1)

Prob.	Range	Zero	Span	Gain	Input	Output (mA)
a	40-200 °C	40 °C	160 °C	0.10 mA/°C	90 °C	9
b	40-120 psi	40 psi	80 psi	0.20 mA/psi	105 psi	17
c	0-80 gpm	0 gpm	80 gpm	0.20 mA/gpm	30 gpm	10

a.

The gain is,

$$K = \frac{(20-4)\ \text{mA}}{(200-40)\ ^\circ\text{C}} = 0.10\ \text{mA}/^\circ\text{C}.$$

By linear interpolation, the output, T_m, is,

$$T_m = 4\ \text{mA} + (0.10\ \text{mA}/^\circ\text{C})(90-40)\ ^\circ\text{C} = 9\ \text{mA}.$$

b.

The upper bound can be solved using linear interpolation,

$$P_{max} = 40\ \text{psi} + \frac{(20-4)\ \text{mA}}{0.20\ \text{mA/psi}} = 120\ \text{psi}.$$

By linear interpolation, the output, P_m, is,

$$P_m = 4\ \text{mA} + (0.20\ \text{mA/psi})(105-40)\ \text{psi} = 17\ \text{mA}.$$

c.

The gain is,

$$K = \frac{(20-4)\ \text{mA}}{(80-0)\ \text{gpm}} = 0.20\ \text{mA/gpm}.$$

By linear interpolation, the input, q, is,

$$q = 0\ \text{gpm} + \frac{(10-4)\ \text{mA}}{0.20\ \text{mA/gpm}} = 30\ \text{gpm}.$$

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2)

Given: $q_{min} = 100$ gpm at $f(\ell) = 0.1$, $q_{max} = 400$ gpm at $f(\ell) = 1$, $\Delta P_{tot} = 35$ psi, $g_s = 0.80$, $\Delta P_f = kq^2$

Using the valve sizing equation,

$$\Delta P_v = \frac{1}{C_v^2}\frac{g_s q^2}{f(\ell)^2}.$$

The total pressure drop is,

$$\Delta P_{tot} = \Delta P_v + \Delta P_f = \frac{1}{C_v^2}\frac{g_s q^2}{f(\ell)^2} + kq^2$$

At the minimum flow condition, $f(\ell) = 0.1$,

$$35 \text{ psi} = \left[\frac{1}{C_v^2}\right]\left[\frac{(0.80)(100^2 \text{ gpm}^2)}{0.1^2 \text{ gpm}^2 \cdot \text{psi}^{-1}}\right] + k\left[100^2 \text{ gpm}^2\right].$$

At the maximum flow condition, $f(\ell) = 1$,

$$35 \text{ psi} = \left[\frac{1}{C_v^2}\right]\left[\frac{(0.80)(400^2 \text{ gpm}^2)}{1^2 \text{ gpm}^2 \cdot \text{psi}^{-1}}\right] + k\left[400^2 \text{ gpm}^2\right].$$

Solving the 2×2 linear system gives,

$$\frac{1}{C_v^2} = 4.143 \times 10^{-5}, \quad k = 1.856 \times 10^{-4} \text{ psi/gpm}^2,$$

$$\implies C_v = \frac{1}{\sqrt{4.143 \times 10^{-5}}}.$$

Solving for the valve coefficient gives, $\boxed{C_v = 155.4}$.

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3)

Given: $q_{des} = 200$ gpm with $\Delta P_f = 35$ psi, $h = 150$ ft, $g_s = 0.75$, $\Delta P_f = kq^2$, $\Delta P_{tot} = 135$ psi

a.

The max flow rate is $q = 1.5q_{des} = 300$ gpm at which $f(\ell) = 1$. Thus,

$$\frac{\Delta P_f}{\Delta P_{f,des}} = \frac{kq^2}{kq_{des}^2} \implies \Delta P_f = (35 \text{ psi})\left(\frac{300}{200}\right)^2 = 78.75 \text{ psi}.$$

The total pressure drop is given by,

$$\Delta P_{tot} = \Delta P_v + \Delta P_f + (\rho g)g_s h,$$

Solving for ΔP_v,

$$\Delta P_v = (135 - 78.75) \text{ psi} - (0.75)\left(62.4 \frac{\text{lb}_\text{f}}{\text{ft}^3}\right)\left(\frac{1 \text{ ft}^2}{144 \text{ in}^2}\right)(150 \text{ ft}) = 7.5 \text{ psi}.$$

Using this pressure drop in the valve design equation,

$$C_v = \frac{q}{f(\ell)}\sqrt{\frac{g_s}{\Delta P_v}} = \frac{300 \text{ gpm}}{1 \text{ gpm} \cdot \text{psi}^{-1/2}}\sqrt{\frac{0.75}{7.5 \text{ psi}}} = 94.87.$$

Thus, $\boxed{C_v = 94.87}$.

b.

Solving for $f(\ell)$ in the valve design equation,

$$f(\ell) = \frac{q}{C_v}\sqrt{\frac{g_s}{\Delta P_f}} = \frac{200 \text{ gpm}}{94.87 \text{ gpm} \cdot \text{psi}^{-1/2}}\sqrt{\frac{0.75}{35 \text{ psi}}} = 0.309.$$

Thus, the fraction flow area is $\boxed{f(\ell) = 0.309}$.