Laplacian Deformation with ICP

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Problem

The problem of non-rigid registration for a template mesh T and a point cloud $\{\mathbf{v}_i\}$ an be formulate as minimizing the registration error and the distortion of the template mesh at the same time:

$$E = w_f E_f + w_d E_d$$

where E_f is fitting error term and E_d is the distortion term.

Fitting Error

Fitting error of non-rigid registration can be written as the weighted sum of two terms, the feature term $E_{feature}$ and the ICP term E_{ICP}.

$$\begin{aligned} E_f &= w_{feature} E_{feature} + w_{ICP} E_{ICP} \\ &= w_{feature} \sum_{\mathbf{v}_i \in \mathcal{F}} \|\mathbf{v}_i - \mathbf{v}'_i\|^2 + w_{ICP} \sum_{\mathbf{v}_i \in \mathcal{M}} \|\mathbf{v}_i - \mathbf{v}'_i\|^2 \end{aligned}$$

where $\mathbf{v}'_i = \alpha \mathbf{v}_{j1} + \beta \mathbf{v}_{j2} + \gamma \mathbf{v}_{j3}$ is the closest point to $\mathbf{v}_i$ on the deformed template mesh T, represented using barycentric coordinates. $\mathbf{F}$ is the set of vertices corresponding to feature points, i.e. initial correspondence, and $\mathcal{M}$ is the set of points in the point cloud.

Mesh Distortion

The distortion of the mesh is measured by per-vertex Laplacian:

$$E_d = \sum_{i=1}^{N} \|\mathcal{L}(\mathbf{v}_i') - \mathbf{T}_i \delta_i\|^2$$

$\mathcal{L}$ is the discrete Laplacian operator and δ_i is the Laplacian at vertex $\mathbf{v}_i$ before deformation. $\mathbf{T}_i$ is a rigid transformation (in fact, rotation and scaling only) matrix that describes the local deformation of $\mathbf{v}_i$ on template mesh.

Algorithm Overview

Input

- Tempalte mesh T
- Point cloud $\mathcal{M}$
- Initial correspondence $\mathcal{F} = \{\mathbf{v}_i, \mathbf{v}'_i\}$

Output

- Deformed mesh T_d

Algorithm Outline

The algorithm for non-rigid registration with ICP is as follows:

initialization;
while *not converged* **do**
    fix ICP correspondence, minimize registration error E;
    update ICP correspondence;
    increase the weight for ICP term, w_{ICP};
end

Algorithm 1: Non-rigid registration.

Appendix: Laplacian Deformation

The local deformation matrix $\mathbf{T}_i(\mathbf{v}_i')$ can be represented as [Sorkine et al. 2004]

$$\mathbf{T}_i(\mathbf{v}_i') = \begin{pmatrix} s & -h_3 & h_2 & t_x \\ h_3 & s & -h_1 & t_y \\ -h_2 & h_1 & s & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

or equivalently, $\mathcal{T}_i(\mathbf{v}_i') = (s_i, \mathbf{h}_i, \mathbf{t}_i)^T$.
Accordingly, the distortion term can be written as

$$\begin{aligned} E_d &= \sum_{i=1}^{N} \|\mathcal{L}(\mathbf{v}_i') - \mathbf{T}_i \delta_i\|^2 \\ &= \sum_{i=1}^{N} \|\mathcal{L}(\mathbf{v}_i') - \mathbf{D}_i \mathcal{T}_i(\mathbf{v}_i')\|^2 \end{aligned}$$

Laplacian Deformation (Cont.)

The matrix $\mathbf{D}_i$ is defined as

$$\mathbf{D}_i = \begin{pmatrix} \delta_{ix} & 0 & \delta_{iz} & -\delta_{iy} & 0 & 0 & 0 \\ \delta_{iy} & -\delta_{iz} & 0 & \delta_{ix} & 0 & 0 & 0 \\ \delta_{iz} & \delta_{iy} & -\delta_{ix} & 0 & 0 & 0 & 0 \end{pmatrix}$$
$$= (\delta_i; \mathbf{S}(\delta_i); \mathbf{0})$$

where $\mathbf{S}$ is a skew matrix defined as $\mathbf{S}(\mathbf{u})\mathbf{v} = \mathbf{u} \times \mathbf{v}$.
$\mathcal{T}_i(\mathbf{v}'_i)$ is defined as

$$(s_i, \mathbf{h}_i, \mathbf{t}_i)^T = (\mathbf{A}_i^T \mathbf{A}_i)^{-1} \mathbf{A}_i^T \mathbf{b}_i$$

where $\mathbf{b}_i = (\mathbf{v}'_i, \mathbf{v}'_{ij_1}, \ldots, \mathbf{v}'_{ij_m})^T$, $\mathbf{v}_{ij_k} \in \mathcal{N}_i$.

Laplacian Deformation (Cont. 2)

$$\mathbf{A}_i = \begin{pmatrix} v_{ix} & 0 & v_{iz} & -v_{iy} & 1 & 0 & 0 \\ v_{iy} & -v_{iz} & 0 & v_{ix} & 0 & 1 & 0 \\ v_{iz} & v_{iy} & -v_{ix} & 0 & 0 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ v_{ij_m x} & 0 & v_{ij_m z} & -v_{ij_m y} & 1 & 0 & 0 \\ v_{ij_m y} & -v_{ij_m z} & 0 & v_{ij_m x} & 0 & 1 & 0 \\ v_{ij_m z} & v_{ij_m y} & -v_{ij_m x} & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \mathbf{v}_i & \mathbf{S}(\mathbf{v}_i) & \mathbf{I} \\ \mathbf{v}_{ij_1} & \mathbf{S}(\mathbf{v}_{ij_1}) & \mathbf{I} \\ \vdots & \vdots & \vdots \\ \mathbf{v}_{ij_m} & \mathbf{S}(\mathbf{v}_{ij_m}) & \mathbf{I} \end{pmatrix}$$

Laplacian Deformation (Cont. 3)

The Laplacian operator $\mathcal{L}$ can be represented as a matrix as well

$$\mathcal{L}(\mathbf{v}_i') = \mathbf{L}_i \mathbf{b}_i$$

where $\mathbf{b}_i = (\mathbf{v}_i', \mathbf{v}_{ij_1}', \ldots, \mathbf{v}_{ij_m}')^T, \mathbf{v}_{ij_k} \in \mathcal{N}_i$ and

$$\mathbf{L}_i = \begin{pmatrix} 1 & 0 & 0 & -\frac{1}{|\mathcal{N}_i|} & 0 & 0 & \ldots & -\frac{1}{|\mathcal{N}_i|} & 0 & 0 \\ 0 & 1 & 0 & 0 & -\frac{1}{|\mathcal{N}_i|} & 0 & \ldots & 0 & -\frac{1}{|\mathcal{N}_i|} & 0 \\ 0 & 0 & 1 & 0 & 0 & -\frac{1}{|\mathcal{N}_i|} & \ldots & 0 & 0 & -\frac{1}{|\mathcal{N}_i|} \end{pmatrix}$$
$$= \left(\mathbf{I}, -\frac{1}{|\mathcal{N}_i|}\mathbf{I}, \ldots, -\frac{1}{|\mathcal{N}_i|}\mathbf{I}\right)$$

References

1. O. Sorkine, D. Cohen-Or, Y. Lipman, M. Alexa, C. Rssl, and H.-P. Seidel. *Laplacian surface editing*. In Proceedings of the 2004 Eurographics/ACM SIGGRAPH symposium on Geometry processing (SGP '04). ACM, New York, NY, USA, 175-184.
2. Ju Tao. CSE 554 Lecture notes. http://www.cse.wustl.edu/~taoju/cse554/lectures/lect08_Deformation.pdf
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