1)

Let 1 denote hexane and 2 denote toluene.
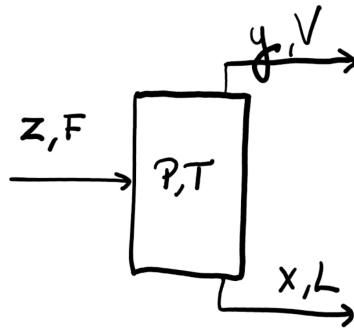
$$z_1 = 0.30$$
$$P = 101.325 \text{ kPa}$$
$$T = 96°\text{C}$$
$$F = 1000 \text{ mol/hr}$$

Solve:

Hexane and toluene are both similar sized hydrocarbons and the operating pressure is low enough to assume an ideal gas mixture. Thus, as a first approximation, we may assume the feed obeys Raoult's law. The P_i^{sat} values can be calculated from Antoine's equation,

$$\ln\left(P_i^{sat}\right) = A_i - \frac{B_i}{T + C_i},$$

where P_i^{sat} is in kPa and T is in °C. For hexane: $A_1 = 13.8193$, $B_1 = 2696.04$, $C_1 = 224.317$ and for toluene: $A_2 = 13.9320$, $B_2 = 3056.96$, $C_2 = 217.625$ [1]. From this, we find,

$$P_1^{sat} = 222.0 \text{ kPa}, \quad P_2^{sat} = 65.68 \text{ kPa}.$$

Raoult's law gives,
$$y_i P = x_i P_i^{sat}.$$

To find the number of phases, we calculate the dew point pressure where $y_i = z_i$, and sum over i,

$$P_{dew} = \frac{1}{y_1/P_1^{sat} + y_2/P_2^{sat}} = \frac{1}{0.30/222.0 + 0.70/65.68} \text{ kPa} = 83.27 \text{ kPa}.$$

The bubble point pressure is given by assuming $x_i = z_i$ and summing over i,

$$P_{bub} = x_1 P_1^{sat} + x_2 P_2^{sat} = [(0.30)(222.0) + (0.70)(65.68)] \text{ kPa} = 112.6 \text{ kPa}.$$

Since $\boxed{P_{dew} < P < P_{bub}}$, we must have $\boxed{2 \text{ phases}}$.

The material balances are,
$$F = L + V,$$
$$z_1 F = x_1 L + y_1 V,$$

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[1]Smith, J.M., Van Ness, H.C., Abbot, M. M., Swihart, M. T., *Introduction to Chemical Engineering Thermodynamics*. 9th Ed., McGraw Hill, 2018.

from which we obtain,

$$\frac{V}{F} = \frac{z_1 - x_1}{y_1 - x_1}.$$

x_1 may be calculated by,

$$P = x_1 P_1^{sat} + (1 - x_1) P_2^{sat},$$

$$\implies x_1 = \frac{P - P_2^{sat}}{P_1^{sat} - P_2^{sat}} = \frac{101.325 - 65.68}{222.0 - 65.68} = 0.228.$$

y_1 is then given by,

$$y_1 = \frac{x_1 P_1^{sat}}{P} = \frac{(0.228)(222.0)}{101.325} = 0.500.$$

Therefore, in total,

$$\boxed{\begin{array}{ll} x_1 = 0.228, & x_2 = 0.772 \\ y_1 = 0.500, & y_2 = 0.500 \end{array}}$$

The vapor fraction, V/F, can thus be calculated as,

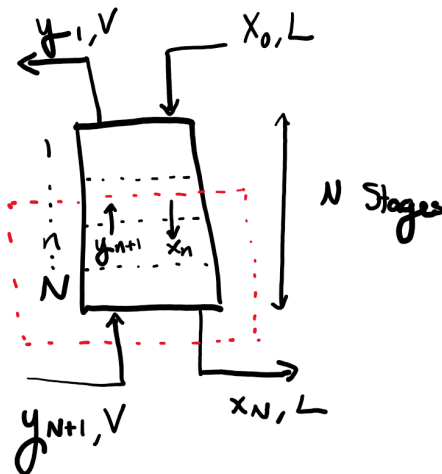$$\frac{V}{F} = \frac{0.30 - 0.228}{0.500 - 0.228} = 0.265,$$

and,

$$\frac{L}{F} = 1 - \frac{V}{F} = 0.735.$$

Since $F = 1000$ mol/hr,

$$\boxed{V = 265 \text{ mol/hr}, \quad L = 735 \text{ mol/hr}}.$$

2)

a.

$$x_0 = 0$$
$$x_N = 0.0605$$
$$y_{N+1} = 0.14$$
$$y_e = 1.20 x_e$$
$$x_N L_N = 0.92 y_{N+1} V_{N+1}$$

Solve:

The material balances around the entire column are,

$$L_0 + V_{N+1} = L_N + V_1,$$

$$x_0 L_0 + y_{N+1} V_{N+1} = x_N L_N + y_1 V_1.$$

Rearranging,

$$V_1 = L_0 + V_{N+1} - L_N,$$

$$y_1 = \frac{x_0 L_0 + y_{N+1} V_{N+1} - x_N L_N}{V_1},$$

$$\implies y_1 = \frac{x_0 L_0 + y_{N+1} V_{N+1} - x_N L_N}{L_0 + V_{N+1} - L_N}.$$

We have the condition that the oil flow rate is constant, i.e.,

$$(1 - x_0)L_0 = (1 - x_N)L_N,$$

or,

$$L_0 = \frac{1 - x_N}{1 - x_0} L_N.$$

Therefore,

$$y_1 = \frac{y_{N+1} V_{N+1} + \left(\dfrac{1 - x_N}{1 - x_0} x_0 - x_N \right) L_N}{V_{N+1} + \left(\dfrac{1 - x_N}{1 - x_0} - 1 \right) L_N},$$

or,

$$y_1 = \frac{y_{N+1} + \left(\dfrac{1 - x_N}{1 - x_0} x_0 - x_N \right) \dfrac{L_N}{V_{N+1}}}{1 + \left(\dfrac{1 - x_N}{1 - x_0} - 1 \right) \dfrac{L_N}{V_{N+1}}}.$$

In this example, $x_0 = 0$, thus,

$$y_1 = \frac{y_{N+1} - \dfrac{x_N L_N}{V_{N+1}}}{1 - \dfrac{x_N L_N}{V_{N+1}}}.$$

$x_N L_N / V_{N+1}$ may be solved for,

$$\frac{x_N L_N}{V_{N+1}} = 0.92 y_{N+1} = (0.92)(0.14) = 0.1288,$$

giving,

$$y_1 = \frac{0.14 - 0.1288}{1 - 0.1288} = 0.0129$$

The operating line may now be plotted since $(x_0, y_1) = (0, 0.0129)$ and $(x_N, y_{N+1}) = (0.0605, 0.14)$ are known.
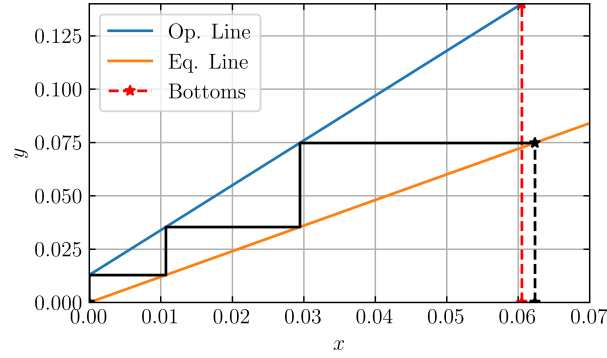
Figure 1: Acetone-Oil Absorption

Thus, at least $\boxed{N = 3}$ stages are required.

b.

Since the acetone output concentration must remain the same, the intercept of the operating line must remain the same while the slope, L_N/V_{N+1}, will change. If the operating line intersects the outlet exactly, we must have,

$$y_{N+1} = 1.20x_N.$$

Rearranging the material balance,

$$y_1\left(1 - \frac{x_N L_N}{V_{N+1}}\right) = y_{N+1} - \frac{x_N L_N}{V_{N+1}},$$

$$\frac{L_N}{V_{N+1}} = \frac{y_{N+1} - y_1}{x_N(1 - y_1)}.$$

Imposing the intersection condition,

$$\frac{L_N}{V_{N+1}} = \frac{Kx_N - y_1}{x_N(1 - y_1)} = \frac{(1.20)(0.0605) - 0.0129}{(0.0605)(1 - 0.0129)} = 1.00.$$

Alternatively, the slope of the minimum operating line can be calculated as $L_N/V_{N+1} = 0.988$.

$$\begin{aligned}
L_0 &= (1 - x_N)L_N \\
&= (1 - x_N)1.00V_{N+1} \\
&= (0.9395)(1.00)(100 \text{ mol/hr}) = 94.0 \text{ mol/hr}.
\end{aligned}$$

Therefore,

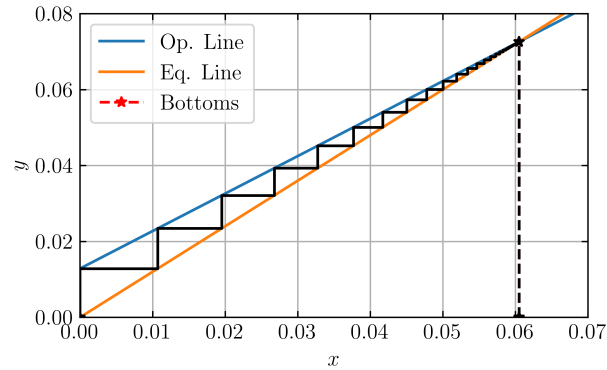$$\boxed{L_{min} = 94.0 \text{ mol/hr}}.$$

Figure 2: Acetone-Oil Absorption with Minimum Oil

c.

The Kremser equation is given as,

$$N = \frac{\ln\left(\left[\dfrac{y_{N+1} - Kx_0}{y_1 - Kx_0}\right]\left[1 - \dfrac{1}{A}\right] + \dfrac{1}{A}\right)}{\ln(A)}.$$

For A, we calculate A_1 and A_N.

$$L_N = \frac{0.92 y_{N+1} V_{N+1}}{x_N} = \frac{(0.92)(0.14)(100 \text{ mol/hr})}{0.0605} = 212.9 \text{ mol/hr},$$

$$L_0 = \frac{1 - x_N}{1 - x_0} L_N = (1 - 0.0605)(212.9 \text{ mol/hr}) = 200.0 \text{ mol/hr},$$

$$V_1 = L_0 + V_{N+1} - L_N = (200.0 + 100 - 212.9) \text{ mol/hr} = 87.12 \text{ mol/hr}.$$

Thus,

$$A_1 = \frac{L_0}{KV_1} = \frac{200.0}{(1.20)(87.12)} = 1.913,$$

$$A_N = \frac{L_N}{KV_{N+1}} = \frac{212.9}{(1.20)(100)} = 1.774,$$

taking the mean,

$$A = \sqrt{A_1 A_N} = \sqrt{(1.913)(1.774)} = 1.842.$$
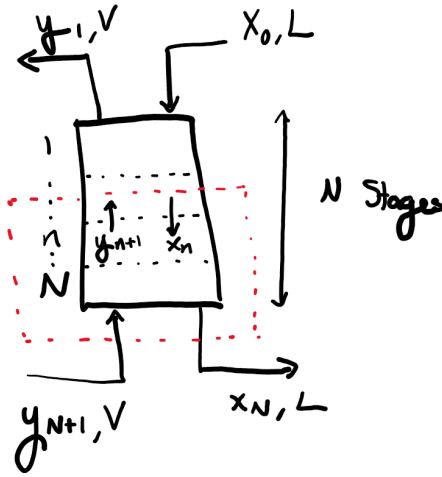
N may now be calculated,

$$N = \frac{\ln\left(\left[\dfrac{0.14}{0.0129}\right]\left[1 - \dfrac{1}{1.842}\right] + \dfrac{1}{1.842}\right)}{\ln(1.842)} = 2.80.$$

To guarantee the desired separation, we must take,

$$\boxed{N = 3}.$$

3)

a.

$$x_0 = 0.005$$
$$x_N L_N = 0.90 y_{N+1} V_{N+1}$$
$$y_{N+1} = 0.15$$
$$y_e = 1.60 x_e$$
$$L_0 = 200 \text{ mol/hr}$$
$$V_{N+1} = 100 \text{ mol/hr}$$

Solve:

The material balance is given in problem 2,

$$(1 - x_0)L_0 = (1 - x_N)L_N,$$

$$y_1 = \frac{y_{N+1} + \left(\dfrac{1 - x_N}{1 - x_0} x_0 - x_N \right) \dfrac{L_N}{V_{N+1}}}{1 + \left(\dfrac{1 - x_N}{1 - x_0} - 1 \right) \dfrac{L_N}{V_{N+1}}}.$$

Solving for L_N,

$$\begin{aligned}
L_N &= (1 - x_0)L_0 + x_N L_N \\
&= (1 - x_0)L_0 + 0.90 y_{N+1} V_{N+1} \\
&= [(1 - 0.005)(200) + (0.90)(0.15)(100)] \text{ mol/hr} = 212.5 \text{ mol/hr}.
\end{aligned}$$

Solve for x_N,

$$x_N = \frac{0.90 y_{N+1} V_{N+1}}{L_N} = \frac{(0.90)(0.15)(100)}{(212.5)} = 0.0635.$$

Let,

$$p = \frac{1 - x_N}{1 - x_0} = \frac{1 - 0.0635}{1 - 0.005} = 0.9412.$$

thus,

$$y_1 = \frac{y_{N+1} + (px_0 - x_N)\dfrac{L_N}{V_{N+1}}}{1 + (p-1)\dfrac{L_N}{V_{N+1}}}$$

$$= \frac{0.15 + [(0.9412)(0.005) - 0.0635](212.5/100)}{1 + (0.9412 - 1)(212.5/100)}$$

$$= 0.0286.$$

The operating line may now be plotted since $(x_0, y_1) = (0.005, 0.0286)$ and $(x_N, y_{N+1}) = (0.0635, 0.15)$ are known.
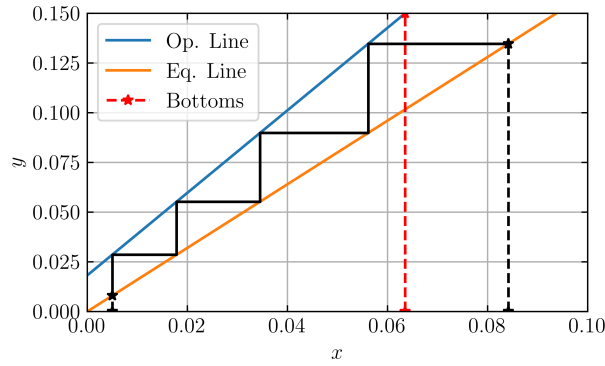

Figure 3: Contaminant Gas Absorption with Water

Thus, at least $\boxed{N = 4}$ stages are required.

Since the contaminant output concentration must remain the same, the y_1 of the operating line must remain the same while the slope, L_N/V_{N+1}, will change. If the operating line intersects the outlet exactly, we must have,

$$y_{N+1} = 1.60x_N.$$

Rearranging the material balance,

$$y_1\left(1 + (p-1)\frac{L_N}{V_{N+1}}\right) = y_{N+1} + (px_0 - x_N)\frac{L_N}{V_{N+1}},$$

$$\frac{L_N}{V_{N+1}} = \frac{y_{N+1} - y_1}{x_N - px_0 + y_1(p-1)}$$

Thus,

$$\frac{L_N}{V_{N+1}} = \frac{(1.60)(0.0635) - 0.0286}{0.0635 - (0.9412)(0.005) + (0.0286)(0.9412 - 1)} = 1.28.$$

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Alternatively, the slope of the minimum operating line can be calculated as $L_N/V_{N+1} = 1.25$.

$$L_0 = pL_N$$
$$= p(1.28V_{N+1})$$
$$= (0.9412)(1.28)(100 \text{ mol/hr}) = 120 \text{ mol/hr}.$$

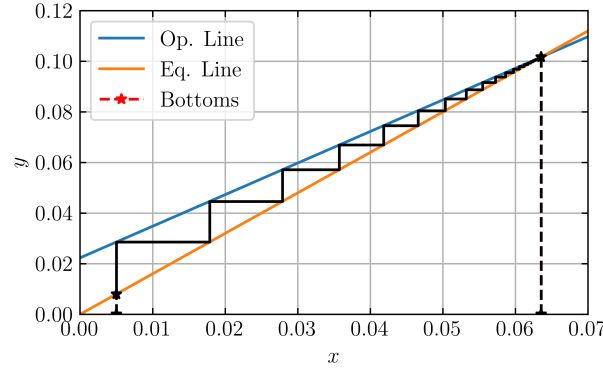Therefore,

$$\boxed{L_{min} = 120 \text{ mol/hr}}.$$

Figure 4: Contaminant Gas Absorption with Minimum Water

b.

For A, we calculate A_1 and A_N.

$$L_N = 212.5 \text{ mol/hr},$$
$$L_0 = 200 \text{ mol/hr},$$
$$V_{N+1} = 100 \text{ mol/hr},$$
$$V_1 = L_0 + V_{N+1} - L_N = (200 + 100 - 212.5) \text{ mol/hr} = 87.5 \text{ mol/hr}.$$

Thus,

$$A_1 = \frac{L_0}{KV_1} = \frac{200}{(1.60)(87.5)} = 1.429,$$

$$A_N = \frac{L_N}{KV_{N+1}} = \frac{212.5}{(1.60)(100)} = 1.328,$$

taking the mean,

$$A = \sqrt{A_1 A_N} = \sqrt{(1.429)(1.328)} = 1.377.$$

N may now be calculated,

$$N = \frac{\ln\left(\left[\frac{0.15 - (1.60)(0.005)}{0.0286 - (1.60)(0.005)}\right]\left[1 - \frac{1}{1.377}\right] + \frac{1}{1.377}\right)}{\ln(1.377)} = 3.005,$$

To guarantee the desired separation, we must take,

$$\boxed{N = 4}.$$

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