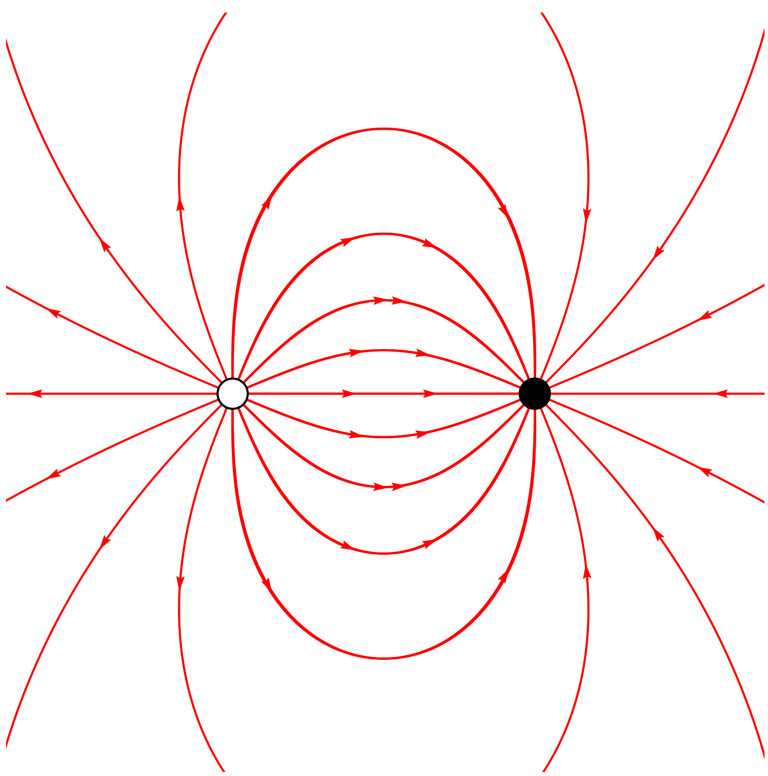
ELECTRICITY & MAGNETISM



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1 Coulomb's Law

Coulomb's law gives the electrostatic force between to charged particles.

$$\vec{F} = k\frac{q_1 q_1}{r^2}\hat{r}.$$

The unit vector $\hat{r}$ points along a radial axis extending through the two particles and $k = 8.99 \times 10^9$ N·m^2/C^2 is Coulomb's constant.

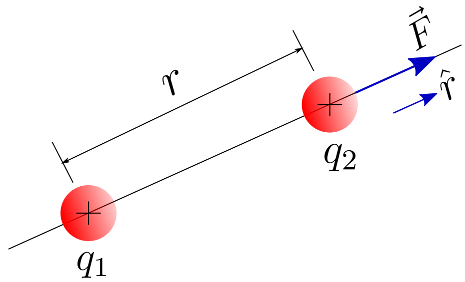

Figure 1: Two particles with charges q_1, q_2 separated by a distance r.

The magnitude of the electrostatic force is given by

$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1||q_2|}{r^2},$$

where ϵ_0 is the permittivity constant and is related to Coulomb's constant by $k = \frac{1}{4\pi\epsilon_0}$.

1.1 Shell Theories

(i) A charged particle outside a shell with charge uniformly distributed on its surface is attracted or repelled as if the shell's charge were concentrated as a particle at its center.

(ii) A charged particle inside a shell with charge uniformly distributed on its surface has no net force acting on it due to the shell.

If excess charge is placed on a spherical shell that is made of conducting material, the excess charge spreads uniformly over the (external) surface.

1.2 Conservation of Charge

Electric charge is quantized; e is the elementary charge (1.602×10^{-19} C) which is the magnitude of the charge on an electron or proton. The net electric charge of any isolated system is always conserved.

2 Electric Field

A charged particle sets up an electric field in the surrounding space. If a second charged particle is located in that space, an electrostatic force acts on it due to the magnitude and direction of the field at its location. Electric field lines originate on positive charges and terminate on negative charges. The electric field vector at any point is tangent to the field line through that point. The density of field lines in that region is proportional to the magnitude of the electric field there.

The electric field $\vec{E}$ at any point is defined in terms of the electrostatic force $\vec{F}$ that would be exerted on a positive test charge q_0 placed there:

$$\vec{E} = \frac{\vec{F}}{q_0}.$$

The magnitude of the electric field $\vec{E}$ set up by a particle with charge q at a distance r from the particle is

$$E = \frac{1}{4\pi\epsilon_0}\frac{|q|}{r^2}.$$

The electric field vectors set up by a positively charged particle all point directly away from the particle. Those set up by a negatively charged particle all point directly toward the particle. If more than one charged particle sets up an electric field at a point, the net electric field is the vector sum of the individual electric fields — electric fields obey the superposition principle.

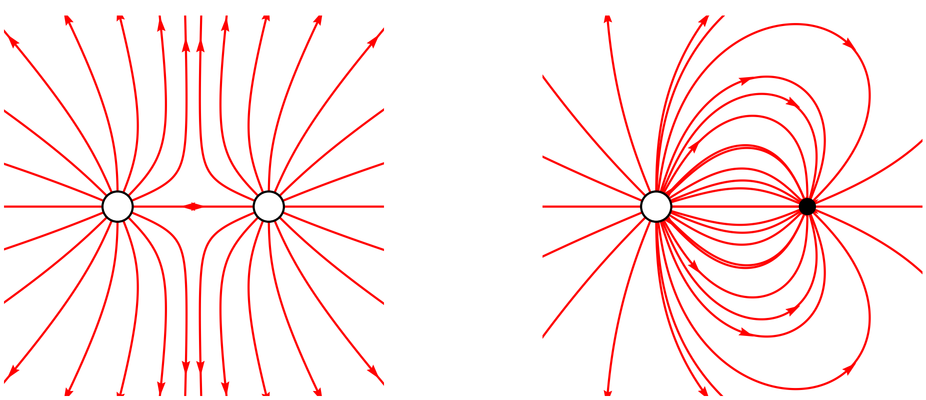

Figure 2: (Left) Two +2 charges. (Right) One +2 and one −1 charge.

2.1 Electric Dipole

An electric dipole is an arrangement of two particles with the same charge magnitude but opposite sign. Suppose the particles are separated by a distance

d, and we want to find the magnitude and direction of the electric field at an arbitrary point P along the dipole axis, a distance z from the dipole center.

$$\begin{aligned}
E &= E_{(+)} - E_{(-)} \\
&= \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2_{(+)}} - \frac{q}{r^2_{(-)}}\right) \\
&= \frac{q}{4\pi\epsilon_0}\left(\frac{1}{(z-\frac{1}{2}d)^2} - \frac{1}{(z+\frac{1}{2}d)^2}\right) \\
&= \frac{q}{2\pi\epsilon_0 z^3}\left(\frac{d}{1-\left(\frac{d}{2z}\right)^2}\right)
\end{aligned}$$

For sufficiently large z, $\frac{d}{2z} \ll 1$, and we approximate the electric field via

$$E = \frac{1}{2\pi\epsilon_0}\frac{p}{z^3},$$

where p is the magnitude of the electric dipole moment $\vec{p} = qd$. The torque on an electric dipole is given by $\vec{\tau} = \vec{p} \times \vec{E}$ and the potential energy is $U = -\vec{p} \cdot \vec{E}$.

2.2 Electric Field from Collections of Charges

2.2.1 Thin Ring

The charge dq of an arc length element ds is $dq = \lambda ds$. Thus, the magnitude of the electric field due to dq is

$$\begin{aligned}
dE &= \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2} \\
&= \frac{1}{4\pi\epsilon_0}\frac{\lambda ds}{z^2 + R^2}
\end{aligned}$$

Note however that all components of the electric field perpendicular to the central axis cancel; hence the electric field at P points directly away from the ring and we only need to sum components parallel to the central axis. Since $\cos\theta = \frac{z}{r}$, the electric field is given by

$$\begin{aligned}
E = \int dE\cos\theta &= \frac{z\lambda}{4\pi\epsilon_0(z^2+R^2)^{3/2}}\int_0^{2\pi R} ds \\
= \frac{z\lambda(2\pi R)}{4\pi\epsilon_0(z^2+R^2)^{3/2}} &= \frac{qz}{4\pi\epsilon_0(z^2+R^2)^{3/2}}
\end{aligned}$$

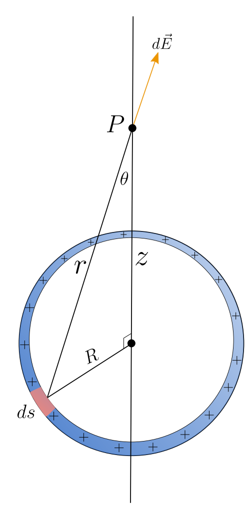

Figure 3: A thing ring of radius R and uniform charge density λ.

2.2.2 Disc

Using the formula for a thin ring, we can integrate over the disc via the radius.

$$dE = \frac{dqz}{4\pi\epsilon_0(z^2+r^2)^{3/2}},$$

but $dq = \sigma(2\pi r)dr$ where σ is the surface charge density. Integrating from 0 to R we have

$$E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right).$$

Letting $R \to \infty$, we see that an infinite plate has $E = \frac{\sigma}{2\epsilon_0}$.

3 Gauss' Law

The electric flux is the amount of electric field which pierces a given patch on a Gaussian surface. Thus for a flat surface with uniform electric field

$$\Delta\Phi = (E\cos\theta)\Delta A$$

since only the x component actually pierces the surface, the y component only skims. The total flux of a surface is then

$$\Phi = \int \vec{E} \cdot d\vec{A}.$$

An inward piercing field is negative flux, an outward piercing field is positive flux, and a skimming field is zero flux. The net flux through a closed surface is

$$\Phi = \oint \vec{E} \cdot d\vec{A}.$$

Gauss' Law states that for a closed surface

$$\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}}.$$

> **Exercise.** Prove Coulomb's Law using Gauss' Law.

3.1 Charged Conductor

The electric field inside an isolated conductor with excess charge q is zero. Thus, all the excess charge is located entirely on the outer surface of the conductor (even if there's a cavity in the conductor). The magnitude of the electric field just outside a conductor is proportional to the surface charge density

$$E = \frac{\sigma}{\epsilon_0}.$$

3.2 Cylindrical Symmetry

Lets calculate the electric field at a distance r from the center of an infinitely long cylindrical plastic rod with uniform charge density λ. The charge distribution has cylindrical symmetry, that is a Gaussian cylinder of radius r and height h has the same electric field about its surface. Note that all electric field lines will point radially outward, so there's zero flux through the end caps. The net flux through the curved surface is $\Phi = EA\cos\theta = E(2\pi rh)$. We also know that $q_{\text{enc}} = \lambda h$. Thus

$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

3.3 Planar Symmetry

Consider a thin, infinite, nonconducting sheet with a uniform (positive) surface charge density σ. Lets model $\vec{E}$ a distance r in front of the sheet. A useful Gaussian surface is a closed cylinder with end caps of area A perpendicular to the sheet. The net flux points outward perpendicular to the sheet, thus the only net flux through the cylinder goes through the end caps. Thus, $\Phi = \epsilon_0(EA + EA) = q_{\text{enc}} = \sigma A$, or

$$E = \frac{\sigma}{2\epsilon_0}$$

3.4 Spherical Symmetry

Consider a uniform spherical shell with total charge q and radius R. Let S_1 be a concentric spherical Gaussian surface with radius $r < R$. Applying Gauss' Law to S_1, which encloses no charge, we see that $E = 0$ within the shell, proving part (ii) of the shell theorem.

Finally, consider the electric field inside a uniform sphere of charge of radius R with charge q and let S_2 be a concentric spherical Gaussian surface with radius $r \geq R$. Then the electric field is directed radially and has magnitude

$$E = \left(\frac{q}{4\pi\epsilon_0 R^3}\right) r.$$

4 Electric Potential

Electric potential is defined to be the amount of electric potential energy per unit charge when a positive test charge is brought in from infinity, i.e.

$$V = \frac{-W_\infty}{q_0} = \frac{U}{q_0}.$$

If a charged particle moves through an electric field with no force acting on it other than the electric force due to the field, then the mechanical energy is conserved. So

$$\Delta K = -\Delta U = -q\Delta V.$$

If an external force is applied then

$$\Delta K = -\Delta U + W_{\text{app}} = -q\Delta V + W_{\text{app}}$$

4.1 Equipotential Surfaces

An equipotential surface is the set of all points that have the same electric potential. No work is done by moving a charged particle along an equipotential surface. The change in potential between two points i and f in an electric field is given by

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

and is independent of path. Further, the electric field is directed perpendicular to corresponding equipotential surfaces.

4.2 Potential from a Charged Particle

Consider a point P a distance R from a fixed particle with charge $+q$. Recall that the electric field extends radially from the fixed particle, thus

$$V_f - V_i = -\int_R^\infty \vec{E} \cdot d\vec{s} = -\int_R^\infty E\ dr.$$

Since $V_f = 0$ at ∞, if we set $V = V_i$ then

$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{R}$$

since $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$. Note that for multiple charged particles, the potentials sum linearly.

4.3 Potential of a Charged Isolated Conductor

When an excess charge is place on an isolated conductor, it will distribute itself on the surface of that conductor so that all points on the conductor come to the same potential, regardless of whether the conductor contains an internal cavity.

This is evident because $\vec{E} = 0$ for all points within a conductor, so since $V_f - V_i = \int \vec{E} \cdot d\vec{s}$, it follows that $V_f = V_i$ for any points i, f in the conductor.

4.4 Potential from a Dipole

Consider the setup in Figure 4. The net potential at P is

$$V = V_{(+)} + V_{(-)} = k\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right).$$

Or equivalently,

$$V = kq\left(\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}\right).$$

However, for sufficiently large r we can approximate the lines to P as parallel. Thus, $r_{(-)} - r_{(+)} \approx d\cos\theta$ and $r_{(-)}r_{(+)} \approx r^2$. Therefore,

$$V = \frac{q}{4\pi\epsilon_0}\frac{d\cos\theta}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2}.$$

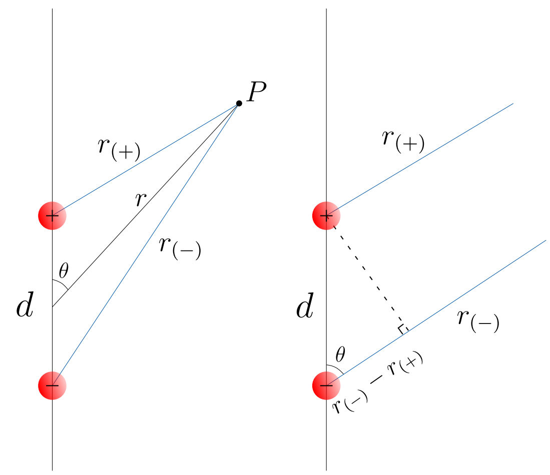

Figure 4: Potential from an electric dipole.

4.5 Potential from a Continuous Charge Distribution

The potential from a continuous charge distribution (e.g. uniformly charged disk) can be determined by considering a differential element of charge dq and integrating over the entire charge distributions, i.e.

$$V = \frac{1}{4\pi\epsilon_0}\int\frac{dq}{r}.$$

Example. A thin nonconducting rod of length L has positive charge of uniform linear density λ. Determine the potential due to the rod at a perpendicular distance d from the rod.

A differential length of the rod dx has charge $dq = \lambda\ dx$ and is a distance $r = (x^2 + d^2)^{1/2}$ from the point in question. Thus, $dV = \frac{1}{4\pi\epsilon_0}\frac{\lambda\ dx}{(x^2+d^2)^{1/2}}$. Integrating over the length of the rod we have

$$V = \frac{\lambda}{4\pi\epsilon_0}\ln\left(\frac{L^2 + (L^2 + d^2)^{1/2}}{d}\right).$$

Exercise. Derive an expression for $V(z)$ the potential at any point along the central axis of a positively charged disk with uniform charge density σ and radius R. *Hint: Consider the potential from a thin ring, then integrate.*

4.6 Electric Field from Potential

Consider a test charge moving through an electric field as in Figure 5. As it moves a distance $d\vec{s}$ between equipotential surfaces, it experiences a potential difference of dV. The work done on the charge is $W = -q_0\ dV = q_o E \cos\theta\ ds$, where θ is the angle between the electric field and $d\vec{s}$.

Thus, $E \cos\theta = -\frac{dV}{ds}$, but since $E\cos\theta$ is simply the component of $\vec{E}$ in the direction of $d\vec{s}$, we have

$$E_s = -\frac{\partial V}{\partial s}.$$

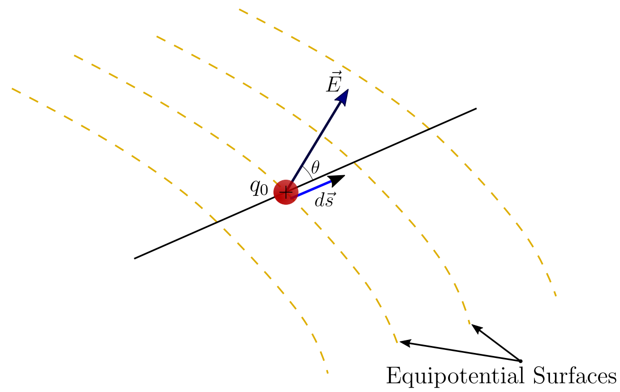

Figure 5: A charge q_0 moves a distance $d\vec{s}$ between two equipotential surfaces.

4.7 Electric Potential Energy of a System of Charged Particles

Consider particles with positive charges q_1 and q_2 each initially infinitely far apart. The potential energy of the two-particle system when these particles are brought together is the negative of the work done to move charge q_2 towards the fixed charge q_1. Thus, $U_f - U_i = q_1(V_f - V_i)$. Since $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$, the potential energy of the system is

$$U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}.$$

The total potential energy of a system of particle is the sum of the potential energies for every pair of particles in the system.

5 Capacitance

The charge stored on a capacitor is given by

$$Q = CV$$

where V is the voltage drop across the capacitor and C is the capacitance, a constant related to the configuration of the capacitor. Capacitance is measured in farads.

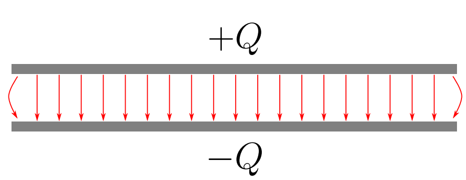

Figure 6: Parallel plate capacitor with charge Q.

5.1 Calculating Capacitance

> **Example.** Consider a parallel plate capacitor with area A and separation d. Assume the plates are large and close together such that we can neglect fringing of the electric field at the edges and $\vec{E}$ is constant.
>
> Using Gauss' Law, the charge on the plate is $q = \epsilon_0 EA$. The voltage between the plates is $V = \int_0^d E\, ds = Ed$. Thus, $C = \frac{\epsilon_0 A}{d}$.

> **Example.** Consider a cylindrical capacitor of length L formed by two coaxial cylinders of radii a and b with $a < b$. Neglect the fringing of the electric field that occurs at the ends of the cylinders. Each plate contains a charge of magnitude q. The charged enclosed by a Gaussian cylinder of radius r and length L is $q = \epsilon_0 E(2\pi rL)$. Using $E = \frac{q}{2\pi\epsilon_0 Lr}$, the voltage drop is $V = \frac{q}{2\pi\epsilon_0 L}\int_a^b \frac{dr}{r} = \frac{q}{2\pi\epsilon_0 L}\ln\left(\frac{b}{a}\right)$. Therefore, $C = 2\pi\epsilon_0 \frac{L}{\ln(b/a)}$.

> **Example.** Consider a capacitor composed of two concentric spherical shells with radii a and b. A Gaussian sphere of radius r encloses $q = \epsilon_0 E(4\pi r^2)$. Hence, $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$ and the voltage drop is $V = \frac{q}{4\pi\epsilon_0}\int_a^b \frac{dr}{r^2} = \frac{q}{4\pi\epsilon_0}\frac{b-a}{ab}$. Finally, $C = 4\pi\epsilon_0 \frac{ab}{b-a}$.

5.2 Capacitors in Parallel and in Series

Parallel capacitors and their equivalent have the same voltage drop V. Thus, capacitors connected in parallel can be replaced with an equivalent capacitor that have the same total charge q and the same potential difference. The equivalent capacitance is therefore $C_{\text{eq}} = \sum_{i=1}^n q_i V = \sum_{i=1}^n C_i$.

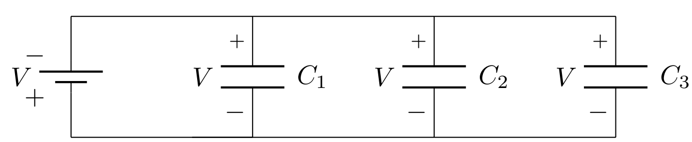

Figure 7: Circuit with capacitors in parallel.

For capacitors in series, when a voltage difference V is applied, the capacitors have identical charge q, and the sum of the potential differences across all the capacitors is equal to V. The total potential difference is $V = \sum_{i=1}^n V_i = q\left(\sum_{i=1}^n \frac{1}{C_i}\right)$. Thus, $C_{\text{eq}} = \frac{q}{V} = \frac{1}{\sum 1/C_i}$ or $\frac{1}{C_{\text{eq}}} = \sum_{i=1}^n \frac{1}{C_i}$.

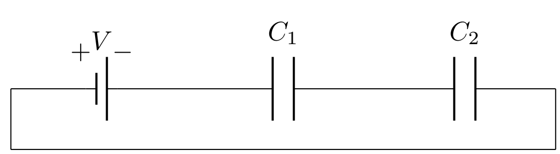

Figure 8: Circuit with capacitors in series.

5.3 Energy

The amount of work to bring a charge dq' across a parallel plate capacitor is $dW = V'\ dq' = \frac{q'}{C}dq'$, where q' charge has already been transfered. Thus the total work is $W = \frac{1}{C}\int_0^q q'dq' = \frac{q^2}{2C}$. Thus the energy stored in a parallel plate capacitor is $E = \frac{1}{2}QV = \frac{1}{2}CV^2$. The energy density is simply the potential energy per unit volume between the plates, hence $u = \frac{1}{2}\epsilon_0 E^2$.

5.4 Dielectrics

A dielectric is a non-conducting material placed between the plates of a capacitor. Dielectrics increase the capacitance by a numerical factor κ, the dielectric constant of a material. Thus, if C is the original capacitance of a capacitor with air between the plates, then $C' = \kappa C$. Gauss' Law generalizes to dielectrics via $\epsilon_0 \oint \kappa\vec{E}\cdot d\vec{A} = q$.

6 Current & Resistance

If a charge dq passes through a hypothetical plane in time dt then the current through the plane is defined to be

$$I = \frac{dq}{dt},$$

and is measured in amperes (A). Current is drawn in the direction which positive charge carriers would move.

Alternatively, we can use current density $\vec{J}$, such that $I = \int \vec{J}\cdot d\vec{A}$.

Drift speed v_d is the average velocity of electrons through a conductor due to an electric field. The total charge of carriers in a length L of conductor with cross sectional area A is $q = (nAL)e$, where n is the number of carrier/unit volume and e is the fundamental charge. The time to move a length L is $t = L/v_d$. Thus, $I = q/t = nAev_d$. Hence, $v_d = I/(nAe) = J/(ne)$.

6.1 Resistance & Resistivity

The resistance of a conductor is defined to be

$$R = \frac{V}{I},$$

and is measured in ohms (Ω) or volt per ampere.

The resistivity of a material, ρ is defined to be $\rho = \frac{E}{J}$, the ratio of the magnitude of the electric field to the magnitude of the current density. It's related to the conductivity of a material, σ by $\rho = \frac{1}{\sigma} = \frac{E}{J}$. The resistance of a conducting wire of length L and uniform cross section is $R = \rho\frac{L}{A}$. *Note: Resistance is a property of an object. Resistivity is a property of a material.*

Note that resistivity varies with temperature via

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0),$$

where ρ_0 is the reference resistivity at temperature T_0.

6.2 Ohm's Law

A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference, that is the relationship between V and I is linear. *Not all materials obey Ohm's law.*

6.3 Power

The power, or rate of energy transfer in an electrical device across which a potential difference V is maintained is $P = IV$. Power is measured in watts (W). For an object with resistance $P = I^2R = V^2/R$.

7 Circuits

Emf ($\mathscr{E}$) is just potential difference.

7.1 Circuit Analysis

- LOOP RULE: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit is zero.
- RESISTANCE RULE: For a move through a resistance in the direction of the current, the potential change is $-IR$; in the opposite direction is $+IR$.
- EMF RULE: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+V$; conversely it's $-V$.
- JUNCTION RULE: The sum of the current entering any junction must be equal to the sum of the currents leaving that junction.

For resistors in series, an equivalent resistor has resistance

$$R_{\text{eq}} = \sum_{i=1}^{n} R_i.$$

For resistors in parallel, an equivalent resistor has resistance

$$\frac{1}{R_{\text{eq}}} = \sum_{i=1}^{n} \frac{1}{R_i}.$$

An ammeter is a device used to measure current and a voltmeter is a device used to measure voltage.

7.2 RC Circuit

7.2.1 Charging

Applying the loop rule to Figure 9, we have

$$V - IR - \frac{q}{C} = 0,$$

where V is the battery's emf, I is the current, and q is the charge on the capacitor. Since $I = \frac{dq}{dt}$, we have

$$R\frac{dq}{dt} + \frac{1}{C}q = V.$$

Since $q(0) = 0$, we have

$$q(t) = CV(1 - e^{-t/RC}).$$

Therefore,

$$I(t) = \frac{dq}{dt} = \left(\frac{V}{R}\right) e^{-t/RC}.$$

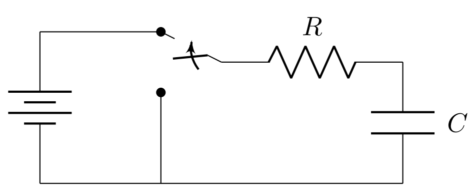

Figure 9: Resistor-Capacitor (RC) circuit.

7.2.2 Discharging

Similarly, for discharging the capacitor, we derive

$$q(t) = q_0 e^{-t/RC}$$

and

$$I(t) = -\frac{q_0}{RC} e^{-t/RC}.$$

8 Magnetic Fields

The magnetic force is

$$\vec{F_B} = q\vec{v} \times \vec{B}$$

where $\vec{v}$ is the velocity of a particle with charge q in a magnetic field $\vec{B}$. Magnetic field strength is measured in $\mathrm{N\,A^{-1}\,m^{-1}}$, or tesla (T). The force $\vec{F_B}$ acting on a charged particle is always perpendicular to $\vec{v}$ and $\vec{B}$.

8.1 Thomson Experiment & The Hall Effect

If a charged particle moves through a region containing both an electric field and a magnetic field, it can be affected by both an electric force and a magnetic force. Thomson's experiment used this fact and his experiment is considered to be the discovery of the electron.

Consider a metal strip of width d and length l, carrying current I. When a uniform magnetic field, $\vec{B}$, is applied perpendicular to the direction of current, a force $\vec{F}_B$ deflects the moving charge carriers to one side of the strip. The separation of charges creates and electric field, $\vec{E}$ and a potential difference, $V = Ed$, is established across the width of the strip. The charge carrier density can be calculated via $n = BI/nle$.

8.2 Magnetism & Circular Motion

Suppose a charged particle with charge magnitude $|q|$ and mass m moves through a magnetic field $\vec{B}$, perpendicular to its velocity $\vec{v}$. Then, applying Newton's 2nd law to the resultant circular motion, we have

$$|q|vB = \frac{mv^2}{r}.$$

Thus, the path has radius $r = \frac{mv^2}{|q|vB}$ and period $T = \frac{2\pi m}{|q|B}$. *Note. If the velocity has a component parallel to the (uniform) magnetic field, the particle will move in a helical path.*

8.2.1 Cyclotrons & Synchrotrons

In a cyclotron, charged particles are accelerated by electric forces as they circle in a magnetic field. A synchrotron is needed for particles accelerated to nearly the speed of light (due to relativity problems).

8.3 Magnetic Force on a Current-carrying Wire

A straight wire carrying a current I in a uniform magnetic field experiences a sideways force $\vec{F}_B = I\vec{L} \times \vec{B}$, where the direction of $\vec{L}$ is that of the current I. The force acting on a current element $Id\vec{L}$ in a magnetic field is $d\vec{F}_B = Id\vec{L} \times \vec{B}$.

The net torque acting on the coil has a magnitude given by $\tau = NIAB \sin u$, where N is the number of turns in the coil, A is the area of each turn, I is the current, B is the field magnitude, and u is the angle between the magnetic field and the normal vector, $\vec{n}$ to the coil. We assign a special name to the quantity, $\mu = NIA$, the magnetic dipole moment. The direction of $\vec{\mu}$ is that of the normal vector to the plane of the coil. Thus we can write $\vec{\tau} = \vec{\mu} \times \vec{B}$, $U(\theta) = -\vec{\mu} \cdot \vec{B}$ (potential energy of dipole), and $W_a = \Delta U$ (work to rotate dipole).

9 Magnetic Fields Due to Currents

9.1 Biot-Savart Law

The magnetic field at a point P a distance r away from a differential element $d\vec{s}$ of an arbitrary wire is given by

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i ds \vec{\times} \hat{r}}{r^2}$$

where i is the current in the wire, $\hat{r}$ points from ds towards P, and $\mu_0 = 1.26 \times 10^{-6}\ \mathrm{T\,m\,A^{-1}}$. We can calculate the magnetic field due to a long straight wire with current I at a radial distance R by integrating along the wire;

$$B = \frac{\mu_0 I}{2\pi R}.$$

The direction is given by the right-hand rule: point the thumb in the direction of the current and the curl the fingers (which represent the magnetic field) naturally around the wire. The magnetic field from at the center of a circular arc with central angle ϕ, current I, and radius R is

$$B = \frac{\mu_0 I \phi}{4\pi R}.$$

To find the force on a current-carrying wire due to a second current-carrying wire, first, find the field due to the second wire at the site of the first wire. Then, find the force on the first wire due to that field. For two wires of length L with currents i_a and i_b, separated by a distance d, we have

$$F_{ba} = \frac{\mu_0 L i_a i_b}{2\pi d}$$

Furthermore, we find that parallel currents attract and antiparallel currents repel.

9.2 Ampere's Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}.$$

Right-hand rule: Curl your right hand around the Amperian loop with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assumed positive. In the other direction it is negative.

9.3 Solenoids & Toroids

Let i be the current and n the number of loops per unit length of a solenoid (helically wrapped wire). Then, assuming the magnetic field outside the solenoid is significantly smaller than inside, $B = \mu_0 in$, Ampere's law

A toroid is a hollow solenoid, curved into the shape of a torus. If N is the total number of turns, i is the current, and r is the radius from the center of the toroid, then $B = \frac{\mu_0 iN}{2\pi}\frac{1}{r}$, by Ampere's law.

Selected Constants

- $\epsilon_0 = 8.85 \times 10^{-12}\ \mathrm{F\,m^{-1}}$
- $k = 8.99 \times 10^{9}\ \mathrm{N\,m^2\,C^{-2}}$
- $e = 1.602 \times 10^{-19}\ \mathrm{C}$
- $\mu_0 = 1.26 \times 10^{-6}\ \mathrm{T\,m\,A^{-1}}$

Selected Equations

Module 1.

- $\vec{F} = \frac{kq_1q_1}{r^2}\hat{r}$
- $\vec{E} = \frac{\vec{F}}{q_0}$

Module 2.

- $E = \frac{qz}{4\pi\epsilon_0(z^2+R^2)^{3/2}}$
- $E = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$
- $E = \frac{\sigma}{2\epsilon_0}$
- $\epsilon_0 \oint \vec{E}\cdot d\vec{A} = q_{\text{enc}}$

Module 3.

- $V = \frac{-W_\infty}{q_0}$
- $V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}$
- $V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$
- $V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$
- $E_s = -\frac{\partial V}{\partial s}$
- $U = W = \frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r}$

Module 4.

- $Q = CV$
- $C = \frac{\epsilon_0 A}{d}$
- $C_{\text{eq}} = \sum C_i$
- $\frac{1}{C_{\text{eq}}} = \sum \frac{1}{C_i}$
- $U = \frac{CV^2}{2}$
- $\tau = \vec{p}\times\vec{E}$
- $\vec{p} = q\vec{d}$
- $C' = \kappa C$

Module 5.

- $V = IR$
- $I = \dot{q}(t)$
- $I = \int J \cdot dA$
- $v_d = I/(nAe)$
- $\rho = E/J$
- $R = \rho L/A$

- $P = I^2 R = V^2/R$
- $\rho \propto T$

Module 6.

- $R_{\text{eq}} = \sum R_i$
- $\dfrac{1}{R_{\text{eq}}} = \sum \dfrac{1}{R_i}$
- $q(t) = C\mathscr{E}(1 - e^{-t/RC})$
- $I(t) = \dfrac{\mathscr{E}}{R} e^{-t/RC}$

Module 7.

- $\vec{F}_B = q\vec{v} \times \vec{B}$
- $|q|vB = \dfrac{mv^2}{r}$
- $\vec{F}_B = I\vec{L} \times \vec{B}$
- $\vec{\mu} = (NIA)\hat{n}$
- $\vec{\tau} = \vec{\mu} \times \vec{B}$

Module 8.

- $d\vec{B} = \dfrac{\mu_0}{4\pi} \dfrac{id\vec{s} \times \hat{r}}{r^2}$
- $B = \dfrac{\mu_0 I}{2\pi R}$
- $B = \dfrac{\mu_0 I \phi}{4\pi R}$
- $F_{ba} = \dfrac{\mu_0 L i_a i_b}{2\pi d}$
- $B = \mu_0 i n$
- $B = \dfrac{\mu_0 i N}{2\pi r}$

Module 9.

- $\Phi_B = \oint \vec{B} \cdot d\vec{A}$
- $\mathcal{E} = \dfrac{-d\Phi_B}{dt}$

Module 10.

- $L = \dfrac{N\Phi_B}{I}$
- $\mathcal{E}_L = -L\dfrac{dI}{dt}$
- $U_B = \dfrac{1}{2} L i^2$
- $u_B = \dfrac{B^2}{2\mu_0}$
- $M_{21} = \dfrac{N_2 \Phi_{21}}{i_1}$
- $\tau = L/R$

Module 11.

- $L\dfrac{d^2q}{dt^2} + R\dfrac{dq}{dt} + \dfrac{1}{C}q = \mathcal{E}$
- $X_C = \dfrac{1}{\omega_d C}$
- $X_L = \omega_D L$
- $Z = \sqrt{R^2 + (X_L - X_C)^2}$
- $\dfrac{V_p}{N_p} = \dfrac{V_s}{N_S}$
- $I_p V_p = I_s V_s$

MODULE 12.

- $\oint \vec{B} \cdot d\vec{l} = \mu_0 i_{enc} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$
- $\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$
- $\oiint \vec{B} \cdot d\vec{A} = 0$
- $\frac{E_0}{B_0} = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = c$
- $\vec{\mathcal{S}} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$
- $\vec{\mathcal{P}} = \frac{1}{c} \vec{\mathcal{S}}$
- $k\lambda = 2\pi = T\omega = \frac{\omega}{f}$
- $c = \lambda f$