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Sampled-Data Based Observer Design for Nonlinear Systems with Output Distributed Delay

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Abstract— Existing observers that accommodate distributed-delays are mainly designed for linear systems with continuous-time output measurements. In this paper, we seek sampled-measurements-based observers for nonlinear systems with strict-feedback globally-Lipschitz dynamics and distributed-delay. Invoking the high-gain principle, we design two observers operating with sampled output measurements. In both, the delay effect is compensated for by using distributed output-predictors. The first observer involves an intersample predictor and its exponential convergence is established using a quadratic Lyapunov function. The second observer involves a zero-order-hold (ZOH) predictor and the exponential convergence is proved using a Lyapunov-Krasovskii functional. To the author's knowledge, it is the first time that an exponentially convergent observer is developed for nonlinear systems with output distributed-delay and sampled output measurements.

Keywords. Observer design, sampled-data, time-delay, distributed delay.

I. INTRODUCTION

Time-delay or dead-time is a common aspect of system dynamics. It has become more crucial in the recent years due to the penetration of digital technology in control systems. Therefore, an intensive research activity has been devoted to the problem of control design for systems with delays, see e.g. (Karafyllis et al., 2016; Karafyllis and Krstic, 2013; Michiels and Niculescu, 2014; Fridman, 2014). In observer design, the delay issue is most crucial when the delay comes in the output of a nonlinear system. Then, the general approach consists in starting with the design of an exponentially convergent observer for the delay-free system and then modify the observer so that exponential convergence is preserved when the delay is present. The key extra components introduced in the observer to compensate for delay effect are output-predictors. This approach has been exemplified with various nonlinear systems and different observer design techniques, see e.g. (Germani et al., 2002; Cacace et al., 2010; Ahmed-Ali et al., 2012). The observer convergence analysis was generally performed using Lyapunov stability tools. It was shown that exponential stability of the corresponding estimation error systems is ensured provided that the delay is sufficiently small. To cope with large delays, a new class of observers, referred to chain observers, has been introduced in (Germani et al., 2002) and developed further (Kazantzis and Wright , 2005; Besançon et al., 2007; Cacace et al., 2014). The chain observers involve a set of predictors operating in cascade, each predictor compensate for a portion of the delay. The above results were essentially established using ODE based models in the observer design. A quite different approach has been developed in (Krstic, 2009) that consists in modeling the delay operator with a first-order hyperbolic PDE. A notable feature of this approach is that it leads to full-order observers proving estimates of both the state of the (finite-dimensional) system and the state of the (infinite-dimensional) delay operator. To cope with nonlinear systems, with possibly large delays, an extension of this approach has been developed in (Ahmed-Ali et al., 2018; 2020). The observers obtained using the extended approach are chain-structure and multi-predictor and their exponential convergence is ensured, for any size of the delay, if the number of predictors is sufficiently large.

As pointed out above, all previous works on observer design for nonlinear systems with output delays have been focused on discrete (or localized) delays. The problem of designing observers in the presence of distributed output delay has yet to be solved. In this respect, the results of (Bekiaris-Liberis and Krstic, 2011) constitute an exception as the observer design developed there applies to linear systems with distributed output delay (the latter is modelled by a first order hyperbolic PDE). The observer exponential convergence was established under an observability condition that underlies a limitation of the delay size. Another point is that the proposed observer was based on continuous-time output measurements.

In the present work, we present a new observer design for a class of nonlinear systems involving strict-feedback (globally) Lipschitz dynamics and output distributed-delay. Furthermore, the (delayed) output measurements are only available at sampling period. We make use of the high-gain observer design principle to cope with the system nonlinearity and compensate for the output delay and sampling by introducing output distributed-predictors. Depending on the predictor type, two observers are developed. The first observer involves intersample predictor while the second involve a ZOH predictor. The resulting state estimation error systems are analysed using a quadratic Lyapunov function, for the first observer, and a Lyapunov-Krasovskii functional for the second observer. It is shown that exponential stability is ensured for not too large delay and sampling interval. The maximal admissible delay and sampling interval depend on the system nonlinearity: the stronger the nonlinearity the smaller the maximal admissible delay. Compared to previous works, the present observer design features several novelties: (i) it applies to a class of nonlinear systems, unlike

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(Bekiaris-Liberis and Krstic, 2011); (ii) it applies to output distributed-delay and sampling, unlike (Germani et al., 2002; Cacace et al., 2010; Ahmed-Ali et al., 2012; Kahelras et al., 2018; Cuny et al., 2019); it only requires sampled-output-measurements; unlike (Germani et al., 2002; Cacace et al., 2010; Ahmed-Ali et al., 2012). A preliminary step of the present study was reported on in the 2019 CDC [20], where the problem of observer design, for output distributed-delay systems, was dealt in the case of continuous-output-measurements. Compared to the conference version, the present paper is quite different as it deals with the observer design problem in the presence of sampled-output-measurements.

The paper is organized as follows: the observer problem is formulated in Section II; the observer design and analysis are dealt with in Sections III, using an intersample-predictor-based observer, and in Section IV using a ZOH-predictor-based observer; simulation results are presented in Section V; a conclusion and a reference list end the paper.

Notation. Throughout the paper, t_k $(k = 0,1,2,3\cdots)$ are sampling time instants, such that $t_0 = 0$ and $h = \max_{0\le k<\infty}(t_{k+1} - t_k)$ is finite. Accordingly, the set $\{t_k\}$ constitutes a partition of the space $\mathbf{R}$ of real numbers and h represents the partition diameter.

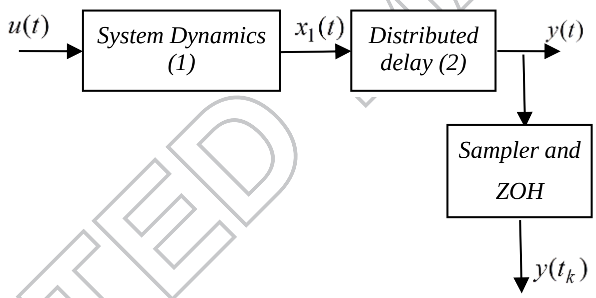

Figure 1. System with output delay and sampling

II. OBSERVER PROBLEM FORMULATION

The class of systems of interest, depicted by Fig. 1, is described by the following model:

$$\dot{x}(t) = Ax(t) + f(x(t),t) + u(t), \text{ for } t > 0 \tag{1}$$

$$y(t) = \int_{-d}^{0} cx(t+s)ds, \text{ for } t > 0 \tag{2}$$

for some $x(0) \in \mathbf{R}^n$, where $x(t) \in \mathbf{R}^n$ denotes the state vector signal, $y(t) \in \mathbf{R}$ the output signal, and $u(t) \in \mathbf{R}^m$ an external input. The input and output signals are accessible to measurements, but the state is not. The state dimension $n > 0$ and the delay (average) size $d > 0$ are known numbers. The function $f:\mathbf{R}^n\times\mathbf{R}\to\mathbf{R}^n$ is known, triangular form, class C^1 and its Jacobian with respect to x is bounded. Specifically, we have:

$$f(x,t)=\begin{bmatrix} f_1(x_1,t) \\ f_2(x_1,x_2,t) \\ \vdots \\ f_n(x) \end{bmatrix} \tag{3}$$

where the functions $f_i:\mathbf{R}^i\times\mathbf{R}\to\mathbf{R}$ are such that, there exists a real constant k_f so that, for all $x\in\mathbf{R}^n$ and all $t\in\mathbf{R}_+$, one has:

$$\| f_x(x,t)\|\le k_f \quad \text{with} \quad f_x(x,t)=\frac{\partial f}{\partial x}(x,t) \tag{4}$$

The remaining components of the model (1)-(2) are defined as follows:

$$A=\begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}\in\mathbf{R}^{n\times n} \tag{5}$$

$$c=[1\,0\ldots 0]\in\mathbf{R}^{1\times n} \tag{6}$$

The problem under study is to design an observer providing online estimates $\hat{x}(t)$ of the state vector $x(t)$ such that the estimation error $\hat{x}(t)-x(t)$ converges exponentially to the origin, whatever the initial condition $\hat{x}(0)$. The observer must only make use of the output measurements $y(t_k)$, $(k=0,1,2,3\ldots)$.

Remark 1. It readily follows from (6) that $v(t)=x_1(t)$ is the first component of $x(t)$. The case of discrete output delay is that when the available output measurement (at a time t) is given by $y(t)=cx_1(t-d)$. Observer design for these systems has been widely studied and the proposed solutions include both ODE-based observers (Germani et al., 2002; Cacace et al., 2010; Ahmed-Ali et al., 2012; Kazantzis and Wright , 2005; Besançon et al., 2007; Cacace et al., 2014) and PDE-based observers (Krstic, 2009; Ahmed-Ali et al., 2018). The distributed delay case underlies output averaging over the interval $[t-d,t]$. So far, quite a few works were devoted to observer design for systems with this output delay and the available results are on linear systems (Bekiaris-Liberis. and M. Krstic) ■

III. INTERSAMPLE-PREDICTOR-BASED OBSERVER DESIGN AND ANALYSIS

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In this section, we propose the following high-gain type observer based on an intersample output predictor:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + f(\hat{x}(t),t) + u(t) - \frac{\theta}{d}\Delta^{-1}L\left(\int_{-d}^{0}\hat{x}_1(t+s)ds - w(t)\right), \text{ for } t>0 \tag{7}$$

$$\dot{w}(t) = c\int_{-d}^{0}(A\hat{x}(t+s) + f(\hat{x}(t+s),t+s) + u(t+s))ds\,,$$

$$\text{for } t_k < t < t_{k+1} \text{ and } k = 0,1,2\cdots \tag{8}$$

$$w(t_k) = y(t_k) \tag{9}$$

with

$$\Delta = diag\left[1,\frac{1}{\theta},\ldots,\frac{1}{\theta^{n-1}}\right] \in \mathbf{R}^{n\times n} \text{ and any } \theta > 1 \tag{10}$$

The gain $L \in \mathbf{R}^n$ in (7) is any vector such that the matrix $A - Lc$ is Hurwitz. This algebraic problem has an infinite number of solutions because the pair (A,c) is observable. In (7), the initial conditions $\hat{x}_1(s)$ $(-d < s \le 0)$ are arbitrarily chosen.

Clearly, (7)-(10) define a high-gain observer featuring delay effect compensation by means of the intersample output predictor $w(t)$ $(t_k \le t < t_{k+1})$. The high-gain nature is resorted to cope with the Lipschitz function $f(x,t)$ involved in the system state equation (Deza et al., 1992). The output predictor defined be (8)-(9) is used to compensate for the delay and sampling effects. Note that, despite the discontinuity (9) in the predictor, the trajectory of the state vector estimate $\hat{x}(s)$ is continuously varying in time.

Now, the main result is stated in the following theorem, using the state estimation error defined by:

$$\tilde{x} = \hat{x} - x \tag{11}$$

Theorem 1. Let the intersample-predictor-based observer (7)-(9) be applied to the system (1)-(2). There exists $1 < \theta^* < \infty$ such that for any $\theta > \theta^*$ there are $0 < d^* < \infty$ and $0 < h^* < \infty$ so that, if $0 < d < d^*$ and $0 < h < h^*$ then the observer (1)-(2) is exponentially convergent in the sense that the state estimation error $\tilde{x}(t)$ exponentially converges to the origin ■

Proof. Subtracting (1) from (7) one gets the following equation that describes the dynamics of the state estimation errors $\tilde{x}(t)$:

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + \tilde{f}(\hat{x}(t),x(t),t) - \frac{\theta}{d}\Delta^{-1}Lc\int_{-d}^{0}\tilde{x}(t+s)ds - \frac{\theta}{d}\Delta^{-1}Le(t)\,, \text{ for all } t>0 \tag{12}$$

with:

$$e(t) = w(t) - y(t) \tag{13}$$

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$$\tilde{f}(\hat{x}(t), x(t), t) = f(\hat{x}(t), t) - f(x(t), t) \tag{14}$$

where we have used (2) and (6). Adding the quantity $\tilde{x}_1(t) - \tilde{x}_1(t)$ within the integral symbol, we obtain rearranging terms:

$$\dot{\tilde{x}}(t) = (A - \theta\Delta^{-1}Lc)\tilde{x}(t) + \tilde{f}(\hat{x}(t), x(t), t) - \frac{\theta}{d}\Delta^{-1}Lc\int_{-d}^{0}(\tilde{x}(t+s) - \tilde{x}(t))ds - \frac{\theta}{d}\Delta^{-1}Le(t) \tag{15}$$

Also, one immediately gets from (1)-(2):

$$\dot{y}(t) = C\int_{-d}^{0}(Ax(t+s) + f(x(t+s), t+s) + u(t+s))ds, \text{ for all } t \geq d \tag{16}$$

Subtracting (16) from (8) gives, using (13):

$$\dot{e}(t) = C\int_{-d}^{0}(A\tilde{x}(t+s) + \tilde{f}(\hat{x}(t+s), \hat{x}(t+s), t+s))ds, \text{ for } t_k < t < t_{k+1} \text{ and } k \geq k_0 \tag{17}$$

$$e(t_k) = 0 \tag{18}$$

where k_0 is the smallest integer such that $t_{k_0} \geq d$ (so that (16) can be used). Introduce the state transformation:

$$z(t) = \Delta\tilde{x}(t) \tag{19}$$

Differentiating $z(t)$ yields using (15):

$$\dot{z}(t) = \theta(A - Lc)z(t) + \Delta\tilde{f}(\hat{x}(t), x(t), t) - \frac{\theta}{d}Lc\int_{-d}^{0}(\tilde{x}(t+s) - \tilde{x}(t))ds - \frac{\theta}{d}Le(t), \quad \text{for all } t > 0 \tag{20}$$

where the last equality is obtained using the identities $\Delta A = \theta A\Delta$ and $c\Delta = c$ which are easily checked using the definitions (5)-(6). The next of the proof is divided in three parts. In the first part, we will show that the mapping $e \to z$ is ISS. In the second part, we will show that the mapping $z \to e$ is also ISS. Then, invoking the small gain theorem, we will show that $z(t)$ is exponentially converging.

To analyze the system (20), consider the following Lyapunov function candidate:

$$V = z^T P z \tag{21}$$

with $P = P^T$ being the unique positive definite matrix satisfying:

$$P(A - Lc) + (A - Lc)^T P = -I \tag{22}$$

where I denotes the identity matrix. The algebraic Lyapunov equation (16) has a unique solution because the matrix $(A - Lc)$ is Hurwitz. Differentiation of V yields, using (20) to (21):

$$\begin{aligned}\dot{V} &= \dot{z}^T P z + z^T P \dot{z} \\ &= -\theta\|z(t)\|^2 + 2z^T P\Delta\tilde{f}(\hat{x}(t), x(t), t) - \frac{2\theta}{d}z^T PLc\int_{-d}^{0}(\tilde{x}(t+s) - \tilde{x}(t))ds - \frac{2\theta}{d}z^T PLe(t)\end{aligned}$$

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$$\leq -\frac{\theta}{\lambda_{\min}(P)}V + 2z^T P\Delta\tilde{f}(\hat{x}(t),x(t),t) - \frac{2\theta}{d}z^T PLc\int_{-d}^{0}(\tilde{x}(t+s)-\tilde{x}(t))ds - \frac{2\theta}{d}z^T PLe(t),$$

$$\text{for all } t>0 \qquad (23)$$

where $\lambda_{\min}(P)$ denotes the smallest eigenvalue of P. The last three terms on the right side of (23) will be handled in order. First, applying the mean-value theorem to (14), the the second term on the right of (23) implies:

$$2z^T P\Delta\tilde{f}(\hat{x}(t),x(t),t) = 2z^T P\Delta\left(\int_0^1 f_x(x(t)+s\tilde{x}(t))ds\right)\tilde{x}(t)$$

$$= 2z^T P\Delta\left(\int_0^1 f_x(x(t)+s\tilde{x}(t))ds\right)\Delta^{-1}z(t)$$

$$\leq 2k_f\lambda_{\max}(P)\|z(t)\|^2, \text{ for } t>0 \qquad (24)$$

with $\lambda_{\max}(P)$ the largest eigenvalue of P, where the last inequality is obtained using the inequality $\left\|\Delta\left(\int_0^1 f_x(x(t)+s\tilde{x}(t))ds\right)\Delta^{-1}\right\| \leq k_f$ which holds for any $\theta>1$, due to (4) and the triangular structure of the function f and the diagonal form of Δ.

Applying Young's and Jensen's inequalities, the second term on the right side of (23) yields:

$$-\frac{2\theta}{d}z^T(t)PLc\int_{-d}^{0}(\tilde{x}(t+s)-\tilde{x}(t))ds \leq z^T(t)Pz(t) + \frac{\theta^2}{d}\|P\|\|L\|^2\int_{t-d}^{t}(c\tilde{x}(s)-c\tilde{x}(t))^2 ds$$

$$\leq z^T(t)Pz(t) + \frac{4d\theta^2}{\pi^2}\|P\|\|Lc\|^2\int_{t-d}^{t}(c\dot{\tilde{x}}(s))^2 ds \qquad (25)$$

for all $t>0$, where the last inequality is obtained applying Wirtinger's inequality (Hardy, 1934). On the other hand, from (6) and (19) we have:

$$c\tilde{x} = \tilde{x}_1 = z_1 \qquad (26)$$

Using (3) (5), (10), (19) and (26), it follows from (12):

$$\dot{\tilde{x}}_1(t+s) = \theta z_2(t+s) + \tilde{f}_1(\hat{x}_1(t+s),x_1(t+s),t+s)$$

$$-\frac{\theta l_1}{d}\int_{-d}^{0}z_1(t+s+\tau)d\tau - \frac{\theta l_1}{d}e(t+s), \quad \text{for all } t\in\mathbf{R}_+ \text{ and } -d<s<0 \qquad (27)$$

where l_1 denotes the first component of L and $\tilde{f}_1(\hat{x}_1(s),x_1(s),s) \overset{def}{=} f_1(\hat{x}_1(s),s) - f_1(x_1(s),s)$, due to (3). Again, applying Young's inequality to (27) one gets, for all $t\in\mathbf{R}_+$ and all $-d<s<0$:

$$\left|\dot{\tilde{x}}_1(t+s)\right|^2 = 2\theta^2(z_2(t+s))^2 + 2k_f^2(z_1(t+s))^2 + 2\frac{\theta^2 l_1^2}{d^2}\left(\int_{-d}^{0}z_1(t+s+\tau)d\tau\right)^2 + \frac{2\theta^2 l_1^2}{d^2}e^2(t+s)$$

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$$\leq 2(\theta^2 + k_f^2)\|z(t+s)\|^2 + 2\frac{\theta^2 l_1^2}{d^2} d\int_{-d}^{0} z_1^2(t+s+\tau)d\tau + \frac{2\theta^2 l_1^2}{d^2} e^2(t+s)$$

$$\leq 2(\theta^2 + k_f^2)\|z(t+s)\|^2 + 2\theta^2 l_1^2 \max_{-2d<s<0} z_1^2(t+s) + \frac{2\theta^2 l_1^2}{d^2} e^2(t+s) \tag{28}$$

where the penultimate inequality is obtained applying Jenssen's inequality [16]. In view of (28) and (26), inequality (25) becomes:

$$-\frac{2\theta}{d} z^T(t)PLc\int_{-d}^{0} (\tilde{x}(t+s) - \tilde{x}(t))ds \leq z^T(t)Pz(t)$$

$$+\frac{8d^2\theta^2(\theta^2 + k_f^2)\|P\|\|Lc\|^2}{\pi^2} \max_{-d<s<0} \|z(t+s)\|^2$$

$$+\frac{8d^2\theta^4 l_1^2\|P\|\|Lc\|^2}{\pi^2} \max_{-2d<s<0} z_1^2(t+s) \qquad +\frac{8\theta^4 l_1^2\|P\|\|Lc\|^2}{\pi^2} \max_{-d<s<0} e^2(t+s),$$

$$\leq V(t) + \frac{8d^2\theta^2\|P\|\|Lc\|^2}{\pi^2\lambda_{\min}(P)} \left(\theta^2(1+l_1^2) + k_f^2\right) \max_{-d<s<0} V(t+s)$$

$$+\frac{8\theta^4 l_1^2\|P\|\|Lc\|^2}{\pi^2} \max_{-d<s<0} e^2(t+s), \text{ for } t \in \mathbf{R}_+ \tag{29}$$

Finally, the last term on the right side of (23) will also be bounded from above as follows, applying Young's inequality:

$$-\frac{2\theta}{d} z^T PLe(t) \leq \frac{\theta}{2\lambda_{\min}(P)} V(t) + \frac{2\theta\|P\|\|L\|^2 \lambda_{\min}(P)}{d^2} e^2(t), \text{ for } t \in \mathbf{R}_+ \tag{30}$$

Using (30), (29) and (24), it follows from (23) that, for all $t \in \mathbf{R}_+$:

$$\dot{V}(t) \leq -\left(\frac{\theta}{2\lambda_{\min}(P)} - 2k_f \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} - 1\right)V(t)$$

$$+\frac{8d^2\theta^2\|P\|\|Lc\|^2}{\pi^2\lambda_{\min}(P)} \left(\theta^2(1+l_1^2) + k_f^2\right) \max_{-d<s<0} V(t+s)$$

$$+\frac{8\theta^4 l_1^2\|P\|\|Lc\|^2}{\pi^2} \max_{-d<s<0} e^2(t+s) + \frac{2\theta\|P\|\|L\|^2 \lambda_{\min}(P)}{d^2} e^2(t) \tag{31}$$

The last couple of terms on the right side of (31) is now focused on. From (17)-(18), it readily follows using (14), (5)-(6) and (3):

$$e(t) = \int_{t_k}^{t}\int_{-d}^{0} (\tilde{x}_2(\tau+s) + \tilde{f}_1(x_1(\tau+s), \hat{x}_1(\tau+s), \tau+s))dsd\tau,$$

$$\text{for } t_k < t < t_{k+1} \text{ and } k \geq k_0 \tag{32}$$

Squaring (32) yields, using (4), (19), and applying Young's inequality and the mean-value theorem:

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$$e^2(t) = 2dh\int_{t_k}^{t}\int_{-d}^{0} \tilde{x}_2^2(\tau+s)dsd\tau + 2dh\int_{t_k}^{t}\int_{-d}^{0} \tilde{f}_1^2(x_1(\tau+s),\hat{x}_1(\tau+s),\tau+s))dsd\tau$$

$$\leq 2dh\theta^2\int_{t_k}^{t}\int_{-d}^{0} z_2^2(\tau+s)dsd\tau + 2dhk_f^2\int_{t_k}^{t}\int_{-d}^{0} z_1^2(\tau+s)dsd\tau,$$

$$\leq 2d^2h^2\left(\theta^2+k_f^2\right)\max_{-d-h<s<0}\|z(t+s)\|^2,$$

$$\leq \frac{2d^2h^2(\theta^2+k_f^2)}{\lambda_{\min}(P)}\max_{-d-h<s<0} V(t+s), \quad \text{for } t \geq t_{k_0} \tag{33}$$

Combining (31) and (33) gives:

$$\dot{V}(t) \leq -\left(\frac{\theta}{2\lambda_{\min}(P)} - 2k_f\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} - 1\right)V(t)$$

$$+\frac{8d^2\theta^2\|P\|\|Lc\|^2}{\pi^2\lambda_{\min}(P)}\left(\theta^2(1+l_1^2)+k_f^2\right)\max_{-d<s<0} V(t+s)$$

$$+\frac{16d^2h^2\theta^4(\theta^2+k_f^2)l_1^2\|P\|\|Lc\|^2}{\pi^2\lambda_{\min}(P)}\max_{-2d-h<s<0} V(t+s)$$

$$+\frac{4d^2h^2\theta(\theta^2+k_f^2)\|P\|\|L\|^2\lambda_{\min}(P)}{d^2\lambda_{\min}(P)}\max_{-d-h<s<0} V(t+s)$$

$$\leq -2\delta(\theta)V(t) + 2\gamma(d,h,\theta)\max_{-2d-h<s<0} V(t+s), \text{ for all } t \geq \max(2d+h, t_{k_0}), \tag{34}$$

with

$$\delta(\theta) = \frac{\theta - 4k_f\lambda_{\max}(P) - 2\lambda_{\min}(P)}{4\lambda_{\min}(P)} \tag{35}$$

$$\gamma(d,h,\theta) = \frac{4d^2\theta^2\|P\|\|Lc\|^2}{\pi^2\lambda_{\min}(P)}\left(\theta^2(1+l_1^2)+k_f^2\right) + \frac{8d^2h^2\theta^4(\theta^2+k_f^2)l_1^2\|P\|\|Lc\|^2}{\pi^2\lambda_{\min}(P)}$$

$$+\frac{2d^2h^2\theta(\theta^2+k_f^2)\|P\|\|L\|^2\lambda_{\min}(P)}{d^2\lambda_{\min}(P)} \tag{36}$$

Letting $\theta^* = \max\{1, 4k_f\lambda_{\max}(P) + 2\lambda_{\min}(P)\}$, it is readily checked that $\delta(\theta) > 0$ for $\theta > \theta^*$. Also, let d and h be sufficiently small so that $\gamma(d,h,\theta) < \delta(\theta)$. This is not an issue because $\gamma(d,h,\theta) \to 0$ as $(d,h) \to (0,0)$. Then, it follows applying Halanay's inequality (Halanay , 1966) to (23) that:

$$V(t) \leq e^{-2\alpha^* t}\sup_{-2d<s<0} V(s) \tag{37}$$

with α^* being the unique positive real number satisfying the equality $\delta(\theta) - 2\alpha = \gamma(d,h,\theta)e^{2\alpha d}$.Then, since $P > 0$, it follows from (37), using (21), that $z(t)$ is exponentially converging to the origin. The same results holds with $\tilde{x}(t) = \Delta^{-1}z(t)$. Owing to α^*, its existence is easily shown by Fig. 2. The largest delay, denoted d^*, for which one can finds a value of $\theta > \theta^*$ so that the observer is

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exponentially convergent is not easy to determine. A lower bound of it is obtained from the condition $\gamma(d,\theta) < \delta(\theta)$ letting there $\theta = \theta^* + \varepsilon$ for $0 < \varepsilon << 1$. This ends the proof of Theorem 1 ■

Remark 2. The condition $\gamma(d,\theta) < \delta(\theta)$ emphasized in the proof of Theorem 1 is a sufficient condition for the observer to be exponentially convergent. In view of (35)-(36), the condition entails a limitation on the admissible values of the system delay and sampling interval. The limitation does hold even in the simpler case of linear systems and applies both for discrete and distributed delays, see e.g. the stability results of linear discrete-delay systems in (Fridman, 2014) and distributed-delay systems in (Fridman, 2018; Seuret et al., 2015). Theorem 1 shows that, for nonlinear distributed-delay systems of the form (1)-(2), the maximal admissible delay d^* and sampling interval h decrease when the system nonlinearity gets stronger (here when the Lipschitz coefficient k_f gets larger). That is, the largest values of the maximal admissible delay and sampling interval are applicable in the linear case and approximate values of them can be found by letting $k_f = 0$ and $\theta = \theta^* + \varepsilon$ (with $0 < \varepsilon << 1$) in the condition $\gamma(d,\theta) < \delta(\theta)$.

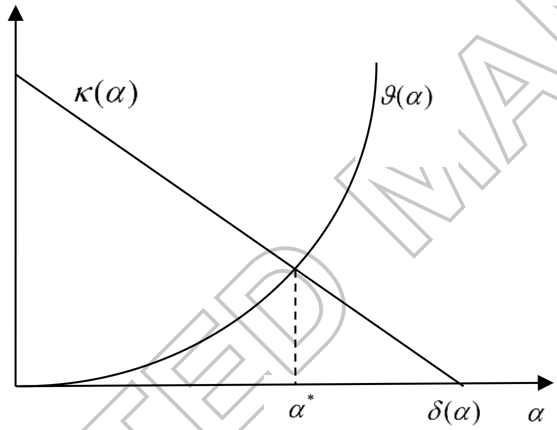

Figure 2. Plots of $\kappa(\alpha) = \delta(\theta) - 2\alpha$ and $\vartheta(\alpha) = \gamma(d,h,\theta)e^{2\alpha d}$

IV. ZOH-PREDICTOR-BASED OBSERVER DESIGN AND ANALYSIS

As in Section III, we seek an observer that continuously-in-time an accurate estimate $\hat{x}(t)$ of $x(t)$, based only on the available output measurements $y(t_k)$. To meet this objective we consider the following ZOH-predictor-based observer:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + f(\hat{x}(t),t) + u(t) - \frac{\theta}{d}\Delta^{-1}L\left(\int_{-d}^{0}\hat{x}_1(t_k+s)ds - y(t_k)\right), \tag{38}$$

for $t_k \le t < t_{k+1}$ and $k = 0,1,2\cdots$ All notations are similar to those of the intersample-predictor-based observer (7)-(9) and most comments made upon the latter still apply to (38).

The main result for the sampled data observer is stated in the next theorem.

Theorem 2. *Let the ZOH-predictor-based observer (38) be applied to the system (1)-(2). There exists* $1 < \theta^* < \infty$ *such that for any* $\theta > \theta^*$ *there are* $0 < d^* < \infty$ *and* $0 < h^* < \infty$ *so that, if* $0 < d < d^*$ *and* $0 < h < h^*$ *then the observer (1)-(2) is exponentially convergent* ■

Proof. Subtracting (1) from (38), yields for all $t_k \le t < t_{k+1}$ and $k = 0,1,2\cdots$:

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + \left(\int_0^1 f_x(x(t)+s\tilde{x}(t))ds\right)\tilde{x}(t) - \frac{\theta}{d}\Delta^{-1}Lc\int_{-d}^{0}\tilde{x}(t_k+s)ds, \tag{39}$$

Again, we introduce the variable change (13) and differentiate $z(t) = \Delta\tilde{x}(t)$. Doing so, it follows from (39) using the equalities $\Delta A = \theta A\Delta$ and $c\Delta = c$:

$$\dot{z}(t) = \theta Az(t) + \Delta\left(\int_0^1 f_x(x(t)+s\tilde{x}(t))ds\right)\Delta^{-1}z(t) - \frac{\theta}{d}Lc\int_{-d}^{0}z(t_k+s)ds,$$

$$= \theta Az(t) + \Delta\left(\int_0^1 f_x(x(t)+s\tilde{x}(t))ds\right)\Delta^{-1}z(t) - \frac{\theta}{d}Lc\int_{-d}^{0}z(t+s)ds$$

$$-\frac{\theta}{d}Lc\int_{-d}^{0}\int_{t_k}^{t}\dot{z}(r+s)drds, \text{ for } t_k \le t < t_{k+1} \text{ and } k = 0,1,2\cdots \tag{40}$$

Note that in the case of no sampling the last term in (40) disappears. Accordingly, we rewrite (40) in the following convenient form:

$$\dot{z}(t) = \eta_{nos}(t) + \eta_{sam}(t), \text{ for } t_k \le t < t_{k+1} \text{ and } k = 0,1,2\cdots \tag{41}$$

where $\eta_{nos}(t)$ denotes the part that remains in the case of no sampling while $\eta_{sam}(t)$ denotes the part due to sampling. Specifically, one has for $t_k \le t < t_{k+1}$ and $k = 0,1,2\cdots$:

$$\eta_{nos}(t) = \theta Az(t) + \Delta\left(\int_0^1 f_x(x(t)+s\tilde{x}(t))ds\right)\Delta^{-1}z(t) - \frac{\theta}{d}Lc\int_{-d}^{0}z(t+s)ds, \text{ for } t \ge 0 \tag{42}$$

$$\eta_{sam}(t) = -\frac{\theta}{d}Lc\int_{-d}^{0}\int_{t_k}^{t}\dot{z}(r+s)drds \tag{43}$$

That is, in the case of no-sampling, $\eta_{sam}(t)$ is removed and the system (41) boils down to system (14) in [20]. To analyze (41), we consider the following Lyapunov function candidate:

$$W = W_{nos} + W_{sam} \tag{44}$$

where $W_{nos} = z^T Pz$, with P is as in (16), and

$$W_{sam} = \int_{-d}^{0}\int_{t_k}^{t}(r-t+h)\dot{z}^2(r+s)drds, \tag{45}$$

for $t_k \le t < t_{k+1}$ and $k = 0,1,2\cdots$ It is readily checked that:

$$0 \le W_{sam} \le h\int_{-d}^{0}\int_{t_k}^{t}\dot{z}^2(r+s)drds, \text{ for } t_k \le t < t_{k+1} \tag{46}$$

Differentiating (44) yields, using (41):

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$$\dot{W} = \dot{z}^T Pz + z^T P\dot{z} + h\int_{-d}^{0} \|\dot{z}(t+s)\|^2 ds - \int_{-d}^{0}\int_{t_k}^{t} \|\dot{z}(r+s)\|^2 drds$$

$$= \eta_{nos}^T Pz + z^T P\eta_{nos} + \eta_{sam}^T Pz + z^T P\eta_{sam} + h\int_{-d}^{0} \|\dot{z}(t+s)\|^2 ds - \int_{-d}^{0}\int_{t_k}^{t} \|\dot{z}(r+s)\|^2 drds \quad (47)$$

It has already noted that the expression of η_{nos}, given by (33), is algebraically identical to the right side of (14) in (Ammeh et al., 2019). Also, W_{nos} is algebraically identically to the Lyapunov function (15) in [20]. Then, we make use of inequality (23) in (Ammeh et al., 2019) and obtain:

$$\eta_{nos}^T Pz + z^T P\eta_{nos} \leq -2\delta(\theta)W_{nos} + \gamma(d,\theta) \max_{t-2d<s<t} W_{nos}(s), \text{ for all } t \geq 0 \quad (48)$$

where $\delta(\theta)$ and $\gamma(d,\theta)$ are respectively defined by the expressions (24) and (25) in (Ammeh et al., 2019). For convenience, there are rewritten:

$$\delta(\theta) = \frac{1}{2}\left(\frac{\theta - 2k_f \lambda_{\max}(P)}{\lambda_{\min}(P)} - 1\right) \quad (49)$$

$$\gamma(d,\theta) = \frac{8d^2\theta^2}{\pi^2}\left(\theta^2(1+l_1^2) + k_f^2\right)\frac{\|P\|\|L\|^2}{\lambda_{\min}(P)} \quad (50)$$

On the other hand, we have applying Young's inequality:

$$\eta_{sam}^T Pz + z^T P\eta_{sam} = 2z^T P\eta_{sam}$$

$$\leq \zeta_1 z^T Pz + \frac{1}{\zeta_1}\eta_{sam}^T P\eta_{sam}$$

$$\leq \zeta_1 V_{nos} + \frac{h\theta^2}{d\zeta_1}\lambda_{\max}(P)\|LC\|^2\left(\int_{-d}^{0}\int_{t_k}^{t} \|\dot{z}(r+s)\|^2 drds\right) \quad (51)$$

for any $\zeta_1 > 0$, where we have used (43) and applied Jenssen's inequality twice to obtain the second term on the right side of (49).

Using (48) and (41), it follows from (37):

$$\dot{W} \leq -(2\delta(\theta) - \zeta_1)W_{nos} + \gamma(d,\theta) \max_{t-2d<s<t} W_{nos}(s)$$

$$-\left(1 - \frac{h\theta^2}{d\zeta_1}\lambda_{\max}(P)\|LC\|^2\right)\int_{-d}^{0}\int_{t_k}^{t} \dot{z}^2(r+s)drds,$$

$$+h\int_{-d}^{0} \dot{z}^2(t+s)ds, \text{ for } t_k \leq t < t_{k+1} \text{ and } k = 0,1,2\cdots \quad (52)$$

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The last term on the right side of (53) needs a separate treatment. First, writing the equality above (40) for $t+s$ with $t_k \le t < t_{k+1}$ and $-d \le s \le 0$, one gets using successively Young's and Jenssen's inequalities:

$$\|\dot{z}(t+s)\|^2 \le 2\theta\|A\|^2\|z(t+s)\|^2 + 2k_f^2\|z(t+s)\|^2 - \frac{2\theta^2}{d}\|Lc\|^2\int_{-d}^{0}\|z(t_k+\tau)\|^2 d\tau,$$

$$\text{for } t_k \le t+s < t_{k+1} \text{ and } -d \le s \le 0 \tag{53}$$

Integrating both sides of (53) gives:

$$\int_{-d}^{0}\|\dot{z}(t+s)\|^2 ds \le 2(\theta^2\|A\|^2 + k_f^2)\int_{-d}^{0}\|z(t+s)\|^2 ds + 2\theta^2\|Lc\|^2\int_{-d}^{0}\|z(t_k+\tau)\|^2 d\tau$$

$$\le \frac{2d(\theta^2\|A\|^2 + k_f^2)}{\lambda_{\min}(P)}\max_{-d<s<0} W_{nos}(t+s) + \frac{2d\theta^2\|Lc\|^2}{\lambda_{\min}(P)}\max_{-d-h<s<0} W_{nos}(t+s),$$

$$\text{for } t_k \le t < t_{k+1} \text{ and } k = 0,1,2\cdots \tag{54}$$

Using (54), inequality (52) gives:

$$\dot{W} \le -\left(2\delta(\theta) - \zeta_1\right)W_{nos} + \bar{\gamma}(d,\theta)\max_{t-\bar{d}<s<t} W_{nos}(s)$$

$$-\left(1 - \frac{h\theta^2}{d\zeta_1}\lambda_{\max}(P)\|Lc\|^2\right)\int_{-d}^{0}\int_{t_k}^{t}\dot{z}^2(r+s)drds,$$

$$\text{for } t_k \le t < t_{k+1} \text{ and } k = 0,1,2\cdots \tag{55}$$

with $\bar{d} = d + h + \max(d,h)$. Let the (so far) free coefficient $\zeta_1 = \dfrac{\theta}{2\lambda_{\min}(P)}$ and define $\theta^* = \max\left(1,\, 4k_f\lambda_{\max}(P) + 2\lambda_{\min}(P) + 4\right)$. Then, using (49), it is readily checked that:

$$2\delta(\theta) - \zeta_1 - \frac{2}{\lambda_{\min}(P)} = \frac{\theta - 4k_f\lambda_{\max}(P) - 2\lambda_{\min}(P) - 4}{2\lambda_{\min}(P)} > 0, \text{ for } \theta > \theta^* \tag{56}$$

Also, we let the couple (d,h) be such that the following inequality holds:

$$\frac{h\theta^2}{d\zeta_1}\lambda_{\max}(P)\|Lc\|^2 = \frac{h\theta}{d}2\lambda_{\min}(P)\lambda_{\max}(P)\|Lc\|^2 < 1 \tag{57}$$

Introduce the notation:

$$2\sigma(d,h,\theta) = \min\left\{\frac{\theta - 4k_f\lambda_{\max}(P) - 2\lambda_{\min}(P) - 4}{2\lambda_{\min}(P)},\, \frac{1}{h}\left(1 - \frac{h\theta}{d}2\lambda_{\min}(P)\lambda_{\max}(P)\|LC\|^2\right)\right\} \tag{58}$$

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$$\bar{\gamma}(d,h,\theta) = \gamma(d,\theta) + \frac{2dh(\theta^2\|A\|^2 + k_f^2)}{\lambda_{\min}(P)} + \frac{2dh\theta^2\|Lc\|^2}{\lambda_{\min}(P)} \tag{59}$$

Then, inequality (55) implies, using (56)-(59) and (44):

$$\dot{W}(t) \leq -2\sigma(d,h,\theta)W(t) + \bar{\gamma}(d,h,\theta)\max_{-\bar{d}<s<0} W(t+s), \text{ for all } t \geq 0 \tag{60}$$

Again, applying Halanay's inequality to (60), it follows that:

$$W(t) \leq e^{-\alpha^* t} \sup_{-\bar{d}<s<0} W(s) \tag{61}$$

with α^* being the unique positive real number satisfying the equality

$$\sigma(d,h,\theta) - \alpha = \frac{\bar{\gamma}(d,h,\theta)e^{2\alpha\bar{d}}}{2} \tag{62}$$

provided that $\bar{\gamma}(d,h,\theta) < 2\sigma(d,h,\theta)$. This is not an issue because, in view of (50), it follows from (59) that $\bar{\gamma}(d,h,\theta) \to 0$ as $d \to 0$ and $h \to 0$, whatever $\theta > \theta^*$. On the other hand, (58) shows that $2\sigma(d,h,\theta) \to \frac{\theta - 4k_f\lambda_{\max}(P) - 2\lambda_{\min}(P) - 4}{2\lambda_{\min}(P)} > 0$, as $d \to 0$ and $h \to 0$. Owing to the existence of a unique solution α^* of (62), this has already been discussed in the proof of Theorem 1 (see the discussion about Fig. 2). From (61) we get that $z(t)$ is exponentially converging to the origin and the same results holds with $\tilde{x}(t) = \Delta^{-1}z(t)$. This ends the proof of Theorem 2 ■

Remark 3. The discussion in Remark 2 (about the consequence of the condition $\gamma(d,\theta) < 2\delta(\theta)$) applies mutatis-mutandis to the condition $\bar{\gamma}(d,h,\theta) < 2\sigma(d,h,\theta)$ emphasized in the proof of Theorem 2. Accordingly, the last condition entails a limitation on the admissible values of the system delay and the sampling interval. The largest admissible values of the pair (d,h) are obtained in the linear case (when $k_f = 0$) letting $\theta = \theta^* + \varepsilon$ (with $0 < \varepsilon << 1$) in the condition $\bar{\gamma}(d,h,\theta) < 2\sigma(d,h,\theta)$.

V. SIMULATION RESULTS

We illustrate the performances of the observers (7)-(8) and (38) through an application to a class of nonlinear systems of the form (1)-(2) with:

$$A(v,z) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad f(x) = \begin{bmatrix} \sin(\frac{Kx_1}{1+x_1^4}) \\ -x_1 - x_2 + \sin(\frac{Kx_1}{\sqrt{1+x_1^2}}) \end{bmatrix}, \quad c = [1\ 0].$$

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where $K > 0$ is a known parameter that will be given different values. The system delay d will also be given various values. Clearly, the parameter K acts as an amplifier of the nonlinear effect in the system and, as such, it is closely related to the Lipschitz coefficient k_f defined by (4). The larger K the larger k_f (and so the stronger the system nonlinearity). In all simulation trials, the system is excited with an input vector of the form $u(t) = [u_1(t)\ 0]$ with $u_1(t) = \sin(2\pi t)$ and the observer gains are given the values $\theta = 10$ and $L = [0.2\ 0.01]$ which results in a matrix $A - Lc$ with eigenvalues $(-0.1, -0.1)$.

Simulation results for $K = 30$*, delay size* $d = 9$*, fixed sampling interval* $t_{k+1} - t_k = 0.7(s)$.

The obtained system and observers' responses are plotted in Fig. 4. Clearly, the state estimates converge to their true values.

The value $0.7(s)$ of sampling interval is in fact the quasi largest than can be used with the considered delay size. Indeed, if we use a bit larger sampling interval, i.e. $0.8(s)$ (while keeping unchanged all the other system and observer parameters) the observer performances deteriorate as illustrated by Fig. 5 with the ZOH-predictor based observer (38). It turns out that for a given size delay, there is a maximal admissible value of the sampling interval which presently is nearly equal to $h = 0.7(s)$.

Simulation results for $K = 40$.

The simulation study also shows that the maximal admissible delay depends on the nonlinearity amplifier gain K confirming the results stated in Theorems 1 and 2. For instance, with $K = 40$ the largest admissible delay is nearly $d = 1.7$ and we then should use a (fixed) sampling interval not larger than $0.6(s)$. This is illustrated by Fig. 6 with the ZOH-predictor-based observer (38). If the delay or the sampling period are beyond those values, the exact asymptotic convergence of the observer is lost.

The above results are clearly described by Table 1 where different values of the system nonlinearity amplifier gain K are considered and, for each value, the maximal admissible values of the delay and the sampling interval, both found by simulation, are indicated for both observers. Clearly, the admissible delay and sampling interval decrease when K (and so when the system nonlinearity Lipschitz coefficient k_f) grows. This confirms the results of Theorems 1and 2 and the comments in Remarks 2 which stipulate that the strongest the system nonlinearity, the smaller the (maximal) admissible delay size d and sampling interval h. It is also noticed that the intersample-predictor-

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based observer performs better in the sense that it compensates for larger delays. The performance is much more significant for for K less than 40.

TABLE I. MAXIMAL ADMISSIBLE DELAY AND SAMPLING INTERVAL FOR VARIOUS VALUES OF K, FOR THE INTERSAMPLE AND ZOH OBSERVERS

	K	30	40	50	60	100
Intersample observer	d	13.1	1.71	1.68	1.51	0.91
	h	0.7	0.45	0.44	0.25	0.02
ZOH observer	d	9	1.7	1.6	1.5	0.9
	h	0.7	0.6	0.28	0.27	0.18

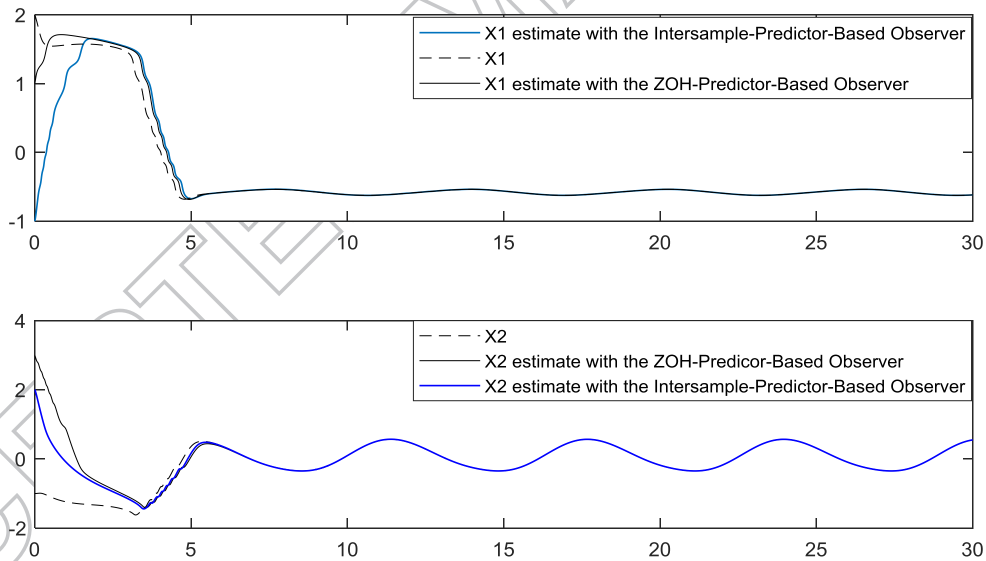

Figure 4. Observer performances in the case $K = 30$ and $d = 9$ and fixed sampling interval $0.7(s)$.

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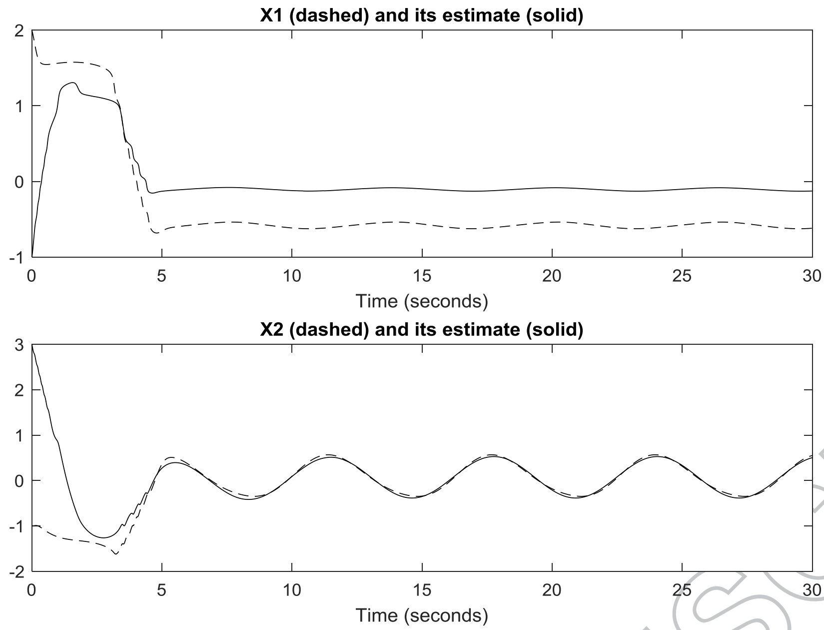

Figure 5. Observer performances of the ZOH-predictor-based observer (38) in the case $K = 30$ and $d = 9$ and (fixed) sampling interval $0.8(s)$. The state estimates provided by the observer (solid) are no longer convergent to their true values provided by the system (dashed).

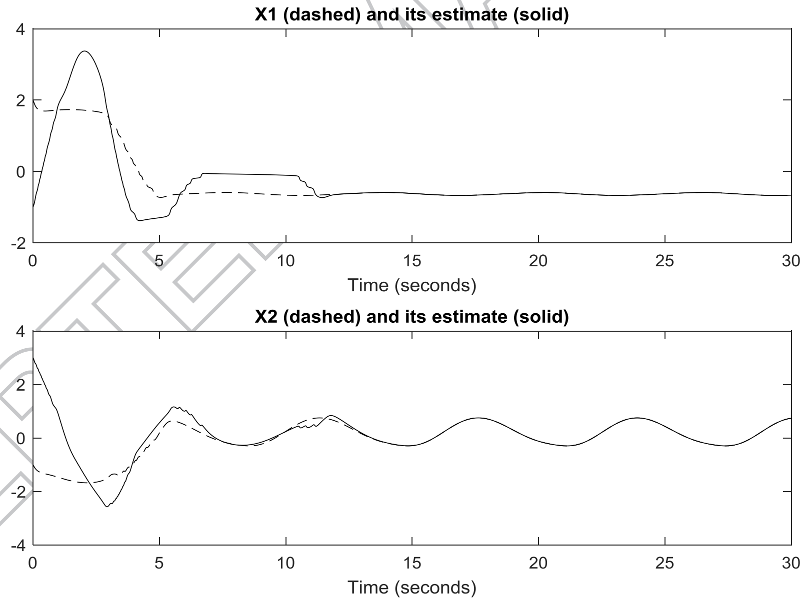

Figure 6. Performances of the ZOH-predictor-based observer (38) in the case $K = 40$ and $d = 1.7$ and (fixed) sampling interval $0.6(s)$. The state estimates provided by the observer (solid) do converge to their true values provided by the system (dashed).

VI. CONCLUDING REMARKS

We have addressed the problem of observer design for strict-feedback systems that involve Lipschitz nonlinear dynamics. The main novelty is this class is that the system output is subject to distributed

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delay. The observer designs we have developed in Sections III and IV rely on two major ideas: (i) we have made use of the high-gain principle to cope with the nonlinear nature of the system: (ii) the delay and sampling effects are compensated for using the intersample predictor (8)-(9) or the ZOH-predictor $\hat{y}(t)=\int_{t_k-d}^{t_k}\hat{x}_1(s)ds\ (t_k\le t<t_{k+1})$ (used in (38)). We have shown that the estimation error systems (12) and (39) are exponentially stable if the delay size d and the sampling interval h are not too large. The extension of the proposed observer design to systems with arbitrarily-large distributed-delay and those with both discrete and distributed delays is under study.

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