Teaching Projections in Classroom and QGIS

Vedran Stojnović

Miljenko Lapaine

21st International Conference on Geoinformation and Cartography

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Who are we?

- **Vedran Stojnović**
- Master of engineering in Geodesy and Geoinformatics by profession.
- I'm experienced with CAD, GIS, SQL, Python
- Currently working in land surveying company
- I like automating tasks using programming and modern tools
- QGIS contributor whenever I have spare time

Who are we?

- **Miljenko Lapaine**
- graduated from the Faculty of Science, University of Zagreb, in the field of Theoretical Mathematics.
- completed postgraduate studies in Geodesy, in the field of Cartography at the Faculty of Geodesy in Zagreb by defending his Master's thesis *A Modern Approach to Map Projections*.
- obtained his PhD from the University of Zagreb with a dissertation *Mapping in the Theory of Map Projections*.
- professor emeritus since 2017.

Who are we?

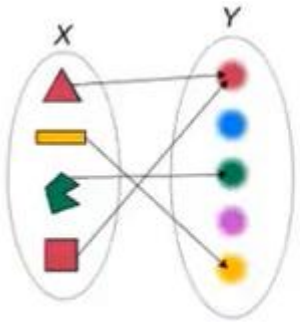
- **Miljenko Lapaine**
- has published more than 1000 papers, several textbooks and monographs - https://www.croris.hr/osobe/profil/18748?lang=en.
- Prof. Lapaine was the Vice-Chair of the ICA Commission on Map Projections - https://mapprojections.icaci.org
- founder and President of the Croatian Cartographic Society - https://www.kartografija.hr
- Managing Editor of the *Cartography and Geoinformation* journal (**published in English** and Croatian). - https://hrcak.srce.hr/en/kig (previously: Editor-in-Chief)

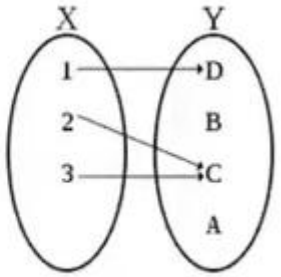
Intro to workshop

- Why are we doing this workshop in the first place?
- Globe?

Intro to workshop

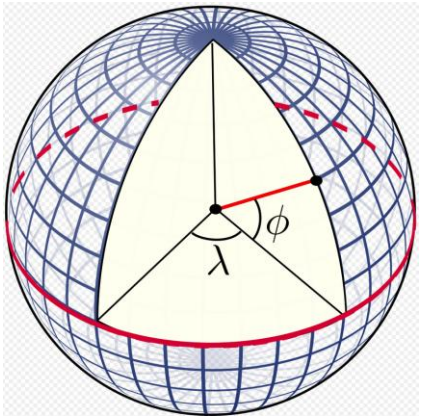
- Map projection in cartography = mapping in math

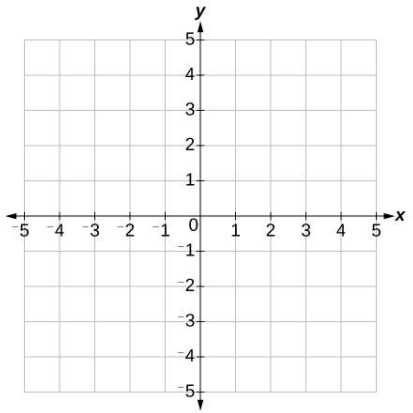




- x = f1(ϕ, λ)
- y = f2(ϕ, λ)

- x = f1(longitude [λ])
- y = f2(latitude [ϕ])





What is a map projection?

- QGIS docs – A Gentle Introduction to GIS

  https://docs.qgis.org/3.40/en/docs/gentle_gis_introduction/coordinate_reference_systems.html#the-three-families-of-map-projections

- The process of creating map projections is best illustrated by positioning a light source inside a transparent globe on which opaque earth features are placed. Then project the feature outlines onto a two-dimensional flat piece of paper. Different ways of projecting can be produced by surrounding the globe in a **cylindrical** fashion, as a **cone**, or even as a **flat surface**. Each of these methods produces what is called a **map projection family**. Therefore, there is a family of **planar projections**, a family of **cylindrical projections**, and another called **conical projections** (see Fig. 8.43)

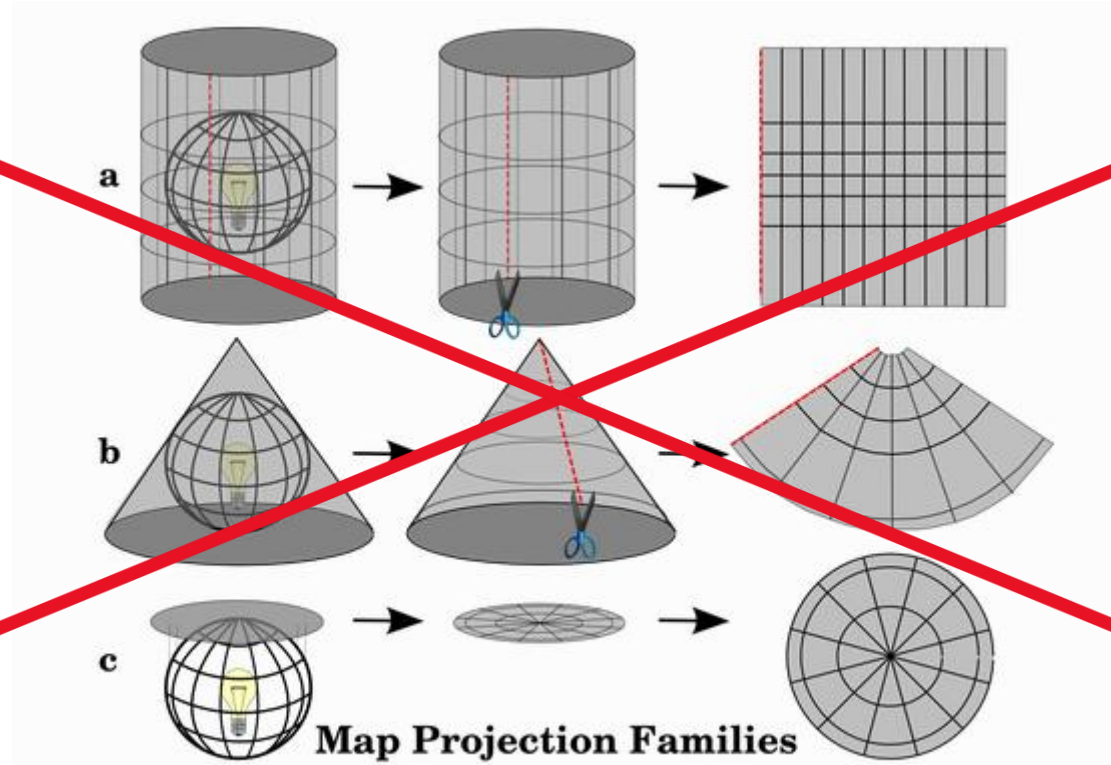
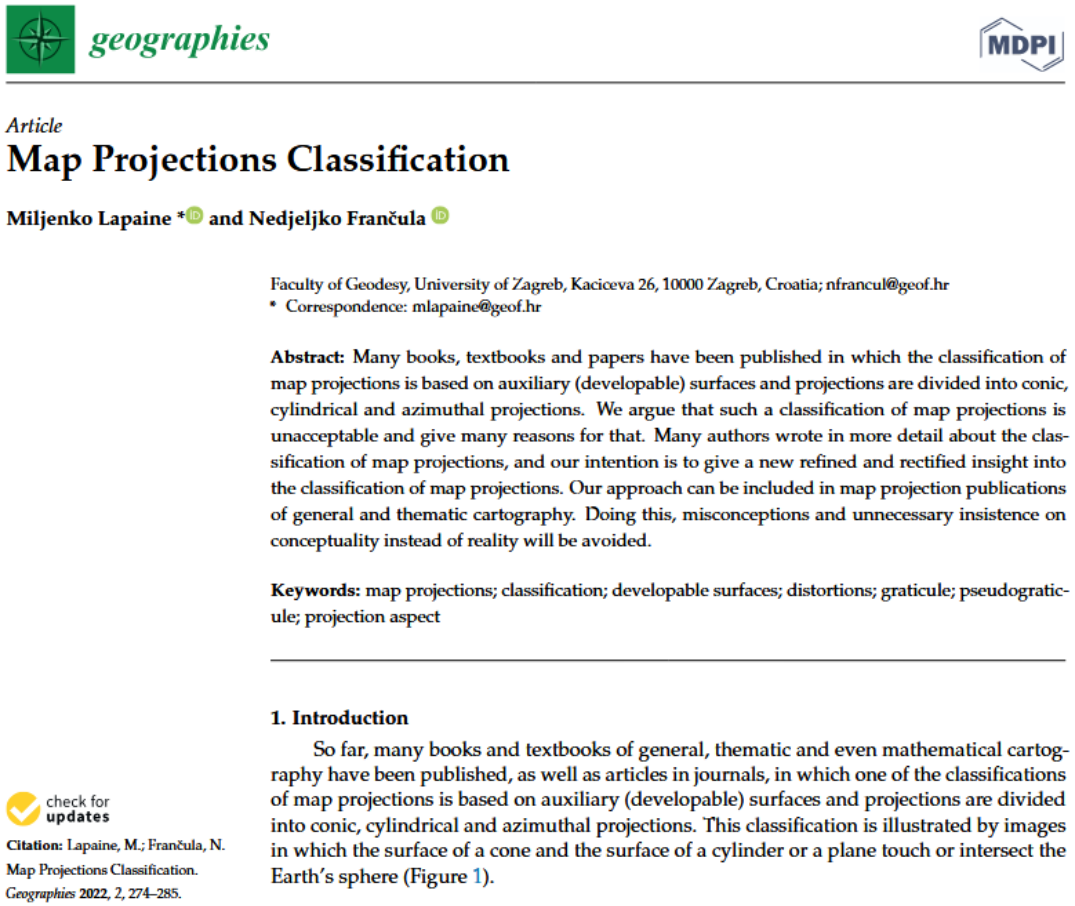


Fig. 8.43 *The three families of map projections. They can be represented by a) cylindrical projections, b) conical projections or c) planar projections.*

Please read! Map Projections Classification (open access)



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Article

Map Projections Classification

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* Correspondence: mlapaine@geof.hr

Abstract: Many books, textbooks and papers have been published in which the classification of map projections is based on auxiliary (developable) surfaces and projections are divided into conic, cylindrical and azimuthal projections. We argue that such a classification of map projections is unacceptable and give many reasons for that. Many authors wrote in more detail about the classification of map projections, and our intention is to give a new refined and rectified insight into the classification of map projections. Our approach can be included in map projection publications of general and thematic cartography. Doing this, misconceptions and unnecessary insistence on conceptuality instead of reality will be avoided.

Keywords: map projections; classification; developable surfaces; distortions; graticule; pseudograticule; projection aspect

1. Introduction

So far, many books and textbooks of general, thematic and even mathematical cartography have been published, as well as articles in journals, in which one of the classifications of map projections is based on auxiliary (developable) surfaces and projections are divided into conic, cylindrical and azimuthal projections. This classification is illustrated by images in which the surface of a cone and the surface of a cylinder or a plane touch or intersect the Earth's sphere (Figure 1).

check for updates

Citation: Lapaine, M.; Frančula, N. Map Projections Classification. *Geographies* **2022**, *2*, 274–285.

- https://www.mdpi.com/2673-7086/2/2/19

Are map projections accurate?

- QGIS docs – A Gentle Introduction to GIS
- https://docs.qgis.org/3.40/en/docs/gentle_gis_introduction/coordinate_reference_systems.html#accuracy-of-map-projections

8.4. Accuracy of map projections

Map projections are never absolutely accurate representations of the spherical earth. As a result of the map projection process, every map shows **distortions of angular conformity, distance and area**. A map projection may combine several of these characteristics, or may be a compromise that distorts all the properties of area, distance and angular conformity, within some acceptable limit. Examples of compromise projections are the **Winkel Tripel projection** and the **Robinson projection** (see Fig. 8.44), which are often used for producing and visualizing world maps.

Are map projections accurate?

In cartography, we are not talking about accuracy, but about distortions that are inherent to every map projection.

If we know the map projection, then we can remove the resulting distortions from the information read from the map.

L. Euler (1707–1783)

He was the first to mathematically prove that a spherical surface cannot be mapped onto a plane with zero distortion.

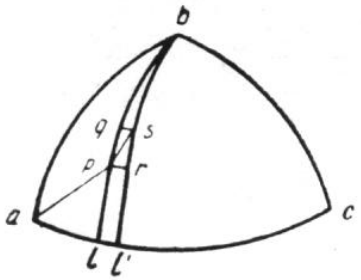


Рис. 1.

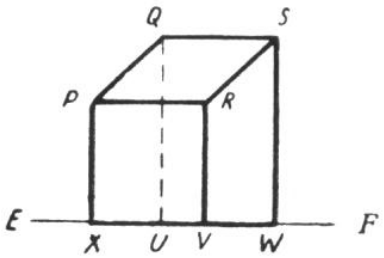


Рис. 2.

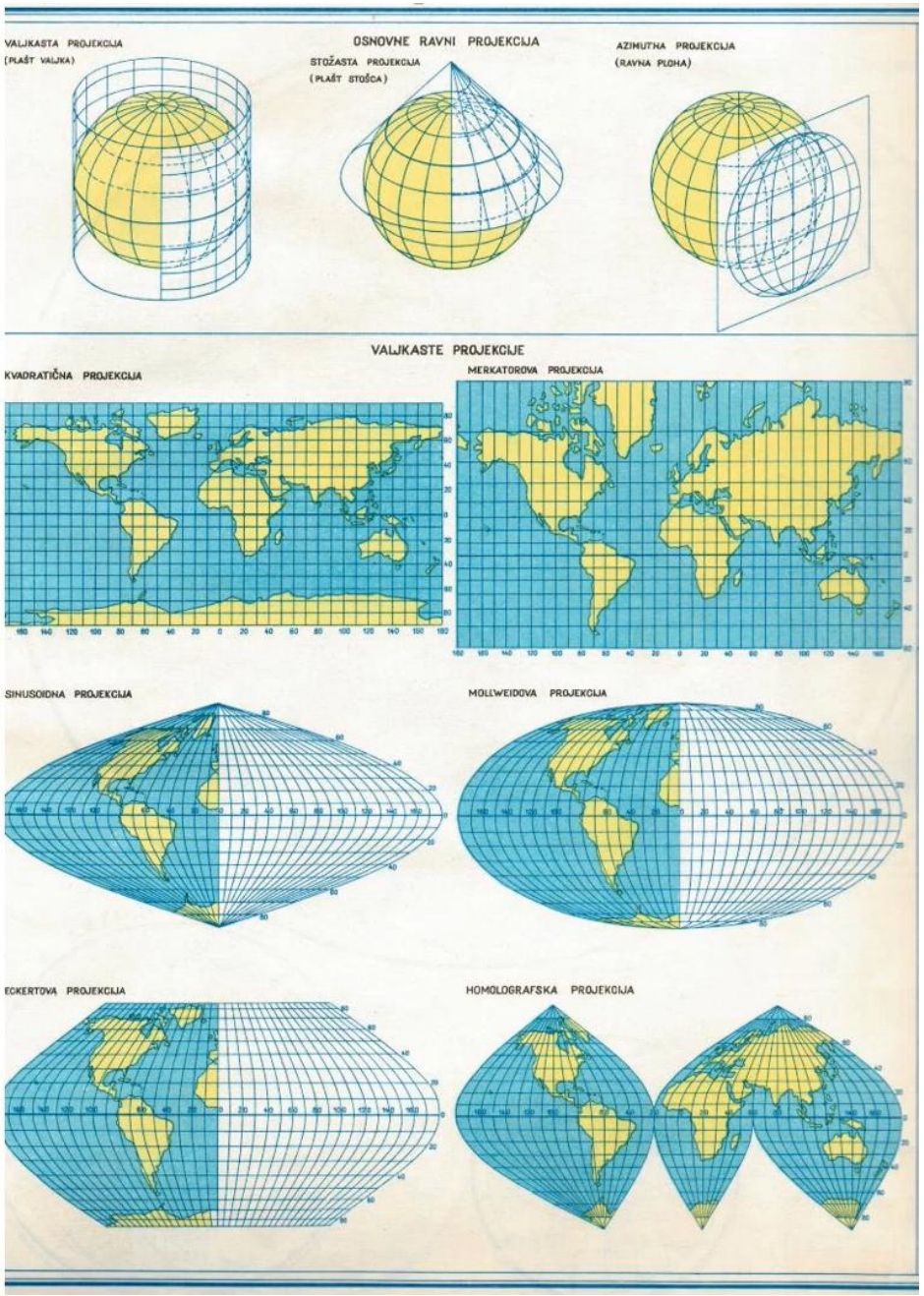
L. Euler (1707–1783)

- In 1778 journal article “De repraesentatione superficiei sphaericae super plano” was published, in which he provided us a proof that sphere’s surface cannot be projected into a plane without distortions: https://archive.org/details/actaacademiaesci01impe/page/106/mode/2up

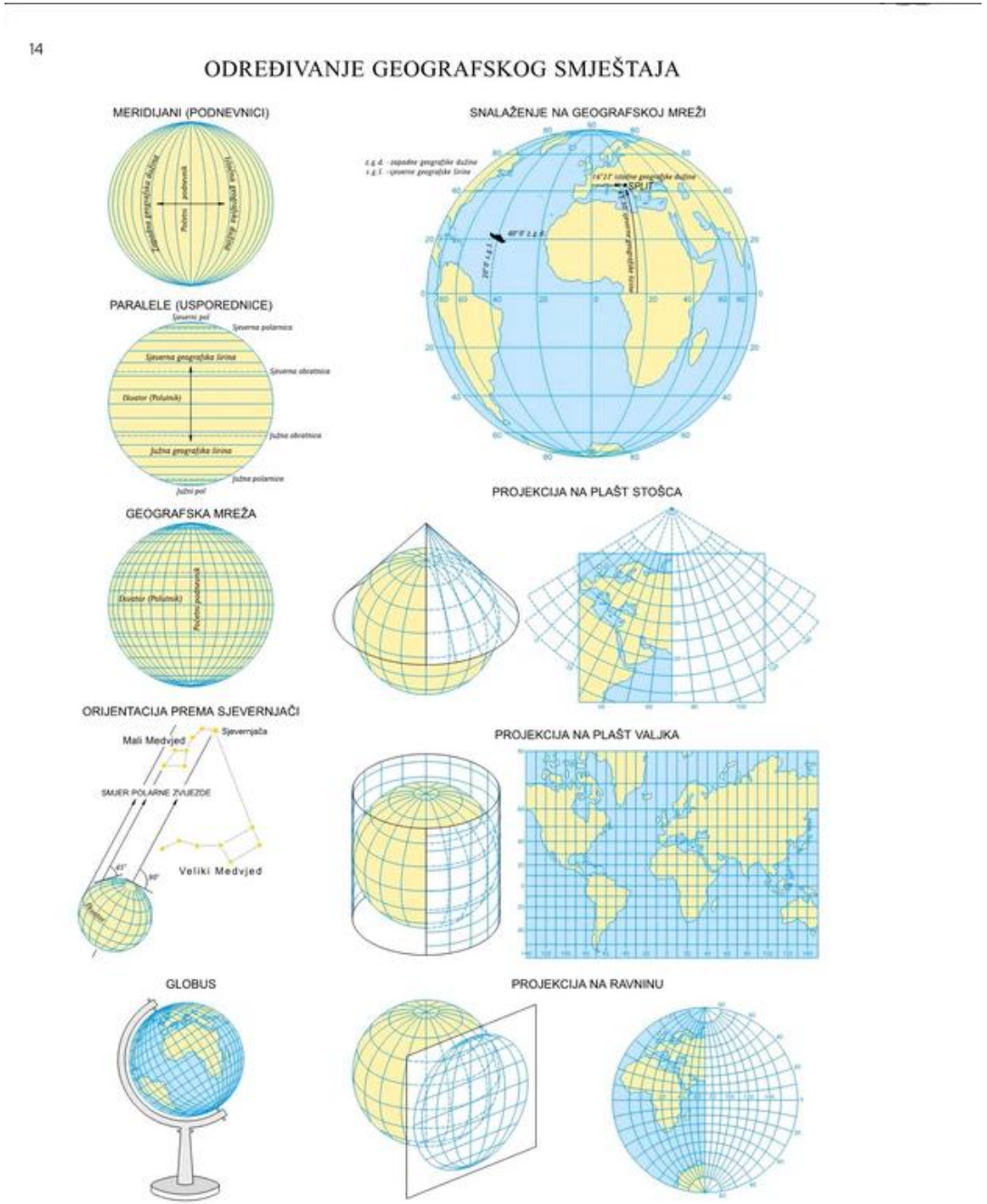
- English and German translation: https://scholarlycommons.pacific.edu/euler-works/490/

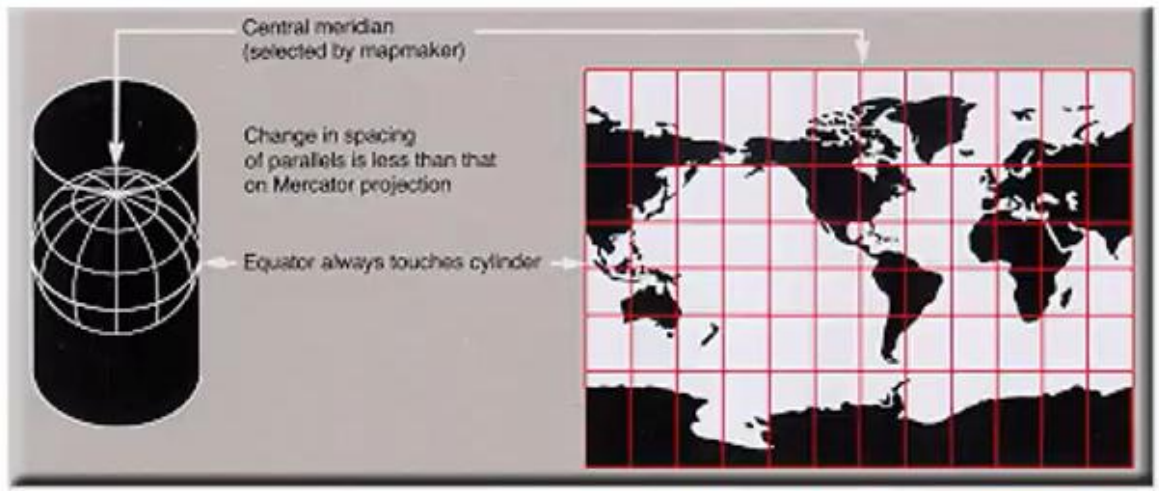
The following is a series of slides that demonstrate a very widespread misconception about map projections.

Croatian school atlas from 1965

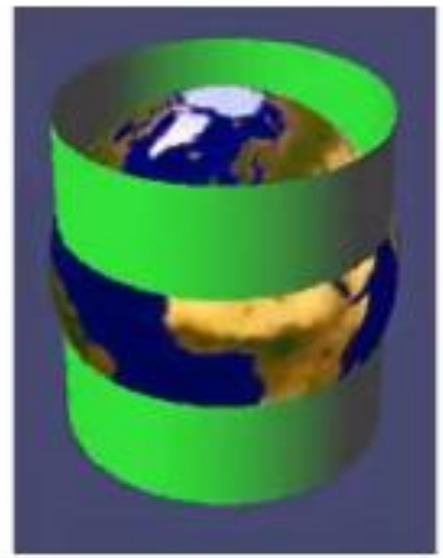


Croatian school atlas from 2021

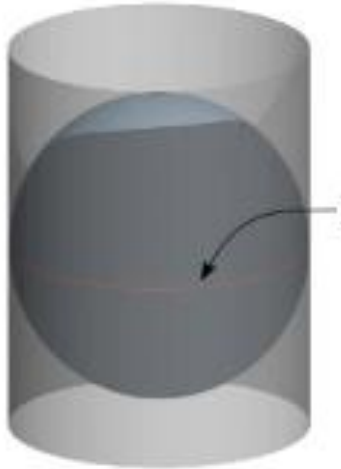


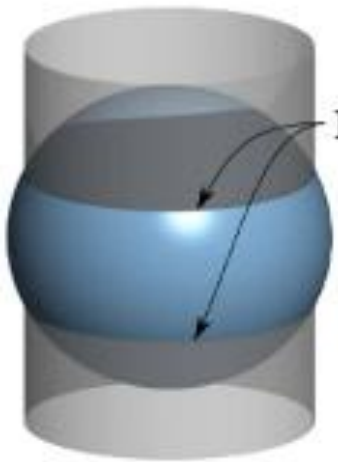


https://hr.wikipedia.org/wiki/Mercatorova_projekcija

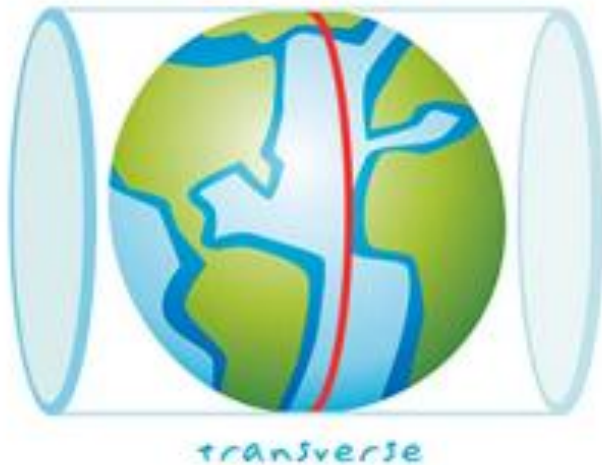


http://lazarus.elte.hu/~guszlev/vet/cylin.htm

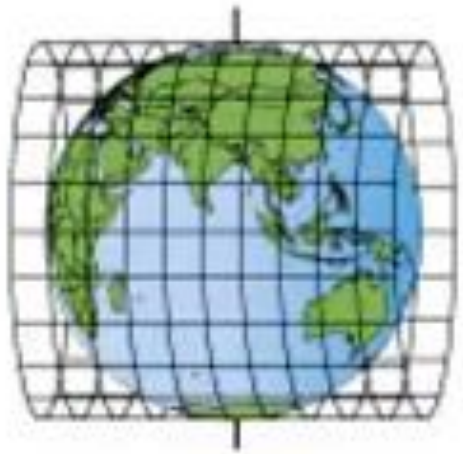


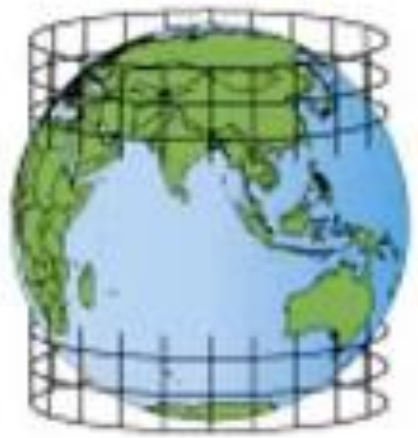


https://mgimond.github.io/Spatial/coordinate-systems.html

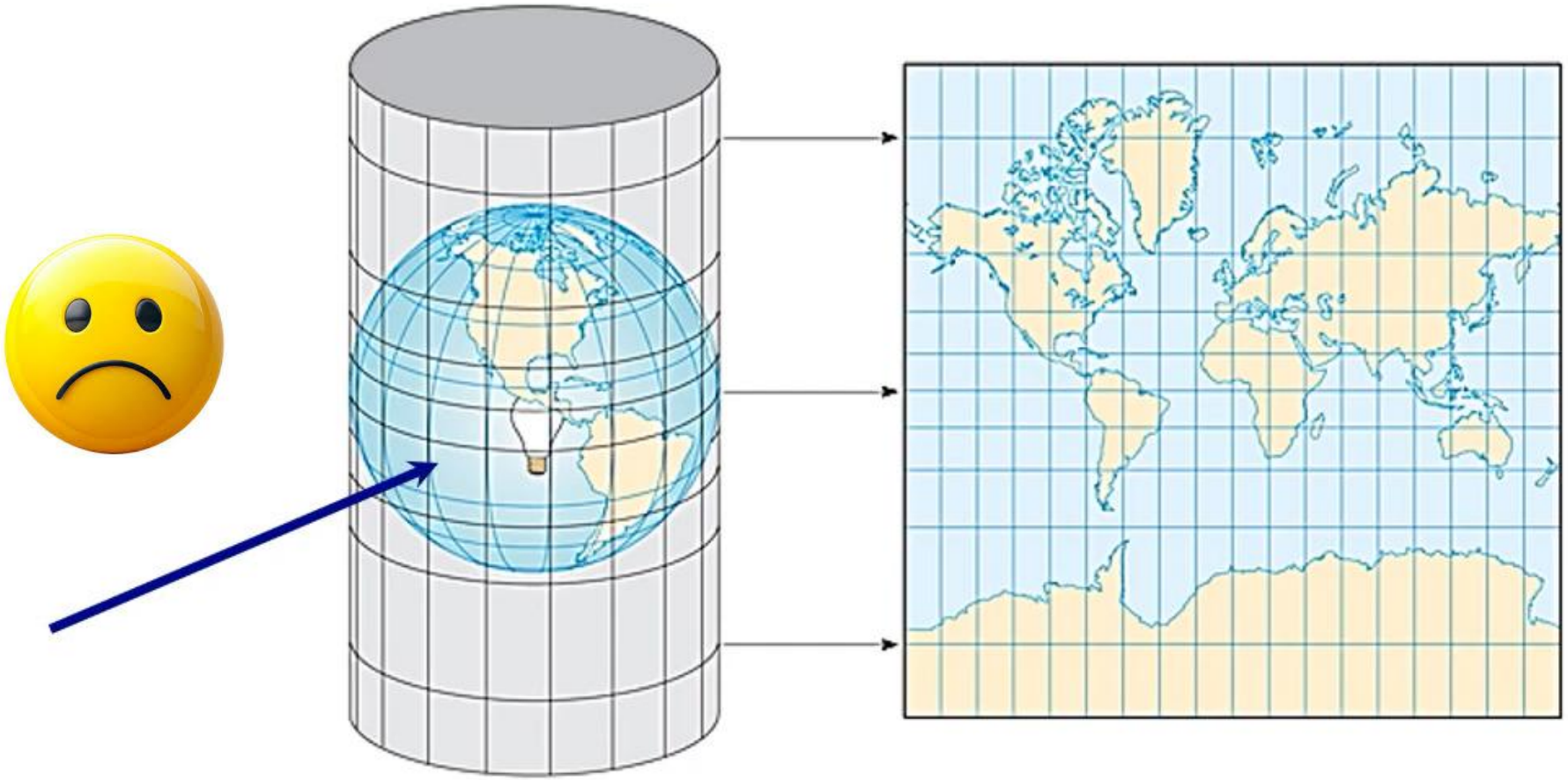


http://support.esri.com/other-resources/gis-dictionary/term/cylindrical%20projection

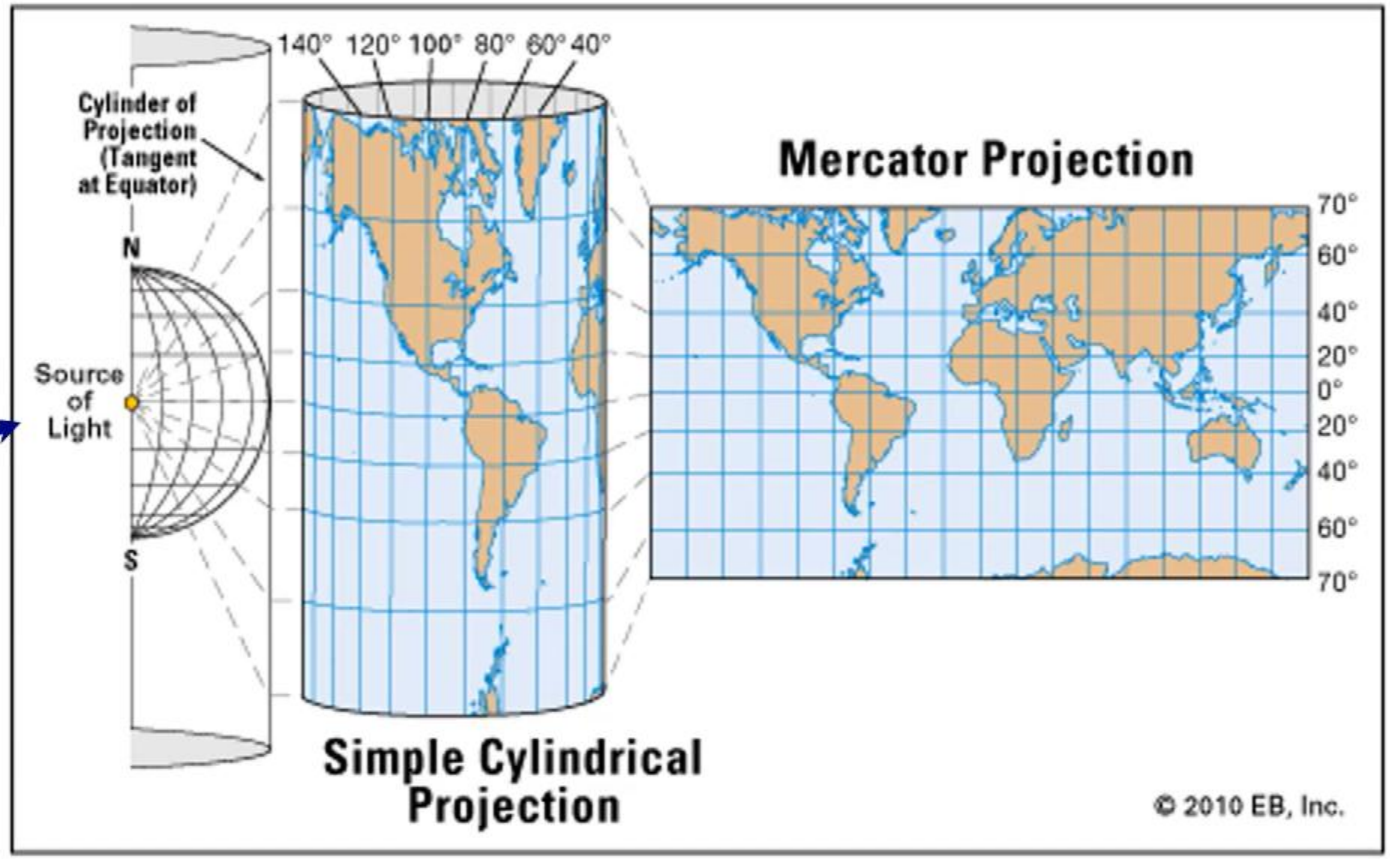




http://geographx.co.nz/map-projections/



http://web.gccaz.edu/~lnewman/gph111/topic_units/systems_grid_proj/systems_time/systems_time2.html



The Mercator projection—a transformation from the simple cylindrical projection—is used for navigation, since lines of constant direction on the Earth appear as straight lines on the map.

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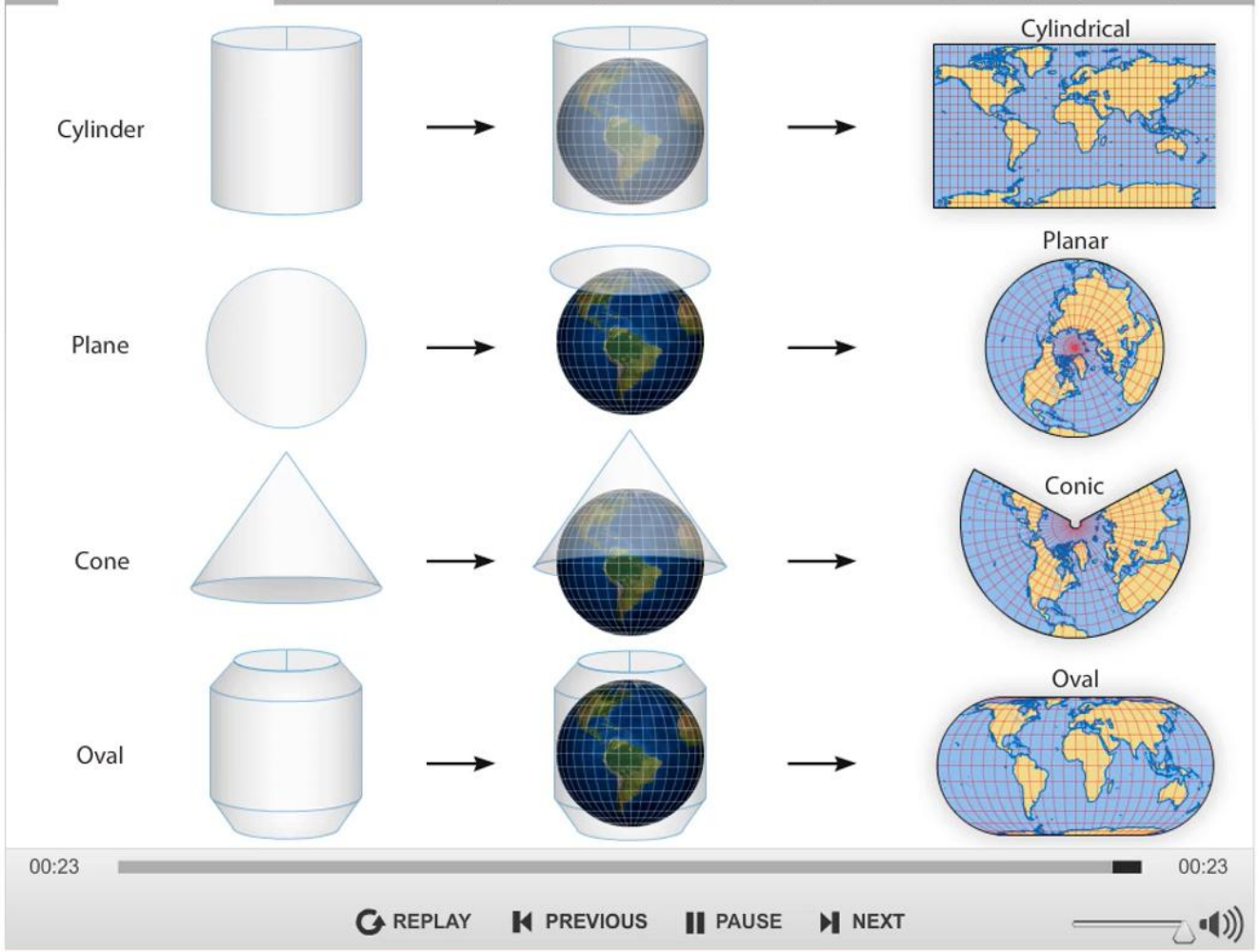
Map Projections

Map Projection Classes: Introduction

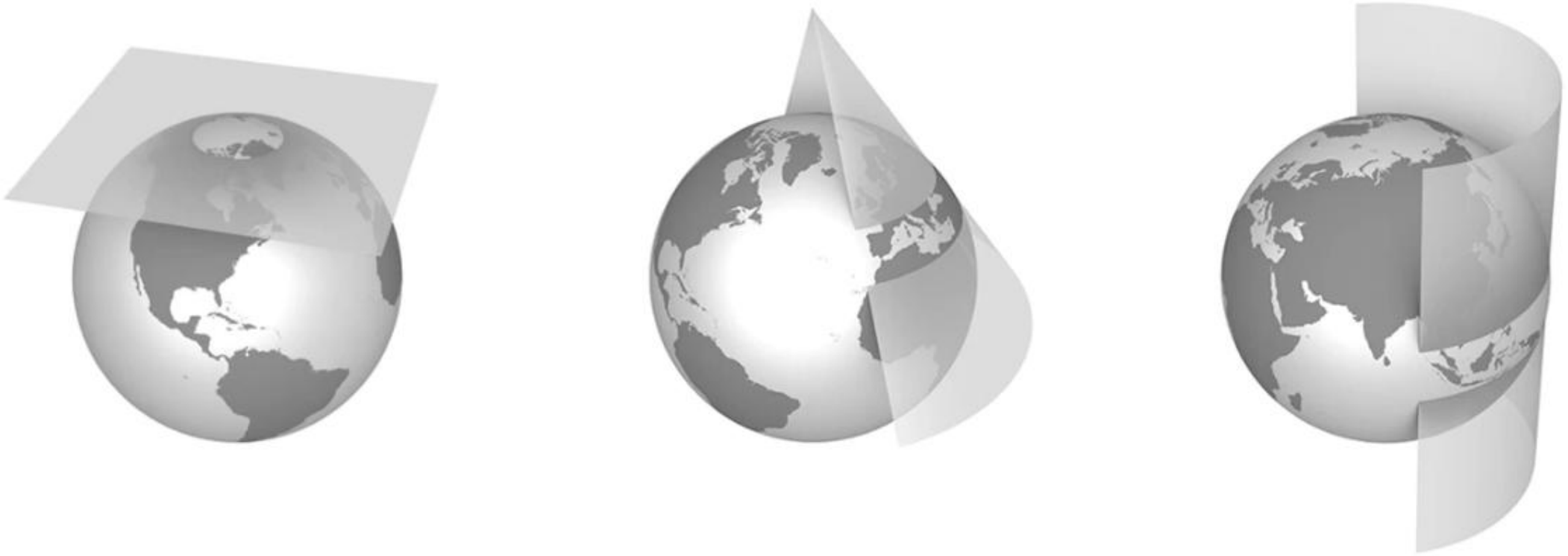
Introduction | Earth's Graticule | Map Projection Properties | Map Projection Classes | Using Map Projections

Introduction | Cylindrical Projections | Planar Projections | Conic Projections | Pseudocylindrical (Oval) Projections

SHOW TEXT

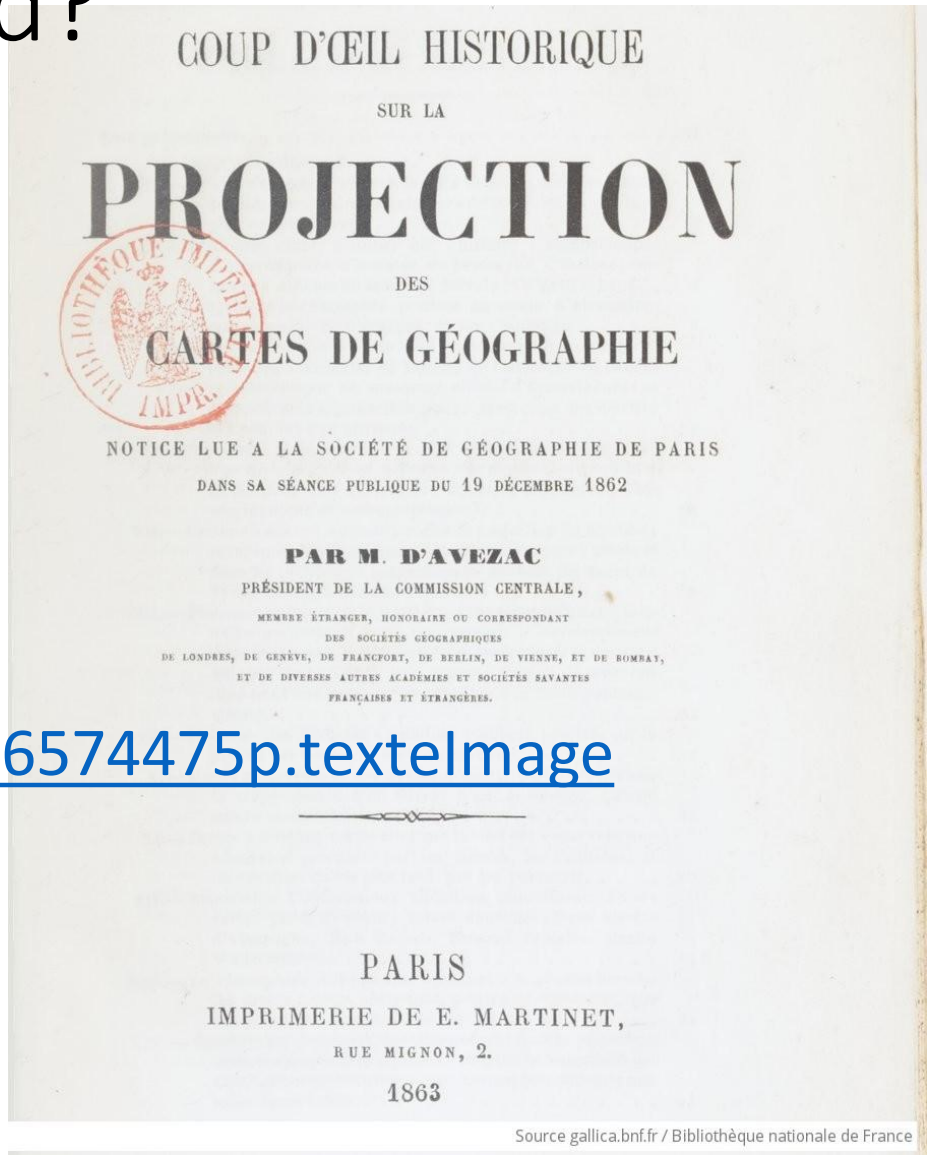


Pandemic in cartography?



Where and when did it started?

- **Coup d'oeil Historique Sur la Projection des Cartes de Géographie: Notice Lue À la Société de Géographie de Paris Dans Sa Séance Publique du 19 Décembre 1862**
- Author: **Avezac, Armand d' (1800-1875)**
- **Original:** https://gallica.bnf.fr/ark:/12148/bpt6k6574475p.texteImage
- Historical Overview of the Projection of Geographical Maps:
  Notice Read to the Geographical Society of Paris in its Public Session of December 19, 1862


COUP D'ŒIL HISTORIQUE

SUR LA

PROJECTION

DES

CARTES DE GÉOGRAPHIE

NOTICE LUE A LA SOCIÉTÉ DE GÉOGRAPHIE DE PARIS

DANS SA SÉANCE PUBLIQUE DU 19 DÉCEMBRE 1862

PAR M. D'AVEZAC

PRÉSIDENT DE LA COMMISSION CENTRALE,

MEMBRE ÉTRANGER, HONORAIRE OU CORRESPONDANT

DES SOCIÉTÉS GÉOGRAPHIQUES

DE LONDRES, DE GENÈVE, DE FRANCFORT, DE BERLIN, DE VIENNE, ET DE BOMBAY,

ET DE DIVERSES AUTRES ACADÉMIES ET SOCIÉTÉS SAVANTES

FRANÇAISES ET ÉTRANGÈRES.

PARIS

IMPRIMERIE DE E. MARTINET,

RUE MIGNON, 2.

1863

Gerardus Mercator (1512-1594)

- Did he reference a cylinder?

Johann Heinrich Lambert

- Notes and Comments on the Composition of Terrestrial and Celestial Maps (Beiträge zum Gebrauche der Mathematik und deren Anwendungen) from 1772 - https://link.springer.com/chapter/10.1007/978-3-031-09570-2_16
- Original: https://gdz.sub.uni-goettingen.de/id/PPN590413880.

Johann Heinrich Lambert

- There are no references to cylinders or cones as projecting surfaces, but opposite:
- **When** he explained (mathematically defined) Conformal conic projection **then** he proposed that this map could be bent to form a cone shape

Johann Heinrich Louis Krüger

- *Konforme Abbildung des Erdellipsoides in der Ebene.* 1912. http://bib.gfz-potsdam.de/pub/digi/krueger2.pdf
- definition of Transverse-Mercator projection

(also known as Gauss-Krüger projection)

- There are no mentions of cylinder

How come no one noticed?

Are there good examples? Yes > No. 1

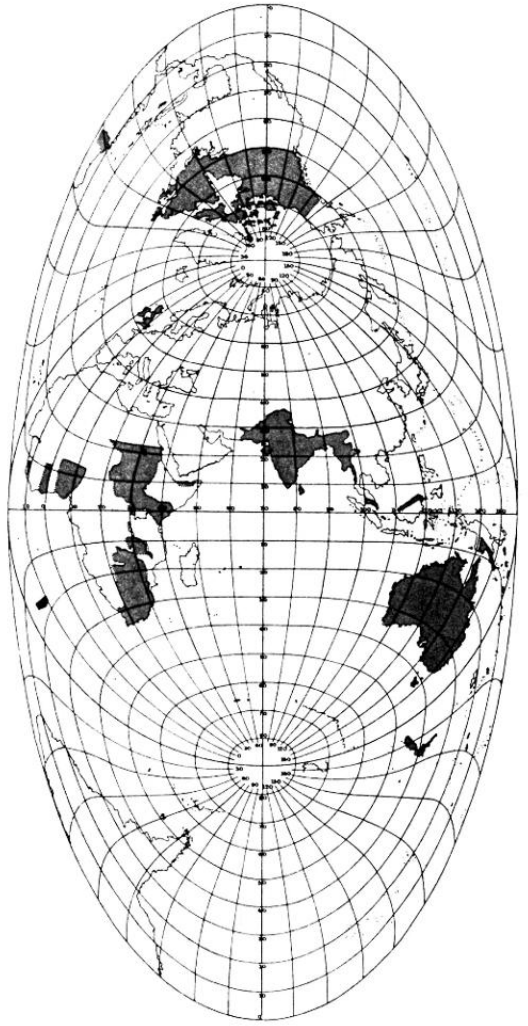
Close, C. F. and Clarke, A. R., (1911): Map projections:
Encyclopaedia Britannica, 11th ed., v. 17, p. 653-663,
reprintings to 1960

Conical projections are those in which the parallels are represented by concentric circles and the meridians by equally spaced radii. There is no necessary connexion between a conical projection and any touching or secant cone.

The name conical is given to the group embraced by the above definition, because, as is obvious, a projection so drawn can be round to form a cone.

Are there good examples? Yes > No. 2

- Map Projections (1912)
  Arthur R. Hinks



MAP PROJECTIONS

BY

ARTHUR R. HINKS, M.A.

CHIEF ASSISTANT, CAMBRIDGE OBSERVATORY, AND
UNIVERSITY LECTURER IN SURVEYING AND CARTOGRAPHY

Cambridge :
at the University Press
1912

is neither equal area nor orthomorphic. But it is more nearly so than the preceding, since the errors of scale are smaller.

It is easy to draw.

As before, the pole is represented by a parallel of finite length, which makes it clear that the projection is useless in extreme latitudes.

This is one of the best of all projections for an Atlas map, but its merits have been very much overlooked until recently, when it was adopted for the original one in a million maps of Great Britain and of India, only to be displaced by the resolutions of the International Map Committee (see page 56).

This projection is commonly, but quite wrongly, called the "secant conic" projection, as distinguished from the simple or "tangent conic" projection described above. But it is clear that the description is false. If we passed a cone through the two selected standard parallels upon the sphere, cut it along a meridian, and laid it out flat, it would not give the same projection as that which we are now considering. The two standard parallels would be of the right length, but they would evidently not be at the right distance apart; in fact the distance between them would be a *chord* of the sphere instead of the corresponding arc. The beginner must take great care to avoid this mistaken idea of the nature of the conical projection with two standard parallels.

The conical equal area projection with one standard parallel.

In all conical projections the scale along the parallels is incorrect, except for the standard parallel or parallels. Hence if we wish to make an equal area conical projection we must suitably modify the distances between the parallels, and make the scale along the meridians incorrect in the inverse proportion.

Suppose, for example, that we wish to modify the simple conic with one standard parallel, to make it equal area.

Let ϕ_0 be the latitude of this parallel. We shall show later (Chap. VIII) that we may attain the desired result by the following means:

Keep the standard parallel its correct length, but describe it with radius

$2R \tan \frac{1}{2}(90^\circ - \phi_0)$ instead of $R \cot \phi_0$ or $R \tan (90^\circ - \phi_0)$.

The cone is no longer the tangent cone at the standard parallel. And the constant of the cone becomes $\cos^2 \frac{1}{2}(90^\circ - \phi_0)$ instead of $\sin \phi_0$.

Instead of marking off the other parallels at their true distances, describe them with radii computed by the formula

$$\text{radius for lat. } \phi = 2R \sec \tfrac{1}{2}(90^\circ - \phi_0) \sin \tfrac{1}{2}(90^\circ - \phi).$$

We shall prove later (*loc. cit.*) that this construction gives an equal area projection.

The scale along both meridians and parallels is wrong, in inverse proportions. It is consequently less orthomorphic than either [illegible] projections, in which the scale along the [illegible] and as easy to draw as other conical proj[illegible]

conical projections, the pole is not repr[illegible]ut by a point, the vertex. This does not, [illegible] more suitable for the representation of [illegible] whole of which are included on the map in a sector of angle $2n\pi = 2\pi \cos^2 \frac{1}{2}(90^\circ - \phi_0)$.

On the side of the standard parallel towards the vertex the scale along the meridians increases quickly; and since the projection is equal area, it follows that the scale along the parallels decreases in inverse proportion. On the opposite side of the parallel the reverse takes place. In the former case the configurations are elongated north and south; in the latter case east and west. Hence the projection is not useful for a map having great extension in latitude; and it has been employed very little.

We should note that, though the above is what is always known as *the* conical equal area projection, it is not by any means the only one possible. Without altering the constant of the cone, or the angles between the meridians, one may so modify the distances between the parallels that the scale along the meridians is inversely proportional to the scale along the

Are there good examples? Yes > No. 3

- Check the definitions from Lee, L. P. – **there is no notion of cylindric or cone surfaces:**

Lee, L. P. (1944): The Nomenclature and Classification of Map Projections, Empire Survey Review, No. 51, Vol. VII, 190–200.

Cylindric: projections in which the meridians are represented as a system of equidistant parallel straight lines, and the parallels by a system of parallel straight lines at right angles to the meridians.

Conic: projections in which the meridians are represented as ...

Azimuthal: projections in which the meridians are represented as ...

Are there good examples? Yes > No. 3

- Check the definitions from Lee, L. P. – **there is no notion of cylindric or cone surfaces:**

"No reference has been made in the above definitions to cylinders, cones or planes. The projections are termed cylindric or conic because they can be regarded as developed on a cylinder or cone, as the case may be, but it is as well to dispense with picturing cylinders and cones, since they have given rise to much misunderstanding.

Particularly is this so with regards to the conic projections with two standard parallels: they may be regarded as developed on cones, but they are cones which bear no simple relationship to the sphere."

Are there good examples? Yes > No. 4

Some projections of the azimuthal, cylindrical and conic families have a direct geometric interpretation as light rays projected from a source intercept the Earth and, according to laws of perspective, "draw" its features on a surface. The latter may be a plane, yielding the map itself, or an intermediate shape like a cylindrical or conical shell.

http://www.progonos.com/furuti/MapProj/

On the other hand, many projections are only distantly inspired by geometric principles. For instance, Mercator's cylindrical projection can't be visualized as a perspective process unless:

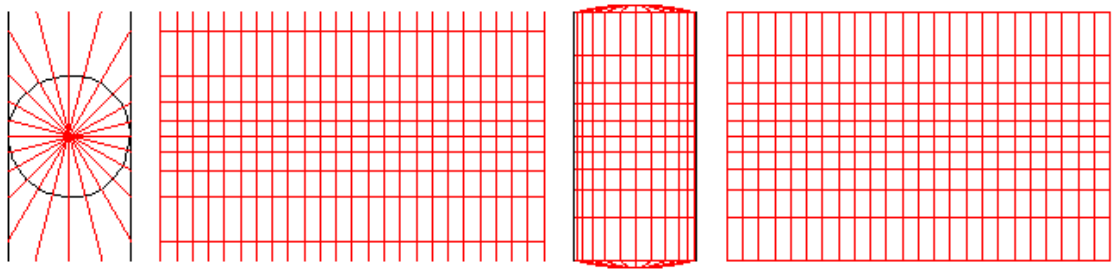
- light rays don't follow straight trajectories, or
- the light source is not a point or straight line, or
- the projection surface is not a simple tube

In all three cases the complexity negates the usefulness of a perspective model. Indeed, many projections have simply no geometric or physical interpretation, and are described purely by mathematical formulae.

- https://web.archive.org/web/20140203200539/http://www.progonos.com/furuti/MapProj/Normal/CartHow/cartHow.html

Are there good examples? Yes > No. 5

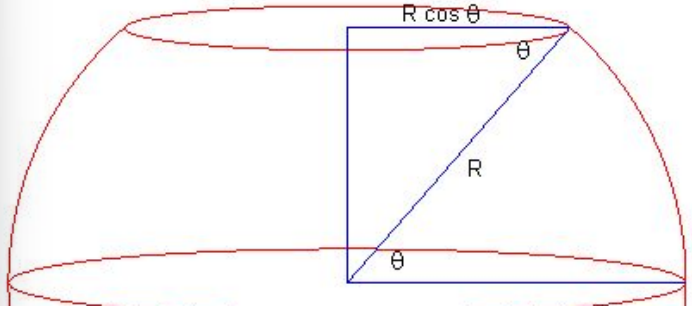
- https://personal.math.ubc.ca/~israel/m103/mercator/mercator.html



Many people have an incorrect idea of how Mercator's projection works. This seems to be commonly taught in high schools. The idea is to imagine a cylinder with the globe inside, and light projected from the centre through the globe onto the cylinder, as in the picture here. Then unroll the cylinder. This cylindrical projection is NOT Mercator's projection.

If you want a physical model of Mercator's projection, let the globe be a spherical balloon that is blown up inside the cylinder, and sticks to the cylinder when it comes into contact with it.

How to do it



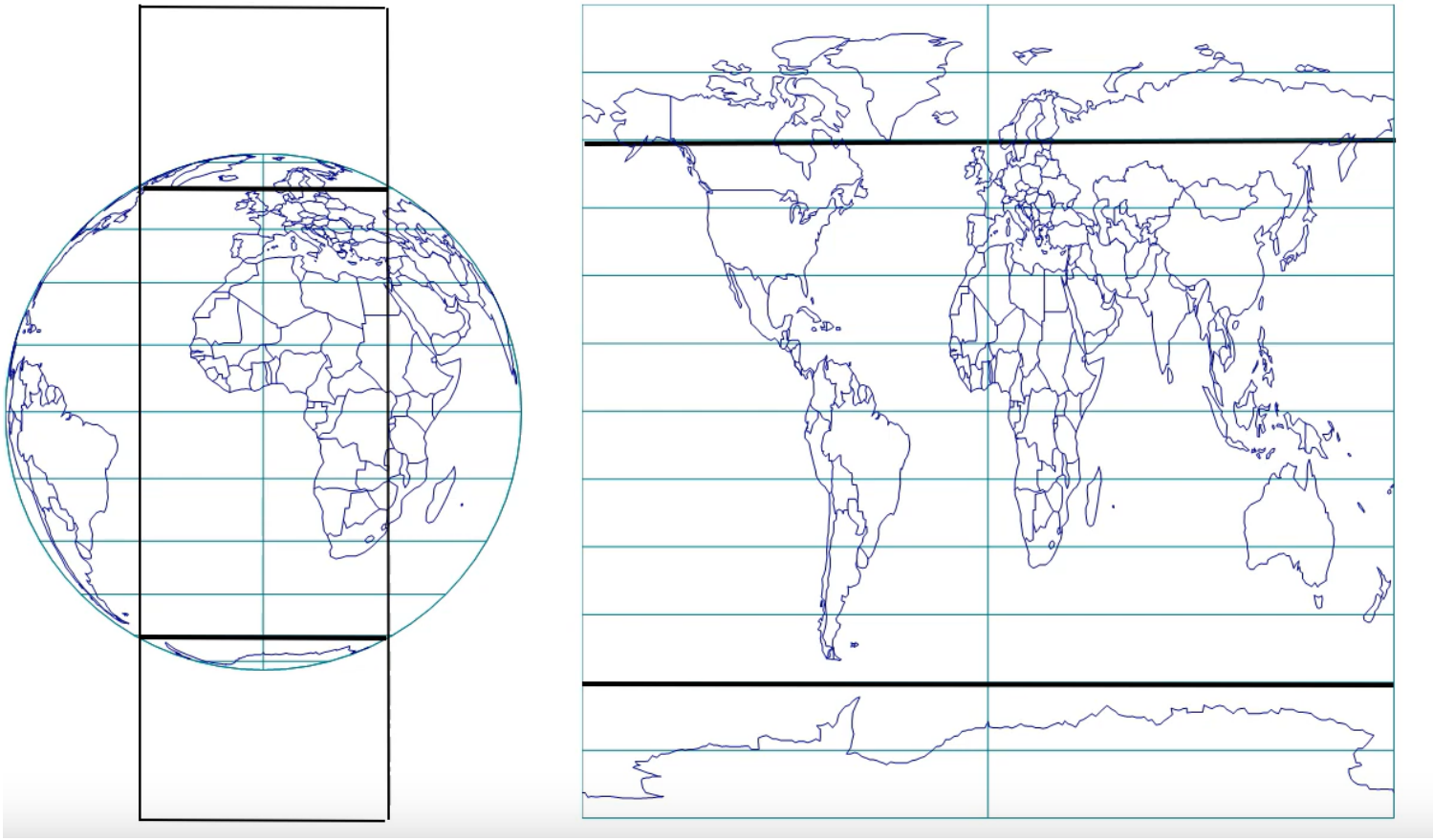
In 1599 the English mathematician Edward Wright explained the mathematics of exactly how Mercator's projection should be done. Let's say that the distance on Mercator's map from the equator to the parallel at latitude θ is $F(\theta)$, in units

where the equator has length 2π on the map.

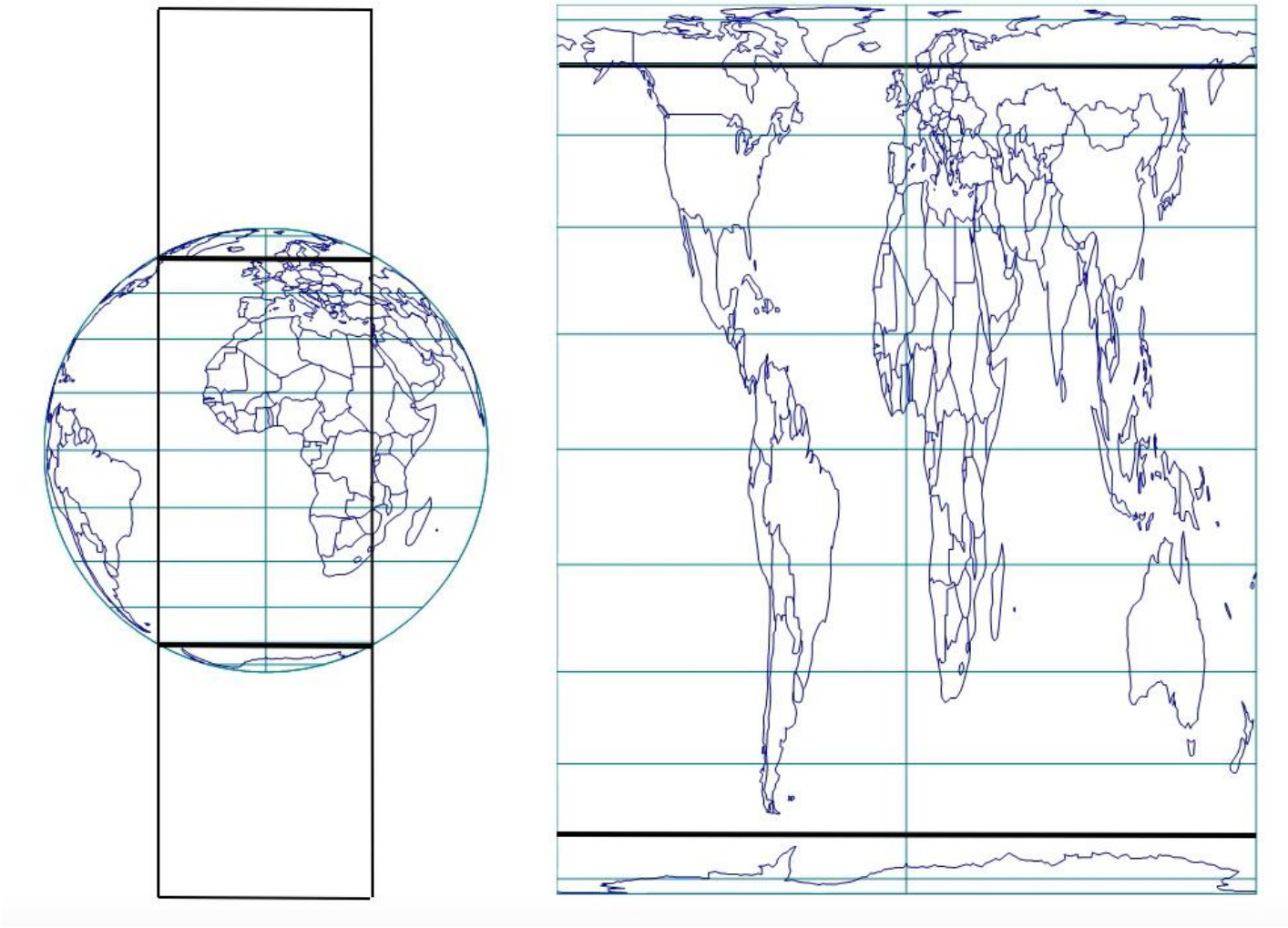
Here is a picture of the Earth with the top sliced off at latitude θ. The equator is a circle with radius R (the radius of the Earth) and circumference $2\pi R$, and the parallel at latitude θ is a circle of radius $R\cos(\theta)$ and circumference

$2\pi R\cos(\theta)$. But on the map the parallels are all the same length. So the

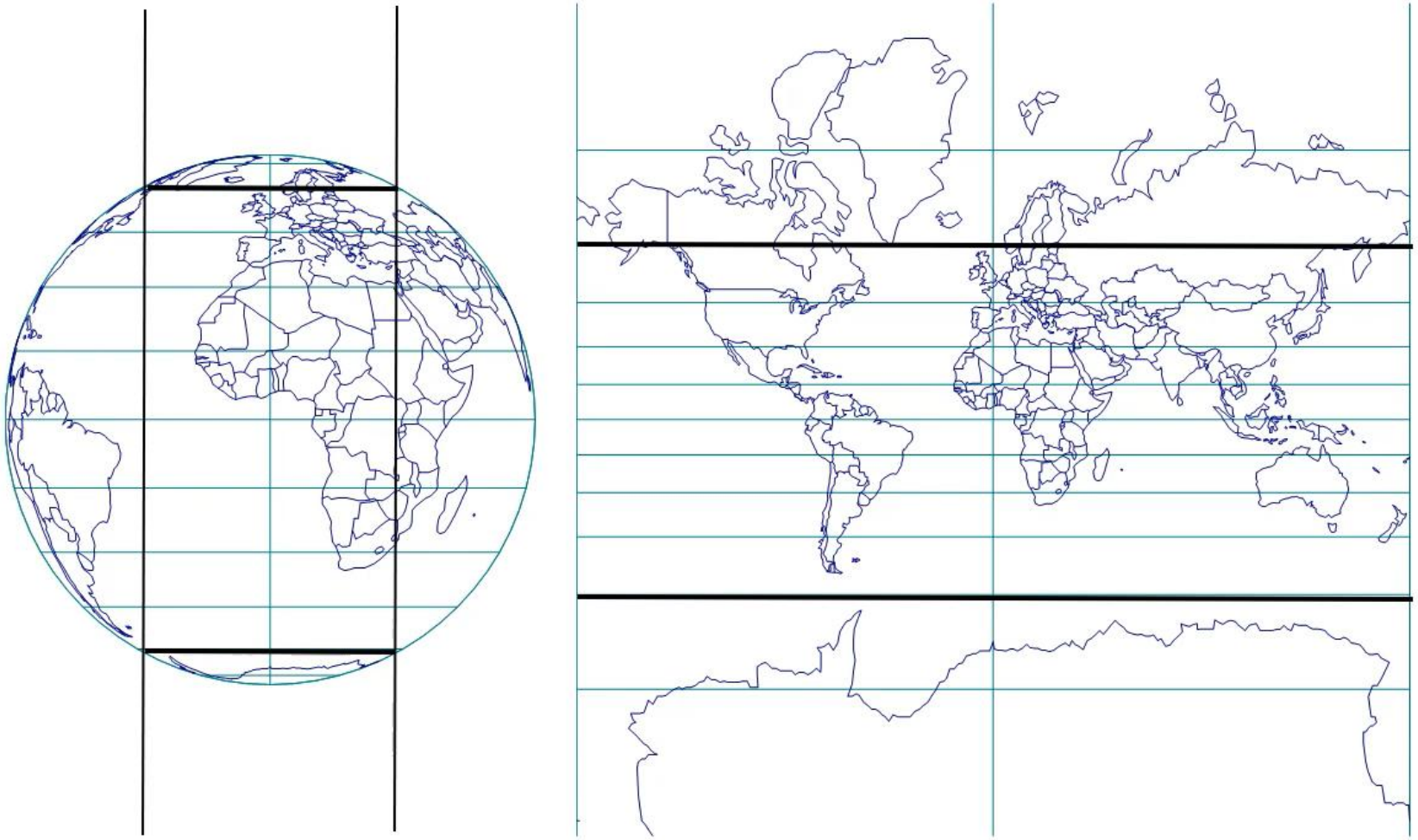
Is equidistant cylindric projection mapping to a cylinder shape?



Is equivalent cylindric projection mapping to a cylinder shape?

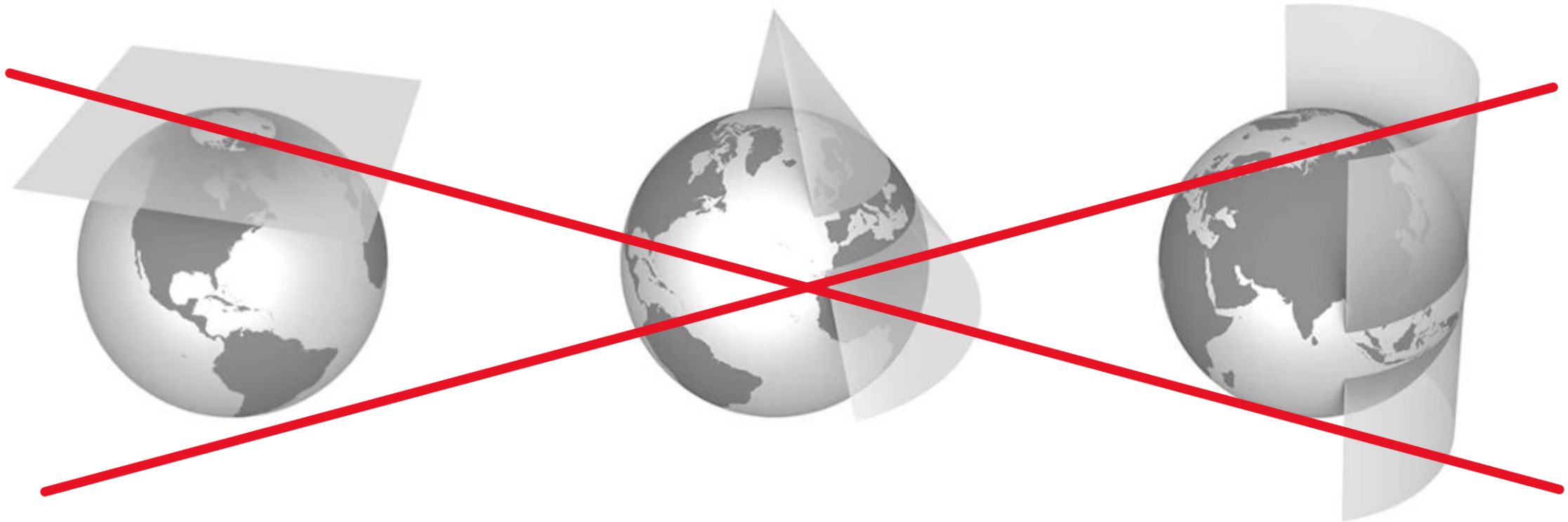


Is Mercator projection (conformal projection) mapping to a cylinder shape?



Let’s remember

- **Generally, there are no cylinders in cylindrical projections.**
- Exceptions are perspective cylindrical projections, which are very rarely used!
- It is not vise to use intermediate surface while describing map projections because:
  - **most map projections in its definition do not have an intermediate surface**
  - visualizing projection with intermediate surface **can lead to wrong conclusions about deformations/distortions in map projections**
- Please, do not call map projections **inaccurate**! They are only distorted and this distortion **must be considered** when reading map.



What can we do about it?

- **We can use QGIS and follow next steps:**

1. Design a custom map in cylindric or conic projection
2. Print a map to a paper
3. Once that we have created an instance of a map projection
   (we have mapped earth surface to a plane!)
4. Then we can fold our map into a shape of a cylinder or a cone!

What can we do about it?

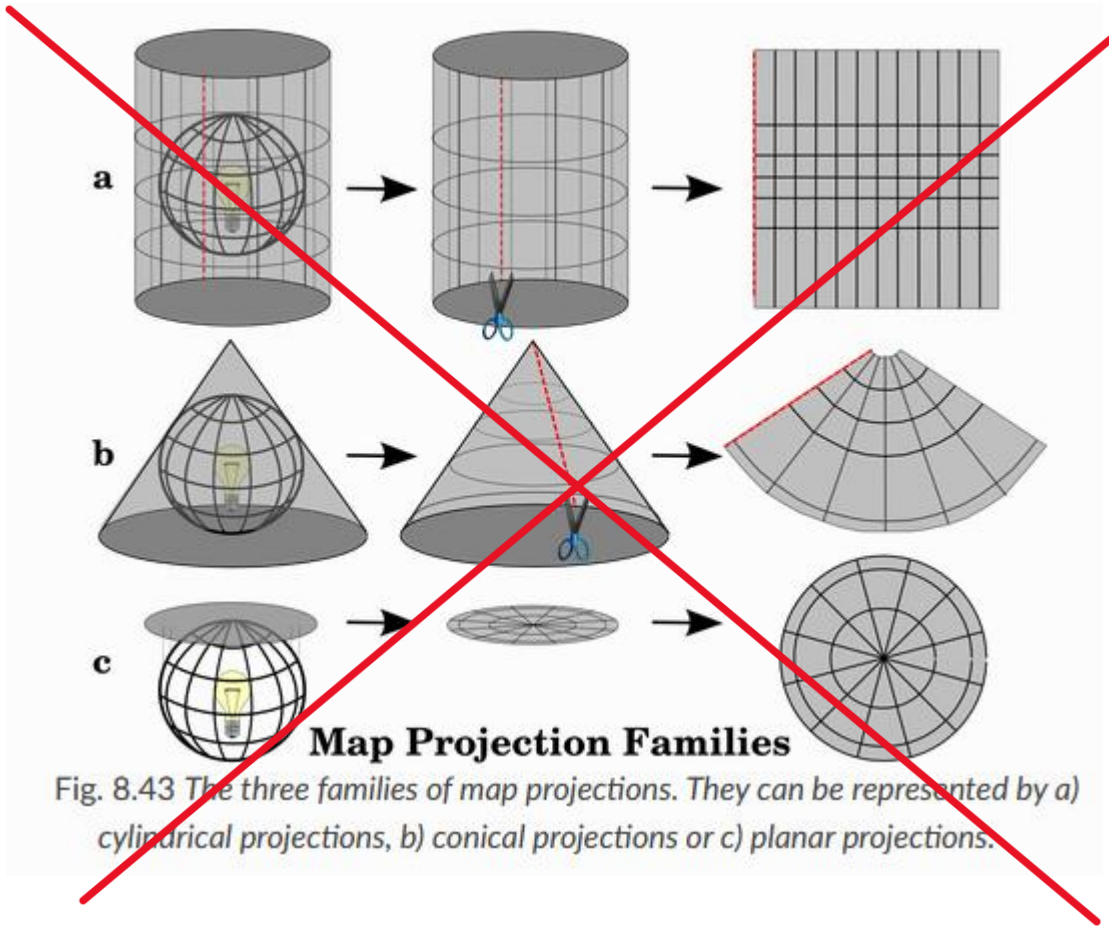
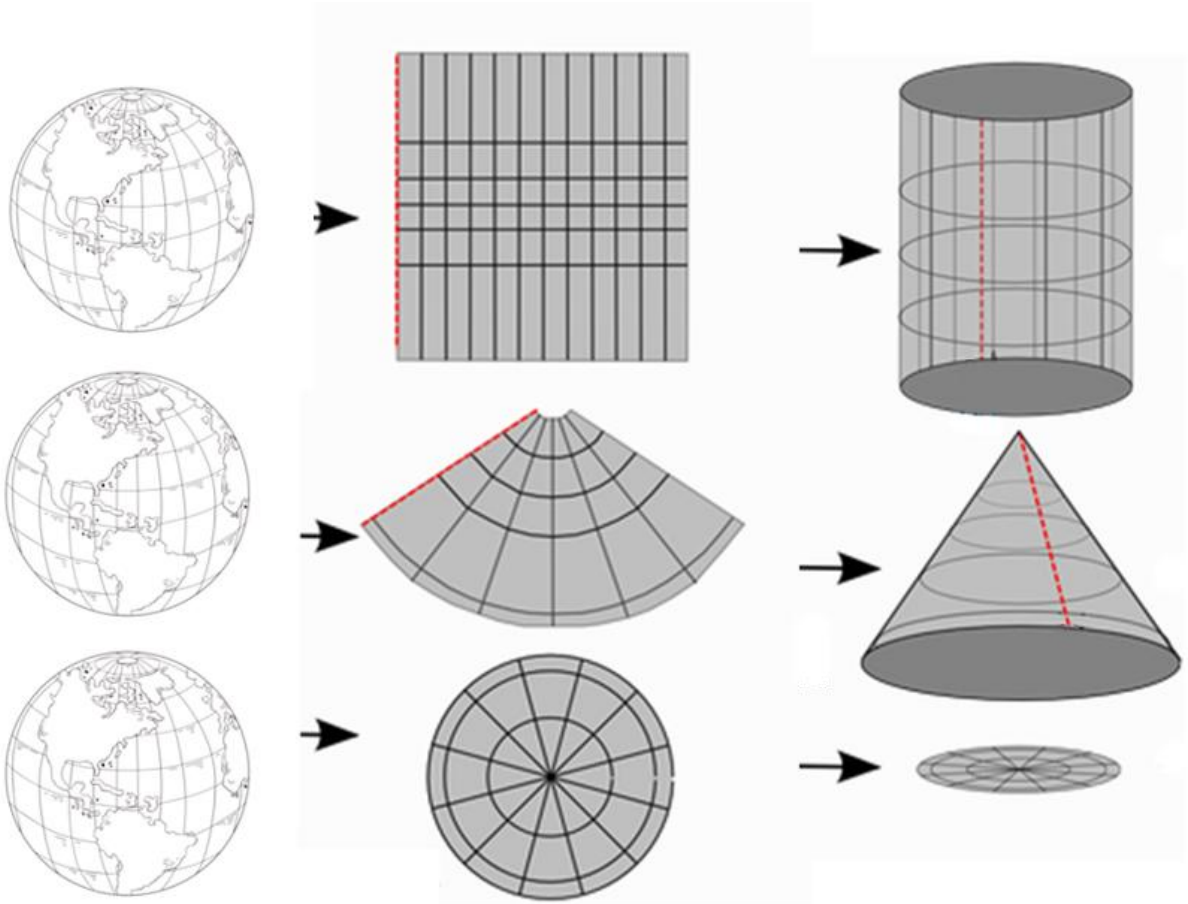


Fig. 8.43 *The three families of map projections. They can be represented by a) cylindrical projections, b) conical projections or c) planar projections.*



Important notes!

- This workshop is intended to properly describe names "cylindrical projections" and "conical projections".

- Cylindrical projections are not suitable for world maps because of the large deformations near the poles. Parallels are circles that get smaller and smaller as we approach the poles, and in cylindrical projections all images of parallels are the same length as the image of the equator.

  Please read: Robinson A. H. (1990): Rectangular world maps – no!, Professional Geographer, v. 42, no. 1, p. 101-104. (Opposes general use of cylindrical projections for world maps).

- Conical projections do not need to be applied to hemisphere - they can be applied to either smaller or bigger part of hemisphere or to entire sphere - for example, one can use equidistant conic projection along all meridians (ESRI:54027).

Additional readings

- **Choosing a Map Projection (Editors: Miljenko Lapaine, E. Lynn Usery)**
- https://link.springer.com/book/10.1007/978-3-319-51835-0

- **Map Projections Classification (Miljenko Lapaine and Nedjeljko Frančula)**
- https://www.mdpi.com/2673-7086/2/2/19

- Map projections (Nedjeljko Frančula - in Croatian [author of Frančula projections])
- https://www.researchgate.net/publication/279885540_Kartografske_projekcije

- A problem in 'Basic Cartography' (Miljenko Lapaine)
- https://www.tandfonline.com/doi/figure/10.1080/23729333.2022.2157106?scroll=top&needAccess=true

Additional readings

- Matching Standard and Secant Parallels in Cylindrical Projections (Miljenko Lapaine)
- https://www.mdpi.com/2220-9964/12/2/63

- Map Projection Aspects (Miljenko Lapaine and Nedjeljko Frančula)
- https://cartography.tuwien.ac.at/eurocarto/wp-content/uploads/2015/09/10_2_ppt.pdf
- https://cartography.tuwien.ac.at/eurocarto/wp-content/uploads/2015/10/4-10.pdf

- Equidistant, Standard and Secant Parallels (Miljenko Lapaine, Paulo Márcio Leal de Menezes)
- https://kig.kartografija.hr/index.php/kig/article/view/909/1670

Data and presentation – 21st ICGC

- https://github.com/phidrho/21_ICGC_materials

Data and presentation – QGIS Open Day

- https://github.com/qgis/QGIS/wiki/QOD-July-2025

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Thank you for your attention and understanding!