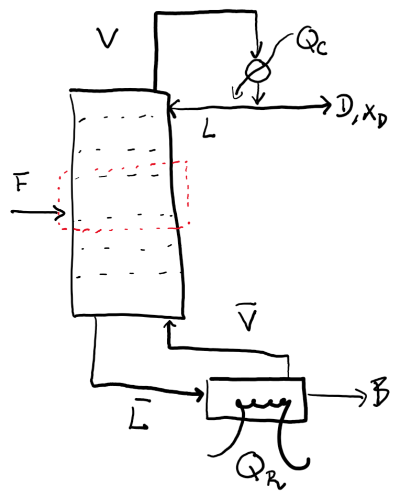
1)

a.



$P = 101.325$ kPa
$x_B = 0.12$
$x_D = 0.92$
$x_F = 0.60$
$F = 100$ mol/hr
$R = 2.4 : 1.0$

Assume: CMO

Solve:

The material balances around the entire column are,

$$F = D + B,$$

$$x_F F = x_D D + x_B B,$$

From which we find,

$$\frac{D}{F} = \frac{x_F - x_B}{x_D - x_B}.$$

Thus,

$$\boxed{D = (100 \text{ kg/hr}) \left(\frac{0.60 - 0.12}{0.92 - 0.12} \right) = 60 \text{ mol/hr}},$$

and,

$$\boxed{B = F - D = 40 \text{ mol/hr}}.$$

b.

For an arbitrary stage in the rectifying section, the material balances around the top of the column are,

$$V = L + D,$$

$$yV = xL + x_D D,$$

dividing by V and using $L = RD$,

$$1 = (R+1)\frac{D}{V},$$

$$y = \left(\frac{L}{V}\right)x + x_D\left(\frac{D}{V}\right),$$

giving,

$$y = \left(\frac{L}{V}\right)x + \frac{x_D}{R+1},$$

whose intercept is given by,

$$b = \frac{0.92}{2.4+1} = 0.271.$$

Since the distillate vapor is totally condensed, we know $y_1 = x_D$ must lie on the rectifying line. Therefore, we now know two points, $(0, 0.271)$ and $(0.92, 0.92)$, and the curve may be drawn on an xy diagram. The equation for the stripping line is,

$$y = \frac{\bar{L}}{\bar{V}}x - \frac{B}{\bar{V}}x_B.$$

Plugging in $x = x_B$ and using $\bar{V} = \bar{L} - B$ implies that the point (x_B, x_B) lies on the stripping line. The second point comes from the material balance around the feed plate which dictates that the rectifying and stripping curves intersect at the q-line. Therefore, we must draw the q-line whose equation is given by,

$$y = -\frac{q}{1-q}x + \frac{x_F}{1-q}.$$

Since the the feed is a saturated liquid, we have $q = 1$, meaning the q-line has an infinite slope and must pass through x_F. Two points of the stripping curve are now known and the line may be drawn on the xy diagram.

The equilibrium line may be estimated by fitting a polynomial to the data [1]. A 4th order polynomial was used to prevent overfitting of the data and the intercept was set to 0. That is,

$$y(x) = ax^4 + bx^3 + cx^2 + dx,$$

for some constants (a, b, c, d). Using scipy's optimize.curve_fit function in Python, this gives,

$$(a, b, c, d) = (-0.5353, 1.738, -2.523, 2.321),$$

with estimated (MLE) relative errors,

$$(0.08, 0.05, 0, 02, 0, 01).$$

By starting at (x_D, x_D) and stepping off the equilibrium and rectifying/stripping lines, the number of stages may be determined.

[1] I tried to fit an activity coefficient model to the curve; its not worth the hassle for such a simple system.

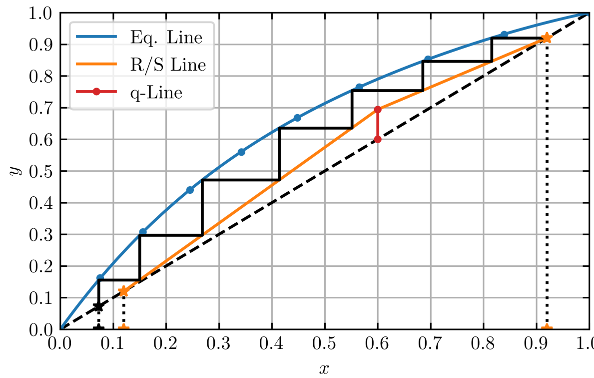


Figure 1: McCabe-Thiele diagram for benzene-toluene distillation, saturated liquid feed

Thus, the feed should be at stage $\boxed{N_F = 3}$ and the total number of stages is $\boxed{N = 6 + 1}$.

c.

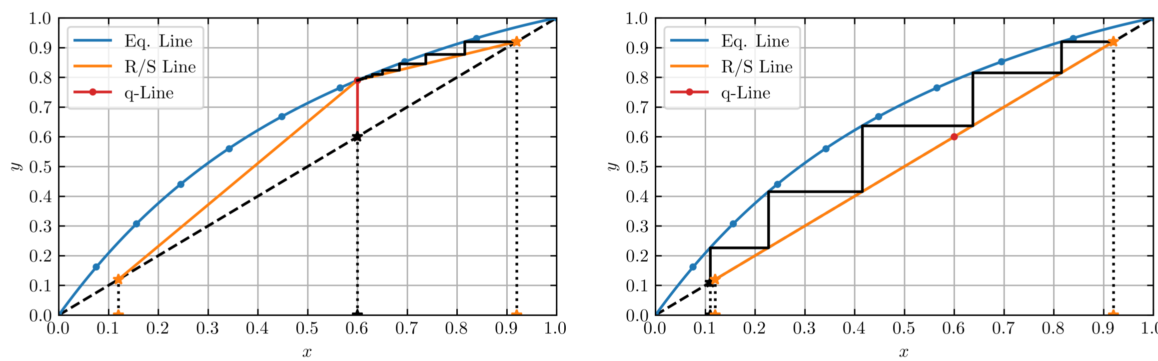
The minimum reflux ratio, R_{min}, is given by generating a pinch point and having the q-line touch the equilibrium curve. The slope of the resulting rectifying line may be used to find R_{min},

$$\frac{R_{min}}{R_{min}+1} = \frac{x_D - y(x_F)}{x_D - x_F} = \frac{0.98 - 0.803}{0.98 - 0.44} = 0.328.$$

Solving for R_{min},

$$\boxed{R_{min} = 0.685}.$$

Total reflux occurs when the column operates without a feed (also implying no outputs). The q-line simply becomes a point at (x_F, x_F) and the rectifying and stripping lines degenerate into the same curve. The minimum number of stages, N_{min}, may be found by stepping off the diagram in this case.



(a) R_{min}, saturated liquid feed (b) Total reflux

Figure 2: McCabe-Thiele diagrams for benzene-toluene distillation with R_{min} and total reflux

Therefore, $\boxed{N_{min} = 4 + 1}$.

d.

The equilibrium and rectifying lines remain the same as in part b, but the q-line changes. Since $q = 0$, the q-line is horizontal and the stripping line may be found by connecting the bottoms composition and q-line's intersection with the rectifying line. This is given graphically in the following figure.

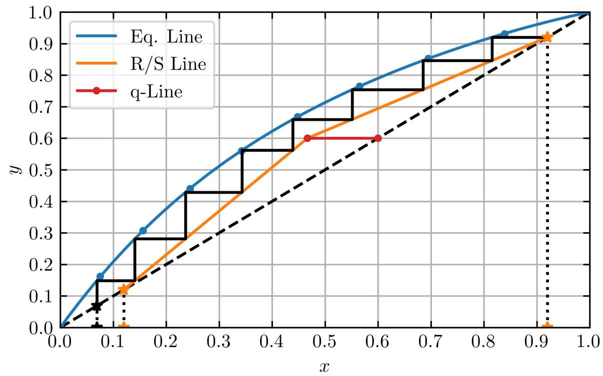


Figure 3: McCabe-Thiele diagram for benzene-toluene distillation, saturated vapor feed

Thus, the feed should be at stage $\boxed{N_F = 4}$ and the total number of stages is $\boxed{N = 7 + 1}$.

e.

The minimum number of stages in total reflux remains the same as in part c. However, the R_{min} changes since the pinch point occurs at a different location.

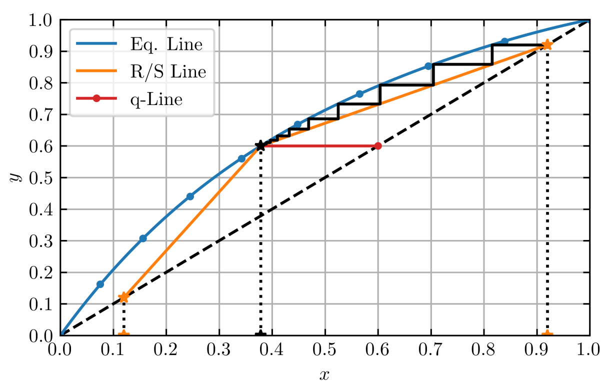


Figure 4: McCabe-Thiele diagram for benzene-toluene distillation, saturated vapor feed with R_{min}

Calculating the slope of the rectifying line,

$$\frac{R_{min}}{R_{min} + 1} = \frac{x_D - x_F}{x_D - x_q} = \frac{0.92 - 0.60}{0.92 - 0.38} = 0.591.$$

Solving for R_{min},

$$\boxed{R_{min} = 1.44}.$$

Appendix

This is a explanation of the q-line for my course notebook, this section is extra and may be skipped.

Let i denote the position of the feed stage. The material balance around the feed stage is,

$$F + L + \bar{V} = \bar{L} + V,$$

which may be rearranged as,

$$1 = \frac{\bar{L} - L}{F} + \frac{V - \bar{V}}{F}.$$

For convenience, q is defined as,

$$q \coloneqq \frac{\bar{L} - L}{F},$$

which also implies,

$$\frac{V - \bar{V}}{F} = 1 - q.$$

The energy balance around the feed stage then gives,

$$F\mathcal{H}_F + Lh_{i-1} + \bar{V}H_{i+1} = \bar{L}h_i + VH_i,$$

where $\mathcal{H}_F$ is the molar enthalpy of the feed whose phase we will not assume. If we solve for $\mathcal{H}_F$,

$$\mathcal{H}_F = \frac{\bar{L}}{F}h_i - \frac{L}{F}h_{i-1} + \frac{V}{F}H_i - \frac{\bar{V}}{F}H_{i+1}.$$

Initially, it seems we have reached a dead end. However, an important constraint to remember is that CMO implies that one mole of vapor is condensed for every mole of liquid vaporized. What does this really mean? It implies that the liquid enthalpy, h, and the vapor enthalpy, H, do not vary significantly with temperature. Therefore, there are no sensible enthalpy changes and energy transfer only occurs between the liquid and vapor phases. It total, this implies $h_i \approx h_{i-1}$ and $H_i \approx H_{i+1}$, meaning,

$$\begin{aligned}\mathcal{H}_F &= \left(\frac{\bar{L} - L}{F}\right) h_i + \left(\frac{V - \bar{V}}{F}\right) H_i \\ &= qh_i + (1-q)H_i.\end{aligned}$$

Solving for q,

$$q = \frac{H_i - \mathcal{H}_F}{H_i - h_i},$$

therefore, these two expressions for q link the energy and material balances. The final piece of the puzzle comes from the component balances on the rectifying and stripping lines,

$$yV = xL + x_D D,$$

$$y\bar{V} = x\bar{L} - x_B B.$$

Since these are two straight lines, they must intersect at some unique (x, y) point [2]. This family of intersection points may be found by subtracting the two equations,

$$\left(V - \bar{V}\right) y = \left(L - \bar{L}\right) x + x_D D + x_B B,$$

dividing by F,

$$(1 - q)\, y = -qx + x_F,$$

which rearranges to the final form of the q-line,

$$y = -\frac{q}{1-q} x + \frac{x_F}{1-q}.$$

Therefore, it may be seen why the rectifying and stripping lines always intersect at the q-line. In the special case that $H_i = \mathcal{H}_F$, we have a saturated vapor and $q = 0$, meaning $y = x_F$ for all x, or the q-line is horizontal. In the limiting case that $h_i = \mathcal{H}_F$, $q = 1$ and the slope of the q-line becomes infinite and appears vertical.

[2] Assuming they are not parallel, as in the case with total reflux.