SIM & Treynor-Black

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Single Index Model (Sharpe)

- Rendement

$$R_i(t) = \alpha_i + \beta_i R_M(t) + e_i(t)$$

- Espérance de rendement

$$E(R_i(t)) = \alpha_i + \beta_i E(R_M(t))$$

- Variance du rendement

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma(e_i)^2$$

- Covariance des rendements

$$\text{cov}(R_i, R_j) = \beta_i \beta_j \sigma_M^2$$

Portefeuille selon le SIM

Soit un portefeuille de n actifs avec $w_i = \frac{1}{n}$.

$$R_P(t) = \alpha_P + \beta_P R_M(t) + e_P(t)$$
$$\sigma_P^2 = \beta_P^2 \sigma_M^2 + \sigma(e_P)^2$$

Division du travail en Gestion de Portefeuille

- Espérance de rendement

$$E(R_i(t)) = \alpha_i + \beta_i E(R_M(t))$$

- Variance du rendement

$$\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma(e_i)^2$$

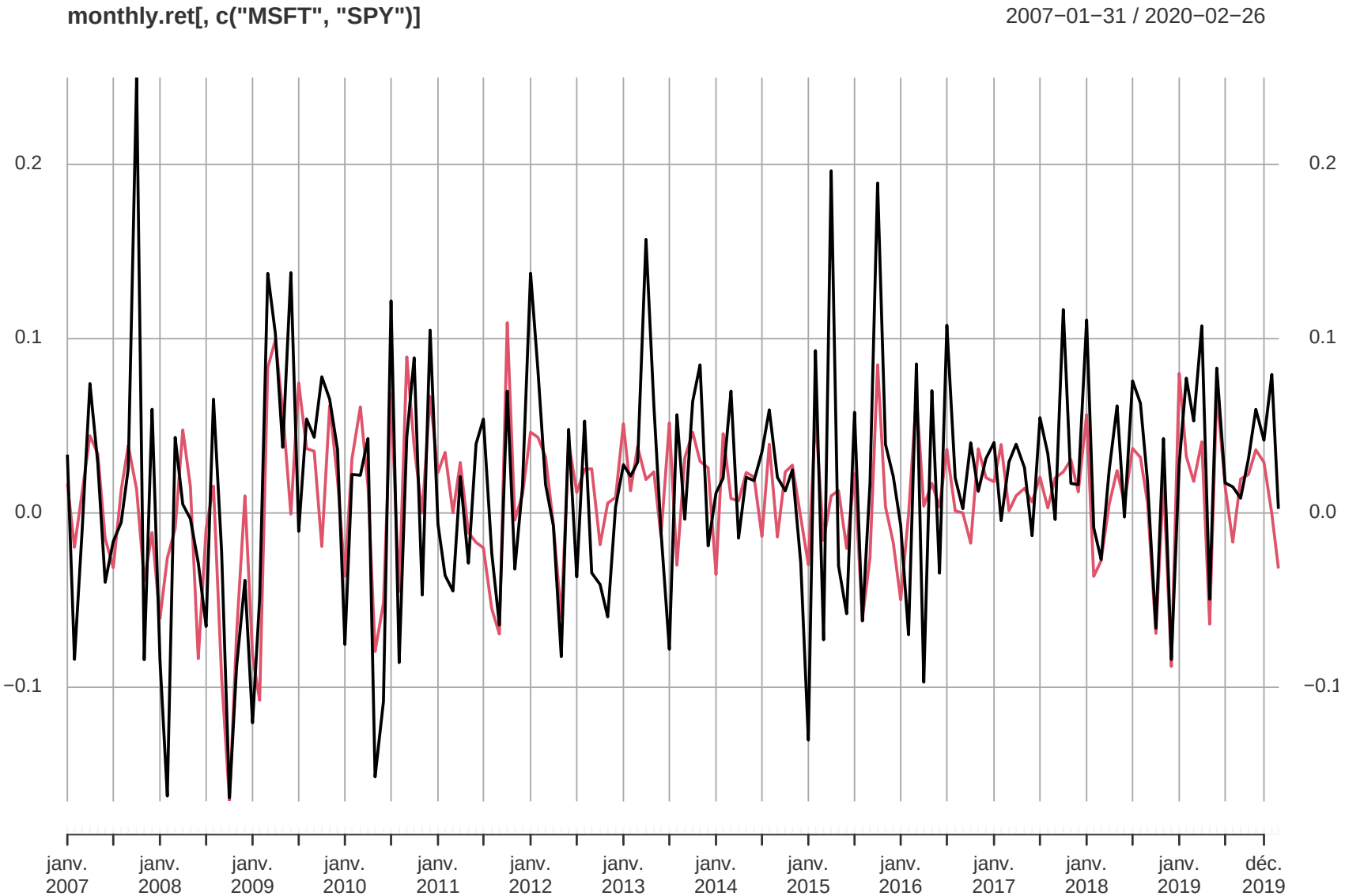
Allocation Treynor-Black

Voir note pour le détail des calculs.

Données

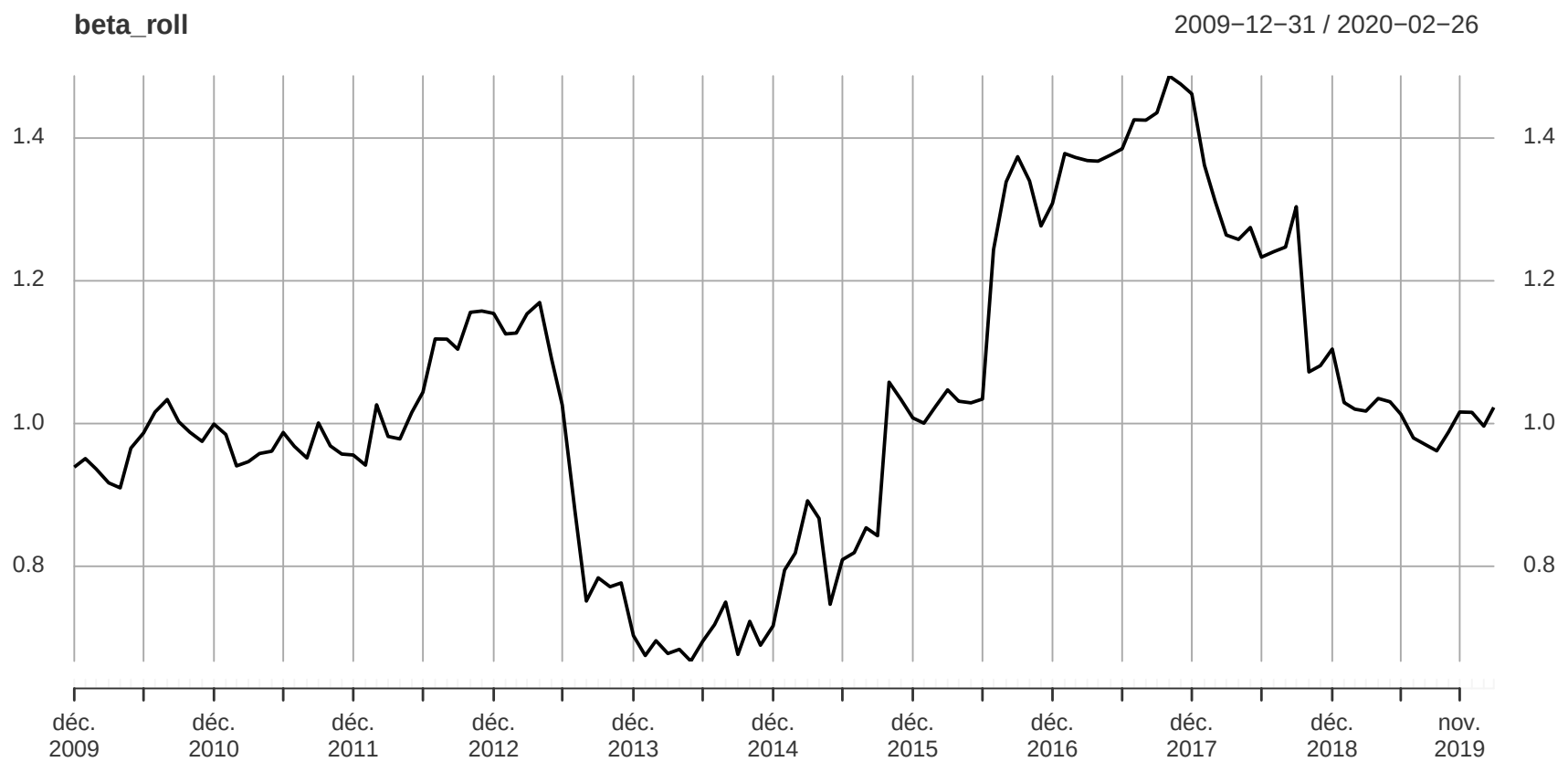
	AAPL	AMZN	MSFT	F	SPY	QQQ	XOM	MMM	HD	PG	KO
Observations	158.0000	158.0000	158.0000	158.0000	158.0000	158.0000	158.0000	158.0000	158.0000	158.0000	158.0000
NAs	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Minimum	-0.3296	-0.2540	-0.1634	-0.5788	-0.1652	-0.1558	-0.1423	-0.1498	-0.1652	-0.1161	-0.1668
Quartile 1	-0.0213	-0.0303	-0.0280	-0.0456	-0.0144	-0.0159	-0.0265	-0.0230	-0.0227	-0.0211	-0.0168
Median	0.0291	0.0256	0.0203	-0.0022	0.0128	0.0192	0.0022	0.0144	0.0178	0.0081	0.0104
Arithmetic Mean	0.0254	0.0298	0.0152	0.0115	0.0076	0.0123	0.0017	0.0079	0.0151	0.0074	0.0100
Geometric Mean	0.0214	0.0252	0.0130	0.0020	0.0067	0.0110	0.0004	0.0063	0.0133	0.0065	0.0090
Quartile 3	0.0792	0.0830	0.0545	0.0470	0.0324	0.0455	0.0411	0.0455	0.0608	0.0361	0.0405
Maximum	0.2377	0.5413	0.2495	1.2738	0.1091	0.1317	0.1128	0.1734	0.1605	0.1161	0.1419
SE Mean	0.0071	0.0079	0.0053	0.0120	0.0033	0.0040	0.0040	0.0045	0.0049	0.0034	0.0035
LCL Mean (0.95)	0.0114	0.0143	0.0047	-0.0121	0.0010	0.0044	-0.0062	-0.0010	0.0055	0.0006	0.0031
UCL Mean (0.95)	0.0394	0.0454	0.0257	0.0352	0.0142	0.0201	0.0095	0.0168	0.0248	0.0141	0.0170
Variance	0.0079	0.0098	0.0045	0.0226	0.0018	0.0025	0.0025	0.0032	0.0037	0.0019	0.0020
Stdev	0.0889	0.0990	0.0670	0.1504	0.0419	0.0502	0.0502	0.0566	0.0612	0.0430	0.0442
Skewness	-0.6572	0.7403	0.0788	3.6175	-0.7421	-0.6247	-0.3969	-0.3662	-0.3033	-0.2344	-0.2589
Kurtosis	1.9491	4.0251	0.8958	31.4460	1.5521	0.9798	0.0867	0.5184	0.4291	0.0619	1.1258

MSFT & SPY


monthly.ret[, c("MSFT", "SPY")]
2007−01−31 / 2020−02−26
0.2
0.1
0.0
−0.1
janv. 2007
janv. 2008
janv. 2009
janv. 2010
janv. 2011
janv. 2012
janv. 2013
janv. 2014
janv. 2015
janv. 2016
janv. 2017
janv. 2018
janv. 2019
déc. 2019

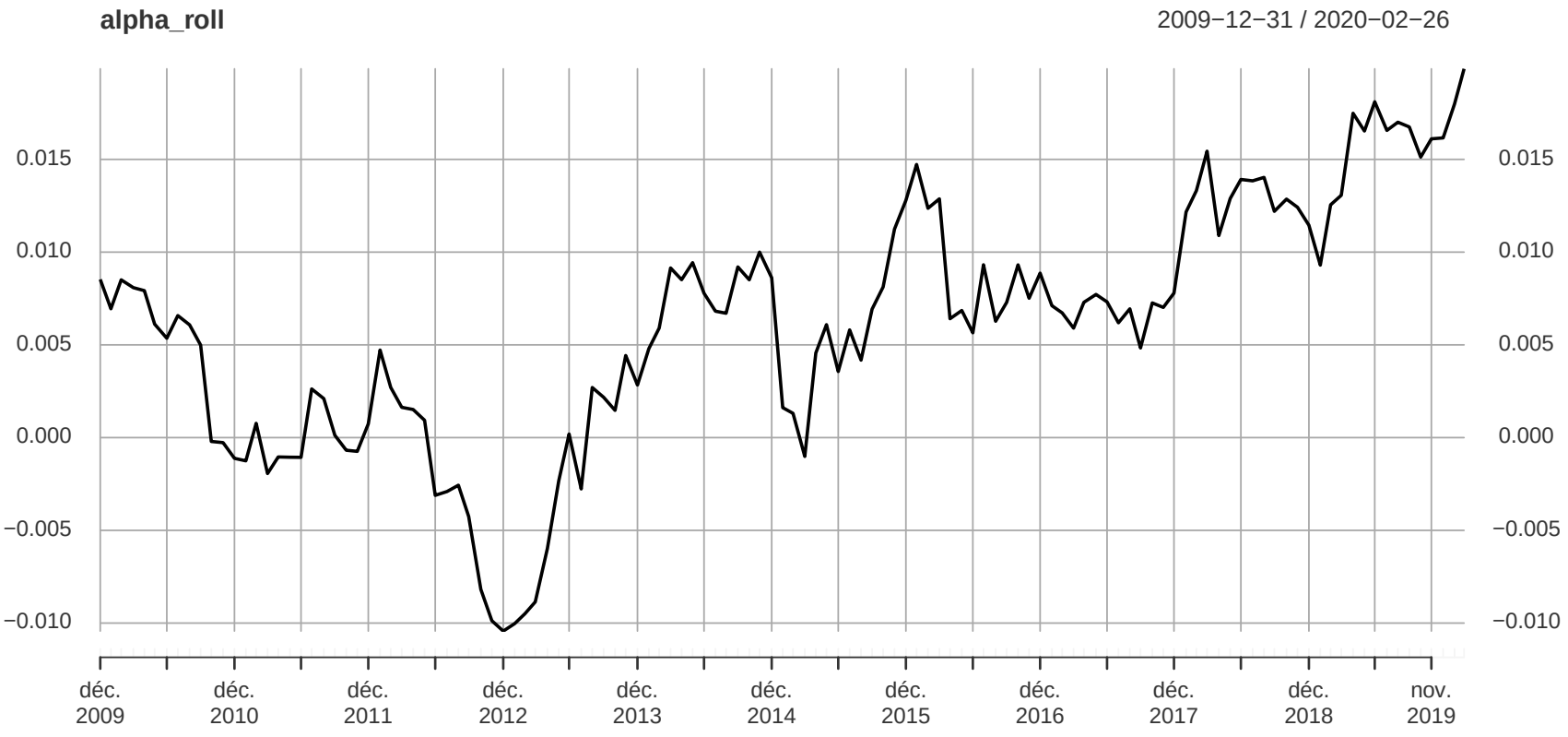
Calcul de β

```
beta_roll <- removeNA(rollapply(data=monthly.ret$MSFT, Rb=m
                                FUN=CAPM.beta, width=36, by
```



Calcul de α

```
alpha_roll <- removeNA(rollapply(data=monthly.ret$MSFT, Rb=
                                 FUN=CAPM.alpha, width=36,
```



Bibliographie

Grinhold, R.C. and Kahn, R. Active Portfolio Management, Mc Graw-Hill, 2000