1)

Given: $\tau = 24$ s, $M = 55$ psi, $K = 1$

Due to the sudden change in pressure, P' is modeled by a step function, $P' = 55/s$. This gives,

$$P'_m = \frac{55}{s(24s+1)}.$$

Taking the inverse transform,

$$P'_m = 55\left(1 - e^{-t/24}\right).$$

To find the alarm time, set $P'_m = (95 - 60)$ psi $= 35$ psi, and solve for t,

$$t = -(24 \text{ s}) \ln\left(1 - \frac{P'_m}{55}\right) = 24.28 \text{ s}.$$

Adding this to the initial time gives $\boxed{02:05:24 \text{ PM}}$. That is, the alarms sounds after about 1 τ, which makes sense since $35/55 = 0.636 \approx 0.632$.

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2)

Given: $\overline{C_F} = 2$ mol/L, C'_F is a 3 mol/L pulse for 2 time units

$$\frac{C'}{C'_F} = \frac{5}{3s+1}, \quad C'_F = \frac{3}{s}\left(1 - e^{-2s}\right)$$

a.

Solve for $C'(s) = G(s)C'_F(s)$,

$$C'(s) = \frac{15}{s(3s+1)}\left(1 - e^{-2s}\right) = \frac{15}{s(3s+1)} - \frac{15e^{-2s}}{s(3s+1)}.$$

The first term is just a first order response while the second term is a first order response with a 2 unit time delay and a negative sign. Therefore, taking the inverse transform,

$$\boxed{c'(t) = 15\left(1 - e^{-t/3}\right) - 15\left(1 - e^{-(t-2)/3}\right) S(t-2)}.$$

b.

The max value must occur at the end of the pulse when $c'(t)$ is the most 'charged', meaning, $\boxed{t_{max} = 2 \text{ units}}$. To find the max concentration, set $t = t_{max}$,

$$c'(t) = 15\left(1 - e^{-2/3}\right) - 15\left(1 - e^{-(2-2)/3}\right)(0) = 7.299.$$

Therefore, $\boxed{c'_{max} = 7.299 \text{ mol/L}}$. The final value is given by taking $t \to \infty$,

$$c'(\infty) = 15(1-0) - 15(1-0) = 0.$$

Therefore, the final value is $\boxed{c'_\infty = 0 \text{ mol/L}}$.

c.

By the definition of deviation variables,

$$c(t) = c(0) + c'(t) = 1 + 15\left(1 - e^{-t/3}\right) - 15\left(1 - e^{-(t-2)/3}\right) S(t-2).$$

Set $c(t) = 2.10$, and since $t > 2$, $S(t-2) = 1$,

$$1.10 = 15\left(1 - e^{-t/3}\right) - 15\left(1 - e^{-(t-2)/3}\right).$$

Algebra...

$$\frac{1.10}{15} = \left(e^{2/3} - 1\right) e^{-t/3},$$

$$\implies t = -3\ln\left(\frac{1.10}{15(e^{2/3}-1)}\right) = 7.677,$$

By subtracting t_{max}, it takes $\boxed{t = 5.677 \text{ units}}$ after t_{max} to return to $c = 2.10$ mol/L.

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3)

Given: $\tau = 1.2$ min, $K = 0.5$ °C/kW, $\zeta = 0.3$, $T(0) = 70$ °C, $U(0) = 20$ kW

$$\frac{T'(s)}{U'(s)} = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1} = \frac{5}{1.44s^2 + 0.72s + 1}$$

a.

$0 < \zeta < 1 \implies$ underdamped, oscillations. $U : 20 \to 26$ kW, meaning $U'(s)$ is a step change with magnitude 6. Solving for $T'(s)$,

$$T'(s) = \frac{6K}{s(\tau^2 s^2 + 2\zeta\tau s + 1)} = \frac{30}{s(1.44s^2 + 0.72s + 1)}.$$

This is of the form of eqn. 22 from the Laplace transform table, giving a solution of,

$$T'(t) = 6K\left[1 - e^{-\zeta t/\tau}\left(\cos\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right) + \frac{\zeta}{\sqrt{1-\zeta^2}}\sin\left(\frac{\sqrt{1-\zeta^2}}{\tau}t\right)\right)\right].$$

Numerically,

$$\boxed{T(t) = 70 + 30\left[1 - e^{-0.25t}\left[\cos(0.7949t) + 0.3145\sin(0.7949t)\right]\right]}.$$

b.

The max temperature should occur at the first peak of the cosine term, therefore, set,

$$\frac{\sqrt{1-\zeta^2}}{\tau}t_p = \pi \implies t_p = \pi\frac{\tau}{\sqrt{1-\zeta^2}}.$$

Numerically,

$$t_p = \frac{\pi}{0.7949} = 3.952.$$

Therefore, $\boxed{t_p = 3.952 \text{ min}}$. To find the max value, set $t = t_p$ in the equation for $T(t)$,

$$T_{max} = 70 + 30\left[1 - e^{-0.25(3.952)}\left[(1) + 0.3145(0)\right]\right] = 111.$$

Therefore, $\boxed{T_{max} = 111\ {}^{\circ}\text{C}}$.