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Brief paper

Event-triggered tracking control for nonlinear systems subject to time-varying external disturbances[☆]

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ABSTRACT

Event-triggered tracking control for a class of nonlinear systems with disturbances is investigated in this paper. Compared to existing related results, the nonlinearities only need to satisfy a generalized Lipschitz condition, and the time-varying external disturbances are allowed to be unmatched. By using finite-time disturbance observers, the finite-time estimation of the steady states is achieved to reduce the complexity of tracking control design. The event-triggered controller is designed by a new feedback domination approach, which can dynamically compensate for both errors caused by disturbances and the sampled-data implementation of the controller. A new Lyapunov stability analysis is given to show that all the signals of the closed-loop system are globally bounded and the tracking error is ensured to converge to a set, which can be made as small as desired by adjusting control parameters. Finally, a numerical example demonstrates the effectiveness of the designed scheme.

1. Introduction

Networked control is attracting much research attention recently, due to its extensive practical application in intelligent vehicle systems, and advanced aircraft and spacecraft, e.g., Gupta and Chow (2008), Lian, Moyne, and Tilbury (2001), Sun and Guo (2017) and Wang, Chen, Lin, Zhang, and Meng (2017). However, due to limited bandwidth, communication and actuation modules, small-scale micro-processor and capability-limited communication and actuation modules are often needed. Motivated by the above-mentioned realities, event-triggered control strategy has been proposed, e.g., (Åström & Bernhardsson, 1999; Garcia & Antsaklis, 2011; Heemels, Sandee, & Van Den Bosch, 2008; Johansson, Egerstedt, Lygeros, & Sastry, 1999; Liu & Jiang, 2015b; Tabuada, 2007). The most appealing feature of event-triggered control is that control tasks are executed only when certain specified conditions are satisfied, which may decrease the signal transmission burden in the communication network. Therefore, the study on event-triggered control has achieved a series of fruitful results, e.g., Borgers and Heemels (2014), Donkers and Heemels (2012), Heemels, Johansson, and Tabuada (2012), Henningsson, Johannesson, and Cervin (2008) and Liu and Jiang (2015a).

In practice, the research on robustness of event-triggered control for systems has become a focus in recent years (Abdelrahim, Postoyan, Daafouz, & Nešić, 2017; Gao, Wang, Wen, & Wang, 2017; Heemels et al., 2012; Henningsson et al., 2008; Liu & Jiang, 2015a; Marchand, Durand, & Castellanos, 2013; Postoyan, Tabuada, Nesic & Anta, 2015; Seuret, Prieur, & Marchand, 2014; Tallapragada & Chopra, 2013; Xing, Wen, Guo, Liu, & Su, 2017; Xing, Wen, Liu, Su, & Cai, 2017a, 2017b). For most related research works, e.g., Abdelrahim et al. (2017), Gao et al. (2017), Heemels et al. (2012), Henningsson et al. (2008), Liu and Jiang (2015a), Marchand et al. (2013), Postoyan, Tabuada et al. (2015), Seuret et al. (2014) and Xing et al. (2017), only robust stabilization of the origin is considered for linear/nonlinear systems. Although event-triggered tracking control is investigated in Postoyan and Bragagnolo et al. (2015), Tallapragada and Chopra (2013) and Xing et al. (2017a, 2017b), the nonlinear functions are required to be smooth or globally Lipschitz when no time-varying disturbances are acting on the system. As a result, how to handle time-varying external disturbances as well as relaxing the hypothesis about nonlinear functions remains to be overcome within the framework of event-triggered tracking control.

In this paper, we aim at addressing the event-triggered tracking control problem of nonlinear systems involving unmatched

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time-varying external disturbances, where the full state is measured. The nonlinearities of considered systems are continuous and satisfy a generalized Lipschitz condition which means that the power of the difference between variables can be extended to take any value in an interval and the bounding gains can be variables, as specified in Assumption 2 given in Section 2. To the best of our knowledge, no results have ever been reported to solve such a problem, mainly due to the following two difficulties: (i) How to design a control strategy to regulate the magnitude of tracking error arbitrarily small for uncertain nonlinear systems with external disturbances by adjusting as few parameters as possible. (ii) Deal with more general nonlinear systems in this paper. Hence, more intricate design and performance analysis should be used to solve such a problem.

Inspired by the idea of relative threshold strategy in Borgers and Heemels (2014), Donkers and Heemels (2012) and Tabuada (2007) an event-triggered controller based on the sign function is proposed to overcome the aforementioned challenges. An intermediate control signal is obtained via the finite-time estimation of the steady states and an adapted feedback domination approach. More specifically, finite-time estimation of the steady states is achieved by designing appropriate finite-time disturbance observers that can be effectively implemented to estimate on-line various kinds of disturbances, e.g., Huang (2004), Isidori (1995), Isidori and Byrnes (1990), Li, Sun, Yang, and Yu (2015), Shtessel, Edwards, Fridman, and Levant (2014) and Yang, Ding, Li, and Zhang (2018). Meanwhile, by recursive design, the improved feedback domination approach is designed to effectively compensate for the sampling-induced errors and counteract generalized disturbances. Furthermore, in order to deal with a wider class of nonlinear systems, a Lyapunov function based on the sign function (Sun, Li, & Yang, 2016) is used to show that the proposed event-triggered controller guarantees the global boundedness of the closed-loop signals, and the tracking error is ensured to converge to a set, which can be made as small as desired by adjusting control parameters.

2. Preliminaries and problem formulation

2.1. Notations and preliminaries

The following notations will be used throughout this paper. $\mathbb{R}^{\geq 1}_{odd}$ denotes the real number set, whose elements are of the form $\frac{q_1}{q_2}$, where q_1 and q_2 are positive odd integers with $q_1 \geq q_2$. $\mathbb{R}_+$ denotes the set of nonnegative real numbers. $D^+F(t)$ and $D^-F(t)$ denote the right derivative and the left derivative of F at $t \in [0, \infty)$, respectively. Let $\mathbb{N}_{j:i} \triangleq \{j, j+1, \dots, i\}$, where j and i are nonnegative integers satisfying $j \leq i$. The symbol C^i denotes the set of all differentiable functions whose ith order derivatives are continuous. The symbol $\mathcal{L}_\infty$ represents the set of all signals whose infinity-norms are bounded. For a real vector $x = [x_1, \dots, x_n]^T \in \mathbb{R}^n$, $\bar{x}_i \triangleq [x_1, \dots, x_i]^T \in \mathbb{R}^i$, $i = 1, \dots, n$, and we let $\bar{x}_n = x$. For real vectors $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^m$, (x, y) stands for $[x^T, y^T]^T$. Define $\bar{z} = [\bar{z}_2, \dots, \bar{z}_{n+1}]^T$. We consider the sign function defined as, for any $y \in \mathbb{R}$, $\mathrm{sign}(y) = 1$ if $y > 0$, $\mathrm{sign}(y) = 0$ if $y = 0$, and $\mathrm{sign}(y) = -1$ if $y < 0$. For a given positive constant a, $\lceil y \rfloor^a \triangleq |y|^a \mathrm{sign}(y)$, $\forall y \in \mathbb{R}$.

Now we present the following definition and two preliminary lemmas which are useful in our design and analysis.

Definition 1 (*See p. 387 in Huang (2004)*)**.** For nonlinear system $\dot{x} = f(x, u)$, where x is the n-dimensional state in a neighborhood U of the origin in $\mathbb{R}^n$, u is the m-dimensional input, $f(0, 0) = 0$. Let $x(t, x(0), u(\cdot))$ denote the value of the solution achieved at a time $t > 0$ under the effect of the input $u(\cdot)$, starting from the initial state $x(0)$. Let $u^*(\cdot)$ be a specific input function and suppose there exists an initial state x^* with the property that $\lim_{t\to\infty} \|x(t, x(0), u^*(\cdot)) - x(t, x^*, u^*(\cdot))\| = 0$ for every $x(0)$ in some neighborhood U^* of x^*. If this is the case, then $x_{ss}(t) = x(t, x^*, u^*(\cdot))$ is called the steady state response of the closed-loop system with the desired control $u^*(\cdot)$.

Lemma 1 (Sun et al., 2016)**.** *For given $r \geq 0$, $\frac{a_1}{a_2} \in \mathbb{R}^{\geq 1}_{odd}$ with $a_2 \geq 1$, $m > 0, n > 0$, $b \in \mathbb{R}$, every $x \in \mathbb{R}, y \in \mathbb{R}$, there exist $c_r \geq 1$ and $c > 0$ such that*

$$|x+y|^r \leq c_r(|x|^r + |y|^r),$$
$$\left|x^{\frac{a_1}{a_2}} - y^{\frac{a_1}{a_2}}\right| \leq 2^{1-\frac{1}{a_2}} \left|\lceil x \rceil^{a_1} - \lceil y \rceil^{a_1}\right|^{\frac{1}{a_2}},$$
$$|bx^m y^n| \leq c|x|^{m+n} + \frac{n}{m+n}\left(\frac{m}{(m+n)c}\right)^{\frac{m}{n}} \cdot |b|^{\frac{m+n}{n}} |y|^{m+n},$$

where $c_r = 2^{r-1}$ if $r > 1$, and $c_r = 1$ if $r \leq 1$.

Lemma 2 (Sun et al., 2016)**.** *The function $f : x \mapsto \lceil x \rfloor^a$ $(a \geq 1)$ is continuously differentiable on $(-\infty, +\infty)$, and its derivative satisfies $f' : x \mapsto a|x|^{a-1}$.*

2.2. Problem formulation

We consider the class of nonlinear systems subject to unmatched time-varying disturbances, depicted by

$$\begin{cases} \dot{x}_i = x_{i+1} + f_i(t, \bar{x}_i) + d_i, i \in \mathbb{N}_{1:n-1}, \\ \dot{x}_n = u + f_n(t, \bar{x}_n) + d_n, \\ y = x_1, \end{cases} \tag{1}$$

where $x = [x_1, \dots, x_n]^T \in \mathbb{R}^n$ is the measured system state with initial value $x(0) = x_0$, $u \in \mathbb{R}$ and $y \in \mathbb{R}$ are system input and output, respectively. For $i \in \mathbb{N}_{1:n}$, the nonlinear functions $f_i(\cdot)$ are continuous in each dependent variable, $d_i(\cdot)$ denotes the external disturbance.

The control objective is to construct an appropriate event-triggered controller such that the output y of system (1) can track a given reference signal y_r. To this end, the following assumptions are needed.

Assumption 1. The external disturbances and reference signal satisfy $d_i \in C^{n+2-i}$, $y_r \in C^{n+1}$. Furthermore, $d_i^{(j)} \in \mathcal{L}_\infty$ and $y_r^{(k)} \in \mathcal{L}_\infty$ are bounded as follows: $\sup_{t\geq 0} |d_i^{(j)}(t)| \leq G_i^+$, $\sup_{t\geq 0} |y_r^{(k)}(t)| \leq G^+$ for $t \geq 0, j \in \mathbb{N}_{0:n+2-i}$, $i \in \mathbb{N}_{1:n}$ and $k \in \mathbb{N}_{0:n}$, where G_i^+ and G^+ are some known positive constants.

Remark 1. Assumption 1 is motivated by Li et al. (2015) and Yang et al. (2018). It means that the external disturbances d_i and their high-order (from first to $(n+2-i)$th order) derivatives are bounded, besides, the reference signal and its high-order (from first to $(n+1)$th) derivatives are bounded. Under this assumption, a disturbance observer and the desired steady-state estimator of system can be designed. □

Assumption 2. For any $t \geq 0$, $\bar{x}_i \in \mathbb{R}^i$ and $\bar{y}_i \in \mathbb{R}^i$, there exists a nonnegative continuous function $h_i : \mathbb{R}^i \times \mathbb{R}^i \to \mathbb{R}_+$ such that the following inequality holds:

$$|f_i(t, \bar{x}_i) - f_i(t, \bar{y}_i)| \leq h_i(\bar{x}_i, \bar{y}_i) \sum_{j=1}^{i} |x_j - y_j|^{\frac{r_i+\tau}{r_j}}, \tag{2}$$

where $\tau \in (-\frac{1}{n}, 0]$, $r_1 = 1$, $r_{i+1} = r_i + \tau$, $i \in \mathbb{N}_{1:n}$.

Remark 2. Assumption 2 relaxes the restriction on nonlinear functions of existing related works including non-event triggered based approaches from two aspects. (i) First, only continuity,

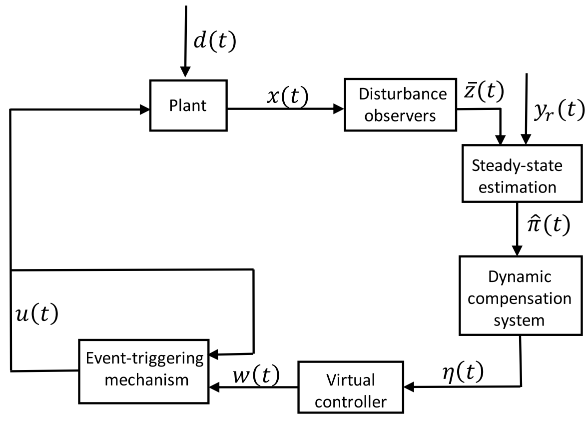

Fig. 1. Event-triggered control schematic.

rather than smoothness, is needed in this paper, contrary to Xing et al. (2017a, 2017b), and Assumption 2 in this paper extends Lipschitz condition in Tabuada (2007) to the generalized form in (2). Actually, Assumption 2 generalizes the classical Lipschitz condition to cover more general engineering applications. For example, the jet engine compressor model in Postoyan, Tabuada et al. (2015) does not satisfy the classical Lipschitz condition but meets Assumption 2. (ii) Second, even for the non-event triggered based results in Lin, Qian, and Huang (2003) and Polendo and Qian (2007), Assumption 2 relaxes their vanishing restriction that requires $f_i(t,0) = 0$ for all t. More specifically, the nonlinear functions in Lin et al. (2003) and Polendo and Qian (2007) are restricted by the inequality

$$|f_i(t,\bar{x}_i)| \le \theta \sum_{j=1}^{i} |x_j|^{\frac{r_i+\tau}{r_j}} \tag{3}$$

with θ being a constant, which implies that $f_i(t,\bar{x}_i)$ should be a vanishing nonlinear function. Note that condition (2) does not have such a requirement. For example, consider a simple system that $\dot{x}_1 = x_2 + x_1^2 + \cos x_1 + d_1(t)$, $\dot{x}_2 = u + x_2 + d_2(t)$. In this case, $f_1(x_1) = x_1^2 + \cos x_1$ and it does not meet inequality (3). However, the inequality in (2) of Assumption 2 is satisfied by choosing $\tau = 0$ and $h_1(x_1,y_1) = |x_1 + y_1| + 1$. □

3. Main results

To better illustrate and understand how to address the event-triggered control for the uncertain nonlinear system (1) by feedback domination approach with dynamic disturbance compensation, the resulting designed system is shown in Fig. 1. In the proposed control method, the finite-time estimation of the steady states is firstly achieved by designing appropriate finite-time disturbance observers. With the help of the estimation of the steady states, a dynamic compensation system is obtained to design event-triggered controller.

3.1. Estimations of steady states

Based on Assumption 1 and the fact that the full state vector x is measured, the following observers are firstly utilized to estimate the disturbances and their derivatives in system (1) by following the approaches in Li et al. (2015) and Shtessel et al. (2014):

$$\dot{z}_0^i = v_0^i + x_{i+1} + f_i(t,\bar{x}_i),\ \ \dot{z}_k^i = v_k^i,$$
$$v_0^i = z_1^i - \lambda_0^i L_i^{\alpha_0^i} \lceil z_0^i - x_i \rfloor^{1-\alpha_0^i},$$
$$v_j^i = z_{j+1}^i - \lambda_j^i L_i^{\alpha_j^i} \lceil z_j^i - v_{j-1}^i \rfloor^{1-\alpha_j^i},$$
$$v_{n+1-i}^i = -\lambda_{n+1-i}^i L_i^{\alpha_{n+1-i}^i} \lceil z_{n+1-i}^i - v_{n-i}^i \rfloor^{1-\alpha_{n+1-i}^i}, \tag{4}$$

where $\alpha_l^i = \frac{1}{n+2-i-l}$, $x_{n+1} = u$, parameters $\lambda_l^i > 0$ and $L_i > 0$ are observer gains, and z_0^i and z_k^i are the estimates of state x_i and disturbance $d_i^{(k-1)}$, respectively, for $i \in \mathbb{N}_{1:n}$, $k \in \mathbb{N}_{1:n+1-i}$, $j \in \mathbb{N}_{1:n-i}$, and $l \in \mathbb{N}_{0:n+1-i}$. Then, it follows from (1) and (4) that

$$\dot{e}_0^i = e_1^i - \lambda_0^i L_i^{\alpha_0^i} \lceil e_0^i \rfloor^{1-\alpha_0^i},$$
$$\dot{e}_j^i = e_{j+1}^i - \lambda_j^i L_i^{\alpha_j^i} \lceil e_j^i - \dot{e}_{j-1}^i \rfloor^{1-\alpha_j^i},$$
$$\dot{e}_{n+1-i}^i \in [-G_i^+, G_i^+] - \lambda_{n+1-i}^i L_i^{\alpha_{n+1-i}^i} \lceil e_{n+1-i}^i - \dot{e}_{n-i}^i \rfloor^{1-\alpha_{n+1-i}^i}, \tag{5}$$

where $e_0^i = z_0^i - x_i$ and $e_k^i = z_k^i - d_i^{(k-1)}$ are the estimation errors for $k \in \mathbb{N}_{1:n+1-i}$.

As stated in Isidori (1995), the steady state response is calculated according to the information of tracking signals and external disturbances, which has nothing to do with disturbance observation and cancelation. Based on Isidori and Byrnes (1990), which addresses the existence of steady state response, we represent the steady state response of system (1) by iterative calculations as follows:

$$\pi_1(t) = \phi_1(\bar{y}_r^1(t)) =: y_r(t),$$
$$\pi_i(t) = \phi_i(t,\bar{d}_i(t),\bar{y}_r^i(t)) =: \frac{\partial \pi_{i-1}(t)}{\partial t} - f_{i-1}(t,\bar{\pi}_{i-1}(t)) - d_{i-1}(t), \tag{6}$$

where $\bar{d}_i = \{\bar{d}_{i-1,0}, \bar{d}_{i-2,1}, \ldots, \bar{d}_{1,i-2}\}$ with $\bar{d}_{p,q} = \{d_1^{(q)}, d_2^{(q)}, \ldots, d_p^{(q)}\}$, and $\bar{y}_r^b = \{y_r, \dot{y}_r, \ldots, y_r^{(b-1)}\}$, $i \in \mathbb{N}_{2:n+1}$, $b \in \mathbb{N}_{1:n+1}$. Define $\bar{z}_i = \{\bar{z}_{i-1,0}, \bar{z}_{i-2,1}, \ldots, \bar{z}_{1,i-2}\}$ with $\bar{z}_{p,q} = \{z_1^{q+1}, z_2^{q+1}, \ldots, z_p^{q+1}\}$. Then, the following set of steady-state estimations of system (1) is obtained:

$$\hat{\pi}_1(t) = \phi_1(\bar{y}_r^1(t)),$$
$$\hat{\pi}_i(t) = \phi_i(t,\bar{z}_i(t),\bar{y}_r^i(t)). \tag{7}$$

Suppose that $\phi_i \in C^1$ with respect to $(t,\bar{z}_i,\bar{y}_r^i)$ for $t \ge 0$, $i \in \mathbb{N}_{1:n+1}$. The finite-time estimation of disturbances and the steady states is guaranteed by the following lemma.

Lemma 3 (*Shtessel et al., 2014*). *If the observer gain L_i is selected such that $L_i \ge G_i^+$, then the following results hold: 1) all the signals e_j^i and z_k^i, $j \in \mathbb{N}_{0:n+1-i}$, $k \in \mathbb{N}_{1:n+1-i}$, are uniformly globally bounded; 2) the observer error dynamics (5) converges to zero in finite time, that is, there exists a time instant T_f such that $e_j^i(t) = 0$ for $t \ge T_f$; 3) $\hat{\pi}_i(t) = \pi_i(t)$ for $i \in \mathbb{N}_{1:n+1}$, $t \ge T_f$.*

Note that the detailed selection of observer parameters λ_l^i and L_i can be found in Shtessel et al. (2014, Remark 6.1) (see p. 229).

3.2. Controller design

Firstly, we introduce the following coordinate transformation based on adding a power integrator method, similar to Yang et al. (2018):

$$\eta_i = x_i - \phi_i(t,\bar{z}_i,\bar{y}_r^i),$$
$$\xi_i = \lceil \eta_i \rfloor^{\frac{1}{r_i}} - \lceil \eta_i^* \rfloor^{\frac{1}{r_i}},$$
$$\eta_{i+1}^* = -K_i^{r_{i+1}}(\bar{\eta}_i) \lceil \xi_i \rfloor^{r_{i+1}}, \tag{8}$$

where $i \in \mathbb{N}_{1:n}$, $K_i(\bar{\eta}_i) > 1$ is a smooth function, $\eta_1^*(t) = 0$ for all $t \ge 0$, ξ_i indicates the deviation of $\lceil \eta_i \rfloor^{\frac{1}{r_i}}$ from its desired value

$\lceil \eta_i^* \rfloor^{\frac{1}{r_i}}$. It then follows from (4), (7) and (8) that

$$\dot{\eta}_i = \eta_{i+1} + f_i(t, \bar{x}_i) - f_i(t, \bar{\hat{\pi}}_i) + e_1^i, \ i \in \mathbb{N}_{1:n-1},$$
$$\dot{\eta}_n = u - \hat{\pi}_{n+1} + f_n(t, \bar{x}_n) - f_n(t, \bar{\hat{\pi}}_n) + e_1^n. \tag{9}$$

Moreover, by (8), Assumption 1 and Lemma 3, we can obtain the boundedness of $\hat{\pi}_i$, which then yields

$$|f_i(t, \bar{x}_i) - f_i(t, \bar{\hat{\pi}}_i)| \le \bar{h}_i(\bar{\eta}_i) \sum_{j=1}^{i} |\eta_j|^{\frac{r_i+\tau}{r_j}}, \tag{10}$$

where $\bar{h}_i(\bar{\eta}_i)$ is a positive continuous function.

We now present the following lemma which is crucial for the controller.

Lemma 4. *For $\tau \in (-\frac{1}{n}, 0]$, the following results hold:*
(i) $0 < r_{i+1} \le r_i \le 1$, $i \in \mathbb{N}_{1:n}$.
(ii) $\xi_1, \ldots, \xi_n$ are differentiable in t.

Proof. See Appendix A. □

Define a sign function based Lyapunov function $V_n : \mathbb{R}^n \mapsto \mathbb{R}$ as

$$V_n(\bar{\eta}_n) = \sum_{i=1}^{n} W_i(\bar{\eta}_i) = \sum_{i=1}^{n} \int_{\eta_i^*}^{\eta_i} \left\lceil \lceil s \rfloor^{\frac{1}{r_i}} - \lceil \eta_i^* \rfloor^{\frac{1}{r_i}} \right\rfloor^{2-r_{i+1}} ds. \tag{11}$$

$W_i = \int_{\eta_i^*}^{\eta_i} \lceil \lceil s \rfloor^{\frac{1}{r_i}} - \lceil \eta_i^* \rfloor^{\frac{1}{r_i}} \rfloor^{2-r_{i+1}} ds$ is nonnegative and differentiable, as summarized by Lemma 5.

Lemma 5 (*Sun et al., 2016*)**.** *For $i = 1, \ldots, n$, $W_i(\cdot)$ is differentiable and satisfies*

$$\begin{cases} \frac{\partial W_i}{\partial \eta_i} = \lceil \xi_i \rfloor^{2-r_i-\tau}, \\ \frac{\partial W_i}{\partial \eta_k} = -(2 - r_i - \tau) \frac{\partial}{\partial \eta_k} \left(\lceil \eta_i^* \rfloor^{\frac{1}{r_i}} \right) \\ \quad \times \int_{\eta_i^*}^{\eta_i} |\lceil s \rfloor^{\frac{1}{r_i}} - \lceil \eta_i^* \rfloor^{\frac{1}{r_i}}|^{1-r_i-\tau} ds, \ k \in \mathbb{N}_{1:i-1}. \end{cases}$$

Moreover, $c_{i1}|\eta_i - \eta_i^*|^{\frac{2-\tau}{r_i}} \le W_i \le c_{i2}|\xi_i|^{2-\tau}$, $c_{i1} = \frac{r_i}{2-\tau} 2^{\frac{(2-r_{i+1})(r_i-1)}{r_i}}$ and $c_{i2} = 2^{1-r_i}$ are positive constants.

Then, we can establish the following key proposition.

Proposition 1. *Suppose that Assumption 2 is satisfied. For the nonlinear system (9) with the initial condition $\eta(0) \in \mathbb{R}^n$ and the estimation errors $e_1^i \in \mathbb{R}$, $i = 1, \ldots, n$, smooth functions $K_i(\bar{\eta}_i)$'s as given in (8) can be determined to achieve, for any $t \ge 0$,*

$$\dot{V}_n \le -\sum_{k=1}^{n} \xi_k^2 + \lceil \xi_n \rfloor^{2-r_n-\tau} (u - \hat{\pi}_{n+1} - \eta_{n+1}^*) + \sum_{k=1}^{n} (e_1^k)^{\frac{2}{r_{k+1}}}. \tag{12}$$

Proof. See Appendix B, in which the design process of $K_i(\bar{\eta}_i)$'s is illustrated clearly. □

3.3. Event-triggered control design

Based on the aforementioned design, we propose the following mix fix and relative thresholds control strategy:

$$u(t) = \omega(t_k), \ \forall t \in [t_k, t_{k+1}), \tag{13}$$

$$t_{k+1} = \inf\{t > t_k | \ |\omega(t) - u(t_k)| \ge \delta |u(t_k)| + m\}, \tag{14}$$

where ω is the continuous control signal and it is designed as

$$\omega = -(1+\delta) \left(\hat{\pi}_{n+1} \tanh\left(\frac{\lceil \xi_n \rfloor^{2-r_{n+1}} \hat{\pi}_{n+1}}{\epsilon} \right) + \bar{\eta}_{n+1}^* \times \tanh\left(\frac{\lceil \xi_n \rfloor^{2-r_{n+1}} \bar{\eta}_{n+1}^*}{\epsilon} \right) + \bar{m} \tanh\left(\frac{\lceil \xi_n \rfloor^{2-r_{n+1}} \bar{m}}{\epsilon} \right) \right) \tag{15}$$

with

$$\bar{\eta}_{n+1}^* = \begin{cases} -K_n(\bar{\eta}_n) \lceil \xi_n \rfloor^{r_{n+1}}, & |\xi_n| > 1, \\ -K_n(\bar{\eta}_n), & |\xi_n| \le 1, \end{cases} \tag{16}$$

ϵ, m, $\bar{m} > \frac{m}{1-\delta}$, $0 \le \delta < 1$ are all positive design parameters, and t_k, $k \in Z^+$, is the control updating time with $t_1 = 0$. Define the sampling-induced error $e = \omega - u$ for $t \ge 0$.

Now, we analyze the designed event-triggered controller and establish the stability of the closed-loop system and its tracking performance, which is constructive in deriving the virtual control law in (15). In addition, we also show that the proposed design meets the requirement of avoiding the Zeno behavior. These results are stated in the following theorem.

Theorem 1. *For the closed-loop system consisting of nonlinear system (9) with the initial condition $\eta(0) \in \mathbb{R}^n$ satisfying Assumptions 1 and 2, the event-triggered controller (13)–(15) renders the following results:*
(i) all the signals are uniformly globally bounded;
(ii) the inter-execution intervals $t_{k+1} - t_k \ge \tilde{t} \ge \frac{m}{M_1} > 0$, $\forall k \in Z^+$, where $\tilde{t}$ is a lower bound of inter-execution intervals and M_1 is a positive constant;
(iii) the tracking error $\eta_1 = y - y_r$ will be regulated to a compact set $\Omega = \{\eta_1 | |\eta_1| \le P\epsilon^{\frac{1}{2}}\}$, where P is a positive constant.

Proof. From (13) and (14), we obtain, for any $t \in [t_k, t_{k+1})$ and $k \in Z^+$,

$$-\delta u(t_k) - m < \omega(t) - u(t_k) < \delta u(t_k) + m \ \text{ if } \ u(t_k) \ge 0,$$
$$\delta u(t_k) - m < \omega(t) - u(t_k) < -\delta u(t_k) + m \ \text{ if } \ u(t_k) < 0, \tag{17}$$

which implies $\omega(t) = (1+\lambda_1(t)\delta)u(t) + \lambda_2(t)m$ for $t \ge 0$, and $\lambda_1(t)$ and $\lambda_2(t)$ are time-varying parameters satisfying $|\lambda_1(t)| < 1$ and $|\lambda_2(t)| < 1$. Then, there holds

$$u(t) = \frac{\omega(t)}{1+\lambda_1(t)\delta} - \frac{\lambda_2(t)m}{1+\lambda_1(t)\delta}, \quad t \ge 0. \tag{18}$$

According to Xing et al. (2017a), the hyperbolic tangent function $\tanh(\cdot)$ satisfies $0 \le |\nu| - \nu \tanh\left(\frac{\nu}{\epsilon}\right) \le 0.2785\epsilon$, where $\nu \in \mathbb{R}$ and $\epsilon > 0$. Therefore, substituting (13)–(18) into (12), we have

$$\begin{aligned} \dot{V}_n \le & -\sum_{k=1}^{n} \xi_k^2 + \lceil \xi_n \rfloor^{2-r_n-\tau} \left(\frac{\omega - \lambda_2 m}{1+\lambda_1\delta} - (\hat{\pi}_{n+1} + \eta_{n+1}^*) \right) \\ & + \sum_{k=1}^{n} (e_1^k)^{\frac{2}{r_{k+1}}} \\ \le & -\sum_{k=1}^{n} \xi_k^2 + \sum_{k=1}^{n} (e_1^k)^2 + |\xi_n|^{2-r_{n+1}} (|\hat{\pi}_{n+1}| + |\eta_{n+1}^*| + |\bar{m}|) \\ & + \frac{\omega}{1+\lambda_1\delta} \lceil \xi_n \rfloor^{2-r_{n+1}} - |\lceil \xi_n \rfloor^{2-r_{n+1}} \bar{m}| + \left| \lceil \xi_n \rfloor^{2-r_{n+1}} \frac{\lambda_2 m}{1+\lambda_1\delta} \right| \\ \le & -\sum_{k=1}^{n} \xi_k^2 + \sum_{k=1}^{n} (e_1^k)^{\frac{2}{r_{k+1}}} + |\lceil \xi_n \rfloor^{2-r_{n+1}} \hat{\pi}_{n+1}| - \lceil \xi_n \rfloor^{2-r_{n+1}} \hat{\pi}_{n+1} \\ & \times \tanh\left(\frac{\lceil \xi_n \rfloor^{2-r_{n+1}} \hat{\pi}_{n+1}}{\epsilon} \right) + |\lceil \xi_n \rfloor^{2-r_{n+1}} \bar{\eta}_{n+1}^*| - \lceil \xi_n \rfloor^{2-r_{n+1}} \bar{\eta}_{n+1}^* \\ & \times \tanh\left(\frac{\lceil \xi_n \rfloor^{2-r_{n+1}} \bar{\eta}_{n+1}^*}{\epsilon} \right) + |\lceil \xi_n \rfloor^{2-r_{n+1}} \bar{m}| - \lceil \xi_n \rfloor^{2-r_{n+1}} \bar{m} \\ & \times \tanh\left(\frac{\lceil \xi_n \rfloor^{2-r_{n+1}} \bar{m}}{\epsilon} \right) - |\lceil \xi_n \rfloor^{2-r_{n+1}} \bar{m}| + \left| \lceil \xi_n \rfloor^{2-r_{n+1}} \frac{m}{1-\delta} \right| \\ \le & -\sum_{k=1}^{n} \xi_k^2 + \sum_{k=1}^{n} (e_1^k)^{\frac{2}{r_{k+1}}} + 0.8355\epsilon. \end{aligned} \tag{19}$$

(i) Due to the fact that $e_i^k \in \mathcal{L}_\infty$ for $k \in \mathbb{N}_{1:n}$ based on Lemma 3, we get a constant $M > 0$ such that $M = \sup_{t\geq 0}\left(\sum_{k=1}^n (e_1^k)^2\right)$.

Case 1. For $\tau = 0$, by the definition of W_i, there holds $W_i = \frac{1}{2}\xi_i^2$, which implies $\dot{V}_n \leq -2V_n + \Delta$, where $\Delta = M + 0.8355\epsilon$. Then,

$$V_n(t) \leq e^{-2t}V_n(0) + \frac{\Delta}{2}(1 - e^{-2t}). \tag{20}$$

It follows from (8) and (20) that $\xi_i \in \mathcal{L}_\infty, \eta_i \in \mathcal{L}_\infty, \quad i \in \mathbb{N}_{1:n}$. *Case* 2. For $\tau \in (-\frac{1}{n}, 0)$, with Lemma 5 in mind, there holds

$$\dot{V}_n \leq -\frac{1}{2}V_n^{\frac{2}{2-\tau}} + \Delta. \tag{21}$$

Case 2.1: $V_n(0) \geq (6\Delta)^{\frac{2-\tau}{2}}$. By the continuity of $V_n(t)$, we can get a finite time T_1 such that $V_n(T_1) = (6\Delta)^{\frac{2-\tau}{2}}$ and $\dot{V}_n \leq -\frac{1}{3}V_n^{\frac{2}{2-\tau}}$, $\forall t \in [0, T_1]$, where

$$T_1 = -\frac{3(2-\tau)\left(V_n^{-\frac{\tau}{2-\tau}}(0) - (6\Delta)^{-\frac{\tau}{2-\tau}}\right)}{\tau}. \tag{22}$$

Moreover, it is possible to see that the trajectories of ξ_i's enter the compact set Ω_ξ and remains in Ω_ξ for all $t \geq T_1$, where $\Omega_\xi = \{(\xi_1, \dots, \xi_n) | V_n(t) \leq (6\Delta)^{\frac{2-\tau}{2}}\}$. If this is not true, there must exist a time $T_2 > T_1$ such that $V_n(T_2) > (6\Delta)^{\frac{2-\tau}{2}}$. By the continuity of V_n, we can find a finite time T_1' satisfying $T_1 \leq T_1' < T_2$ such that $V_n(T_1') = (6\Delta)^{\frac{2-\tau}{2}}$, and $V_n(t) > (6\Delta)^{\frac{2-\tau}{2}}$ for $t \in (T_1', T_2]$. From (21), we know $\dot{V}_n(t) < -\frac{1}{3}V_n^{\frac{2}{2-\tau}}(t) < 0, \quad \forall t \in (T_1', T_2]$, which implies

$$\begin{aligned} V_n(T_2) = &\int_{T_1'}^{T_2} \dot{V}_n(t)\mathrm{d}t + V_n(T_1') = \int_{T_1'}^{T_2}\left(-\frac{1}{3}V_n^{\frac{2}{2-\tau}}(t)\right)\mathrm{d}t \\ &+ V_n(T_1') < V_n(T_1) = (6\Delta)^{\frac{2-\tau}{2}}. \end{aligned} \tag{23}$$

This is clearly a contradiction. Therefore, $(\xi_1, \dots, \xi_n) \in \Omega_\xi$.

Case 2.2: $V_n(0) < (6\Delta)^{\frac{2-\tau}{2}}$. By the similar technique to that in *Case* 2.1, the conclusion that $(\xi_1, \dots, \xi_n) \in \Omega_\xi$ for $t \geq 0$ follows.

To sum up, for $\tau \in (-\frac{1}{n}, 0]$, there holds $\xi_i \in \mathcal{L}_\infty, i \in \mathbb{N}_{1:n}$. Therefore, all the signals are bounded, and the closed-loop system will not escape to infinity during the transient process of the observer.

(ii) From Lemma 4, (15) and (16), we know that $\omega(t)$ is differentiable when $|\xi_n| < 1$. For the case that $|\xi_n| > 1$, the differentiability of $\omega(t)$ is guaranteed by the smoothness of $K_n(\cdot)$ and the differentiability of $\hat{\pi}_{n+1}$, $\lceil\cdot\rfloor^{r_{n+1}}$ and ξ_n. Therefore, $\dot{\omega}(t) = D^+\omega(t) = D^-\omega(t)$ if $|\xi_n| \neq 1$. Suppose that there exists $t^0 \in [t_k, t_{k+1})$ (or $t^1 \in [t_k, t_{k+1})$) such that $\xi_n(t^0) = 1$ (or $\xi_n(t^1) = -1$). Although $\omega(t)$ may be not differentiable at $t = t^0$ (or $t = t^1$), $D^+\omega(t^0)$ (or $D^+\omega(t^1)$) and $D^-\omega(t^0)$ (or $D^-\omega(t^1)$) exist and are bounded. Since $\phi_{n+1} \in C^1$, from the continuity and differentiability of's ξ_n, and the boundedness of ξ_i's and η_i's, $D^+\omega(t)$ and $D^-\omega(t)$ are bounded; that is, there exists a constant M_1 such that $|D^+\omega(t)| \leq M_1$ and $|D^-\omega(t)| \leq M_1$. From $u(t) = \omega(t_k)$ for $t \in [t_k, t_{k+1})$ and the definition that $e(t) = \omega(t) - u(t)$, it can be concluded that $|D^+e(t)| = |D^+\omega(t)| \leq M_1$ and $|D^-e(t)| = |D^-\omega(t)| \leq M_1$ for $t \in [t_k, t_{k+1})$. If $\lim_{t\to t_{k+1}} e(t) = \delta|u(t_k)| + m > 0$, by (13), (14) and the continuity of $\omega(t)$, there must exist an interval $[t_{k1}, t_{k+1}) \subset [t_k, t_{k+1})$ such that $e(t_{k1}) = 0$ and $e(t) \geq 0$ for $t \in [t_{k1}, t_{k+1})$. By Comparison Lemma in Khalil (2002), there holds

$$\delta|u(t_k)| + m \leq \int_{t_{k1}}^{t_{k+1}} M_1\mathrm{d}t \leq M_1(t_{k+1} - t_k), \tag{24}$$

which implies $t_{k+1} - t_k \geq \frac{\delta|u(t_k)|+m}{M_1} > 0$. If $\lim_{t\to t_{k+1}} e(t) = -\delta|u(t_k)| - m < 0$, by a proof similar to that in the previous case, we can still obtain that $t_{k+1} - t_k \geq \frac{\delta|u(t_k)|+m}{M_1} > 0$. In summary, the lower bound of inter-execution intervals $\widetilde{t}$ satisfies $\widetilde{t} \geq \frac{m}{M_1} > 0$, which implies the Zeno-behavior is avoided.

(iii) Consider the case that $\tau = 0$. From the fact that $\eta_i \in \mathcal{L}_\infty$, $e_1^i \in \mathcal{L}_\infty$, and $e_1^i = 0$ for $t \geq T_f$, $i \in \mathbb{N}_{1:n}$, we have $\dot{V}_n \leq -2V_n + 0.8355\epsilon$ for all $t \geq T_f$. Then, the tracking error η_1 satisfies

$$\frac{1}{2}\eta_1^2 = \frac{1}{2}\xi_1^2 \leq V_n \leq e^{-2t}V_n(0) + \frac{0.8355\epsilon}{2}(1 - e^{-2t}) \tag{25}$$

for $t \geq T_f$. So, for $t \geq T_f$, η_1^2 is bounded by a function and converges exponentially towards a compact set

$$\Omega_1 = \{\eta_1 \big| |\eta_1| \leq 0.9141\epsilon^{\frac{1}{2}}\}. \tag{26}$$

Now we consider that $\tau \in (-\frac{1}{n}, 0)$. From Lemma 5 and *Case* 2, it can be concluded that η_1 enters the compact set

$$\Omega_2 = \{\eta_1 \big| \, |\eta_1| \leq c_{11}^{-\frac{1}{2-\tau}}(5.013\epsilon)^{\frac{1}{2}}\} \tag{27}$$

and remains in Ω_2 for all $t \geq T_3$, where

$$T_3 = \max\left\{T_f, T_f - \frac{3(2-\tau)\left(V_n^{-\frac{\tau}{2-\tau}}(0) - (5.013\epsilon)^{-\frac{\tau}{2-\tau}}\right)}{\tau}\right\}. \tag{28}$$

In conclusion, the tracking error $\eta_1(t)$ will be regulated to the compact set $\Omega = \{\eta_1 \big| |\eta_1| \leq P\epsilon^{\frac{1}{2}}\}$, where $P = \max\{0.9141, 2.239c_{11}^{-\frac{1}{2-\tau}}\}$. □

Table 1
Correlations of control parameters with the ultimate bound and ratio of tracking error, and the triggering frequency of control action. (Symbols ⋆, + and − denote no, positive and negative correlations, respectively.).

Controller parameters		Ultimate bound	Regulation rate	Triggering frequency
Adjustable	ϵ	+	−	−
design	$\bar{m}$	⋆	+	+
parameters	$K_n(\eta)$	⋆	+	+
Observer	L_i	⋆	+	+
gains	λ_i	⋆	⋆	⋆

Remark 3. By the coordinate transformation (8), the event-triggered tracking problem of system (1) is converted into the event-triggered stabilization problem of tracking error system (9) by considering a difference between nonlinear system (1) and its steady-state estimation system (7) which is derived from disturbance observer (4) and steady state response (6). It then can be concluded from Lemma 3 and (1) that (i)–(iii) in Theorem (1) still hold for the closed-loop system consisting of uncertain system (1), disturbance observer (4), steady state response (6) and the event-triggered controller (13)–(15). □

Remark 4. It is worthy noting that the proposed method guarantees that the tracking error is regulated to an arbitrarily small set by only adjusting the value of ϵ as shown by (26) and (27). The effects of the controller parameters on the ultimate bound and regulation ratio of the tracking error, and the trigger frequency of control action are given in Table 1. A heuristic guideline for choosing controller parameters is summarized based on Table 1 as follows: (1) the greater the value of ϵ, the bigger the ultimate bound of tracking error, the slower the regulation rate of tracking error, and the lower the triggering frequency; (2) the greater the values of $\bar{m}$, $K_n(\eta)$ and L_i, the faster the regulation rate of tracking error, and the higher the triggering frequency. □

Remark 5. Note that the lower bound on $t_{k+1} - t_k$ depends on M_1. Based on (15), and the inequalities $|D^+\omega(t)| \leq M_1$ and

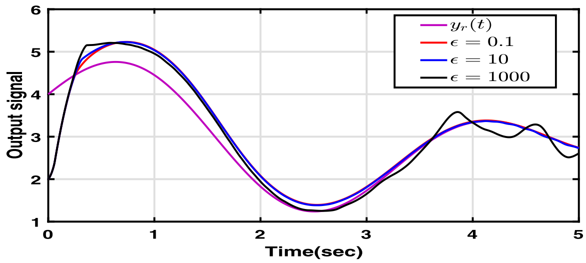

Fig. 2. Reference signal $y_r(t)$ and output signal $y(t)$.

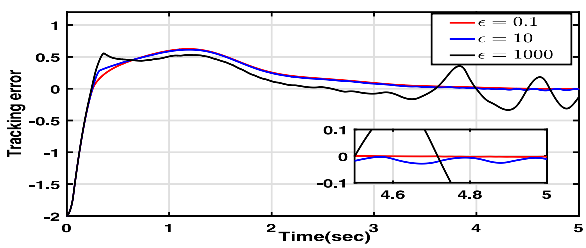

Fig. 3. Tracking error $\eta_1(t)$.

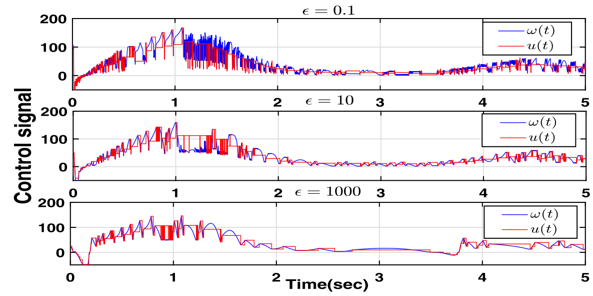

Fig. 4. Signals $\omega(t)$ and $u(t)$.

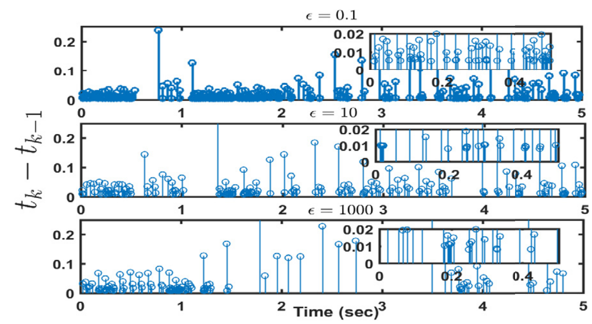

Fig. 5. Time interval of triggering event.

$|D^{-}\omega(t)| \leq M_1$, one can obtain that the selected parameter M_1 depends on the initial conditions as well as the transient bounds of system states. Therefore, we conclude that the lower bound on $t_{k+1} - t_k$ depends on the initial conditions. However, it is tricky to give an explicit relationship since it is difficult to evaluate the transient bounds of system states. Similar to Davó, Prieur, and Fiacchini (2017), only a qualitative relationship can be concluded that the larger the magnitude of initial state errors between the system states and their desired steady state responses, the larger the lower bound on $t_{k+1} - t_k$. □

4. Simulation example

We consider the following numerical example system

$$\dot{x}_1 = x_2 + \sin(t)x_1^{\frac{3}{5}} + 2 + d_1(t),$$
$$\dot{x}_2 = u + x_1^2 x_2 + d_2(t) \tag{29}$$

with the reference signal $y_r(t) = \sin(2t) + \cos(t) + 3$. Due to the need for ISS conditions where a well designed controller is needed to make the closed-loop system ISS with certain sampling-induced error and the limitation that only the stabilization problem is taken into account, the event-triggered design strategies in Abdelrahim et al. (2017) and Borgers and Heemels (2014) are still invalid for the tracking control problem of nonlinear system (29). Clearly, $f_1(t, x_1) = \sin(t)x_1^{\frac{3}{5}} + 2$ and $f_2(x_1, x_2) = x_1^2 x_2$ satisfy Assumption 2 with $\tau = -\frac{2}{5}$. Followed by the design procedure in Section 4, $\eta_3^* = (7 + 0.5(\eta_1^2+1)^{\frac{3}{2}} + (\eta_2^2+1)^{\frac{3}{2}})(\eta_2^{\frac{5}{3}} + 6.23\eta_1)^{\frac{1}{5}}$. In simulation, we choose the disturbances as $d_1(t) = 1 + \sin(\frac{\pi t}{4})$, $d_2(t) = \cos(\pi t)$, the observer parameters $\lambda_0^1 = 2$, $\lambda_1^1 = 1.5$, $\lambda_0^2 = 1.1$, $L_1 = L_2 = 7$ and the control parameters $\delta = 0.5$, $m = 4$, $\bar{m} = 20$, and set the initial conditions as $x_1(0) = 2$, $x_2(0) = z_0^1(0) = z_1^1(0) = z_2^1(0) = z_0^2(0) = z_1^2(0) = 0$. In order to demonstrate the effectiveness of design scheme, we then investigate the tracking performance of the closed-loop system under three cases ($\epsilon = 0.1$, 10, 1000).

Fig. 2 shows the tracking performance of the output signals $y(t)$, with tracking errors presented in Fig. 3. The initial value of system output is set to have a large deviation from its reference (see Fig. 2). This makes it easier to observe the regulation process of output tracking errors. Moreover, we zoom in the response curves of tracking errors in Fig. 3, which clearly demonstrates the difference of ultimate bounds of the tracking errors under different ϵ. In Fig. 4, the control signals $u(t)$ given in (13) together with signals $\omega(t)$ in (15) are illustrated. The triggered time intervals of each case are presented in Fig. 5. From the simulation results, it is observed that the tracking error can be sufficiently small by tuning the parameter ϵ at the expense of smaller inter-execution interval. Based on the simulation results, we obtain that the minimum inter-event time intervals under the three cases $\epsilon = 0.1$, 10, and 1000 are 0.00508, 0.00795, and 0.00809 s, respectively.

5. Conclusions

Event-triggered tracking control for nonlinear systems with time-varying external disturbances has been investigated in this paper. Based on disturbance observation techniques, finite-time estimation of the steady states is first proposed to reduce the complexity of traditional tracking control. Then an event-triggered controller is designed by virtual of the feedback domination approach, which can dynamically compensate for sampling-induced error as well as external disturbances. The feasibility of the presented control design has been proved by the developed Lyapunov stability analysis method and illustrated by simulation examples. We believe that the results in this paper could inspire further investigations on the event-triggered control for nonlinear systems, for example addressing issues related to unknown nonlinear functions and external disturbance.

Appendix A

(i) Because $r_{i+1} = r_i + \tau$, $\tau \in (-\frac{1}{n}, 0]$, and $r_1 = 1$, it is easy to get that $0 < r_{i+1} \leq r_i \leq 1$.

(ii) Based on the expression of η_i^* in (8), it follows that $\mathrm{sign}(\eta_i^*) = -\mathrm{sign}(\xi_i)$. Hence, (8) ensures

$$\lceil\eta_i^*\rfloor^{\frac{1}{r_i}} = -\sum_{l=1}^{i-1}\Big(\prod_{j=l}^{i-1}K_j(\bar\eta_j)\Big)\lceil\eta_l\rfloor^{\frac{1}{r_l}},\ i\in\mathbb{N}_{2:n+1}. \tag{30}$$

With the help of (i) and Lemma 2, $\lceil\eta_2^*\rfloor^{\frac{1}{r_2}},\ldots,\lceil\eta_{n+1}^*\rfloor^{\frac{1}{r_{n+1}}}$ are differentiable with respect to t, and its derivative can be calculated as follows by applying the chain rule

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\big(\lceil\eta_i^*\rfloor^{\frac{1}{r_i}}\big) &= \sum_{l=1}^{i-1}\frac{\partial}{\partial\eta_l}\big(\lceil\eta_i^*\rfloor^{\frac{1}{r_i}}\big)\frac{\mathrm{d}\eta_l}{\mathrm{d}t}\\ &= -\sum_{l=1}^{i-1}\Bigg(\sum_{k=1}^{i-1}\Big(\frac{\partial}{\partial\eta_l}\big(\prod_{j=k}^{i-1}K_j(\bar\eta_j)\big)\lceil\eta_k\rfloor^{\frac{1}{r_k}}\Big)+\frac{1}{r_l}\big(\prod_{j=l}^{i-1}K_j(\bar\eta_j)\big)|\eta_l|^{\frac{1}{r_l}-1}\Bigg)\dot\eta_l\\ &= -\sum_{l=1}^{i-1}\Big(\sum_{k=1}^{i-1}\bar g_{l,k}(\bar\eta_{i-1})\lceil\eta_k\rfloor^{\frac{1}{r_k}}+g_{l,2}(\bar\eta_{i-1})|\eta_l|^{\frac{1}{r_l}-1}\Big)\dot\eta_l,\end{aligned} \tag{31}$$

where $\bar g_{l,k}(\bar\eta_{i-1}) = \frac{\partial}{\partial\eta_l}\big(\prod_{j=k}^{i-1}K_j(\bar\eta_j)\big)$ and $g_{l,2}(\bar\eta_{i-1}) = \frac{1}{r_l}\big(\prod_{j=l}^{i-1}K_j(\bar\eta_j)\big)$ are smooth functions. From (8), (9), (31) and Lemma 3, we further deduce that $\xi_1(t),\ldots,\xi_n(t)$ are differentiable. Thus Lemma 4 is proved. □

Appendix B

For $V_1 = W_1$, there holds

$$\begin{aligned}\dot V_1 &= \lceil\xi_1\rfloor^{2-r_1-\tau}(\eta_2-\eta_2^*)+\lceil\xi_1\rfloor^{2-r_1-\tau}\eta_2^*+\lceil\xi_1\rfloor^{2-r_1-\tau}\\ &\quad\times(f_1(t,x_1)-f_1(t,\hat\pi_1))+\lceil\xi_1\rfloor^{2-r_1-\tau}e_1^1\\ &\le -n\xi_1^2+\lceil\xi_1\rfloor^{2-r_1-\tau}(\eta_2-\eta_2^*)+\lceil\xi_1\rfloor^{2-r_1-\tau}(\eta_2^*\\ &\quad+(n+\bar h_1(\eta_1)+\rho_1)\lceil\xi_1\rfloor^{r_2})+\frac{(e_1^1)^{\frac{2}{r_2}}}{n},\end{aligned}$$

where ρ_1 is a positive constant. Then, the choice of the smooth function $K_1(\eta_1)\ge n+\bar h_1(\eta_1)+\rho_1$ leads to

$$\dot V_1 \le -n\xi_1^2+\lceil\xi_1\rfloor^{2-r_1-\tau}(\eta_2-\eta_2^*)+\frac{(e_1^1)^{\frac{2}{r_2}}}{n}.$$

If the choices of $K_1,K_2,\ldots,K_{i-1}$, $i\in\mathbb{N}_{2:n-1}$, ensure that the differentiable function $V_{i-1}=\sum_{k=1}^{i-1}W_k$ satisfies

$$\begin{aligned}\dot V_{i-1} &\le -(n+2-i)\sum_{k=1}^{i-1}\xi_k^2+\lceil\xi_{i-1}\rfloor^{2-r_i}(\eta_i-\eta_i^*)\\ &\quad+\frac{1}{n+2-i}\sum_{k=1}^{i-1}(e_1^k)^{\frac{2}{r_{k+1}}}.\end{aligned}$$

Then, for $V_i = V_{i-1}+W_i$, we have

$$\begin{aligned}\dot V_i &\le -(n+2-i)\sum_{k=1}^{i-1}\xi_k^2+\lceil\xi_{i-1}\rfloor^{2-r_i}(\eta_i-\eta_i^*)\\ &\quad+\frac{1}{n+2-i}\sum_{k=1}^{i-1}(e_1^k)^{\frac{2}{r_{k+1}}}\\ &\quad+\lceil\xi_i\rfloor^{2-r_{i+1}}\big(\eta_{i+1}+f_i(t,\bar\eta_i)-f_i(t,\bar{\hat\pi}_i)+e_1^i\big)+\sum_{k=1}^{i-1}\frac{\partial W_i}{\partial\eta_k}\dot\eta_k.\end{aligned} \tag{32}$$

It can be deduced from Lemma 1 and (8) that

$$\lceil\xi_{i-1}\rfloor^{2-r_i}(\eta_i-\eta_i^*)\le\frac{1}{3}\xi_{i-1}^2+\rho_i\xi_i^2, \tag{33}$$

where ρ_i is a positive constant. According to (10),

$$\begin{aligned}&\big|\lceil\xi_i\rfloor^{2-r_{i+1}}(f_i(t,\bar\eta_i)-f_i(t,\bar{\hat\pi}_i)+e_1^i)\big|\\ &\le\frac{1}{3}\sum_{k=1}^{i-1}\xi_k^2+\frac{1}{n+1-i}(e_1^i)^{\frac{2}{r_{i+1}}}+\widetilde h_{i,1}(\bar\eta_i)\xi_i^2,\end{aligned} \tag{34}$$

where $\widetilde h_{i,1}(\bar\eta_i)$ is a smooth function. By the definition of W_i, we have

$$\left|(2-r_i-\tau)\int_{\eta_i^*}^{\eta_i}\left|\lceil s\rfloor^{\frac{1}{r_i}}-\lceil\eta_i^*\rfloor^{\frac{1}{r_i}}\right|^{1-r_i-\tau}\mathrm{d}s\right|\le 4|\xi_i|^{1-\tau}. \tag{35}$$

It follows from (31) that

$$\begin{aligned}&\left|\frac{\partial}{\partial\eta_k}\big(\lceil\eta_i^*\rfloor^{\frac{1}{r_i}}\big)\right| = \left|\sum_{l=1}^{i-1}\bar g_{k,l}(\bar\eta_{i-1})\lceil\eta_l\rfloor^{\frac{1}{r_l}}+g_{i,2}(\bar\eta_{i-1})|\eta_k|^{\frac{1}{r_k}-1}\right|\\ &\le\widetilde g_{i,k}(\bar\eta_{i-1})\Big(\sum_{l=1}^{i-1}|\eta_l|^{\frac{1}{r_l}}+|\eta_k|^{\frac{1}{r_k}-1}\Big)\le G_{i,k}(\bar\eta_{i-1})\sum_{l=1}^{i-1}|\xi_l|^{1-r_k},\end{aligned} \tag{36}$$

where $G_{i,k}(\bar\eta_{i-1})$ and $\widetilde g_{i,k}(\bar\eta_{i-1})$ are nonnegative smooth functions. By (9), it is not difficult to show that

$$\begin{aligned}|\dot\eta_k| &\le |\xi_{k+1}|^{r_{k+1}}+K_k^{r_{k+1}}(\bar\eta_k)|\xi_k|^{r_{k+1}}+\bar h_k(\bar\eta_k)\sum_{j=1}^{k}\Big(|\xi_j|^{r_{k+1}}\\ &\quad+K_{j-1}^{r_{k+1}}(\bar\eta_{j-1})|\xi_{j-1}|^{r_{k+1}}\Big)+|e_1^k|\le\psi_k(\bar\eta_k)\sum_{j=1}^{k+1}|\xi_j|^{r_{k+1}}+|e_1^k|,\end{aligned} \tag{37}$$

where $\psi_k(\bar\eta_k)>0$ is a smooth function, $k\in\mathbb{N}_{1:i-1}$. From (35)–(37) and Lemma 1, one can conclude

$$\begin{aligned}\left|\frac{\partial W_i}{\partial\eta_k}\dot\eta_k\right| &\le 4\Big(G_{i,k}(\bar\eta_{i-1})\sum_{l=1}^{i-1}|\xi_l|^{1-r_k}\Big)\\ &\quad\times\Big(\psi_k(\bar\eta_k)\sum_{j=1}^{k+1}|\xi_j|^{r_{k+1}}+|e_1^k|\Big)|\xi_i|^{1-\tau}\\ &\le\frac{1}{3(i-1)}\sum_{j=1}^{i-1}\xi_j^2+B_i(e_1^k)^{\frac{2}{r_{k+1}}}+\widetilde\psi_k(\bar\eta_{i-1})\xi_i^2,\end{aligned} \tag{38}$$

where $\widetilde\psi_k(\bar\eta_{i-1})>0$ is a smooth function, $B_i=\frac{1}{(n+2-i)(n+1-i)}$ is a constant, $k\in\mathbb{N}_{1:i-1}$. Therefore,

$$\left|\sum_{k=1}^{i-1}\frac{\partial W_i}{\partial\eta_k}\dot\eta_k\right|\le\frac{1}{3}\sum_{k=1}^{i-1}\xi_j^2+B_i\sum_{k=1}^{i-1}(e_1^k)^{\frac{2}{r_{k+1}}}+\widetilde h_{i,2}(\bar\eta_{i-1})\xi_i^2, \tag{39}$$

where $\widetilde h_{i,2}(\bar\eta_{i-1})>0$ is a smooth function. Substituting (33), (34) and (39) into (32) yields

$$\begin{aligned}\dot V_i &\le -(n+1-i)\sum_{k=1}^{i}\xi_k^2+\lceil\xi_i\rfloor^{2-r_{i+1}}(\eta_{i+1}-\eta_{i+1}^*)\\ &\quad+\lceil\xi_i\rfloor^{2-r_{i+1}}\big(\eta_{i+1}^*+\big((n+1\\ &\quad-i)+\widetilde h_{i,1}(\bar\eta_i)+\widetilde h_{i,2}(\bar\eta_{i-1})+\rho_i\big)\lceil\xi_i\rfloor^{r_{i+1}}\big)\\ &\quad+\frac{1}{n+1-i}\sum_{k=1}^{i}(e_1^k)^{\frac{2}{r_{k+1}}}.\end{aligned} \tag{40}$$

Then, the choice of the smooth function $K_i(\bar\eta_i)\ge(n+1-i)+\widetilde h_{i,1}(\bar\eta_i)+\widetilde h_{i,2}(\bar\eta_{i-1})+\rho_i$ leads to

$$\dot V_i\le-(n+1-i)\sum_{k=1}^{i}\xi_k^2+\lceil\xi_i\rfloor^{2-r_{i+1}}(\eta_{i+1}-\eta_{i+1}^*)$$

$$+\frac{1}{n+1-i}\sum_{k=1}^{i}(e_1^k)^{\frac{2}{r_{k+1}}}. \tag{41}$$

For $V_n = V_{n-1} + W_n$, there holds

$$\begin{aligned}\dot{V}_n \le\ & -2\sum_{k=1}^{n-1}\xi_k^2 + \lceil \xi_{n-1} \rfloor^{2-r_n}(\eta_n - \eta_n^*) \\ & + \frac{1}{2}\sum_{k=1}^{n-1}(e_1^k)^{\frac{2}{r_{n+1}}} + \lceil \xi_n \rfloor^{2-r_{n+1}} \\ & \times \left(u - \hat{\pi}_{n+1} + f_n(t, \bar{\eta}_n) - f_n(t, \bar{\hat{\pi}}_n) + e_1^n\right) + \sum_{k=1}^{n-1}\frac{\partial W_n}{\partial \eta_k}\dot{\eta}_k. \end{aligned} \tag{42}$$

Through the proof process similar to (33), (34) and (39), we obtain a smooth function $K_n > 0$ such that

$$\dot{V}_n \le -\sum_{k=1}^{n}\xi_k^2 + \lceil \xi_n \rfloor^{2-r_{n+1}}(u - \hat{\pi}_{n+1} - \eta_{n+1}^*) + \sum_{k=1}^{n}(e_1^k)^{\frac{2}{r_{k+1}}}.$$

Therefore, Proposition 1 is shown. □

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