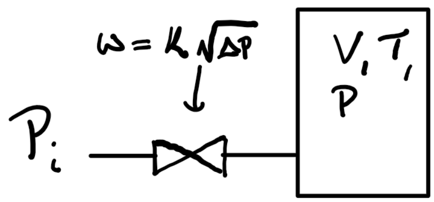
System: Tank with contents and inlet valve



- $V = 500$ ft^3 (Const.)
- $P(t = 0) = 1$ atm $= 14.7$ psi
- $P_i = 300$ psi (Const.)
- $T = 60$ °F $= 519.67$ °R (Const.)
- $w = k\sqrt{P_i - P}$, $k = 40$ lb$_\text{m}$/min/psi$^{1/2}$

Further Assumptions:

- Ideal gas: $P = \frac{\rho R T}{M}$ (M = Molar mass of air)

Variables:

Var.	Units
P	psi
w	lb$_\text{m}$/min

Parameters (Constant):

Var.	Units
P_i	psi
V	ft^3
T	°R
M	lb$_\text{m}$/lbmol
R	psi $\cdot$ ft^3/lbmol $\cdot$° R
k	lb$_\text{m}$/min $\cdot$ psi$^{1/2}$

Conservation of Mass:

$$\frac{d(\rho V)}{dt} = w$$

By the ideal gas law, $V, T, M, R =$ const, and the $w = k\sqrt{\Delta P}$ equation,

$$\boxed{\frac{MV}{RT}\frac{dP}{dt} = k\sqrt{P_i - P}}.$$

DOF Analysis:

There is 1 equation for P with 1 variable, P, and initial condition, $P(t = 0) = 14.7$ psi. The system is solvable.

Solve:

Integrate via separation of variables,

$$\frac{MV}{kRT}\int_{P_0}^{P} \frac{dP}{\sqrt{P_i - P}} = \int_0^t dt$$

$$\implies t = \frac{2MV}{kRT}\left(\sqrt{P_i - P_0} - \sqrt{P_i - P}\right)$$

$$\frac{2MV}{kRT} = \frac{2\left(28.97\ \frac{\text{lb}_\text{m}}{\text{lbmol}}\right)(500\ \text{ft}^3)}{\left(40\ \frac{\text{lb}_\text{m}}{\text{min}\cdot\text{psi}^{1/2}}\right)\left(10.731\ \frac{\text{psi}\cdot\text{ft}^3}{\text{lbmol}\cdot{}^\circ\text{R}}\right)(519.67\ {}^\circ\text{R})} = 0.12987\ \text{min}\cdot\text{psi}^{-1/2}$$

Therefore,

$$t = \left(0.12987 \text{ min} \cdot \text{psi}^{-1/2}\right)\left(\sqrt{300 - 14.7} - \sqrt{300 - 280}\right) \text{psi}^{1/2},$$

which means the time to fill the tank to the desired pressure is,

$$\boxed{t = 1.61 \text{ min} = 96.8 \text{ s}}.$$