1)

a.

The change in the input is specified as $\Delta u = 20$ gpm. The change from the initial value to the new steady state is,

$$\Delta y = (182 - 190)\ ^\circ\text{F} = -8\ ^\circ\text{F}.$$

Therefore, the gain is,

$$\boxed{K = \frac{\Delta y}{\Delta u} = \frac{-8\ ^\circ\text{F}}{20\ \text{gpm}} = -0.4\ ^\circ\text{F/gpm}}.$$

For the Smith method, calculate 63.2% of Δy and 28.3% of Δy,

$$y_{63.2} = [190 - (0.632)(8)]\ ^\circ\text{F} = 184.94\ ^\circ\text{F}.$$

$$y_{28.3} = [190 - (0.283)(8)]\ ^\circ\text{F} = 187.74\ ^\circ\text{F}.$$

Use linear interpolation to find $t_{63.2}$ and $t_{28.3}$,

$$t_{63.2} = \left[6 + (184 - 184.94)\,\frac{(8-6)}{(184.0 - 185.2)}\right] \text{min} = 6.427\ \text{min}.$$

$$t_{28.3} = \left[2 + (186.8 - 187.74)\,\frac{(4-2)}{(186.8 - 188.8)}\right] \text{min} = 3.064\ \text{min}.$$

Therefore,

$$\boxed{\tau = 1.5\,(6.427 - 3.064)\ \text{min} = 5.044\ \text{min}},$$

$$\boxed{\theta = (6.427 - 5.044)\ \text{min} = 1.383\ \text{min}}.$$

This is gives the final FOPDT model,

$$\boxed{\text{Smith} : G(s) = \frac{-0.4e^{1.383s}}{5.044s + 1}}.$$

For the S&K method, the gain is the same. Calculate 85.3% of Δy and 35.3% of Δy,

$$y_{85.3} = [190 - (0.853)(8)]\ ^\circ\text{F} = 183.18\ ^\circ\text{F}.$$

$$y_{35.3} = [190 - (0.353)(8)]\ ^\circ\text{F} = 187.18\ ^\circ\text{F}.$$

Use linear interpolation to find $t_{85.3}$ and $t_{35.3}$,

$$t_{85.3} = \left[10 + (183.18 - 183.2)\,\frac{(12-10)}{(182.8 - 183.2)}\right] \text{min} = 10.12\ \text{min}.$$

$$t_{35.3} = \left[2 + (186.8 - 187.18)\,\frac{(4-2)}{(186.8 - 188.8)}\right] \text{min} = 3.624\ \text{min}.$$

Therefore,

$$\boxed{\tau = 0.67\,(10.12 - 3.624)\text{ min} = 4.352\text{min}},$$

$$\boxed{\theta = [(1.3)(3.624) - (0.29)(10.12)]\text{ min} = 1.776\text{ min}}.$$

This is gives the final FOPDT model,

$$\boxed{\text{S\&K}: G(s) = \frac{-0.4e^{1.776s}}{4.352s+1}}.$$

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b.

The FOPDT models in the time domain are given by,

$$T(t) = T_0 + K\Delta u\left(1 - e^{-(t-\theta)/\tau}\right)S(t-\theta),$$

where $S(t)$ is the step function. Plotting the results in MATLAB gives the following figure.

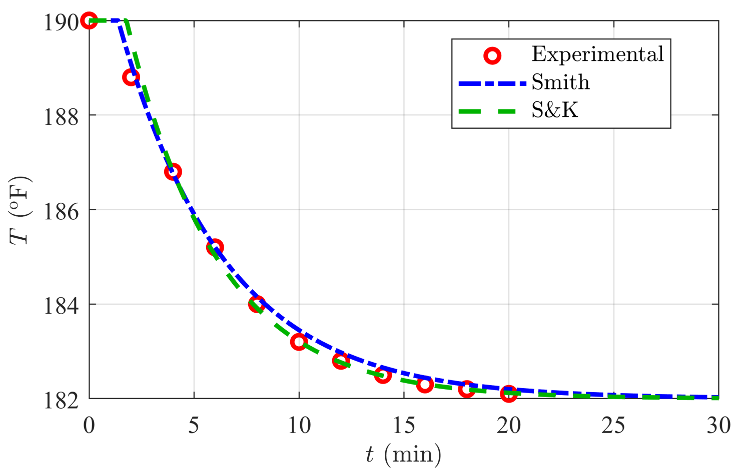

Figure 1: Comparison of FOPDT models for shell and tube heat exchanger data for a 20 gpm step input

Both models are visually very similar, but Smith model seems to be better at earlier times while the S&K model seems to fit better at later times. The early error of the S&K model is likely due to the larger dead time. Calculating the sum of squared errors gives the following table.

Method	SSE
Smith	0.259
S&K	0.677

This implies that for the given data, the Smith method has the smaller sum of squared errors and is thus the better model.