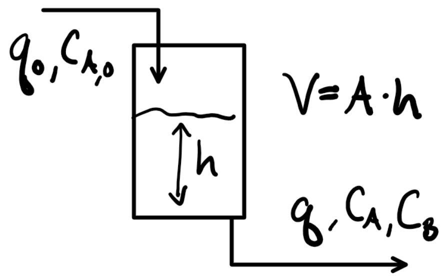
System: Tank with contents and valves



- 2A → B
- $-r_A = kC_A^{1/2}$
- $\rho =$ const.
- $T =$ const.
- Perfect mixing

Further Assumptions:

- $\boxed{q_0 = f(t)}$, $\boxed{C_{A,0} = f(t)}$, $\boxed{q = f(h)}$, $C_{B,0} = 0$
- Stoichiometry: $-r_A = (1/2)r_B$ (2 B produced per A consumed)

Variables:

Inputs	Units
q_0, q	m^3/s
$C_{A,0}$	mol/m^3
Outputs	**Units**
h	m
C_A, C_B	mol/m^3

Parameters (Constant):

Var.	Units
A	m^2
k	$(m^3/mol)^{1/2}s^{-1}$
ρ	kg/m^3

Conservation of Mass:

$$\frac{d(\rho V)}{dt} = \rho\left(q_0 - q\right)$$

Since $V = Ah$ and $\rho, A =$ const.,

$$\boxed{A\frac{dh}{dt} = q_0 - q}.$$

Component Balances:

$$\frac{d\left(C_A V\right)}{dt} = q_0 C_{A,0} - qC_A - kC_A^{1/2}V,$$

$$\frac{d\left(C_B V\right)}{dt} = q_0 C_{B,0} - qC_B + 2kC_A^{1/2}V.$$

Since $V = Ah$, $C_{B,0} = 0$, and $A =$ const.,

$$\boxed{A\frac{d\left(C_A h\right)}{dt} = q_0 C_{A,0} - qC_A - kC_A^{1/2}Ah},$$

$$\boxed{A\frac{d\left(C_B h\right)}{dt} = -qC_B + 2kC_A^{1/2}Ah}.$$

Alternatively, the derivatives can be expanded by the product rule,

$$Ah\frac{dC_A}{dt} + A\frac{dh}{dt}C_A = q_0 C_{A,0} - qC_A - kC_A^{1/2}Ah,$$

$$Ah\frac{dC_B}{dt} + A\frac{dh}{dt}C_B = -qC_B + 2kC_A^{1/2}Ah.$$

with the dh/dt equation,

$$Ah\frac{dC_A}{dt} + (q_0 - q)\,C_A = q_0 C_{A,0} - qC_A - kC_A^{1/2}Ah,$$

$$Ah\frac{dC_B}{dt} + (q_0 - q)\,C_B = -qC_B + 2kC_A^{1/2}Ah,$$

giving the final equations,

$$Ah\frac{dC_A}{dt} = q_0\,(C_{A,0} - C_A) - kC_A^{1/2}Ah,$$

$$Ah\frac{dC_B}{dt} = -q_0 C_B + 2kC_A^{1/2}Ah.$$

DOF Analysis:

There are 3 differential equations for h, C_A, and C_B and 3 algebraic equations for q_0, $C_{A,0}$, and q. There are 6 total variables (h, C_A, C_B, q_0, $C_{A,0}$, q). Therefore, there are no degrees of freedom left and the system is solvable if given initial conditions.

Possible Control Strategy:

A possible control objective is to keep C_A and C_B at given set points by manipulating the outlet flow rate, q. In this case, a feedback or feedforward control loop can be used where C_A and C_B are the controlled variables and q is the manipulated variable. If using feedback control, there needs to be a composition sensor (AT) at the outlet, connected to an outlet valve (final control element), which is controlled by AC. If using feedforward control, there needs to be a composition sensor (AT) and/or a flow sensor (FT) at the inlet, connected to an outlet valve (final control element), which is controlled by AC and/or FC whose control laws are dictated by the above theoretical model. An example of feedback control is shown in the following P&ID.