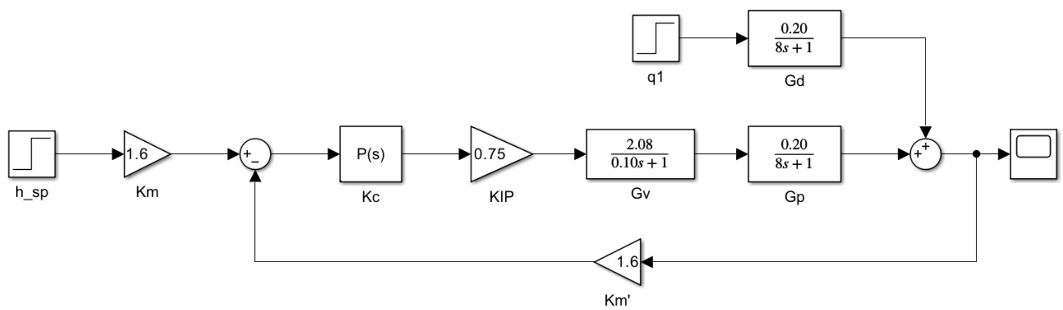
1)

Given: $\overline{q}_1 = 7.5$ ft^3/min, $\overline{q}_2 = 12.5$ ft^3/min, $\overline{h} = 4$ ft, $q_3 = kh$, MV $= q_2$, DV $= q_1$

a.

To prevent the tank from overflowing, the q_2 control valve should fail close or be $\boxed{\text{air to open}}$. If h increases, q_2 must decrease, and since the valve is fail close, the controller should be $\boxed{\text{reverse acting}}$.

b.



To find G_d and G_p, perform a mass balance on the tank,

$$A\frac{dh}{dt} = q_1 + q_2 - q_3.$$

By using $q_3 = kh$ and rearrangement,

$$\frac{A}{k}\frac{dh}{dt} + h = \frac{1}{k}q_1 + \frac{1}{k}q_2.$$

Let $\tau = A/k$ and $K = 1/k$. Putting in deviation form and taking the Laplace transform,

$$\boxed{h(s) = \underbrace{\left[\frac{K}{\tau s + 1}\right]}_{G_d} q_1(s) + \underbrace{\left[\frac{K}{\tau s + 1}\right]}_{G_p} q_2(s)}.$$

k can be found by a steady mass balance,

$$k = \frac{\overline{q}_1 + \overline{q}_2}{\overline{h}} = \frac{(7.5 + 12.5)\text{ft}^3/\text{min}}{4 \text{ ft}} = 5 \text{ ft}^2/\text{min}.$$

Thus,

$$\boxed{K = 1/k = 0.20 \text{ min/ft}^2, \quad \tau = A/k = 8 \text{ min}},$$

and,

$$\boxed{G_p(s) = G_d(s) = \frac{0.20}{8s+1}}.$$

Since the valve is air to open, the valve gain, K_v, is,

$$\boxed{K_v = \frac{(25-0)\text{ ft}^3/\text{min}}{(15-3)\text{ psi}} = 2.0833\text{ ft}^3/\text{min}\cdot\text{psi}}.$$

The valve time constant is given as $\tau_v = 0.10$ min, giving the valve transfer function,

$$\boxed{G_v(s) = \frac{K_v}{\tau_v s+1} = \frac{2.08}{0.10s+1}}.$$

The current to pressure gain is,

$$\boxed{K_{IP} = \frac{(15-3)\text{ psi}}{(20-4)\text{ mA}} = 0.75\text{ psi/mA}}.$$

The controller is a P controller with gain, K_c. The measurement gain, K_m, is finally given by,

$$\boxed{K_m = \frac{(20-4)\text{ mA}}{(10-0)\text{ ft}} = 1.6\text{ mA/ft}}.$$

c.

The closed loop transfer function for a setpoint change is,

$$\frac{h(s)}{h_{sp}(s)} = \frac{K_mK_cK_{IP}G_vG_p}{1+K_mK_cK_{IP}G_vG_p} = \frac{(1.6)(K_c)(0.75)\left(\frac{2.08}{0.10s+1}\right)\left(\frac{0.20}{8s+1}\right)}{1+(1.6)(K_c)(0.75)\left(\frac{2.08}{0.10s+1}\right)\left(\frac{0.20}{8s+1}\right)}.$$

Multiply the top and bottom by $(8s+1)(0.10s+1)$ and simplify,

$$\boxed{\frac{h(s)}{h_{sp}(s)} = \frac{0.5K_c}{0.8s^2+8.1s+(1+0.5K_c)} = \frac{\left(\frac{0.5K_c}{1+0.5K_c}\right)}{\left(\frac{0.8}{1+0.5K_c}\right)s^2+\left(\frac{8.1}{1+0.5K_c}\right)s+1}}.$$

Thus,

$$\boxed{K = \frac{0.5K_c}{1+0.5K_c}},$$

$$\boxed{\tau = \sqrt{\frac{0.8}{1+0.5K_c}}},$$

$$\boxed{\zeta = \frac{1}{2\tau}\frac{8.1}{1+0.5K_c} = \frac{4.528}{\sqrt{1+0.5K_c}}}.$$

The disturbance change is given by,

$$\frac{h(s)}{q_1(s)} = \frac{G_d}{1+K_mK_cK_{IP}G_vG_p} = \frac{\left(\frac{0.20}{8s+1}\right)\left(\frac{0.20}{8s+1}\right)}{1+(1.6)(K_c)(0.75)\left(\frac{2.08}{0.10s+1}\right)\left(\frac{0.20}{8s+1}\right)}.$$

Simplifying,

$$\frac{h(s)}{q_1(s)} = \frac{G_d}{1+K_mK_cK_{IP}G_vG_p} = \frac{\left(\frac{0.20}{8s+1}\right)}{1+(1.6)(K_c)(0.75)\left(\frac{2.08}{0.10s+1}\right)\left(\frac{0.20}{8s+1}\right)}.$$

Multiply the top and bottom by $(8s+1)(0.10s+1)$ and simplify,

$$\boxed{\frac{h(s)}{q_1(s)} = \frac{0.20(0.10s+1)}{\left(\frac{0.8}{1+0.5K_c}\right)s^2+\left(\frac{8.1}{1+0.5K_c}\right)s+1}}.$$

Thus,

$$\boxed{K = 0.20},$$

$$\boxed{\tau = \sqrt{\frac{0.8}{1+0.5K_c}}},$$

$$\boxed{\zeta = \frac{1}{2\tau}\frac{8.1}{1+0.5K_c} = \frac{4.528}{\sqrt{1+0.5K_c}}},$$

and an additional time constant,

$$\boxed{\tau_a = 0.10}.$$

d.

Rearranging the equation for ζ,

$$\boxed{K_c = 2\left[\left(\frac{4.528}{\zeta}\right)^2 - 1\right] = 48.625}.$$

2)

The block diagram for this process has the same structure as in problem 1, just that T_i is the disturbance and W_s is the manipulated variable. Therefore, the closed loop transfer function for a set point change is,

$$\frac{T(s)}{W_s(s)} = \frac{K_c K_{IP} G_v G_p G_m}{1 + K_c K_{IP} G_v G_p G_m}.$$

For stability,

$$\begin{aligned} 0 &= 1 + K_c K_{IP} G_v G_p G_m \\ &= 1 + K_c(0.75)\left(\frac{42}{0.2s+1}\right)\left(\frac{0.50}{2s+1}\right)\left(\frac{0.08}{0.5s+1}\right) \\ &= (2s+1)(0.5s+1)(0.2s+1) + (1 + 1.26K_c) \\ &= 0.2s^3 + 1.5s^2 + 2.7s + (1 + 1.26K_c). \end{aligned}$$

Let $s = i\omega$,

$$0 = -0.2\omega^3 i - 1.5\omega^2 + 2.7\omega i + 1 + 1.26K_c.$$

This gives the relations,

$$0i = \omega\left(2.7 - 0.2\omega^2\right) i,$$

$$0 = 1.26K_c - 1.5\omega^2 + 1.$$

If $\omega = 0$, then,

$$K_c = -\frac{1}{1.26} = -0.79365.$$

If $\omega^2 = 2.7/0.2$, then,

$$1.26K_c = 1.5\left(\frac{2.7}{0.2}\right) - 1 \implies K_c = 15.2\overline{77}.$$

The controller should be reverse acting ($K_c > 0$), thus,

$$\boxed{0 < K_c < 15.2\overline{77}}.$$