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# A note on the existence of a solution and stability for Lipschitz discrete-time descriptor systems[☆]

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ABSTRACT

This paper addresses the existence and uniqueness of a solution and stability for Lipschitz discrete-time descriptor systems. By means of the fixed point principle, a criterion is presented via a matrix inequality, and the criterion guarantees the existence and uniqueness of a solution. In addition, a sufficient condition is also presented to ensure that the solution exists for any compatible initial condition and the system is globally exponentially stable simultaneously. It is shown that the criterion presented in this paper is easy to verify and independent of the choices of decomposition matrices for the given system. Finally, the approach is illustrated by a numerical example.



## 1. Introduction

Recently, a considerable amount of research concerning descriptor systems (singular systems or generalized state–space systems or implicit systems) has been addressed in the literature, see Dai (1989), Fang and Chang (1996), Fang, Lee, and Chang (1994), Gao and Ding (2007), Ho and Lu (2003), Lin and Chen (1999), Lin, Lam, Wang, and Yang (2001), Lu and Ho (2006), Lu and Ho (2006), Lu and Ho (2006), Xia, Zhang, and Boukas (2008), Xiong, Ho, and Cao (2008), Xu, Lam, and Yang (2001), Xu, Yang, Niu, and Lam (2001), Xu and Lam (2004a), Xu and Lam (2004b), Xu and Yang (2000), Zhou and Lu (2009) and Zuo, Ho, and Wang (2010). A linear descriptor system may possess impulsive modes and a non-singular transfer function, which is remarkably different from a non-singular system. Some attention has been focused on the significant problems of robust stability and stabilization for linear discrete-time descriptor systems (Fang & Chang, 1996; Fang et al., 1994; Lin & Chen, 1999; Lin et al., 2001; Xu & Lam, 2004a,b; Xu, Lam et al., 2001; Xu, Yang et al., 2001).

It is interesting to find that all uncertainties are *time-invariant* for uncertain discrete-time descriptor systems presented in the aforementioned literatures. However, there is limited work in the literature for discrete-time descriptor systems with *time-varying* uncertainties. The main difficulty is that the existence of a solution has not been fully studied for *time-varying* uncertain discrete-time descriptor systems. It should be mentioned that the existence of a solution is a fundamental issue for descriptor systems. Certain classes of linear time-varying (or time-invariant) uncertain discrete-time descriptor systems can be regarded as a special case of Lipschitz discrete-time descriptor systems (LDDS). To the best of the authors' knowledge, the issue of existence of solution for LDDS has not been fully investigated yet and remains important and challenging.

In Lu and Ho (2006), generalized quadratic stability for continuous-time descriptor systems with nonlinear perturbation is investigated. It provides a sufficient condition for the existence and uniqueness of the solution. The work in Lu and Ho (2006) provides a fundamental basis for the descriptor systems studied in Gao and Ding (2007), Xiong et al. (2008) and Zuo et al. (2010). The objective of this paper is to present a criterion to guarantee the existence and uniqueness of the solution for LDDS. The criterion is independent of the choices of decomposition and only dependent of the system matrices. In addition, the criterion is in a simple form and can easily be checked. The main tool used is the fixed point principle. Our approach can be directly extended to discuss the existence of a solution for the LDDS with time-delay. Furthermore, existence and uniqueness of the solution and the global exponential stability for the LDDS are established via LMIs simultaneously. Finally, a numerical example is presented to

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show the effectiveness of the proposed approach. The result of this work is considered as a counterpart of that in Lu and Ho (2006), however, the mathematical approach is more difficult and the proof is nontrivial to establish. It is expected that the contribution of this work will also be useful for development of further results pertaining to discrete-time descriptor systems.

*Notation*: $\mathbb{R}, \mathbb{R}^n, \mathbb{R}^{n\times m}$: the set of real numbers, $n$-vectors and $n$ by $m$ matrices, respectively; $\mathbb{N} = \{0, 1, 2, \ldots\}$; $W^T$: transpose of matrix $W \in \mathbb{R}^{n\times m}$; $\overline{\sigma}(M)$: $[\lambda_{\max}(W^TW)]^{\frac{1}{2}}$, i.e. the square root of the maximal eigenvalue of $W^TW$; $X^{-T}$: transpose of matrix $X^{-1}$; $\det(X)$: the determinant of the square matrix $X$; $\deg(\cdot)$: the degree of a polynomial; $I$ $(I_r)$: identity matrix of appropriate dimensions (of $\mathbb{R}^{r\times r}$); $\|x\| = \sqrt{x^Tx}$, where $x = \begin{pmatrix} x_1 & x_2 & \cdots & x_n \end{pmatrix}^T \in \mathbb{R}^n$. Throughout this paper, for symmetric matrices $X$ and $Y$, $X \geq Y$ (respectively, $X > Y$): $X - Y$ is positive semi-definite (respectively, positive definite); $X \leq Y$ (respectively, $X < Y$): $X - Y$ is negative semi-definite (respectively, negative definite). Matrices, if not explicitly stated, are assumed to have compatible dimensions.

## 2. Preliminaries

Consider the following class of Lipschitz discrete-time descriptor systems

$$Ex_{k+1} = Ax_k + B\Phi(k, x_k), \tag{1}$$

where $x_k \in \mathbb{R}^n$ ($n \in \mathbb{N}$, with $n > 0$) is the system state, constant matrices $A, E \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times s}$; matrix $E$ is singular. It is assumed that $0 < \mathrm{rank}(E) = r < n$; $\Phi = \Phi(k, x_k) \in \mathbb{R}^s$ is a vector-valued nonlinear time-varying function with $\Phi(k, 0) = 0$ for all $k \in \mathbb{N}$ and satisfies the following Lipschitz condition for all $(k, x_k, \tilde{x}_k) \in \mathbb{N} \times \mathbb{R}^n \times \mathbb{R}^n$:

$$\|\Phi(k, x_k) - \Phi(k, \tilde{x}_k)\| \leq \|F(x_k - \tilde{x}_k)\|, \tag{2}$$

where $F$ is a constant matrix with appropriate dimension. It follows from (2) that

$$\|\Phi(k, x_k)\| \leq \|Fx_k\|. \tag{3}$$

**Remark 2.1.** Consider the following linear uncertain descriptor systems

$$Ex_{k+1} = Ax_k + BG(k)Fx_k, \tag{4}$$

where $G(k)$ is a time-varying uncertainty with appropriate dimension satisfying $G^T(k)G(k) \leq I$. Let $\Phi(k, x_k) = G(k)Fx_k$, then it is easy to show that $\Phi(k, x_k)$ satisfies constraints (2) and (3), which implies that system (4) is a special case of system (1).

**Definition 2.1.** (1) The pair $(E, A)$ is said to be regular if $\det(zE - A)$ is not identical zero. (2) The pair $(E, A)$ is said to be causal if deg $(\det(zE - A)) = \mathrm{rank}(E)$, where $z \in \mathbb{C}$.

Suppose the pair $(E, A)$ is regular and causal, then the descriptor system (1) can be partitioned into two parts (see Dai, 1989 for more details), namely a dynamic subsystem and an algebraic constraint (equation). If an initial condition satisfies the algebraic constraint, then the initial condition is called a compatible initial condition.

It should be mentioned that even though the pair $(E, A)$ is regular and causal, a solution for system (1) may not exist for some compatible initial conditions; see the following two examples.

**Example 1.** Consider the following LDDS

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} x_{k+1} = \begin{pmatrix} -0.5 & 0 \\ 0 & 1 \end{pmatrix} x_k + \begin{pmatrix} 0 \\ \phi(k, x_k) \end{pmatrix}, \tag{5}$$

where $x_k = \begin{pmatrix} v_k \\ w_k \end{pmatrix} \in \mathbb{R}^2$, $\phi(k, x_k) = |v_k + 2w_k|$. It is clear that $\phi(k, x_k)$ satisfies Lipschitz condition. For compatible initial condition $v_0 = 1$ and $w_0 = -1$, it follows from (5) that $v_1 = -0.5$, but one cannot get any solution from $w_1 + |v_1 + 2w_1| = 0$.

In addition, a solution for the uncertain system (4) also may not exist for some compatible initial condition.

**Example 2.** Consider system (5) with

$$\phi(k, x_k) = v_k \cos(k\pi) + w_k \sin\left(\frac{k}{2}\pi\right).$$

It is clear that for compatible initial condition $v_0 = 1$ and $w_0 = -1$, $w_3$ does not exist.

## 3. Existence of a solution

Suppose that the pair $(E, A)$ is regular and causal, then there exist two nonsingular matrices

$$M = \begin{pmatrix} M_1 \\ M_2 \end{pmatrix}, \qquad N = \begin{pmatrix} N_1 & N_2 \end{pmatrix} \tag{6}$$

such that

$$MEN = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}, \qquad MAN = \begin{pmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{pmatrix}, \tag{7}$$

where $M_1 \in \mathbb{R}^{r\times n}$, $M_2 \in \mathbb{R}^{(n-r)\times n}$, $N_1 \in \mathbb{R}^{n\times r}$, $N_2 \in \mathbb{R}^{n\times(n-r)}$ and $A_1 \in \mathbb{R}^{r\times r}$, see Dai (1989).

The following theorem is one of the main results in this paper, which presents a simple criterion to verify the existence and uniqueness of the solution for system (1).

**Theorem 3.1.** *Suppose that the pair $(E, A)$ is regular and causal, then a solution of system (1) exists uniquely to any compatible initial condition if the following inequality holds for some decompositions (6) and (7):*

$$FN_2M_2BB^TM_2^TN_2^TF^T < I. \tag{8}$$

**Proof.** Inequality (8) implies that there exists a sufficiently small $\epsilon > 0$ such that the following inequality also holds.

$$FN_2M_2(BB^T + \epsilon I)M_2^TN_2^TF^T < I. \tag{9}$$

Nonsingularity of $M$ implies that $M_2$ is full row rank, then $M_2M_2^T$ is positive definite and $M_2(BB^T + \epsilon I)M_2^T > 0$, so we can choose $X_{22} = [M_2(BB^T + \epsilon I)M_2^T]^{-\frac{1}{2}}$ and let

$$\tilde{M} = \begin{pmatrix} M_1 \\ \tilde{M}_2 \end{pmatrix}, \qquad \tilde{N} = \begin{pmatrix} N_1 & \tilde{N}_2 \end{pmatrix},$$

where $\tilde{M}_2 = X_{22}M_2$ and $\tilde{N}_2 = N_2X_{22}^{-1}$. It follows that

$$\tilde{M}E\tilde{N} = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}, \qquad \tilde{M}A\tilde{N} = \begin{pmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{pmatrix}, \tag{10}$$

and $\tilde{M}_2BB^T\tilde{M}_2^T < I$ holds for the sufficiently small $\epsilon > 0$, that is, $\overline{\sigma}(\tilde{M}_2B) < 1$.

In addition, inequality (9) implies

$$F(N_2X_{22}^{-1})(N_2X_{22}^{-1})^TF^T = F\tilde{N}_2\tilde{N}_2^TF^T < I, \tag{11}$$

which also implies that there exists a sufficiently small $\mu > 0$ such that

$$\overline{\sigma}(F\tilde{N}_2) < 1 - \mu. \tag{12}$$

Introduce a change of coordinates

$$\tilde{N}^{-1}x_k = \begin{pmatrix} v_k \\ w_k \end{pmatrix}, \quad v_k \in \mathbb{R}^r,\ w_k \in \mathbb{R}^{n-r}, \tag{13}$$

then from (10), system (1) is equivalent to

$$\begin{aligned} v_{k+1} &= A_1 v_k + M_1 B\Phi(k, N_1 v_k + \tilde{N}_2 w_k), \\ 0 &= w_k + \tilde{M}_2 B\Phi(k, N_1 v_k + \tilde{N}_2 w_k). \end{aligned} \tag{14}$$

Therefore, it is sufficient to show that $w_k$ exists uniquely for any given $v_k$ from the algebraic constraint in (14). Note that for any given $w_k,\ \tilde{w}_k \in \mathbb{R}^{n-r}$,

$$\begin{aligned} &\|\tilde{M}_2 B\Phi(k, N_1 v_k + \tilde{N}_2 w_k) - \tilde{M}_2 B\Phi(k, N_1 v_k + \tilde{N}_2 \tilde{w}_k)\| \\ &\quad \le \overline{\sigma}(\tilde{M}_2 B)\|\Phi(k, N_1 v_k + \tilde{N}_2 w_k) - \Phi(k, N_1 v_k + \tilde{N}_2 \tilde{w}_k)\| \\ &\quad \le \overline{\sigma}(F\tilde{N}_2)\|w_k - \tilde{w}_k\| \\ &\quad \le (1-\mu)\|w_k - \tilde{w}_k\|. \end{aligned} \tag{15}$$

It follows from the well-known fixed point principle (Curtain & Protchard, 1977) that (15) implies that there exists a unique solution $w_k = \phi(k, v_k)$ for any given $v_k$ from the second equation of (14), which completes the proof. □

**Remark 3.1.** Inequality (8) in Theorem 3.1 is independent of any decomposition of pair $(E, A)$, that is, condition (8) is independent of the choice $M_2$ and $N_2$. The reason is shown as follows: for any nonsingular matrices $\tilde{M}$ and $\tilde{N}$ such that

$$\tilde{M}E\tilde{N} = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}, \qquad \tilde{M}A\tilde{N} = \begin{pmatrix} \tilde{A}_1 & 0 \\ 0 & I_{n-r} \end{pmatrix}, \tag{16}$$

where $\tilde{A}_1 \in \mathbb{R}^{r\times r}$, then there exist two nonsingular matrices $X$ and $Y$ such that $\tilde{M} = XM$ and $\tilde{N} = NY$. Decompose $X$ and $Y$ with appropriate dimensions as

$$X = \begin{pmatrix} X_{11} & X_{12} \\ X_{21} & X_{22} \end{pmatrix}, \qquad Y = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix}. \tag{17}$$

From (7) and (16), it easily follows that $Y_{11} = X_{11}^{-1}$, $Y_{22} = X_{22}^{-1}$, $X_{12} = Y_{12} = 0$ and $X_{21} = Y_{21} = 0$. Then

$$\tilde{M} = \begin{pmatrix} X_{11}M_1 \\ X_{22}M_2 \end{pmatrix}, \qquad \tilde{N} = \begin{pmatrix} N_1 X_{11}^{-1} & N_2 X_{22}^{-1} \end{pmatrix}, \tag{18}$$

which implies that $\tilde{N}_2\tilde{M}_2 = N_2 M_2$. Therefore inequality (8) is independent of the choice $M_2$ and $N_2$.

**Remark 3.2.** If the conditions in Theorem 3.1 hold, then we can get the same result as in Theorem 3.1 for the following time-delay system

$$Ex_{k+1} = Ax_k + A_d x_{k-d} + B\Phi(k, x_k, x_{k-d}), \tag{19}$$

where positive $d \in \mathbb{N}$, with $d > 0$, is a time-delay, $A_d \in \mathbb{R}^{n\times n}$, $\Phi = \Phi(k, x_k, x_{k-d}) \in \mathbb{R}^s$ is a vector-valued nonlinear time-varying function with $\Phi(k, 0, 0) = 0$ for all $k \in \mathbb{N}$ and satisfies the following Lipschitz condition for all $(k, x_k, x_{k-d}), (k, \tilde{x}_k, x_{k-d}) \in \mathbb{N} \times \mathbb{R}^n \times \mathbb{R}^n$:

$$\|\Phi(k, x_k, x_{k-d}) - \Phi(k, \tilde{x}_k, x_{k-d})\| \le \|F(x_k - \tilde{x}_k)\|. \tag{20}$$

## 4. Stability

The following theorem is the main result of this section, which presents a sufficient condition via LMIs, to guarantee both the existence and uniqueness of solution, and global exponential stability of system (1).

**Theorem 4.1.** *System (1) is globally exponentially stable if there exists a symmetric matrix P such that the following linear matrix inequalities (LMIs) hold:*

$$E^T PE \ge 0, \tag{21}$$

$$\Omega = \begin{pmatrix} A^T PA - E^T PE + F^T F & A^T PB \\ B^T PA & B^T PB - I \end{pmatrix} < 0. \tag{22}$$

**Proof.** The proof is divided into two parts. The first part is to show that LMIs (22) imply that inequality (8) holds, which guarantees the existence and uniqueness of a solution for the given system (1) from Theorem 3.1.

Consider decomposition (7) and let $M^{-T}PM^{-1} = \begin{pmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{pmatrix}$, where $P_1 \in \mathbb{R}^{r\times r}$, $P_2 \in \mathbb{R}^{r\times(n-r)}$, $P_3 \in \mathbb{R}^{(n-r)\times(n-r)}$. Then LMIs (21) and (22) are equivalent to the following LMIs.

$$N^T E^T PEN \ge 0, \qquad \mathrm{diag}(N^T, I)\Omega\,\mathrm{diag}(N, I) < 0, \tag{23}$$

which are also equivalent to

$$P_1 \ge 0, \qquad \begin{pmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{12}^T & \Omega_{22} \end{pmatrix} < 0, \tag{24}$$

where

$$\Omega_{11} = \begin{pmatrix} A_1^T P_1 A_1 - P_1 & A_1^T P_2 \\ P_2^T A_1 & P_3 \end{pmatrix} + N^T F^T FN,$$

$$\Omega_{12} = \begin{pmatrix} A_1^T P_1 & A_1^T P_2 \\ P_2^T & P_3 \end{pmatrix} MB, \tag{25}$$

$$\Omega_{22} = B^T M^T \begin{pmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{pmatrix} MB - I.$$

By pre- and post-multiplying the second LMI in (24) with $\begin{pmatrix} 0 & I_r & 0 \\ 0 & 0 & I_s \end{pmatrix}$ and its transpose, respectively, we obtain the following inequality,

$$\begin{pmatrix} P_3 + N_2^T F^T FN_2 & P_2^T M_1 B + P_3 M_2 B \\ B^T M_1^T P_2 + B^T M_2^T P_3 & \Gamma \end{pmatrix} < 0, \tag{26}$$

where

$$\begin{aligned} \Gamma &= B^T M_1^T P_1 M_1 B + B^T M_1^T P_2 M_2 B \\ &\quad + B^T M_2^T P_2^T M_1 B + B^T M_2^T P_3 M_2 B - I. \end{aligned} \tag{27}$$

Using the Schur Complement Lemma (Boyd, Ghaoui, Feron, & Balakrishnan, 1994), inequality (26) is equivalent to the inequalities

$$P_3 + N_2^T F^T FN_2 < 0 \tag{28}$$

and

$$\begin{aligned} &\Gamma - (B^T M_1^T P_2 + B^T M_2^T P_3)(P_3 + N_2^T F^T FN_2)^{-1} \\ &\quad \times (P_2^T M_1 B + P_3 M_2 B) < 0. \end{aligned} \tag{29}$$

Apply the completing square technique to the LHS of inequality (29), inequality (29) is also equivalent to

$$\begin{aligned} &B^T M_2^T N_2^T F^T FN_2 M_2 B - I + B^T M_1^T P_1 M_1 B \\ &\quad < (Z_1 + Z_2)^T (P_3 + N_2^T F^T FN_2)^{-1}(Z_1 + Z_2), \end{aligned} \tag{30}$$

where

$$Z_1 = P_2^T M_1 B + P_3 M_2 B, \qquad Z_2 = -(P_3 + N_2^T F^T FN_2)M_2 B.$$

Noticing that $P_1 \ge 0$, therefore inequalities (28) and (30) yield

$$B^T M_2^T N_2^T F^T FN_2 M_2 B < I, \tag{31}$$

which is equivalent to inequality (8). Therefore, the solution of system (1) exists uniquely.

The second part is to show the stability. To this end, choose the Lyapunov function candidate as follows:

$$V_k = x_k^T E^T P E x_k. \tag{32}$$

Then, the difference of $V_k$ along system (1) yields

$$\begin{aligned} V_{k+1} - V_k &= (Ax_k + B\Phi)^T P(Ax_k + B\Phi) - x_k^T E^T PEx_k \\ &\le (Ax_k + B\Phi)^T P(Ax_k + B\Phi) - x_k^T E^T PEx_k \\ &\quad + x_k^T F^T F x_k - \Phi^T \Phi \\ &= \begin{pmatrix} x_k^T & \Phi^T \end{pmatrix} \Omega \begin{pmatrix} x_k \\ \Phi \end{pmatrix}. \end{aligned}$$

Let $\lambda_{\min}(-\Omega) = \lambda_0$, then $\lambda_0 > 0$ and we have

$$V_{k+1} - V_k \le -\lambda_0(\|x_k\|^2 + \|\Phi\|^2) \le -\lambda_0 \|x_k\|^2. \tag{33}$$

In addition, it follows from inequality (24) with (25) that $\Omega_{11} < 0$, which implies that $A_1^T P_1 A_1 - P_1 < 0$, that is, $P_1 > A_1^T P_1 A_1 \ge 0$. Then, we have

$$0 \le N^T E^T PEN = \begin{pmatrix} P_1 & 0 \\ 0 & 0 \end{pmatrix} \neq 0. \tag{34}$$

Denote $\lambda_m = \lambda_{\max}(E^T PE)$, then $\lambda_m > 0$ and $V_k \le \lambda_m \|x_k\|^2$. In this case, we have $V_{k+1} - V_k \le -\lambda_0 \lambda_m^{-1} V_k \le -\lambda_0 \lambda_m^{-1} V_{k+1}$, that is, $V_{k+1} \le \beta V_k$, where $\beta = (1 + \lambda_0 \lambda_m^{-1})^{-1} < 1$. Then we have $V_k \le \beta^k V_0$. From this, (33) implies that

$$\|x_k\|^2 \le -\lambda_0^{-1} V_{k+1} + \lambda_0^{-1} V_k \le \lambda_0^{-1} V_k \le \lambda_0^{-1} \beta^k V_0. \tag{35}$$

Therefore system (1) is globally exponentially stable. This completes the proof. □

**Remark 4.1.** The first LMI of (21) is not a strict LMI. In order to use MATLAB® LMI Toolbox to compute numerical examples, we can use a decomposition to transform (21) into a strict LMI. From the proof of the above theorem, we have the following result. System (1) is globally exponentially stable if there exist decompositions (7), positive-definite matrix $P_1 \in \mathbb{R}^{r\times r}$, $P_2 \in \mathbb{R}^{r\times(n-r)}$ and negative-definite matrix $P_3 \in \mathbb{R}^{(n-r)\times(n-r)}$ such that strict LMI (24) with (25) holds.

**Remark 4.2.** It should be mentioned that the feasibility of $P_1, P_2$ and $P_3$ in Remark 4.1 is independent of the choices of decomposition matrix $M$ and $N$. That is, if we choose decomposition matrices $\tilde{M}$ and $\tilde{N}$ in the form of (16), we can show the feasibility for the following LMI is equivalent to that for (24) and (25):

$$\begin{pmatrix} \tilde{\Omega}_{11} & \tilde{\Omega}_{12} \\ \tilde{\Omega}_{12}^T & \tilde{\Omega}_{22} \end{pmatrix} < 0,$$

where

$$\tilde{\Omega}_{11} = \begin{pmatrix} \tilde{A}_1^T P_1 \tilde{A}_1 - P_1 & \tilde{A}_1^T P_2 \\ P_2^T \tilde{A}_1 & P_3 \end{pmatrix} + \tilde{N}^T F^T F \tilde{N},$$

$$\tilde{\Omega}_{12} = \begin{pmatrix} \tilde{A}_1^T P_1 & \tilde{A}_1^T P_2 \\ P_2^T & P_3 \end{pmatrix} \tilde{M} B,$$

$$\tilde{\Omega}_{22} = B^T \tilde{M}^T \begin{pmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{pmatrix} \tilde{M} B - I,$$

$\tilde{M}$ and $\tilde{N}$ are given by (18), $\tilde{A}_1 = X_{11}^{-1} A_1 X_{11}$.

**Remark 4.3.** From the main result in this paper, it is straightforward to obtain the stability results for linear discrete-time descriptor systems with time-varying uncertainties (and time-delays). These stability results can be regarded as extensions of those works presented in Fang and Chang (1996), Lin and Chen (1999), Lin et al. (2001), Xu, Lam et al. (2001), Xu, Yang et al. (2001), Xu and Lam (2004a) and Xu and Lam (2004b), where all uncertainties addressed are *time-invariant* for uncertain discrete-time descriptor systems.

**Remark 4.4.** The main result in this section can be viewed as a counterpart for continuous-time systems addressed in Lu and Ho (2006), however, the proof for the existence and uniqueness of the solution is more difficult and is nontrivial to establish. If $E = I$ in system (1), Theorem 4.1 is similar to Lemma 2 in Ho and Lu (2003), then Theorem 4.1 can be regarded as a nontrivial extension for a class of nonlinear discrete-time systems in Ho and Lu (2003).

**Remark 4.5.** Although the conditions presented in Xu and Yang (2000) and this paper seem to be similar, the research topics are different. Also the system discussed in Xu and Yang (2000) is linear and time-invariant while the system (1) is nonlinear and time-varying. In addition, in this paper we discuss the existence and uniqueness of a solution by means of the fixed point theorem, which is certainly different from those for the linear case in Xu and Yang (2000). The difficulties can be seen from Examples 1 and 2 in Section 2 above. Furthermore, it is interesting to find out that the existence and uniqueness condition is implicitly included in the stability condition.

## 5. Numerical example

Consider the system (1) with the following matrices.

$$\begin{pmatrix} 5 & 0 \\ -5 & 0 \end{pmatrix} x_{k+1} = \begin{pmatrix} -2.5 & 0 \\ 2.5 & 2.5 \end{pmatrix} x_k + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ |\begin{pmatrix} 1 & 1 \end{pmatrix} x_k| \end{pmatrix}. \tag{36}$$

Then $B = F = I_2$. Choose $M = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ and $N = \begin{pmatrix} 0.2 & 0 \\ 0 & 0.4 \end{pmatrix}$, then $MAN = \mathrm{diag}(-0.5, 1)$ and $MEN = \mathrm{diag}(1, 0)$. In this case, inequality (8) holds, which implies that solution for system (36) exists for any compatible initial condition. By means of LMI Toolbox, we can get following solution from LMI (24).

$$P_1 = 0.5284, \qquad P_2 = -0.0592, \qquad P_3 = -8.7918.$$

Therefore system (36) is globally exponentially stable.

## 6. Conclusions

The existence and uniqueness of solution for LDDS and its stability are investigated in this paper. By means of the fixed point principle, a criterion for existence and uniqueness of solution is given via matrix inequality for the given LDDS. If the existence and uniqueness of solution can be guaranteed, a global exponential stability criterion is obtained by means of LMI. All conditions presented in this paper are independent of the decomposition of system matrices. By using the approach developed in this paper, further study will be made on robustness issues for time-varying and nonlinear discrete-time descriptor systems in the future.

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