

# Controllability and Observability of Input /Output Delayed Discrete Systems

Cosmin Ionete, *Member, IEEE*, Arben Cela, Mongi Ben Gaid

**Abstract**— This paper proposes the study of network-delayed systems controllability and observability. Since the controllability and observability are the first conditions we impose in control practice, we study if a controllable/observable non-delayed system can loose those properties if we augment the model with pure input-output delays due to network control architecture. We started our approach with a classical discrete-time multi-input/output linear system with non-equal network-induced delays on command signals (inputs) and measures (outputs). We will prove that if the initial (non-delayed) system is controllable /observable, the new network-delayed system is controllable/observable despite the delay values in each input/output channel. This general result ensures further implementation of model-based predictive control strategies based on state observers, especially LQG/LTR,  $H_2$  or loop-shaping  $H_\infty$  methods for networked control systems.

## I. INTRODUCTION

A control system is called a *networked control system* (NCS) if its feedback loop(s) are closed via a shared communication medium. In other words, a control system where sensors, actuators, and controllers are interconnected over a shared communication network is a networked control system (NCS). The main advantage of this NCS configuration is flexibility. It allows placing controllers on the network, separated from multiple controlled plants.

In the literature we find models for different communication constraints such as network induced delays ([10], [12]), data rate limitations ([11], [13], [15] and [16]), quantization effects ([6], [7] and [11]) and medium access constraints ([7], [8], [9], [14], [17] and [18]).

The use of networked control systems in industrial environment is rapidly spreading. In the case of spatially distributed control systems, transmission delays of command or measure scalar values are different, it is very useful the study of controllability and observability of the global

control system. As presented in [24] and [25], various aspects of the controllability of linear systems with delay were considered by several authors [19]-[25]. The discrete cases have been considered by [19], [20], [21], but the mathematical conditions given for investigating the controllability are not suitable for real time verification and application. The results from [24] and [25] are strong and general (controllability of linear discrete-time systems with time-delay in state and control), despite high computational demands: in order to verify the controllability conditions we need approximately  $(p^2 n^4)$  matrix multiplications where 'p' is the maximum state delay time and n is the system degree.

Controllability and observability are structural properties of systems. It seems that the input/output delay induced by a network do not change those properties. We will systematically prove this. It is true that for a given delay structure the system can be put into the standard linear system (A,B,C) format with modified A,B,C matrices obtained by extending the state space. It is not straightforward to see that under fixed delay the controllability and observability properties of the original system will be preserved because the dimension of the augmented state vector is the dimension of the original state vector plus the sum of each input channel delay. Since the computational complexity increases and due to the fact that input channels delays can be variable we propose a different approach based on the use of old but powerful mathematical results: *Popov-Belevitch-Hautus* criterion and Artstein transform. Comparing to non-delayed systems [2], the controllability and observability of delayed systems presents differences related with state variables, time variable and realization of control law. The state variables depend on a function defined on a time interval whose length is equal to the maximum input-channel delay the future behavior of the system depends not only on the present state, but also on entire state trajectory over the above mentioned time interval. Delays introduce a required *reaching time*, since during a time period equal to (at least) the minimum output-channel delay the system state cannot be predicted and during a time period equal to (at least) the minimum input-channel delay the system has a free, uncontrollable evolution. In this article we will prove that the structural properties (controllability and observability) of the non-delayed system are preserved under network input/output

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C. Ionete, is with University of Craiova, Romania, Faculty of Automation, Computers and Electronics (phone:0040-251-432724, e-mails: [cosmin@automation.ucv.ro](mailto:cosmin@automation.ucv.ro), [ionetec@esiee.fr](mailto:ionetec@esiee.fr)). He is now with the Control Systems (COSI) Laboratory, ESIEE (Ecole Supérieure d'Ingenieurs en Electronique et Electrotechnique) Noisy-le-Grand Cedex, 93162, France.

A. Cela is the Responsible of Control Systems (COSI) Laboratory, ESIEE Noisy-le-Grand Cedex, 93162, France. (e-mail: [celaa@esiee.fr](mailto:celaa@esiee.fr)).

M. Ben Gaid is with the Control Systems (COSI) Laboratory, ESIEE Noisy-le-Grand Cedex, 93162, France. (e-mail: [bengaidm@esiee.fr](mailto:bengaidm@esiee.fr)).

delays. In Fig. 1 is presented the networked control architecture.

Using notations from [3], we can define

$$\begin{cases} z^{-G} = \text{diag}(z^{-g_1}, z^{-g_2}, \dots, z^{-g_m}) \\ z^{-H} = \text{diag}(z^{-h_1}, z^{-h_2}, \dots, z^{-h_p}) \end{cases}$$

where  $z^{-1}$  is the shift (one step delay) operator.

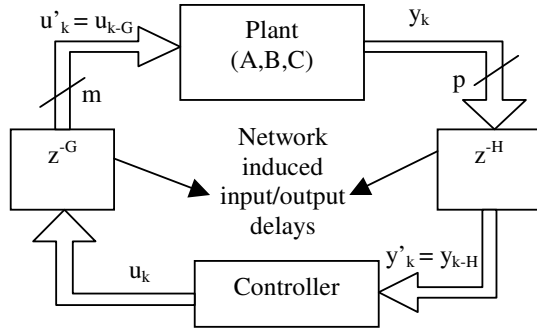


Fig. 1. Networked control architecture

The paper is structured as follows: In the second section are presented the controllability problem for systems with input delays and the problem solution. In the third section the dual problem, observability, is treated in the same manner but relative to output delays. The conclusions represent the final section.

## II. CONTROLLABILITY PROBLEM STATEMENT

### A. Network input-delayed controllability

Given the original non-delayed discrete-time linear system

$$\begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k \end{cases} \quad (1)$$

$$u_k = [u_k^1 \ u_k^2 \ \dots \ u_k^m]^T, \quad A_{n \times n}, B_{n \times m}, C_{p \times n}$$

The controller command is sent to the plant actuators via a spatially distributed network, which induces different delays in each transmission channel. The non-uniform delay is measured in multiples of sampling period. The network input-delayed system is

$$\begin{cases} x_{k+1} = Ax_k + Bu_{k-G} \\ y_k = Cx_k \end{cases} \quad (2)$$

$$u_{k-G} = [u_{k-G}^1 \ u_{k-G}^2 \ \dots \ u_{k-G}^m]^T$$

The column representation of command matrix B is

$$B = [c_B^1 \ c_B^2 \ \dots \ c_B^m] \quad (3)$$

$c_B^j$  is the "j"-th column of B,  $j \in \{1, 2, \dots, m\}$

$$B_i = \begin{bmatrix} 0 & 0 \dots & c_B^i & 0 \dots & 0 \end{bmatrix}$$

↑ "i"-th column

The original system from (2) can be wrote

$$B = \sum_{i=1}^m B_i \quad (4)$$

$$\begin{cases} x_{k+1} = Ax_k + \sum_{i=1}^m B_i \cdot u_{k-g_i} \\ y_k = Cx_k \end{cases} \quad (5)$$

Matrix A is the state matrix for a system with no delays in the state, so matrix A is invertible:

$$\det(A) \neq 0 \Leftrightarrow (\exists) A^{-1} \quad (6)$$

Some inputs could be without delays

$$(g_i = 0, i \in \{i_1, i_2, \dots, i_r\}, r < m)$$

so we can separate the inputs in delayed/non-delayed.

$$\begin{cases} G_0 = \{i | g_i = 0\} = \{i_1, i_2, \dots, i_r\} \\ G_p = \{1, 2, \dots, m\} \setminus G_0 \end{cases}$$

In [4] is presented so called "Artstein transform" for continuous linear time-invariant systems with delays in command with the discrete-time variant

$$z_k = x_k + \sum_{i \in G_p} \sum_{j=k-g_i}^{k-1} A^{k-g_i-j-1} \cdot B_i \cdot u_j$$

Using the above transform, we obtain the system

$$z_{k+1} = Az_k + \left( \sum_{i=1}^m A^{-g_i} \cdot B_i \right) \cdot u_k$$

$$\begin{cases} \bar{B} = \left( \sum_{i=1}^m A^{-g_i} \cdot B_i \right) \\ \bar{B} = [A^{-g_1} c_B^1 \ A^{-g_2} c_B^2 \ \dots \ A^{-g_m} c_B^m] \end{cases} \quad (7)$$

$$\begin{cases} z_{k+1} = Az_k + \bar{B}u_k \\ y_k = C \left( z_k - \sum_{i \in G_p} \sum_{j=k-g_i}^{k-1} A^{k-g_i-j-1} \cdot B_i \cdot u_j \right) \end{cases} \quad (8)$$

The un-delayed system from (8) is equivalent with input-delayed system (5) from the control point of view. It was defined in [5] the *absolute controllability* where the discrete-time variant is:

**Definition:** The linear system (5) is absolutely controllable if, for any initial condition  $\{x_0, u(k)_{k \in [-g, 0]}\}$   $\{g = \max\{g_i\}\}$

there is a time  $k_1 > 0$  and a bounded control law  $u(k)$  such

$$\text{that } \begin{cases} x_{k_1} = 0 \\ u(k) = 0, k \in [k_1 - g, k_1] \end{cases}$$

In [5] we find also the following important result:

*Proposition 1:* A necessary and sufficient condition for absolute controllability is

$$\text{rank}[\bar{B}, A\bar{B}, \dots, A^{n-1}\bar{B}] = n \quad (9)$$

The rank condition from (9) is good for representation, but not from computational point of view. For verifying (9) we shall use the *Popov-Belevitch-Hautus* (PBH) criterion:

$$\text{rank}[\bar{B}, A\bar{B}, \dots, A^{n-1}\bar{B}] = n \Leftrightarrow \text{rank}[\lambda I - A, \bar{B}] = n, \forall \lambda \in \mathbb{R} \quad (10)$$

Let  $\Lambda(A)$  be the spectrum of  $A$

$$\Lambda(A) = \text{eig}(A) = \{\lambda_1, \lambda_2, \dots, \lambda_r\}$$

$$\partial(\lambda_1) = n_1, \partial(\lambda_2) = n_2, \dots, \partial(\lambda_r) = n_r, \quad \sum_{i=1}^r n_i = n$$

Relation (6) is equivalent to

$$\lambda_i \neq 0, i = 1, \dots, r \quad (11)$$

Since  $\text{rank}[\lambda I - A] = n, (\forall) \lambda \notin \Lambda(A)$

it is sufficient to study the controllability only for the spectrum of  $A$  matrix, so the PBH criterion (10) becomes

$$\text{rank}[\lambda I - A, \bar{B}] = n, \forall \lambda \in \Lambda(A)$$

### B. Controllability problem solution

Suppose that un-delayed system (1) is controllable. The network-induced delay system is represented by (5) with the restriction (4). Since the command matrices of the network-induced delay system are obtained from the initial command matrix  $B$ , is useful to study relation between the controllability of non-delayed system (1) and the absolute controllability of the delayed system from (5) with restriction (4). The main result is represented by the following theorem:

*Theorem 1:* The non-delayed system (1) is controllable if and only if the network-delayed system (5) with restriction (4) is absolute controllable.

*Proof:* We have to prove that:

$$\text{rank}[\lambda I - A, B] = n \Leftrightarrow \text{rank}[\lambda I - A, \bar{B}] = n, (\forall) \lambda \in \Lambda(A) \quad (12)$$

From (7), relation (12) is equivalent with

$$(\forall) \lambda \in \Lambda(A), \text{rank}[\lambda I - A, c_B^1, c_B^2, \dots, c_B^p] = n \Leftrightarrow$$

$$\Leftrightarrow \text{rank}[\lambda I - A, A^{-g_1} c_B^1, A^{-g_2} c_B^2, \dots, A^{-g_p} c_B^p] = n$$

$(\Rightarrow)$  Suppose first that (1) is controllable, so

$$\text{rank}[\lambda I - A, c_B^1, c_B^2, \dots, c_B^m] = n, (\forall) \lambda \in \Lambda(A) \quad (13)$$

Suppose (by absurd) that

$$\begin{cases} (\exists) \lambda \in \Lambda(A) \\ \text{rank}[\lambda I - A, \bar{B}] < n \end{cases} \quad (14)$$

Relation (14) is equivalent to

$$(\exists) q \in \mathbb{R}_{1 \times n} \text{ i.e. } \begin{cases} qA = \lambda q \\ q\bar{B} = 0_{1 \times m}, q \neq 0_{1 \times n} \end{cases} \quad (15)$$

Using repeatedly the first equality in (15) we obtain:

$$\begin{cases} qA = \lambda q \Rightarrow q = \lambda q A^{-1} \Rightarrow q A^{-1} = \frac{1}{\lambda} q \\ qA^{-2} = \frac{1}{\lambda} q A^{-1} = \frac{1}{\lambda^2} q \\ \dots\dots\dots \\ qA^{-g} = \frac{1}{\lambda} q A^{-g+1} = \frac{1}{\lambda^g} q \end{cases} \quad (16)$$

From the second equality in (15) we obtain

$$q\bar{B} = 0_{1 \times m} \Leftrightarrow q[A^{-g_1} c_B^1, A^{-g_2} c_B^2, \dots, A^{-g_m} c_B^m] = 0_{1 \times m} \quad (17)$$

Replacing (16) in (17), we obtain:

$$q\bar{B} = \left[ \frac{1}{\lambda^{g_1}} q \cdot c_B^1, \frac{1}{\lambda^{g_2}} q \cdot c_B^2, \dots, \frac{1}{\lambda^{g_m}} q \cdot c_B^m \right] = 0_{1 \times m} \quad (18)$$

From (11) and (18) results that

$$\begin{cases} q[c_B^1, c_B^2, \dots, c_B^2] = 0_{1 \times m} \\ q[\lambda I - A, c_B^1, c_B^2, \dots, c_B^2] = 0_{1 \times (n+m)} \\ q \neq 0_{1 \times n} \end{cases} \quad (19)$$

Relation (19) is in contradiction with hypothesis (13), which concludes the demonstration.

$$(\Leftarrow) \begin{cases} \text{rank}[\lambda I - A, \bar{B}] = n \\ (\forall) \lambda \in \Lambda(A) \end{cases} \quad (20)$$

Suppose (by absurd) that

$$\begin{cases} (\exists) \lambda \in \Lambda(A) \\ \text{rank}[\lambda I - A, B] < n \end{cases} \quad (21)$$

The above condition is equivalent with

$$\begin{cases} (\exists) q \neq 0_{1 \times n} \\ qA = \lambda A \Rightarrow qB = 0_{1 \times m} \Rightarrow q[c_B^1, c_B^2, \dots, c_B^m] = 0_{1 \times m} \\ qB = 0_{1 \times m} \end{cases}$$

From (11) we obtain

$$\left[ \frac{1}{\lambda_i^{g_1}} q \cdot c_B^1, \frac{1}{\lambda_i^{g_2}} q \cdot c_B^2, \dots, \frac{1}{\lambda_i^{g_m}} q \cdot c_B^m \right] = 0_{1 \times m}$$

which is equivalent to

$$\begin{cases} (\exists) q \neq 0_{1 \times n} \\ q\bar{B} = 0_{1 \times m} \end{cases} \quad (22)$$

Relation (22) contradicts hypothesis (20), so (21) is false, which concludes demonstration.

## III. OBSERVABILITY PROBLEM STATEMENT

### A. Network output-delayed observability

The measured plant outputs are sent to the controller via a spatially distributed network, which induces different delays in each transmission channel. Consider the system

$$\begin{cases} x_{k+1} = Ax_k + Bu_k \\ y_k = Cx_k \\ y'_k = y_{k-H} \end{cases} \quad (23)$$

$$y_{k-H} = \begin{bmatrix} y_{k-h_1}^1 \\ y_{k-h_2}^2 \\ \dots \\ y_{k-h_p}^p \end{bmatrix} = \begin{bmatrix} y_{k-h_1}^1 \\ 0 \\ \dots \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ y_{k-h_2}^2 \\ \dots \\ 0 \end{bmatrix} + \dots + \begin{bmatrix} 0 \\ 0 \\ \dots \\ y_{k-h_p}^p \end{bmatrix}$$

$$C = \begin{bmatrix} l_C^1 T & l_C^2 T & \dots & l_C^p T \end{bmatrix}^T \quad (24)$$

$$C_i^T = \begin{bmatrix} 0_{n \times 1} & 0_{n \times 1} \dots & l_C^i T & 0_{n \times 1} \dots & 0_{n \times 1} \end{bmatrix}^T$$

↑ "i"-th column

$$y_{k-H} = \begin{bmatrix} y_{k-h_1}^1 \\ y_{k-h_2}^2 \\ \dots \\ y_{k-h_p}^p \end{bmatrix} = C_1 x_{k-h_1} + C_2 x_{k-h_2} + \dots + C_p x_{k-h_p} \quad (25)$$

Relation (25) shows that the non-uniformly delayed output can be wrote as a combination of previous states of the non-delayed system. Since every state at moment "k" can be represented using the past state (at moment "k-p") and all the commands in the interval [k-p, k), we will consider the maximum time-delay over all communication channels:

$$\begin{cases} h = \max\{h_i\} \\ d_i = h - h_i, \quad x_{k-h_i} = x_{k-h} + d_i \\ 0 \leq d_i \leq h \end{cases}$$

Every state at the moment 'k-h' can be represented as a final state of state evolution starting at initial moment 'k-h'.

$$\begin{cases} x_{k-h} = Ix_{k-h} \\ x_{k-h+1} = Ax_{k-h} + Bu_{k-h} \\ \dots \\ x_{k-h+d_i} = A^{d_i} x_{k-h} + \sum_{i=1}^{d_i} A^{d_i-i} Bu_{k-h-1+i} \end{cases}$$

$$y'_k = y_{k-H} = \sum_{i=1}^p C_i x_{k-h_i} = \sum_{i=1}^p C_i A^{d_i} x_{k-h} + \dots$$

$$\dots + \sum_{i=1}^p C_i \left( \sum_{i=1}^{d_i} A^{d_i-i} Bu_{k-h-1+i} \right) \quad (26)$$

Introducing the matrix function  $\sigma(A^r B) = \begin{cases} A^r B, r \geq 0 \\ 0_{n \times m}, r < 0 \end{cases}$ , we

can write (26) as

$$y'_k = \begin{bmatrix} l_C^1 A^{d_1} \\ l_C^2 A^{d_2} \\ l_C^3 A^{d_3} \\ \dots \\ l_C^p A^{d_p} \end{bmatrix} x_{k-h} + \dots$$

$$+ \begin{bmatrix} l_C^1 \cdot \sigma(A^{d_1-1} B) & l_C^1 \cdot \sigma(A^{d_1-2} B) & \dots & \dots & \dots \\ l_C^2 \cdot \sigma(A^{d_2-1} B) & l_C^2 \cdot \sigma(A^{d_2-2} B) & \dots & \dots & \dots \\ l_C^3 \cdot \sigma(A^{d_3-1} B) & l_C^3 \cdot \sigma(A^{d_3-2} B) & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ l_C^p \cdot \sigma(A^{d_p-1} B) & l_C^p \cdot \sigma(A^{d_p-2} B) & \dots & \dots & \dots \end{bmatrix} \begin{bmatrix} u_{k-h} \\ u_{k-h+1} \\ u_{k-h+2} \\ \dots \\ u_{k-1} \end{bmatrix}$$

Defining

$$\bar{U} = \begin{bmatrix} u_{k-h}^T & u_{k-h+1}^T & \dots & u_{k-2}^T & u_{k-1}^T \end{bmatrix}^T \quad (27)$$

$$\bar{B} = \begin{bmatrix} B & 0 & 0 \dots & 0 \end{bmatrix}$$

$$\bar{C}^T = \begin{bmatrix} (l_C^1 A^{d_1})^T & (l_C^2 A^{d_2})^T & \dots & (l_C^p A^{d_p})^T \end{bmatrix}^T \quad (28)$$

$$\bar{D} = \begin{bmatrix} l_C^1 \cdot \sigma(A^{d_1-1} B) & l_C^1 \cdot \sigma(A^{d_1-2} B) & \dots & \dots & \dots \\ l_C^2 \cdot \sigma(A^{d_2-1} B) & l_C^2 \cdot \sigma(A^{d_2-2} B) & \dots & \dots & \dots \\ l_C^3 \cdot \sigma(A^{d_3-1} B) & l_C^3 \cdot \sigma(A^{d_3-2} B) & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots \\ l_C^p \cdot \sigma(A^{d_p-1} B) & l_C^p \cdot \sigma(A^{d_p-2} B) & \dots & \dots & \dots \end{bmatrix}$$

we obtain the system

$$\begin{cases} x_{k-h+1} = A \cdot x_{k-h} + \bar{B} \cdot \bar{U} \\ y'_k = \bar{C} \cdot x_{k-h} + \bar{D} \cdot \bar{U} \end{cases} \quad (29)$$

### B. Observability problem solution

Despite the fact that controllability and observability are dual concepts, we give the observability problem solution because we use a different system representation for observability: model (29) is different from model (8). The output-delayed system is represented by (27) ... (29). As presented in [3], this representation is more adequate for prediction because realizes a clear decoupling between the state to be estimated and the deterministic variables (commands). If we know the measured outputs  $y_{k-H}$ , we can estimate the state  $x_{k-h}$  at moment "k-h" if the linear system in (29) is observable. The observability property for system (29) has a slightly modified interpretation comparing to observability of non-delayed system: we may say that output-delayed system (23) is observable if and only if system (29) is observable. Both systems are equivalent, but system (29) has a 'classical' input-state-output representation using the state  $x_{k-h}$ . As a consequence, the observability is referred with respect to state  $x_{k-h}$  not  $x_k$  in the same manner as for a non-delayed system:

*Proposition 2:* The system (29) is observable if and only if

$$\text{rank} \begin{bmatrix} \bar{C} \\ \lambda I - A \end{bmatrix} = n, (\forall) \lambda \in \Lambda(A)$$

The main result is represented in next theorem:

*Theorem 2:* The non-delayed system (1) is observable if and only if the network-delayed system (29) is observable.

*Proof:* We have to prove that:

$$\text{rank} \begin{bmatrix} C \\ \lambda I - A \end{bmatrix} = n \Leftrightarrow \text{rank} \begin{bmatrix} \bar{C} \\ \lambda I - A \end{bmatrix} = n, (\forall) \lambda \in \Lambda(A)$$

The above equivalence can be wrote as

$$\text{rank} \begin{bmatrix} l_C^1 \\ l_C^2 \\ \dots \\ l_C^p \\ \lambda I - A \end{bmatrix} = n \Leftrightarrow \text{rank} \begin{bmatrix} l_C^1 A^{d_1} \\ l_C^2 A^{d_2} \\ \dots \\ l_C^p A^{d_p} \\ \lambda I - A \end{bmatrix} = n, (\forall) \lambda \in \Lambda(A)$$

$$(\Rightarrow) \text{rank} \begin{bmatrix} l_C^1 \\ l_C^2 \\ \dots \\ l_C^p \\ \lambda I - A \end{bmatrix} = n, (\forall) \lambda \in \Lambda(A) \quad (30)$$

Suppose (by absurd) that

$$(\exists) \lambda \in \Lambda(A), \text{rank} \begin{bmatrix} l_C^1 A^{d_1} \\ l_C^2 A^{d_2} \\ \dots \\ l_C^p A^{d_p} \\ \lambda I - A \end{bmatrix} < n \quad (31)$$

If we suppose that, for the given  $\lambda \in \Lambda(A)$

$$(\exists) q \in \mathbb{R}_{nx1} \text{ i.e. } \begin{cases} Aq = \lambda q \\ \bar{C}q = 0_{px1} \\ q \neq 0_{nx1} \end{cases} \quad (32)$$

then we have, from the first equality in (32):

$$\begin{cases} A^2 q = \lambda Aq = \lambda^2 q \\ \dots \\ A^{n-1} q = \lambda A^{n-2} q = \dots = \lambda^{n-1} q \end{cases}$$

We obtain using (33):

$$\begin{bmatrix} l_C^1 A^{d_1} \\ l_C^2 A^{d_2} \\ \dots \\ l_C^p A^{d_p} \\ \lambda I - A \end{bmatrix} \cdot q = \begin{bmatrix} l_C^1 A^{d_1} \cdot q \\ l_C^2 A^{d_2} \cdot q \\ \dots \\ l_C^p A^{d_p} \cdot q \\ (\lambda I - A) \cdot q \end{bmatrix} = \begin{bmatrix} l_C^1 \cdot \lambda^{d_1} \\ l_C^2 \cdot \lambda^{d_2} \\ \dots \\ l_C^p \cdot \lambda^{d_p} \\ 0_{nxn} \end{bmatrix} \cdot q = 0_{(n+p) \times 1}$$

From (11)

$$\begin{bmatrix} l_C^1 \\ l_C^2 \\ \dots \\ l_C^p \\ \lambda I - A \end{bmatrix} \cdot q = 0_{px1} \Leftrightarrow \text{rank} \begin{bmatrix} l_C^1 \\ l_C^2 \\ \dots \\ l_C^p \\ \lambda I - A \end{bmatrix} < n \quad (34)$$

The relation (34) contradicts the observability condition from (30), so the supposition from (31) is false, which concludes demonstration.

( $\Leftarrow$ ) The starting hypothesis is

$$(\forall) \lambda \in \Lambda(A), \text{rank} \begin{bmatrix} l_C^1 A^{d_1} \\ l_C^2 A^{d_2} \\ \dots \\ l_C^p A^{d_p} \\ \lambda I - A \end{bmatrix} = n \quad (35)$$

Suppose (by absurd) that

$$(\exists) \lambda \in \Lambda(A), \text{rank} \begin{bmatrix} l_C^1 \\ l_C^2 \\ \dots \\ l_C^p \\ \lambda I - A \end{bmatrix} < n \quad (36)$$

The above relation is equivalent with

$$(\exists) q \in \mathbb{R}_{nx1} \text{ i.e. } \begin{cases} Aq = \lambda q \\ Cq = 0_{px1} \\ q \neq 0_{nx1} \end{cases} \quad (37)$$

Using the second equality from (66) and (11), we can write

$$\begin{bmatrix} l_C^1 \\ l_C^2 \\ \dots \\ l_C^p \end{bmatrix} \cdot q = 0_{px1} \Leftrightarrow \begin{bmatrix} l_C^1 \cdot \lambda^{d_1} \\ l_C^2 \cdot \lambda^{d_2} \\ \dots \\ l_C^p \cdot \lambda^{d_p} \end{bmatrix} \cdot q = 0_{px1} \quad (38)$$

$$\text{Using (33) we obtain } \begin{bmatrix} l_C^1 A^{d_1} \\ l_C^2 A^{d_2} \\ \dots \\ l_C^p A^{d_p} \\ \lambda I - A \end{bmatrix} \cdot q = 0_{(n+p) \times 1} \quad (39)$$

that contradicts the starting hypothesis from (35), so supposition (36) is false, which concludes demonstration.

#### IV. CONCLUSIONS

In the present paper we have studied the structural properties of systems with constant network-induced delays in command (input) and measure (output). The main results are synthesized in two theorems, which state that, if the non-delayed system is controllable and/or observable, the same system is controllable and/or observable using networked control architecture. The key point in proving the above

results is the fact that both input-delayed system (8) and output-delayed system (29) have the same state matrix,  $A$ , as the original system in (1). Perhaps the most important consequence is the fact that we can use the *model predictive control* methods in network-based architectures. Many usual and well-known control methods (as is stated in [6]) have observer-based architectures: LQG, LQG/LTR,  $H_2$  or  $H_\infty$  (loop-shaping).

Despite the fact that the results from [24] and [25] are more general, we are forced to use matrix computation in order to accomplish the controllability test. Our results need no computation and, since the network model has usually no dynamics, the initial system dynamic is also shared by the network control system, so our model is quite general for NCS framework.

In the proofs of controllability and observability theorems, we used the hypothesis of constant values of delays on different input/output channels. Despite the fact that Artstein transform and Popov-Belevitch-Hautus criterion are not so recent results, they prove to be valuable tools for the study of structural properties of networked control systems. Future work will be focused in generalizing our results in the more general case of linear *time-variable* systems with different constant input/output network induced delays and also for the general frame of linear time-variable systems with different *variable* input/output network induced delays. It seems that bounded variation of different channels network-induced delays cannot affect the structural properties of initial non-delayed system. The delay values and variations on different channels are taken into account only in the control law synthesis.

As a final and global conclusion, the system structural properties of controllability and observability are not modified under the network-induced constant delays: an initially controllable/observable system remains controllable/observable with different but constant input/output network induced delays. We consider our result important due to the fact that the networked control framework is general and the control engineer needs no preliminary conditions to verify. The control engineer can focus only on the control law design or on other real-time networked control aspects as presented in [26], where the joined state-feedback controller and scheduling strategies design strategy is considered in the optimal criteria.

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