1

A real number as unfinished process. A function from $\mathbb{N}$ to $\mathbb{Q} \times \mathbb{Q}$. Intervals with rational endpoints.

But they must shrink toward 0 width. One of many rules that would work is:

$|f(n+1)| \leq \frac{1}{2}|f(n)|$.

For instance, $f(n) = [0, 2^{-n}]$ would be a zero-like rule-specified real number.

But we also want to allow choice sequences. Let's assume only that $f(1) = [1, 2]$. Then $f(2) \subset f(1)$ and $|f(2)| \leq \frac{1}{2}$. This allows for an infinite number of choices of $f(2)$. This choice sequence f does not have to exist all at once. It is a becoming rather than a being.

2

Now we consider real-valued functions of real-valued inputs. If the input is a becoming, so is the output. Let's switch to x_n as notation for a real numbers, reserving f for functions.

We need to constrain the definition of such functions. We need to demand that $n < N \implies x_n = y_n$ insures that $n < N \implies f_n(x_n) = f_n(y_n)$.

So a function always treats equal initial segments equally. The function can't "see ahead."

3

We can focus on the time issue by forgetting real numbers and thinking instead of choice sequences of bits. For instance, $x = 0100101...$, where the ellipses indicate that x has only developed this far. If $y = 0100101...$, we cannot say that $x = y$. But for any f, we can say that $f(0100101...)$ has a unique value, which would itself be an unfinished sequence. For instance, f may just invert bits.

We can also consider rule-based sequences like $z = \overline{01} = 0101010101....$ This function is "fully defined." But this simple, complete definition encourages us to think of its rule-producible bits as already here. We might consider a more complicated example like the infinite decimal expression of $\sqrt{2}$. In this more complicated case, the effort required to produce the bits is greater.

We might also ask whether a Turing machine will halt. Some will say that it definitely halts or doesn't halt. Although there is no upper bound on how long we'll have to wait, the fact of the matter is "out there." The Turing machine executes a rule. It is or is not the case that this rule leads to one of two outcomes.

This is where the deflation of truth can help us. If we don't reify belief, we can question whether this unknown future state is nevertheless genuinely present.

4

Returning to our original function approach to real numbers, we might loosen the constraint, requiring only that

$f(n+1) \subset f(n)$ and $|f(n)| \to 0$.

This is computationally less convenient, but it avoids unmotivated choices like halving the length at each click forward. It even further emphasizes the work-in-progress essence of choice sequences, for now we can't even predict when we'll get enough precision for a practical application. Of course this wide definition allows still for rule-based sequences and for intermediate sequences like those with a predictable increase in precision.

It's also clear that $|f(n)g(n)| \to 0$ with the usual interval arithmetic multiplication. Is it clear that $f(n+1)g(n+1) \subset f(n)g(n)$?