A Port-Hamiltonian Model of Power Networks including the Transmission Lines

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Motivation

- We want to simulate power networks to foresee unexpected occurrences
- We consider three components: generators, loads and transmission lines
- Each component of a power network can be presented with different models
- For the transmission line we want to use the telegraph equations

Port-Hamiltonian descriptor system

Let $m, n, p \in \mathbb{N}$ and $\mathcal{I} \subseteq \mathbb{R}$ be a time interval, $\mathcal{X} \subseteq \mathbb{R}^n$ a state space and $\mathcal{Z} = \mathcal{I} \times \mathcal{X}$. A system of DAEs of the form,

pHDAE [MehM19]

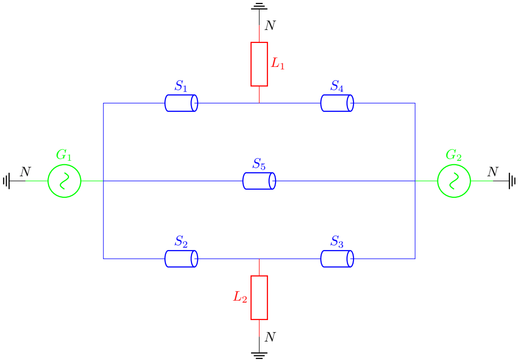
$$
\begin{aligned}
E(t,x)\dot{x} &= \Big[J(t,x) - R(t,x)\Big] z(t,x) + B(t,x)u, \\
y &= B(t,x)^T z(t,x),
\end{aligned}
$$

with a Hamiltonian function $\mathcal{H} \in C^1(\mathcal{X}, \mathbb{R})$ and properties $\nabla\mathcal{H} = E^T z$, $J = -J^T$ and $R = R^T \geq 0$ pointwise as well as

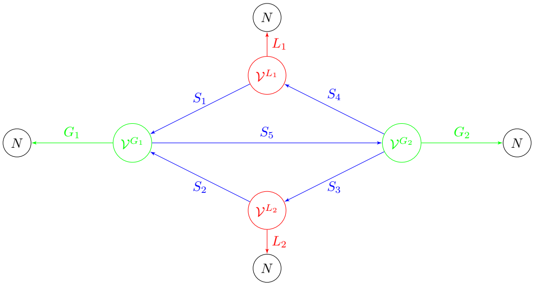
$$
\begin{aligned}
&E \in C(\mathcal{Z}, \mathbb{R}^{p,n}), && J \in C(\mathcal{Z}, \mathbb{R}^{p,p}), \\
&R \in C(\mathcal{Z}, \mathbb{R}^{p,p}), && B \in C(\mathcal{Z}, \mathbb{R}^{p,m}), \\
&x \in C^1(\mathcal{I}, \mathcal{X}), && u : \mathcal{I} \to \mathbb{R}^m, \\
&y : \mathcal{I} \to \mathbb{R}^m && \text{and } z \in C(\mathcal{Z}, \mathbb{R}^p)
\end{aligned}
$$

is called a port-Hamiltonian system of DAEs.

Power Network



A power network example with two generators G_1, G_2, two loads L_1, L_2, five transmission lines S_1, S_2, S_3, S_4, S_5 and one reference bus N



Graph model with generator bus nodes $\mathcal{V}^{G_1}$, $\mathcal{V}^{G_2}$, load bus nodes $\mathcal{V}^{L_1}, \mathcal{V}^{L_2}$, a reference bus node N, generator edges G_1, G_2, load edge L_1, L_2 and transmission line edges S_1, S_2, S_3, S_4, S_5

Generator Edge

Let $J_r \in \mathbb{R}$, $d \in \mathbb{R}$ the

Swing Equation

$$J_r \frac{\partial}{\partial t}(\omega(t) - \tilde{\omega}) = -d(\omega(t) - \tilde{\omega}) + \tau_e(t) + \tau(t),$$
$$\frac{\partial}{\partial t}\theta(t) = \omega(t),$$

with $\tilde{\omega}$,$\omega(t)$, $\tau(t) \in \mathbb{R}$ and $\tau_e = \frac{V^T I}{\omega}$, where $I(t) \in \mathbb{R}^3$ and

$$V(t) := V(\theta(t)) = v \begin{bmatrix} \cos(\theta(t) + \alpha) \\ \cos(\theta(t) + \alpha - \frac{2\pi}{3}) \\ \cos(\theta(t) + \alpha + \frac{2\pi}{3}) \end{bmatrix} \in \mathbb{R}^3,$$

v, α, $\theta \in \mathbb{R}$.

Port-Hamiltonian Formulation of a Generator Edge

Suppose $d \in \mathbb{R}_+$.

Current-Controlled pHDAE

$$\begin{bmatrix} J_r & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\omega} \\ \dot{\theta} \end{bmatrix} = \left(\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} d & 0 \\ 0 & 0 \end{bmatrix} \right) \begin{bmatrix} \omega \\ 0 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{V(\theta)^T}{\omega} & 1 \\ 0 & 0 \end{bmatrix}}_{B(x)} \begin{bmatrix} I \\ \tilde{\tau} \end{bmatrix},$$

$$\begin{bmatrix} V \\ \omega \end{bmatrix} = \begin{bmatrix} \frac{V(\theta)^T}{\omega} & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \omega \\ 0 \end{bmatrix}$$

with Hamiltonian $\mathcal{H}(\omega) = \frac{1}{2} J_r (\omega)^2$ and $\tilde{\tau} = d\tilde{\omega} + \tau$.

Note that B depends on $x = \begin{bmatrix} \omega \\ \theta \end{bmatrix}$.

Port-Hamiltonian Formulation of a Load Edge

Load Equation

$$0_{3,1} = -\hat{R}I(t) + \mathbb{I}_3 V(t)$$

with $\hat{R} := \mathrm{diag}(r_1, r_2, r_3) \in \mathbb{R}^{3,3}$, $I(t), V(t) \in \mathbb{R}^3$. Suppose $r_1, r_2, r_3 \in \mathbb{R}_+$

Current-Controlled pHDAE

$$\begin{aligned} 0_{3,1} &= \left(\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} \frac{1}{r_1} & 0 & 0 \\ 0 & \frac{1}{r_2} & 0 \\ 0 & 0 & \frac{1}{r_3} \end{bmatrix} \right) V + \mathbb{I}_3 I \\ V &= \mathbb{I}_3 V \end{aligned}$$

with Hamiltonian $\mathcal{H}(I) = 0$

Transmission Line

Let $C := \operatorname{diag}(c_1, c_2, c_3)$, $L := \operatorname{diag}(\ell_1, \ell_2, \ell_3) \in \mathbb{R}^{3,3}$, $G := \operatorname{diag}(g_1, g_2, g_3)$, $\tilde{R} := \operatorname{diag}(\tilde{r}_1, \tilde{r}_2, \tilde{r}_3) \in \mathbb{R}^{3,3}$, $i : \mathcal{I} \times [0,1] \to \mathbb{R}^3$ and $v : \mathcal{I} \times [0,1] \to \mathbb{R}^3$

Telegraph Equations

$$\frac{\partial}{\partial \mathrm{x}} i(t,\mathrm{x}) + C\frac{\partial}{\partial t} v(t,\mathrm{x}) + Gv(t,\mathrm{x}) = 0,$$
$$\frac{\partial}{\partial \mathrm{x}} v(t,\mathrm{x}) + L\frac{\partial}{\partial t} i(t,\mathrm{x}) + \tilde{R}i(t,\mathrm{x}) = 0,$$

Boundary Condition

$$\left.\begin{aligned} v(t,0) &= \mathrm{v}_0(t), \\ v(t,1) &= \mathrm{v}_1(t) \end{aligned}\right. \quad \text{, for all } t \in \mathcal{I},$$

with $\mathrm{v}_0, \mathrm{v}_1 : \mathcal{I} \to \mathbb{R}^3$

Initial Conditions

$$\left.\begin{aligned} i(0,\mathrm{x}) &= i_0(\mathrm{x}), \\ v(0,\mathrm{x}) &= v_0(\mathrm{x}) \end{aligned}\right. \quad \text{, for all } \mathrm{x} \in [0,1],$$

with $i_0, v_0 : [0,1] \to \mathbb{R}^3$.

Port-Hamiltonian Formulation of Transmission Line Edges

Suppose $G = \operatorname{diag}(g_1, g_2, g_3)$, $\tilde{R} = \operatorname{diag}(\tilde{r}_1, \tilde{r}_2, \tilde{r}_3) \in \mathbb{R}^{3,3}_{+}$

Voltage-controlled port-Hamiltonian Form

$$\begin{bmatrix} C & 0 \\ 0 & L \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \frac{\partial}{\partial t} \begin{bmatrix} v \\ i \end{bmatrix} = \begin{bmatrix} -G & -\frac{\partial}{\partial \mathrm{x}} & 0 & 0 \\ -\frac{\partial}{\partial \mathrm{x}} & -\tilde{R} & 0 & 0 \\ -\delta_0 & 0 & 0 & 0 \\ \delta_1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} v \\ i \\ \delta_0^3 i \\ \delta_1^3 i \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \mathbb{I}_3 & 0 \\ 0 & -\mathbb{I}_3 \end{bmatrix} \begin{bmatrix} \mathrm{v}_0 \\ \mathrm{v}_1 \end{bmatrix}$$

$$\begin{bmatrix} \delta_0^3 i \\ -\delta_1^3 i \end{bmatrix} = \begin{bmatrix} 0 & 0 & \mathbb{I}_3 & 0 \\ 0 & 0 & 0 & -\mathbb{I}_3 \end{bmatrix} \begin{bmatrix} v \\ i \\ \delta_0^3 i \\ \delta_1^3 i \end{bmatrix}$$

with $\delta_{\mathrm{x}}^3 = \operatorname{diag}(\delta_{\mathrm{x}}, \delta_{\mathrm{x}}, \delta_{\mathrm{x}})$, $\delta_{\mathrm{x}} : H^1(0,1) \to \mathbb{R}$ the Dirac-δ-function for $\mathrm{x} \in [0,1]$, such that $\delta_{\mathrm{x}}^3 i = i(\mathrm{x})$ and Hamiltonian $\mathcal{H}$

$$\mathcal{H}(x) = \mathcal{H}(v,i) := \frac{1}{2} \int_0^1 \sum_{j=1}^{3} (c_j v_j(t,\mathrm{x})^2 + \ell_j i_j(t,\mathrm{x})^2) d\mathrm{x}.$$

Connection of the same kind of edges

Let $j \in 1, 2$

Port-Hamiltonian System

$$E_j \dot{x}_j = \Big(J_j - R_j\Big) z_j(x) + B_j(x) u_j$$
$$y_j = (B_j(x))^T z_j(x).$$

Combined Port-Hamiltonian System

$$\begin{bmatrix} E_1 & 0 \\ 0 & E_2 \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \left(\begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} - \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix} \right) \begin{bmatrix} z_1(x_1) \\ z_2(x_2) \end{bmatrix} + \begin{bmatrix} B_1(x) & 0 \\ 0 & B_2(x) \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \left(\begin{bmatrix} B_1(x) & 0 \\ 0 & B_2(x) \end{bmatrix} \right)^T \begin{bmatrix} z_1(x_1) \\ z_2(x_2) \end{bmatrix}.$$

$$\mathcal{H} := \mathcal{H}_1 + \mathcal{H}_2$$

Interconnection Laws

Let

$$I^{\mathrm{I}} := \left[(I^{G_1})^T, \ldots, (I^{G_g})^T, (I^{L_1})^T, \ldots, (I^{L_\ell})^T\right]^T,$$

$$V^{\mathrm{I}} := \left[(V^{G_1})^T, \ldots, (V^{G_g})^T, (V^{L_1})^T, \ldots, (V^{L_\ell})^T\right]^T,$$

$$I^{\mathrm{V}} := \left[i^{S_1}(\cdot, 0), -i^{S_1}(\cdot, 1), \ldots, i^{S_s}(\cdot, 0), -i^{S_s}(\cdot, 1)\right]^T,$$

$$V^{\mathrm{V}} := \left[v^{S_1}(\cdot, 0), v^{S_1}(\cdot, 1), \ldots, v^{S_s}(\cdot, 0), v^{S_s}(\cdot, 1)\right]^T,$$

Kirchhoff's laws

$$I^{\mathrm{I}} = MI^{\mathrm{V}}, \tag{KCL}$$

$$V^{\mathrm{V}} = -M^T V^{\mathrm{I}} \tag{KVL}$$

where $M \in \mathbb{R}^{3g+3l,6s}$ is the oriented incidence matrix

Global model

Current-controlled form

$$E^{\mathrm{I}}\dot{x}^{\mathrm{I}} = (J^{\mathrm{I}} - R^{\mathrm{I}})z^{\mathrm{I}}(x^{\mathrm{I}}) + B_1^{\mathrm{I}}(x^{\mathrm{I}})I^{\mathrm{I}} + B_2^{\mathrm{I}}u_2^{\mathrm{I}},$$
$$V^{\mathrm{I}} = (B_1^{\mathrm{I}}(x^{\mathrm{I}}))^T z^{\mathrm{I}}(x^{\mathrm{I}}),$$
$$y_2^{\mathrm{I}} = (B_2^{\mathrm{I}})^T z^{\mathrm{I}}(x^{\mathrm{I}})$$

Voltage-controlled form

$$E^{\mathrm{V}}\dot{x}^{\mathrm{V}} = (J^{\mathrm{V}} - R^{\mathrm{V}})z^{\mathrm{V}}(x^{\mathrm{V}}) + B_1^{\mathrm{V}}V^{\mathrm{V}} + B_2^{\mathrm{V}}u_2^{\mathrm{V}}$$
$$I^{\mathrm{V}} = (B_1^{\mathrm{V}})^T z^{\mathrm{V}}(x^{\mathrm{V}})$$
$$y_2^{\mathrm{V}} = (B_2^{\mathrm{V}})^T z^{\mathrm{V}}(x^{\mathrm{V}})$$

Kirchhoff's laws : $I^{\mathrm{I}} = MI^{\mathrm{V}}$ and $V^{\mathrm{V}} = -M^T V^{\mathrm{I}}$

Global Model

$$\begin{bmatrix} E^{\mathrm{I}} & 0 \\ 0 & E^{\mathrm{V}} \end{bmatrix} \begin{bmatrix} \dot{x}^{\mathrm{I}} \\ \dot{x}^{\mathrm{V}} \end{bmatrix} = \begin{bmatrix} J^{\mathrm{I}} - R^{\mathrm{I}} & B_1^{\mathrm{I}}(x^{\mathrm{I}})M(B_1^{\mathrm{V}})^T \\ -B_1^{\mathrm{V}}M^T(B_1^{\mathrm{I}}(x^{\mathrm{I}}))^T & J^{\mathrm{V}} - R^{\mathrm{V}} \end{bmatrix} \begin{bmatrix} z^{\mathrm{I}}(x^{\mathrm{I}}) \\ z^{\mathrm{V}}(x^{\mathrm{V}}) \end{bmatrix} + \begin{bmatrix} B_2^{\mathrm{I}} & 0 \\ 0 & B_2^{\mathrm{V}} \end{bmatrix} \begin{bmatrix} u_2^{\mathrm{I}} \\ u_2^{\mathrm{V}} \end{bmatrix}$$
$$\begin{bmatrix} y_2^{\mathrm{I}} \\ y_2^{\mathrm{V}} \end{bmatrix} = \begin{bmatrix} (B_2^{\mathrm{I}})^T & 0 \\ 0 & (B_2^{\mathrm{V}})^T \end{bmatrix} \begin{bmatrix} z^{\mathrm{I}}(x^{\mathrm{I}}) \\ z^{\mathrm{V}}(x^{\mathrm{V}}) \end{bmatrix}$$

$$\mathcal{H} := \mathcal{H}^{\mathrm{I}} + \mathcal{H}^{\mathrm{V}}$$

Space Discretization

$$\frac{\partial}{\partial \mathrm{x}} f(t,\mathrm{x}) \approx \frac{f(t,\mathrm{x}+\frac{1}{2}h) - f(t,\mathrm{x}-\frac{1}{2}h)}{h} =: \frac{d}{d\mathrm{x}} f(t,\mathrm{x})$$

with $h := \frac{1}{k+1}$ and $k \in \mathbb{N}$. We can now use this type of discretization to represent i and v. Hence, we define

$$\begin{aligned} V_h(t) &:= \left[V_1^T(t), V_2^T(t), \ldots, V_k^T(t)\right]^T \\ &\approx \left[v^T(t,h), v^T(t,2h), \ldots, v^T(t,1-h)\right]^T, \\ I_h(t) &:= \left[I_1^T(t), I_2^T(t), \ldots I_{k+1}^T(t)\right]^T \\ &\approx \left[i^T(t,\tfrac{1}{2}h), i^T(t,\tfrac{1}{2}h+h), \ldots, i^T(t,1-\tfrac{1}{2}h)\right]^T. \end{aligned}$$

Space Discretization

We can now represent our discretization in matrix form

$$\frac{d}{d\mathrm{x}} v_h(t) = D_h^T v_h(t), \qquad D_h := \frac{1}{h}\begin{bmatrix} -\mathbb{I}_3 & \mathbb{I}_3 & & & & \\ & -\mathbb{I}_3 & \mathbb{I}_3 & & & \\ & & \ddots & \ddots & & \\ & & & -\mathbb{I}_3 & \mathbb{I}_3 & \\ & & & & -\mathbb{I}_3 & \mathbb{I}_3 \end{bmatrix} \in \mathbb{R}^{3k,3(k+1)}$$

$$\frac{d}{d\mathrm{x}} i_h(t) = -D_h i_h(t)$$

Space Discretization

The telegraph equation are now in the discretized form

$$0 = -D_h I_h(t) + C_k \frac{\partial}{\partial t} V_h(t) + G_k V_h(t),$$

$$0 = D_h^T V_h(t) + L_{k+1} \frac{\partial}{\partial t} I_h(t) + \tilde{R}_{k+1} I_h(t) - \frac{1}{h} \begin{bmatrix} \mathbb{I}_3 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & -\mathbb{I}_3 \end{bmatrix}^T \begin{bmatrix} v(t,0) \\ v(t,1) \end{bmatrix},$$

with $C_k := \text{blockdiag}\underbrace{(C, \ldots, C)}_{k-times}$, $G_k := \text{blockdiag}\underbrace{(G, \ldots, G)}_{k-times} \in \mathbb{R}^{3k,3k}$,

$L_{k+1} := \text{blockdiag}\underbrace{(L, \ldots, L)}_{(k+1)-times}$ and $\tilde{R}_{k+1} := \text{blockdiag}\underbrace{(\tilde{R}, \ldots, \tilde{R})}_{(k+1)-times} \in \mathbb{R}^{3(k+1),3(k+1)}$.

Space Discretization and Index

Global Model

$$\begin{bmatrix} E^{\mathrm{I}} & 0 \\ 0 & E_h^{\mathrm{V}} \end{bmatrix} \begin{bmatrix} \dot{x}^{\mathrm{I}} \\ \dot{x}_h^{\mathrm{V}} \end{bmatrix} = \begin{bmatrix} J^{\mathrm{I}} - R^{\mathrm{I}} & B_1^{\mathrm{I}}(x^{\mathrm{I}}) M {B_{1,h}^{\mathrm{V}}}^T \\ -B_{1,h}^{\mathrm{V}} M^T B_1^{\mathrm{I}}(x^{\mathrm{I}})^T & J_h^{\mathrm{V}} - R_h^{\mathrm{V}} \end{bmatrix} \begin{bmatrix} z^{\mathrm{I}}(x^{\mathrm{I}}) \\ z^{\mathrm{V}}(x_h^{\mathrm{V}}) \end{bmatrix} + \begin{bmatrix} B_2^{\mathrm{I}} & 0 \\ 0 & B_{2,h}^{\mathrm{V}} \end{bmatrix} \begin{bmatrix} u_2^{\mathrm{I}} \\ u_2^{\mathrm{V}} \end{bmatrix}$$

$$\begin{bmatrix} y_2^{\mathrm{I}} \\ y_2^{\mathrm{V}} \end{bmatrix} = \begin{bmatrix} B_2^{\mathrm{I}} & 0 \\ 0 & B_{2,h}^{\mathrm{V}} \end{bmatrix}^T \begin{bmatrix} z^{\mathrm{I}}(x^{\mathrm{I}}) \\ z^{\mathrm{V}}(x_h^{\mathrm{V}}) \end{bmatrix}$$

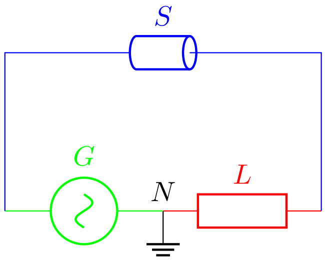
$$\mathcal{H} := \mathcal{H}^{\mathrm{I}} + \mathcal{H}_h^{\mathrm{V}}$$

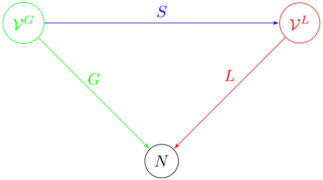
Theorem

Let $J_r^G, c_1^S, c_2^S, c_3^S, \ell_1^S, \ell_2^S, \ell_3^S, r_1^L, r_2^L, r_3^L \in \mathbb{R}$ not equal to zero for all edges $G \in \mathcal{G}, S \in \mathcal{S}$ and $L \in \mathcal{L}$. Then the discret global pHDAE is of differentiation index one and strangness-free.

Numerical Results

- programming language Python with an object-orientated approach
- DAE solver "IDA" of the interface scikits.odes
- Sundials package , which provides a BDF linear multistep method for stiff problems and an Adams-Moulton linear multistep method for non-stiff problems
- strangeness-free problem to converge and starting variables to fulfill algebraic equations



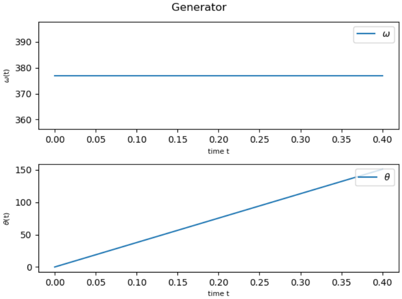


Numerical Results

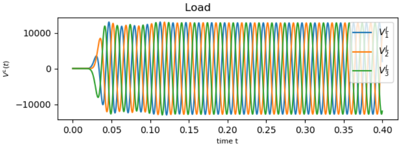
We choose the parameters

$$
\begin{aligned}
&v = 24,000, && \alpha = 0, && J_r = 10,000,\\
&d = 10, && r_1^L = r_2^L = r_3^L = 1, && \ell_1 = \ell_2 = \ell_3 = \frac{1}{120\pi},\\
&c_1 = c_2 = c_3 = \frac{1}{120\pi}, && g_1 = g_2 = g_3 = 0, && r_1^S = r_2^S = r_3^S = 1,\\
&\text{and } \tilde{\tau}(t) = 0 && \text{for all } t \in \mathbb{R}.
\end{aligned}
$$

Numerical Results



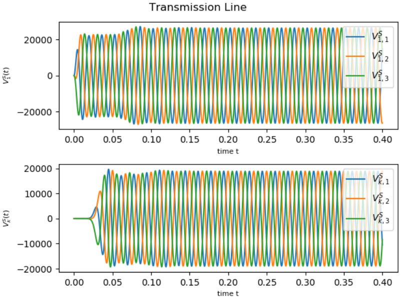
(a) The angular velocity ω and angular displacement θ of the generator edge G



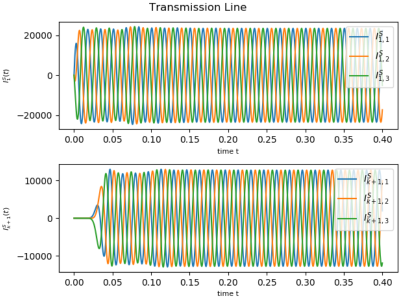
(b) The voltage V^L of the load edge L

Results of the program if we choose $k = 10$. The time is displayed on the x-axis and the functional values are displayed on the y-axis.

Numerical Results



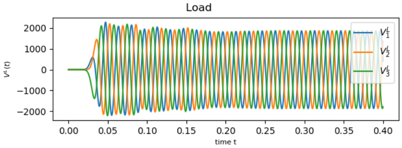
(a) The voltage V_1^S, V_k^S of the transmission line edge S



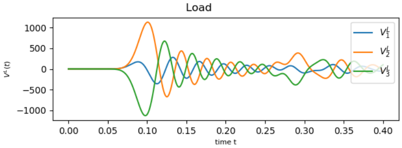
(b) The current I_1^S, I_{k+1}^S of the transmission line edge S

Results of the program if we choose $k = 10$. The time is displayed on the x-axis and the functional values are displayed on the y-axis.

Numerical Results



(a) The voltage V^L of the load edge L with the resistance $r_1^L = r_2^L = r_3^L = 0.1$

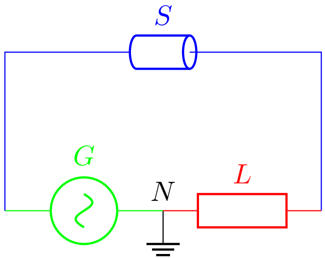


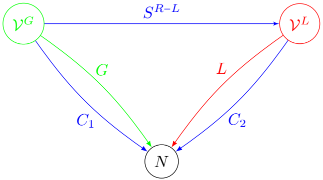
(b) The voltage V^L of the load edge L with the inductance $\ell_1 = \ell_2 = \ell_3 = \frac{10}{120\pi}$

Results of the program if we choose $k = 10$. The time is displayed on the x-axis and the functional values are displayed on the y-axis.

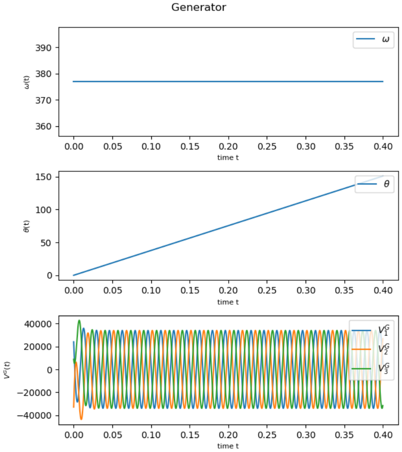
Different Powernetwork Example

- model based on S.Fiaz, D. Zonetti, R. Ortega, J.M.A. Scherpen and A.J. van der Schaft 2013
- *R-L edge*, describes the resistance and inductance of a transmission line
- *capacitor edge*, describes the capacitance of a transmission line

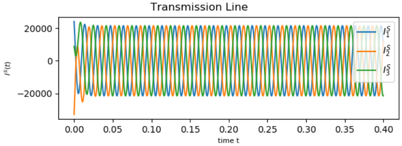




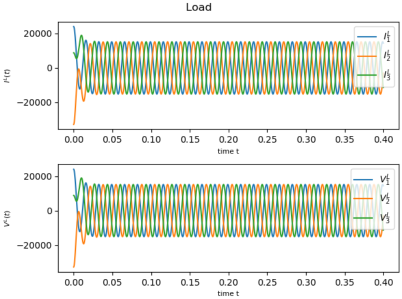
Numerical Results



(a) The angular velocity ω, angular displacement θ and the voltage $V^G = V_1^C$ of the generator capacitor edge G^C



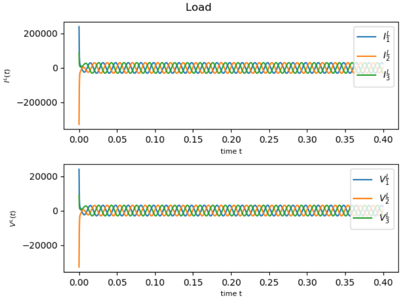
(b) The current I^S of the R-L edge



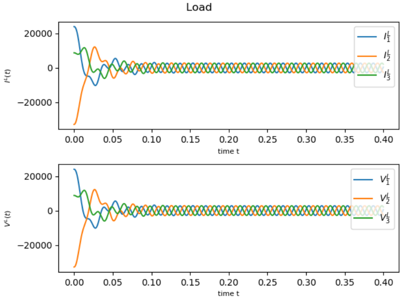
(a) The current I^L and the voltage $V^L = V_2^C$ of the load capacitor edge L^C

Results of the program without telegraph equation. The time is displayed on the x-axis and the functional values are displayed on the y-axis.

Numerical Results



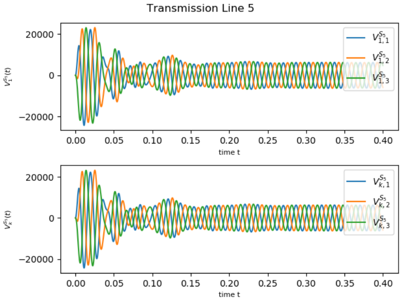
(a) The current I^L and the voltage V_2^C of the load capacitor edge L^C with the resistance $r_1^L = r_2^L = r_3^L = 0.1$



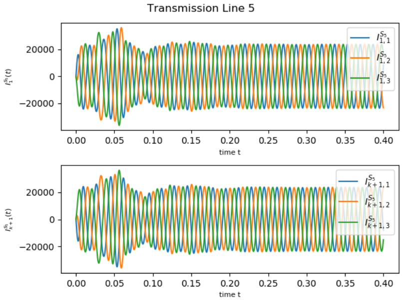
(b) The current I^L and the voltage V_2^C of the load capacitor edge L^C with the inductance $l_1 = l_2 = l_3 = \frac{10}{120\pi}$

Results of the program without telegraph equation. The time is displayed on the x-axis and the functional values are displayed on the y-axis.

Numerical Results



(a) The voltage $V_1^{S_5}$, $V_k^{S_5}$ of the transmission line edge S_5



(b) The current $I_1^{S_5}$, $I_{k+1}^{S_5}$ of the transmission line edge S_5

Results of the program if we choose $k = 10$. The time is displayed on the x-axis and the functional values are displayed on the y-axis.

Summary:

- We presented a port-Hamiltonian model of a power network
- We have three components: generators, loads and transmission lines, where the transmission lines are modeled by telegraph equations
- The components are connected through the Kirchhoff's laws
- The strangeness index of the space discretized global model is 0
- Numerical simulation

Main References

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