Structure Preserving Model Order Reduction by Parameter Optimization

Scope

We illustrate the method proposed in P. Schwerdtner and M. Voigt, *Structure Preserving Model Order Reduction by Parameter Optimization* (https://arxiv.org/abs/2011.07567) on the basis of a port-Hamiltionian system. For further explanations, references, and all technical details we refer to our paper.
A brief talk on the method can be found at: https://www.youtube.com/watch?v=kzUr1GdA5T0

Setup – Large Scale Port Hamiltionian Systems

We are given a dynamical system

$$\Sigma_{\mathrm{pH}}: \begin{cases} \dot{x}(t) = (J-R)Qx(t) + Bu(t), \\ y(t) = B^{\mathsf{T}}Qx(t), \end{cases}$$

where J, R, $Q \in \mathbb{R}^{n_x \times n_x}$ and $B \in \mathbb{R}^{n_x \times n_u}$ with the structural constraints $J = -J^{\mathsf{T}}$, $R \geq 0$, and $Q \geq 0$. Furthermore, we have that $n_x \gg n_u \geq 1$.

The system is described in *frequency domain* by its transfer function

$$G_{\mathrm{pH}}(s) = B^{\mathsf{T}}Q(sI - (J-R)Q)^{-1}B.$$

G_{pH} describes the frequency-wise amplification of the input, i.e. $Y(s) = G_{\mathrm{pH}}(s)U(s)$, where U and Y are the Laplace transformed input and output, respectively.

Task – Model Order Reduction

We must find an approximation to the system Σ_{pH} with the same structure, i.e.

$$\Sigma_{\mathrm{pH},r}: \begin{cases} \dot{x}_r(t) = (J_r - R_r)Q_r x_r(t) + B_r u(t), \\ y_r(t) = B_r^{\mathsf{T}} Q_r x_r(t), \end{cases}$$

where J_r, R_r, $Q_r \in \mathbb{R}^{r \times r}$, $B_r \in \mathbb{R}^{r \times n_u}$ with $J_r = -J_r^{\mathsf{T}}$, $R_r \geq 0$, and $Q_r \geq 0$ and with $r \ll n_x$.

When approximating a dynamical system, we want to achieve that for any given input u, the outputs y and y_r are similar. The approximation quality of a *reduced order model* is often measured in terms of the $\mathcal{H}_\infty$ norm of the error transfer function $G_{\mathrm{pH}} - G_{\mathrm{pH},r}$. It is defined as

$$\|G_{\mathrm{pH}} - G_{\mathrm{pH},r}\|_{\mathcal{H}_\infty} := \sup_{\omega \in \mathbb{R}} \|G_{\mathrm{pH}}(\mathrm{i}\omega) - G_{\mathrm{pH},r}(\mathrm{i}\omega)\|_2$$

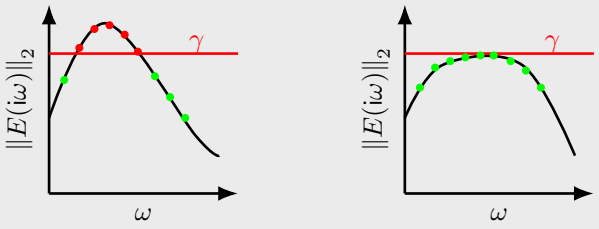
Our Approach – Approximation by Nonlinear Optimization

We define a parametrized port-Hamiltionian system

$$\Sigma_{\mathrm{pH}}(\theta): \begin{cases} \dot{x}(t) = (J(\theta) - R(\theta))\, Q(\theta)x(t) + B(\theta)u(t), \\ y(t) = B(\theta)^{\mathsf{T}} Q(\theta)x(t). \end{cases}$$

The matrices $J(\theta), R(\theta), Q(\theta) \in \mathbb{R}^{r \times r}$ and $B(\theta) \in \mathbb{R}^{r \times n_u}$ depend on the parameter vector θ such that for all $\theta \in \mathbb{R}^{n_\theta}$ the structural constraints are satisfied and $\Sigma_{\mathrm{pH}}(\theta)$ is a port-Hamiltionian system.
Therefore, we can use unconstrained optimization algorithms to minimize $\|G_{\mathrm{pH}} - G_{\mathrm{pH}}(\cdot, \theta)\|_{\mathcal{H}_\infty}$.
Note that we do not minimize $\|G_{\mathrm{pH}} - G_{\mathrm{pH}}(\cdot, \theta)\|_{\mathcal{H}_\infty}$ directly since $\|\cdot\|_{\mathcal{H}_\infty}$ is hard to compute.

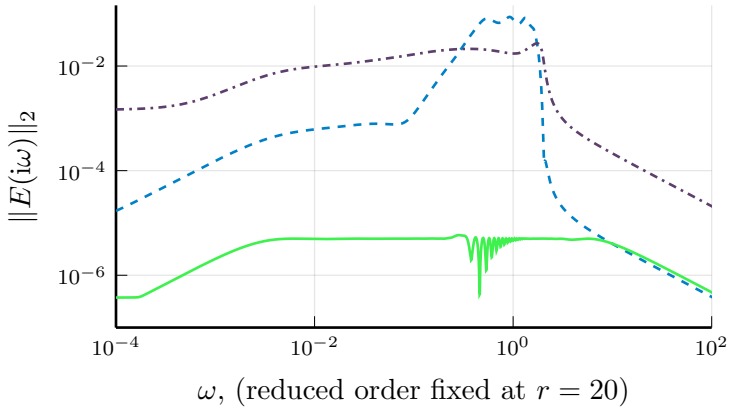
We minimize the sum of the squared error at test frequencies s_i, if the error $\|E(s_i)\|_2$ is larger than a prescribed value γ. This yields the following behavior.

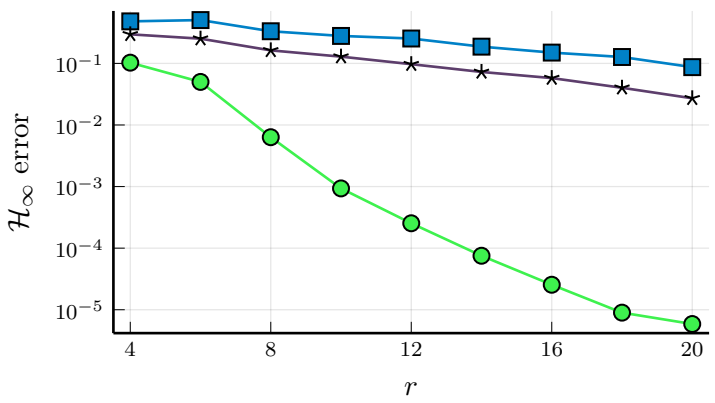


(a) Before (b) After

We iteratively reduce γ until the accuracy of $\Sigma_{\mathrm{pH}}(\theta)$ is satisfactory.

Results





pH balanced truncation — pH IRKA — our method

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