

Supplementary Documents for ICAPS 2023

Proposition 1 (Ontic Action) *Let (M, s) be a state and \mathbf{a} be an ontic action instance that is executable in (M, s) and $\omega(\mathbf{a})$ be given in Definition 4. It holds that:*

1. *For every agent $x \in \mathcal{AG}$, $[x \textbf{ observes } \mathbf{a} \textbf{ if } \delta_x]$ and $[\mathbf{a} \textbf{ causes } \ell \textbf{ if } \varphi]$ belong to D , if $(M, s) \models \delta_x$, $(M, s) \models \mathbf{B}_x \varphi$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \sigma)$ then $(M', s') \models \mathbf{B}_x \ell$.*
2. *For every pair of agents $x, y \in \mathcal{AG}$, $[\mathbf{a} \textbf{ causes } \ell \textbf{ if } \varphi]$, $[x \textbf{ observes } \mathbf{a} \textbf{ if } \delta_x]$ and $[y \textbf{ observes } \mathbf{a} \textbf{ if } \delta_y]$ belong to D , if $(M, s) \models \delta_x$, $(M, s) \models \mathbf{B}_x \delta_y$, $(M, s) \models \mathbf{B}_x \mathbf{B}_y \varphi$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \sigma)$ then $(M', s') \models \mathbf{B}_x \mathbf{B}_y \ell$.*
3. *For every pair of agents $x, y \in \mathcal{AG}$, a belief formula η , $[x \textbf{ observes } \mathbf{a} \textbf{ if } \delta_x]$ and $[y \textbf{ observes } \mathbf{a} \textbf{ if } \delta_y]$ belong to D , if $(M, s) \models \delta_x$, $(M, s) \models \mathbf{B}_x \neg \delta_y$, $(M, s) \models \mathbf{B}_x \mathbf{B}_y \eta$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \sigma)$ then $(M', s') \models \mathbf{B}_x \mathbf{B}_y \eta$.*

Proof. We have that $s' = (s, \sigma)$. Assume that the fluent in ℓ is p , i.e., $\ell = p$ or $\ell = \neg p$. Let $\Psi^+(p, \mathbf{a}) = \bigvee \{\varphi \mid [\mathbf{a} \textbf{ causes } p \textbf{ if } \varphi] \in D\}$ and $\Psi^-(p, \mathbf{a}) = \bigvee \{\varphi \mid [\mathbf{a} \textbf{ causes } \neg p \textbf{ if } \varphi] \in D\}$ and $\gamma = \Psi^+(p, \mathbf{a}) \vee \Psi^-(p, \mathbf{a})$. By Definition 4, $p \rightarrow \gamma \in \text{sub}(\sigma)$.

1. *Proof of the first item:* For every $u' \in M'[S]$ such that $(s', u') \in M'[x]$, it holds that $u' = (u, \sigma)$ for some $u \in M[S]$, $(M, u) \models \psi$ and $(s, u) \in M[x]$. Because $(M, s) \models \mathbf{B}_x \varphi$, we have $(M, u) \models \varphi$. Consider two cases:

- $\ell = p$. Then, $(M, u) \models \Psi^+(p, \mathbf{a})$, and, $(M, u) \models \gamma$. So, $M'[\pi]((u, \sigma)) \models p$.
- $\ell = \neg p$. Then, because $(M, u) \models \varphi$, the consistency of D implies that $(M, u) \not\models \gamma$. Therefore, $M'[\pi]((u, \sigma)) \not\models p$, i.e., $M'[\pi]((u, \sigma)) \models \neg p$.

Both cases imply that $M'[\pi]((u, \sigma)) \models \ell$. This holds for every $u' \in M'[S]$ such that $(s', u') \in M'[x]$, which implies $(M', s') \models \mathbf{B}_x \ell$.

2. *Proof of the second item:* Consider $u', v' \in M'[S]$ such that $(s', u') \in M'[x]$, $(u', v') \in M'[y]$. Since $(M, s) \models \delta_x$ and $(M, s) \models \mathbf{B}_x \delta_y$, it holds that $v' = (v, \sigma)$, $u' = (u, \sigma)$ for some $u, v \in M[S]$, $(s, u) \in M[x]$, $(u, v) \in M[y]$ and $(M, v) \models \psi$. Because $(M, s) \models \mathbf{B}_x \mathbf{B}_y \varphi$, we have $(M, v) \models \varphi$. Consider two cases:

- $\ell = p$. Then, $(M, v) \models \Psi^+(p, \mathbf{a})$, and, $(M, v) \models \gamma$. So, $M'[\pi]((v, \sigma)) \models p$.
- $\ell = \neg p$. Then, because $(M, v) \models \varphi$, the consistency of D implies that $(M, v) \not\models \gamma$. Therefore, $M'[\pi]((v, \sigma)) \not\models p$, i.e., $M'[\pi]((v, \sigma)) \models \neg p$.

Both cases imply that $M'[\pi]((v, \sigma)) \models \ell$. This holds for every $v', u' \in M'[S]$ such that $(s', u') \in M'[x]$, $(u', v') \in M'[y]$, which implies $(M', s') \models \mathbf{B}_x \mathbf{B}_y \ell$.

3. *Proof of the third item:* By the construction of M' , we have the following observations:

- For every $u \in M[S]$ iff $(u, \epsilon) \in M'[S]$;
- For every $z \in \mathcal{AG}$, $(u, v) \in M[z]$ iff $((u, \epsilon), (v, \epsilon)) \in M'[z]$;
- For every $u \in M[S]$ and $p \in \mathcal{F}$, $M'[\pi]((u, \epsilon)) \models p$ iff $(M', (u, \epsilon)) \models p$ because $\text{sub}(\epsilon) = \emptyset$.

These observations allow us to conclude for every formula η , $(M, u) \models \eta$ iff $(M', (u, \epsilon)) \models \eta$. Consider $u', v' \in M'[S]$ such that $(s', u') \in M'[x]$, $(u', v') \in M'[y]$. Since $(M, s) \models \delta_x$ and $(M, s) \models \mathbf{B}_x \neg \delta_y$, it holds that $v' = (v, \epsilon)$, $u' = (u, \sigma)$ for some $u, v \in M[S]$, $(s, u) \in M[x]$ and $(u, v) \in M[y]$. Assume that $(M, s) \models \mathbf{B}_x \mathbf{B}_y \eta$. This implies $(M, v) \models \eta$, means that $(M', (v, \epsilon)) \models \eta$, i.e., which implies $(M', s') \models \mathbf{B}_x \mathbf{B}_y \eta$. \square

Proposition 2 (Sensing Action) *Let (M, s) be a state and \mathbf{a} be a sensing action instance senses φ that is executable in (M, s) and $\omega(\mathbf{a})$ be given in Definition 5. It holds that:*

1. *For every agent $x \in \mathcal{AG}$, $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$ belong to D , if $(M, s) \models \delta_x$, $(M, s) \models \varphi$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \{\sigma, \tau\})$ then $(M', s') \models \mathbf{B}_x \varphi$.*
2. *For every agent $x \in \mathcal{AG}$, $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$ belong to D , if $(M, s) \models \delta_x$, $(M, s) \models \neg \varphi$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \{\sigma, \tau\})$ then $(M', s') \models \mathbf{B}_x \neg \varphi$.*
3. *For every agent $x \in \mathcal{AG}$, $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$ and $[x \text{ aware_of } \mathbf{a} \text{ if } \theta_x]$ belong to D , if $(M, s) \models \neg \delta_x \wedge \theta_x$, $(M, s) \not\models (\mathbf{B}_x \varphi \vee \mathbf{B}_x \neg \varphi)$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \{\sigma, \tau\})$ then $(M', s') \not\models (\mathbf{B}_x \varphi \vee \mathbf{B}_x \neg \varphi)$.*
4. *For every pair of agents $x, y \in \mathcal{AG}$, $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$, $[y \text{ observes } \mathbf{a} \text{ if } \delta_y]$ and $[y \text{ aware_of } \mathbf{a} \text{ if } \theta_y]$ belong to D , if $(M, s) \models \mathbf{B}_y \delta_x$, $(M, s) \models \delta_y \vee \theta_y$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \{\sigma, \tau\})$ then $(M', s') \models \mathbf{B}_y (\mathbf{B}_x \varphi \vee \mathbf{B}_x \neg \varphi)$.*
5. *For every pair of agents $x, y \in \mathcal{AG}$, a belief formula η , $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$, $[x \text{ aware_of } \mathbf{a} \text{ if } \theta_x]$, $[y \text{ observes } \mathbf{a} \text{ if } \delta_y]$ and $[y \text{ aware_of } \mathbf{a} \text{ if } \theta_y]$ belong to D , if $(M, s) \models \mathbf{B}_x \neg (\delta_y \vee \theta_y)$, $(M, s) \models \delta_x \vee \theta_x$, $(M, s) \models \mathbf{B}_x \mathbf{B}_y \eta$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \{\sigma, \tau\})$ then $(M', s') \models \mathbf{B}_x \mathbf{B}_y \eta$.*

Proof. We will prove for the case $(M, s) \models \varphi$, the proof when $(M, s) \models \neg \varphi$ is similar and is omitted here. We have that $s' = (s, \sigma)$.

1. *Proof of the first item:* For every $u' \in M'[S]$ such that $(s', u') \in M'[x]$, since $(M, s) \models \delta_x$ it holds that $u' = (u, \sigma)$ for some $u \in M[S]$ and $(s, u) \in M[x]$. Which means $(M', s') \models \mathbf{B}_x \varphi$.
2. *Proof of the third item:* For every $u' \in M'[S]$ such that $(s', u') \in M'[x]$, since $(M, s) \models \neg \delta_x \wedge \theta_x$ it holds that $u' = (u, \sigma)$ or $u' = (u, \tau)$ for some $u \in M[S]$ and $(s, u) \in M[x]$. We have that $(M, s) \not\models (\mathbf{B}_x \varphi \vee \mathbf{B}_x \neg \varphi)$, which mean $\exists u_1, u_2 \in M[S]$ such that, $(M, u_1) \models \varphi$, $(M, u_2) \models \neg \varphi$ and $(s, u_1), (s, u_2) \in M[x]$. From this we have $(M', (u_1, \sigma)) \models \varphi$, $(M', (u_2, \tau)) \models \neg \varphi$ and $(s', (u_1, \sigma)), (s', (u_2, \tau)) \in M'[x]$. Which implies $(M', s') \not\models (\mathbf{B}_x \varphi \vee \mathbf{B}_x \neg \varphi)$.
3. *Proof of the fourth item:* Consider $u', v' \in M'[S]$ such that $(s', u') \in M'[y]$, $(u', v') \in M'[x]$. Since $(M, s) \models \delta_y \vee \theta_y$ and $(M, s) \models \mathbf{B}_y \delta_x$, it holds that $u' = (u, \sigma)$ and $v' = (v, \sigma)$ (or $u' = (u, \tau)$ and $v' = (v, \tau)$) for some $u, v \in M[S]$, $(s, u) \in M[y]$ and $(u, v) \in M[x]$. Which means $(M', s') \models \mathbf{B}_y (\mathbf{B}_x \varphi \vee \mathbf{B}_x \neg \varphi)$.
4. *Proof of the fifth item:* By the construction of M' , we have the following observations:
 - For every $u \in M[S]$ iff $(u, \epsilon) \in M'[S]$;
 - For every $z \in \mathcal{AG}$, $(u, v) \in M[z]$ iff $((u, \epsilon), (v, \epsilon)) \in M'[z]$;
 - For every $u \in M[S]$ and $p \in \mathcal{F}$, $M'[\pi]((u, \epsilon)) \models p$ iff $(M', (u, \epsilon)) \models p$ because $\text{sub}(\epsilon) = \emptyset$.

These observations allow us to conclude for every formula η , $(M, u) \models \eta$ iff $(M', (u, \epsilon)) \models \eta$. Consider $u', v' \in M'[S]$ such that $(s', u') \in M'[x]$, $(u', v') \in M'[y]$. Since $(M, s) \models \delta_x \vee \theta_x$ and $(M, s) \models \mathbf{B}_x \neg (\delta_y \wedge \theta_y)$, it holds that $v' = (v, \epsilon)$, $u' = (u, \sigma)$ (or $u' = (u, \tau)$) for some $u, v \in M[S]$, $(s, u) \in M[x]$ and $(u, v) \in M[y]$. Assume that $(M, s) \models \mathbf{B}_x \mathbf{B}_y \eta$. This implies $(M, v) \models \eta$, means that $(M', (v, \epsilon)) \models \eta$, i.e., which implies $(M', s') \models \mathbf{B}_x \mathbf{B}_y \eta$. \square

Proposition 3 *Let (M, s) be a state and \mathbf{a} be an announcement action instance announces φ that is executable in (M, s) and $\omega(\mathbf{a})$ be given in Definition 5. If $(M, s) \models \varphi$ then it holds that:*

1. *For every agent $x \in \mathcal{AG}$, $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$ belong to D , if $(M, s) \models \delta_x$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \sigma)$ then $(M', s') \models \mathbf{B}_x \varphi$.*
2. *For every agent $x \in \mathcal{AG}$, $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$ and $[x \text{ aware_of } \mathbf{a} \text{ if } \theta_x]$ belong to D , if $(M, s) \models \neg \delta_x \wedge \theta_x$, $(M, s) \not\models (\mathbf{B}_x \varphi \vee \mathbf{B}_x \neg \varphi)$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \sigma)$ then $(M', s') \not\models (\mathbf{B}_x \varphi \vee \mathbf{B}_x \neg \varphi)$.*

3. For every pair of agents $x, y \in \mathcal{AG}$, $[x \text{ observes } a \text{ if } \delta_x]$, $[y \text{ observes } a \text{ if } \delta_y]$ and $[y \text{ aware_of } a \text{ if } \theta_y]$ belong to D , if $(M, s) \models \mathbf{B}_y \delta_x$, $(M, s) \models \delta_y \vee \theta_y$ and $(M', s') = (M, s) \otimes (\omega(a), \sigma)$ then $(M', s') \models \mathbf{B}_y(\mathbf{B}_x \varphi \vee \mathbf{B}_x \neg \varphi)$.
4. For every pair of agents $x, y \in \mathcal{AG}$, a belief formula η , $[x \text{ observes } a \text{ if } \delta_x]$, $[x \text{ aware_of } a \text{ if } \theta_x]$, $[y \text{ observes } a \text{ if } \delta_y]$ and $[y \text{ aware_of } a \text{ if } \theta_y]$ belong to D , if $(M, s) \models \mathbf{B}_x \neg(\delta_y \vee \theta_y)$, $(M, s) \models \delta_x \vee \theta_x$, $(M, s) \models \mathbf{B}_x \mathbf{B}_y \eta$ and $(M', s') = (M, s) \otimes (\omega(a), \sigma)$ then $(M', s') \models \mathbf{B}_x \mathbf{B}_y \eta$.

Proof. The proof of this proposition is similar to the proof of Proposition 2 and is omitted. \square

Proposition 4 (Lying Announcement Action) Let $\mathbf{a} = a\langle\alpha\rangle$ be the occurrence of a lying announcement of φ in (M, s) . Let us denote F (resp. P and O) as $F(\mathbf{a}, M, s)$ (resp. $P(\mathbf{a}, M, s)$ and $O(\mathbf{a}, M, s)$). Assume that $(M', s') = (M, s) \otimes (\omega(\mathbf{a}, (M, s)), \sigma)$. We have

1. $(M', s') \models \mathbf{C}_{F_t} \neg \varphi$ where $F_t = \{i \in F \mid (M, s) \models \mathbf{B}_i \neg \varphi\}$;
2. $(M', s') \models \mathbf{C}_{F_{uf}} \varphi$ where $F_{uf} = \{i \in F \mid (M, s) \models \neg K_i \neg \varphi\}$;
3. $(M', s') \models \mathbf{C}_P(\mathbf{C}_{F_u} \varphi \vee \mathbf{C}_{F_u} \neg \varphi)$ where $F_u = \{i \in F \mid (M, s) \models \neg(K_i \varphi \vee K_i \neg \varphi)\}$;
4. $(M', s') \models \mathbf{C}_F(\mathbf{C}_P(\mathbf{C}_{F_u} \varphi \vee \mathbf{C}_{F_u} \neg \varphi))$;
5. $(M', s') \models \mathbf{B}_j \eta$ iff $(M, s) \models \mathbf{B}_j \eta$ for a formula η and $j \in O$;
6. $(M', s') \models \mathbf{B}_i \mathbf{B}_j \eta$ iff $(M, s) \models \mathbf{B}_i \mathbf{B}_j \eta$ for a formula η , $i \in F \cup P$ and $j \in O$.

Proof. (Sketch). The first item is true because for any F_t -reachable¹ world u' in M' must satisfy that $u' \models \neg \varphi$ because of the preconditions of χ and σ , the definition of \otimes with edge-conditions, and Lemma 2.2.1 of [FHMV95]. Similarly, any F_{uf} -reachable world in M' must satisfy φ and this proves the second item. The third and fourth item are correct because any P -reachable world u' in M' satisfies $\mathbf{C}_{F_u} \varphi \vee \mathbf{C}_{F_u} \neg \varphi$ and any F -reachable world u' satisfies $\mathbf{C}_P(\mathbf{C}_{F_u} \varphi \vee \mathbf{C}_{F_u} \neg \varphi)$. For agent j such that $j \in O$, her belief stays the same because of the edge-condition from σ to ϵ . Similarly, it is easy to verify that the last item is correct because there exist s_1 and s_2 in $M[S]$ such that $(s, s_1) \in M[i]$ and $(s_1, s_2) \in M[j]$ iff there exists $x_1 \in \{\sigma, \tau, \zeta, \chi, \mu\}$ such that (s, σ) , (s_1, x_1) and (s_2, ϵ) belong to $M'[S]$ and $(s_1, x_1) \in M'[i]$, $(s_2, \epsilon) \in M'[j]$. \square

Proposition 5 (Misleading Announcement Action) Let $\mathbf{a} = a\langle\alpha\rangle$ be an occurrence of a misleading announcement of φ in (M, s) . Let us denote F (resp. P and O) as $F(\mathbf{a}, M, s)$ (resp. $P(\mathbf{a}, M, s)$ and $O(\mathbf{a}, M, s)$) and F_t, F_{uf} and F_u are defined in Prop.4. Assume that $(M', s') = (M, s) \otimes (\omega(\mathbf{a}, (M, s)), \sigma)$. It holds that:

1. $(M', s') \models \neg(\mathbf{B}_i \varphi \wedge \neg \mathbf{B}_i \neg \varphi)$ for $i \in \alpha$;
2. $(M', s') \models \mathbf{C}_{F_t} \neg \varphi$;
3. $(M', s') \models \mathbf{C}_{F'_{uf}} \varphi$ where $F'_{uf} = F_{uf} \setminus \alpha$;
4. $(M', s') \models \mathbf{C}_{P_u}(\mathbf{C}_{F_u} \varphi \vee \mathbf{C}_{F_u} \neg \varphi)$ where $P_u = \{i \in P \mid (M, s) \models \neg K_i \neg(K_\alpha \varphi \vee K_\alpha \neg \varphi)\}$;
5. $(M', s') \models \mathbf{C}_F(\mathbf{C}_{P_u}(\mathbf{C}_{F_u} \varphi \vee \mathbf{C}_{F_u} \neg \varphi))$;
6. $(M', s') \models \mathbf{B}_j \eta$ iff $(M, s) \models \mathbf{B}_j \eta$ for a formula η and $j \in O$; and
7. $(M', s') \models \mathbf{B}_i \mathbf{B}_j \eta$ iff $(M, s) \models \mathbf{B}_i \mathbf{B}_j \eta$ for a formula η , $i \in F \cup P$ and $j \in O$.

Proof. (Sketch). The first item is true because of the definition of \otimes and the edge-conditioned update model for misleading announcements, we have that for any world u' in M' that is α -reachable from s' , is of the form (x, σ) or (y, ζ) or (z, τ) . Because of the precondition of σ, ζ and τ ; and $(M, s) \models \neg(K_i \varphi \wedge K_i \neg \varphi)$ for $i \in \alpha$, we can conclude that $(M', s') \models \neg(\mathbf{B}_i \varphi \wedge \neg \mathbf{B}_i \neg \varphi)$ for $i \in \alpha$. The fourth and fifth item are correct because any P_u -reachable world u' in M' satisfies $\mathbf{C}_{F_u} \varphi \vee \mathbf{C}_{F_u} \neg \varphi$ and any F -reachable world u' satisfies $\mathbf{C}_{P_u}(\mathbf{C}_{F_u} \varphi \vee \mathbf{C}_{F_u} \neg \varphi)$. For other items, the proofs are similar to the Proof of Proposition 4. \square

¹A world u is G -reachable from s in M if there exists a sequence $s = s_0, s_1, \dots, s_k = u$ such that for each $i = 0, \dots, k-1$, there exists some $j \in G$ such that $(s_j, s_{j+1}) \in M[j+1]$.

References

- [FHMV95] R. Fagin, J. Halpern, Y. Moses, and M. Vardi. *Reasoning about Knowledge*. MIT press, 1995.