1 Markov Chains

1.1 Regular Markov Chains

Definition: A transition probability matrix, P, is *regular* if some power of P has only positive entries. A Markov chain is a regular Markov chain if its transition matrix is regular. For example,

$$P = \begin{array}{c} \\ \\ \\ \end{array}\begin{array}{cc} \begin{array}{ccc} A & B & C \end{array} & \\ \begin{pmatrix} 0.1 & 0.3 & 0.6 \\ 0.4 & 0.4 & 0.2 \\ 0.0 & 0.8 & 0.2 \end{pmatrix} & \begin{array}{c} A \\ B \\ C \end{array} \end{array}$$

Properties of Regular MC:

- Has a unique stationary matrix, S, which satisfies $SP = S$.
- Given, S_0, the initial state matrix, successive state matrices will approach the stationary matrix, S.
- P^k approaches a limiting matrix $\bar{P}$, where each row of $\bar{P}$ is equal to S.

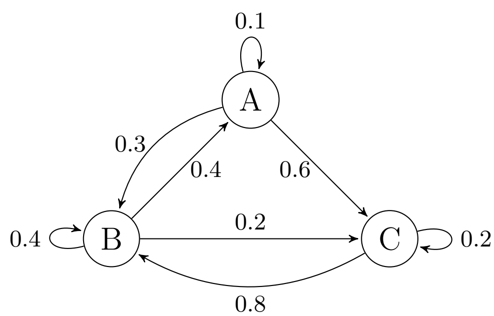

Figure 1: Transition diagram showing probabilities from a transition matrix.

1.2 Absorbing Markov Chains

Definition: A state in a Markov chain is called an *absorbing state*, if once the state is entered it's impossible to leave.

A Markov chain is an absorbing Markov chain if:

1. There's at least one absorbing state.
2. It's possible to go from each non-absorbing state to at least one absorbing state in finitely many steps.

1.2.1 Standard Form for an Absorbing Markov Chain

An absorbing Markov chain is in standard form if the rows and columns are labeled such that absorbing states precede all non-absorbing states.

A standard form can be partitioned into 4 submatrices.

$$P = \left[\begin{array}{c|c} I & O \\ \hline R & Q \end{array}\right] \qquad \begin{array}{l} I = \text{Identity Matrix} \\ O = \text{Zero Matrix} \end{array}$$

As k increases, P^k approaches a limiting matrix, $\bar{P}$.

$$\bar{P} = \left[\begin{array}{c|c} I & O \\ \hline FR & O \end{array}\right]$$

$F = (I - Q)^{-1}$
F is the *fundamental matrix* for P.