Supplementary Documents for ICAPS 2023

Proposition 1 (Ontic Action) *Let (M, s) be a state and a be an ontic action instance that is executable in (M, s) and $\omega(\mathsf{a})$ be given in Definition 4. It holds that:*

1. *For every agent $x \in \mathcal{AG}$, $[x$ **observes** a **if** $\delta_x]$ and $[\mathsf{a}$ **causes** ℓ **if** $\varphi]$ belong to D, if $(M, s) \models \delta_x$, $(M, s) \models \mathbf{B}_x\varphi$ and $(M', s') = (M, s) \otimes (\omega(\mathsf{a}), \sigma)$ then $(M', s') \models \mathbf{B}_x\ell$.*
2. *For every pair of agents $x, y \in \mathcal{AG}$, $[\mathsf{a}$ **causes** ℓ **if** $\varphi]$, $[x$ **observes** a **if** $\delta_x]$ and $[y$ **observes** a **if** $\delta_y]$ belong to D, if $(M, s) \models \delta_x$, $(M, s) \models \mathbf{B}_x\delta_y$, $(M, s) \models \mathbf{B}_x\mathbf{B}_y\varphi$ and $(M', s') = (M, s) \otimes (\omega(\mathsf{a}), \sigma)$ then $(M', s') \models \mathbf{B}_x\mathbf{B}_y\ell$.*
3. *For every pair of agents $x, y \in \mathcal{AG}$, a belief formula η, $[x$ **observes** a **if** $\delta_x]$ and $[y$ **observes** a **if** $\delta_y]$ belong to D, if $(M, s) \models \delta_x$, $(M, s) \models \mathbf{B}_x\neg\delta_y$, $(M, s) \models \mathbf{B}_x\mathbf{B}_y\eta$ and $(M', s') = (M, s) \otimes (\omega(\mathsf{a}), \sigma)$ then $(M', s') \models \mathbf{B}_x\mathbf{B}_y\eta$.*

Proof. We have that $s' = (s, \sigma)$. Assume that the fluent in ℓ is p, i.e., $\ell = p$ or $\ell = \neg p$. Let $\Psi^+(p, \mathsf{a}) = \bigvee\{\varphi \mid [\mathsf{a}$ **causes** p **if** $\varphi] \in D\}$ and $\Psi^-(p, \mathsf{a}) = \bigvee\{\varphi \mid [\mathsf{a}$ **causes** $\neg p$ **if** $\varphi] \in D\}$ and $\gamma = \Psi^+(p, \mathsf{a}) \vee \Psi^-(p, \mathsf{a})$. By Definition 4, $p \rightarrow \gamma \in sub(\sigma)$.

1. *Proof of the first item:* For every $u' \in M'[S]$ such that $(s', u') \in M'[x]$, it holds that $u' = (u, \sigma)$ for some $u \in M[S]$, $(M, u) \models \psi$ and $(s, u) \in M[x]$. Because $(M, s) \models \mathbf{B}_x\varphi$, we have $(M, u) \models \varphi$. Consider two cases:

   - $\ell = p$. Then, $(M, u) \models \Psi^+(p, \mathsf{a})$, and, $(M, u) \models \gamma$. So, $M'[\pi]((u, \sigma)) \models p$.
   - $\ell = \neg p$. Then, because $(M, u) \models \varphi$, the consistency of D implies that $(M, u) \not\models \gamma$. Therefore, $M'[\pi]((u, \sigma)) \not\models p$, i.e., $M'[\pi]((u, \sigma)) \models \neg p$.

   Both cases imply that $M'[\pi]((u, \sigma)) \models \ell$. This holds for every $u' \in M'[S]$ such that $(s', u') \in M'[x]$, which implies $(M', s') \models \mathbf{B}_x\ell$.

2. *Proof of the second item:* Consider $u', v' \in M'[S]$ such that $(s', u') \in M'[x]$, $(u', v') \in M'[y]$. Since $(M, s) \models \delta_x$ and $(M, s) \models \mathbf{B}_x\delta_y$, it holds that $v' = (v, \sigma)$, $u' = (u, \sigma)$ for some $u, v \in M[S]$, $(s, u) \in M[x]$, $(u, v) \in M[y]$ and $(M, v) \models \psi$. Because $(M, s) \models \mathbf{B}_x\mathbf{B}_y\varphi$, we have $(M, v) \models \varphi$. Consider two cases:

   - $\ell = p$. Then, $(M, v) \models \Psi^+(p, \mathsf{a})$, and, $(M, v) \models \gamma$. So, $M'[\pi]((v, \sigma)) \models p$.
   - $\ell = \neg p$. Then, because $(M, v) \models \varphi$, the consistency of D implies that $(M, v) \not\models \gamma$. Therefore, $M'[\pi]((v, \sigma)) \not\models p$, i.e., $M'[\pi]((v, \sigma)) \models \neg p$.

   Both cases imply that $M'[\pi]((v, \sigma)) \models \ell$. This holds for every $v', u' \in M'[S]$ such that $(s', u') \in M'[x]$, $(u', v') \in M'[y]$, which implies $(M', s') \models \mathbf{B}_x\mathbf{B}_y\ell$.

3. Proof of the third item: By the construction of M', we have the following observations:

   - For every $u \in M[S]$ iff $(u, \epsilon) \in M'[S]$;
   - For every $z \in \mathcal{AG}$, $(u, v) \in M[z]$ iff $((u, \epsilon), (v, \epsilon)) \in M'[z]$;
   - For every $u \in M[S]$ and $p \in \mathcal{F}$, $M'[\pi]((u, \epsilon)) \models p$ iff $(M', (u, \epsilon)) \models$ because $sub(\epsilon) = \emptyset$.

   These observations allow us to conclude for every formula η, $(M, u) \models \eta$ iff $(M', (u, \epsilon)) \models \eta$. Consider $u', v' \in M'[S]$ such that $(s', u') \in M'[x]$, $(u', v') \in M'[y]$. Since $(M, s) \models \delta_x$ and $(M, s) \models \mathbf{B}_x\neg\delta_y$, it holds that $v' = (v, \epsilon)$, $u' = (u, \sigma)$ for some $u, v \in M[S]$, $(s, u) \in M[x]$ and $(u, v) \in M[y]$. Assume that $(M, s) \models \mathbf{B}_x\mathbf{B}_y\eta$. This implies $(M, v) \models \eta$, means that $(M', (v, \epsilon)) \models \eta$, i.e., which implies $(M', s') \models \mathbf{B}_x\mathbf{B}_y\eta$. □

Proposition 2 (Sensing Action) *Let (M, s) be a state and* a *be a sensing action instance senses* φ *that is executable in (M, s) and $\omega(\mathsf{a})$ be given in Definition 5. It holds that:*

1. *For every agent $x \in \mathcal{AG}$, $[x$ **observes** a **if** $\delta_x]$ belong to D, if $(M, s) \models \delta_x$, $(M, s) \models \varphi$ and $(M', s') = (M, s) \otimes (\omega(\mathsf{a}), \{\sigma, \tau\})$ then $(M', s') \models \mathbf{B}_x\varphi$.*

2. *For every agent $x \in \mathcal{AG}$, $[x$ **observes** a **if** $\delta_x]$ belong to D, if $(M, s) \models \delta_x$, $(M, s) \models \neg\varphi$ and $(M', s') = (M, s) \otimes (\omega(\mathsf{a}), \{\sigma, \tau\})$ then $(M', s') \models \mathbf{B}_x\neg\varphi$.*

3. *For every agent $x \in \mathcal{AG}$, $[x$ **observes** a **if** $\delta_x]$ and $[x$ **aware_of** a **if** $\theta_x]$ belong to D, if $(M, s) \models \neg\delta_x \wedge \theta_x$, $(M, s) \not\models (\mathbf{B}_x\varphi \vee \mathbf{B}_x\neg\varphi)$ and $(M', s') = (M, s) \otimes (\omega(\mathsf{a}), \{\sigma, \tau\})$ then $(M', s') \not\models (\mathbf{B}_x\varphi \vee \mathbf{B}_x\neg\varphi)$.*

4. *For every pair of agents $x, y \in \mathcal{AG}$, $[x$ **observes** a **if** $\delta_x]$, $[y$ **observes** a **if** $\delta_y]$ and $[y$ **aware_of** a **if** $\theta_y]$ belong to D, if $(M, s) \models \mathbf{B}_y\delta_x$, $(M, s) \models \delta_y \vee \theta_y$ and $(M', s') = (M, s) \otimes (\omega(\mathsf{a}), \{\sigma, \tau\})$ then $(M', s') \models \mathbf{B}_y(\mathbf{B}_x\varphi \vee \mathbf{B}_x\neg\varphi)$.*

5. *For every pair of agents $x, y \in \mathcal{AG}$, a belief formula η, $[x$ **observes** a **if** $\delta_x]$, $[x$ **aware_of** a **if** $\theta_x]$, $[y$ **observes** a **if** $\delta_y]$ and $[y$ **aware_of** a **if** $\theta_y]$ belong to D, if $(M, s) \models \mathbf{B}_x\neg(\delta_y \vee \theta_y)$, $(M, s) \models \delta_x \vee \theta_x$, $(M, s) \models \mathbf{B}_x\mathbf{B}_y\eta$ and $(M', s') = (M, s) \otimes (\omega(\mathsf{a}), \{\sigma, \tau\})$ then $(M', s') \models \mathbf{B}_x\mathbf{B}_y\eta$.*

Proof. We will prove for the case $(M, s) \models \varphi$, the proof when $(M, s) \models \neg\varphi$ is similar and is omitted here. We have that $s' = (s, \sigma)$.

1. *Proof of the first item:* For every $u' \in M'[S]$ such that $(s', u') \in M'[x]$, since $(M, s) \models \delta_x$ it holds that $u' = (u, \sigma)$ for some $u \in M[S]$ and $(s, u) \in M[x]$. Which means $(M', s') \models \mathbf{B}_x\varphi$.

2. *Proof of the third item:* For every $u' \in M'[S]$ such that $(s', u') \in M'[x]$, since $(M, s) \models \neg\delta_x \wedge \theta$ it holds that $u' = (u, \sigma)$ or $u' = (u, \tau)$ for some $u \in M[S]$ and $(s, u) \in M[x]$. We have that $(M, s) \not\models (\mathbf{B}_x\varphi \vee \mathbf{B}_x\neg\varphi)$, which mean $\exists u_1, u_2 \in M[S]$ such that, $(M, u_1) \models \varphi$, $(M, u_2) \models \neg\varphi$ and $(s, u_1), (s, u_2) \in M[x]$. From this we have $(M', (u_1, \sigma)) \models \varphi$, $(M', (u_2, \tau)) \models \neg\varphi$ and $(s', (u_1, \sigma)), (s', (u_2, \tau)) \in M'[x]$. Which implies $(M', s') \not\models (\mathbf{B}_x\varphi \vee \mathbf{B}_x\neg\varphi)$.

3. *Proof of the fourth item:* Consider $u', v' \in M'[S]$ such that $(s', u') \in M'[y]$, $(u', v') \in M'[x]$. Since $(M, s) \models \delta_y \vee \theta_y$ and $(M, s) \models \mathbf{B}_y\delta_x$, it holds that $u' = (u, \sigma)$ and $v' = (v, \sigma)$ (or $u' = (u, \tau)$ and $v' = (v, \tau)$) for some $u, v \in M[S]$, $(s, u) \in M[y]$ and $(u, v) \in M[x]$. Which means $(M', s') \models \mathbf{B}_y(\mathbf{B}_x\varphi \vee \mathbf{B}_x\neg\varphi)$

4. Proof of the fifth item: By the construction of M', we have the following observations:

   - For every $u \in M[S]$ iff $(u, \epsilon) \in M'[S]$;
   - For every $z \in \mathcal{AG}$, $(u, v) \in M[z]$ iff $((u, \epsilon), (v, \epsilon)) \in M'[z]$;
   - For every $u \in M[S]$ and $p \in \mathcal{F}$, $M'[\pi]((u, \epsilon)) \models p$ iff $(M', (u, \epsilon)) \models$ because $sub(\epsilon) = \emptyset$.

   These observations allow us to conclude for every formula η, $(M, u) \models \eta$ iff $(M', (u, \epsilon)) \models \eta$. Consider $u', v' \in M'[S]$ such that $(s', u') \in M'[x]$, $(u', v') \in M'[y]$. Since $(M, s) \models \delta_x \vee \theta_x$ and $(M, s) \models \mathbf{B}_x\neg(\delta_y \wedge \theta_y)$, it holds that $v' = (v, \epsilon)$, $u' = (u, \sigma)$ (or $u' = (u, \tau)$) for some $u, v \in M[S]$, $(s, u) \in M[x]$ and $(u, v) \in M[y]$. Assume that $(M, s) \models \mathbf{B}_x\mathbf{B}_y\eta$. This implies $(M, v) \models \eta$, means that $(M', (v, \epsilon)) \models \eta$, i.e., which implies $(M', s') \models \mathbf{B}_x\mathbf{B}_y\eta$. □

Proposition 3 *Let (M, s) be a state and* a *be an announcement action instance announces φ that is executable in (M, s) and $\omega(\mathsf{a})$ be given in Definition 5. If $(M, s) \models \varphi$ then it holds that:*

1. *For every agent $x \in \mathcal{AG}$, $[x$ **observes** a **if** $\delta_x]$ belong to D, if $(M, s) \models \delta_x$ and $(M', s') = (M, s) \otimes (\omega(\mathsf{a}), \sigma)$ then $(M', s') \models \mathbf{B}_x\varphi$.*

2. *For every agent $x \in \mathcal{AG}$, $[x$ **observes** a **if** $\delta_x]$ and $[x$ **aware_of** a **if** $\theta_x]$ belong to D, if $(M, s) \models \neg\delta_x \wedge \theta_x$, $(M, s) \not\models (\mathbf{B}_x\varphi \vee \mathbf{B}_x\neg\varphi)$ and $(M', s') = (M, s) \otimes (\omega(\mathsf{a}), \sigma)$ then $(M', s') \not\models (\mathbf{B}_x\varphi \vee \mathbf{B}_x\neg\varphi)$.*

3. *For every pair of agents $x, y \in \mathcal{AG}$, $[x$ **observes** a **if** $\delta_x]$, $[y$ **observes** a **if** $\delta_y]$ and $[y$ **aware_of** a **if** $\theta_y]$ belong to D, if $(M,s) \models \mathbf{B}_y\delta_x$, $(M,s) \models \delta_y \vee \theta_y$ and $(M',s') = (M,s) \otimes (\omega(\mathsf{a}), \sigma)$ then $(M',s') \models \mathbf{B}_y(\mathbf{B}_x\varphi \vee \mathbf{B}_x\neg\varphi)$.*

4. *For every pair of agents $x, y \in \mathcal{AG}$, a belief formula η, $[x$ **observes** a **if** $\delta_x]$, $[x$ **aware_of** a **if** $\theta_x]$, $[y$ **observes** a **if** $\delta_y]$ and $[y$ **aware_of** a **if** $\theta_y]$ belong to D, if $(M,s) \models \mathbf{B}_x\neg(\delta_y \vee \theta_y)$, $(M,s) \models \delta_x \vee \theta_x$, $(M,s) \models \mathbf{B}_x\mathbf{B}_y\eta$ and $(M',s') = (M,s) \otimes (\omega(\mathsf{a}), \sigma)$ then $(M',s') \models \mathbf{B}_x\mathbf{B}_y\eta$.*

Proof. The proof of this proposition is similar to the proof of Proposition 2 and is omitted. □

Proposition 4 (Lying Announcement Action) *Let $\mathsf{a} = a\langle\alpha\rangle$ be the occurrence of a lying announcement of φ in (M,s). Let us denote F (resp. P and O) as $F(\mathsf{a}, M, s)$ (resp. $P(\mathsf{a}, M, s)$ and $O(\mathsf{a}, M, s)$). Assume that $(M',s') = (M,s) \otimes (\omega(\mathsf{a},(M,s)), \sigma)$. We have*

1. *$(M',s') \models \mathbf{C}_{F_t}\neg\varphi$ where $F_t = \{i \in F \mid (M,s) \models \mathbf{B}_i\neg\varphi\}$;*
2. *$(M',s') \models \mathbf{C}_{F_{uf}}\varphi$ where $F_{uf} = \{i \in F \mid (M,s) \models \neg\mathbf{B}_i\neg\varphi\}$;*
3. *$(M',s') \models \mathbf{C}_P(\mathbf{C}_{F_u}\varphi \vee \mathbf{C}_{F_u}\neg\varphi)$ where $F_u = \{i \in F \mid (M,s) \models \neg(\mathbf{B}_i\varphi \vee \mathbf{B}_i\neg\varphi)\}$;*
4. *$(M',s') \models \mathbf{C}_F(\mathbf{C}_P(\mathbf{C}_{F_u}\varphi \vee \mathbf{C}_{F_u}\neg\varphi))$;*
5. *$(M',s') \models \mathbf{B}_j\eta$ iff $(M,s) \models \mathbf{B}_j\eta$ for a formula η and $j \in O$;*
6. *$(M',s') \models \mathbf{B}_i\mathbf{B}_j\eta$ iff $(M,s) \models \mathbf{B}_i\mathbf{B}_j\eta$ for a formula η, $i \in F \cup P$ and $j \in O$.*

Proof. (Sketch). The first item is true because for any F_t-reachable[1] world u' in M' must satisfy that $u' \models \neg\varphi$ because of the preconditions of χ and σ, the definition of $\otimes$ with edge-conditions, and Lemma 2.2.1 of [FHMV95]. Similarly, any F_{uf}-reachable world in M' must satisfy φ and this proves the second item. The third and fourth item are correct because any P-reachable world u' in M' satisfies $\mathbf{C}_{F_u}\varphi \vee \mathbf{C}_{F_u}\neg\varphi$ and any F-reachable world u' satisfies $\mathbf{C}_P(\mathbf{C}_{F_u}\varphi \vee \mathbf{C}_{F_u}\neg\varphi)$. For agent j such that $j \in O$, her belief stays the same because of the edge-condition from σ to ϵ. Similarly, it is easy to verify that the last item is correct because there exist s_1 and s_2 in $M[S]$ such that $(s, s_1) \in M[i]$ and $(s_1, s_2) \in M[j]$ iff there exists $x_1 \in \{\sigma, \tau, \zeta, \chi, \mu\}$ such that (s, σ), (s_1, x_1) and (s_2, ϵ) belong to $M'[S]$ and $(s_1, x_1) \in M'[i]$, $(s_2, \epsilon) \in M'[j]$. □

Proposition 5 (Misleading Announcement Action) *Let $\mathsf{a} = a\langle\alpha\rangle$ be an occurrence of a misleading announcement of φ in (M,s). Let us denote F (resp. P and O) as $F(\mathsf{a}, M, s)$ (resp. $P(\mathsf{a}, M, s)$ and $O(\mathsf{a}, M, s)$) and F_t, F_{uf} and F_u are defined in Prop.4. Assume that $(M',s') = (M,s) \otimes (\omega(\mathsf{a},(M,s)), \sigma)$. It holds that:*

1. *$(M',s') \models \neg(\mathbf{B}_i\varphi \wedge \neg\mathbf{B}_i\neg\varphi)$ for $i \in \alpha$;*
2. *$(M',s') \models \mathbf{C}_{F_t}\neg\varphi$;*
3. *$(M',s') \models \mathbf{C}_{F'_{uf}}\varphi$ where $F'_{uf} = F_{uf} \setminus \alpha$;*
4. *$(M',s') \models \mathbf{C}_{P_u}(\mathbf{C}_{F_u}\varphi \vee \mathbf{C}_{F_u}\neg\varphi)$ where $P_u = \{i \in P \mid (M,s) \models \neg\mathbf{B}_i\neg(\mathbf{B}_\alpha\varphi \vee \mathbf{B}_\alpha\neg\varphi)\}$;*
5. *$(M',s') \models \mathbf{C}_F(\mathbf{C}_{P_u}(\mathbf{C}_{F'_u}\varphi \vee \mathbf{C}_{F'_u}\neg\varphi))$;*
6. *$(M',s') \models \mathbf{B}_j\eta$ iff $(M,s) \models \mathbf{B}_j\eta$ for a formula η and $j \in O$;*
7. *$(M',s') \models \mathbf{B}_i\mathbf{B}_j\eta$ iff $(M,s) \models \mathbf{B}_i\mathbf{B}_j\eta$ for a formula η, $i \in F \cup P$ and $j \in O$.*

Proof. (Sketch). The first item is true because of the definition of $\otimes$ and the edge-conditioned update model for misleading announcements, we have that for any world u' in M' that is α-reachable from s', is of the form (x, σ) or (y, ζ) or (z, τ). Because of the precondition of σ, ζ and τ; and $(M,s) \models \neg(\mathbf{B}_i\varphi \wedge \mathbf{B}_i\neg\varphi)$ for $i \in \alpha$, we can conclude that $(M',s') \models \neg(\mathbf{B}_i\varphi \wedge \neg\mathbf{B}_i\neg\varphi)$ for $i \in \alpha$. The fourth and fifth item are correct because any P_u-reachable world u' in M' satisfies $\mathbf{C}_{F_u}\varphi \vee \mathbf{C}_{F_u}\neg\varphi$ and any F-reachable world u' satisfies $\mathbf{C}_{P_u}(\mathbf{C}_{F_u}\varphi \vee \mathbf{C}_{F_u}\neg\varphi)$. For other items, the proofs are similar to the Proof of Proposition 4. □

[1]A world u is G-reachable from s in M if there exists a sequence $s = s_0, s_1, \ldots, s_k = u$ such that for each $i = 0, \ldots, k-1$, there exists some $j \in G$ such that $(s_j, s_{j+1}) \in M[j+1]$.

References

[FHMV95] R. Fagin, J. Halpern, Y. Moses, and M. Vardi. *Reasoning about Knowledge*. MIT press, 1995.