REFEREE REPORT ON THE PAPER: EXPONENTIAL DICHOTOMY AND STABLE MANIFOLDS FOR DIFFERENTIAL-ALGEBRAIC EQUATIONS ON THE HALF-LINE BY N.T. HUY AND H. PHI

In this paper the authors consider differential algebraic equations of the form

$$E(t)x'(t) = A(t)x(t) + f(t, x(t)),\ \ x(t) \in \mathbb{R}^n, \tag{1}$$

where the matrix functions $A(t), E(t)$ are continuous in $t \in [0, \infty)$. Under reasonable conditions on $E(t)$ as in [1], the equations could be transformed to the simple decoupled form

$$\dot{y}_1(t) = A_1(t)y_1(t) + f_1(t, y_1(t), y_2(t)) \tag{2}$$

$$y_2(t) = A_3(t)y_1(t) + f_2(t, y_1(t), y_2(t)). \tag{3}$$

This justifies the name "differential algebraic equations". The authors assume similar conditions as in [1] for the equations to have an exponential dichotomy. The main topic discussed in this paper is the existence of (local and global) invariant manifolds to the above equations under the condition that f_1, f_2's Lipschitz coefficients are small in the some sense. The proofs of the existence of invariant manifolds follow the well known Perron-Lyapunov method as in the literature on the subject with a little modification similar to what the first author and his/her collaborators often do in their previous works.

While this paper formally presents a "new result" I would say it is not originally new. In fact, with the Assumption 3.1 as claimed by Lemma 3.2, given any y_1 there exists a unique $y_2 = g(y_1)$. That means the equation (3) above is no longer a challenge. Everything now depends on Eq. (2) with y_2 replaced with $g(y_1)$. The problem is reduced to the study of the ODE

$$\dot{y}_1(t) = A_1(t)y_1(t) + f_1(t, y_1(t), g(y_1(t))). \tag{4}$$

The nature of the paper's results is to repeat the well known results for this ODE (4).

In conclusion, I do not see any new contributions the authors would make in this paper to the area. Because of that I am reserved to recommend this paper for publication.

References

1. Vu Hoang Linh · Volker Mehrmann:
Lyapunov, Bohl and Sacker-Sell Spectral Intervals for Differential-Algebraic Equations. *J Dyn Diff Equat* (2009) **21**, p.153-194