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Impulsive natural observers for vector second-order Lipschitz non-linear systems

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Xiaomei Lu[1], Wu-Hua Chen[1] ✉, Feifei Xue[1]

[1]*College of Mathematics and Information Science, Guangxi University, Nanning, Guangxi 530004, People's Republic of China*

✉ *E-mail: wuhua_chen@163.com*

Abstract: Many mechanical systems can be modelled by vector second-order differential equations. In this study, the observation problem of a class of vector second-order Lipschitz non-linear systems with discrete measurements is addressed. Under the premise that the discrete measurements of position and velocity vectors are available, an impulsive observer in the second-order framework is designed. The proposed observation scheme ensures that the second-order structure of the observed system can be retained, and allows the sampling on the system output to be aperiodic. The stability analysis of the observation error system is performed by employing an impulse-time-dependent discretised Lyapunov function based method. The novelty of the introduced Lyapunov function is that its structure relies on the partition on the impulse intervals. With the increase of the partition number, the existence condition of impulsive natural observers can be relaxed. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed design method.

1 Introduction

Vector second-order systems are widely used to model the dynamical processes arising in many areas such as flexible structure, vibration analysis, electromagnetic, fluid dynamics, and so on. For example, when modelling a mechanical system, a vector second-order differential equation appears due to applying the Newton's second law of motion, where the second time derivatives of state (accelerations) are expressed in terms of state displacements and velocities. On the other hand, many structural systems are represented by partial differential equations with an infinite number of degrees of freedom. After a finite-dimensional approximation, the resultant model of discretised structure is a vector second-order system. Traditionally, by introducing new state variables, the vector second-order systems can be transformed into augmented vector first-order forms. This implies that the methods for vector first-order systems can be utilised to handle the estimation and control problems of vector second-order systems. However, this augmentation of the vector second-order system leads to the lose of physical insight enjoyed by the original system, and adds the complexity of design due to the increase of dimension. Further, when applying the augmentation approach to design first-order Luenberger observers for second-order mechanical systems, the derivative of the estimated position vector is not necessarily equal to the value of the estimated velocity vector. Therefore, it is of importance to develop new control/observation methods fitting in with the characteristics of vector second-order systems.

In order to overcome the disadvantages of observers design in the first-order state-space framework for vector second-order systems, Balas [1] proposed the so-called natural observer to reconstruct the state of the observed vector second-order system. Different from the first-order observer, the natural observer preserves the second-order algebraic structure of the observed system. As a result, the mismatch of the estimated values of position vector and velocity vector entailed in first-order observers can be avoid. Inspired by the designing idea of natural observers, several design techniques in the second-order framework for different types of vector second-order systems have been developed. In [2], some sufficient conditions for the existence of natural observers for different measurement information were presented and the corresponding design methods were provided therein. In [3], a class of natural observers for two time-scale systems were constructed in the singular perturbation framework. In [4], natural observers were designed for vector second-order system with unknown inputs and also were applied to fault detection. In [5], the observation problem of vector second-order systems by taking measurement noise into account was investigated, and a natural observer for optimal state estimation was constructed. In [6, 7], the results of natural observers for second-order finite dimensional systems were extended to second-order infinite dimensional systems. The control design problem for vector second-order systems in the second-order framework was studied in [8–12].

On the other hand, owing to the applications of digital sensors, the outputs of many control systems are only available at discrete time instants as an outcome of discrete-time sampling processes. For this reason, much attention has been paid to the study of observers design for dynamical systems using sampled measurements [13–15]. In [16], Raff and Allgöwer proposed a so-called impulsive observer to estimate the state of Lipschitz non-linear systems with equidistant discrete-time measurements, by which the observer state is updated in an impulsive fashion. It was pointed out in [16] that the impulsive update can be easily implemented by a microcontroller. In [17], a systematic procedure for designing impulsive functional observers for linear systems was presented, where the sampling times are allowed to be aperiodic. In [18], an additional impulsive feedback element was introduced in the continuous adaptive observer to weaken the persistence-of-excitation condition. By employing time-dependent Lyapunov function based methods, impulsive observers were constructed in [19] for Lipschitz non-linear time-delay systems. In [20], an adaptive impulsive observer was designed to treat the problem of state and parameter estimation for Lipschitz non-linear systems with unknown parameters. Recently, impulsive observers based control methods have been developed in [21, 22]. However, application of these impulsive observers in vector second-order systems has not been investigated in the literature. Due to the usefulness of impulsive observers in the background of broad applications of digital control technology, further study should be needed to develop effective impulsive observer design method for vector second-order systems.

The purpose of this paper is to propose a novel approach for designing impulsive observers for vector second-order Lipschitz non-linear systems. Under the premise that the discrete measurements of position and velocity vectors are available, a new type of impulsive observers in the second-order framework is constructed. Different from the continuous-time natural observers, the velocity vector of the introduced impulsive natural observer (INO) is needed to be updated only at certain discrete instants. Such structure here proposed is capable of preserving the physical insight of the observed system and has the potential to improve practicability in digital control systems, relative to continuous natural observers. The stability analysis of the observation error system is carried out using an impulse-time-dependent discretised Lyapunov function based method in which the construction of the piecewise Lyapunov function is based on the partition on the impulse intervals. As the the partition number increases, the existence condition of INOs can be relaxed.

The rest of this paper is organised as follows. Section 2 presents the description of the proposed INOs and the stability definition of the observation error dynamics. A sufficient condition for the existence of INOs for vector second-order Lipschitz non-linear systems is described in Section 3. Whereas, a numerical simulation is provided to illustrate the proposed design method in Section 4, and Section 5 concludes this paper.

Notations. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\|\cdot\|$ refers to the vector 2-norm or the spectral norm of matrices. I_n and 0_n represent the $n \times n$ identity matrix and zero matrix, respectively. The notation $X > 0$ (respectively, $X \geq 0$) means that the matrix is real symmetric positive definite (respectively, positive semi-definite). For two integers n_1 and n_2 satisfying $n_1 \leq n_2$, set $\overline{n_1, n_2} = \{n_1, n_1 + 1, \ldots, n_2\}$.

2 Problem formulations and preliminaries

Consider the following class of vector second-order non-linear systems:

$$\begin{cases} M\ddot{x} + D\dot{x} + Kx + Gf(x) = Bu \\ y = C\begin{bmatrix} x \\ \dot{x} \end{bmatrix} \end{cases} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^{n_1}$, and $y \in \mathbb{R}^{n_2}$, are the state vector, the input vector, and the output vector, respectively. The system model (1) can capture the dynamical behaviour of many physical systems, such as n-freedom rigid-bodies, power systems, circuits, acoustics, and fluid dynamics [23–25]. In the rest of the paper, we will refer to x as the state position vector and $\dot{x}$ as the state velocity vector. Correspondingly, the $n \times n$ matrices $M > 0$, $D \geq 0$, and $K > 0$ represent the mass, damping, and stiffness matrices, respectively. G is a $n \times p$ constant matrix. The non-linear function $f: \mathbb{R}^n \to \mathbb{R}^p$ indicates the non-linear stiffness, which satisfies the following global Lipschitz condition:

(H1) There exists a positive scalar κ_f such that

$$\| f(X) - f(Y) \| \leq \kappa_f \| X - Y \|, \forall X, Y \in \mathbb{R}^n.$$

We assume that the output matrix $C \in \mathbb{R}^{n_2 \times 2n}$ takes the following form:

$$C = \begin{bmatrix} C_1 & 0 \\ 0 & C_2 \end{bmatrix}, \tag{2}$$

where $C_1 \in \mathbb{R}^{m \times n}$ and $C_2 \in \mathbb{R}^{(n_2 - m) \times n}$ are known matrices.

Remark 1: According to the structure of C, $C_1 \neq 0$ means that the displacement measurement is available, while $C_2 \neq 0$ means that the velocity measurement is available.

With $\xi_1 = x$ and $\xi_2 = \dot{x}$, system (1) can be rewritten in state-space form as

$$\begin{cases} \dot{\xi}_1 = \xi_2 \\ \dot{\xi}_2 = -M^{-1}Gf(\xi_1) - M^{-1}D\xi_2 - M^{-1}K\xi_1 + M^{-1}Bu, \\ y = C\xi \end{cases} \tag{3}$$

where $\xi = \mathrm{col}(\xi_1, \xi_2)$.

When the output vector $y(t)$ is measured continuously, the following natural observer is proposed for system (3) in [2]:

$$\begin{cases} \dot{\hat{\xi}}_1 = \hat{\xi}_2 \\ \dot{\hat{\xi}}_2 = -M^{-1}Gf(\hat{\xi}_1) - M^{-1}D\hat{\xi}_2 - M^{-1}K\hat{\xi}_1 + M^{-1}Bu \\ \qquad + \sum_{i=1}^{2} L_i(y_i - \hat{y}_i) \end{cases}, \tag{4}$$

where $\hat{\xi}_1$ is the estimate of the position ξ_1, $\hat{\xi}_2$ is the estimate of the velocity ξ_2, L_1 and L_2 are the observer gain matrices, $y_i = C_i\xi_i$, and $\hat{y}_i = C_i\hat{\xi}_i$, $i = 1, 2$.

Remark 2: Different from the standard Luenberger estimator, the natural observer (4) can guarantee the derivative of the position estimate $\dot{\hat{\xi}}_1(t)$ to be indeed the estimate of the derivative of the position $\xi_1(t)$.

When the output vector $y(t)$ is only available at sampling times t_k, $k \in \mathbb{N}$, the continuous natural observer (4) is not applicable. To obtain an estimate of $(x, \dot{x})$ using the samples $y(t_k)$, $k \in \mathbb{N}$, we introduce the following INO:

$$\begin{cases} \dot{\hat{\xi}}_1(t) = \hat{\xi}_2(t) \\ \dot{\hat{\xi}}_2(t) = -M^{-1}Gf(\hat{\xi}_1(t)) - M^{-1}D\hat{\xi}_2(t) - M^{-1}K\hat{\xi}_1(t) \\ \qquad + M^{-1}Bu(t),\ t \neq t_k \\ \Delta\hat{\xi}_2(t) = \sum_{i=1}^{2} L_iC_i\left(\xi_i(t) - \hat{\xi}_i(t^-)\right),\ t = t_k,\ k \in \mathbb{N} \end{cases}, \tag{5}$$

where $\Delta\hat{\xi}_2(t_k) = \hat{\xi}_2(t_k) - \hat{\xi}_2(t_k^-)$, $\hat{\xi}_2(t_k^-) = \lim_{h \to 0^+} \hat{\xi}_2(t_k - h)$, and

$$\hat{\xi}_2(t_k) = \hat{\xi}_2(t_k^+) = \lim_{h \to 0^+} \hat{\xi}_2(t_k + h), \quad k \in \mathbb{N}.$$

We assume that the sampling may be aperiodic, but satisfies a ranged-dwell time constraint, i.e. $\{t_k\} \in \mathcal{S}(\sigma_1, \sigma_2) \triangleq \{\{t_k\}: \sigma_1 \leq t_k - t_{k-1} \leq \sigma_2, k \in \mathbb{N}\}$, where σ_1 and σ_2 are positive scalars satisfying $\sigma_1 \leq \sigma_2$.

Remark 3: Unlike the continuous natural observer (4), the update of the velocity estimator $\hat{\xi}_2(t)$ in the INO (5) takes place only at the discrete instants t_k, $k \in \mathbb{N}$. This means that the INO (5) is capable of reconstructing the state variables of system (1) with discrete time measurements, and thus can be easily implemented in digital devices.

Defining the observation error estimate $e_i = \xi_i - \hat{\xi}_i$, $i = 1, 2$, the resulting error dynamics is given by

$$\begin{cases} \dot{e}_1(t) = e_2(t) \\ \dot{e}_2(t) = -M^{-1}G[f(\xi_1) - f(\hat{\xi}_1)] - M^{-1}Ke_1(t) \\ \qquad - M^{-1}De_2(t),\ t \neq t_k \\ \Delta e_2(t) = -\sum_{i=1}^{2} L_iC_ie_i(t),\ t = t_k,\ k \in \mathbb{N} \end{cases}. \tag{6}$$

Definition 1: For a given class $\mathcal{S}(\sigma_1, \sigma_2)$ of sampling time sequences, the zero solution of the error dynamics (6) is said to be uniformly globally exponentially stable (*UGES*), if there exist two positive scalars c and γ such that, for any $\{t_k\} \in \mathcal{S}(\sigma_1, \sigma_2)$, and for any initial value $e_0 \in \mathbb{R}^{2n}$, the solution $e(t, t_0, e_0)$ satisfies

$$\| e(t, t_0, e_0) \| \le c \| e_0 \| e^{-\gamma(t - t_0)}, \quad \forall t \ge t_0. \tag{7}$$

Furthermore, γ is called the exponential convergence rate. In this case, we say that system (1) has an exponentially convergent INO (5) over $\mathcal{S}(\sigma_1, \sigma_2)$ with convergence rate γ.

Remark 4: According to [19], the standard impulsive observer (SIO) for system (3) takes the following form:

$$\begin{cases} \dot{\tilde{\xi}}_1(t) = \tilde{\xi}_2(t) \\ \dot{\tilde{\xi}}_2(t) = -M^{-1}Gf(\tilde{\xi}_1(t)) - M^{-1}D\tilde{\xi}_2(t) - M^{-1}K\tilde{\xi}_1(t) \\ \qquad + M^{-1}Bu(t),\ t \ne t_k \\ \Delta\tilde{\xi}_j(t) = \sum_{i=1}^{2} \tilde{L}_{ij}C_i\big(\xi_i(t) - \tilde{\xi}_i(t^-)\big), \\ \qquad j \in \overline{1,2},\ t = t_k,\ k \in \mathbb{N} \end{cases}. \tag{8}$$

It can be seen from (5) and (8) that the INOs are a special class of SIOs with $\tilde{L}_{i1} = 0$, $i \in \overline{1,2}$. One of the advantages of the proposed INOs over SIOs is that the system dimension of the discrete dynamics is reduced and thus can be easily implemented in practice. Another benefit is the reduced effort in the calculation of observer gains.

It is important to realise that the existence criteria of SIOs does not guarantee the existence of INOs. To elaborate this point clearly, let us consider a simple second-order linear system:

$$\begin{cases} \ddot{x} + x = u \\ y = x, \end{cases} \tag{9}$$

where $x \in \mathbb{R}$ is the state, u is the known input, and $y \in \mathbb{R}$ is the output. The corresponding INO is given by

$$\begin{cases} \dot{\hat{\xi}}_1(t) = \hat{\xi}_2(t) \\ \dot{\hat{\xi}}_2(t) = -\hat{\xi}_1(t) + u(t),\ t \ne t_k \\ \Delta\hat{\xi}_2(t) = L_1\big(\xi_1(t) - \hat{\xi}_1(t^-)\big),\ t = t_k,\ k \in \mathbb{N} \end{cases}, \tag{10}$$

while its SIO takes the form of

$$\begin{cases} \dot{\tilde{\xi}}_1(t) = \tilde{\xi}_2(t) \\ \dot{\tilde{\xi}}_2(t) = -\tilde{\xi}_1(t) + u(t),\ t \ne t_k \\ \Delta\tilde{\xi}_j(t) = \tilde{L}_{1j}\big(\xi_1(t) - \tilde{\xi}_1(t^-)\big),\ j \in \overline{1,2},\ t = t_k,\ k \in \mathbb{N} \end{cases}. \tag{11}$$

In the above, $\xi_1 = x$, and $L_1, \tilde{L}_{1j} \in \mathbb{R}$, $j \in \overline{1,2}$, are the observer gains to be designed.

Proposition 1: Assume that the sampling time $\{t_k\} \in \mathcal{S}(\sigma, \sigma)$. Then, for any $\sigma \in (0, \pi)$, the second-order linear system (9) has an exponentially convergent SIO (11) but no convergent INO (10).

Proof: The error dynamics of SIO (11) is

$$\begin{cases} \dot{\tilde{e}}(t) = \bar{A}\tilde{e}(t),\ t \ne t_k \\ \tilde{e}(t) = \tilde{J}\tilde{e}(t),\ t = t_k,\ k \in \mathbb{N} \end{cases}, \tag{12}$$

where

$$\tilde{e}(t) = \mathrm{col}(\xi_1(t) - \tilde{\xi}_1(t), \xi_2(t) - \tilde{\xi}_2(t)),$$

$$\bar{A} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad \text{and} \quad \tilde{J} = \begin{bmatrix} 1 - \tilde{L}_{11} & 0 \\ -\tilde{L}_{12} & 1 \end{bmatrix}.$$

Due to $\{t_k\} \in \mathcal{S}(\sigma, \sigma)$, the error dynamics (12) is exponentially stable if and only if $\boldsymbol{\rho}(\tilde{J}\exp(\bar{A}\sigma)) < 1$, where $\boldsymbol{\rho}(\cdot)$ denotes the spectral radius of a square matrix. The characteristic polynomial of $\tilde{J}\exp(\bar{A}\sigma)$ is

$$\lambda^2 - ((2 - \tilde{L}_{11})\cos\sigma - \tilde{L}_{12}\sin\sigma)\lambda + 1 - \tilde{L}_{11} = 0.$$

For any given $\sigma \in (0, \pi)$, choosing $\tilde{L}_{11} \in (0, 2)$, and $\tilde{L}_{12} = (\cos\sigma/\sin\sigma)(2 - \tilde{L}_{11})$, then $\boldsymbol{\rho}(\tilde{J}\exp(\bar{A}\sigma)) = \sqrt{|1 - \tilde{L}_{11}|} < 1$. This means that for any $\sigma \in (0, \pi)$, system (9) has an exponentially convergent SIO (11).

On the other hand, the error dynamics of INO (10) can also be represented by (12) in which $\tilde{J}$ is replaced by

$$\bar{J} = \begin{bmatrix} 1 & 0 \\ -L_1 & 1 \end{bmatrix}.$$

The characteristic polynomial of $\bar{J}\exp(\bar{A}\sigma)$ is

$$\lambda^2 + (-2\cos\sigma + L_1\sin\sigma)\lambda + 1 = 0.$$

It follows that for any $\sigma > 0$, $\boldsymbol{\rho}(\bar{J}\exp(\bar{A}\sigma)) \ge 1$. This fact shows that for any $\sigma > 0$, system (9) does not have convergent INO (10). □

Proposition 1 indicates that the previous existence condition of SIOs [17, 19] cannot guarantee the existence of INOs. This has motivated our research in the INO design problem for vector second-order systems.

In order to exploit the characteristics of the Lipschitz non-linear, we apply the LPV approach proposed in [26] to deal with the non-linear function f. To describe the LPV approach formally, we need to introduce several notations. Consider two vectors $X = (x_1, x_2, \cdots, x_n)^{\mathrm{T}} \in \mathbb{R}^n$ and $Y = (y_1, y_2, \cdots, y_n)^{\mathrm{T}} \in \mathbb{R}^n$, define $n + 1$ vectors $X^{y_i} \in \mathbb{R}^n$, $i \in \overline{0,n}$, related to X and Y as follows:

$$X^{y_0} = x,\ X^{y_i} = (y_1, \ldots, y_i, x_{i+1}, \ldots, x_n),\ i \in \overline{0,n}.$$

Further, given a scalar function $g: \mathbb{R}^n \to \mathbb{R}$, the associated functions $\bar{g}_j: \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$, $j \in \overline{1,n}$, are defined as follows:

$$\bar{g}_j(X, Y) = \begin{cases} 0, & x_j = y_j \\ \dfrac{g(X^{y_{j-1}}) - g(X^{y_j})}{x_j - y_j}, & x_j \ne y_j \end{cases}.$$

The following lemma describes the characteristic of the Lipschitz non-linearities.

Lemma 1 [26]: Consider a vector function $f: \mathbb{R}^n \to \mathbb{R}^p$, where $f(x) = (f_1(x), f_2(x), \ldots, f_p(x))^{\mathrm{T}}$. The following items are equivalent:

(i) $f(x)$ satisfies the assumption (**H1**);
(ii) $f(x)$ satisfies the following assumption (**H2**):

(**H2**) There exist constants $\underline{\kappa}_{f_{ij}}$ and $\bar{\kappa}_{f_{ij}}$, $i \in \overline{1,p}$, $j \in \overline{1,n}$, such that for any $X, Y \in \mathbb{R}^n$,

$$\underline{\kappa}_{f_{ij}} \le \bar{f}_{ij}(X, Y) \le \bar{\kappa}_{f_{ij}}, \quad i \in \overline{1,p}, \quad j \in \overline{1,n}.$$

Using the notations of $\bar{f}_{ij}$, we can write

$$f(X) - f(Y) = \bar{F}(X, Y)(X - Y), \tag{13}$$

where $\bar{F}(X, Y) = \big(\bar{f}_{ij}(X, Y)\big)_{p \times n}$.

Let $\mathcal{S}_f$ denote the bounded closed convex set whose vertex set is $\mathscr{V}_f = \{(v_{ij})_{p \times n}: v_{ij} \in \{\underline{\kappa}_{f_{ij}}, \bar{\kappa}_{f_{ij}}\}\}$. Suppose that the number of distinct vertices of $\mathscr{V}_f$ is n_0. We denote these distinct vertices of

$\mathscr{V}_f$ by H_v, $v \in \overline{1, n_0}$. According to assumption (**H2**), $\bar{F}(X, Y) \in \mathscr{S}_f$, for any $X, Y \in \mathbb{R}^n$, and thus can be expressed as

$$\bar{F}(X, Y) = \sum_{v=1}^{n_0} \beta_v(X, Y) H_v, \tag{14}$$

where $\beta_v(X, Y) \in [0, 1]$, $v \in \overline{1, n_0}$, satisfy $\sum_{v=1}^{n_0} \beta_v(X, Y) = 1$. Substituting (13) and (14) into (6) yields the new form of the error dynamic (6)

$$\begin{cases} \dot{e}_1(t) = e_2(t) \\ \dot{e}_2(t) = -M^{-1}\left(G \sum_{v=1}^{n_0} \beta_v H_v + K\right) e_1(t) \\ \qquad - M^{-1} D e_2(t),\ t \neq t_k \\ \Delta e_2(t) = -\sum_{i=1}^{2} L_i C_i e_i(t),\ t = t_k,\ k \in \mathbb{N} \end{cases}, \tag{15}$$

where $\beta_v = \beta_v(\xi_1, \hat{\xi}_1)$.

For notational brevity, we define

$$\mathscr{I}_1 = [I_n \quad 0_n],\ \mathscr{I}_2 = [0_n \quad I_n],\ \hat{\mathscr{I}}_1 = [I_m \quad 0_{m \times (n_2 - m)}],$$
$$\hat{\mathscr{I}}_2 = [0_{(n_2 - m) \times m} \quad I_{n_2 - m}],\ L = [L_1 \quad L_2],\ J = I_{2n} - \mathscr{I}_2^{\mathrm{T}} L C,$$
$$A_v = \begin{bmatrix} 0 & I_n \\ -M^{-1}(G H_v + K) & -M^{-1} D \end{bmatrix}.$$

Using the above notations, the observation error system (15) can be further expressed into the following compact form:

$$\begin{cases} \dot{e}(t) = \sum_{v=1}^{n_0} \beta_v A_v e(t),\ t \neq t_k, \\ e(t) = J e(t_k^-),\ t = t_k. \end{cases} \tag{16}$$

where $e(t) = \mathrm{col}(e_1(t), e_2(t))$.

Lemma 2: Consider the observation error system (16). Suppose that the impulse time sequence $\{t_k\} \in \mathscr{S}(\sigma_1, \sigma_2)$. Then

$$\| e(t) \| \le \| J \| e^{(\lambda_A \sigma_2)/2} \| e(t_k^-) \|, \quad t \in [t_k, t_{k+1}), \quad k \in \mathbb{N}, \tag{17}$$

where $\lambda_A = \max\{\max\{0, \lambda_{\max}(A_v + A_v^{\mathrm{T}})\};\ v \in \overline{1, n_0}\}$.

Proof: Choose a Lyapunov function $V_0(t) = e^{\mathrm{T}}(t) e(t)$. Then, along the solution of system (16), $V(t)$ fulfils the next relation for $t \in (t_k, t_{k+1})$, $k \in \mathbb{N}$

$$\begin{aligned} \dot{V}_0(t) &= \sum_{v=1}^{n_0} \beta_v e^{\mathrm{T}}(t)(A_v + A_v^{\mathrm{T}}) e(t) \\ &\le \lambda_A e^{\mathrm{T}}(t) e(t) \\ &= \lambda_A V_0(t). \end{aligned}$$

It follows that

$$V_0(t) \le V_0(t_k) e^{\lambda_A (t - t_k)} \le V_0(t_k) e^{\lambda_A \sigma_2},\ \forall t \in [t_k, t_{k+1}). \tag{18}$$

From the second equation of (16), we get $V_0(t_k) \le \| J \|^2 V_0(t_k^-)$. Substituting the inequality into (18) leads to (17). This completes the proof. □

Remark 5: According to Lemma 2, for each $k \in \mathbb{N}$, the norm of $e(t)$ over the impulse interval $[t_{k-1}, t_k)$ is dominated by the norm of $e(t_k^-)$.

3 Main result

In this section, we first establish an exponential stability criterion for the observation error system (16) using an impulse-time-dependent Lyapunov function based approach. For this purpose, we introduce several functions associated with the impulse time sequence. Divide the impulse interval $[t_{k-1}, t_k)$ into N subintervals $[t_{k,l-1}, t_{k,l})$ of equal length $\delta_k \triangleq (t_k - t_{k-1})/N$, where $t_{k,0} = t_{k-1}$, and $t_{k,N} = t_k$, $k \in \mathbb{N}$. Define

$$\rho_{10}(t) = \frac{t - t_{k,l-1}}{t_{k,l} - t_{k,l-1}}, \quad t \in [t_{k,l-1}, t_{k,l}), \quad l \in \overline{1, N}, \quad k \in \mathbb{N}.$$

Then

$$\rho_{10}(t_{k,l-1}) = 0, \quad \rho_{10}(t_{k,l}^-) = 1, \quad l \in \overline{1, N}, \quad k \in \mathbb{N}. \tag{19}$$

and

$$\dot{\rho}_{10}(t) = N \varrho(t), \quad \text{for } t \neq t_{k,l-1}, \quad l \in \overline{1, N}, \quad k \in \mathbb{N},$$

where

$$\varrho(t) = \frac{1}{t_k - t_{k-1}}, \quad t \in [t_{k-1}, t_k), \quad k \in \mathbb{N}.$$

Let $\rho_{11}(t) = 1 - \rho_{10}(t)$. Using the impulse-time-dependent functions $\rho_{10}(t)$ and $\varrho(t)$, we construct the following time-dependent Lyapunov function for system (16):

$$V(t) = e^{2\gamma t} \varphi(t) e^{\mathrm{T}}(t) P(t) e(t), \tag{20}$$

where $\gamma > 0$, $\varphi(t) = \prod_{i=1}^{l-1} \mu_i \mu_l^{\rho_{10}(t)}$, and $P(t) = \rho_{10}(t) P_l + \rho_{11}(t) P_{l-1}$, for $t \in [t_{k,l-1}, t_{k,l})$, $l \in \overline{1, N}$, $k \in \mathbb{N}$, in which μ_l, $l \in \overline{1, N}$, are positive scalars, P_l, $l \in \overline{0, N}$, are $2n \times 2n$ real symmetric matrices.

Set $\tilde{\mu}_0 = 1$, $\tilde{\mu}_l = \prod_{i=1}^{l} \mu_i$, and $\hat{\mu}_{lj} = \tilde{\mu}_{l-1} \mu_l^{1-j}$, for $j \in \overline{0, 1}$, and $l \in \overline{1, N}$. It is easy to see that $V(t)$ is continuous inside $(t_{k,l-1}, t_{k,l})$, $l \in \overline{1, N}$, $k \in \mathbb{N}$. Furthermore, from (19), we have

$$\begin{aligned} V(t_{k,0}) &= e^{2\gamma t_{k-1}} e^{\mathrm{T}}(t_{k-1}) P_0 e(t_{k-1}), \\ V(t_{k,N}^-) &= \tilde{\mu}_N e^{2\gamma t_k} e^{\mathrm{T}}(t_k^-) P_N e^{\mathrm{T}}(t_k^-), \\ V(t_{k,l}) &= V(t_{k,l}^-) = \tilde{\mu}_l e^{2\gamma t_{k,l}} e^{\mathrm{T}}(t_{k,l}) P_l e(t_{k,l}), \quad l \in \overline{1, N-1}. \end{aligned}$$

It follows that $V(t)$ is continuous inside (t_{k-1}, t_k), $k \in \mathbb{N}$.

Theorem 1: Given $n \times m$ matrix L_1, $n \times (n_2 - m)$ matrix L_2, and a class $\mathscr{S}(\sigma_0, \sigma_1)$ of impulse time sequences, considering the observation error system (16). If for given positive integer N, positive scalars γ, and μ_l, $l \in \overline{1, N}$, there exist $2n \times 2n$ matrices $P_l = P_l^{\mathrm{T}}$, $l \in \overline{0, N-1}$, and $P_N > 0$, such that the following matrix inequalities hold:

$$\begin{aligned} &(2\gamma + (N/\sigma_j) \ln \mu_l) P_{l-i} + (N/\sigma_j)(P_l - P_{l-1}) + P_{l-i} A_v \\ &\quad + A_v^{\mathrm{T}} P_{l-i} < 0, \quad v \in \overline{1, n_0}, \quad l \in \overline{1, N}, \quad i \in \overline{0, 1}, \end{aligned} \tag{21}$$

$$J^{\mathrm{T}} P_0 J \le \tilde{\mu}_N P_N, \tag{22}$$

then the zero solution of the observation error system (16) is UGES over $\mathscr{S}(\sigma_0, \sigma_1)$ with convergence rate γ.

Proof: Define

$$\rho_{20}(t) = \begin{cases} \dfrac{\varrho(t) - 1/\sigma_1}{1/\sigma_0 - 1/\sigma_1}, & \text{if } \sigma_0 < \sigma_1 \\ 1/\sigma_0, & \text{if } \sigma_0 = \sigma_1 \end{cases}.$$

According to the assumption of $\{t_k\} \in \mathcal{S}(\sigma_0, \sigma_1)$, $0 \le \rho_{20}(t) \le 1$ for all $t \ge 0$. Moreover, $\varrho(t)$ can be written as

$$\varrho(t) = (1/\sigma_0)\rho_{20}(t) + (1/\sigma_1)\rho_{21}(t),$$

where $\rho_{21}(t) = 1 - \rho_{20}(t)$.

For $t \in (t_{k,l-1}, t_{k,l})$, $l \in \overline{1,N}$, $k \in \mathbb{N}$,

$$\dot{\varphi}(t) = \varphi(t)N\varrho(t)\ln\mu_l, \quad \dot{P}(t) = N\varrho(t)(P_l - P_{l-1}).$$

It follows that the derivative of $V(t)$ along the solution of system (16) is

$$\begin{aligned}\dot{V}(t) = &\sum_{v=1}^{n_0}\beta_v e^{2\gamma t}\varphi(t)e^{\mathrm{T}}(t)[(2\gamma + N\ln\mu_l\varrho(t))P(t)\\ &+N\varrho(t)(P_l - P_{l-1}) + P(t)A_v + A_v^{\mathrm{T}}P(t)]e(t)\\ = &\sum_{v=1}^{n_0}\sum_{i,j=0}^{1}\beta_v\rho_{1i}(t)\rho_{2j}(t)e^{2\gamma t}\varphi(t)e^{\mathrm{T}}(t)[(2\gamma\\ &+\ln\mu_l(N/\sigma_j))P_{l-i} + (N/\sigma_j)(P_l - P_{l-1})\\ &+P_{l-i}A_v + A_v^{\mathrm{T}}P_{l-i}]e(t).\end{aligned} \tag{23}$$

Then, with condition (21), we get $\dot{V}(t) \le 0$, $t \in (t_{k,l-1}, t_{k,l})$, $l \in \overline{1,N}$, $k \in \mathbb{N}$. This together with the continuity of $V(t)$ in (t_{k-1}, t_k) implies that

$$V(t) \le V(t_k), \quad \forall\, t \in (t_{k-1}, t_k), \quad k \in \mathbb{N}. \tag{24}$$

On the other hand, at the impulse instants t_k, $k \in \mathbb{N}$, we have

$$V(t_k) = e^{2\gamma t_k}e^{\mathrm{T}}(t_k)P_0e(t_k) = e^{2\gamma t_k}e^{\mathrm{T}}(t_k^-)J^{\mathrm{T}}P_0Je(t_k).$$

It follows from condition (22) that

$$V(t_k) \le V(t_k^-), \quad k \in \mathbb{N}. \tag{25}$$

Combining the relations (24) and (25) together, we could deduce that

$$V(t) \le V(t_0), \quad \forall t \ge t_0.$$

Specifically, we have $V(t_k^-) \le V(t_0)$, for all $k \in \mathbb{N}$, which implies

$$\| e(t_k^-) \| \le \sqrt{\lambda_{01}/(\lambda_{N0}\tilde{\mu}_N)}\, \| e(0) \|\, e^{-\gamma t_k}, \quad k \in \mathbb{N}. \tag{26}$$

where $\lambda_{01} = |\lambda_{\max}(P_0)|$, and $\lambda_{N0} = \lambda_{\min}(P_N)$.

For any given $t > 0$, there exists $k_0 \in \mathbb{N}$ such that $t \in [t_{k_0}, t_{k_0+1})$. Using Lemma 2 and (26), we have

$$\begin{aligned}e(t) \le &\ \| J \|\, e^{(\lambda_A\sigma_2)/2}\, \| e(t_{k_0}^-) \|\\ \le &\ \| J \|\, e^{(\lambda_A\sigma_2)/2}\sqrt{\lambda_{01}/(\lambda_{N0}\tilde{\mu}_N)}\, \| e(0) \|\, e^{\gamma(t - t_k)}e^{-\gamma t}\\ \le &\ \| J \| \sqrt{\lambda_{01}/(\lambda_{N0}\tilde{\mu}_N)}e^{\sigma_2(\lambda_A/2 + \gamma)}\, \| e(0) \|\, e^{-\gamma t},\end{aligned}$$

which implies that the zero solution of the observation error system (16) is UGES over $\mathcal{S}(\sigma_0, \sigma_1)$ with convergence rate γ. □

Remark 6: Theorem 1 provides a new exponential stability criterion for impulsive systems by applying a novel impulse-time-dependent Lyapunov function (20). The jump behaviour of (20) at impulse instants makes it more suitable for capturing the dynamic characteristics of impulsive systems. The idea of the construction of Lyapunov function (20) is borrowed from [17, 19]. However, there are two new features that are worth mentioning. First, different from [17, 19], Lyapunov function (20) is only required to be positive definite at left side of impulse instant t_k, $k \in \mathbb{N}$, which allows to reduce the conservatism of the impulse-time-dependent Lyapunov function based method. Second, Lyapunov function (20) relies the partition number N on the impulse intervals. As the partition number N increases, the stability result becomes less and less conservative. As a tradeoff, the number of decision variables used to test the stability is $(n/2)(n+1)(N+1) + N$. So large partition number N would lead to high computational cost.

Based on Theorem 1, we can now give an existence condition of INOs in the framework of LMI.

Theorem 2: Consider the INO (5) with $\{t_k\} \in \mathcal{S}(\sigma_0, \sigma_1)$. Suppose that the non-linear function f satisfies assumption (**H2**). If for given positive integer N, positive scalars γ, κ, and μ_l, $l \in \overline{1,N}$, there exist $2n \times 2n$ matrices $P_l = P_l^{\mathrm{T}}$, $l \in \overline{1,N-1}$, $P_N > 0$, and $0 \le P_0$ with the following structure:

$$P_0 = \begin{bmatrix} P_{01} & \kappa P_{02} \\ * & P_{02} \end{bmatrix}, \tag{27}$$

where $0 < P_{02} \in \mathbb{R}^{n \times n}$, $i = 1, 2$, and a $n \times n_2$ matrix $\bar{L}$, such that (21) and the following matrix inequality hold:

$$\begin{bmatrix} -\tilde{\mu}_N P_N & P_0 - C^{\mathrm{T}}\bar{L}^{\mathrm{T}}(\kappa\mathcal{I}_1 + \mathcal{I}_2) \\ * & -P_0 \end{bmatrix} \le 0, \tag{28}$$

then the INO (5) with $L_i = P_{02}^{-1}\bar{L}_i\hat{\mathcal{I}}_i^{\mathrm{T}}$, $i = 1, 2$, uniformly exponentially estimates the component $(x, \dot{x})$ of system (1) over $\mathcal{S}(\sigma_1, \sigma_2)$ with convergence rate γ.

Proof: According to (27), we have

$$\begin{aligned}P_0 = &\ \mathcal{I}_1^{\mathrm{T}}P_{01}\mathcal{I}_1 + \left(\kappa\mathcal{I}_1 + \frac{1}{2}\mathcal{I}_2\right)^{\mathrm{T}}P_{02}\mathcal{I}_2\\ &+\mathcal{I}_2^{\mathrm{T}}P_{02}\left(\kappa\mathcal{I}_1 + \frac{1}{2}\mathcal{I}_2\right),\end{aligned}$$

Then, applying the change of variable $\bar{L} = P_{02}L$ yields

$$P_0 - C^{\mathrm{T}}\bar{L}^{\mathrm{T}}(\kappa\mathcal{I}_1 + \mathcal{I}_2) = J^{\mathrm{T}}P_0.$$

It follows from Schur complement that (28) is equivalent to (22). Then the desired conclusion follows from Theorem 1. □

Remark 7: In order to provide a LMI-based sufficient condition for designing the observer gains L_i, $i = 1, 2$, the decision matrix P_0 is selected as the form of (27). The benefit of such treatment is that the observer design problem can be casted into the LMI framework. As a tradeoff, it may bring certain additional conservatism in the existence condition of INOs.

As a byproduct of Theorem 1, an existence condition of the SIO (5) for system (1) can be obtained as follows.

Theorem 3: Consider the SIO (5) with $\{t_k\} \in \mathcal{S}(\sigma_0, \sigma_1)$. Suppose that the non-linear function f satisfies assumption (**H2**). If for given positive integer N, positive scalars γ, and μ_l, $l \in \overline{1,N}$, there exist $2n \times 2n$ matrices $P_l = P_l^{\mathrm{T}}$, $l \in \overline{1,N-1}$, $P_0 \ge 0$, and $P_N > 0$, and a $2n \times n_2$ matrix $\mathcal{L}$, such that (21) and the following matrix inequality hold:

$$\begin{bmatrix} -\tilde{\mu}_N P_N & P_0 - C^{\mathrm{T}}\mathcal{L}^{\mathrm{T}} \\ * & -P_0 \end{bmatrix} \le 0, \tag{29}$$

then the SIO (8) with

$$\begin{bmatrix} \tilde{L}_{11} & \tilde{L}_{21} \\ \tilde{L}_{12} & \tilde{L}_{22} \end{bmatrix} = P_0^{-1}\mathcal{L},$$

Table 1 Estimates of $\sigma_{\max}$ for the existence of INOs/SIOs for different N

N_0	1	3	5	7	9	12
$\sigma_{\max}$ for INOs	0.17	0.66	1.07	1.24	1.31	1.37
$\sigma_{\max}$ for SIOs	2.05	3.28	3.81	4.10	4.30	4.50

uniformly exponentially estimates the component $(x, \dot{x})$ of system (1) over $\mathcal{S}(\sigma_1, \sigma_2)$ with convergence rate γ.

Remark 8: In system (1), it is assumed that the input $u(t)$ is known. In practical applications, however, the input $u(t)$ may be unknown due to additive disturbance and modelling uncertainties. It is well known that observer design for linear or non-linear systems with unknown inputs is an important research topic in the field of robust control, fault detection and diagnosis [27–30]. How to design INOs for vector second-order systems with unknown inputs presents an interesting topic for future research.

4 Numerical example

The usefulness of the proposed INOs is illustrated through the following example.

Example 1: Consider an example given in [31], which can be described by second-order system (1) with the following data:

$$M = \begin{bmatrix} 1.4685 & 0.7177 & 0.4757 & 0.4311 \\ 0.7177 & 2.6938 & 1.2660 & 0.9676 \\ 0.4757 & 1.2660 & 2.7061 & 1.3918 \\ 0.4311 & 0.9676 & 1.3918 & 2.1876 \end{bmatrix},$$

$$D = \begin{bmatrix} 1.3525 & 1.2695 & 0.7967 & 0.8160 \\ 1.2695 & 1.3274 & 0.9144 & 0.7325 \\ 0.7967 & 0.9144 & 0.9456 & 0.8310 \\ 0.8160 & 0.7325 & 0.8310 & 1.1536 \end{bmatrix},$$

$$K = \begin{bmatrix} 1.7824 & 0.0076 & -0.1359 & -0.7290 \\ 0.0076 & 1.0287 & -0.0101 & -0.0493 \\ -0.1359 & -0.0101 & 2.8360 & -0.2564 \\ -0.7290 & -0.0493 & -0.2564 & 1.9130 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.3450 & 0.4578 \\ 0.0579 & 0.7630 \\ 0.5967 & 0.9990 \\ 0.2853 & 0.3063 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad G^{\mathrm{T}} = [0 \quad 0 \quad 0 \quad 3.33], \quad f(x) = \sin x_1 .$$

One can see that the non-linearity $f(x)$ satisfies (**H2**) with $\mathcal{V}_f = \{H_1, H_2\}$, where $H_1 = (-1, 0, 0, 0)$, and $H_2 = -H_1$. We apply Theorem 2 to design INOs. First, we consider the case that the impulse time sequence $\{t_k\}$ is periodic, i.e. $\{t_k\} \in \mathcal{S}(\sigma, \sigma)$ for some $\sigma > 0$. For prescribed convergence rare $\gamma = 0.001$, and for different number N of partition on impulse intervals, by solving LMIs (21) and (28) with tuning parameters κ, and μ_l, $l \in \overline{1, N_0}$, the obtained maximum values of $\sigma_{\max}$ for the existence of INOs are listed in Table 1. Using Theorem 3, the resulting $\sigma_{\max}$ for the existence of SIOs is also included in Table 1. Table 1 clearly shows that increasing the partition number N on the impulse intervals can efficiently relax the existence conditions of INOs and SIOs. On the other hand, in comparison with SIOs, the INOs require higher update frequency.

Next, we consider the case that the impulse time sequence $\{t_k\}$ may be aperiodic, i.e. $\{t_k\} \in \mathcal{S}(\sigma_1, \sigma_2)$. Assume that $\sigma_1 = 0.5$. For prescribed convergence rare $\gamma = 0.001$, choosing $N = 12$, by solving LMIs (21) and (28) with $\kappa = 0.05$, and $\mu_l = 1$ for $l \in \overline{1, 12}$, the obtained maximum value of $\sigma_{\max}$ for which an INO can be constructed is $\sigma_{2\max} = 1.03$. The corresponding gain matrices are given by

$$L_1 = \begin{bmatrix} -0.0162 & -0.0581 \\ -0.0678 & -0.0346 \\ 0.0008 & -0.0046 \\ 0.1089 & 0.0266 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0.6813 & 0.1631 \\ -0.8629 & -0.5322 \\ 0.9185 & -0.0674 \\ 0.2372 & 1.1735 \end{bmatrix}.$$

For simulation studies, the initial condition for system (1) is taken as

$$\mathrm{col}(x(0), \dot{x}(0)) = (0.1, 0.2, 0.3, 0.3, 0.2, 0.1, 0.2, 0.4)^{\mathrm{T}},$$

while the initial condition for the INO (5) is chosen by

$$\mathrm{col}(\hat{\xi}_1(0), \hat{\xi}_2(0)) = (1.3, 1.2, 1.5, 1.3, 1.4, 1.7, 1.7, 2.2)^{\mathrm{T}} .$$

Let the impulse instants t_k, $k \in \mathbb{N}$, be generated randomly with the constraint of $0.5 \le t_k - t_{k-1} \le 1.03$ for all $k \in \mathbb{N}$. Select $u(t) = (\cos t, \sin t)^{\mathrm{T}}$. Fig. 1 shows the time evolution of the state variables and their estimates. The position estimation error $\| e_1(t) \|$ and the velocity estimation error $\| e_2(t) \|$ are plotted in Fig. 2*a*. The simulation results show that both the position and velocity vectors can be estimated by the designed INO.

In order to compare the performance of INOs and SIOs, considering the case of $\{t_k\} \in \mathcal{S}(0.5, 1.03)$, the obtained SIO gain matrix by applying Theorem 3 with $N = 1$ is

$$\tilde{L}_{11} = \begin{bmatrix} 0.9996 & -0.0006 \\ -0.0001 & 0.9994 \\ 0.8431 & -0.1325 \\ -0.1631 & -0.1134 \end{bmatrix},$$

$$\tilde{L}_{21} = \begin{bmatrix} -0.0012 & -0.0000 \\ -0.0008 & -0.0000 \\ -0.4428 & -0.2447 \\ 0.7154 & 0.2793 \end{bmatrix},$$

$$\tilde{L}_{12} = \begin{bmatrix} -0.2299 & 0.2092 \\ -0.2102 & 0.0464 \\ -0.0150 & -0.0021 \\ 0.0105 & 0.0016 \end{bmatrix},$$

$$\tilde{L}_{22} = \begin{bmatrix} 0.7788 & 0.1962 \\ -0.6424 & -0.4171 \\ 0.9979 & 0.0007 \\ -0.0004 & 0.9994 \end{bmatrix}.$$

The position estimation error $\| \tilde{e}_1(t) \|$ and the velocity estimation error $\| \tilde{e}_2(t) \|$ resulting from the the designed SIO are plotted in Fig. 2*b*. We can see that the SIO can achieve faster response than the INO. However, as was discussed in Remark 4, the INO has simpler structure than the SIO and can be easily implemented in practice. As a tradeoff, the convergence performance of the INOs is not as good as the SIOs.

5 Conclusion

An INO has been proposed for a class of vector second-order Lipschitz non-linear systems whose outputs are only available at certain discrete-time instants. In order to capture the dynamical properties of the resulting impulsive observation error system, an impulse-time-dependent discretised Lyapunov function has been constructed based on the partition on the impulse intervals. By employing the introduced Lyapunov function, a sufficient condition for the existence of INOs has been derived. The existence condition is dependent on the partition number on the impulse intervals. As the partition number increases, the conservatism of the existence condition can be weakened. A numerical example has been given to illustrate the applicability of the proposed INOs. The

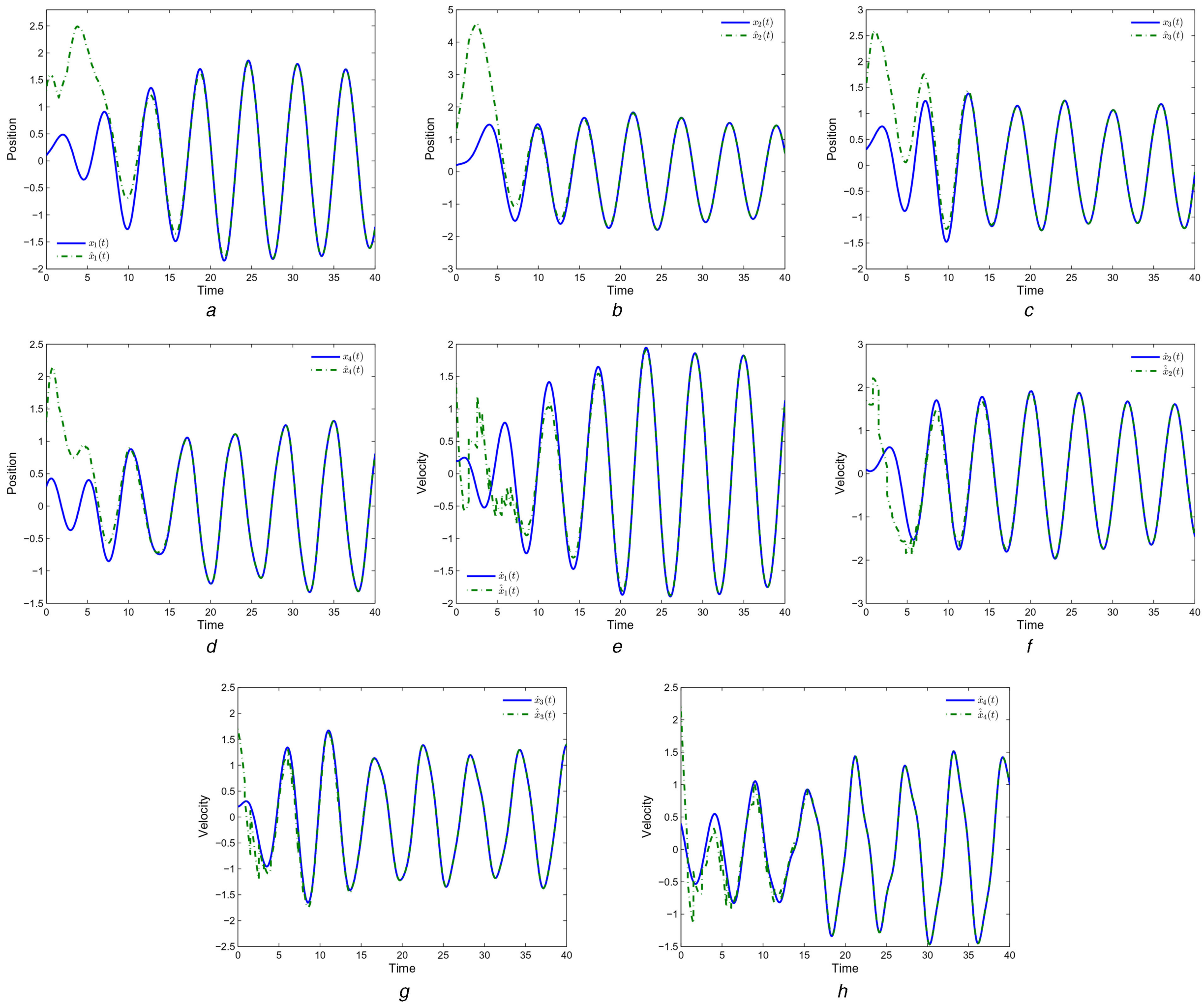

Fig. 1 *Simulation results using the designed INO*
(a) $x_1(t)$ and its estimate $\hat{x}_1(t)$, ***(b)*** $x_2(t)$ and its estimate $\hat{x}_2(t)$, ***(c)*** $x_3(t)$ and its estimate $\hat{x}_3(t)$, ***(d)*** $x_4(t)$ and its estimate $\hat{x}_4(t)$, ***(e)*** $\dot{x}_1(t)$ and its estimate $\hat{\dot{x}}_1(t)$, ***(f)*** $\dot{x}_2(t)$ and its estimate $\hat{\dot{x}}_2(t)$, ***(g)*** $\dot{x}_3(t)$ and its estimate $\hat{\dot{x}}_3(t)$, ***(h)*** $\dot{x}_4(t)$ and its estimate $\hat{\dot{x}}_4(t)$

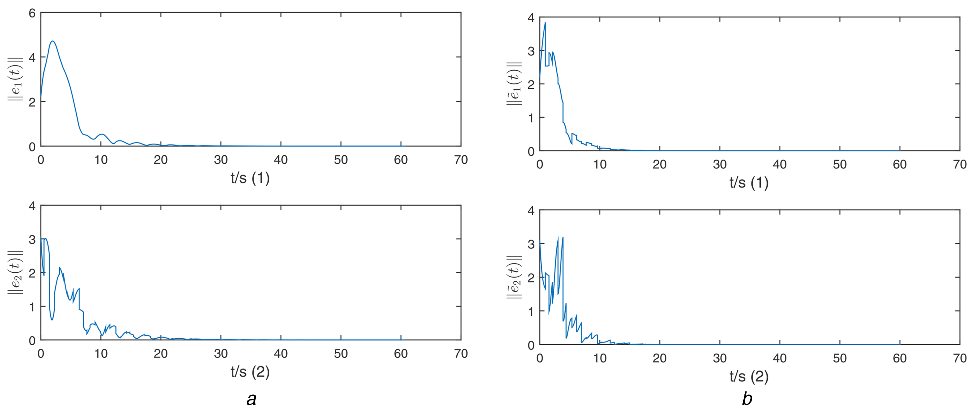

Fig. 2 *Simulation results of estimation errors*
(a.1) $\| e_1(t) \|$ resulting from the INO, ***(a.2)*** $\| e_2(t) \|$ resulting from the INO, ***(b.1)*** $\| \tilde{e}_1(t) \|$ resulting from the SIO, ***(b.2)*** $\| \tilde{e}_2(t) \|$ resulting from the SIO

derived result is based on the assumption that the matrix M is non-singular. However for some mechanical systems with high degrees-of-freedom, this assumption might not be satisfied. An interesting future work will be to develop new techniques to design INOs for singular vector second-order systems. Further, studying the fault detection problem for vector second-order systems with time-varying actuator faults by using INOs is another task for future work.

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7 References

[1] Balas, M.J.: 'Do all linear flexible structures have convergent second-order observers?', *AIAA J. Guid. Control Dyn.*, 1999, **22**, (6), pp. 905–908

[2] Kmk, S.-K., Yedavalli, R.K.: 'New approaches for observer design in linear matrix secand order sysm'. Proc. 2000 American Control Conf., Chicago, USA, June 2000, pp. 2311–2315

[3] Demetriou, M.A., Kazantzis, N.: 'Natural observer design for singularly perturbed vector second-order systems', *ASME J. Dyn. Syst. Meas. Control*, 2004, **127**, (4), pp. 648–655
[4] Demetriou, M.A.: 'Using unknown input observers for robust adaptive fault detection in vector second-order systems', *Mech. Syst. Signal Process.*, 2005, **19**, (2), pp. 291–309
[5] Hernandez, E.M.: 'A natural observer for optimal state estimation in second order linear structural systems', *Mech. Syst. Signal Process.*, 2011, **25**, (8), pp. 2938–2947
[6] Demetriou, M.A.: 'Natural observers for second-order distributed parameter systems', *Syst. Control Lett.*, 2004, **51**, (3-4), pp. 225–234
[7] Demetriou, M.A.: 'Natural consensus filters for second order infinite dimensional systems', *Syst. Control Lett.*, 2009, **58**, (12), pp. 826–833
[8] Diwekar, A.M., Yedavalli, R.K.: 'Smart structure control in matrix second-order form', *Smart Mater. Struct.*, 1996, **5**, pp. 429–436
[9] Karimi, H.R., Moshiri, B., Lohmann, B., *et al.*: 'Haar wavelet-based approach for optimal control of second-order linear systems in time domain', *J. Dyn. Control Syst.*, 2005, **11**, (2), pp. 237–252
[10] Karimi, H.R., Gao, H.: 'LMI-based delay-dependent mixed H_2/H_∞ control of second-order neutral systems with time-varying state and input delays', *ISA Trans.*, 2008, **47**, (3), pp. 311–324
[11] Abdelaziz, T.H.S.: 'Robust pole placement for second-order linear systems using velocity-plus-acceleration feedback', *IET Control Theory Appl.*, 2013, **7**, (14), pp. 1843–1856
[12] Adegas, F.D., Stoustrup, J.: 'Linear matrix inequalities for analysis and control of linear vector second-order systems', *Int. J. Robust Nonlinear Control*, 2015, **25**, pp. 2939–2964
[13] Ahmed-Ali, T., Assche, V.V., Massieu, J.-F., *et al.*: 'Continuous-discrete observer for state affine systems with sampled and delayed measurements', *IEEE Trans. Autom. Control*, 2013, **58**, (4), pp. 1085–1091
[14] Farza, M., M'Saad, M., Fall, M.L., *et al.*: 'Continuous-discrete time observers for a class of mimo nonlinear systems', *IEEE Trans. Autom. Control*, 2014, **59**, (4), pp. 1060–1065
[15] Mazenc, F., Dinh, T.N.: 'Construction of interval observers for continuous-time systems with discrete measurements', *Automatica*, 2014, **50**, pp. 2555–2560
[16] Raff, T., Allgöwer, F.: 'Observers with impulsive dynamical behavior for linear and nonlinear continuous-time systems'. Proc. 46th IEEE Conf. on Decision and Control, New Orleans, LA, December 2007, pp. 4287–4292
[17] Chen, W.-H., Li, D.-X., Lu, X.: 'Impulsive functional observers for linear systems', *Int. J. Control Autom.*, 2011, **9**, (5), pp. 987–992
[18] Efimov, D., Fradkov, A.: 'Impulsive adaptive observers: improving persistency of excitation'. Proc. 18th IFAC World Congress, Milano, Italy, August 2011, pp. 2326–2331
[19] Chen, W.-H., Li, D.-X., Lu, X.: 'Impulsiver observers with variable update intervals for Lipschitz nonlinear time delays systerms', *Int. J. Syst. Sci.*, 2013, **44**, (10), pp. 1934–1947
[20] Chen, W.-H., Yang, W., Zheng, W.X.: 'Adaptive impulsive observers for nonlinear systems: revisited', *Automatica*, 2015, **61**, pp. 232–240
[21] Chen, W.-H., Yang, W., Lu, X.: 'Impulsive observer-based stabilisation of uncertain linear systems', *IET Control Theory Appl.*, 2014, **8**, (3), pp. 149–159
[22] Etienne, L., Gennaro, S.D., Barbot, J.-P.: 'Periodic event-triggered observation and control for nonlinear Lipschitz systems using impulsive observers', *Int. J. Robust Nonlinear Control*, 2017, **27**, (18), pp. 4363–4380
[23] Chiang, H.-D., Wu, F.F.: 'Stability of nonlinear systems described by a second-order vector differential equation', *IEEE Trans. Circuits Syst.*, 1988, **35**, (6), pp. 703–711
[24] Diwekar, A.M., Yedavalli, R.K.: 'Stability of matrix second-order systems: new conditions and perspectives', *IEEE Trans. Autom. Control*, 1999, **44**, (9), pp. 1773–1777
[25] Benner, P., Saak, J.: 'Efficient balancing-based MOR for large-scale second-order systems', *Math. Comp. Model. Dyn. Syst.*, 2011, **17**, (2), pp. 123–143
[26] Zemouche, A., Boutayeb, M.: 'On LMI conditions to design observers for Lipschitz nonlinear systems', *Automatica*, 2013, **49**, (2), pp. 585–591
[27] Karimi, H.R.: 'Observer-based mixed H_2/H_∞ control design for linear systems with time-varying delays: an LMI approach', *Int. J. Control Autom. Syst.*, 2008, **6**, (1), pp. 1–14
[28] Chadli, M., Karimi, H.R.: 'Robust observer design for unknown inputs Takagi–Sugeno models', *IEEE Trans. Fuzzy Syst.*, 2013, **21**, (1), pp. 158–164
[29] Karimi, H.R.: 'Robust synchronization and fault detection of uncertain master-slave systems with mixed time-varying delays and nonlinear perturbations', *Int. J. Control Autom. Syst.*, 2011, **9**, (4), pp. 671–680
[30] Sakthivel, R., Selvaraj, P., Lim, Y., *et al.*: 'Adaptive reliable output tracking of networked control systems against actuator faults', *J. Franklin Inst.*, 2017, **354**, (9), pp. 3813–3837
[31] Qian, J., Xu, S.: 'Robust partial eigenvalue assignment problem for the second-order system', *J. Sound Vib.*, 2005, **282**, (3–5), pp. 937–948