2. Theory

It is assumed that the reader of this paper already has basic knowledge of the mathematical principles of time series analysis. Therefore, this section will only briefly describe the mathematical models and processes.

2.1. Time-Series

Almost anything with a data point to a given timestamp can be named a time-series. The monthly gross domestic product, the weekly US gasoline production, or daily stock price movements. In this paper lies the focus of the analysis of financial time series. Due to trades often only take place during the week, there are gaps in the time series on the weekends, an exception would be the trading of cryptocurrencies like Bitcoin which are also tradeable at the weekends.

A series of data points with more or less equidistant time points t with the sample length of T, is called a time-series $x_t, t = 1, ..., T$ [@eco1]. The analysis of a time-series x_t involves creating a reasonable model that can be utilized to perform forecast predictions.

2.1.1. Stationarity

In order to fit a suitable model with a given time series x_t, the assumptions of stationarity must be met. In this practical application, only the following weak-stationarity properties are required.

$$E[x_t] = \mu \tag{1}$$

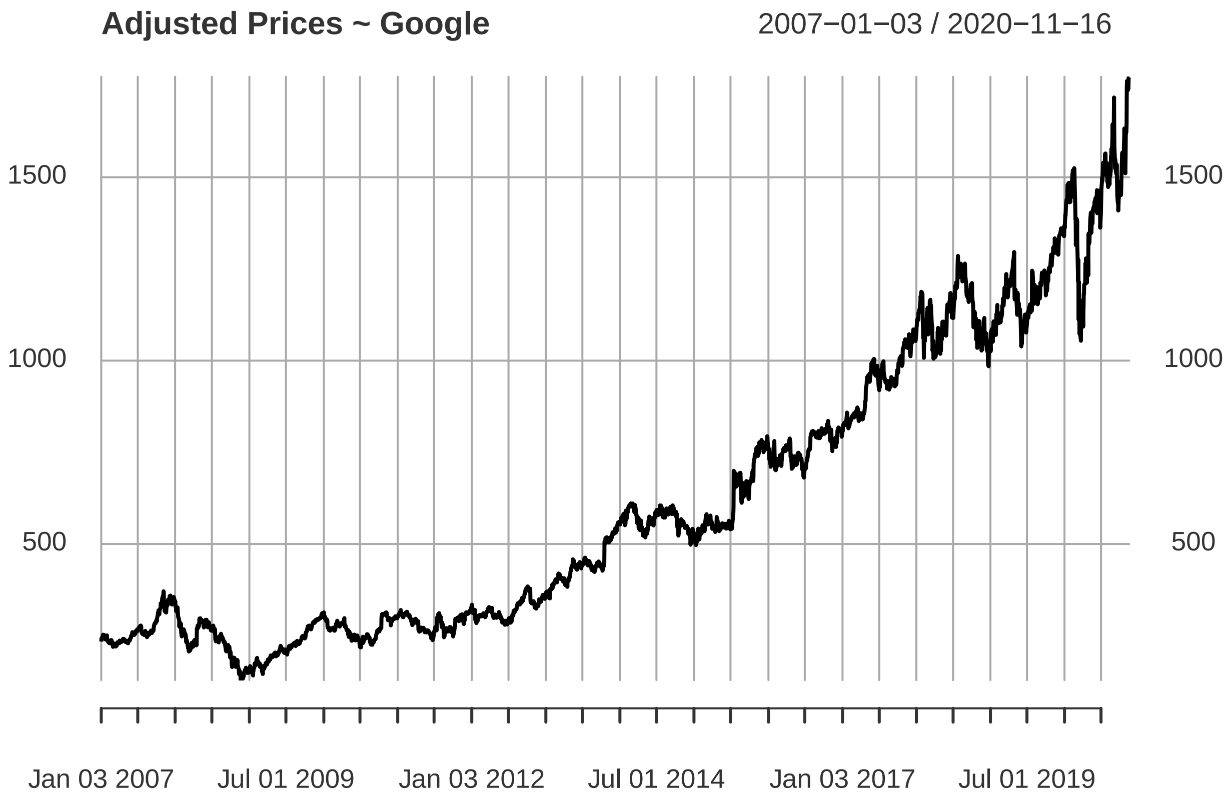
$$Var(x_t) = \sigma_x^2 \tag{2}$$

$$Cov(x_t, x_{t-k}) = R(k) \tag{3}$$

Many financial time-series are subject to shift, trends or changing volatility. In figure **??** are the stock prices of Alphabet Inc Class A (Google) visualized. This time-series shows a clear upwards drift and towards the end the volatility increases.

```
## Warning: package 'quantmod' was built under R version 4.0.3

## Loading required package: xts

## Warning: package 'xts' was built under R version 4.0.3

## Loading required package: zoo

## Warning: package 'zoo' was built under R version 4.0.3

##
## Attaching package: 'zoo'

## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

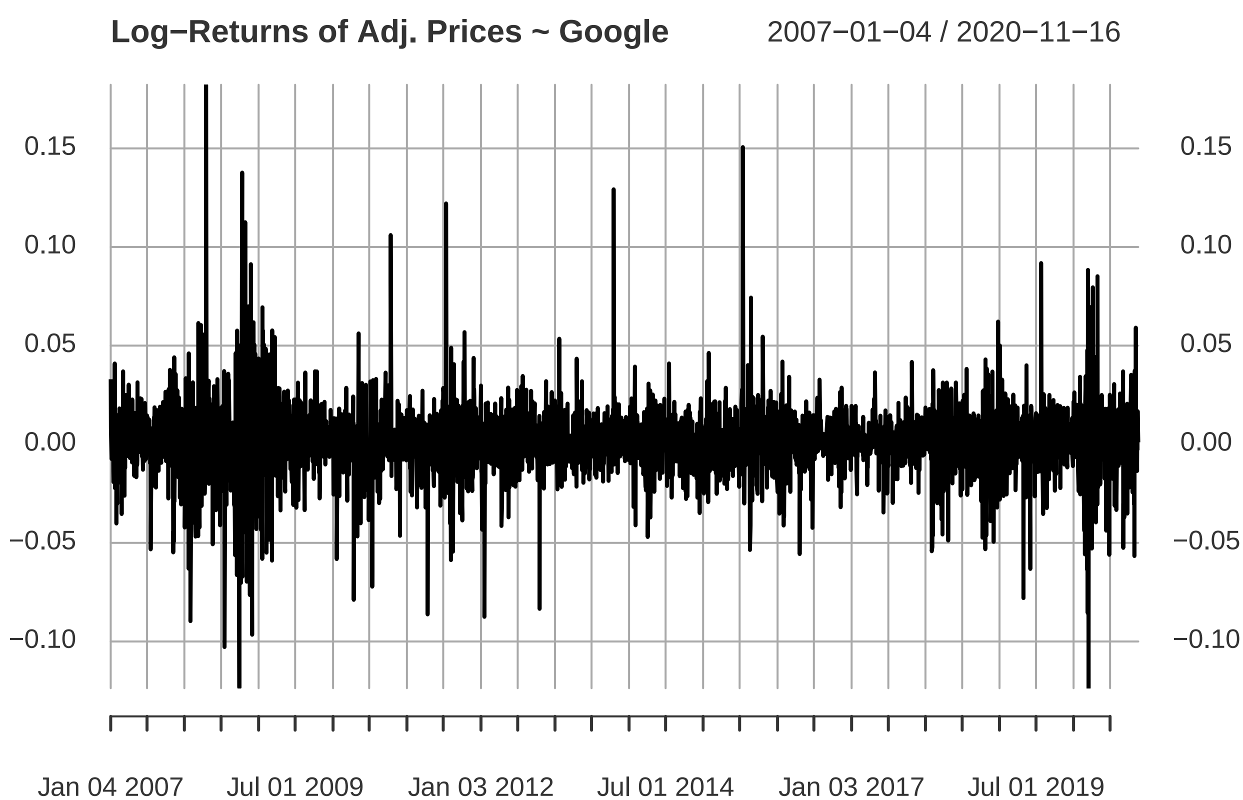
```
## Loading required package: TTR

## Warning: package 'TTR' was built under R version 4.0.3

## Registered S3 method overwritten by 'quantmod':
##   method            from
##   as.zoo.data.frame zoo

## Version 0.4-0 included new data defaults. See ?getSymbols.

## Warning: package 'PerformanceAnalytics' was built under R version 4.0.3

##
## Attaching package: 'PerformanceAnalytics'

## The following object is masked from 'package:graphics':
##
##     legend

## Warning: package 'forecast' was built under R version 4.0.3

## 'getSymbols' currently uses auto.assign=TRUE by default, but will
## use auto.assign=FALSE in 0.5-0. You will still be able to use
## 'loadSymbols' to automatically load data. getOption("getSymbols.env")
## and getOption("getSymbols.auto.assign") will still be checked for
## alternate defaults.
##
## This message is shown once per session and may be disabled by setting
## options("getSymbols.warning4.0"=FALSE). See ?getSymbols for details.
```


Adjusted Prices ~ Google
2007−01−03 / 2020−11−16
1500
1000
500
1500
1000
500
Jan 03 2007
Jul 01 2009
Jan 03 2012
Jul 01 2014
Jan 03 2017
Jul 01 2019

To improve the violated properties the first difference can be applied and additionally a logarithmic transformation can be performed [@slide_eco3_1]. The log-returns transformation can only performed to strict positive data.

$$\text{LogReturn} = \log(x_t) - \log(x_{t-1})$$

The result is the so-called log-returns.



Applying the transformation to the data causes the drift to disappear, but the series still contains stronger and weaker volatile phases. This effect often occurs in non-stationary financial data and is called volatility cluster. This special property is used for the modelling of forecast models, which will be discussed in chapter 2.2.

2.1.2. Autocorrelation

The autocorrelation function (ACF) reveals how the correlation between any two data points of the time series changes as their separation changes [@acf]. More precisely, acf measures the dependence between x_t and $x_{t\pm k}$ at lag k. The partial autocorrelation (PACF) measures the dependency between x_t and x_{t-k} at lag k [@eco1]. For stationary time series, ACF can be used to identify the model order of a MA-process, PACF for AR-processes.

In the following figure **??** are ACF and PACF of the non-stationary adjusted Google stock visualized. Both graphics show the typical pattern of a non-stationary time series. The plot above shows the dependence structure of the time series. This means that it takes a long time until the series changes. Often a large value is followed by another large value, which indicates a strong trend. This property of the series can be seen in figure **??** as the long upward drift. The plot below indicates a significant partial autocorrelation at lag $k = 1$.

In the following section 2.2. the characteristics of the autocorrelation function can be used for the verification of ARIMA and ARCH-processes.

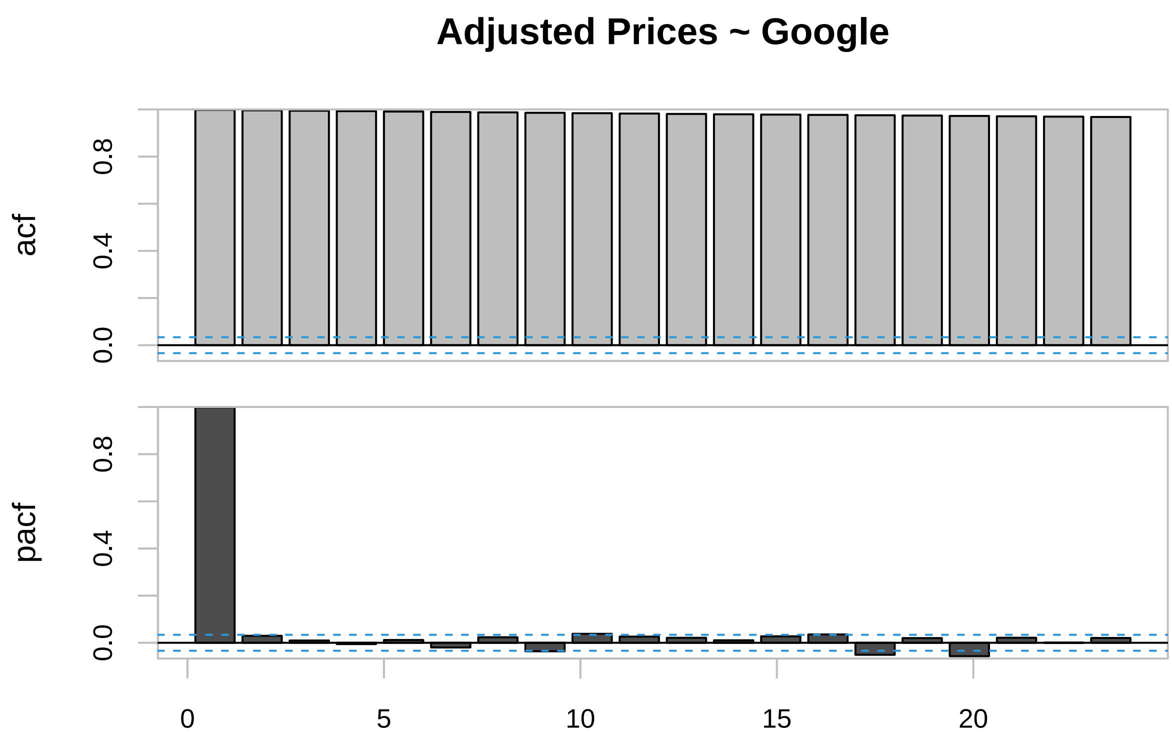


Figure 1: Acf and Pacf of the Adjusted Prizes of Google.

2.2. Models

The following processes are used to determine certain properties and characteristics of a time series so that they are transformed into a model. The goal is to fit the time series as well as possible in order to create reliable forecasts.

2.2.1. ARIMA

An ARIMA(p,d,q) process is defined as follows.

$$x_t = c + a_1 x_{t-1} + ... + a_p x_{t-p} + \epsilon_t + b_1 \epsilon_{t-1} + ... + b_q \epsilon_{t-q} \tag{4}$$

- p and q are the AR- and MA-model orders
- a and b are the AR- and MA-model parameters
- d is the differential parameter
- ϵ_t is a white noise sequence
- x_t is the given data $x_1, ..., x_T$

The mean of an ARIMA-process can be computed as:

$$\mu = \frac{c}{1 - a_1 - ... - a_p}$$

ARIMA processes can be divided into 4 different models. Choosing a model that best represents the time series is a difficult task. The goal is to find the best possible model with as few parameters as possible.

The previously introduced ACF and PACF can help to determine the orders of simple models. Provided that the time series is stationary, the model orders can be determined directly. For an AR(p)-process (ARIMA(p,*0*,*0*)), the ACF plot will gradually decrease and simultaneously the PACF should have a sharp drop after p significant lags. For an MA(q)-process (ARIMA(*0*,*0*.q)) the opposite is true, the ACF should show a sharp drop after a certain q number of lags while PACF should show a gradual decreasing trend. If both ACF and PACF show a gradual decreasing pattern, then the ARIMA(p,*0*,q)-process should be considered for modeling [@arima]. If the time series is not stationary, differentiation can be considered (ARIMA(p,d,q)).

The application of an analysis method to Google prices finds an ARIMA($1,1,0$) as the optimal model. This makes sense if you look back at figure **??**. Long dependency structures in the ACF plot indicating an AR(p) process and at the same time after lag=1=p the PACF has a strong drop. The differential operator d=1 transforms the non-stationary series into a stationary one.

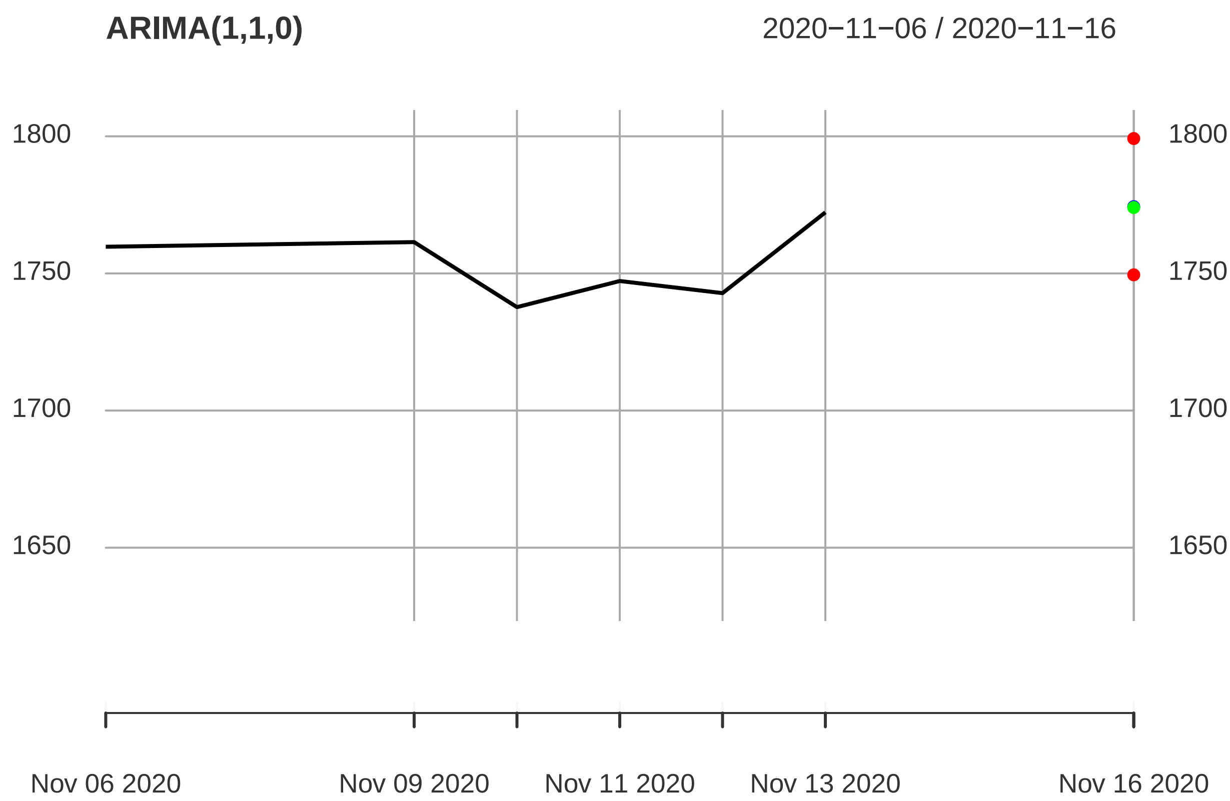


Figure 2: ARIMA-Forecast

2.2.2. ARCH & GARCH

The volatility clustering mentioned in section 2.1.1 can be handled with an auto-regressive conditional heteroscedastic process.

$$\begin{aligned} \epsilon_t &= \log(x_t) - \log(x_{t-1}) \\ \epsilon_t &= \sigma_t u_t \\ \sigma_t^2 &= c\sigma^2 + \sum_{k=1}^{m} \beta_k \epsilon_{t-k}^2 \end{aligned} \tag{5}$$

with:

- x_t is the original data (often non-stationary)
- ϵ_t is the stationary log-return
- u_t is independent and identically distributed (iid) and standardized random variable
- σ^2 is the unconditional variance of the process ϵ_t.

- σ_t^2 is the conditional variance of the process ϵ_t.

The ARCH-process can be generalized by adding the lagged conditional variances to the equation 5.

$$
\begin{aligned}
\epsilon_t &= \log(x_t) - \log(x_{t-1}) \\
\epsilon_t &= \sigma_t u_t \\
\sigma_t^2 &= c\sigma^2 + \sum_{j=1}^{n} \alpha_j \sigma_{t-j}^2 + \sum_{k=1}^{m} \beta_k \epsilon_{t-k}^2
\end{aligned} \tag{6}
$$

2.2.3. ARIMA-GARCH

Another process is the combination of ARIMA and GARCH processes.

$$
y_t = \mu + a_1 y_{t-1} + ... + a_p y_{t-p} + \epsilon_t + b_1 \epsilon_{t-1} + ... + b_q \epsilon_{t-q} \tag{7}
$$

$$
\epsilon_t = \sigma_t u_t
$$

$$
\sigma_t^2 = c\sigma^2 + \sum_{j=1}^{n} \alpha_j \sigma_{t-j}^2 + \sum_{k=1}^{m} \beta_k \epsilon_{t-k}^2 \tag{8}
$$

Is called the mean-equation 7

Is called the variance-equation 8

[@eco2]

2.3. Moving Average Filters

moving average filters are basically used to identify trends and smooth out price fluctuations. As a commonly used tool moving average filters are very simple in its usage, historical data was summarized and divided bi the length of the filter. The actual challenge in using Moving average filters is to figure out which length of the filter brings the most useful information.

2.3.1. Equally-weighted Moving Average

SMA stands for Simple Moving Average which. depending on the length of the filter(l) l obervations since the last noted observation will be considered. theyre getting summarized and divided by the filterlength equals the EqMA. For every timestep a new observation is considered and the last one eliminated.

EqMA

$$y_t = \frac{1}{L}\sum_{k=0}^{L-1} x_{t-k} \tag{9}$$

- L = filterlength

2.3.2. Exponentially-weighted Moving Average

Since not all observations are having the same influence of future value we can apply a weight to past observations. One method will be exponentially weighted Moving average. So we chose an optimal parameter to give past observations weights decreasing by alpha

A skillfull trader chose an optimal to increase the performace of the measurement. Weights could also be given individually by adding a weight vector to the filter.

EMA

$$y_t = \frac{1}{\sum_{k=0}^{m} \alpha^k} \sum_{k=0}^{m} \alpha^k x_{t-k} \tag{10}$$

- m = filterlength
- α = Parameter to weigh the observations

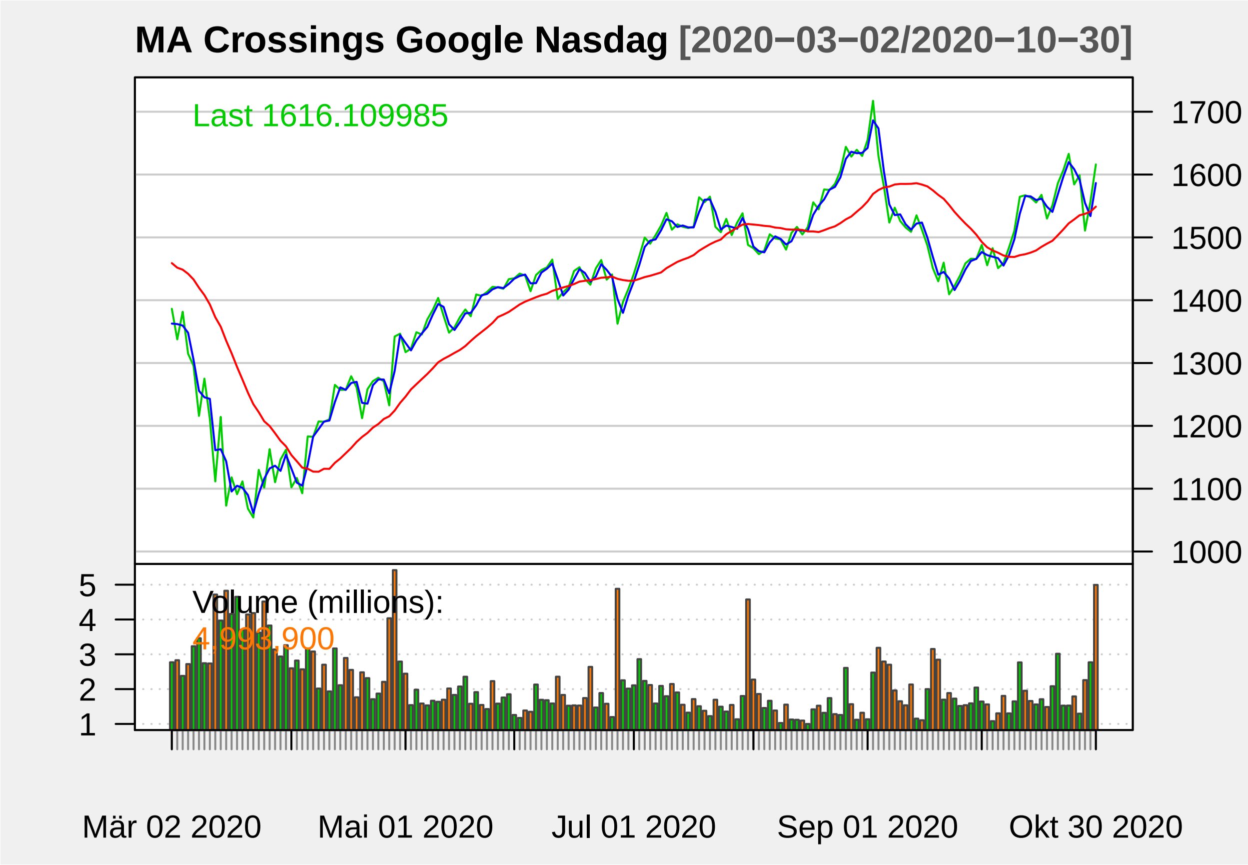
2.3.3. Moving Average Crossings

$$y_t = \frac{1}{L_1}\sum_{k=0}^{L_1-1} x_{t-k} - \frac{1}{L_2}\sum_{k=0}^{L_2-1} x_{t-k} \tag{11}$$

- L_1 = filterlength 1
- L_2 = filterlength 2
- $0 < \alpha < 1$ = Parameter to weigh the observations

Moving average crossings are basically just different MA's with different lengths applied to a time-series. The points the filters then cross will be used as a trading signal to go long, short or hold.

An easy example of Ma average crossings with 2 mas of different length is visualized in # **??**.


MA Crossings Google Nasdaq [2020−03−02/2020−10−30]
Last 1616.109985
1700
1600
1500
1400
1300
1200
1100
1000
Volume (millions):
4,993,900
5
4
3
2
1
Mär 02 2020
Mai 01 2020
Jul 01 2020
Sep 01 2020
Okt 30 2020

2.4. Real Strength Index

2.5. Sharpe Ratio

Sharpe ratio is a very powerful and widely used ratio to measure performance. It describes return per risk

$$\text{SharpeRatio} = \frac{R_p - R_f}{\sigma} \tag{12}$$

- R_p = Return of Portfolio
- R_p = Risk free Rate, mostly treasury bonds
- σ_p = standard deviation of portfolios excess return (risk)

2.6. Carry

carry trades are trading strategies where usually money is borrowed at a lower intrest rate than the investment is gioving in return. the risk of this strategy is based in the currency risk.

2.7. Value

2.8 Bollinger bands

Bollinger bands are a analysis tool founded by John Bollinger. It contains a moving average and an upper and lower band . The bands are defined by adding a constant$K$ times a standard deviation σ_t to the *Moving Average* for the upper , and subtracting it for the lower band.

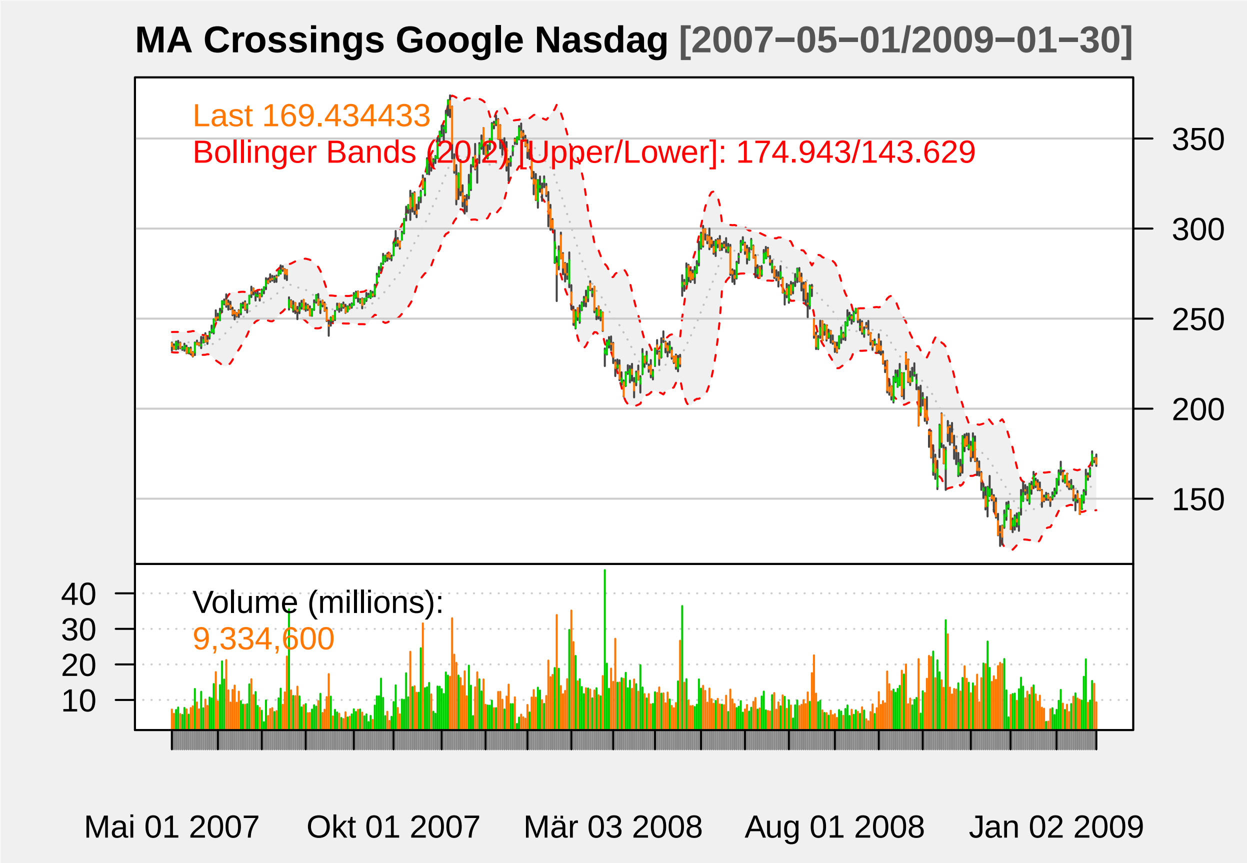
$$\mathrm{U_t} = MA_t + K\sigma, L_t = MA_t - K\sigma \tag{13}$$

the variance from bollingers theory is calculated by:

$$\sigma_{\mathrm{t}}^2 = \frac{1}{N}\sum_{k=0}^{N-1}(x_{t-k} - MA_t)^2 \tag{14}$$

the calculated σ_t could be problematic because its derived from the original series and increases with the level. Therefore an other method to calculate the standart deviation could be used.

- N = usually the filterlength and the lentgh considered for σ the same
- K = Constant usually equals 2
- σ_p = standard deviation of the series
- U_t = upper band
- L_t = lower band


MA Crossings Google Nasdag [2007−05−01/2009−01−30]
Last 169.434433
Bollinger Bands (20,2) [Upper/Lower]: 174.943/143.629
350
300
250
200
150
Volume (millions):
9,334,600
40
30
20
10
Mai 01 2007
Okt 01 2007
Mär 03 2008
Aug 01 2008
Jan 02 2009