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Author(s):	Chaibi Noreddine, Belamfedel Alaoui Sadek, Tissir El Houssaine, Ben-salem Boukili
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Improved Results on Stability Criteria for Singular Systems with Interval Time-varying Delays

Chaibi Noreddine[1]; Belamfedel Alaoui Sadek[1]; Tissir El Houssaine[1]; Bensalem Boukili[1]
[1]*LESSI, Faculty of Sciences Dhar el Mahraz B.P. 1796 Fes-Atlas, Universite Sidi Mohamed Ben Abdellah, Morocco*
E-mail: chaibi.noreddine@gmail.com

ABSTRACT: The purpose of this paper is to address the problem of assessing the stability of singular time-varying delay systems. In order to highlight the relations between the delay and the state, the singular system is transformed into a neutral form. Then, a model transformation using a three-terms approximation of the delayed state is exploited. Based on the lifting method and the Lyapunov-Krasovskii functional method (LKF), new linear matrix inequality (LMI) is obtained, allowing conclusions on stability to be drawn using the Scaled Small Gain Theorem (SSG). The use of SSG theorem for stability of singular systems with time varying delay has not been investigated elsewhere in the literature. This represents the main novelty of this paper. The result is applicable for assessing stability of both singular systems and neutral systems with time-varying delays. The less conservativeness of the stability test is illustrated by comparison with recent literature results.

Keywords: singular system; neutral system; time-varying delays; delay-dependent stability.

1. INTRODUCTION

It is well known that the existence of time delay is one of many causes of instability and degradation of system performances. The time delay can be found in various practical systems such as transmission networks, networked control, chemical processes and long transmission lines in pneumatic systems; e.g. see [1-5]. The stability analysis problem of time delay systems is very important, e.g. see [3-9]. For an overview of recent advances on time delay stability, we refer the reader to the paper [10]. Besides, singular systems are also called descriptor systems, implicit systems, generalized state-space systems, differential-algebraic systems or semi-state systems [11]. This particular class of systems has been extensively studied in the past years due to the fact that singular systems preserve the structure of physical systems and it describes both dynamic and static behaviors. Therefore, the stability analysis and synthesis of singular systems are more complex than that of regular ones. Recently, these systems have received increasing research attention [12-23].

The existing stability results for singular delayed systems can be divided into two categories: delay independent and delay-dependent conditions. Delay independent stability conditions don't consider the delay, and they turn out to be conservative even for systems with small delays. On the delay-dependent side, several approaches have been explored, one of which is the delay partitioning approach among many others, e.g. see [24, 25]. This approach relies on decomposing the time delay in several parts, and thus, by constructing a Lyapunov Krasovkii Functional (LKF) that considers each part, it was proven that the stability conditions obtained are effective in reducing the conservatism by increasing the number of partitions. However, this results in a heavier computational burden while increasing the number of delay partitions. In the paper [26], a neutral system approach was carried out for singular time-invariant-delay system which has been extended to the case of singular time-variant-delay systems in [27]. Even if those results were advantageous over the existing ones in the literature. They were developed by using the Jensen based integral inequality [28]. Wirtinger-based integral inequality introduced in [29] has been shown to be more effective than the Jensen based integral inequality in providing an upper bound of the integral term appearing in the LKF derivative. That benefit is principally correlated with the choice of augmented Lyapunov Krasovkii Functional [30]. On the same line of research, much attention has been devoted to the input/output approach, see [4]. The main idea is to convert the original system into two interconnected subsystems. One of the keys to addressing time varying delay with this approach is to find an appropriate approximation of the delayed state so that the approximation error satisfies the SSG's first condition. The studies [31-32] have introduced the two-term approximation of the delayed state for continuous-time delayed system, that is the state $x(t-\tau(t))$ is expressed in terms of $x(t-\tau_1)$ and $x(t-\tau_2)$ where τ_1 and τ_2 are the lower and upper bounds of $\tau(t)$. The same approximation is used in [33] for the class of T-S fuzzy systems with time-varying delay. Recently, [34, 35, 20] has developed the three-terms approximation of the delayed state for regular dynamical systems, nonlinear quadratic systems and delta operator systems respectively. It is pointed out there that the approximation model of the delayed state with three terms is better than that with only one or two terms. We emphasise here that all the above references interested in input/output approach don't consider the singular systems. Thus, there is potential for improvement when projecting that approach to the class of singular systems. Based on the above observation, this paper is motivated by analyzing stability of singular systems via the three-terms approximation of the delayed state along with the make use of the Wirtinger based integral inequality.

In this paper, the problem of stability analysis for a class of singular systems with time varying delay is investigated. Firstly, the considered system is represented as a neutral type. This enables to address the stability problem for both singular and neutral systems with time varying delay. Thus, by using the three-terms approximation of the delayed state, the closed-loop system is transformed into two interconnected sub-systems. Then, through the lifting method it is proven that the first subsystem satisfies the SSG condition while the Lyapunov–krasovskii method is used to obtain - Linear Matrix Inequality (LMI) - condition under which the complementary SSG condition is satisfied. These two results with the SSG Theorem allow to conclude on the stability of the original closed-loop system. The contributions of this paper are stated as follows:

(1) The choice of the Lyapunov Krasovkii Functional is harmonic with the three-terms used to approximate the delayed state, that is the three-terms used to approximate the delayed state appear in the derivative of the LKF. Moreover, the chosen LKF contains terms that outcome from the use of the Wirtinger based integral inequality, which leads to a tighter estimation of the LKF derivative.

(2) The proposed method is also applicable in testing stability of both singular and neutral systems with interval time-varying delays.

(3) The results are more general than those previously established in the literature. This is because singular systems are more general than regular systems. Thus, by setting E=I, the established conditions can be used as a tool to evaluate the stability problem of regular systems.

(4) The less conservativeness aspect of the obtained stability test is demonstrated through a comparison with some recent results.

Notations: R^n and $R^{m\times n}$ represent the set of real n−vectors and m×n matrices, respectively. $H_1 \circ H_2$ represents the series connection of mapping H_1 and H_2. Notation $P > 0$ (≥ 0) means that matrix P is positive (semi) definite. $\|x(t)\|_P^2$ denotes the vector norm weighted by symmetric definite matrix P, that is , $x(t)^T P x(t)$. $\|H\|_\infty$ denotes the l_2 −induced norm of transfer function matrix or a general operator.

2. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the linear singular system with time-varying delay described by

$$\begin{cases} E\dot{x}(t) = Ax(t) + A_d x(t-\tau(t)), \quad t > 0 \\ x(t) = \varphi(t), t \in [-\tau_2, 0] \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector. $\tau(t)$ is the time-varying delays satisfying

$$\tau_1 \le \tau(t) \le \tau_2, \ \dot{\tau}(t) \le d \tag{2}$$

The matrix $E \in R^{n\times n}$ may be singular and $\text{rank}E = r \le n$ is assumed. A and A_d are known real constant matrices, $\varphi(t)$ is a compatible vector valued continuous initial function.

In this section, we will transform system (1) into a neutral form under certain constraint criterion. To proceed, the following definitions and lemmas are needed.

Definition 1 [11]: The pair(E, A) is said to be regular if $\det(sE - A)$ is not identically zero; The pair (E, A) is said to be impulse free if $\deg(\det(\lambda E - A)) = \text{rank}(E)$.

Lemma 1 [36]: If the pair (E, A) is regular and impulse free, then the solution to the singular time-delay system (1) exists and is impulse free and unique on $[0, \infty)$.

By Lemma1, the following definition is naturally introduced.

Definition 2 [36]:The singular time delay system (1) is said to be regular and impulse free if the pairs (E, A) is regular and impulse free.

Lemma 2 [11]: If the pair (E, A) is regular and impulse free, there exist two invertible matrices $F \in R^{n\times n}$ and $G \in R^{n\times n}$ such that

$$FEG = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}; \ FAG = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix}. \tag{3}$$

According to Lemma 2, if the pair (E, A) is regular and impulse free, invertible matrices $F, G \in R^{n\times n}$ can always be found such that

$$\bar{E} := FEG = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}; \ \bar{A} := FAG = \begin{bmatrix} A_1 & 0 \\ 0 & I_{n-r} \end{bmatrix}; \tag{4}$$

Let

$$\bar{A}_d := FA_dG = \begin{bmatrix} A_{d1} & A_{d2} \\ A_{d3} & A_{d4} \end{bmatrix}; \ \mu(t) = G^{-1}x(t) = \begin{bmatrix} \mu_1(t) \\ \mu_2(t) \end{bmatrix} \tag{5}$$

where the partitions are compatible with the structure of $\bar{E}$, the system (1) is thus equivalent to

$$\bar{E}\dot{\mu}(t) = \bar{A}\mu(t) + \bar{A}_d\mu(t - \tau(t)) \tag{6}$$

which is of the form

$$\dot{\mu}_1(t) = A_1\mu_1(t) + A_{d1}\mu_1(t - \tau(t)) + A_{d2}\mu_2(t - \tau(t)) \tag{7a}$$

$$0 = \mu_2(t) + A_{d3}\mu_1(t - \tau(t)) + A_{d4}\mu_2(t - \tau(t)) \tag{7b}$$

Then we rewrite the second equation, by differentiating (7b), as

$$\dot{\mu}_2(t) + (1 - \dot{\tau}(t))A_{d3}\dot{\mu}_1(t - \tau(t)) + (1 - \dot{\tau}(t))A_{d4}\dot{\mu}_2(t - \tau(t)) = 0. \tag{8}$$

Combing (7a)- (8)we have

$$\begin{bmatrix} \dot{\mu}_1(t) \\ \dot{\mu}_2(t) \end{bmatrix} = \begin{bmatrix} A_1\mu_1(t) + A_{d1}\mu_1(t - \tau(t)) + A_{d2}\mu_2(t - \tau(t)) \\ -\mu_2(t) - A_{d3}\mu_1(t - \tau(t)) - A_{d4}\mu_2(t - \tau(t)) \end{bmatrix} + (1 - \dot{\tau}(t)) \begin{bmatrix} 0 & 0 \\ -A_{d3} & -A_{d4} \end{bmatrix} \begin{bmatrix} \dot{\mu}_1(t - \tau(t) \\ \dot{\mu}_2(t - \tau(t) \end{bmatrix} \tag{9}$$

Let

$$\tilde{A} = \begin{bmatrix} A_1 & 0 \\ 0 & -I_{n-r} \end{bmatrix}; \ \tilde{A}_d = \begin{bmatrix} A_{d1} & A_{d2} \\ -A_{d3} & -A_{d4} \end{bmatrix}; \ C = \begin{bmatrix} 0 & 0 \\ -A_{d3} & -A_{d4} \end{bmatrix}; \ \tilde{C}(t) = (1 - \dot{\tau}(t))C \tag{10}$$

Then the system (9) is of the form of neutral type system,

$$\begin{cases} \dot{\mu}(t) - \tilde{C}(t)\dot{\mu}(t - \tau(t)) = \tilde{A}\mu(t) + \tilde{A}_d\mu(t - \tau(t)) \\ \mu(t) = \varphi(t); \qquad t \in [-\tau_2 \quad 0] \end{cases} \tag{11}$$

Note that systems (11) and (1) are not equivalent, but the asymptotic stability of (11) will guarantee the admissibility of (1), and vice versa [26]. With respected to the stability analysis of system (11), we assume that all the eigenvalues of matrix $\tilde{C}(t)$ are inside the unit circle, i.e., $\rho(\tilde{C}(t)) = \max\{|(1-d)\rho(C)|\} < 1$, where the symbol ρ denotes the spectral radius of the matrix. So the following assumption is made.

Lemma 3 [29]: For any matrix R>0 and a differentiable signal μ in $[\alpha,\ \beta] \to R^n$, the following inequality holds:

$$-\int_{\alpha}^{\beta} \dot{\mu}^T(s) R \dot{\mu}(s) ds \le \frac{1}{\alpha-\beta} \chi^T \hat{\theta} \chi$$

Where

$$\hat{\theta} = \begin{bmatrix} -4R & -2R & 6R \\ * & -4R & 6R \\ * & * & -12R \end{bmatrix}$$

$$\chi = \begin{bmatrix} \mu^T(\beta) & \mu^T(\alpha) & \frac{1}{\beta-\alpha}\int_{\alpha}^{\beta} \mu^T(s) ds \end{bmatrix}^T$$

In this section, the main objective is to develop a less conservative stability condition for time delay system (1) via the neutral system (11) using the scaled small gain theorem (SSG) proposed in [33]. To apply this theorem, we need to transform system (11) into the two following subsystems:

$$(S_1):\ z_\Delta(t) = H\omega_\Delta(t) \qquad (S_2):\ \omega_\Delta(t) = \Delta z_\Delta(t) \tag{12}$$

Where the forward S_1 is a known LTI system with operator H mapping $\omega_\Delta(t)$ to $z_\Delta(t)$, and feedback S_2 is an unknown linear time-varying one with operator $\Delta \in D \stackrel{\Delta}{=} \{\Delta : \|\Delta\|_\infty \le 1\}$ and $z_\Delta(t) \in \mathbb{R}^z,\ \omega_\Delta(t) \in \mathbb{R}^\omega$. The Direct result of the small-gain theorem given in [33] concerns the sufficient condition of the robust asymptotic stability of the interconnected system in (12).

Lemma 4 (SSG Theorem [33])**:** Consider (12), and assume that S_1 is internally stable. The closed loop system formed by S_1 and S_2 is robustly asymptotically stable for all $\Delta \in D$ if there exist matrices $\{T_\omega,\ T_z\} \in T$ with

$$T \triangleq \left\{\{T_\omega,\ T_z\} \in \mathbb{R}^{\omega\times\omega} \times \mathbb{R}^{z\times z} : T_\omega,\ T_z \text{ nonsingular},\ \left\|T_\omega \circ \Delta \circ T_z^{-1}\right\|_\infty \le 1\right\}$$

such that the following SSG condition holds:

$$\left\|T_z \circ H \circ T_\omega^{-1}\right\|_\infty < 1 \tag{13}$$

3. Main Results

This section start by introducing the model transformation method of neutral system (11), then we present our stability condition-based SSG Theorem.

3.1. Model transformation

Considering system (11), inspired by the work presented in [34], we propose an approximation of $\mu(t-\tau(t))$ using the lower, upper bounds $\tau_2,\ \tau_1$ and the average value $\tau_a = (\tau_1+\tau_2)/2$. Then, the delayed state is approximated by the following equation:

$$\mu(t-\tau(t)) = \frac{1}{3}\left[\mu(t-\tau_1) + \mu(t-\tau_2) + \mu(t-\tau_a)\right] + \frac{\tau_{12}}{3}\tilde{\omega}(t) \tag{14}$$

where $\tau_{12} = \tau_2 - \tau_1;\ \tau_a = \dfrac{\tau_2+\tau_1}{2};$

$\frac{1}{3}\left[\mu(t-\tau_1) + \mu(t-\tau_2) + \mu(t-\tau_a)\right]$ is the approximation of $\mu(t-\tau(t))$ and $\frac{\tau_{12}}{3}\tilde{w}(t)$ is the approximation error. From (14), system (11) can be written as a forward and a feedback interconnection system (S_1) and (S_2), respectively

$$(S_1): \begin{cases} \dot{\mu}(t) - \tilde{C}(t)\dot{\mu}(t-\tau(t)) = \tilde{A}\mu(t) + \frac{1}{3}\tilde{A}_d\mu(t-\tau_1) + \frac{1}{3}\tilde{A}_d\mu(t-\tau_2) + \frac{1}{3}\tilde{A}_d\mu(t-\tau_a) + \frac{\tau_{12}}{3}\tilde{A}_d\tilde{\omega}(t) \\ z_\Delta(t) = \dot{\mu}(t); \end{cases}$$

$$(S_2):\ \omega_\Delta(t) = \Delta z_\Delta(t)$$

The approximation error in (14) can be written as follows:

Case 1: $\tau_1 \le \tau(t) \le \tau_a$

$$\frac{\tau_{12}}{3}\tilde{w}(t) = \mu(t-\tau(t)) - \frac{1}{3}\mu(t-\tau_1) + \frac{1}{3}\mu(t-\tau_2) + \frac{1}{3}\mu(t-\tau_a)$$
$$= \frac{1}{3}\left[-\int_{t-\tau(t)}^{t-\tau_1} z_\Delta(s)ds + 2\int_{t-\tau_a}^{t-\tau(t)} z_\Delta(s)ds + \int_{t-\tau_2}^{t-\tau_a} z_\Delta(s)ds\right]$$

Case 2: $\tau_a \le \tau(t) \le \tau_2$

$$\frac{\tau_{12}}{3}\tilde{w}(t) = \mu(t-\tau(t)) - \frac{1}{3}\mu(t-\tau_1) + \frac{1}{3}\mu(t-\tau_2) + \frac{1}{3}\mu(t-\tau_a)$$
$$= \frac{1}{3}\left[-\int_{t-\tau_a}^{t-\tau_1} z_\Delta(s)ds - 2\int_{t-\tau(t)}^{t-\tau_a} z_\Delta(s)ds + \int_{t-\tau_2}^{t-\tau(t)} z_\Delta(s)ds\right]$$

Lemma 5: The operator Δ satisfies the SSG condition $\left\|X \circ \Delta \circ X^{-1}\right\|_\infty \le 1$, where X is a general invertible matrix.

Proof : If we can prove that $\|\Delta\|_\infty \le 1$, holds for two cases, $\tau_1 \le \tau(t) \le \tau_a$ and $\tau_a \le \tau(t) \le \tau_2$, then $\|\Delta\|_\infty \le 1$, is true.

Case 1: $\tau_1 \le \tau(t) \le \tau_a$

We have by Jensen (Cauchy–Schwartz) inequality for all $t \ge 0$

$$\frac{\tau_{12}{}^2}{9}\|\tilde{\omega}(t)\|^2 = \frac{1}{9}\left\|-\int_{t-\tau(t)}^{t-\tau_1} z_\Delta(s)ds + 2\int_{t-\tau_a}^{t-\tau(t)} z_\Delta(s)ds + \int_{t-\tau_2}^{t-\tau_a} z_\Delta(s)ds\right\|^2$$
$$\le \frac{3}{9}\left\{\left\|\int_{t-\tau(t)}^{t-\tau_1} z_\Delta(s)ds\right\|^2 + \left\|2\int_{t-\tau_a}^{t-\tau(t)} z_\Delta(s)ds\right\|^2 + \left\|\int_{t-\tau_2}^{t-\tau_a} z_\Delta(s)ds\right\|^2\right\}$$

We continue the proof for each term separately. The function $s = p(t) = t - \tau(t)$ is strongly increasing. Hence, inverse $t = p^{-1}(s) = q(s)$ is well-defined and satisfies $|q(s) - (s+\tau_1)| \le \tau_{12}/2$. Then, integrating $\left\|\int_{t-\tau(t)}^{t-\tau_1} z_\Delta(s)ds\right\|^2$ between 0 and ∞, changing the order of the integration and taking into account that $z_\Delta(s) = 0,\ s \le 0$, we find that

$$\int_0^\infty \left\|\int_{t-\tau(t)}^{t-\tau_1} z_\Delta(s)ds\right\|^2 dt \le \int_0^\infty (\tau(t)-\tau_1)\int_{t-\tau(t)}^{t-\tau_1} \|z_\Delta(s)\|^2 ds\, dt = \left|\int_0^\infty (\tau(q(s))-\tau_1)\int_{q(s)}^{s+\tau_1} \|z_\Delta(s)\|^2 dt\, ds\right|$$
$$= \left|\int_0^\infty (\tau(q(s))-\tau_1)(s+\tau_1-q(s)\|z_\Delta(s)\|^2 ds\right|$$
$$\le \frac{\tau_{12}}{2}\frac{\tau_{12}}{2}\|z_\Delta(s)\|_{l_2}^2$$

We follow the same process for the other terms, and we obtain

$$\int_0^\infty \left\|2\int_{t-\tau_a}^{t-\tau(t)} z_\Delta(s)ds\right\|^2 dt \le 4\frac{\tau_{12}{}^2}{4}\|z_\Delta(s)\|_{l_2}^2$$
$$\int_0^\infty \left\|\int_{t-\tau_2}^{t-\tau_a} z_\Delta(s)ds\right\|^2 dt \le \frac{\tau_{12}{}^2}{4}\|z_\Delta(s)\|_{l_2}^2$$

Then the addition of the three terms together gives

$$\frac{\tau_{12}{}^2}{9}\|\tilde{\omega}(t)\|_{l_2}^2 \le \frac{1}{3}\left(\frac{\tau_{12}{}^2}{4} + 4\frac{\tau_{12}{}^2}{4} + \frac{\tau_{12}{}^2}{4}\right)\|z_\Delta(t)\|_{l_2}^2$$
$$= \frac{\tau_{12}{}^2}{2}\|z_\Delta(t)\|_{l_2}^2$$

For $\tilde{\omega}(t) = \frac{3}{\sqrt{2}}\omega_\Delta(t)$ we obtain $\|\omega_\Delta(t)\|_{l2}{}^2 \le \|z_\Delta(t)\|_{l2}{}^2$. For case 2, using similar proof process, we obtain the same results as in case 1. This completes the proof. □

Remark 1: $\{X, X\} \in T$ are the matrices for the SSG theorem given in Lemma 4, to ensure that the system (11) is Input Output stable. It is necessary to verify that (S1) is internally stable and there exists X such that the SSG condition $\left\|X \circ H \circ X^{-1}\right\|_\infty < 1$ holds.

Remark 2: The prime object of this paper is to put forward a new stability criterion of the considered systems and reduce the conservatism of existing results by developing the idea of SSG to the singular systems with interval time-varying delays and derive some new delay-dependent results.

3.2. Stability analysis

The SSG theorem analysis can be performed in many transformations. The following theorem presents a new delay dependent condition of system (S_1)

Theorem 1:

Given τ_1, τ_2 and d, system (11) is asymptotically stable, if there exist positive definite matrices

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} \\ * & P_{22} & P_{23} & P_{24} & P_{25} \\ * & * & P_{33} & P_{34} & P_{35} \\ * & * & * & P_{44} & P_{45} \\ * & * & * & * & P_{55} \end{bmatrix}, \ Q_i (i = 1,\ 2,\ 3),\ R_j (j = 1,\ 2,\ 3),\ \begin{bmatrix} Q_{41} & Q_{42} \\ * & Q_{43} \end{bmatrix} \text{ and } S = X^T X \tag{15}$$

such that the following LMI is feasible

$$\Psi = \begin{bmatrix} \Psi_1 & \Psi_2 & \Psi_3 & \Psi_4 & \Psi_5 \\ * & -12R_1 & 0 & 0 & 0 \\ * & * & -12R_2 & 0 & 0 \\ * & * & * & -12R_3 & 0 \\ * & * & * & * & -\Psi_6 \end{bmatrix} < 0 \tag{16}$$

where

$$\Psi_1 = \begin{bmatrix} \Psi_{11} & (P_{11} + Q_{42})\Psi_{12} + \Psi_{13} \\ * & \Psi_{22} \end{bmatrix}$$

$$\Psi_{11} = P_{11}\tilde{A} + \tilde{A}^T P_{11} + P_{13} + P_{13}^T + P_{14} + P_{14}^T + P_{15} + P_{15}^T + Q_1 + Q_2 + Q_3 + Q_{41} + Q_{42}\tilde{A} + \tilde{A}^T Q_{42}^T - 4R_1 - 4R_2 - 4R_3$$

$$\Psi_{12} = \begin{bmatrix} \frac{1}{3}\tilde{A}_d & \frac{1}{3}\tilde{A}_d & \frac{1}{3}\tilde{A}_d & \tilde{C} & \frac{\tau_{12}}{3}\tilde{A}_d \end{bmatrix}$$

$$\Phi_1 = \frac{1}{3}(\tilde{A}^T P_{12} + P_{23}^T + P_{24}^T + P_{25}^T)$$

$$\Psi_{13} = \begin{bmatrix} \Phi_1 - 2R_1 - P_{13} & \Phi_1 - 2R_2 - P_{14} & \Phi_1 - 2R_3 - P_{15} & (1-d)P_{12} & \tau_{12}\Phi_1 \end{bmatrix}$$

$$\Phi_2 = \frac{1}{9}\tilde{A}_d^T P_{12} + \frac{1}{9}P_{12}^T \tilde{A}_d - \frac{(1-d)}{9}Q_{41}$$

$$\Phi_3 = \frac{1}{3}P_{12}^T \tilde{C} + \frac{(1-d)}{3}P_{22} - \frac{(1-d)}{3}Q_{42}$$

$$\Psi_{22} = \begin{bmatrix} \Phi_2 - \frac{1}{3}(P_{23} + P_{23}^T) - Q_1 - 4R_1 & \Phi_2 - \frac{1}{3}(P_{24} + P_{23}^T) & \Phi_2 - \frac{1}{3}(P_{25} + P_{23}^T) & \Phi_3 & \tau_{12}(\Phi_2 - \frac{1}{3}P_{23}^T) \\ * & \Phi_2 - \frac{1}{3}(P_{24} + P_{24}^T) - Q_2 - 4R_2 & \Phi_2 - \frac{1}{3}(P_{25} + P_{24}^T) & \Phi_3 & \tau_{12}(\Phi_2 - \frac{1}{3}P_{24}^T) \\ * & * & \Phi_2 - \frac{1}{3}(P_{25} + P_{25}^T) - Q_3 - 4R_3 & \Phi_3 & \tau_{12}(\Phi_2 - \frac{1}{3}P_{25}^T) \\ * & * & * & -(1-d)Q_{43} & \tau_{12}\Phi_3^T \\ * & * & * & * & \tau_{12}^2\Phi_2 - \frac{2}{9}S \end{bmatrix}$$

$$\Omega = \begin{bmatrix} \tilde{A} & \Psi_{12} \end{bmatrix}$$

$$\Psi_2 = \tau_1 \Omega^T P_{13} + \Phi_4^T$$

$$\Phi_4 = \begin{bmatrix} \tau_1(P_{33} + P_{34} + P_{35}) + 6R_1 & -\tau_1 P_{33} + 6R_1 & -\tau_1 P_{34} & -\tau_1 P_{35} & \tau_1(1-d)P_{23}^T & 0 \end{bmatrix}$$

$$\Psi_3 = \tau_a \Omega^T P_{14} + \Phi_5^T$$

$$\Phi_5=\begin{bmatrix}\tau_a(P_{34}{}^T+P_{34}+P_{35})+6R_2 & -\tau_a P_{34}{}^T & -\tau_a P_{44}+6R_2 & -\tau_a P_{45} & \tau_a(1-d)P_{24}{}^T & 0\end{bmatrix}$$

$$\Psi_4=\tau_2\Omega^T P_{15}+\Phi_6{}^T$$

$$\Phi_6=\begin{bmatrix}\tau_2(P_{35}{}^T+P_{45}{}^T+P_{55})+6R_3 & -\tau_2 P_{35}{}^T & -\tau_2 P_{45}{}^T & -\tau_2 P_{55}+6R_3 & \tau_2(1-d)P_{25}{}^T & 0\end{bmatrix}$$

$$\Psi_5=\begin{bmatrix}\tau_1\Omega^T R_1 & \tau_a\Omega^T R_2 & \tau_2\Omega^T R_3 & \Omega^T S & \Omega^T Q_{43}\end{bmatrix}$$

$$\Psi_6=\mathrm{diag}\{R_1 \quad R_2 \quad R_3 \quad S \quad Q_{43}\}$$

Proof of theorem 1:

Consider the following Lyapuno Krasovskii functional:

$$V(\mu(t))=V_1(\mu(t))+V_2(\mu(t))+V_3(\mu(t)) \tag{17}$$

where

$$V_1(\mu(t))=\eta^T(t)P\eta(t)$$

$$V_2(\mu(t))=\int_{t-\tau_1}^{t}\mu^T(\theta)Q_1\mu(\theta)d\theta+\int_{t-\tau_2}^{t}\mu^T(\theta)Q_2\mu(\theta)d\theta+\int_{t-\tau_a}^{t}\mu^T(\theta)Q_3\mu(\theta)d\theta+\int_{t-\tau(t)}^{t}\begin{bmatrix}\mu(\theta)\\ \dot{\mu}(\theta)\end{bmatrix}^T Q_4\begin{bmatrix}\mu(\theta)\\ \dot{\mu}(\theta)\end{bmatrix}d\theta$$

$$V_3(\eta(t))=\tau_1\int_{-\tau_1}^{0}\int_{s}^{t}\dot{\mu}^T(\theta)R_1\dot{\mu}(\theta)d\theta ds+\tau_a\int_{-\tau_a}^{0}\int_{s}^{t}\dot{\mu}^T(\theta)R_2\dot{\mu}(\theta)d\theta ds+\tau_2\int_{-\tau_2}^{0}\int_{s}^{t}\dot{\mu}^T(\theta)R_3\dot{\mu}(\theta)d\theta ds$$

with

$$\eta^T(t)=\begin{bmatrix}\mu^T(t) & \mu^T(t-\tau(t)) & \int_{t-\tau_1}^{t}\mu^T(s)ds & \int_{t-\tau_a}^{t}\mu^T(s)ds & \int_{t-\tau_2}^{t}\mu^T(s)ds\end{bmatrix}$$

Then, the time-derivative of $V(\mu(t))$ along the trajectory of system (S1) gives

$$\begin{aligned}\dot{V}_1(\mu(t))&=2\mu^T(t)\{P_{11}\dot{\mu}(t)+P_{12}(1-d)\dot{\mu}(t-\tau(t))-P_{13}\mu(t-\tau_1)-P_{14}\mu(t-\tau_a)-P_{15}\mu(t-\tau_2)+(P_{13}+P_{14}+P_{15})\mu(t)\}\\ &+2\mu^T(t-\tau(t))\{P_{12}{}^T\dot{\mu}(t)+P_{22}(1-d)\dot{\mu}(t-\tau(t))-P_{23}\mu(t-\tau_1)-P_{24}\mu(t-\tau_a)-P_{25}\mu(t-\tau_2)+(P_{23}+P_{24}+P_{25})\mu(t)\}\\ &+2\int_{t-\tau_1}^{t}\mu^T(s)ds\{P_{13}{}^T\dot{\mu}(t)+P_{23}{}^T(1-d)\dot{\mu}(t-\tau(t))-P_{33}\mu(t-\tau_1)-P_{34}\mu(t-\tau_a)-P_{35}\mu(t-\tau_2)+(P_{33}+P_{34}+P_{35})\mu(t)\}\\ &+2\int_{t-\tau_a}^{t}\mu^T(s)ds\{P_{14}{}^T\dot{\mu}(t)+P_{24}{}^T(1-d)\dot{\mu}(t-\tau(t))-P_{34}{}^T\mu(t-\tau_1)-P_{44}\mu(t-\tau_a)-P_{45}\mu(t-\tau_2)+(P_{34}{}^T+P_{44}+P_{45})\mu(t)\}\\ &+2\int_{t-\tau_2}^{t}\mu^T(s)ds\{P_{15}{}^T\dot{\mu}(t)+P_{25}{}^T(1-d)\dot{\mu}(t-\tau(t))-P_{35}{}^T\mu(t-\tau_1)-P_{45}{}^T\mu(t-\tau_a)-P_{55}\mu(t-\tau_2)+(P_{35}{}^T+P_{45}{}^T+P_{55})\mu(t)\}\end{aligned} \tag{18}$$

$$\begin{aligned}\dot{V}_2(\mu(t))&=\mu^T(t)\{Q_1+Q_2+Q_3\}\mu(t)-\mu^T(t-\tau_1)Q_1\mu(t-\tau_1)-\mu^T(t-\tau_a)Q_2\mu(t-\tau_a)-\mu^T(t-\tau_2)Q_3\mu(t-\tau_2)\\ &+\begin{bmatrix}\mu(t)\\ \dot{\mu}(t)\end{bmatrix}^T\begin{bmatrix}Q_{41} & Q_{42}\\ * & Q_{43}\end{bmatrix}\begin{bmatrix}\mu(t)\\ \dot{\mu}(t)\end{bmatrix}-(1-\dot{\tau}(t))\begin{bmatrix}\mu(t-\tau(t))\\ \dot{\mu}(t-\tau(t))\end{bmatrix}^T\begin{bmatrix}Q_{41} & Q_{42}\\ * & Q_{43}\end{bmatrix}\begin{bmatrix}\mu(t-\tau(t))\\ \dot{\mu}(t-\tau(t))\end{bmatrix}\end{aligned} \tag{19}$$

Applying Lemma 3 to the integral terms in $\dot{V}_3(\mu(t))$, we have

$$\begin{aligned}\dot{V}_3(\mu(t))&=\dot{\mu}^T(t)\{\tau_1{}^2R_1+\tau_a{}^2R_2+\tau_2{}^2R_3\}\dot{\mu}(t)-\int_{t-\tau_1}^{t}\dot{\mu}^T(s)\tau_1R_1\dot{\mu}(s)ds-\int_{t-\tau_a}^{t}\dot{\mu}^T(s)\tau_aR_2\dot{\mu}(s)ds-\int_{t-\tau_2}^{t}\dot{\mu}^T(s)\tau_2R_3\dot{\mu}(s)ds\\ &\le\dot{\mu}^T(t)\{\tau_1{}^2R_1+\tau_a{}^2R_2+\tau_2{}^2R_3\}\dot{\mu}(t)+\begin{bmatrix}\mu(t)\\ \mu(t-\tau_1)\\ \frac{1}{\tau_1}\int_{t-\tau_1}^{t}\mu^T(s)ds\end{bmatrix}^T\begin{bmatrix}-4R_1 & -2R_1 & 6R_1\\ * & -4R_1 & 6R_1\\ * & * & -12R_1\end{bmatrix}\begin{bmatrix}\mu(t)\\ \mu(t-\tau_1)\\ \frac{1}{\tau_1}\int_{t-\tau_1}^{t}\mu^T(s)ds\end{bmatrix}\\ &+\begin{bmatrix}\mu(t)\\ \mu(t-\tau_a)\\ \frac{1}{\tau_a}\int_{t-\tau_a}^{t}\mu^T(s)ds\end{bmatrix}^T\begin{bmatrix}-4R_2 & -2R_2 & 6R_2\\ * & -4R_2 & 6R_2\\ * & * & -12R_2\end{bmatrix}\begin{bmatrix}\mu(t)\\ \mu(t-\tau_a)\\ \frac{1}{\tau_a}\int_{t-\tau_a}^{t}\mu^T(s)ds\end{bmatrix}+\begin{bmatrix}\mu(t)\\ \mu(t-\tau_2)\\ \frac{1}{\tau_2}\int_{t-\tau_2}^{t}\mu^T(s)ds\end{bmatrix}^T\begin{bmatrix}-4R_3 & -2R_3 & 6R_3\\ * & -4R_3 & 6R_3\\ * & * & -12R_3\end{bmatrix}\begin{bmatrix}\mu(t)\\ \mu(t-\tau_2)\\ \frac{1}{\tau_2}\int_{t-\tau_2}^{t}\mu^T(s)ds\end{bmatrix}\end{aligned} \tag{20}$$

Let

$$\xi^T(t)=\left[\mu^T(t) \quad \mu^T(t-\tau_1) \quad \mu^T(t-\tau_a) \quad \mu^T(t-\tau_2) \quad \dot{\mu}^T(t-\tau(t)) \quad \tilde{\omega}(t) \quad \frac{1}{\tau_1}\int_{t-\tau_1}^{t}\mu^T(s)ds \quad \frac{1}{\tau_a}\int_{t-\tau_a}^{t}\mu^T(s)ds \quad \frac{1}{\tau_2}\int_{t-\tau 2}^{t}\mu^T(s)ds\right]$$

Let us consider the following performance index,

$$J_\Delta=\int_0^\infty\left[z_\Delta{}^T(s)Sz_\Delta(s)-\omega_\Delta{}^T(s)S\omega_\Delta(s)\right]ds$$

under the zero initial condition, it follows that,

$$J_\Delta\le\int_0^\infty\left[\dot{V}(\mu(s))+z_\Delta{}^T(s)Sz_\Delta(s)-\tilde{\omega}^T(s)\frac{2}{9}S\tilde{\omega}(s)\right]ds \tag{21}$$

Substituting (14) into (18) and (19) and taking into account (21), we get

$$J_\Delta\le\int_0^\infty\left[\dot{V}(\mu(s))+z_\Delta{}^T(s)Sz_\Delta(s)-\tilde{\omega}^T(s)\frac{2}{9}S\tilde{\omega}(s)\right]ds\le\int_0^\infty\xi^T(s)\tilde{\Psi}\xi(s)ds \tag{22}$$

with

$$\tilde{\Psi}=\begin{bmatrix}\Psi_1 & \Psi_2 & \Psi_3 & \Psi_4\\ * & -12R_1 & 0 & 0\\ * & * & -12R_2 & 0\\ * & * & * & -12R_3\end{bmatrix}+\Psi_5\Psi_6{}^{-1}\Psi_5{}^T$$

since (16) holds, using the Schur complement leads to $\xi(t)^T\tilde{\Psi}\xi(t)<0$, which implies that $J_\Delta<0$. This also implies that, $\dot{V}(x(t))+z_\Delta{}^T(t)Sz_\Delta(t)-\tilde{\omega}^T(t)\frac{2}{9}S\tilde{\omega}(t)<0\ \forall\tilde{\omega}$. When $\tilde{\omega}=0$, it is clear that $\dot{V}<0$ which implies that system (S_1) is asymptotically stable. The condition $J_\Delta<0$ implies that, $\int_0^\infty z_\Delta{}^T(s)Sz_\Delta(s)ds<\int_0^\infty\omega_\Delta{}^T(s)S\omega_\Delta(s)ds.$ Letting $S=X^TX$, this inequality guarantees that there exists X such that $\left\|X\circ H\circ X^{-1}\right\|_\infty<1$. This along with lemma 5 allow us to conclude according to lemma 4 that the system (11) is asymptotically stable. □

Remark 3: Theorem 1 presents an admissibility condition in terms of LMI for singular system (1) with time varying delays, and then we use the a tree-terms based approximation of delayed state. The improvements are summarized as follows:

(i) Motivated by the neutral system approach, we model the singular system (1) with time varying delays as a class of neutral systems by using tree-terms approximation of the delayed state. Compared with [35-36], the proposed method considers more information by highlighting the relations among the delay, its variation and the state, which help to achieve less conservativeness. Furthermore, the SSG theorem provides a higher bound compared with the recent work [22, 23, 27]. This will be shown in Example 1.

(ii) The stability of singular systems with time-varying delays via tree-terms based approximation of the delayed state has not been investigated yet in the literature. The literature results are limited to normal dynamical systems.

(iii) In the existing literature results [26], the time delay considered is constant. If the delay parameter is time varying, the extension of results of systems with constant delay to systems with time varying delay is not obvious. Generally the results of time varying delay tend to be conservative than the time invariant ones. In most papers dealing with time varying delay, it is assumed that $\tau_1=0$ see, e.g., [19, 21, 24].

(iv) In our study, we have analyzed the stability by using augmented Lyapunov-Krasovskii functional with the three-terms used to approximate the delayed state, i.e. the three-terms used to approximate the delayed state appear in the derivative of the Lyapunov-Krasovskii functional and the Wirtinger-based integral inequality which according to [37] conducts to less conservative conditions. This comes from the fact that the Wirtingers inequality [29] is more accurate than the Jensen inequality [28] in approximating the resultant quadratic term appearing in the derivative of LKF.

(v) It is noting that the proposed method is also applicable in testing stability of both singular and neutral systems with interval time-varying delays.

(vi) The results are more general than those previously established in the literature. This is because singular systems are more general than regular systems. Thus, by setting E=I, the established conditions can be used as a tool to evaluate the stability problem of regular systems.

4. NUMERICAL EXAMPLES

In this section, examples are provided to illustrate the effectiveness and the less conservatism of the obtained results.

Example 1 [27]

Consider the singular system (1) with the following parameters borrowed from the literature $E=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\ A=\begin{bmatrix}-0.5 & 0\\ 0 & -1\end{bmatrix},\ A_d=\begin{bmatrix}-1 & -1\\ 0 & 0.5\end{bmatrix},$

and the time-varying delays satisfying (2). Obviously, the pair (E, A) is regular and impulse free and there exist two invertible matrices

$F=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $G=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$, such that $FEG=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\ FAG=\begin{bmatrix}-0.5 & 0\\ 0 & 1\end{bmatrix},\ FA_dG=\begin{bmatrix}-1 & 1\\ 0 & -0.5\end{bmatrix}.$

According to Theorem 1, let

$$\tilde{C}(t)=(1-\dot{\tau}(t))\begin{bmatrix}0 & 0\\ 0 & 0.5\end{bmatrix},\ \text{and}\ \tilde{A}=\begin{bmatrix}-0.5 & 0\\ 0 & -1\end{bmatrix},\ \tilde{A}_d=\begin{bmatrix}-1 & 1\\ 0 & 0.5\end{bmatrix}.$$

In order to guarantee $\rho(\tilde{C}(t))<1$, we choose $|\dot{\tau}(t)|<1$.

To prove the advantages and the merit of our approach, comparisons with some recent results will be presented via this example.

Table 1 gives the comparison of the maximum allowed delay τ_2 for various values of τ_1 and d. It is clear that the conditions in this paper give better results than those in [22, 23, 27], showing the advantage of the stability result in this paper and the importance of the obtained improvements.

To verify the obtained results, we plot the trajectories of the retarded system with time-varying delays. Let $0.5\le\tau(t)\le 7.6$. The response of the state $\mu(t)$ of the retarded system is depicted in Fig. 1 under initial condition $\mu(0)=\begin{bmatrix}1 & -1\end{bmatrix}^T$, from which it is clear that the system is asymptotically stable.

In the papers [22, 23, 27], the results depend only on the upper and lower bounds of time delay. In this paper, the derived conditions are dependent on both τ_1, τ_a and τ_2, that is our results take account of more information on the time varying delay. This has the advantage to lead to less restrictive conditions.

Table 1: The comparison of the maximum allowed delay τ_2 for various values of τ_1 and d

τ_1	Method	d=0	d=0.1	d=0.3	d=0.5
0.5	Corollary [22]	2.071	1.988	1.829	1.675
	Theorem 1[23]	2.000	1.906	1.798	1.751
	Theorem 1 [27]	2.402	2.254	2.038	1.912
	Theorem 1	**7.6237**	**6.9792**	**5.8124**	**5.1390**
1.5	Corollary [22]	2.117	2.001	1.810	1.810
	Theorem 1[23]	2.000	1.917	1.842	1.842
	Theorem 1 [27]	2.402	2.256	2.055	1.940
	Theorem 1	**6.4732**	**5.9299**	**4.9586**	**4.2582**

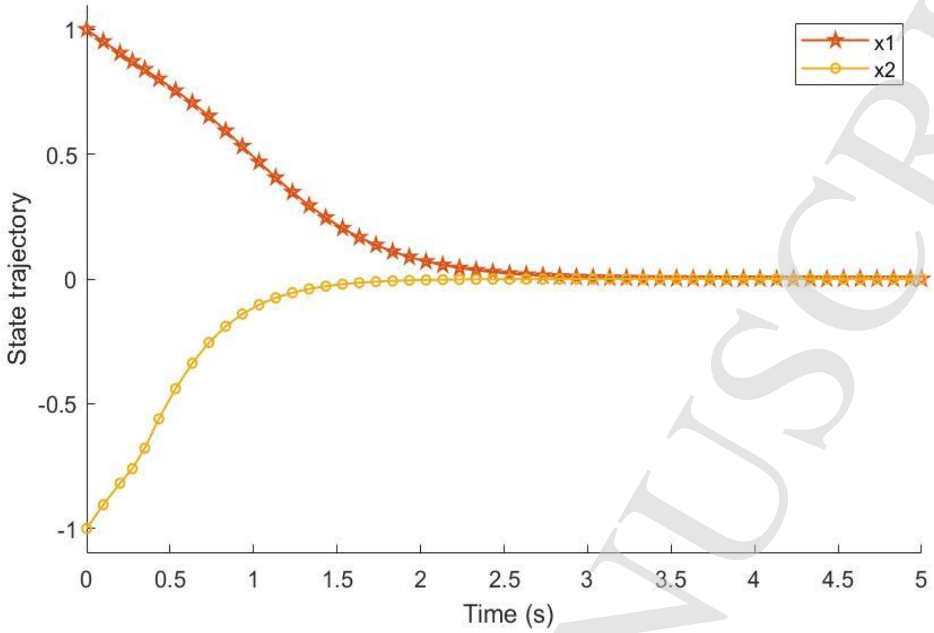

Fig. 1. State trajectories of the system described in Example 1.

Example 2 [27]
Consider the neutral system in [27] with following parameters borrowed from the literature:

$$A=\begin{bmatrix}-2 & 0\\ 0 & -0.9\end{bmatrix},\ A_d=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix},\ C=\begin{bmatrix}c & 0\\ 0 & c\end{bmatrix},$$

In this example, we present another powerful point of the proposed method in this paper. To compare our results with those in [27, 38, 39, 40, 41], we assume that $\tau_1=0$ and we consider various values of c, in which N denotes the delay-partitioning number. Applying Theorem 1 in this paper, Tables 2 gives the comparison of the maximal allowable upper bounds of τ_2. The table 2 shows that the results obtained with our method are less conservative than those obtained with the existing methods. Note that LMI conditions in Theorem 1 gives the best maximum bound of delay with respect to conditions in [27, 38, 39, 40, 41].
The gain provided by this approach is principally correlated with the use of the three term approximation of the delayed state and the use of the Wirtinger based integral inequality with augmented LKF.

Table 2: The comparison of the maximum allowed delay τ_2 for various values of c

c	0.1	0.3	0.5	0.7	0.9
Theorem 1 [38]	4.42	4.17	3.69	2.87	1.41
Theorem 1 [39]	5.21	4.85	3.19	4.2	1.49
Theorem 3.1 [40] N=2	5.61	5.18	4.46	3.34	1.52
Theorem 3.3 [41] N=2	5.822	5.367	4.600	3.425	1.549
Theorem 1 [27]	5.933	5.468	4.685	3.483	1.567
Theorem 1	**7.9108**	**7.0435**	**5.6654**	**3.7911**	**1.8935**

Example 3 [24]
Consider the singular system (1) with the following parameters borrowed from the literature

$$E=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\ A=\begin{bmatrix}0.150 & 0.216\\ -0.897 & -0.295\end{bmatrix},\ A_d=\begin{bmatrix}-0.018 & 0.363\\ 0.211 & 0.252\end{bmatrix},$$

Asymptotic stability of the singular system (1) has been tested with LMI conditions in [12, 15, 24], and Theorem1 of this work. Table 3 lists the comparison of the maximum admissible upper bounds τ_2 of the stability criteria between Theorem 1 and previous literature results. It can be seen from Table 3 that Theorem 1 in this paper provides the largest τ_2 allowed, which refines these previous results and leads to less conservatism.

With the use of the three terms approximation of the delayed state, the results obtained by Theorem 1 have shown very a good superiority compared with Theorem 3.1 in [12], Theorem 3.1, Theorem 3.2 and Corollary 3.1 in [15], Theorem 3.1 in [24]. It verifies that the three terms approximation of the delayed state can make a great progress for the stability criteria for singular time delay systems.

Table 3: The comparison of the maximum allowed delay τ_2 for $\tau_1 = 0$

Method	Upper bound τ_2
Theorem [12] N=2	1.1547
Theorem 3.1 [15]	0.9472
Corollary 3.1 [15]	1.1394
Theorem 3.2 [15]	1.1694
Theorem 3.1 [24] N=4	1.1653
Theorem 3.1 [24] N=5	1.1665
Theorem 1	**2.7270**

5. Conclusion

In this paper, by using the SSG theorem, we have studied the problems of delay dependent stability of singular systems with interval time-varying delay. Firstly, the singular system is represented as a neutral system. By constructing an appropriated Lyapunov–Krasovskii functionnal and employing Wirtinger-based integral inequality and singular analysis technique used in the recent literature, some new sufficient conditions have been derived in terms of LMIs, which can be efficiently solved. Note that the proposed method can also be used in testing stability of singular and neutral systems with time-varying delays. Finally, two numerical examples are given to illustrate the effectiveness of the proposed method and to show that our criteria give less conservative results.

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