1)

a.

$$G(s) = \frac{1}{s^2+s+1} = \frac{1}{(s-p_1)(s-p_2)}$$

By the quadratic formula,

$$p_1, p_2 = -\frac{1}{2} \pm \frac{\sqrt{1-4}}{2} = -\frac{1}{2} \pm i\frac{\sqrt{3}}{2}.$$

- $Re(p_1, p_2) < 0 \implies$ Response is stable
- $Im(p_1, p_2) \neq 0 \implies$ Oscillatory response
- No zeros $\implies$ No inverse response

b.

$$G(s) = \frac{1}{s^2-s+1} = \frac{1}{(s-p_1)(s-p_2)}$$

By the quadratic formula,

$$p_1, p_2 = \frac{1}{2} \pm \frac{\sqrt{1-4}}{2} = \frac{1}{2} \pm i\frac{\sqrt{3}}{2}.$$

- $Re(p_1, p_2) > 0 \implies$ Response is unstable
- $Im(p_1, p_2) \neq 0 \implies$ Oscillatory response
- No zeros $\implies$ No inverse response

c.

$$G(s) = \frac{8s+1}{s^2+5s+4} = \frac{K(s-z_1)}{(s-p_1)(s-p_2)} = \frac{8(s+1/8)}{(s+1)(s+4)}$$

- $Re(p_1, p_2) < 0 \implies$ Response is stable
- $Im(p_1, p_2) = 0 \implies$ No oscillations
- Negative zero $\implies$ No inverse response

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2)

$$G(s) = \frac{2e^{-3s}}{(12s+1)(7s+1)(3s+1)(s+1)}$$

a.

For a first order approximation, the largest time constant is 12 which will be kept. For the Taylor approximation, lump the rest of the time constants into an effective time delay,

$$\theta = 3 + 7 + 3 + 1 = 14.$$

This gives,

$$\boxed{\text{FOPDT Taylor} : G(s) = \frac{2e^{-14s}}{12s+1}}.$$

For Skogestad,

$$\tau = 12 + 7/2 = 15.5, \quad \theta = 3 + 7/2 + 3 + 1 = 10.5.$$

This gives,

$$\boxed{\text{FOPDT Skogestad} : G(s) = \frac{2e^{-10.5s}}{15.5s+1}}.$$

b.

For a second order approximation, the two largest time constant are 12 and 7. For the Taylor approximation, lump the rest of the time constants into an effective time delay,

$$\theta = 3 + 3 + 1 = 7.$$

This gives,

$$\boxed{\text{SOPDT Taylor} : G(s) = \frac{2e^{-7s}}{(12s+1)(7s+1)}}.$$

For Skogestad,

$$\tau_2 = 7 + 3/2 = 8.5, \quad \theta = 3 + 3/2 + 1 = 5.5.$$

This gives,

$$\boxed{\text{SOPDT Skogestad} : G(s) = \frac{2e^{-5.5s}}{(12s+1)(8.5s+1)}}.$$