1)

The sensor gain, K_m, is,

$$K_m = \frac{(20-4)\text{ mA}}{(140-40)\ ^{\text{o}}\text{F}} = 0.16\text{ mA}/^{\text{o}}\text{F}.$$

The open loop transfer function, G_{OL}, is,

$$G_{OL}(s) = K_m K_{IP} G_v G_p = \frac{0.368e^{-0.15s}}{1.5s+1},$$

where $K = 0.368$, $\tau = 1.5$ min, and $\theta = 0.15$ min.

a.

The IMC relations for a FOPDT and PID controller are ($\tau_c = \theta$),

$$\boxed{K_c = \frac{1}{K}\left[\frac{\tau+\theta/2}{3\theta/2}\right] = \frac{1}{0.368}\left[\frac{1.5+(0.15)(0.5)}{(1.5)(0.15)}\right] = 19.0},$$

$$\boxed{\tau_I = \tau + \theta/2 = [1.5+(0.15)(0.5)]\ \text{min} = 1.575\text{ min}},$$

$$\boxed{\tau_D = \frac{\tau\theta}{2\tau+\theta} = \left[\frac{(1.5)(0.15)}{(2)(1.5)+0.15}\right]\ \text{min} = 0.0714\text{ min}}.$$

b.

For ITAE set point, calculate the Y's,

$$\theta/\tau = 0.1,$$

$$Y_K = A\left(\frac{\theta}{\tau}\right)^B = (0.965)\,(0.1)^{-0.85} = 6.83,$$

$$Y_D = A\left(\frac{\theta}{\tau}\right)^B = (0.308)\,(0.1)^{0.929} = 0.0363.$$

Back calculating the controller settings,

$$\boxed{K_c = Y_K/K = 6.83/0.368 = 18.6},$$

$$\boxed{\tau_I = \frac{\tau}{A+B(\theta/\tau)} = \frac{1.5\text{ min}}{0.796+(-0.1465)(0.1)} = 1.92\text{ min}},$$

$$\boxed{\tau_D = \tau Y_D = (1.5\text{ min})(0.0363) = 0.0544\text{ min}}.$$

c.

For ITAE disturbance, calculate the Y's,

$$\theta/\tau = 0.1,$$

$$Y_K = A\left(\frac{\theta}{\tau}\right)^B = (1.357)\,(0.1)^{-0.947} = 12.0,$$

$$Y_I = A\left(\frac{\theta}{\tau}\right)^B = (0.842)\,(0.1)^{-0.738} = 4.61,$$

$$Y_D = A\left(\frac{\theta}{\tau}\right)^B = (0.381)\,(0.1)^{0.995} = 0.0385.$$

Back calculating the controller settings,

$$\boxed{K_c = Y_K/K = 12.0/0.368 = 32.6},$$

$$\boxed{\tau_I = \tau/Y_I = (1.5\text{ min})/4.61 = 0.326\text{ min}},$$

$$\boxed{\tau_D = \tau Y_D = (1.5\text{ min})(0.0385) = 0.0578\text{ min}}.$$

d.

For AMIGO with FOPDT and PID controller,

$$\boxed{K_c = \frac{1}{K}\left[0.2 + 0.45\frac{\tau}{\theta}\right] = \frac{1}{0.368}\left[0.2 + 0.45\frac{1.5}{0.15}\right] = 12.8},$$

$$\boxed{\tau_I = \left[\frac{0.4\theta + 0.8\tau}{\theta + 0.1\tau}\right]\theta = \left[\frac{(0.4)(0.15) + (0.8)(1.5)}{0.15 + (0.1)(1.5)}\right](0.15\text{ min}) = 4.2\text{ min}},$$

$$\boxed{\tau_D = \frac{0.5\theta\tau}{0.3\theta + \tau} = \frac{(0.5)(0.15)(1.5)}{(0.3)(0.15) + 1.5}\text{ min} = 0.0728\text{ min}}.$$

e.

For Tyreus-Luyben, calculate K_{cu} and P_u. First find the stability limit,

$$\begin{aligned} 0 &= 1 + G_{OL} \\ &= 1 + K_c\left[\frac{0.368e^{-0.15s}}{1.5s+1}\right] \\ &= 1.5s + 1 + 0.368e^{-0.15s}K_c. \end{aligned}$$

Let $s = i\omega$,

$$\begin{aligned} 0 &= 1.5\omega i + 1 + 0.368K_c e^{-0.15\omega i} \\ &= \left[1 + 0.368K_c\cos(0.15\omega)\right] + \left[1.5\omega - 0.368K_c\sin(0.15\omega)\right] i \end{aligned}$$

From the real equation,

$$0.368K_c = -\frac{1}{\cos(0.15\omega)}.$$

Using this in the imaginary equation,

$$0 = 1.5\omega + \tan(0.15\omega).$$

There are multiple roots to the equation, take the smallest non-zero root,

$$\omega = 10.88 \text{ rad/min}.$$

This gives an ultimate period of,

$$P_u = \frac{2\pi}{\omega} = 0.5775 \text{ min}.$$

Use the real equation to solve for K_c,

$$K_{cu} = 44.4.$$

Thus, the PID settings are given by,

$$\boxed{K_c = 0.45K_{cu} = 20.0},$$

$$\boxed{\tau_I = 2.2P_u = 1.27 \text{ min}},$$

$$\boxed{\tau_D = P_u/6.3 = 0.0917 \text{ min}}.$$

2)

The Simulink model for the system is shown as follows.

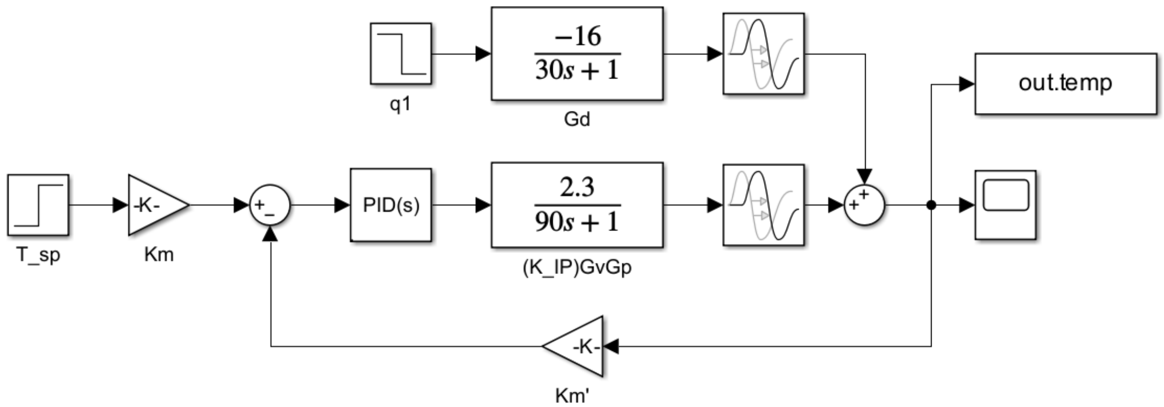

Figure 1: Simulink model of heat exchanger system (time units converted to seconds)

Plotting the response for all PID tuning methods yields the following results.

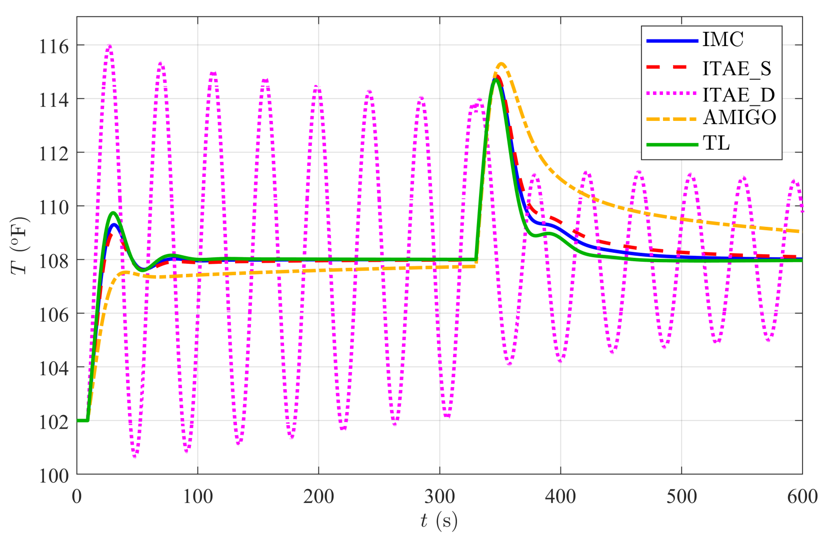

Figure 2: Closed loop HX response for various PID tuning methods

IMC, ITAE set point, and Tyreus-Luyben (TL) all give similar results with good set point tracking, but large deviations[1] when the disturbance hits. TL is a bit better at disturbance rejection as the offset dies out quicker. AMIGO is much slower, not even reaching the new steady date before the disturbance hits. Conversely, ITAE disturbance is too aggressive and is approaching sustained oscillations.

[1] Mostly because the gain for the disturbance is huge.