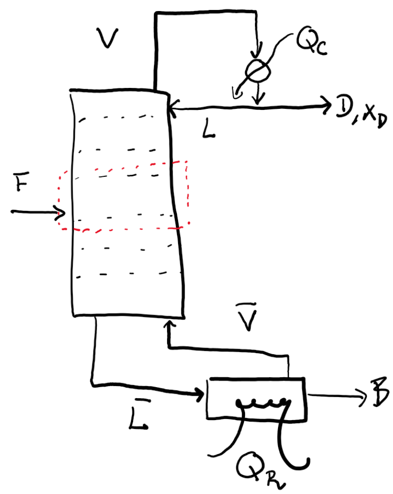
1)

a.



$P = 101.325$ kPa
$x_B = 0.06$
$x_D = 0.98$
$x_F = 0.44$
$F = 30000$ mol/hr
$R = 2.2 : 1.0$

Assume: CMO

Solve:

The equilibrium xy data may be generated by assuming an ideal solution and applying Raoult's law,

$$P = x_1 P_1^{sat} + (1 - x_1) P_2^{sat},$$

which implies,

$$x_1 = \frac{P - P_2^{sat}}{P_1^{sat} - P_2^{sat}} \quad \text{and} \quad y_1 = \frac{x_1 P_1^{sat}}{P}.$$

The material balances around the entire column are,

$$F = D + B,$$

$$x_F F = x_D D + x_B B,$$

from which we find,

$$\frac{D}{F} = \frac{x_F - x_B}{x_D - x_B}.$$

Thus,

$$\boxed{D = (30000 \text{ mol/hr}) \left(\frac{0.44 - 0.06}{0.98 - 0.06} \right) = 12391 \text{ mol/hr}},$$

and,

$$\boxed{B = F - D = 17609 \text{ mol/hr}}.$$

b.

For an arbitrary stage in the rectifying section, the material balances around the top of the column imply,

$$y = \left(\frac{R}{R+1}\right)x + \frac{x_D}{R+1},$$

whose intercept is given by,

$$\frac{x_D}{R+1} = \frac{0.98}{2.2+1} = 0.306.$$

Since the distillate vapor is totally condensed, we know $y_1 = x_D$ must lie on the rectifying line. Therefore, we now know two points, $(0, 0.306)$ and $(0.98, 0.98)$, and the curve may be drawn on an xy diagram. The equation for the stripping line is,

$$y = \frac{\bar{L}}{\bar{V}}x - \frac{B}{\bar{V}}x_B.$$

Plugging in $x = x_B$ and using $\bar{V} = \bar{L} - B$ implies that the point (x_B, x_B) lies on the stripping line. The second point comes from the intersection with the q-line whose equation is given by,

$$y = -\frac{q}{1-q}x + \frac{x_F}{1-q}.$$

Since the the feed is a saturated liquid, we have $q = 1$, meaning the q-line has an infinite slope and must pass through x_F. Two points of the stripping curve are now known and the line may be drawn on the xy diagram.

The equilibrium line may be estimated by fitting a polynomial to the data. A 5th order polynomial was used to prevent overfitting and the intercept was set to 0. That is,

$$y(x) = a_5x^5 + a_4x^4 + a_3x^3 + a_2x^2 + a_1x,$$

for some constants $(a_5, a_4, a_3, a_2, a_1)$. Using scipy's optimize.curve_fit function in Python, this gives,

$$(a_5, a_4, a_3, a_2, a_1) = (3.407, -10.90, 14.05, -9.744, 4.193),$$

with estimated $\mathrm{R}^2 = 1.000$. By starting at (x_D, x_D) and stepping off the equilibrium and rectifying/stripping lines, the number of stages may be determined.

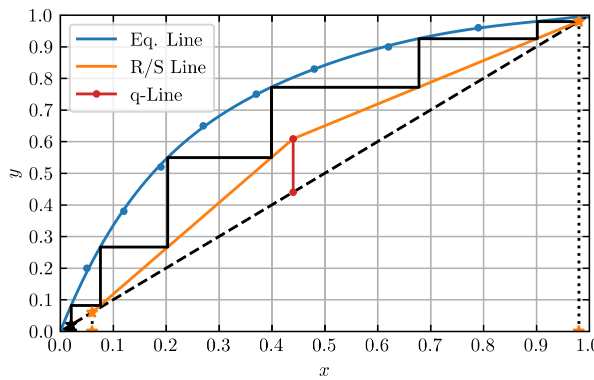


Figure 1: McCabe-Thiele diagram for benzene-ethylbenzene distillation, saturated liquid feed

Thus, the feed should be at stage $\boxed{N_F = 3}$ and the total number of stages is $\boxed{N = 5 + 1}$.

c.

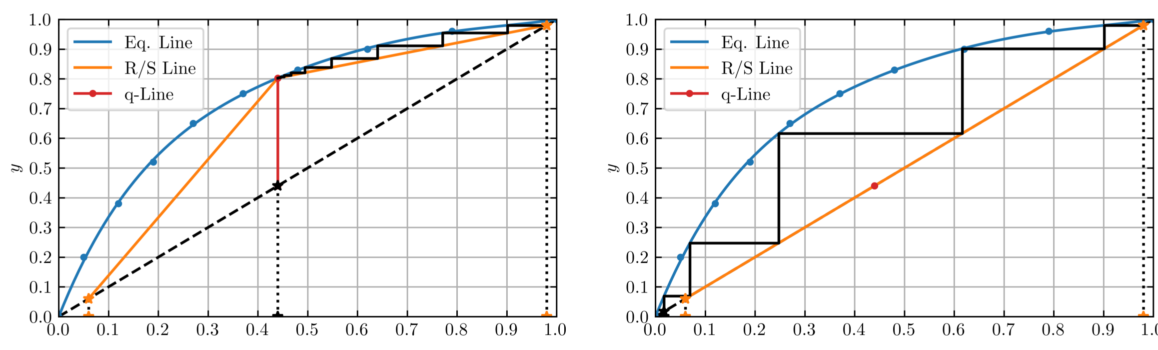
The minimum reflux ratio, R_{min}, is given by generating a pinch point and having the q-line touch the equilibrium curve. The slope of the resulting rectifying line may be used to find R_{min},

$$\frac{R_{min}}{R_{min}+1} = \frac{x_D - y(x_F)}{x_D - x_F} = \frac{0.98 - 0.80}{0.98 - 0.44} = 0.323.$$

Solving for R_{min},

$$\boxed{R_{min} = 0.489}.$$

Total reflux occurs when the column operates without a feed (also implying no outputs). The q-line simply becomes a point at (x_F, x_F) and the rectifying and stripping lines degenerate into the same curve. The minimum number of stages, N_{min}, may be found by stepping off the diagram in this case.



(a) R_{min}, saturated liquid feed

(b) Total reflux

Figure 2: McCabe-Thiele diagrams for benzene-ethylbenzene distillation with R_{min} and total reflux

Therefore, $\boxed{N_{min} = 4+1}$.

d.

The stripping line may be expressed as,

$$y = \frac{\bar{L}}{\bar{V}}x - \frac{x_B}{R_B},$$

where R_B is the reboil ratio. The slope of the line is known from the two points,

$$\frac{\bar{L}}{\bar{V}} = \frac{y_q - x_B}{x_F - x_B} = \frac{0.609 - 0.06}{0.44 - 0.06} = 1.44.$$

We may plug in $(x, y) = (x_B, x_B)$ and solve for R_B,

$$\boxed{R_B = \frac{1}{(\bar{L}/\bar{V}) - 1} = \frac{1}{1.44 - 1} = 2.25}.$$

e.

The equilibrium and rectifying lines remain the same as in part b, but the q-line changes. Since $q = 0$, the q-line is horizontal and the stripping line may be found by connecting the bottoms composition and q-line's intersection with the rectifying line. This is given graphically in the following figure.

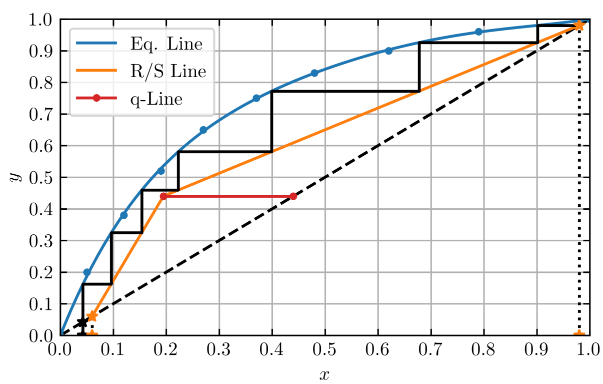


Figure 3: McCabe-Thiele diagram for benzene-ethylbenzene distillation, saturated vapor feed

Thus, the feed should be at stage $\boxed{N_F = 5}$ and the total number of stages is $\boxed{N = 6+1}$.

f.

The minimum number of stages in total reflux remains the same as in part c. However, the R_{min} changes since the pinch point occurs at a different location.

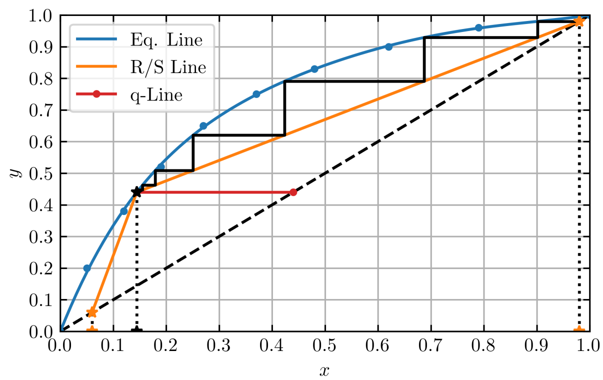


Figure 4: McCabe-Thiele diagram for benzene-ethylbenzene distillation, saturated vapor feed with R_{min}

Calculating the slope of the rectifying line,

$$\frac{R_{min}}{R_{min}+1} = \frac{x_D - x_F}{x_D - x_q} = \frac{0.98 - 0.56}{0.98 - 0.144} = 0.646.$$

Solving for R_{min},

$$\boxed{R_{min} = 1.83}.$$

g.

The slope of the stripping line is known from the two points,

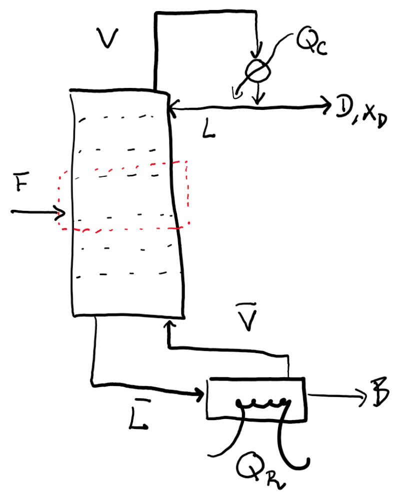
$$\frac{\bar{L}}{\bar{V}} = \frac{x_F - x_B}{x_q - x_B} = \frac{0.44 - 0.06}{0.195 - 0.06} = 2.82.$$

We may plug in $(x, y) = (x_B, x_B)$ into the stripping line and solve for R_B,

$$\boxed{R_B = \frac{1}{(\bar{L}/\bar{V}) - 1} = \frac{1}{2.82 - 1} = 0.548}.$$

2)

a.



$P = 101.325$ kPa
$x_B = 0.10$
$x_D = 0.96$
$x_F = 0.72$
$F = 250$ mol/hr
$R = 6.0 : 1.0$ AND $18 : 1.0$

Assume: CMO

Solve:

From problem 1,

$$\boxed{D = (250 \text{ mol/hr})\left(\frac{0.72 - 0.10}{0.96 - 0.10}\right) = 180 \text{ mol/hr}},$$

and,

$$\boxed{B = F - D = 70 \text{ mol/hr}}.$$

b.

Fitting the equilibrium data to a 5th order polynomial with a fixed intercept yields,

$$(a_5, a_4, a_3, a_2, a_1) = (0.1844, -0.4951, 0.5804, -0.6996, 1.430),$$

with an estimated $\mathrm{R}^2 = 1.000$.

Additionally, the rectifying line is given by,

$$y = \left(\frac{R}{R+1}\right)x + \frac{x_D}{R+1},$$

where for $R = 6$,

$$\frac{R}{R+1} = \frac{6}{7} = 0.857,$$

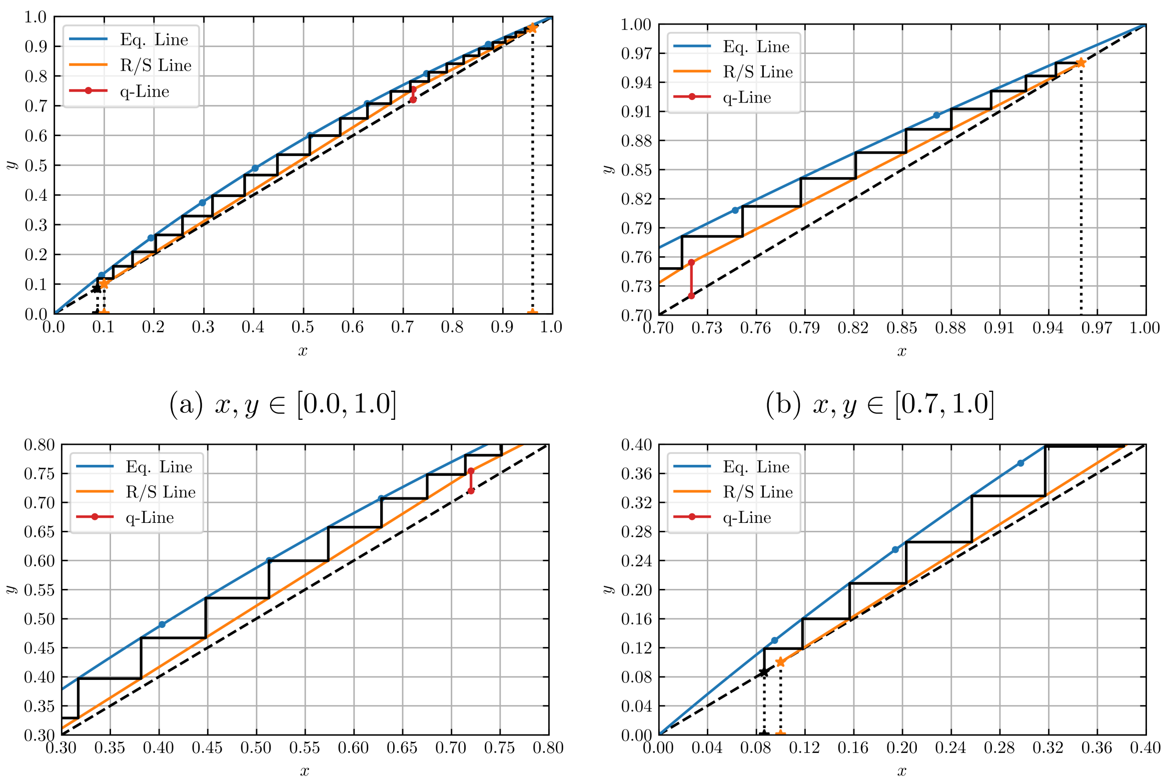
and,

$$\frac{x_D}{R+1} = \frac{0.96}{6+1} = 0.137.$$

By observing the narrow equilibrium line in the following figure, we notice that this system will be *much* harder to separate. Thus, to practically perform a graphical solution, we must either print the xy plot on a large paper or break the plot into multiple sections; we will take the later approach. To this end, it is convenient to get the explicit equation for the stripping line, this may be done by using the point $(x_B, x_B) = (0.10, 0.10)$ and the q-line intersection, $(x_q, y_q) = (0.72, 0.754)$, from which we may calculate the slope and intercept,

$$\frac{\bar{L}}{\bar{V}} = 1.055, \quad -\frac{B}{\bar{V}}x_B = -0.0055.$$

Plotting these curves and stepping yields the following figures.



(a) $x, y \in [0.0, 1.0]$

(b) $x, y \in [0.7, 1.0]$

(c) $x, y \in [0.3, 0.8]$

(d) $x, y \in [0.0, 0.4]$

Figure 5: McCabe-Thiele diagrams for n-heptane-toluene distillation with $R = 6$ and saturated liquid feed

Thus, we may (carefully) count that the feed stage should be at $\boxed{N_F = 9}$ and the total number of stages is $\boxed{N = 20 + 1}$ [1] for $R = 6$.

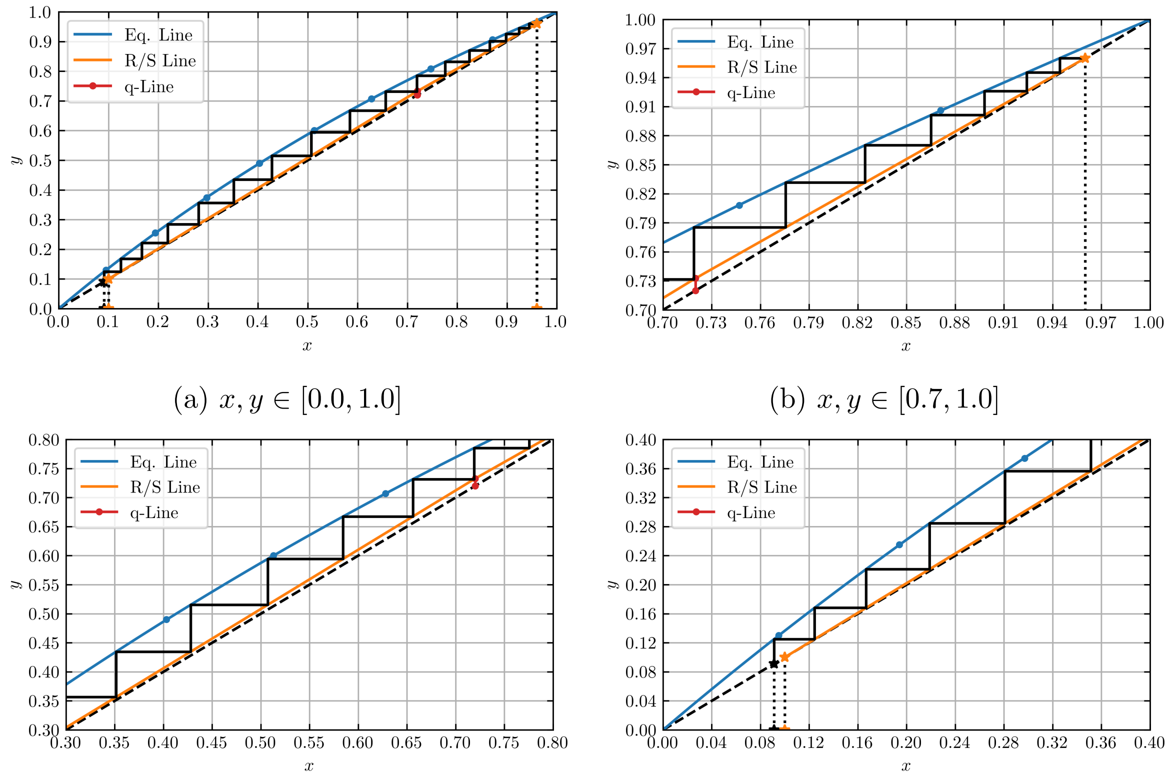
[1] I suspect this number is quite sensitive to equation used to fit the equilibrium curve.

A nearly identical analysis may be performed for $R = 18$, using,

$$\frac{R}{R+1} = 0.947, \quad \frac{x_D}{R+1} = 0.0505,$$

$$\frac{\bar{L}}{\bar{V}} = 1.020, \quad -\frac{B}{\bar{V}}x_B = -0.002.$$

This yields the following figures.



(a) $x, y \in [0.0, 1.0]$

(b) $x, y \in [0.7, 1.0]$

(c) $x, y \in [0.3, 0.8]$

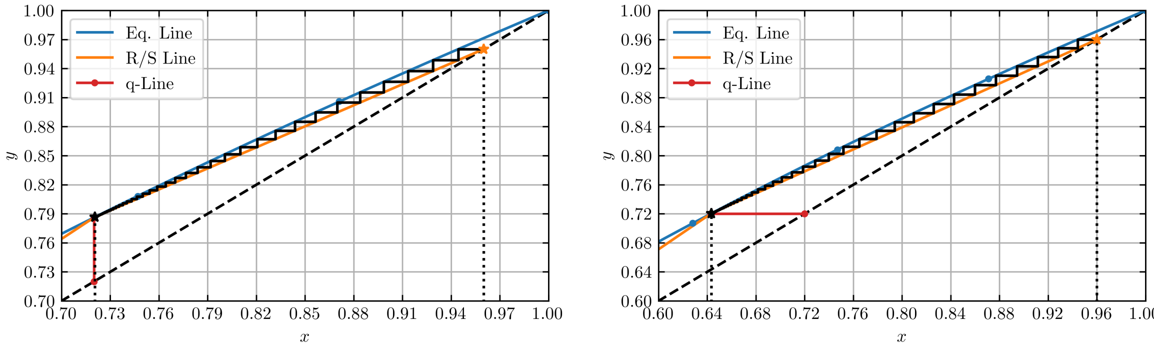
(d) $x, y \in [0.0, 0.4]$

Figure 6: McCabe-Thiele diagrams for n-heptane-toluene distillation with $R = 18$ and saturated liquid feed

From this, we may count that the feed stage should be at $\boxed{N_F = 7}$ and the total number of stages is $\boxed{N = 16 + 1}$ for $R = 18$.

c. & f.

We may find the R_{min} values by generating pinch points between the q-line and equilibrium curves.



(a) Saturated liquid feed (b) Saturated vapor feed

Figure 7: McCabe-Thiele diagrams for n-heptane-toluene distillation with R_{min}

For the saturated liquid feed,

$$\frac{R_{min}}{R_{min}+1} = \frac{x_D - y(x_F)}{x_D - x_F} = \frac{0.96 - 0.786}{0.96 - 0.72} = 0.724,$$

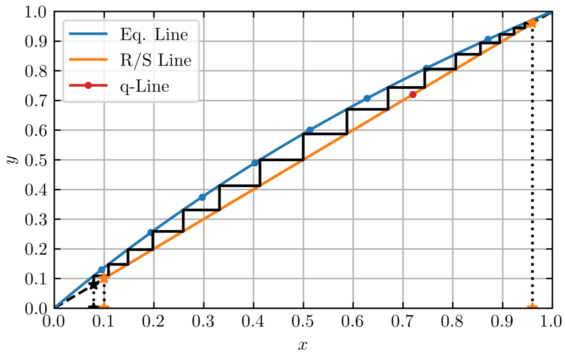
$$\implies \boxed{R_{min} = 2.63}.$$

For the saturated vapor feed,

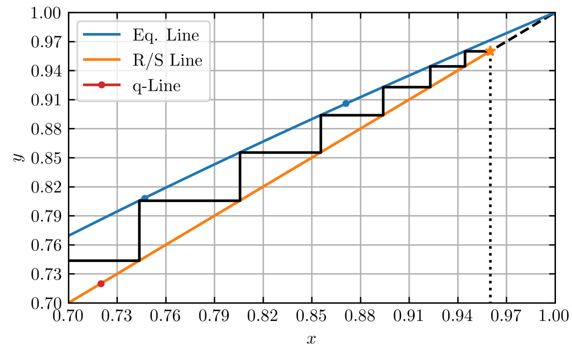
$$\frac{R_{min}}{R_{min}+1} = \frac{x_D - x_F}{x_D - x_q} = \frac{0.96 - 0.72}{0.96 - 0.643} = 0.757,$$

$$\implies \boxed{R_{min} = 3.11}.$$

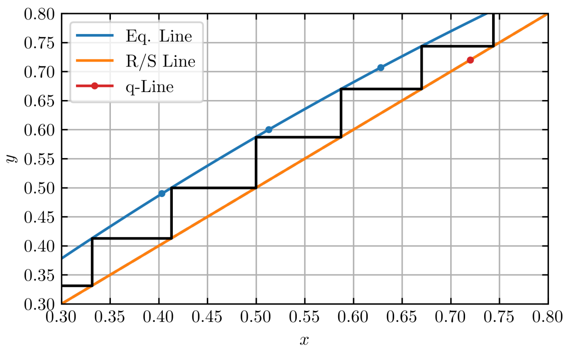
The stages from total reflux remains the same for both the saturated vapor and liquid. This is simply found by stepping down the $y = x$ line.



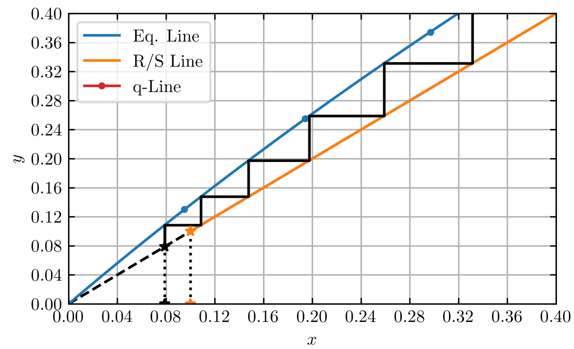
(a) $x, y \in [0.0, 1.0]$



(b) $x, y \in [0.7, 1.0]$



(c) $x, y \in [0.3, 0.8]$



(d) $x, y \in [0.0, 0.4]$

Figure 8: McCabe-Thiele diagrams for n-heptane-toluene distillation with total reflux

Thus, $\boxed{N_{min} = 15 + 1}$.

d.

For the saturated liquid, $R = 6$ requires 21 total stages while $R = 18$ requires 17 total stages, which is only 1 off from the minimum number of stages. Physically and graphically, this makes sense; such a high reflux ratio means that most of the condensed vapor is sent back to the column, making separation 'wider' and easier to achieve. This is also confirmed mathematically since,

$$\frac{R}{R+1} \to 1 \quad \text{and} \quad \frac{x_D}{R+1} \to 0$$

as R becomes large, meaning the rectifying line approaches $y = x$, or the total reflux condition. Thus, higher R leads to lower N.

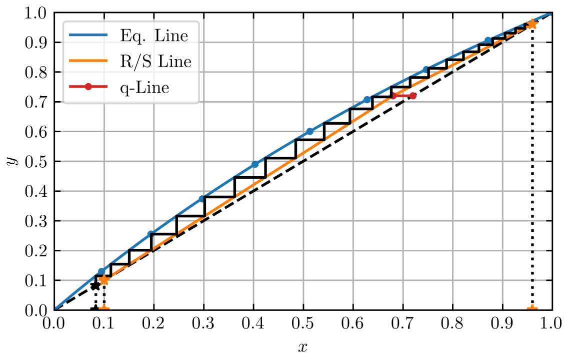
e.

For a saturated vapor feed with $R = 6$, the slopes and intercepts of the rectifying and stripping lines are given by,

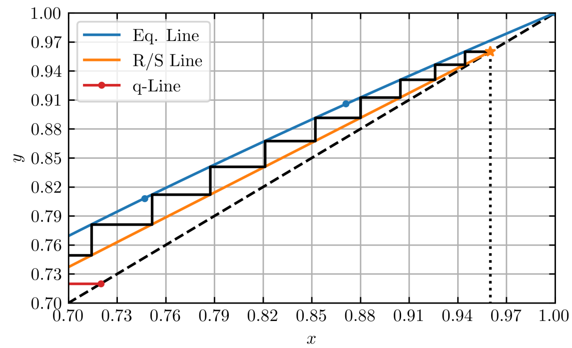
$$\frac{R}{R+1} = 0.857, \quad \frac{x_D}{R+1} = 0.137,$$

$$\frac{\bar{L}}{\bar{V}} = 1.069, \quad -\frac{B}{\bar{V}}x_B = -0.007.$$

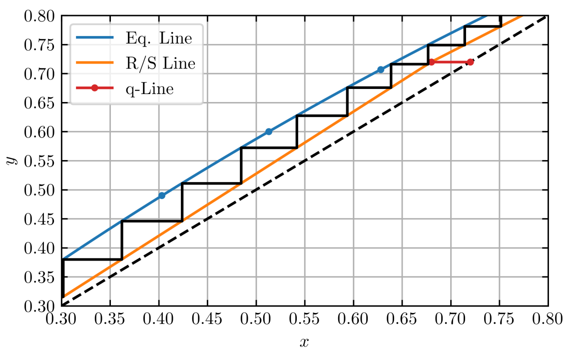
Which yields the following plots.



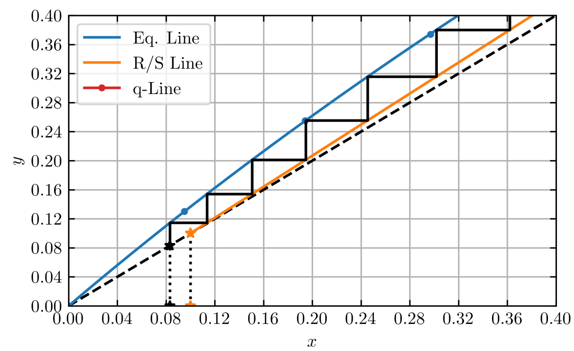
(a) $x, y \in [0.0, 1.0]$



(b) $x, y \in [0.7, 1.0]$



(c) $x, y \in [0.3, 0.8]$



(d) $x, y \in [0.0, 0.4]$

Figure 9: McCabe-Thiele diagrams for n-heptane-toluene distillation with $R = 6$ and saturated vapor feed

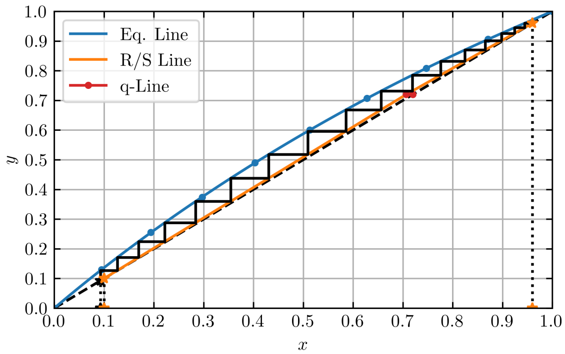
Therefore, the feed stage should be at $\boxed{N_F = 10}$ and the total number of stages is $\boxed{N = 21 + 1}$ for $R = 6$.

With $R = 18$, the slopes and intercepts of the rectifying and stripping lines are given by,

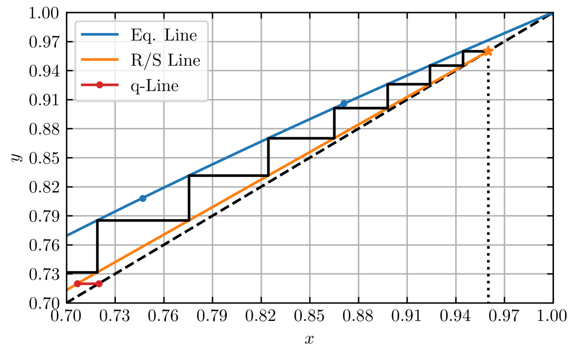
$$\frac{R}{R+1} = 0.947, \quad \frac{x_D}{R+1} = 0.0505,$$

$$\frac{\bar{L}}{\bar{V}} = 1.022, \quad -\frac{B}{\bar{V}}x_B = -0.002.$$

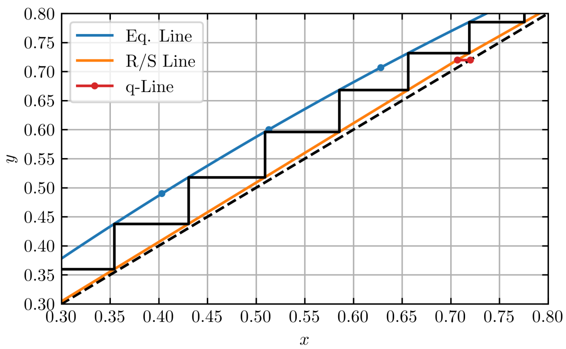
Which yields the following plots.



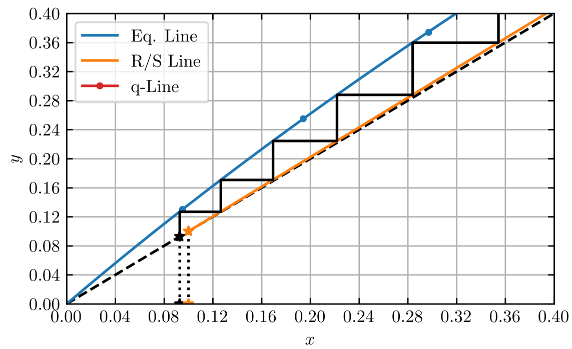
(a) $x, y \in [0.0, 1.0]$



(b) $x, y \in [0.7, 1.0]$



(c) $x, y \in [0.3, 0.8]$



(d) $x, y \in [0.0, 0.4]$

Figure 10: McCabe-Thiele diagrams for n-heptane-toluene distillation with $R = 18$ and saturated vapor feed

Therefore, the feed stage should be at $\boxed{N_F = 8}$ and the total number of stages is $\boxed{N = 16 + 1}$ for $R = 18$.

g.

For the saturated vapor, $R = 6$ requires 22 total stages while $R = 18$ requires 17 total stages, which is only 1 off from the minimum number of stages. This follows the same trend as the saturated liquid: higher R implies lower N.