

1)

$$3 \frac{dy}{dt} + y = 6, \quad y(0) = 1$$

Taking the Laplace transform of both sides,

$$\begin{aligned} 3[sY(s) - y(0)] + Y(s) &= \frac{6}{s}, \\ \implies Y(s)[3s + 1] &= \frac{6}{s} + 3, \\ \implies Y(s) &= 6 \left[\frac{1}{s(3s + 1)} \right] + 3 \left[\frac{1}{3s + 1} \right]. \end{aligned}$$

For the inverse of first factor, use eq. 13 with $\tau = 3$. For the second factor, use eq. 6 with $\tau = 3$, giving,

$$y(t) = 6 \left[1 - e^{-t/3} \right] + 3 \left[\frac{1}{3} e^{-t/3} \right].$$

Simplifying,

$$\boxed{y(t) = 6 - 5e^{-t/3}}.$$

2)

$$\frac{d^2y}{dt^2} + 6 \frac{dy}{dt} + 8y = 2, \quad y(0) = y'(0) = 0$$

Taking the Laplace transform of both sides,

$$\begin{aligned} s^2[Y(s) + 0 + 0] + 6[sY(s) + 0] + 8Y(s) &= \frac{2}{s}, \\ \implies Y(s)[s^2 + 6s + 8] &= \frac{2}{s}, \\ \implies Y(s) &= \frac{2}{s(s^2 + 6s + 8)} = \frac{2}{s(s+2)(s+4)}. \end{aligned}$$

By partial fraction expansion,

$$\frac{2}{s(s+2)(s+4)} = \frac{a_1}{s} + \frac{a_2}{s+2} + \frac{a_3}{s+4}.$$

Multiplying both sides by $s(s+2)(s+4)$,

$$2 = a_1(s+2)(s+4) + a_2s(s+4) + a_3s(s+2).$$

For $s = 0$,

$$2 = a_1(2)(4) + 0 + 0 \implies a_1 = 1/4.$$

For $s = -2$,

$$2 = 0 + a_2(-2)(2) + 0 \implies a_2 = -1/2.$$

For $s = -4$,

$$2 = 0 + 0 + a_3(-4)(-2) \implies a_3 = 1/4.$$

Therefore,

$$Y(s) = \frac{1}{4} \left[\frac{1}{s} \right] - \frac{1}{2} \left[\frac{1}{s+2} \right] + \frac{1}{4} \left[\frac{1}{s+4} \right].$$

All of the factors may be inverted using eq. 5, giving,

$$y(t) = \frac{1}{4} - \frac{1}{2}e^{-2t} + \frac{1}{4}e^{-4t}.$$

3)

$$\frac{d^2y}{dt^2} + 4\frac{dy}{dt} + 4y = 2e^{-t}, \quad y(0) = y'(0) = 0$$

Taking the Laplace transform of both sides,

$$\begin{aligned} s^2 [Y(s) + 0 + 0] + 4[sY(s) + 0] + 4Y(s) &= \frac{2}{s+1}, \\ \implies Y(s)[s^2 + 4s + 4] &= \frac{2}{s+1}, \\ \implies Y(s) &= \frac{2}{(s+1)(s^2 + 4s + 4)} = \frac{2}{(s+1)(s+2)^2}. \end{aligned}$$

By partial fraction expansion,

$$\frac{2}{(s+1)(s+2)^2} = \frac{a_1}{s+1} + \frac{a_2}{s+2} + \frac{a_3}{(s+2)^2}.$$

Multiplying both sides by $(s+1)(s+2)^2$,

$$2 = a_1(s+2)^2 + a_2(s+1)(s+2) + a_3(s+1).$$

For $s = -1$,

$$2 = a_1(1)^2 + 0 + 0 \implies a_1 = 2.$$

For $s = -2$,

$$2 = 0 + 0 + a_3(-1) \implies a_3 = -2.$$

For $s = 0$,

$$\begin{aligned} 2 = a_1(2)^2 + a_2(1)(2) + a_3(1) &\implies 2 = (2)(4) + 2a_2 - 2, \\ \implies 2a_2 &= -4 \implies a_2 = -2. \end{aligned}$$

Therefore,

$$Y(s) = 2 \left[\frac{1}{s+1} \right] - 2 \left[\frac{1}{s+2} \right] - 2 \left[\frac{1}{(s+2)^2} \right].$$

The first 2 factors can be inverted using eq. 5 while the repeated factor uses eq. 7, giving,

$$y(t) = 2e^{-t} - 2e^{-2t} - 2te^{-2t}.$$