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Sampled-Data Output Feedback Control for a Class of Nonlinear Differential-Algebraic Equations Systems

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Abstract: For nonlinear differential-algebraic equations system which satisfies linear growth condition and is of index one, this paper shows that there exists an appropriate sampling period such that the problem of sampled-data output feedback stabilization control can be solved. A linear explicit non-initialized observer is proposed, which means that the initial states of the observer are not necessarily restricted to the algebraic equations and the nonlinear terms of the controlled system are not needed to be known exactly. Then by feedbacking the state estimates, a linear sampled-data controller is designed, through which the whole closed-loop systems are asymptotically stable.

Key Words: Differential-algebraic equations system, output feedback, sampled-data control, linear growth condition, explicit non-initialized observer

1 Introduction

From the view of conservation of energy or mass, many physical systems, such as robot systems with movement constraint, chemical engineering processes and power systems ,etc., usually yield mathematical descriptions that are a mixture of ordinary differential equations and algebraic equations (DAE). Among the existing research, DAE systems of index one occupy important positions [1-4]. Great progress has been made in the research about DAE system of index one. In [2] the problem of multi-index control for SISO DAE model of power systems is studied. In [3], the concept of uniform relative degree for nonlinear DAE system of index one is put forward and the closed-loop stability is achieved through Backstepping control method.

Most existing control research about nonlinear DAE system is based on continuous-time state feedback. However on one hand, more and more controllers are being implemented using digital computers in practice[5]. On the other hand, in many cases the system output is the only measurable signal that can be used in controller design. Therefore, to design a sampled-data controller based on output feedback for nonlinear DAE is meaningful and imperative. In [6] a local observer design for nonlinear DAE system is presented. In [7] a high gain observer is proposed for nonlinear DAE system of index one, which is in fact a "Luenberger-like" nonlinear observer. In [8] for nonlinear DAE system which satisfies linear growth condition and is of index one, a non-initialized linear high gain state observer is proposed. In [9] both implicit and explicit observers are presented for nonlinear DAE system of index one and the sufficient conditions for the existence of the observers are given.

For nonlinear DAE systems which satisfy linear growth condition and is of index one, this paper shows that there exists an appropriate sampling period such that the problem of sampled-data output feedback stabilization control can be solved. A linear explicit non-initialized high gain observer is constructed for the equivalent system, which is no longer the form of a copy of equivalent system plus an error correction term. The initial states of the observer do not need to constraint to the algebraic equation and the nonlinear terms do not need to be known exactly, so the observer bears good robust performance. A linear sampled-data control law is constructed through which the closed-loop systems are asymptotically stable.

2 System Description and Problem Formulation

Consider following nonlinear DAE system

$$\begin{aligned}\dot{x} &= f_1(x,z)+g(x,z,u)\\ 0 &= f_2(x,z)\\ y &= h(x,z)\end{aligned} \tag{1}$$

where $x=(x_1,\cdots,x_n)^T\in R^n$, $z=(z_1,\cdots,z_m)^T\in R^m$, $u\in R$, $y\in R$ are the vector of differential variables, vector of algebraic variables, control input and system output respectively, $f_1\in R^n, f_2\in R^m, g\in R^n$ and $h\in R$ are smooth vector maps. Denote Ω the set of zeros of $f_2:\Omega=\{(x,z)\in R^n\times R^m : f_2(x,z)=0\}$. Without loss of generality, we assume that system (1) has an isolated equilibrium in Ω which we regard to be the origin and $(x(0),z(0))$ is the compatible initial state of system (1)[3].

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Compared with the usual affine form in most existing results, system (1) is nonlinear with respect to the control input.

Throughout this paper, following basic hypotheses are made for (1).

H1. The Jacobian matrix of $f_2(x,z)$ with respect to z has constant full rank on Ω:

$$rank(\frac{\partial f_2}{\partial z}) = m \tag{2}$$

i.e., system (1) is of index one[2-3].

H2. There exists a local diffeomorphism $\Psi:(\zeta,\chi)=\Psi(x,z), 0=\Psi(0)$ at an open subset $\Omega_\kappa=\{(x,z)\in\Omega \,|\, \|f_2(x,z)\|<\kappa, \kappa>0\}$ such that system (1) can be equivalently transformed into following form :

$$\begin{aligned}
&\dot{\zeta}_1 = \zeta_2 + \phi_1(\zeta,\chi,u)\\
&\quad\vdots\\
&\dot{\zeta}_{n-1} = \zeta_n + \phi_{n-1}(\zeta,\chi,u)\\
&\dot{\zeta}_n = u + \phi_n(\zeta,\chi,u)\\
&\dot{\chi} = 0\\
&y = \zeta_1
\end{aligned} \tag{3}$$

where $\zeta(t)=[\zeta_1(t),\cdots,\zeta_n(t)]^T$, $\chi\in R^m$.

H3. For $i=1,\mathrm{L}\ ,n$, there exists a constant $c\ge 0$ such that

$$|\phi_i(\zeta,\chi,u)|\le c(|\zeta_1|+\cdots+|\zeta_i|), i=1,\cdots,n \tag{4}$$

i.e., $\phi_i(\cdot)$ satisfies so-called "linear growth condition".

The objective of this paper is to design following dynamic sampled-data output feedback controller

$$\begin{aligned}
&\vartheta(t_{k+1}) = \Upsilon(\vartheta(t_k), y(t_k))\\
&u(t) = u(t_k) = U(\vartheta(t_k)), k=0,1,2,\cdots
\end{aligned} \tag{5}$$

for nonlinear DAE system (1) satisfying hypotheses H1~H3, such that the whole closed-loop systems (1) and (5) are asymptotically stable at the zero equilibrium for $\forall t\in[t_k,t_{k+1})$,where $t_k=kT$ is the sampling point and T is the sampling period.

3 Main Results

Definition 1. If there exists following system for nonlinear DAE system (1):

$$\begin{aligned}
&\dot{\hat{x}} = X(\hat{x},\hat{z},u,y)\\
&\dot{\hat{z}} = Z(\hat{x},\hat{z},u,y), (\hat{x},\hat{z})\in\Omega_\kappa
\end{aligned} \tag{6}$$

such that $\lim_{t\to+\infty}\|(x,z)-(\hat{x},\hat{z})\|=0$ for $(x(0),z(0))\in\Omega, (\hat{x}(0),\hat{z}(0))\in\Omega_\kappa$, then system (6) is called an explicit non-initialized state observer of nonlinear DAE system (1), where "non-initialized" means that the initial state $(\hat{x}(0),\hat{z}(0))$ of the observer (6) can be non-compatible and does not need to satisfy the algebraic equation: $f_2(\hat{x}(0),\hat{z}(0))=0$.

Theorem 1. If choose sampling period T as

$$T<\frac{c^*}{\sigma^2\gamma} \tag{7}$$

then the problem of sampled-data output feedback control for nonlinear DAE system (1) can be solved. $c^*>0,\gamma>0,\sigma\ge 1$ are design parameter, constant and gain respectively which will be determined in next proof.

Proof: The proof will be divided into two parts.

3.1 Design of linear explicit non-initialized state observer for (3)

For the sake of concision, we re-write equivalent system (3) into following form:

$$\begin{aligned}
&\dot{\zeta}(t) = A\zeta(t) + Bu(t) + \Phi(t,\zeta(t),\chi(t),u(t))\\
&\dot{\chi} = 0\\
&y = C\zeta
\end{aligned} \tag{8}$$

where

$$A=\begin{bmatrix}0&1&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\ 0&0&\cdots&1\\ 0&0&\cdots&0\end{bmatrix}_{n\times n}, B=\begin{bmatrix}0\\ \vdots\\ 0\\ 1\end{bmatrix}_{n\times 1},$$
$$C=\begin{bmatrix}1\\ \vdots\\ 0\\ 0\end{bmatrix}^T_{1\times n}, \Phi(t,\zeta(t),u(t))=\begin{bmatrix}\phi_1(\cdot)\\ \phi_2(\cdot)\\ \vdots\\ \phi_n(\cdot)\end{bmatrix}_{n\times 1} \tag{9}$$

For system (8), define change of coordinates as follows

$$v=\frac{u}{\sigma^n}, \xi_i=\frac{\zeta_i}{\sigma^{i-1}}, i=1,\cdots,n \tag{10}$$

where v is the new control input. From (10), system (8) can be further transformed into following form:

$$\begin{aligned}
&\dot{\xi}(t) = \sigma A\xi(t) + \sigma Bv(t) + \bar{\Phi}(t,\xi(t),\chi(t),v(t))\\
&\dot{\chi} = 0\\
&y = C\xi
\end{aligned} \tag{11}$$

where $\xi=(\xi_1,\cdots,\xi_n)^T, \bar{\Phi}(\cdot)=[\bar{\phi}_1(\cdot),\cdots,\bar{\phi}_n(\cdot)]^T$, $\bar{\phi}_i(\xi,\chi,u)=\frac{\phi_i(\zeta,\chi,u)}{\sigma^{i-1}}, i=1,\cdots,n$. By virtue of (4) and (10), it can be verified that following inequality holds

$$|\bar{\phi}_i(\xi,\chi,u)|\le c(|\xi_1|+\cdots+|\xi_i|), i=1,\cdots,n \tag{12}$$

Now we first design a continuous-time linear explicit non-initialized high gain state observer for (11) as follows:

$$\begin{aligned}
\dot{\hat{\xi}}(t) &= \sigma A\hat{\xi}(t) + \sigma Bv(t_k) + \sigma H(\xi_1(t_k) - \hat{\xi}_1(t))\\
&= \sigma\bar{A}\hat{\xi}(t) + \sigma Bv(t_k) + \sigma H\xi_1(t_k)\\
\dot{\hat{\chi}}(t) &= -\Lambda\hat{\chi}(t), \forall t\in[t_k,t_{k+1})
\end{aligned} \tag{13}$$

where $H=[h_n,\cdots,h_1]^T, h_i, i=1,\cdots,n$ are coefficients of some Hurwitz polynominal $s^n+h_n s^{n-1}+\cdots+h_1$, $\hat{\xi}=(\hat{\xi}_1,\cdots,\hat{\xi}_n)^T$, $\bar{A}=A-HC$, and Λ is a $m\times m$ symmetric positive definite matrix.

From (13), we can also obtain discrete-time linear explicit non-initialized high gain state observer:

$$\begin{aligned}\hat{\xi}(t_{k+1})&=e^{\sigma\bar{A}T}\hat{\xi}(t_k)+\int_0^T e^{\sigma\bar{A}s}ds(\sigma Bv(t_k)+\sigma H\xi_1(t_k))\\&=M_1\hat{\xi}(t_k)+Gv(t_k)+N\xi_1(t_k)\\\hat{\chi}(t_{k+1})&=e^{-\Lambda T}\hat{\chi}(t_k)=M_2\hat{\chi}(t_k)\end{aligned}\tag{14}$$

where $M_1=e^{\sigma\bar{A}T}, G=\sigma\int_0^T e^{\sigma\bar{A}s}dsB, N=\sigma\int_0^T e^{\sigma\bar{A}s}dsH$, $,M_2=e^{-\Lambda T}$.

Obviously, (13) and (14) produce the same estimate $(\hat{\xi}(t_k),\hat{\chi}(t_k))$.

3.2 Construction of linear sampled-data output feedback controller

The sampled-data output feedback controller is constructed as follows for $\forall t\in[t_k,t_{k+1})$:

$$v(t)=v(t_k)=-k_1\hat{\xi}_1(t_k)-k_2\hat{\xi}_2(t_k)-\cdots-k_n\hat{\xi}_n(t_k)\tag{15}$$

Define $K=[k_1,\mathrm{L}\ ,k_n]$ where $k_i, i=1,\mathrm{L}\ ,n$ are coefficients of some Hurwitz polynominal $s^n+k_n s^{n-1}+\cdots+k_1$. Submit (15) into (11) and (13), we get following closed-loop systems:

$$\begin{aligned}\begin{bmatrix}\dot{\xi}(t)\\\dot{\hat{\xi}}(t)\end{bmatrix}&=\sigma\begin{bmatrix}A&0\\0&\bar{A}\end{bmatrix}\begin{bmatrix}\xi(t)\\\hat{\xi}(t)\end{bmatrix}-\sigma\begin{bmatrix}B\\B\end{bmatrix}K\hat{\xi}(t_k)\\&+\sigma\begin{bmatrix}0\\H\end{bmatrix}\xi_1(t_k)+\begin{bmatrix}\bar{\Phi}(t,\xi(t),\chi(t),v(t))\\0\end{bmatrix}\\\dot{\chi}(t)&=0\\\dot{\hat{\chi}}(t)&=-\Lambda\hat{\chi}(t),\forall t\in[t_k,t_{k+1})\end{aligned}\tag{16}$$

Since

$$\begin{aligned}\xi_1(t_k)&=C\xi(t_k)=C\xi(t)+C\xi(t_k)-C\xi(t)\\\hat{\xi}(t_k)&=\xi(t)+\hat{\xi}(t_k)-\xi(t)\end{aligned}\tag{17}$$

then submit (17) into (16), we can get

$$\begin{aligned}\begin{bmatrix}\dot{\xi}(t)\\\dot{\hat{\xi}}(t)\end{bmatrix}&=\sigma\begin{bmatrix}A&-BK\\HC&\bar{A}-BK\end{bmatrix}\begin{bmatrix}\xi(t)\\\hat{\xi}(t)\end{bmatrix}\\&-\sigma\begin{bmatrix}B\\B\end{bmatrix}K(\hat{\xi}(t_k)-\hat{\xi}(t))\\&+\sigma\begin{bmatrix}0\\H\end{bmatrix}C(\xi(t_k)-\xi(t))+\begin{bmatrix}\bar{\Phi}(\cdot)\\0\end{bmatrix}\\\dot{\chi}(t)&=0\\\dot{\hat{\chi}}(t)&=-\Lambda\hat{\chi}(t),\forall t\in[t_k,t_{k+1})\end{aligned}\tag{18}$$

It can be verified that following equation holds:

$$\begin{aligned}\Xi&=\begin{bmatrix}A&-BK\\HC&\bar{A}-BK\end{bmatrix}\\&=\begin{bmatrix}I&0\\-I&I\end{bmatrix}^{-1}\begin{bmatrix}A-BK&-BK\\0&\bar{A}\end{bmatrix}\begin{bmatrix}I&0\\-I&I\end{bmatrix}\end{aligned}\tag{19}$$

Obviously matrices $A-BK$ and $\bar{A}$ are both Hurwitz, so from (19) we know that matrix Ξ is Hurwitz. Therefore, there is a symmetric positive definite matrix $P\in R^{2n\times 2n}$ for Ξ such that $\Xi^TP+P\Xi=-I$.

Define $\eta(t)=(\xi(t)^T,\hat{\xi}(t)^T)^T$ and choose following Lyapunov function for (18)

$$V(\eta(t),\hat{\chi}(t))=V_0(\eta(t))+W(\hat{\chi}(t))\tag{20}$$

where

$$\begin{aligned}V_0(\eta(t))&=\eta(t)^TP\eta(t)\\W(\hat{\chi}(t))&=\hat{\chi}(t)^T\hat{\chi}(t)\end{aligned}\tag{21}$$

By virtue of (18) and (21), we have

$$\begin{aligned}\dot{V}_0&=-\sigma\|\eta(t)\|^2+2\eta(t)^TP\begin{bmatrix}\bar{\Phi}(\xi(t),\chi(t),v(t))\\0\end{bmatrix}\\&+2\sigma\eta(t)^TP\begin{bmatrix}B\\B\end{bmatrix}K(\hat{\xi}(t_k)-\hat{\xi}(t))\\&+2\sigma\eta(t)^TP\begin{bmatrix}0\\H\end{bmatrix}(\xi_1(t_k)-\xi_1(t))\end{aligned}\tag{22}$$

and

$$\dot{W}(\hat{\chi})=-2\hat{\chi}^T\Lambda\hat{\chi}\le-2\lambda_{\min}(\Lambda)W(\hat{\chi})\tag{23}$$

where $\|\cdot\|$ is the Euclidean norm of vector and $\lambda_{\min}(\Lambda)$ is the minimum eigenvalue of Λ.

Now we will check the terms in the right side of (22) one by one.

(I) First investigate $2\eta(t)^TP\begin{bmatrix}\bar{\Phi}(\xi(t),\chi(t),v(t))\\0\end{bmatrix}$ of (22). From (12) we can get

$$\begin{aligned}\|\bar{\Phi}(\cdot)\|&=\sqrt{\bar{\phi}_1^2(\cdot)+\bar{\phi}_2^2(\cdot)+\cdots+\bar{\phi}_n^2(\cdot)}\\&\le c\sqrt{\xi_1^2+(|\xi_1|+|\xi_2|)^2+\cdots+(|\xi_1|+\cdots+|\xi_n|)^2}\\&\le c_1\|\eta(t)\|\end{aligned}\tag{24}$$

where $c_1=c\sqrt{1+2+\mathrm{L}\ +n}$. By virtue of (24) we have

$$2\eta(t)^TP\begin{bmatrix}\bar{\Phi}(\xi(t),\chi(t),v(t))\\0\end{bmatrix}\le 2c_1\lambda_{\max}(P)\|\eta(t)\|^2\tag{25}$$

where $\|P\|$ is the 2-norm of matrix P, i.e., $\|P\|$ is the maximum eigenvalue of $P:\|P\|=\lambda_{\max}(P)$.

(II) Next investigate $2\sigma\eta(t)^TP\begin{bmatrix}B\\B\end{bmatrix}K(\hat{\xi}(t_k)-\hat{\xi}(t))$ of (22). Notice that

$$\left|K(\hat{\xi}(t_k)-\hat{\xi}(t))\right| \le \int_{t_k}^{t}\left|K\dot{\hat{\xi}}(\tau)\right|d\tau \qquad (26)$$

From (21) it is obvious that

$$\sqrt{\frac{V_0(\eta(t))}{\lambda_{\max}(P)}} \le \|\eta(t)\| \le \sqrt{\frac{V_0(\eta(t))}{\lambda_{\min}(P)}} \qquad (27)$$

Then from (13), (15) and (27), for $\forall \tau \in [t_k, t)$ we have

$$\begin{aligned}\left|K\dot{\hat{\xi}}(\tau)\right| &= \left|\sigma K\bar{A}\hat{\xi}(\tau) + \sigma KBv(t_k) + \sigma KH\xi_1(t_k)\right| \\ &\le \sigma\left\|K\bar{A}\right\|\left\|\hat{\xi}(\tau)\right\| + \sigma\left\|KBK\right\|\left\|\hat{\xi}(t_k)\right\| \\ &\quad + \sigma\left\|KHC\right\|\left\|\xi(t_k)\right\| \\ &\le \sigma c_2\sqrt{V_{\max}^0(t)}\end{aligned} \qquad (28)$$

where $c_2 = \dfrac{\|K\bar{A}\| + \|KBK\| + \|KHC\|}{\sqrt{\lambda_{\min}(P)}}$,

$V_{\max}^0(t) = \max\limits_{\forall\tau\in[t_k,t]} V_0(\eta(\tau)), t \in [t_k, t_{k+1})$. From (26) and (28) we have

$$\left|K(\hat{\xi}(t_k)-\hat{\xi}(t))\right| \le \sigma c_2 (t-t_k)\sqrt{V_{\max}^0(t)} \qquad (29)$$

From (27) and (29), we have

$$\begin{aligned}&2\sigma\eta(t)^T P\begin{bmatrix}B\\B\end{bmatrix}K(\hat{\xi}(t_k)-\hat{\xi}(t)) \\ &\le 2c_2\sigma^2\|\eta(t)\|\left\|P\begin{bmatrix}B\\B\end{bmatrix}\right\|(t-t_k)\sqrt{V_{\max}^0(t)} \qquad (30) \\ &\le 2c_2\sigma^2\left\|P\begin{bmatrix}B\\B\end{bmatrix}\right\|\sqrt{\frac{V_0(\eta(t))}{\lambda_{\min}(P)}}(t-t_k)\sqrt{V_{\max}^0(t)}\end{aligned}$$

(III)Finally investigate $2\sigma\eta(t)^T P\begin{bmatrix}0\\H\end{bmatrix}(\xi_1(t_k)-\xi_1(t))$ of (22). Form (11) we have $\dot{\xi}_1 = \sigma\xi_2 + \bar{\phi}_1(\xi,\chi,u)$. If we set $\sigma \ge c$, then we can get

$$\left|\dot{\xi}_1(t)\right| \le \sigma\left|\xi_2(t)\right| + c\left|\xi_1(t)\right| \le \sigma\sqrt{\frac{2}{\lambda_{\min}(P)}}\sqrt{V_{\max}^0(t)} \qquad (31)$$

Define $c_3 = \sqrt{\dfrac{2}{\lambda_{\min}(P)}}$, and from (31) it can be verified that following inequality holds:

$$\left|\xi_1(t_k)-\xi_1(t)\right| \le \sigma c_3 (t-t_k)\sqrt{V_{\max}^0(t)} \qquad (32)$$

From (27), (29) and (32), we have

$$\begin{aligned}&2\sigma\eta(t)^T P\begin{bmatrix}0\\H\end{bmatrix}(\xi_1(t_k)-\xi_1(t)) \\ &\le 2c_3\sigma^2\|\eta(t)\|\left\|P\begin{bmatrix}0\\H\end{bmatrix}\right\|(t-t_k)\sqrt{V_{\max}^0(t)}\end{aligned} \qquad (33)$$

Then submit (25), (30) and (33) into (22), we have

$$\begin{aligned}\dot{V}_0 &\le -(\sigma - 2c_1\lambda_{\max}(P))\|\eta(t)\|^2 \\ &\quad + 2c_2\sigma^2\left\|P\begin{bmatrix}B\\B\end{bmatrix}\right\|\sqrt{\frac{V_0(\eta(t))}{\lambda_{\min}(P)}}(t-t_k)\sqrt{V_{\max}^0(t)} \\ &\quad + 2c_3\sigma^2\left\|P\begin{bmatrix}0\\H\end{bmatrix}\right\|\sqrt{\frac{V_0(\eta(t))}{\lambda_{\min}(P)}}(t-t_k)\sqrt{V_{\max}^0(t)}\end{aligned} \qquad (34)$$

Define $\gamma = \left(2c_2\left\|P\begin{bmatrix}B\\B\end{bmatrix}\right\| + 2c_3\left\|P\begin{bmatrix}0\\H\end{bmatrix}\right\|\right)/\sqrt{\lambda_{\min}(P)}$, then (34) can be re-wrote as follows

$$\begin{aligned}\dot{V}_0 &\le -(\sigma - 2c_1\lambda_{\max}(P))\|\eta(t)\|^2 \\ &\quad + \sigma^2\gamma(t-t_k)\sqrt{V_0 V_{\max}^0(t)}\end{aligned} \qquad (35)$$

By virtue of (23) and (35), we have

$$\begin{aligned}&\dot{V}(\eta(t),\hat{\chi}(t)) \le -(\sigma - 2c_1\lambda_{\max}(P))\|\eta(t)\|^2 \\ &+\sigma^2\gamma(t-t_k)\sqrt{V_0 V_{\max}^0(t)} - 2\lambda_{\min}(\Lambda)W(\hat{\chi})\end{aligned} \qquad (36)$$

If we choose such symmetric positive definite matrix Λ of (13) that satisfes

$$\lambda_{\min}(\Lambda) \ge \frac{\sigma - 2c_1\lambda_{\max}(P)}{2\lambda_{\max}(P)} \qquad (37)$$

Then from (36) we can get

$$\begin{aligned}\dot{V}(\eta(t),\hat{\chi}(t)) &\le -(\sigma - 2c_1\lambda_{\max}(P))\|\eta(t)\|^2 \\ &\quad - 2\lambda_{\min}(\Lambda)W(\hat{\chi}(t)) \\ &\quad + \sigma^2\gamma\sqrt{V_0 V_{\max}(t)}(t-t_k) \qquad (38) \\ &\le -(\frac{\sigma}{\lambda_{\max}(P)} - 2c_1)V(\eta(t),\hat{\chi}(t)) \\ &\quad + \sigma^2\gamma(t-t_k)\sqrt{V_0 V_{\max}^0(t)}\end{aligned}$$

Now we have following conclusion:

If choose the gain σ to satisfy

$$\sigma > \max\{1, c, \lambda_{\max}(P)(2c_1 + c^*)\} \qquad (39)$$

and sampling period T to satisfy

$$T < \frac{c^*}{\sigma^2\gamma} \qquad (40)$$

where $c^* > 0$ is a given constant, then following equality holds:

$$\max_{\forall\tau\in[t_k,t_{k+1})} V(\eta(t),\hat{\chi}(t)) = V(\eta(t_k),\hat{\chi}(t_k)) \qquad (41)$$

We will prove this conclusion by *Contradiction*. If (41) does not hold, then there exists some time instant $t' \in [t_k, t_{k+1})$ such that $V(\eta(t'),\hat{\chi}(t')) > V(\eta(t_k),\hat{\chi}(t_k))$. When $t = t_k$, we have $\dot{V}(\eta(t_k),\hat{\chi}(t_k)) < 0, \forall(\eta(t_k),\hat{\chi}(t_k)) \ne 0$ from (38). So $V(\eta(t),\hat{\chi}(t))$ will decrease in a short time starting from t_k. Therefore, there exists a time instant $t'' \in [t_k, t']$ such that following equations hold:

$$(i)V(\eta(t^{"}),\hat{\chi}(t^{"}))=V(\eta(t_k),\hat{\chi}(t_k))$$
$$(ii)V(\eta(t),\hat{\chi}(t))\le V(\eta(t^{"}),\hat{\chi}(t^{"}),\forall t\in[t_k,t^{"}] \quad (42)$$
$$(iii)\dot{V}(\eta(t^{"}),\hat{\chi}(t^{"}))>0$$

But from (37),(38),(39) and (40), together with $\max_{\forall\tau\in[t_k,t]} V_0(\eta(\tau))\le \max_{\forall\tau\in[t_k,t]} V(\eta(\tau),\hat{\chi}(\tau)),\forall t\in[t_k,t_{k+1})$ and $V_0(\eta(t))\le V(\eta(t),\hat{\chi}(t))$ in mind, we have

$$\dot{V}(\eta(t^{"}),\hat{\chi}(t^{"}))\le -(\frac{\sigma}{\lambda_{\max}(P)}-2c_1)V(\eta(t^{"}),\hat{\chi}(t^{"})) + \sigma^2\gamma T\sqrt{V_0(\eta(t^{"}))V^0_{\max}(t^{"})} < 0 \quad (43)$$

which contradicts to the (iii) of (42). Thus (41) is true.

Similar to (43), from (38) and (41), for $\forall t\in[t_k,t_{k+1})$ we have

$$\dot{V}(\eta(t),\hat{\chi}(t))\le -(\frac{\sigma}{\lambda_{\max}(P)}-2c_1)V(\eta(t),\hat{\chi}(t)) + \sigma^2\gamma T\sqrt{V(\eta(t),\hat{\chi}(t))V(\eta(t_k),\hat{\chi}(t_k))} \quad (44)$$

By virtue of (44), we have

$$\frac{d}{dt}\sqrt{V(\eta(t),\hat{\chi}(t))/V(\eta(t_k),\hat{\chi}(t_k))}\le \frac{1}{2}\sigma^2\gamma T - \frac{1}{2}(\frac{\sigma}{\lambda_{\max}(P)}-2c_1)\sqrt{V(\eta(t),\hat{\chi}(t))/V(\eta(t_k),\hat{\chi}(t_k))} \quad (45)$$

Noting that $\sqrt{V(\eta(t),\hat{\chi}(t))/V(\eta(t_k),\hat{\chi}(t_k))}=1$ when $t=t_k$, then from (45) and *Integral Inequality*, we have

$$\sqrt{V(\eta(t),\hat{\chi}(t))/V(\eta(t_k),\hat{\chi}(t_k))} \le e^{-\frac{1}{2}((\sigma/\lambda_{\max}(P))-2c_1)(t-t_k)} + \frac{\sigma^2 c_4 T}{(\sigma/\lambda_{\max}(P))-2c_1}(1-e^{-\frac{1}{2}((\sigma/\lambda_{\max}(P))-2c_1)(t-t_k)}) \quad (46)$$

From (7), when $t=t_{k+1}$ we have

$$\sqrt{V(\eta(t_{k+1}),\hat{\chi}(t_{k+1}))/V(\eta(t_k),\hat{\chi}(t_k))} \le \frac{\sigma^2 c_4 T}{(\sigma/\lambda_{\max}(P))-2c_1} + (1-\frac{\sigma^2 c_4 T}{(\sigma/\lambda_{\max}(P))-2c_1})e^{-\frac{1}{2}((\sigma/\lambda_{\max}(P))-2c_1)T} \triangleq \vartheta \quad (47)$$

From(39) and (40), we know $\frac{\sigma^2 c_4 T}{(\sigma/\lambda_{\max}(P))-2c_1}<1$ and $e^{-\frac{1}{2}((\sigma/\lambda_{\max}(P))-2c_1)T}<1$, so from (47) we have

$$\sqrt{V(\eta(t_{k+1}),\hat{\chi}(t_{k+1}))/V(\eta(t_k),\hat{\chi}(t_k))}\le\vartheta<1 \quad (48)$$

So $V(\eta(t_{k+1}),\hat{\chi}(t_{k+1}))\le \upsilon^2 V(\eta(t_k),\hat{\chi}(t_k))$ As a result of (48), further we have $\lim_{k\to\infty} V(\eta(t_k),\hat{\chi}(t_k))=0$. From definition 1, (13) is a linear explicit non-initialized observer of (8) and the closed-loop systems (8) and (13) are asymptotically stable. From the property of Ψ defined in H2, the original state (x,z) of DAE system (1) is asymptotically stable too.

As a conclusion, for equivalent system (3), if we choose appropriate gain σ as (39) and sampling period T as (7), then we can construct continuous-time linear explicit non-initialized observer (13) and linear sampled-data controller (15), through which the whole closed-loop systems are asymptotically stable. This ends the proof.

4 An Illustrative Example

Consider following nonlinear DAE system

$$\begin{aligned} \dot{x}_1 &= x_2 + x_1\sin(u\cdot x_2) \\ \dot{x}_2 &= u + x_2\sin((1-z)^{1/3}) \\ 0 &= f_2(x,z) = z^2+x_1^2+x_2^2-r^2 \\ y &= x_1 \end{aligned} \quad (49)$$

Define

$$\Omega=\{(x_1,x_2,z)\in R^3 : z^2+x_1^2+x_2^2-r^2=0, z>0\} \quad (50)$$

Obviously nonlinear DAE system (49) is of index one. From (3), the diffeomorphism Ψ can be chosen as follows

$$(\varsigma_1,\varsigma_2,\chi)=(x_1,x_2,z^2+x_1^2+x_2^2-r^2) \quad (51)$$

With (51), nonlinear DAE system (49) can be equivalently transformed into:

$$\begin{aligned} \dot{\varsigma}_1 &= \varsigma_2+\varsigma_1\sin(u\cdot\varsigma_2) \\ \dot{\varsigma}_2 &= u+\varsigma_2\sin((1-z)^{1/3}) \\ \dot{\chi} &= 0 \\ y &= \xi_1 \end{aligned} \quad (52)$$

Define $\phi_1(\cdot)=\varsigma_1\sin(u\varsigma_2)$, $\phi_2(\cdot)=\varsigma_2\sin((1-z)^{1/3})$. Obviously $|\phi_1(\cdot)|\le|\varsigma_1|,|\phi_2(\cdot)|\le|\varsigma_2|$, i.e., ϕ_1,ϕ_2 satisfy lin -ear growth condition (4) with $c=1$.

With following change of coordinates

$$\xi_1=\varsigma_1,\xi_2=\frac{\varsigma_2}{\sigma},v=\frac{u}{\sigma^2} \quad (53)$$

we can get equivalent systems of (49):

$$\begin{aligned} \dot{\xi}_1 &= \sigma\xi_2+\xi_1\sin(\sigma^3 u\xi_2) \\ \dot{\xi}_2 &= \sigma v+\xi_2\sin(1-z)^{1/3} \\ \dot{\chi} &= 0 \end{aligned} \quad (54)$$

According to (15), the linear controller v is constructed as follows:

$$v(t)=v(t_k)=-k_1\hat{\xi}_1(t_k)-k_2\hat{\xi}_2(t_k)=-K\hat{\xi}(t_k) \quad (55)$$

where $k_1=0.5,k_2=2,\hat{\xi}(t_k)=(\hat{\xi}_1(t_k),\hat{\xi}_2(t_k))^T$, i.e., $K=[-0.5,-2]$. According to (13), we can choose

$H = [2, 0.5]^T$ and construct the linear explicit non-initialized high gain state observer as follows

$$\dot{\hat{\xi}}_1(t) = \sigma\hat{\xi}_2(t) + 0.5\sigma(\xi_1(t_k) - \hat{\xi}_1(t))$$
$$\dot{\hat{\xi}}_2(t) = \sigma v(t) + 2\sigma(\xi_1(t_k) - \hat{\xi}_1(t)) \quad (56)$$
$$\dot{\hat{\chi}}(t) = -\Lambda\hat{\chi}(t), \forall t \in [t_k, t_{k+1})$$

In this example, $c^* = 1$ and gain σ can be chosen as $\sigma = 50$, sampling period T can be chosen as $T = 0.01s$, the compatible initial state of nonlinear DAE systems(49) can be set as $(x_1(0), x_2(0), z(0)) = (0,1,0)$, the initial state of observer (56) can be set as $(\hat{\xi}_1(0), \hat{\xi}_2(0), \hat{\chi}(0)) = (0, 1/\sigma, 0.2)$. it can be easily verified that

$$\xi_1 = x_1, \xi_2 = \frac{x_2}{\sigma}, \hat{\xi}_1 = \hat{x}_1, \hat{\xi}_2 = \frac{\hat{x}_2}{\sigma} \quad (57)$$

The simulation results are shown as Fig.1-3, where the solid line on behalf of states of nonlinear DAE system (49), and dot line on behalf of estimations produced by observer (56).

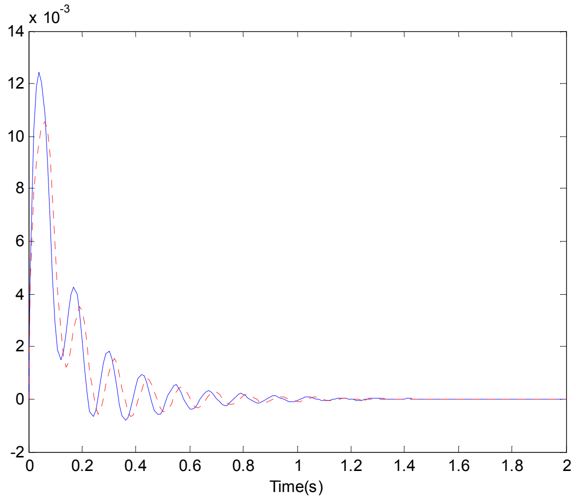

Fig.1: Transient response of state x_1 and its estimate $\hat{x}_1$

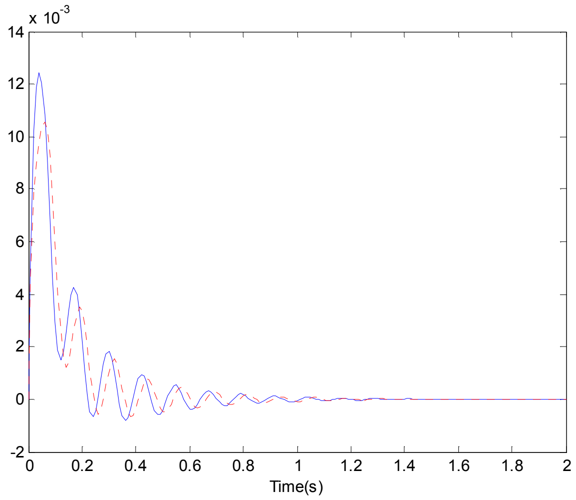

Fig.2: Transient response of state x_2 and its estimate $\hat{x}_2$

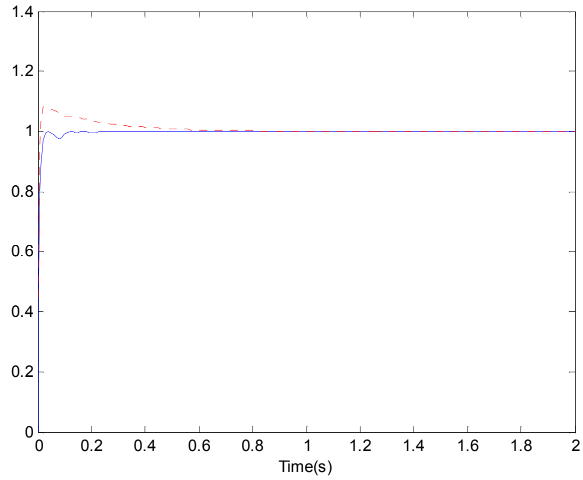

Fig.3: Transient response of state z and its estimate z

As shown in Fig.1-3, the whole closed-loop systems(49) and (56) are asymptotically stable.

5 CONCLUSIONS

For nonlinear DAE system which satisfies linear growth condition and is of index one, this paper shows that its sampled-data output feedback control problem can be solved. The initial state of the observer does not need to constraint to the algebraic equation. A suitable gain is chosen to dominate the uncertain nonlinear terms. After fixing the gain, A sampling period is selected such that the asymptotical stability of whole closed-loop systems is achieved through a linear sampled-data output feedback controller.

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