Stability Analysis for Time-Delay Hamiltonian Systems

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***Abstract*— This paper investigates the asymptotically stability and robust stability for time-delay Hamiltonian systems. First, the asymptotical stability of two classes of time-delay Hamiltonian systems is studied and some sufficient conditions are derived based on the Lyapunov-Krasovskii (L-K) approach. Then, the robust stability is investigated for polytypic uncertain time-delay Hamiltonian systems, which possesses time-invariant uncertainties belonging to some convex bounded polytypic domain. Finally, several illustrative examples are studied to support the new results proposed in this paper.**

***Keywords*—Time-delay Hamiltonian systems, Asymptotic stability, Uncertainty, Robust stability, Sufficient condition.**

I. Introduction

In recent years, Port-Controlled Hamiltonian (PCH) systems have been well investigated in a series works [1], [3]–[6]. The Hamiltonian function, the sum of potential energy (excluding gravitational potential energy) and kinetic energy in physical systems, is a good candidate of Lyapunov function for the system. Due to this and its nice structure with clear physical meaning, the PCH system has drawn a good deal of attention in practical control designs. Up to now, the energy-based approach has been used in various control problems [2], [3], [5]. In particular, it has been successfully applied to the control of power systems in a series of works [1], [4], [6], and based on the PCH system several effective controllers have been designed for power systems.

Time-delay phenomena is often encountered in many control systems, which include communication systems, engineering systems and process control systems, etc. During the past two decades, the study on linear time-delay systems has drawn considerable attention and many important stability and stabilization results have been obtained [8]–[16]. However, for general nonlinear time-delay systems, the stability analysis is far more difficult than that of linear time-delay systems, and accordingly there are relatively fewer results for nonlinear time-delay systems [17]–[19]. Delay-independent stability for nonlinear time-delay systems has been investigated deeply under Lyapunov-Razumikhin conditions [17]. In [19], a methodology to construct L-K functionals for time delay systems based on the Sum of Square decomposition was presented. The construction is entirely algorithmic and is done through the solution of a set of Linear Matrix Inequalities(LMIs).

For Hamiltonian systems, there are, to the authors' best knowledge, fewer works on the stability or stabilization of time-delay Hamiltonian systems. The time-delay Hamiltonian system is a kind of important nonlinear time-delay system. Such a system not only plays an important role in the development of time-delay control theory, but also is believed to find many applications in control design for practical systems, including power systems and robotic systems [4], [7], in order to obtain better control performances. So, it is significant to study the analysis and synthesis for time-delay Hamiltonian systems.

In this paper, we mainly investigate the stability of several kinds of time-delay Hamiltonian systems. Firstly, the asymptotical stability of two classes of time-delay Hamiltonian systems is studied and some sufficient conditions are derived based on L-K approach. Then, the robust stability is investigated for a class of time-delay Hamiltonian systems: polytypic uncertain time-delay Hamiltonian systems, in which there are time-invariant uncertainties belonging to some convex bounded polytypic domain. Based on Lyapunov stability theory and L-

K approach, we can show that, the obtained delay-independent criteria on the stabilities of the systems can effectively reduce the conservatism of quadratic stability. We illustrate the results with several numerical examples from population dynamics.

The paper is organized as follows. Section 2 investigates the asymptotical stability of two classes of time-delay Hamiltonian systems. In Section 3, polytypic uncertain time-delay Hamiltonian systems are studied and some stability results are proposed, which is followed by the conclusion in Section 4.

II. Stability of time-delay Hamiltonian systems

In this section, we investigate the stability of two classes of time-delay Hamiltonian systems, and propose some new results on the stability. First, we give a lemma, which will be used in sequel.

Lemma 1: Consider the following dissipative Hamiltonian system

$$\dot{x} = [J(x) - R(x)]\frac{\partial H(x)}{\partial x}, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $J(x) \in \mathbb{R}^{n\times n}$ is a skew-symmetric matrix and $R(x) \in \mathbb{R}^{n\times n}$ is positive semi-definite, $H(x)$ is Hamiltonian function of the system. Letting $y = \psi(x)$ be a coordinate transformation, then under the new coordinate y, system (4) takes the form of

$$\dot{y} = [\bar{J}(y) - \bar{R}(y)]\frac{\partial \bar{H}(y)}{\partial y}, \tag{2}$$

where

$$\bar{J}(y) = \frac{\partial \psi(x)}{\partial x} J(x)\Big(\frac{\partial \psi(x)}{\partial x}\Big)^T|_{x=\psi^{-1}(y)}$$

is skew-symmetric,

$$\bar{R}(y) = \frac{\partial \psi(x)}{\partial x} R(x)\Big(\frac{\partial \psi(x)}{\partial x}\Big)^T|_{x=\psi^{-1}(y)}$$

is positive semi-definite, and $\bar{H}(y) = H(x)|_{x=\psi^{-1}(y)}$.

Proof: A straightforward computation can show the lemma. □

Lemma 1 shows that, under coordinate transformations, the dissipativity of Hamiltonian systems remains unchanged.

In the following, we investigate the stability of time-delay Hamiltonian systems. Consider the following time-delay Hamiltonian system

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\frac{\partial H(x(t))}{\partial x} \\ &\quad + \alpha[J(x) - R(x)]\frac{\partial H(x(t-\tau))}{\partial x}, \end{aligned} \tag{3}$$

where $x(t) \in \mathbb{R}^n$ is the state, τ is the time delay (a constant), $J(x) \in \mathbb{R}^{n\times n}$ is a skew-symmetric matrix and $R(x) \in \mathbb{R}^{n\times n}$ is positive definite, $H(x(t))$ is Hamiltonian function of the system, and $\alpha \in \mathbb{R}^1$ is a constant satisfying $0 < \alpha < 1$.

For the stability of system (3), we have the following result.

Theorem 1: The time-delay Hamiltonian system (3) is asymptotically stable for given $\tau \geq 0$, if there exists a constant symmetric matrix $0 < Q \in \mathbb{R}^{n\times n}$ such that $\Pi(J,R,Q) > 0$ holds, where

$$\Pi(J,R,Q) := \begin{bmatrix} R - Q & A_1 \\ A_1^T & \alpha R + Q \end{bmatrix}, \tag{4}$$

and

$$A_1 = \frac{1}{2}[(1-\alpha)J + (1+\alpha)R].$$

Proof: Construct a Lyapunov function candidate as follows

$$\begin{aligned} V &= H(x(t)) + H(x(t-\tau)) \\ &\quad + \int_{-\tau}^{0} \frac{\partial H^T(x(t+\theta))}{\partial x} Q \frac{\partial H(x(t+\theta))}{\partial x} d\theta. \end{aligned} \tag{5}$$

Obviously, $V > 0$.

Denote $\nabla H = \frac{\partial H(x(t))}{\partial x}$, $\nabla H_\tau = \frac{\partial H(x(t-\tau))}{\partial x}$. The derivative of V along the trajectory of system (3) is as follows

$$\begin{aligned} \dot{V} &= \nabla H^T \dot{x} + \nabla H_\tau^T \dot{x} + \nabla H^T Q \nabla H - \nabla H_\tau^T Q \nabla H_\tau \\ &= \nabla H^T (J(x) - R(x))\nabla H + \nabla H^T \alpha (J(x) \\ &\quad - R(x))\nabla H_\tau + \nabla H_\tau^T (J(x) - R(x))\nabla H \\ &\quad + \nabla H_\tau^T \alpha (J(x) - R(x))\nabla H_\tau + \nabla H^T Q \nabla H \\ &\quad - \nabla H_\tau^T Q \nabla H_\tau \\ &= -\nabla H^T R(x) \nabla H + \nabla H^T \alpha (J(x) - R(x))\nabla H_\tau \\ &\quad + \nabla H_\tau^T (J(x) - R(x))\nabla H - \nabla H_\tau^T \alpha R(x) \nabla H_\tau \\ &\quad + \nabla H^T Q \nabla H - \nabla H_\tau^T Q \nabla H_\tau \\ &= \begin{bmatrix} \nabla H \\ \nabla H_\tau \end{bmatrix}^T \begin{bmatrix} -R + Q & \alpha(J - R) \\ J - R & -\alpha R - Q \end{bmatrix} \begin{bmatrix} \nabla H \\ \nabla H_\tau \end{bmatrix} \\ &= \begin{bmatrix} \nabla H \\ \nabla H_\tau \end{bmatrix}^T \Bigg(\begin{bmatrix} 0 & A_2 \\ -A_2^T & 0 \end{bmatrix} \\ &\quad - \begin{bmatrix} R - Q & A_1 \\ A_1^T & \alpha R + Q \end{bmatrix} \Bigg) \begin{bmatrix} \nabla H \\ \nabla H_\tau \end{bmatrix}, \end{aligned} \tag{6}$$

where

$$A_2 = \frac{1}{2}[(1+\alpha)J + (1-\alpha)R].$$

Since the matrix $\begin{bmatrix} 0 & A_2 \\ -A_2^T & 0 \end{bmatrix}$ is skew-symmetric, and $\Pi(J,R,Q) > 0$, it is easy to see that

$$\begin{bmatrix} 0 & A_2 \\ -A_2^T & 0 \end{bmatrix} - \begin{bmatrix} R - Q & A_1 \\ A_1^T & \alpha R + Q \end{bmatrix}$$

is strictly dissipative. From (6) and Lemma 1, we know that $\dot{V} < 0$ if neither $\nabla H(x(t))$ nor $\nabla H(x(t-\tau))$ is equal to zero.

Therefore, system (3) is asymptotically stable. The proof is completed. $\square$

Remark 1: If $R(x) \equiv 0$ in the Hamiltonian system (3), the Lyapunov function (5) can be chosen as $V = H(x(t)) + H(x(t-\tau))$, i.e., let $Q = 0$ in (5). In this case, we can easily show that system (3) is Lyapunov stable.

Example 1 Consider a simple example with

$$J(x) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix},\ R(x) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix},$$

$$H(x) = \frac{1}{2}(x_1^2 + x_2^2),\ \alpha = \frac{1}{5}.$$

The system is delay-independent stable, and we prove this by constructing a Lyapunov functional with the form of (5). So we choose $Q = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ which satisfies the condition of Theorem 1. Thus, the system is asymptotic stable.

In system (3), if the structural matrix also involves time delay, then the system becomes

$$\begin{aligned} \dot{x} &= \Big[J(x(t-\tau)) - R(x(t-\tau))\Big]\frac{\partial H(x(t))}{\partial x} \\ &\quad + \alpha\Big[J(x(t-\tau)) - R(x(t-\tau))\Big]\frac{\partial H(x(t-\tau))}{\partial x} \end{aligned} \tag{7}$$

For system (7), we have the following stability result.

Corollary 1: The time-delay Hamiltonian system (7) is asymptotically stable for given $\tau \geq 0$, if there exists a constant matrix $0 < Q \in \mathbb{R}^{n\times n}$ satisfying $\Pi(J, R, Q) > 0$.

The proof is omitted. $\square$

Next, we consider another class of time-delay Hamiltonian systems described as

$$\begin{aligned} \dot{x} &= \Big[J(x(t)) - R(x(t))\Big]\frac{\partial H(x(t))}{\partial x} \\ &\quad + \alpha\Big[J(x(t-\tau)) - R(x(t-\tau))\Big]\frac{\partial H(x(t-\tau))}{\partial x} \end{aligned} \tag{8}$$

where $x(t)$, $J(x)$, $R(x)$, $H(x)$ and τ are the same as system (3) above. For simplicity, we denote $J_\tau := J(x(t-\tau))$, $R_\tau := R(x(t-\tau))$ in the following.

For this kind of time-delay Hamiltonian systems given in (8), we have the following result.

Theorem 2: The time-delay Hamiltonian system (8) is asymptotically stable for given $\tau \geq 0$, if there exists a constant symmetric matrix $0 < Q \in \mathbb{R}^{n\times n}$ such that $\Pi(J, J_\tau, R, R_\tau, \bar{Q}) > 0$ holds, where

$$\Pi(J, J_\tau, R, R_\tau, \bar{Q}) := \begin{bmatrix} R - \bar{Q} & A_3 \\ A_3^T & \alpha R_\tau + \bar{Q} \end{bmatrix}, \tag{9}$$

and

$$A_3 = \frac{1}{2}(-\alpha J_\tau + J + \alpha R_\tau + R).$$

Proof: Construct a Lyapunov function candidate as follows

$$\begin{aligned} V &= H(x(t)) + H(x(t-\tau)) \\ &\quad + \int_{-\tau}^{0} \frac{\partial H^T(x(t+\theta))}{\partial x}\bar{Q}\frac{\partial H(x(t+\theta))}{\partial x}d\theta. \end{aligned} \tag{10}$$

Obviously, $V > 0$.

The derivative of V along the trajectory of system (8) is as follows

$$\begin{aligned} \dot{V} &= \nabla H^T\dot{x} + \nabla H_\tau^T\dot{x} + \nabla H^T\bar{Q}\nabla H - \nabla H_\tau^T\bar{Q}\nabla H_\tau \\ &= -\nabla H^T R(x)\nabla H + \nabla H^T\alpha(J_\tau - R_\tau)\nabla H_\tau \\ &\quad + \nabla H_\tau^T(J(x) - R(x))\nabla H - \nabla H_\tau^T\alpha R_\tau\nabla H_\tau \\ &\quad + \nabla H^T\bar{Q}\nabla H - \nabla H_\tau^T\bar{Q}\nabla H_\tau \\ &= \begin{bmatrix} \nabla H \\ \nabla H_\tau \end{bmatrix}^T \begin{bmatrix} -R + \bar{Q} & \alpha(J_\tau - R_\tau) \\ J - R & -\alpha R_\tau - \bar{Q} \end{bmatrix} \begin{bmatrix} \nabla H \\ \nabla H_\tau \end{bmatrix} \\ &= \begin{bmatrix} \nabla H \\ \nabla H_\tau \end{bmatrix}^T \left(\begin{bmatrix} 0 & A_4 \\ -A_4^T & 0 \end{bmatrix} \right. \\ &\quad \left. - \begin{bmatrix} R - \bar{Q} & A_3 \\ A_3^T & \alpha R_\tau + \bar{Q} \end{bmatrix} \right) \begin{bmatrix} \nabla H \\ \nabla H_\tau \end{bmatrix}, \end{aligned} \tag{11}$$

where

$$A_4 = \frac{1}{2}(\alpha J_\tau + J - \alpha R_\tau + R).$$

Since the matrix $\begin{bmatrix} 0 & A_4 \\ -A_4^T & 0 \end{bmatrix}$ is skew-symmetric, and $\Pi(J, J_\tau, R, R_\tau, \bar{Q})$, it is easy to see that

$$\begin{bmatrix} 0 & A_4 \\ -A_4^T & 0 \end{bmatrix} - \begin{bmatrix} R - \bar{Q} & A_3 \\ A_3^T & \alpha R_\tau + \bar{Q} \end{bmatrix}$$

is strictly dissipative.

From (11) and Lemma 1, we know that $\dot{V} < 0$ whenever neither $\nabla H(x(t))$ nor $\nabla H(x(t-\tau))$ is zero, which implies that system (8) is asymptotically stable. The proof is completed. $\square$

Example 2 Consider time-delay Hamiltonian system (8) with

$$J(x) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},\ R(x) = \begin{bmatrix} x^2 + 1 & 0 \\ 0 & 2 \end{bmatrix},$$

$$R(x(t-\tau))) = \begin{bmatrix} x^2(t-\tau) + 1 & 0 \\ 0 & 2 \end{bmatrix},$$

$$H(x) = \frac{1}{2}(x_1^2 + x_2^2),\quad \alpha = \frac{1}{2}$$

Choose $Q = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}$ which satisfies the condition of Theorem 2. Thus, the system is asymptotic stable.

III. Stability of polytypic uncertain time-delay Hamiltonian systems

In this section, we investigate a class of uncertain time delay Hamiltonian systems whose structural matrix contain time invariant uncertainties. We name it as Polytypic Uncertain Time-delay Hamiltonian system, which is described as follows

$$\dot{x} = (J^* - R^*)\frac{\partial H(x(t))}{\partial x} + \alpha(J^* - R^*)\frac{\partial H(x(t-\tau))}{\partial x} \quad (12)$$

where $x \in \mathbb{R}^n$, $0 < \alpha < 1$, $J^* \in \mathbb{R}^{n\times n}$ is skew-symmetric matrix, $R^* \in \mathbb{R}^{n\times n}$ is positive definite matrix; J^* and R^* are not precisely known, but belong to the convex domains respectively given by

$$\Omega_J = \Omega(J_i, \lambda_i) = \Big\{J^* \Big| \ J^* = \sum_{i=1}^{m} \lambda_i J_i, \quad J_i^T = -J_i, \quad \forall\ \lambda_i \geq 0, \quad \sum_{i=1}^{m} \lambda_i = 1\Big\} \quad (13)$$

and

$$\Omega_R = \Omega(R_i, \lambda_i) = \Big\{R^* \Big| \ R^* = \sum_{i=1}^{m} \lambda_i R_i, \quad R_i > 0, \quad \forall\ \lambda_i \geq 0, \quad \sum_{i=1}^{m} \lambda_i = 1\Big\}. \quad (14)$$

Define one convex polytypic uncertainty parametric vector set as

$$\Omega_\Lambda = \Big\{\Lambda \Big| \ \Lambda = [\lambda_1 \ \lambda_2 \ \cdots \ \lambda_m]^T \otimes I_n \in \mathbb{R}^{mn\times n}, \quad \sum_{i=1}^{m} \lambda_i = 1, \quad \lambda_i \geq 0\Big\}, \quad (15)$$

where $\otimes$ stands for the Kronecker product. Note that this definition can show the structural matrix of the system is affinely dependent on the uncertainty parameters $\lambda_i (i = 1, 2, \cdots, m)$.

With (15), system (12) can be rewritten as

$$\dot{x} = (J - R)\Lambda\frac{\partial H(x(t))}{\partial x} + \alpha(J - R)\Lambda\frac{\partial H(x(t-\tau))}{\partial x}, \quad (16)$$

where $J := [J_1 \ J_2 \ \cdots J_m]$ and $R := [R_1 \ R_2 \ \cdots R_m]$ are the vertex vectors of uncertain domain Ω_J and Ω_R. Thus, the convex bounded uncertain domain Ω_J and Ω_R can be then redefined as follows

$$\Omega_J := \{J\Lambda : \lambda \in \Omega_\Lambda\}, \quad \Omega_R := \{R\Lambda : \lambda \in \Omega_\Lambda\}.$$

Our main objective is to provide stability conditions under which all systems belonging to (Ω_J, Ω_R) are stable. Motivated by [15], we obtained the following result.

Proposition 1: The uncertain time-delay Hamiltonian system (12) is robustly stable if there exists a constant symmetric matrix $0 < Q \in \mathbb{R}^{n\times n}$ satisfying the inequality $\Pi(J\Lambda, R\Lambda, Q) > 0$, where

$$\Pi(J\Lambda, R\Lambda, Q) := \begin{bmatrix} R\Lambda - Q & A_5 \\ A_5^T & \alpha R\Lambda + Q \end{bmatrix}, \quad (17)$$

and

$$A_5 = \frac{1}{2}[(1-\alpha)J + (1+\alpha)R]\Lambda.$$

This robust stability condition comes from the L-K stability theory based on the Lyapunov functional as (5). The proof is similar to that of Theorem 1, and thus omitted. □

Because Λ is uncertain, Proposition 1 can only ensure an efficient numerical solution. In general, the quadratic stability is comparatively conservative, since the common matrix Q must be such that $\Pi(J_i, R_i, Q) > 0$ hold for all vertices of (Ω_J, Ω_R). In the following, we use the so-called Single Polyhedral Lyapunov Functional (SPLF) [15], i.e., the following form of Lyapunov function candidate, to investigate the stability of the system (12)

$$\begin{aligned} V(\hat{Q}, t) \quad := \quad & H(x(t)) + H(x(t-\tau)) \\ & + \int_{-\tau}^{0} \frac{\partial H^T(x(t+\theta))}{\partial x}\hat{Q}\frac{\partial H(x(t+\theta))}{\partial x} d\theta, \end{aligned} \quad (18)$$

where $\hat{Q} := \sum_{i=1}^{m} \lambda_i Q_i = \tilde{Q}\Lambda > 0$ and $\tilde{Q} = [Q_1 \ Q_2 \ \cdots Q_m]$.

Using SPLF $V(\hat{Q}, t)$, we can reduce the conservatism of Proposition 1 by increasing the degrees of freedom of Q to $\tilde{Q}$, and thus the stability domain can be enlarged. Based on SPLF $V(\hat{Q}, t)$, we obtained the following result.

Theorem 3: The time-delay Hamiltonian system (12) is robustly stable for any $\Lambda \in \Omega_\Lambda$, if there exist positive definite matrices $Q_i > 0$ $(i = 1, 2, ..., m)$ such that the following matrices inequalities hold

$$\Pi_{ij} + \Pi_{ji} > 0, \quad 1 \leq i \leq j \leq m, \quad (19)$$

where

$$\Pi_{ij} = \begin{bmatrix} R_i - Q_i & A_6 \\ A_6^T & \alpha R_i + Q_i \end{bmatrix}, \quad (20)$$

and

$$A_6 = \frac{1}{2}[(1-\alpha)J_j + (1+\alpha)R_j].$$

Proof: Choose SPLF $V(\hat{Q},t)$ given in (18) as the Lyapunov function candidate. Since

$$\begin{aligned}
&\sum_{i=1}^{m}\lambda_i\Big[\sum_{i=1,i\leq j}^{m}\lambda_i(\Pi_{ij}+\Pi_{ji})\Big] \\
=&\sum_{i=1}^{m}\lambda_i\Big[\sum_{i=1,j=1}^{m}\lambda_i\Pi_{ij}+\sum_{i=1,i<j}^{m}\lambda_i\Pi_{ij}+\sum_{i=1,i>j}^{m}\lambda_i\Pi_{ij}\Big] \\
=&\sum_{i=1}^{m}\lambda_i\sum_{j=1}^{m}\lambda_i\Pi_{ij}=\Pi(J\Lambda,R\Lambda,\tilde{Q}\Lambda), \qquad (21)
\end{aligned}$$

it is easy to see from the theorem's condition that $\Pi(J\Lambda,R\Lambda,\tilde{Q}\Lambda)>0$. With this, we can show that $\dot{V}(\hat{Q},t)<0$, which implies that system (12) is robustly stable. Thus, the proof is completed. □

Example 3 Consider a two-vertices uncertain parameterized time-delay Hamiltonian system

$$\dot{x}=[J^*-R^*]\nabla H(x(t))+\alpha[J^*-R^*]\nabla H(x(t-\tau)), \quad (22)$$

where $x\in\mathbb{R}^2$, $\alpha=\frac{1}{2}$, $\forall\,\tau\geq 0$,

$$J^*-R^*=[J_1(x)-R_1(x)\quad J_2(x)-R_2(x)]([\lambda_1\quad\lambda_2]^T\otimes I_2),$$

$$J_1(x)=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix},\quad J_2(x)=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix},$$

$$R_1(x)=\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix},\quad R_2(x)=\begin{bmatrix}2 & 0\\ 0 & 1\end{bmatrix},$$

and $H(x)=\frac{1}{2}(x_1^2+x_2^2)$, $\lambda_1+\lambda_2=1$.

Choose

$$Q_1=\begin{bmatrix}\frac{1}{2} & 0\\ 0 & \frac{1}{2}\end{bmatrix},\quad Q_2=\begin{bmatrix}1 & 0\\ 0 & \frac{1}{2}\end{bmatrix}$$

It is easy to check that Q_1 and Q_2 are such that the condition of Theorem 3 hold for $1\leq i\leq j\leq 2$. Thus, the system (22) is robustly stable.

To test the correctness of our conclusion above, numerical simulation has investigated with the following choices. Initial Condition: $x(0)=[0.5\ ;-1.0]$, Time delay: $\tau=1.0$. The simulation result is shown in Fig 1, which confirms the correctness of our result.

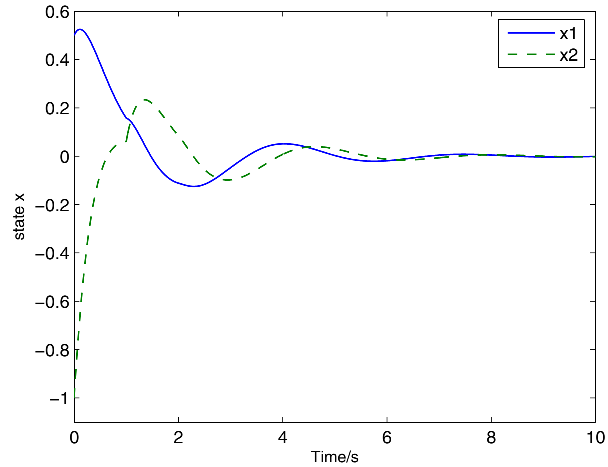

Fig 1: Response of state x

IV. Conclusion

We have investigated the asymptotically stability and robust stability of time-delay Hamiltonian systems, and proposes a number of new results on the stabilities of the systems. Based on Lyapunov-Krasovskii approach, some sufficient conditions have been presented for the asymptotical stability of several classes of time-delay Hamiltonian systems. For polytypic uncertain time-delay Hamiltonian systems, the robust stability has been investigated and several new results have been obtained in this paper.

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