

A New Semantics for The Action Language $\mathbf{m}\mathcal{A}^*$

Loc Pham, Yusuf Izmirliloglu, Tran Cao Son, and Enrico Pontelli

New Mexico State University, Las Cruces NM, USA,
{locpham,yizmir}@nmsu.edu, {tson,epontelli}@cs.nmsu.edu

Abstract. High-level action languages have been shown to be important tools for research in reasoning about actions and change (RAC) as well as planning in single-agent environments. The interest in RAC in multi-agent settings led to the development of the action language $\mathbf{m}\mathcal{A}^*$ with a transition function based semantics. $\mathbf{m}\mathcal{A}^*$ employs the notion of update models in defining transitions between states, represented by pointed Kripke structures. Given an action occurrence and a state, the update model of the action occurrence is automatically constructed from the given state and the observability of agents. A main criticism of this approach is that it cannot deal with situations when agents' have false/incorrect beliefs about the observability of other agents. The present paper addresses this shortcoming by defining a new semantics for $\mathbf{m}\mathcal{A}^*$. The new semantics addresses the aforementioned problem of $\mathbf{m}\mathcal{A}^*$ while maintaining the simplicity of its semantics; the new definitions continue to employ simple update models, with at most three events for all types of actions, which can be constructed given the action specification, independently from the state in which the action occurs. Properties of the new semantics are also discussed.

Keywords: Epistemic Reasoning · Update Models · Action Language.

1 Introduction

In multi-agent environments, agents not only need to reason about properties of the world, but also about their knowledge and beliefs about the world and about other agents' knowledge and beliefs. Among the various formalisms for reasoning about actions in Multi-Agent Systems (MAS), a commonly used one is the *action model*, introduced in [1, 2] and later extended to *update model* [5, 9].¹ For example, update models have been employed in the study of epistemic planning problems in MAS [6, 11, 3].

Intuitively, an update model encodes different views of agents about an action occurrence, which ultimately affects the beliefs of agents about the state of the world and the state of beliefs of other agents after the action occurrence. Formally, given a pointed Kripke structure (M, s) representing the current state

¹ For the sake of simplicity, we will use the generic term “*update model*” to refer to both “action models” and “update models.” Furthermore, we will frequently use the statement “*execution of an action*” to indicate the result of applying an update model.

of the world and of the knowledge/beliefs of agents, the result of the execution of an action \mathbf{a} in (M, s) is determined by a cross product operation of (M, s) and the update model $U(\mathbf{a})$ — corresponding to the occurrence of \mathbf{a} in (M, s) .

The action language $\mathbf{m}\mathcal{A}^*$ and its earlier versions, proposed in [4], are among the first action languages that utilize update models in defining a transition function based semantics for multi-agent domains. A similar idea has recently been developed by [12]. Given an action occurrence, a corresponding update model is automatically derived and used for computing the resulting state. This construction is simple, in that it only uses update models with at most three events. However, as discussed by several researchers (e.g., [4, 8]), the simplicity of $\mathbf{m}\mathcal{A}^*$ presents some challenges for its application. The next example, modified from [4], highlights this issue:

Example 1. Three agents, A, B and C , are in a room with a box containing a coin. It is common knowledge that:

- The agents do not know whether the coin lies heads or tails up;
- The box is locked and only A knows how to open it;
- In order to learn the position of the coin, an agent should peek into the box, which requires the box to be open;
- If one agent is looking at the box and a second agent peeks into the box, then the first agent will observe this fact and will be able to conclude that the second agent knows the status of the coin; nevertheless, the first agent's knowledge about which face of the coin is up does not change.

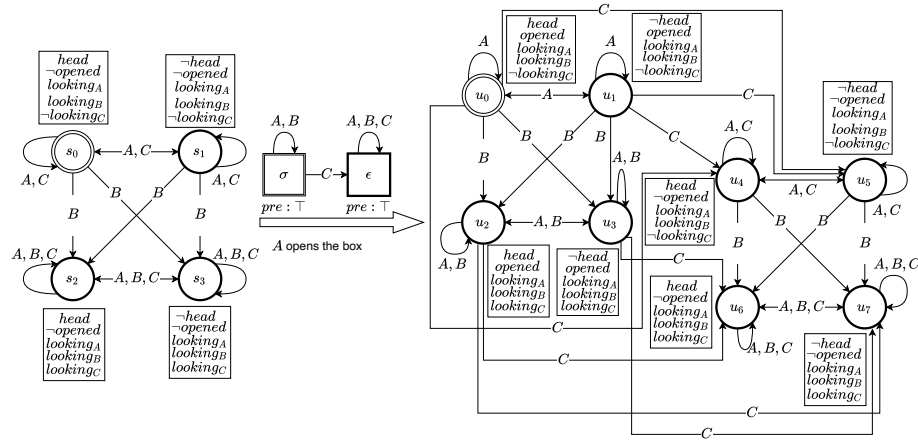


Fig. 1. A opens the box

Suppose that only A and B are looking at the box. However, B believes that all three agents have their eyes on the box. The situation is illustrated by the pointed Kripke structure on top of Figure 1, where circles represent possible

worlds, labeled links encode the accessibility relations of agents, and the boxed texts next to the circles encode the valuations associated with the worlds. Double circle represents the true state of the world.

Assume that A opens the box and anyone who is looking at the box will observe this action. Intuitively, after A opens the box, B should believe that C is also aware of the box being open. The design of $\mathbf{m}\mathcal{A}^*$ produces the pointed Kripke structure at the bottom of Figure 1, leading to the conclusion that B thinks that C considers the box still closed (because of the paths with the labels B and C from u_0 : $u_0 \xrightarrow{B} u_2/u_3 \xrightarrow{C} u_6/u_7$). The reason for this is that the current update models in $\mathbf{m}\mathcal{A}^*$ assume that all full observers (A and B in this example) know about the observability of all other agents. While this assumption is reasonable in many situations, it implies that the use of $\mathbf{m}\mathcal{A}^*$ requires very careful considerations from the domain designers, as pointed out in [4].

In this paper, we propose an extension of the language $\mathbf{m}\mathcal{A}^*$ that can handle situations where agents' have false/incorrect beliefs about the observability of other agents, using *edge-conditioned event update models*, originally introduced by [7]. We begin with a short review of the language $\mathbf{m}\mathcal{A}^*$, then show how to apply the edge-conditioned event update models to help $\mathbf{m}\mathcal{A}^*$ solve the type of problems mentioned in the previous example. We prove relevant properties and provide some final considerations.

2 Background

Belief Formulae. A multi-agent domain $\langle \mathcal{AG}, \mathcal{F} \rangle$ includes a finite and non-empty set of agents \mathcal{AG} and a set of fluents (atomic propositions) \mathcal{F} encoding properties of the world. *Belief formulae* over $\langle \mathcal{AG}, \mathcal{F} \rangle$ are defined by the BNF:

$$\varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid \mathbf{B}_i\varphi$$

where $p \in \mathcal{F}$ is a fluent and $i \in \mathcal{AG}$. We refer to a belief formula which does not contain any occurrence of \mathbf{B}_i as a *fluent formula*. In addition, for a formula φ and a non-empty set $\alpha \subseteq \mathcal{AG}$, $\mathbf{B}_\alpha\varphi$ and $\mathbf{C}_\alpha\varphi$ denote $\bigwedge_{i \in \alpha} \mathbf{B}_i\varphi$ and $\bigwedge_{k=1}^\infty \mathbf{B}_\alpha^k\varphi$, where $\mathbf{B}_\alpha^1\varphi = \mathbf{B}_\alpha\varphi$ and $\mathbf{B}_\alpha^k\varphi = \mathbf{B}_\alpha^{k-1}\mathbf{B}_\alpha\varphi$ for $k > 1$, respectively. $\mathcal{L}_{\mathcal{AG}}$ denotes the set of belief formulae over $\langle \mathcal{AG}, \mathcal{F} \rangle$.

Satisfaction of belief formulae is defined over *pointed Kripke structures* [10]. A Kripke structure M is a tuple $\langle S, \pi, \{\mathcal{B}_i\}_{i \in \mathcal{AG}} \rangle$, where S is a set of worlds (denoted by $M[S]$), $\pi : S \mapsto 2^{\mathcal{F}}$ is a function that associates an interpretation of \mathcal{F} to each element of S (denoted by $M[\pi]$), and for $i \in \mathcal{AG}$, $\mathcal{B}_i \subseteq S \times S$ is a binary relation over S (denoted by $M[i]$). For convenience, we will often draw a Kripke structure M as a directed labeled graph, whose set of labeled nodes represents S and whose set of labeled edges contains $s \xrightarrow{i} t$ iff $(s, t) \in \mathcal{B}_i$; the label of each node is the name of the world and its interpretation is displayed as a text box next to it (see, e.g., Figure 1). For $u \in S$ and a fluent formula φ , $M[\pi](u)$ and $M[\pi](u)(\varphi)$ denote the interpretation associated to u via π and the truth value of φ with respect to $M[\pi](u)$. For a world $s \in M[S]$, (M, s) is a *pointed Kripke structure*, hereafter called a *state*.

The satisfaction relation \models between belief formulae and a state (M, s) is defined as follows:

1. $(M, s) \models p$ if p is a fluent and $M[\pi](s)(p)$ is true;
2. $(M, s) \models \neg\varphi$ if $(M, s) \not\models \varphi$;
3. $(M, s) \models \varphi_1 \wedge \varphi_2$ if $(M, s) \models \varphi_1$ and $(M, s) \models \varphi_2$;
4. $(M, s) \models \varphi_1 \vee \varphi_2$ if $(M, s) \models \varphi_1$ or $(M, s) \models \varphi_2$;
5. $(M, s) \models \mathbf{B}_i\varphi$ if $\forall t. [(s, t) \in \mathcal{B}_i \Rightarrow (M, t) \models \varphi]$.

Edge-Conditioned Update Models. The formalism of *update models* has been used to describe transformations of states according to a predetermined transformation pattern (see, e.g., [1, 5]). This formalism makes use of the notion of \mathcal{L}_{AG} -substitution, which is a set $\{p_1 \rightarrow \varphi_1, \dots, p_k \rightarrow \varphi_k\}$, where each p_i is a distinct fluent in \mathcal{F} and each $\varphi_i \in \mathcal{L}_{AG}$. $SUB_{\mathcal{L}_{AG}}$ denotes the set of all \mathcal{L}_{AG} -substitutions. To handle the nested belief about agents' observability problem, in this extension of \mathbf{mA}^* , we will utilize the *edge-conditioned event update models* as proposed by [7]. In edge-conditioned event update models, the assumption that full and partial observers know observability of all agents no longer holds. An edge-conditioned event update model Σ is a tuple $\langle \Sigma, \{R_i\}_{i \in AG}, pre, sub \rangle$ where Σ is a set of events, $R_i \subseteq \Sigma \times \mathcal{L}_{AG} \times \Sigma$ is the accessibility relation of agent i between events, $pre : \Sigma \rightarrow \mathcal{L}_{AG}$ is a function mapping each event $e \in \Sigma$ to a formula in \mathcal{L}_{AG} , $sub : \Sigma \rightarrow SUB_{\mathcal{L}_{AG}}$ is a function mapping each event $e \in \Sigma$ to a substitution in $SUB_{\mathcal{L}_{AG}}$. Elements of R_i are of the form (e_1, γ, e_2) where γ is a belief formula. In the graph representation, such an accessibility relation is shown by a directed edge from e_1 to e_2 with the label $i : \gamma$. We will omit γ and write simply i as label of the edge when $\gamma = \top$. Given an edge-conditioned update model Σ , an *update instance* ω is a pair (Σ, e) where e is an event in Σ , referred to as a *designated event* (or *true event*). For simplicity of presentation, we often draw an update instance as a graph whose events are rectangles, whose links represent the accessibility relations between events, and a double square represents the *designated event* (see, e.g., Figure 3).

Given a Kripke structure M and an edge-conditioned update model $\Sigma = \langle \Sigma, \{R_i\}_{i \in AG}, pre, sub \rangle$, the *update* of M induced by Σ results in a new Kripke structure M' , denoted by $M' = M \otimes \Sigma$, defined by:

- (i) $M'[S] = \{(s, \tau) \mid \tau \in \Sigma, s \in M[S], (M, s) \models pre(\tau)\}$;
- (ii) $((s, \tau), (s', \tau')) \in M'[i]$ iff $(s, \tau), (s', \tau') \in M'[S], (s, s') \in M[i], (\tau, \gamma, \tau') \in R_i$ and $(M, s) \models \gamma$;
- (iii) For all $(s, \tau) \in M'[S]$ and $f \in \mathcal{F}$, $M'[\pi]((s, \tau)) \models f$ if $f \rightarrow \varphi \in sub(\tau)$ and $(M, s) \models \varphi$.

An *update template* is a pair (Σ, Γ) , where Σ is an update model with the set of events Σ and $\Gamma \subseteq \Sigma$. The update of a pointed Kripke structure (M, s) given an update template (Σ, Γ) is a set of pointed Kripke structures, denoted by $(M, s) \otimes (\Sigma, \Gamma)$, where $(M, s) \otimes (\Sigma, \Gamma) = \{(M \otimes \Sigma, (s, \tau)) \mid \tau \in \Gamma, (M, s) \models pre(\tau)\}$.

Syntax of $\mathbf{m}\mathcal{A}^$.* An action theory in the language $\mathbf{m}\mathcal{A}^*$ over $\langle \mathcal{AG}, \mathcal{F} \rangle$ consists of a set of action instances \mathcal{AI} of the form $a\langle \alpha \rangle$, representing that a set of agents α performs action a , and a collection of statements of the following forms:

$$\mathbf{a} \text{ executable_if } \psi \quad (1)$$

$$\mathbf{a} \text{ causes } \ell \text{ if } \varphi \quad (2)$$

$$\mathbf{a} \text{ determines } \varphi \quad (3)$$

$$\mathbf{a} \text{ announces } \varphi \quad (4)$$

$$z \text{ observes } \mathbf{a} \text{ if } \delta_z \quad (5)$$

$$z \text{ aware_of } \mathbf{a} \text{ if } \theta_z \quad (6)$$

$$\text{initially } \psi \quad (7)$$

where ℓ is a fluent literal (a fluent $f \in \mathcal{F}$ or its negation $\neg f$), ψ is a belief formula, φ , δ_z and θ_z are fluent formulae, $\mathbf{a} \in \mathcal{AI}$, and $z \in \mathcal{AG}$. (1) encodes the executability condition of \mathbf{a} and ψ is referred as the *precondition* of \mathbf{a} . (2) describes the effect of the ontic action \mathbf{a} , i.e., if ψ is true then ℓ will be true after the execution of \mathbf{a} . (3) enables the agents who execute \mathbf{a} to learn the value of the formula φ . (4) encodes an *announcement* action, whose owner announces that φ is true. Statements of the forms (5)–(6) encode the observability of agents given an occurrence of \mathbf{a} . (5) indicates that agent z is a full observer of \mathbf{a} if δ_z holds. (6) states that agent z is a partial observer of \mathbf{a} if θ_z holds. z , \mathbf{a} , and δ_z (resp. θ_z) are referred to as the observing agent, the action instance, and the condition of (5) (resp. (6)). It is assumed that the sets of ontic actions, sensing actions, and announcement actions are pairwise disjoint. Furthermore, for every pair of \mathbf{a} and z , if z and \mathbf{a} occur in a statement of the form (5) then they do not occur in any statement of the form (6) and vice versa. Also, we assume that $z \text{ observes } \mathbf{a} \text{ if } False$ (or $z \text{ aware_of } \mathbf{a} \text{ if } False$) is in the action theory if the information about δ_z (or θ_z) is not given. Statements of the form (7) indicate that ψ is true in the initial state. An action domain is a collection of statements (1)–(6). An action theory is a pair of an action domain and a set of statements of the form (7). By this definition, action domains are deterministic in that each ontic action, when executed in a world, results in a unique world.

3 Edge-Conditioned Event Update Models

In this section, we will show how to define the transition function $\mathbf{m}\mathcal{A}^*$ using edge-conditioned event update models. We will use Example 1 as a running example to illustrate the application of edge-conditioned event update models. We follow the same notation and rules as in Section 2.

Let us denote the multi-agent domain described in Example 1 by D_{coin} . For this domain, we have that $\mathcal{AG} = \{A, B, C\}$. The set of fluents \mathcal{F} for this domain consists of *head* (the coin is heads up), *looking_x* (agent x is looking at the box where $x \in \{A, B, C\}$), and *opened* (the box is open). D_{coin} has two actions: *open* and *peek*. These two actions can be represented by the following $\mathbf{m}\mathcal{A}^*$

statements:

$$\text{open}\langle x \rangle \text{ causes } \text{opened} \quad (8)$$

$$x \text{ observes } \text{open}\langle x \rangle \quad (9)$$

$$y \text{ observes } \text{open}\langle x \rangle \text{ if } \text{looking}_y \quad (10)$$

$$\text{peek}\langle x \rangle \text{ executable_if } \text{opened} \quad (11)$$

$$\text{peek}\langle x \rangle \text{ determines } \text{head} \quad (12)$$

$$x \text{ observes } \text{peek}\langle x \rangle \quad (13)$$

$$y \text{ aware_of } \text{peek}\langle x \rangle \text{ if } \text{looking}_y \quad (14)$$

where $x, y \in \{A, B, C\}$ and $x \neq y$.

Initially, the coin is heads up, the box is closed and A, B are looking at it; however, B thinks that all three agents are looking at the box. The initial state of D_{coin} is (M_0, s_0) in Figure 2.

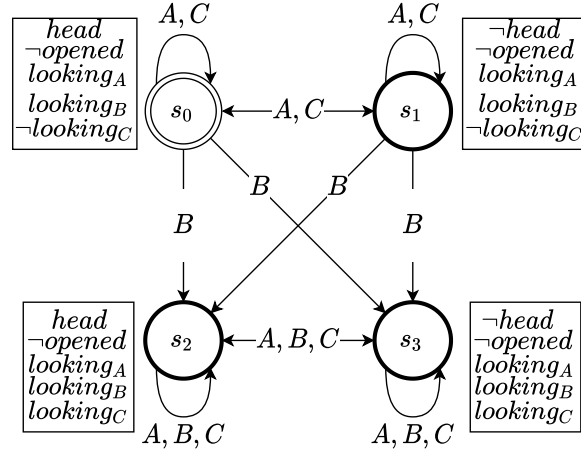


Fig. 2. The initial state (M_0, s_0)

Suppose that agent A would like to know whether the coin lies heads or tails up. She would also like to let agent B know that she knows this fact. However, she would like to make B also thinks that agent C is aware of this fact. Intuitively, because B has already believed that C is looking at the box, agent A could achieve her goals by:

1. Opening the box; and
2. Peeking into the box.

Observe that under the current semantics of mA^* [4], A could not achieve her goal by executing the above sequence of actions. This is because B believes that C does not know that the box is open (as showed in Figure 1) which is

because C does not look at the box in the true world (s_0), B would conclude that C is not observing the execution of the action of A opening the box. When A peeks into the box, B reasons that C is still thinking that A knows nothing because, according to B , C still believes that the box is closed. Therefore, B will think that C 's belief about A 's belief about the state of the coin does not change. This is not intuitive.

A more intuitive outcome with respect to B 's beliefs after the execution of the plan $[open(A); peek(A)]$ is as follows: B should believe that C knows that the box is open and that A knows the value of the coin after the execution of the plan.

The main reason for the above inadequacy of $\mathbf{m}\mathcal{A}^*$ lies in the fact that the construction of the update models in $\mathbf{m}\mathcal{A}^*$ assumes that full observers have the correct observability of *all agents*, which is not the case for B , who is a full observer, and C : B believes that C is a full observer while C is not. One possible way to address the above issue is to create different update models, whose set of events depends on the effects of actions, as done in [12], or to define transition functions by directly manipulating the accessibility relations and the worlds in the resulting Kripke structure as in [8]. In this paper, we introduce a different approach to this problem, through the use of edge-conditioned update models.

3.1 Edge-Conditioned Update Models for Ontic Actions

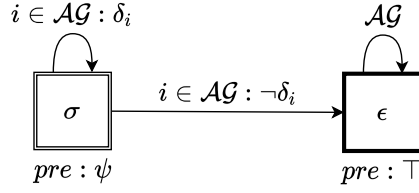
We assume that an action domain D is given. As in $\mathbf{m}\mathcal{A}^*$, we assume that an agent can either observe or not observe the execution of an ontic action, i.e., for an ontic action \mathbf{a} , there exists no statement of the form (6) whose action is \mathbf{a} .

Definition 1 (Ontic Actions). Let \mathbf{a} be an ontic action with the precondition ψ . The update model for \mathbf{a} , denoted by $\omega(\mathbf{a})$, is defined by $\langle \Sigma, \{R_i\}_{i \in \mathcal{AG}}, pre, sub \rangle$ where

- $\Sigma = \{\sigma, \epsilon\}$;
- $R_i = \{(\sigma, \delta_i, \sigma), (\sigma, \neg\delta_i, \epsilon), (\epsilon, \top, \epsilon)\}$ where “ i observes \mathbf{a} if δ_i ” belongs to D ;
- $pre(\sigma) = \psi$ and $pre(\epsilon) = \top$; and
- $sub(\epsilon) = \emptyset$ and $sub(\sigma) = \{p \rightarrow \Psi^+(p, \mathbf{a}) \vee (p \wedge \neg\Psi^-(p, \mathbf{a})) \mid p \in \mathcal{F}\}$, where $\Psi^+(p, \mathbf{a}) = \bigvee \{\varphi \mid [\mathbf{a} \text{ causes } p \text{ if } \varphi] \in D\}$ and $\Psi^-(p, \mathbf{a}) = \bigvee \{\varphi \mid [\mathbf{a} \text{ causes } \neg p \text{ if } \varphi] \in D\}$.

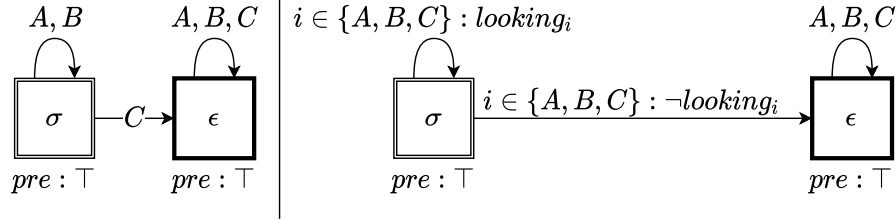
When an ontic action occurs, an agent may or may not observe its occurrence. As such, $\omega(\mathbf{a})$ has two events. σ is the designated event representing the true occurrence of the action whereas ϵ denotes the null event representing that the action does not occur. σ is the event full observers believe occurring and ϵ is the event seen by oblivious agents. Figure 3 shows the edge-conditioned update model of an ontic action \mathbf{a} . In the figure, we use $i \in X : \delta_i$ as a shorthand for the set of links with labels $\{i : \delta_i \mid i \in X\}$.

Observe that the presence of the condition attached to the link and the definition of the cross product between a Kripke structure and the update model enable a flexible update of the accessibility relations, allowing us to eliminate

**Fig. 3.** Edge-Conditioned Update Model for an Ontic Action

the problem of the definition in [4]. For example, given a state (M, s) , the link $(\sigma, i : \delta_i, \sigma)$ in $\omega(\mathbf{a})$ indicates that $((\sigma, s), i, (\sigma, s))$ is an element in the accessibility relation of i in the state resulting from the execution of \mathbf{a} in (M, s) only if $(M, s) \models \delta_i$.

The difference between the edge-conditioned update model of the occurrence of $\text{open}\langle A \rangle$ and the update model defined in [4] is shown in Figure 4.

**Fig. 4.** Old (left) vs. Edge-conditioned (right) update model for $\text{open}\langle A \rangle$

The update induced by the edge-conditioned update model for $\text{open}\langle A \rangle$ on the pointed Kripke structure in Figure 2 is shown in Figure 5. In this figure, the worlds and their interpretations are the same as in the pointed Kripke structure at the bottom of Figure 2. The differences lie in the removal of the links labeled C from u_2/u_3 to u_6/u_7 and the addition of the loops labeled C at u_2/u_3 . The loops labeled C at u_2 and u_3 , denoting the worlds (s_2, σ) and (s_3, σ) , respectively, are added because $(M_0, s_2) \models \text{looking}_C$ and $(M_0, s_3) \models \text{looking}_C$ hold. This is also the reason for the removal of the links labeled C from u_2 and u_3 to u_6 and u_7 .

3.2 Edge-Conditioned Update Models for Sensing/Announcement Actions

An agent can either observe, partially observe, or not observe the occurrence of a sensing or announcement action occurrence. Therefore, the update models for sensing or announcement actions are different from that of update models for ontic actions. They are defined as follows.

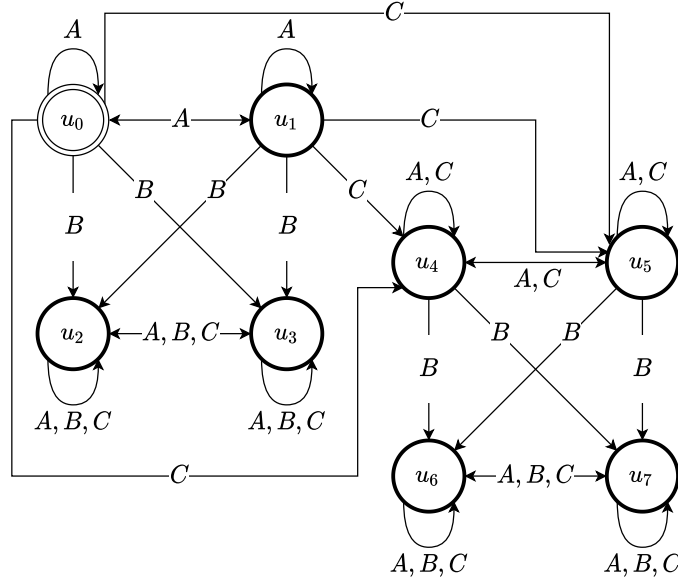


Fig. 5. (M_1, v_0) after A opened the box using edge-conditioned update model

Definition 2 (Sensing and Announcement Actions). Let \mathbf{a} be a sensing action that senses φ or an announcement action that announces φ with the precondition ψ . The update model for \mathbf{a} , denoted by $\omega(\mathbf{a})$, is defined by $\langle \Sigma, \{R_i\}_{i \in \mathcal{AG}}, pre, sub \rangle$ where

- $\Sigma = \{\sigma, \tau, \epsilon\}$;
- $R_i = \{(\sigma, \delta_i \vee \theta_i, \sigma), (\tau, \delta_i \vee \theta_i, \tau), (\sigma, \neg\delta_i \wedge \theta_i, \tau), (\tau, \neg\delta_i \wedge \theta_i, \sigma), (\sigma, \neg\delta_i \wedge \neg\theta_i, \epsilon), (\tau, \neg\delta_i \wedge \neg\theta_i, \epsilon), (\epsilon, \top, \epsilon)\}$ where “ i observes \mathbf{a} if δ_i ” and “ i aware_of \mathbf{a} if θ_i ” belong to D ;
- $pre(\sigma) = \psi \wedge \varphi$, $pre(\tau) = \psi \wedge \neg\varphi$ and $pre(\epsilon) = \top$;
- $sub(x) = \emptyset$ for each $x \in \Sigma$.

Observe that an update model of a sensing or announcement action has three events. However in sensing actions, the true event can be σ or τ whereas in announcement actions, the true event is σ .² As for ontic actions, ϵ is the “null event” representing that the action does not occur. Sensing and announcement actions do not alter the state of the world and thus sub is empty for every event. Full observers learn the value of the formula while partial observers only know that the action has taken place without learning the actual outcome. Fig. 6 illustrates the edge-conditioned update model for an announcement \mathbf{a} that truthfully announces φ (right) and a sensing action \mathbf{a} that determines φ (left).

A comparison between the update models for the execution of the action $peek\langle A \rangle$ after the execution of $open\langle A \rangle$ in the formalization by [4] and the present

² Assuming that announcements are truthful.

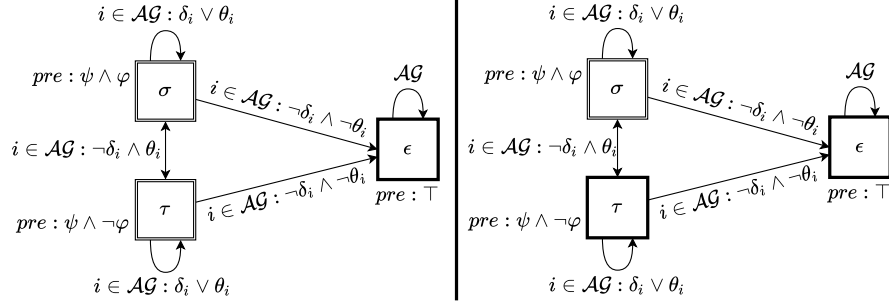


Fig. 6. Edge-Conditioned Update Models for Sensing Action (left) and Truthful Announcement Action (right)

paper is shown in Figure 7. Similar to construction of edge-conditioned update models for ontic actions, the edge-conditioned update models for this action (or generally, for sensing and announcement actions) is *independent* from the state in which the action occurs (i.e., (M, s)).

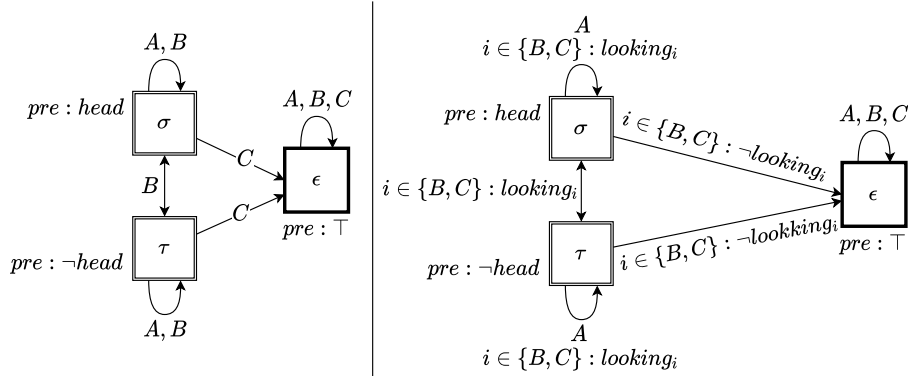


Fig. 7. Old (left) vs. Edge-conditioned (right) update model for $peek\langle A \rangle$

The application of the edge-conditioned update model for $peek\langle A \rangle$ in the state (M_1, u_0) from Figure 5 is given in Figure 8. In this figure, w_0 - w_3 have the same interpretation as u_0 - u_3 ; and w_4 - w_{11} have the same interpretation as u_0 - u_7 . Observe that the execution of $peek\langle A \rangle$ after $open\langle A \rangle$ in the original semantics of $m\mathcal{A}^*$, i.e., the execution of $peek\langle A \rangle$ in the pointed Kripke structure at the bottom of Figure 2, also results in a similar structure with 12 worlds w_0 - w_{11} . The key difference is that it does not have the links labeled C from w_2/w_3 and w_6/w_7 to w_{10}/w_{11} and the addition of the loop labeled C at w_2/w_3 and w_{10}/w_{11} . The

loop labeled C at w_2/w_3 , denoting (u_2, σ) and (u_3, τ) , respectively, are added because $(M_1, v_2) \models \text{looking}_C$ and $(M_1, v_3) \models \text{looking}_C$ hold. This is also the reason for the removal of the links labeled C from w_2/w_3 to w_{10}/w_{11} .

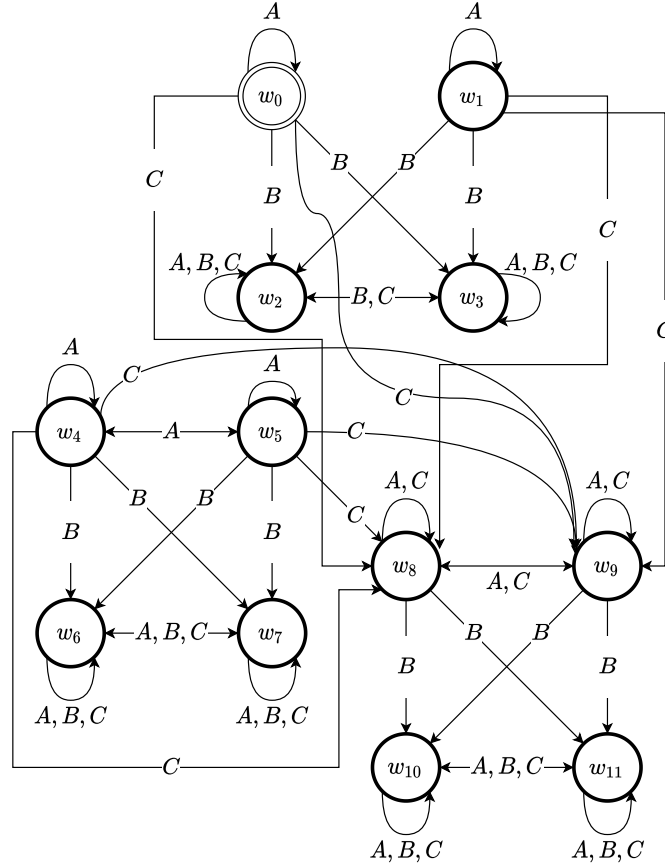


Fig. 8. (M_2, w_0) after A peeked into the box

As we can observe from (M_2, w_0) , A now achieves her goals: not only does A realize that the coin lies head up ($(M_2, w_0) \models \mathbf{B}_A \text{head}$) but B also believes that C knows the fact that A knows the value of the coin now ($(M_2, w_0) \models \mathbf{B}_B \mathbf{B}_C (\mathbf{B}_A \text{head} \vee \mathbf{B}_A \neg \text{head})$). This example shows that the use of edge-conditioned update models enables \mathbf{mA}^* to avoid the side problem discussed in [4].

Having defined the update models for actions in a multi-agent domain D , we can define the transition function Φ_D in D in the similar fashion as in [4]. We omit the details for brevity.

3.3 Properties of Edge-Conditioned Update Models

As illustrate in the above example, the use of edge-conditioned update models enables the modification of the semantics of $\mathbf{m}\mathcal{A}^*$ that takes into consideration the observability of the agents at the local level (i.e., at each possible world). A consequence of this treatment is that the belief of an agent i , who is a full observer, about the belief of another agent j with respect to the action occurrence will change in accordance to the belief of i about j in the true state of the world. The following proposition indicates these properties of edge-conditioned update models.

Proposition 1. *Let (M, s) be a state and \mathbf{a} be an ontic action instance that is executable in (M, s) and $\omega(\mathbf{a})$ be given in Definition 1. It holds that:*

1. *For every agent $x \in \mathcal{AG}$, $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$ and $[\mathbf{a} \text{ causes } \ell \text{ if } \varphi]$ belong to D , if $(M, s) \models \delta_x$, $(M, s) \models \mathbf{B}_x \varphi$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \sigma)$ then $(M', s') \models \mathbf{B}_x \ell$.*
2. *For every pair of agents $x, y \in \mathcal{AG}$, $[\mathbf{a} \text{ causes } \ell \text{ if } \varphi]$, $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$ and $[y \text{ observes } \mathbf{a} \text{ if } \delta_y]$ belong to D , if $(M, s) \models \delta_x$, $(M, s) \models \mathbf{B}_x \delta_y$, $(M, s) \models \mathbf{B}_x \mathbf{B}_y \varphi$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \sigma)$ then $(M', s') \models \mathbf{B}_x \mathbf{B}_y \ell$.*
3. *For every pair of agents $x, y \in \mathcal{AG}$, a belief formula η , $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$ and $[y \text{ observes } \mathbf{a} \text{ if } \delta_y]$ belong to D , if $(M, s) \models \delta_x$, $(M, s) \models \mathbf{B}_x \neg \delta_y$, $(M, s) \models \mathbf{B}_x \mathbf{B}_y \eta$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \sigma)$ then $(M', s') \models \mathbf{B}_x \mathbf{B}_y \eta$.*

Proof. We have that $s' = (s, \sigma)$. Assume that the fluent in ℓ is p , i.e., $\ell = p$ or $\ell = \neg p$. Let $\Psi^+(p, \mathbf{a}) = \bigvee \{\varphi \mid [\mathbf{a} \text{ causes } p \text{ if } \varphi] \in D\}$ and $\Psi^-(p, \mathbf{a}) = \bigvee \{\varphi \mid [\mathbf{a} \text{ causes } \neg p \text{ if } \varphi] \in D\}$ and $\gamma = \Psi^+(p, \mathbf{a}) \vee \Psi^-(p, \mathbf{a})$. By Definition 1, $p \rightarrow \gamma \in \text{sub}(\sigma)$.

1. *Proof of the first item:* For every $u' \in M'[S]$ such that $(s', u') \in M'[x]$, it holds that $u' = (u, \sigma)$ for some $u \in M[S]$, $(M, u) \models \psi$ and $(s, u) \in M[x]$. Because $(M, s) \models \mathbf{B}_x \varphi$, we have $(M, u) \models \varphi$. Consider two cases:
 - $\ell = p$. Then, $(M, u) \models \Psi^+(p, \mathbf{a})$, and, $(M, u) \models \gamma$. So, $M'[\pi]((u, \sigma)) \models p$.
 - $\ell = \neg p$. Then, because $(M, u) \models \varphi$, the consistency of D implies that $(M, u) \not\models \gamma$. Therefore, $M'[\pi]((u, \sigma)) \not\models p$, i.e., $M'[\pi]((u, \sigma)) \models \neg p$.
 Both cases imply that $M'[\pi]((u, \sigma)) \models \ell$. This holds for every $u' \in M'[S]$ such that $(s', u') \in M'[x]$, which implies $(M', s') \models \mathbf{B}_x \ell$.
2. *Proof of the second item:* Consider $u', v' \in M'[S]$ such that $(s', u') \in M'[x]$, $(u', v') \in M'[y]$. Since $(M, s) \models \delta_x$ and $(M, s) \models \mathbf{B}_x \delta_y$, it holds that $v' = (v, \sigma)$, $u' = (u, \sigma)$ for some $u, v \in M[S]$, $(s, u) \in M[x]$, $(u, v) \in M[y]$ and $(M, v) \models \psi$. Because $(M, s) \models \mathbf{B}_x \mathbf{B}_y \varphi$, we have $(M, v) \models \varphi$. Consider two cases:
 - $\ell = p$. Then, $(M, v) \models \Psi^+(p, \mathbf{a})$, and, $(M, v) \models \gamma$. So, $M'[\pi]((v, \sigma)) \models p$.
 - $\ell = \neg p$. Then, because $(M, v) \models \varphi$, the consistency of D implies that $(M, v) \not\models \gamma$. Therefore, $M'[\pi]((v, \sigma)) \not\models p$, i.e., $M'[\pi]((v, \sigma)) \models \neg p$.
 Both cases imply that $M'[\pi]((v, \sigma)) \models \ell$. This holds for every $v', u' \in M'[S]$ such that $(s', u') \in M'[x]$, $(u', v') \in M'[y]$, which implies $(M', s') \models \mathbf{B}_x \mathbf{B}_y \ell$.

3. Proof of the third item: By the construction of M' , we have the following observations:

- For every $u \in M[S]$ iff $(u, \epsilon) \in M'[S]$;
- For every $z \in \mathcal{AG}$, $(u, v) \in M[z]$ iff $((u, \epsilon), (v, \epsilon)) \in M'[z]$;
- For every $u \in M[S]$ and $p \in \mathcal{F}$, $M'[\pi]((u, \epsilon)) \models p$ iff $(M', (u, \epsilon)) \models p$ because $\text{sub}(\epsilon) = \emptyset$.

These observations allow us to conclude for every formula η , $(M, u) \models \eta$ iff $(M', (u, \epsilon)) \models \eta$. Consider $u', v' \in M'[S]$ such that $(s', u') \in M'[x]$, $(u', v') \in M'[y]$. Since $(M, s) \models \delta_x$ and $(M, s) \models \mathbf{B}_x \neg \delta_y$, it holds that $v' = (v, \epsilon)$, $u' = (u, \sigma)$ for some $u, v \in M[S]$, $(s, u) \in M[x]$ and $(u, v) \in M[y]$. Assume that $(M, s) \models \mathbf{B}_x \mathbf{B}_y \eta$. This implies $(M, v) \models \eta$, means that $(M', (v, \epsilon)) \models \eta$, i.e., which implies $(M', s') \models \mathbf{B}_x \mathbf{B}_y \eta$. \square

The first item of Proposition 1 illustrates the fact that full observers will update their beliefs about the true world since they see that the action occur. The second and third item of Proposition 1 show that a full observer will update her beliefs about another agent's beliefs, if she thinks that the agent is also a full observer, or her own beliefs about the other agent will not change if she believes that such agent is unaware of the action occurrence. These two items do not hold w.r.t. the old semantics of $\mathbf{m}\mathcal{A}^*$. Similar propositions can be established for sensing actions and announcement actions and are shown as follows:

Proposition 2. *Let (M, s) be a state and \mathbf{a} be a sensing action instance senses φ that is executable in (M, s) and $\omega(\mathbf{a})$ be given in Definition 2. It holds that:*

1. *For every agent $x \in \mathcal{AG}$, $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$ belong to D , if $(M, s) \models \delta_x$, $(M, s) \models \varphi$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \{\sigma, \tau\})$ then $(M', s') \models \mathbf{B}_x \varphi$.*
2. *For every agent $x \in \mathcal{AG}$, $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$ belong to D , if $(M, s) \models \delta_x$, $(M, s) \models \neg \varphi$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \{\sigma, \tau\})$ then $(M', s') \models \mathbf{B}_x \neg \varphi$.*
3. *For every agent $x \in \mathcal{AG}$, $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$ and $[x \text{ aware_of } \mathbf{a} \text{ if } \theta_x]$ belong to D , if $(M, s) \models \neg \delta_x \wedge \theta_x$, $(M, s) \not\models (\mathbf{B}_x \varphi \vee \mathbf{B}_x \neg \varphi)$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \{\sigma, \tau\})$ then $(M', s') \not\models (\mathbf{B}_x \varphi \vee \mathbf{B}_x \neg \varphi)$.*
4. *For every pair of agents $x, y \in \mathcal{AG}$, $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$, $[y \text{ observes } \mathbf{a} \text{ if } \delta_y]$ and $[y \text{ aware_of } \mathbf{a} \text{ if } \theta_y]$ belong to D , if $(M, s) \models \mathbf{B}_y \delta_x$, $(M, s) \models \delta_y \vee \theta_y$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \{\sigma, \tau\})$ then $(M', s') \models \mathbf{B}_y (\mathbf{B}_x \varphi \vee \mathbf{B}_x \neg \varphi)$.*
5. *For every pair of agents $x, y \in \mathcal{AG}$, a belief formula η , $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$, $[x \text{ aware_of } \mathbf{a} \text{ if } \theta_x]$, $[y \text{ observes } \mathbf{a} \text{ if } \delta_y]$ and $[y \text{ aware_of } \mathbf{a} \text{ if } \theta_y]$ belong to D , if $(M, s) \models \mathbf{B}_x \neg (\delta_y \vee \theta_y)$, $(M, s) \models \delta_x \vee \theta_x$, $(M, s) \models \mathbf{B}_x \mathbf{B}_y \eta$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \{\sigma, \tau\})$ then $(M', s') \models \mathbf{B}_x \mathbf{B}_y \eta$.*

Proof. We will prove for the case $(M, s) \models \varphi$, the proof when $(M, s) \models \neg \varphi$ is similar and is omitted here. We have that $s' = (s, \sigma)$.

1. *Proof of the first item:* For every $u' \in M'[S]$ such that $(s', u') \in M'[x]$, since $(M, s) \models \delta_x$ it holds that $u' = (u, \sigma)$ for some $u \in M[S]$ and $(s, u) \in M[x]$. Which means $(M', s') \models \mathbf{B}_x \varphi$.

2. *Proof of the third item:* For every $u' \in M'[S]$ such that $(s', u') \in M'[x]$, since $(M, s) \models \neg\delta_x \wedge \theta$ it holds that $u' = (u, \sigma)$ or $u' = (u, \tau)$ for some $u \in M[S]$ and $(s, u) \in M[x]$. We have that $(M, s) \not\models (\mathbf{B}_x\varphi \vee \mathbf{B}_x\neg\varphi)$, which mean $\exists u_1, u_2 \in M[S]$ such that, $(M, u_1) \models \varphi$, $(M, u_2) \models \neg\varphi$ and $(s, u_1), (s, u_2) \in M[x]$. From this we have $(M', (u_1, \sigma)) \models \varphi$, $(M', (u_2, \tau)) \models \neg\varphi$ and $(s', (u_1, \sigma)), (s', (u_2, \tau)) \in M'[x]$. Which implies $(M', s') \not\models (\mathbf{B}_x\varphi \vee \mathbf{B}_x\neg\varphi)$.
3. *Proof of the fourth item:* Consider $u', v' \in M'[S]$ such that $(s', u') \in M'[y]$, $(u', v') \in M'[x]$. Since $(M, s) \models \delta_y \vee \theta_y$ and $(M, s) \models \mathbf{B}_y\delta_x$, it holds that $u' = (u, \sigma)$ and $v' = (v, \sigma)$ (or $u' = (u, \tau)$ and $v' = (v, \tau)$) for some $u, v \in M[S]$, $(s, u) \in M[y]$ and $(u, v) \in M[x]$. Which means $(M', s') \models \mathbf{B}_y(\mathbf{B}_x\varphi \vee \mathbf{B}_x\neg\varphi)$.
4. *Proof of the fifth item:* By the construction of M' , we have the following observations:
 - For every $u \in M[S]$ iff $(u, \epsilon) \in M'[S]$;
 - For every $z \in \mathcal{AG}$, $(u, v) \in M[z]$ iff $((u, \epsilon), (v, \epsilon)) \in M'[z]$;
 - For every $u \in M[S]$ and $p \in \mathcal{F}$, $M'[\pi]((u, \epsilon)) \models p$ iff $(M', (u, \epsilon)) \models$ because $sub(\epsilon) = \emptyset$.

These observations allow us to conclude for every formula η , $(M, u) \models \eta$ iff $(M', (u, \epsilon)) \models \eta$. Consider $u', v' \in M'[S]$ such that $(s', u') \in M'[x]$, $(u', v') \in M'[y]$. Since $(M, s) \models \delta_x \vee \theta_x$ and $(M, s) \models \mathbf{B}_x\neg(\delta_y \wedge \theta_y)$, it holds that $v' = (v, \epsilon)$, $u' = (u, \sigma)$ (or $u' = (u, \tau)$) for some $u, v \in M[S]$, $(s, u) \in M[x]$ and $(u, v) \in M[y]$. Assume that $(M, s) \models \mathbf{B}_x\mathbf{B}_y\eta$. This implies $(M, v) \models \eta$, means that $(M', (v, \epsilon)) \models \eta$, i.e., which implies $(M', s') \models \mathbf{B}_x\mathbf{B}_y\eta$. \square

Proposition 3. *Let (M, s) be a state and \mathbf{a} be an announcement action instance announces φ that is executable in (M, s) and $\omega(\mathbf{a})$ be given in Definition 2. If $(M, s) \models \varphi$ then it holds that:*

1. *For every agent $x \in \mathcal{AG}$, $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$ belong to D , if $(M, s) \models \delta_x$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \sigma)$ then $(M', s') \models \mathbf{B}_x\varphi$.*
2. *For every agent $x \in \mathcal{AG}$, $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$ and $[x \text{ aware_of } \mathbf{a} \text{ if } \theta_x]$ belong to D , if $(M, s) \models \neg\delta_x \wedge \theta_x$, $(M, s) \not\models (\mathbf{B}_x\varphi \vee \mathbf{B}_x\neg\varphi)$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \sigma)$ then $(M', s') \not\models (\mathbf{B}_x\varphi \vee \mathbf{B}_x\neg\varphi)$.*
3. *For every pair of agents $x, y \in \mathcal{AG}$, $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$, $[y \text{ observes } \mathbf{a} \text{ if } \delta_y]$ and $[y \text{ aware_of } \mathbf{a} \text{ if } \theta_y]$ belong to D , if $(M, s) \models \mathbf{B}_y\delta_x$, $(M, s) \models \delta_y \vee \theta_y$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \sigma)$ then $(M', s') \models \mathbf{B}_y(\mathbf{B}_x\varphi \vee \mathbf{B}_x\neg\varphi)$.*
4. *For every pair of agents $x, y \in \mathcal{AG}$, a belief formula η , $[x \text{ observes } \mathbf{a} \text{ if } \delta_x]$, $[x \text{ aware_of } \mathbf{a} \text{ if } \theta_x]$, $[y \text{ observes } \mathbf{a} \text{ if } \delta_y]$ and $[y \text{ aware_of } \mathbf{a} \text{ if } \theta_y]$ belong to D , if $(M, s) \models \mathbf{B}_x\neg(\delta_y \vee \theta_y)$, $(M, s) \models \delta_x \vee \theta_x$, $(M, s) \models \mathbf{B}_x\mathbf{B}_y\eta$ and $(M', s') = (M, s) \otimes (\omega(\mathbf{a}), \sigma)$ then $(M', s') \models \mathbf{B}_x\mathbf{B}_y\eta$.*

Proof. The proof of this proposition is similar to the proof of Proposition 2 and is omitted. \square

The first/second item of Proposition 2 and the first item of Proposition 3 describe the fact that full observers will update their belief about φ since they fully observe the outcome of action. The third item of Proposition 2 and the second item of Proposition 3 show that partial observers still uncertain about φ after

the execution if they uncertain about it before. The fourth item of Proposition 2 and the third item of Proposition 3 illustrate that full/partial observers think the agent that they consider fully observant the action would know the true value of φ now. The fifth item of Proposition 2 and the fourth item of Proposition 3 emphasize that full/partial observers would not update their belief about other agents' belief whose they think that was unaware of the action occurrence.

4 Discussion and Related Work

Update models have been used in formalizing actions in multi-agent domains by several authors [1, 2, 5, 9, 6, 11]. However, the automatic generation of update models from an action description has only been discussed with the introduction of actions languages for multi-agent domains in [3] and subsequent versions of languages like $\mathbf{m}\mathcal{A}^*$. To the best of our knowledge, the present work is among the first that attempts to use edge-conditioned update models, introduced by [7], in an action language. As shown in Proposition 1, the use of edge-conditioned update models eliminates the problem encountered by other semantics of $\mathbf{m}\mathcal{A}^*$. We note that [7] discussed only edge-conditioned update models for world-altering actions (ontic actions), while we use it in modeling other types of actions as well.

The use of event models for reasoning about effects of actions in multi-agent domains is also studied in [12] within the language DER. In this language, the observability of agents is encoded by an observations set \mathcal{O} and no distinction between ontic, sensing, and announcement actions is made. Comparing with the update models used in [12], we can see that updated models used in the present paper have a fixed number of events, given the type of the action: two events for ontic actions and three events for sensing/announcement actions. On the other hand, the number of events in DER can vary given the number of statements specifying its effects and observations. We believe that this feature might bring some advantages if update models are used for planning, where efficient construction of update models is critical (in the new $\mathbf{m}\mathcal{A}^*$, the model need to be constructed only once).

5 Conclusion

We define a new semantics for the high-level action language $\mathbf{m}\mathcal{A}^*$ using edge-conditioned update models. In this method, edge-conditioned update models are constructed directly from the domain specification and are independent from the states in which the action occurs. We prove that the new semantics satisfies a desirable property that second order beliefs of agents about other agents' beliefs change consistently with its first-order beliefs about observability of action occurrence, i.e., the new semantics overcomes a problem of the earlier semantics of $\mathbf{m}\mathcal{A}^*$. This makes the use of the new $\mathbf{m}\mathcal{A}^*$ easier, comparing with its earlier versions, which require a careful domain designation to reason with some interested questions [4].

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