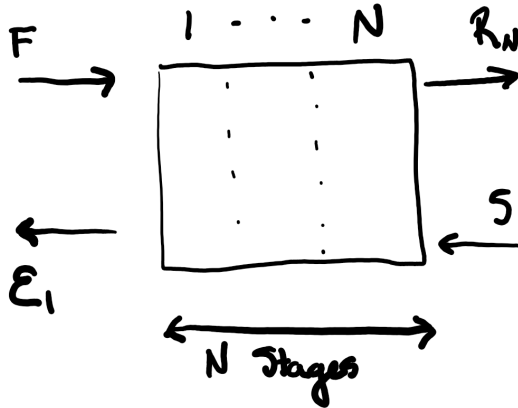# 1)

Let A denote acetone, W denote water, and T denote 1,2,2-trichloroethane.

a.

$$x_A^F = 0.40$$
$$x_W^F = 0.20$$
$$x_A^R = 0.05$$
$$F = 1000 \text{ lb/hr}$$
$$S = 400 \text{ lb/hr}$$
$$\text{(pure)}$$

Solve:

Solving for the mixture composition ($[m_i] = \text{lb/hr}$),

Comp.	F		S		M	
	m_i	x_i	m_i	x_i	m_i	x_i
A	400	0.40	0.00	0.00	400	0.286
W	600	0.60	0.00	0.00	600	0.429
T	0	0.00	400	1.00	400	0.286
Tot.	1000	1.00	400	1.00	1400	1.00

From this, the F, S, and M composition points may be plotted on the Gibbs triangle for the system along with the desired R_N composition. The material balances around the column dictate that E_1, R_N, and M compositions lie on the same line. Thus, we draw a line through $R_N M$ and find its intersection with the equilibrium curve (E_1 exits in equilibrium). Additionally, consider the material balance around the entire column,

$$F + S = E_1 + R_N.$$

Meaning,

$$\Delta = F - E_1 = R_N - S = R_i - E_{i+1},$$

for any stage, i. Therefore, all passing streams *must* intersect at Δ. By drawing the lines through FE_1 and $R_N S$, we may extract Δ. From this, the composition is found by stepping between the tie lines and operating point.
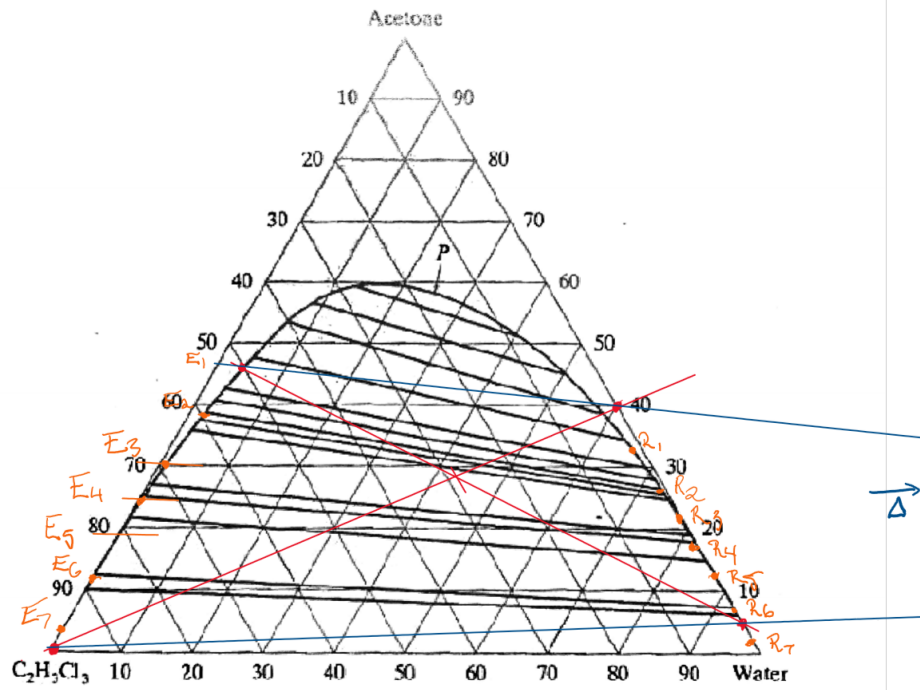
Figure 1: Gibbs triangle for a A-W-T system

From the diagram, we require 7 tie lines to get past R_N, thus, at least $\boxed{N = 7}$ stages are required.

b.

The compositions may be read off the diagram. The raffinate compositions are read from the right side.

				x_i^R			
N	1	2	3	4	5	6	7
A	0.32	0.26	0.22	0.17	0.13	0.07	0.02
W	0.66	0.73	0.78	0.83	0.87	0.93	0.98
T	0.02	0.01	0.00	0.00	0.00	0.00	0.00
Tot.	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Likewise, the extract compositions are read from the left.

				x_i^E			
N	1	2	3	4	5	6	7
A	0.46	0.38	0.30	0.245	0.19	0.12	0.03
W	0.04	0.02	0.01	0.005	0.00	0.00	0.00
T	0.50	0.60	0.69	0.75	0.81	0.88	0.97
Tot.	1.00	1.00	1.00	1.00	1.00	1.00	1.00

2

c.

Since the compositions are known from the Gibbs triangle. We may solve for R_N and E_1 using any component balance. This forms a linear system,

$$M = E_1 + R_N,$$

$$x_i^M M = x_i^E E_1 + x_i^R R_N,$$

whose solution gives a lever rule like expression,

$$\frac{R_N}{M} = \frac{x_i^M - x_i^E}{x_i^R - x_i^E}.$$

Applying this to all of the components gives 3 different values for R_N,

$$(A, W, T) \rightarrow R_N = (555, 579, 600) \text{ lb/hr},$$

Taking the average gives,

$$\boxed{R_N = 578 \text{ lb/hr}}.$$

Since $E_1 = M - R_N$,

$$\boxed{E_1 = 822 \text{ lb/hr}}.$$

The flow rates of the exit streams are summarized below ($[m_i] = \text{lb/hr}$).

Comp.	M		E_1		R_N	
	m_i	x_i	m_i	x_i	m_i	x_i
A	400	0.286	378	0.460	12	0.020
W	600	0.429	32.9	0.040	566	0.980
T	400	0.286	411.1	0.500	0	0.000
Tot.	1400	1.00	822	1.00	578	1.00

d.

The material balance of any component around the feed to an arbitrary stage, j, is,

$$F + E_{j+1} = R_j + E_1,$$

$$x^F F + x_{j+1}^E E_{j+1} = x_j^R R_j + x_1^E E_1.$$

This can be reformulated in terms of Δ,

$$\Delta = F - E_1 = R_j - E_{j+1},$$

or,

$$R_j = \Delta + E_{j+1}.$$

Therefore,

$$x_{j+1}^E E_{j+1} = x_j^R (\Delta + E_{j+1}) + x_1^E E_1 - x^F F,$$

$$\implies \left(x_{j+1}^E - x_j^R\right) E_{j+1} = x_j^R \Delta + x_1^E E_1 - x^F F,$$

$$\implies E_{j+1} = \frac{x_j^R \Delta + x_1^E E_1 - x^F F}{x_{j+1}^E - x_j^R}$$
$$= \frac{\left(x_1^E - x_j^R\right) E_1 + \left(x_j^R - x^F\right) F}{x_{j+1}^E - x_j^R}.$$

The above equation along with,

$$R_j = \Delta + E_{j+1},$$

implies that any stage flow rate may be calculated. However, in practice, these calculated values can be *very* sensitive to the mole fractions that are used. In the following table, the acetone mole fractions lead to quite different answers than the other two components when calculating E at every stage [1]. Therefore, in the average values, the acetone results were omitted ($[E] = [R] = \text{lb/hr}$).

Stream	N	1	2	3	4	5	6	7
	A	822	585	611	694	423	-134	233
E	W	822	703	607	553	506	474	432
	T	822	715	607	548	508	467	424
	Avg.	822	709	607	551	507	471	428
R	Avg.	887	785	728	684	648	606	578

After 2 stages, the extract is leaving as E_6, the acetone flow rate is thus,
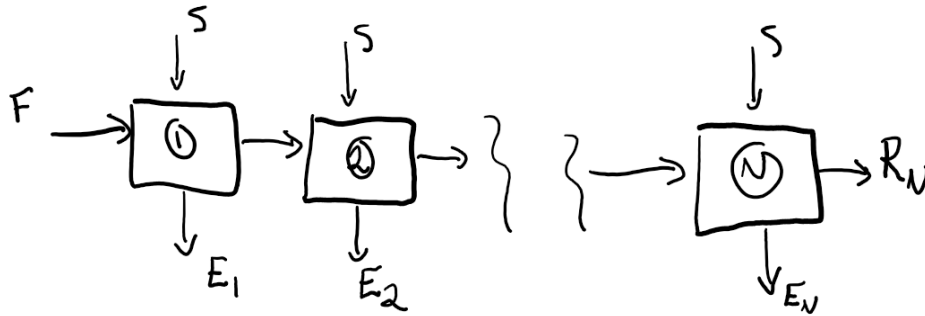
$$x_6^E E_6 = (0.12)(471 \text{ lb/hr}) = 56.5 \text{ lb/hr}$$

Therefore, the percent recovery after 2 stages is,

$$\boxed{\% \text{ Recovery} = \frac{56.5}{400} = 14\%}.$$

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[1] I don't know why the formula doesn't work for acetone, but I'd guess that it's probably due to acetone being the solute and its concentration in the extract and raffinate being similar. Any errors are amplified by the $x_{j+1}^E - x_j^R$ term in the denominator.

2)

Same F conditions as problem 1, S at each stage is 100 kg/hr = 220 lb/hr.

Solve:

The stage compositions are found by iteratively applying single stage balances and using the composition of the raffinate from the previous stage as the feed to the next stage. This means a new mixing point is calculated at each stage to find the new extract composition.
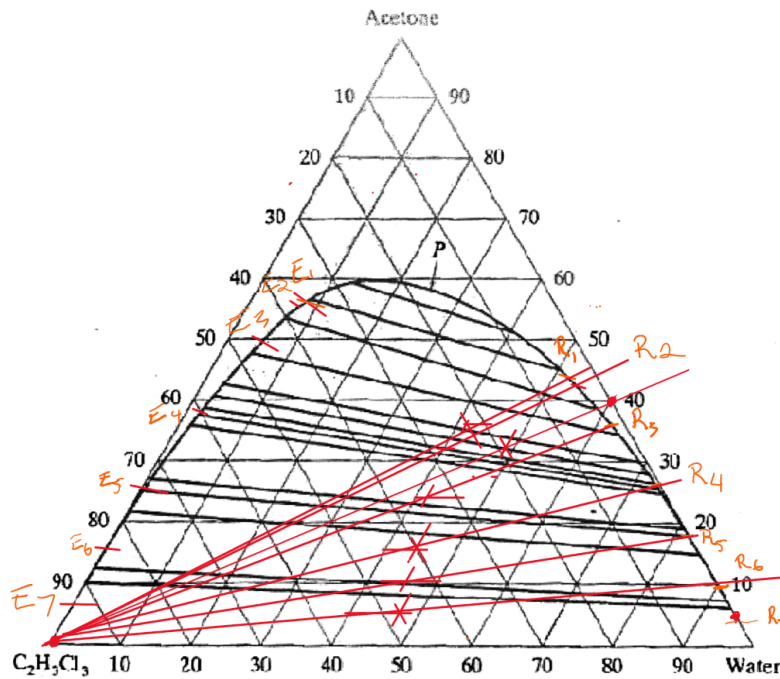

Figure 2: Gibbs triangle for a cross current A-W-T system

a.

The mass flow rates were constructed by applying,

$$\frac{R_N}{M} = \frac{x_i^M - x_i^E}{x_i^R - x_i^E},$$

on the W and T compositions and averaging the results. The mass flow rates follow from multiplying the mass fractions at that stage by the mass flow rate. All exiting flow rates are shown in the following table[2].

Stream	N	1		2		3		4		5		6		7	
	Comp.	m_i	x_i	m_i	x_i	m_i	x_i	m_i	x_i	m_i	x_i	m_i	x_i	m_i	x_i
R (lb/hr)	A	409	0.445	281	0.420	166	0.360	89	0.26	51	0.18	25	0.10	9	0.04
	W	474	0.515	361	0.540	286	0.620	251	0.73	233	0.82	227	0.90	227	0.96
	T	37	0.04	27	0.040	9	0.020	3	0.01	0	0.00	0	0.00	0	0.00
	Tot.	919	1.00	668	1.00	461	1.00	344	1.00	284	1.00	252	1.00	237	1.00
E (lb/hr)	A	169	0.56	264	0.56	209	0.49	128	0.38	73	0.26	38	0.15	14	0.06
	W	24	0.08	33	0.07	21	0.05	7	0.02	0	0.00	0	0.00	0	0.00
	T	108	0.36	175	0.37	197	0.46	203	0.6	207	0.74	215	0.85	221	0.94
	Tot.	301	1.00	472	1.00	427	1.00	338	1.00	280	1.00	253	1.00	235	1.00

The percent stage extraction and overall extraction were calculated according to,

$$(\% \text{ Stage Ext.})_i = \frac{x_{i-1}^R R_{i-1} - x_i^R R_i}{x_{i-1}^R R_{i-1}} \times 100\%,$$

$$(\% \text{ Overall Ext.})_i = \frac{x^F F - \left(x_{i-1}^R R_{i-1} - x_i^R R_i\right)}{x^F F} \times 100\%,$$

where x is the acetone mass fraction. These results were calculated in Excel and are presented in the following table [3].

N	2	3	4	5	6	7
Stage Ext. (%)	31.4	40.8	46.2	42.8	50.7	62.4
Overall Ext. (%)	32.1	28.7	19.2	9.6	6.5	3.9

b.

Cross current requires a lot more solvent to achieve a similar composition at R_N, however, since it uses more solvent, it is able to extract more acetone in the earlier stages. Therefore, cross current staging seems to be better to remove a lot of solute in a few stages, but becomes inefficient when there is only a small amount of solute left. In that case switching to countercurrent seems better when the solute concentration is very low.

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[2] Somehow I didn't notice that the R_1 acetone flow rate is greater than 400 lb/hr, this is physically impossible. I'm not redoing the problem, but a 'quick fix' would be to just ignore the first stage as it is very similar in composition to the second stage.

[3] The first stage is omitted due to the nonphysical acetone flow rate from above.