hypothesis of independence and normality. We find:

1. no evidence of linear autocorrelation of return, however,
2. there is an observable autocorrelation of $|r_t|$ and r_t^2, suggesting autocorrelation in the volatility of return,
3. we also observe large excess kurtosis, which is incompatible with normal density,
4. The rank of a broad stock market such as the NASDAQ is probably much lower than the number of stocks.

Bibliography

Chen, Kan, C Jayaprakash, and Baosheng Yuan. 2008. "Conditional Probability as a Measure of Volatility Clustering in Financial Time Series." *Physica A*, 1–5. https://arxiv.org/abs/0503157v2.