Journal Pre-proof

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Please cite this article as: Wen-bin Chen, Fang Gao, New results on stability analysis for a kind of neutral singular systems with mixed delays, *European Journal of Control* (2019), doi: https://doi.org/10.1016/j.ejcon.2019.10.001

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New results on stability analysis for a kind of neutral singular systems with mixed delays

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Abstract

In this paper, the problem of the stability analysis for neutral singular systems with mixed delays is revisited. By the Lyapunov-Krasovskill functional approach and the singular system methodology, novel stability criteria are acquired, which can be easily expressed by linear matrix inequalities. First of all, in terms of Leibniz-Newton formula, zero equation with free-weighting matrices is fully constructed, which plays a key role in the field of stability analysis and makes the stability criterion feasible. Secondly, the asymptotically stability for neutral singular systems is strictly shown, which is different from other existing results. Thirdly, the regularity, non-impulsiveness and stability can be guaranteed based on some sufficient conditions. Finally, three numerical examples and simulation studies are presented to demonstrate the validity and feasibility of our method.

Keywords: Neutral system; Mixed delays; Singular system; Stability.

1. Introduction

Neutral systems with mixed delays both in their state and in their derivatives of state are frequently encountered in plenty of applications, such as population ecology, rigid environments, distributed networks including lossless transmission lines, etc. Moreover, the mixed delays with constant or time-varying can cause poor performance, oscillations, and even instability in different control systems [1–12]. Hence, the stability analysis for neutral type systems has been investigated and obtained a quantity of achievements [13–25]. However, it is not hard to find some neutral systems, e.g. the partial element equivalent circuit model [26], DC motor [27] and oil catalytic cracking process [28], etc. are not normal neutral ones, which are seen as more general neutral systems, i.e. neutral singular systems. Accordingly, it is of great practical and theoretical significance to study the stability of neutral singular systems.

In recent years, there are few results reported in stability analysis for neutral singular systems based on the fact the conditions of regularity, non-impulsiveness and stability are relatively difficult to be obtained due to the existence of singular matrix and state derivatives terms with time delay. In [29], the stability problems of neutral type descriptor system with mixed delays were studied. New delay-independent stability and robust stability criteria were derived. In [30, 31], robust stability analysis for uncertain neutral type singular systems with time delays were revisited and the corresponding stability criteria are obtained. However, it should be pointed out that the regularity and non-impulsiveness are not discussed by stability criteria in [29–31], which may cause multiple or no solution to given singular neutral system and appear infinite dynamical modes; and the solution $x(t) \rightarrow 0$ are not strictly shown which may lead to a certain degree of conservatism. To this end, very recently, delay-dependent criteria for absolute stability of uncertain Lurie singular systems with neutral type and time-varying delays were acquired in [32]. Nevertheless, the inequality $-(1 - v)\dot{y}(t - h(t))\bar{E}^T \bar{Q}_2 \bar{E}\dot{y}(t - h(t)) \leq -(1 - v)\dot{y}(t - h(t))(\bar{E}^T + \bar{E} + 2mI + m^2\bar{Q}_2)\dot{y}(t - h(t))$ may not be established, which lead to an invalid stability criterion. Furthermore, in [33], the scenario with constant time delays

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for neutral singular systems was also examined. It is worth mentioning that the stability criteria in Corollary 2 are probably not obtained by using the similar method of Theorem 1 due to $ES \neq C$ wherein C satisfying the equation (77) in [33] and the integral term $\int_{t-\tau}^{t} \dot{x}^T(\alpha)E^T U E \dot{x}(\alpha)d\alpha$ is not easy to be processed since the matrix term $E^T U E$ may cause stability criteria with linear matrix inequalities infeasibility. To summarize, many theoretical issues for neutral singular systems remain open, which motivate our current work.

Inspired by the above statement, this paper focuses on the stability analysis for neutral singular systems with mixed delays. By applying a simple Lyapunov-Krasovskill functional and a zero equation with free-weighting matrices, sufficient conditions are given to ensure the neutral singular systems to be regular, impulse free and asymptotically stable. Moreover, the stability criterion are prone to be processed by employing MATLAB LMI control toolbox. However, it should be noted that a rigorous theoretical proof for the asymptotically stability of neutral singular systems is provided, which is different from other existing results. Finally, numerical examples and simulation studies are presented to show the admissibility of the neutral singular systems and the effectiveness of the proposed method.

Notations : Throughout this paper, the symmetric term in a symmetric matrix is denoted by $\star$. $\mathbb{R}^n$ and $\mathbb{R}^{m \times n}$ are respectively the $n-$dimensional Euclidean space and the set of $m \times n$ real matrix. P^T and P^{-1} mean the transpose and the inverse of the matrix P; $\mathbb{S}^n$ and $\mathbb{S}_+^n$ represent the sets of symmetric and symmetric positive definite matrices of $\mathbb{R}^{n \times n}$, respectively. $det(P)$ means the determinant of the matrix P. $\lambda(.)$ means the eigenvalue of a matrix. $\|.\|$ stands for the Euclidean norm of a vector and its induced norm of a matrix. $\|x(t)\|_{\bar{h}} = \sup_{-\bar{h} \leq t \leq 0} \|x(t)\|$. For symmetric matrices P and Q, $P > Q$ ($P \geq Q$) denotes that the matrix $P - Q$ is positive definite matrix (nonnegative). $I_m, 0_n, I_{m \times n}$ and $0_{m \times n}$ mean, respectively, $m \times m$ identity matrix, $n \times n$ zero matrix, $m \times n$ identity matrix, and $m \times n$ zero matrix. $diag\{\cdots\}$ denotes a block-diagonal matrix, $sym\{P\} = P + P^T$, and $col\{p_1, p_2, ..., p_m\} = [p_1^T, p_2^T, ..., p_m^T]^T$.

2. Problem statements

Consider the following neutral singular systems with mixed delays:

$$\begin{cases} \frac{d}{dt}(Ex(t) - Cx(t - h_2)) = Ax(t) + Bx(t - h_1), \\ x(t) = \phi(t), \dot{x}(t) = \theta(t), t \in [-\bar{h}, 0], \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $A, B, C, E \in \mathbb{R}^{n \times n}$ are known constant matrices that characterize the systems. Singular matrix E satisfies $rank(E) = r < n$. $h_1 > 0$ and $h_2 > 0$ are discrete and neutral delays, respectively. The initial conditions $\phi(t), \theta(t)$ are continuous vector-valued function and $\bar{h} = \max\{h_1, h_2\}$. Since $rank(E) = r < n$, let's assume for the sake of argument that there exist two nonsingular constant matrices $\vec{M}$ and $\vec{N}$ such that

$$\vec{M}E\vec{N} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \vec{M}C\vec{N} = \begin{bmatrix} \vec{C}_1 & 0 \\ 0 & \vec{C}_2 \end{bmatrix}, \tag{2}$$

where $I_r \in \mathbb{R}^{r \times r}$ is a identity matrix. $\vec{C}_1 \in \mathbb{R}^{r \times r}, \vec{C}_2 \in \mathbb{R}^{(n-r) \times (n-r)}$ are constant matrices, respectively. Define the neutral singular operator

$$\mathcal{D} : \mathbb{C}([-h_2, 0], \mathbb{R}^n) \to \mathbb{R}^n, (\mathcal{D}x)(t) = Ex(t) - Cx(t - h_2). \tag{3}$$

To better illustrate our results, the following instrumental definitions and lemmas need to be given.

Definition 1. *[1, 34]*
(i) The pair (E, A) is said to be a regular, if $det(sE - A)$ is not identically zero;
(ii) The pair (E, A) is said to be impulse-free, if $deg(det(sE-A))=rank(E)$.

Definition 2. *[28, 34]*
(i) System (1) is said to be regular and impulse-free, if the pair (E, A) is regular and impulse free based on the stable operator $\mathcal{D}$ in (3) ;
(ii) System (1) is said to be stable if for any scalar $\varepsilon > 0$, there exists a scalar $\delta(t)$ satisfying $\sup_{-\max\{h_1, h_2\} \leq t \leq 0} \|\delta(t)\| \leq$
$\delta(\varepsilon)$, the solution $x(t)$ of system (1) satisfies $\|x(t)\| \leq \varepsilon$ for any $t \geq 0$. Furthermore, the singular system (1) is said to be asymptotically stable if it is stable and $x(t) \to 0$ as $t \to \infty$;
(iii) System (1) is said to be admissible if it is regular, impulse free and asymptotically stable.

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71 **Remark 1.** *Definitions 1-2 are feasible and reasonable. Combining with the stable operator $\mathcal{D}$ in (3) and applying the*
72 *similar decomposition method and skill of [28], the regularity and non-impulsiveness of the pair (E, A) can guarantee*
73 *the existence, uniqueness of the solution and no infinite dynamical modes for the neutral singular system (1).*

74 **Lemma 1.** *[1, 34] Consider the function $\omega : \mathbb{R}^+ \to \mathbb{R}$. If $\dot{\omega}$ is bounded on $[0, \infty)$; that is, there exists a scalar $\alpha > 0$*
75 *such that $|\dot{\omega}(t)| \leq \alpha$ for all $t \in [0, \infty)$, then ω is uniformly continuous on $[0, \infty)$.*

Lemma 2. *[1, 34] Consider the function $\omega : \mathbb{R}^+ \to \mathbb{R}$. If ω is uniformly continuous and*

$$\int_0^\infty \omega(s)ds < \infty,$$

then

$$\lim_{t \to \infty} \omega(t) = 0.$$

76 **Lemma 3.** *[30] The operator $\mathcal{D}$ is stable if $\|\vec{C}_1\| < 1$ and $\det(\vec{C}_2) \neq 0$, where $\vec{C}_1, \vec{C}_2$ are defined as in (2).*

Lemma 4. *[30] For any vectors a and b of appropriate dimensions, there exists a matrix $X = X^T > 0$ satisfying $X^T X < I$, we have*

$$-2a^T b < a^T X^{-1} a + b X^T X b.$$

77 ## 3. Main results

Theorem 1. *For given matrices C, B in system (1) satisfying $\|\tilde{C}_1\| < 1, \det(\tilde{C}_2) \neq 0, \lambda(\tilde{B}_3^T \tilde{B}_3) \neq 0$ (see (10)), system (1) is admissible if there exist matrices $P \in \mathbb{S}^n, Q_i, X_j \in \mathbb{S}^n_+, i = 1, 2, j = 1, 2, 3$ and any matrices $M_k \in \mathbb{R}^{n \times n}$ with $\bar{A}_2^T \bar{M}_{12} = 0$ (see (6)), $k = 1, 2, 3, 4$ such that the following LMIs hold:*

$$\begin{bmatrix} -I & X_i \\ \star & -I \end{bmatrix} < 0, j = 1, 2, 3, \tag{4}$$

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \star & \Gamma_{22} \end{bmatrix} < 0, \tag{5}$$

where

$$\Gamma_{11} = \begin{bmatrix} A^T PE + E^T PA + Q_1 + Q_2 + A^T M_1 + M_1^T A & E^T PB + M_2^T & -A^T PC + M_4^T & M_3 \\ \star & -Q_1 - M_2 - M_2^T & -B^T PC - M_4^T & -M_3^T \\ \star & \star & -Q_2 & -M_4 \\ \star & \star & \star & -M_3 - M_3^T \end{bmatrix},$$

$$\Gamma_{12} = \begin{bmatrix} A^T M_1 & 0 & 0 & 0 & A^T M_1 & 0 \\ 0 & M_2 & 0 & 0 & 0 & X_3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & X_1 & X_2 & 0 & 0 \end{bmatrix},$$

$$\Gamma_{22} = diag\{-X_1, -X_2, -I, -I, -X_3, -I\}.$$

Proof. *Firstly, we prove that the system (1) has the regularity and non-impulsiveness properties. Since $rank(E) = r < n$, there exist two invertible matrices $\bar{M}$ and $\bar{N}$ such that*

$$\bar{M}E\bar{N} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \bar{M}A\bar{N} = \begin{bmatrix} \bar{A}_1 & \bar{A}_2 \\ \bar{A}_3 & \bar{A}_4 \end{bmatrix}, \bar{M}^{-T} M_1 \bar{N} = \begin{bmatrix} \bar{M}_{11} & \bar{M}_{12} \\ \bar{M}_{13} & \bar{M}_{14} \end{bmatrix}, \tag{6}$$

$$\bar{N}^T Q_k \bar{N} = \begin{bmatrix} \bar{Q}_{k1} & \bar{Q}_{k2} \\ \bar{Q}_{k3} & \bar{Q}_{k4} \end{bmatrix}, \bar{M}^{-T} P \bar{M}^{-1} = \begin{bmatrix} \bar{P}_1 & \bar{P}_2 \\ \bar{P}_3 & \bar{P}_4 \end{bmatrix}, \bar{N}^T M_j \bar{N} = \begin{bmatrix} \bar{M}_{j1} & \bar{M}_{j2} \\ \bar{M}_{j3} & \bar{M}_{j4} \end{bmatrix}, \tag{7}$$

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where $j = 2, 3, 4, k = 1, 2, \bar{A}_2^T \bar{M}_{12} = 0$ and the partition is compatible with that of A in (6). From (5), we know

$$A^T PE + E^T PA + Q_1 + Q_2 + A^T M_1 + M_1^T A < 0. \tag{8}$$

Pre- and post-multiplying (8) by $\bar{N}^T$ and its transpose, respectively, we get

$$\begin{bmatrix} \bigotimes & \bigotimes \\ \bigotimes & \bar{A}_4^T \bar{M}_{14} + \bar{M}_{14}^T \bar{A}_4 + \bar{Q}_{14} + \bar{Q}_{24} \end{bmatrix} < 0, \tag{9}$$

where $\bigotimes$ is irrelevant terms for the following discussion. Since Q_1 and Q_2 are positive definite symmetric matrices, it is easy to obtain

$$\bar{A}_4^T \bar{M}_{14} + \bar{M}_{14}^T \bar{A}_4 < 0.$$

Therefore, $\bar{A}_4$ is nonsingular such that the pair (E, A) is regular and impulse free. According to Definition 2, the neutral singular system (1) is regular and impulse free. So there always another two nonsingular matrices $\tilde{M}$ and $\tilde{N}$ such that

$$\tilde{E} = \tilde{M}E\tilde{N} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \tilde{A} = \tilde{M}A\tilde{N} = \begin{bmatrix} \tilde{A}_1 & 0 \\ 0 & I \end{bmatrix}, \tilde{B} = \tilde{M}B\tilde{N} = \begin{bmatrix} \tilde{B}_1 & \tilde{B}_2 \\ \tilde{B}_3 & \tilde{B}_4 \end{bmatrix}, \tilde{C} = \tilde{M}C\tilde{N} = \begin{bmatrix} \tilde{C}_1 & 0 \\ 0 & \tilde{C}_2 \end{bmatrix}, \tag{10}$$

$$\tilde{P} = \tilde{M}^{-T}P\tilde{M}^{-1} = \begin{bmatrix} \tilde{P}_1 & \tilde{P}_2 \\ \tilde{P}_3 & \tilde{P}_4 \end{bmatrix}, \tilde{Q}_i = \tilde{N}^T Q_i \tilde{N}, \tilde{M}_1 = \tilde{M}^{-T} M_1 \tilde{N}, \tilde{M}_k = \tilde{N}^T M_k \tilde{N}, \tilde{X}_i = \tilde{N}^T X_i \tilde{N}, \tag{11}$$

where $i = 1, 2, k = 2, 3, 4$. All partition are compatible with that of A in (10).

Now, let $y(t) = \tilde{N}^{-1}x(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$, where $y_1(t) \in \mathbb{R}^r, y_2(t) \in \mathbb{R}^{n-r}$. From (10), we can rewrite (1) as

$$\begin{cases} \tilde{E}\dot{y}(t) - \tilde{C}\dot{y}(t - h_2) = \tilde{A}y(t) + \tilde{B}y(t - h_1), \\ y(t) = \varphi(t) = \tilde{N}^{-1}\phi(t), \dot{y}(t) = \Theta(t) = \tilde{N}^{-1}\theta(t), t \in [-\bar{h}, 0]. \end{cases} \tag{12}$$

It is not hard to see system (1) is stable if system (12) is stable. Next, the stability of the system (12) will be shown.

Choose an augmented LKFs as

$$V(y_t) = ((\mathcal{D}y)(t))^T \tilde{P}((\mathcal{D}y)(t)) + \int_{t-h_1}^t y^T(s)\tilde{Q}_1 y(s)ds + \int_{t-h_2}^t y^T(s)\tilde{Q}_2 y(s)ds,$$

where $y_t = y(t + \theta), -2\bar{h} \le \theta \le 0$. Calculating the time derivative of $V(y_t)$ along the solutions of systems (12) yields:

$$\dot{V}(y_t) = 2(\tilde{A}y(t) + \tilde{B}y(t - h_1))^T \tilde{P}(\tilde{E}y(t) - \tilde{C}y(t - h_2)) + y^T(t)\tilde{Q}_1 y(t) - y^T(t - h_1)\tilde{Q}_1 y(t - h_1) + y^T(t)\tilde{Q}_2 y(t) - y^T(t - h_2)\tilde{Q}_2 y(t - h_2). \tag{13}$$

Noting

$$y(t) - y(t - h_1) = \int_{t-h_1}^t \dot{y}(s)ds$$

and for any matrices $\tilde{M}_i, i = 1, 2, 3, 4$ with appropriate dimensions, we can get

$$0 = 2\Big[y^T(t)\tilde{A}^T\tilde{M}_1 + y^T(t - h_1)\tilde{M}_2 + \int_{t-h_1}^t \dot{y}^T(s)ds\tilde{M}_3 + y^T(t - h_2)\tilde{M}_4\Big]\Big[y(t) - y(t - h_1) - \int_{t-h_1}^t \dot{y}(s)ds\Big]. \tag{14}$$

Let $\mathfrak{y}(t) = \int_{t-h_1}^t \dot{y}(s)ds$ and combining with (13)-(14) yields

$$\begin{aligned} \dot{V}(y_t) = &sym\{y^T(t)\tilde{A}^T\tilde{P}\tilde{E}y(t) + y^T(t - h_1)\tilde{B}^T\tilde{P}\tilde{E}y(t) - y^T(t)\tilde{A}^T\tilde{P}\tilde{C}y(t - h_2) - y^T(t - h_1))\tilde{B}^T\tilde{P}\tilde{C}y(t - h_2)\} \\ &+ y^T(t)\tilde{Q}_1 y(t) - y^T(t - h_1)\tilde{Q}_1 y(t - h_1) + y^T(t)\tilde{Q}_2 y(t) - y^T(t - h_2)\tilde{Q}_2 y(t - h_2) \\ &+ 2y^T(t)\tilde{A}^T\tilde{M}_1 y(t) + 2y^T(t)\tilde{M}_2^T y(t - h_1) - 2y^T(t)\tilde{A}^T\tilde{M}_1 y(t - h_1) - 2y^T(t - h_1)\tilde{M}_2 y(t - h_1) \\ &- 2y^T(t - h_1)\tilde{M}_3^T \mathfrak{y}(t) - 2y^T(t - h_1)\tilde{M}_2^T \mathfrak{y}(t) - 2\mathfrak{y}^T(t)\tilde{M}_3 \mathfrak{y}(t) + 2y^T(t)\tilde{M}_3 \mathfrak{y}(t) - 2y^T(t)\tilde{A}^T\tilde{M}_1 \mathfrak{y}(t) \\ &+ 2y^T(t)\tilde{M}_4^T y(t - h_2) - 2y^T(t - h_1)\tilde{M}_4^T y(t - h_2) - 2\mathfrak{y}^T(t)\tilde{M}_4^T y(t - h_2). \end{aligned} \tag{15}$$

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In addition, by Lemma 4, there exist three matrices $\tilde{X}_i \in \mathbb{S}_+^n, i = 1, 2, 3$ satisfying $\tilde{X}_i^T \tilde{X}_i < I$ such that

$$-2y^T(t)\tilde{A}^T \tilde{M}_1 \mathfrak{y}(t) < y^T(t)\tilde{A}^T \tilde{M}_1 \tilde{X}_1^{-1} \tilde{M}_1^T \tilde{A}y(t) + \mathfrak{y}^T(t)\tilde{X}_1^T \tilde{X}_1 \mathfrak{y}(t), \tag{16}$$

$$-2y^T(t-h_1)\tilde{M}_2 \mathfrak{y}(t) < y^T(t-h_1)\tilde{M}_2 \tilde{X}_2^{-1} \tilde{M}_2^T y(t-h_1) + \mathfrak{y}^T(t)\tilde{X}_2^T \tilde{X}_2 \mathfrak{y}(t), \tag{17}$$

$$-2y^T(t)\tilde{A}^T \tilde{M}_1 y(t-h_1) < y^T(t)\tilde{A}^T \tilde{M}_1 \tilde{X}_3^{-1} \tilde{M}_1^T \tilde{A}y(t) + y^T(t-h_1)\tilde{X}_3^T \tilde{X}_3 y(t-h_1). \tag{18}$$

Substituting (16)-(18) into (15), yields

$$\dot{V}(y_t) \leq \xi^T(t) \prod \xi(t), \tag{19}$$

where

$$\prod = \begin{bmatrix} \Pi_1 & \tilde{E}^T \tilde{P}\tilde{B} + \tilde{M}_2^T & -\tilde{A}^T \tilde{P}\tilde{C} + \tilde{M}_4^T & \tilde{M}_3 \\ \star & -\tilde{Q}_1 - \tilde{M}_2 - \tilde{M}_2^T + \tilde{M}_2 \tilde{X}_2^{-1} \tilde{M}_2^T + \tilde{X}_3^T \tilde{X}_3 & -\tilde{B}^T \tilde{P}\tilde{C} - \tilde{M}_4^T & -\tilde{M}_3^T \\ \star & \star & -\tilde{Q}_2 & -\tilde{M}_4 \\ \star & \star & \star & -\tilde{M}_3 - \tilde{M}_3^T + \tilde{X}_1^T \tilde{X}_1 + \tilde{X}_2^T \tilde{X}_2 \end{bmatrix},$$

$$\Pi_1 = \tilde{A}^T \tilde{P}\tilde{E} + \tilde{E}^T \tilde{P}\tilde{A} + \tilde{Q}_1 + \tilde{Q}_2 + \tilde{A}^T \tilde{M}_1 + \tilde{M}_1^T \tilde{A} + \tilde{A}^T \tilde{M}_1 \tilde{X}_1^{-1} \tilde{M}_1^T \tilde{A} + \tilde{A}^T \tilde{M}_1 \tilde{X}_3^{-1} \tilde{M}_1^T \tilde{A},$$

$$\xi(t) = col\{y(t), y(t-h_1), y(t-h_2), \mathfrak{y}\}.$$

Combining with (4), (5) and (19), and utilizing Schur complement lemma, we know $\prod < 0$, which implies there exists a scalar $\lambda_0 > 0$ such that

$$\dot{V}(y_t) \leq -\lambda_0(\|y(t)\|^2 + \|y(t-h_2)\|^2). \tag{20}$$

Moreover, we have

$$\begin{aligned} \tau_1 \|y_2(t-h_2)\|^2 - V(y_0) &\leq (\mathfrak{D}y)^T(t)\tilde{P}(\mathfrak{D}y)(t) - V(y_0) \\ &\leq V(y_t)) - V(y_0) \\ &= \int_0^t \dot{V}(y_s)ds \\ &< 0, \end{aligned} \tag{21}$$

where $\tau_1 = \min\{|\lambda(\tilde{P})\lambda^2(\tilde{C}_2)|\}$. It is not hard to see (21) yields

$$\|y_2(t)\| < \sqrt{\frac{1}{\tau_1}V(y_0)} < \infty. \tag{22}$$

On the other hand, (12) can be rewritten as

$$\begin{cases} \dot{y}_1(t) = \tilde{A}_1 y_1(t) + \tilde{B}_1 y_1(t-h_1) + \tilde{B}_2 y_2(t-h_1) + \tilde{C}_1 \dot{y}_1(t-h_2)), \\ 0 = y_2(t) + \tilde{B}_3 y_1(t-h_1) + \tilde{B}_4 y_2(t-h_1) + \tilde{C}_2 \dot{y}_2(t-h_2), \\ y(t) = \varphi(t) = \tilde{N}^{-1}\phi(t), \dot{y}(t) = \Theta(t) = \tilde{N}^{-1}\theta(t), t \in [-\bar{h}, 0]. \end{cases} \tag{23}$$

By using the similar analysis method of [35], we can define a function as $W(y_t, t) = e^{\varepsilon t}V(y_t), t \geq \bar{h}$ wherein the scalar $\varepsilon > 0$. Noting that the definition of $V(y_t)$, there exist scalars $\lambda_1, \bar{\lambda}_1, \lambda_2$ such that for any $t \geq \bar{h}$,

$$V(y_t) \leq \lambda_1 \|y(t)\|^2 + \bar{\lambda}_1 \|y(t-h_2)\|^2 + \lambda_2 \int_{t-\bar{h}}^t \|y(s)\|^2 ds. \tag{24}$$

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Thus, for any $t \geq \bar{h}$ yields

$$W(y_t, t) - W(y_{\bar{h}}, \bar{h})$$

$$\leq \int_{\bar{h}}^{t} e^{\varepsilon s} \Big[\varepsilon V(y_s) - \lambda_0 \|y(s)\|^2 - \lambda_0 \|y(s - h_2)\|^2 \Big] ds$$

$$\leq \int_{\bar{h}}^{t} e^{\varepsilon \alpha} \Big[\varepsilon \lambda_1 \|y(\alpha)\|^2 + \varepsilon \bar{\lambda}_1 \|y(\alpha - h_2)\|^2 + \varepsilon \lambda_2 \int_{\alpha - \bar{h}}^{\alpha} \|y(s)\|^2 ds - \lambda_0 \|y(\alpha)\|^2 - \lambda_0 \|y(\alpha - h_2)\|^2 \Big] d\alpha$$

$$\leq \int_{\bar{h}}^{t} e^{\varepsilon \alpha} \varepsilon \lambda_1 \|y(\alpha)\|^2 d\alpha + \int_{\bar{h}}^{t} e^{\varepsilon \alpha} \varepsilon \bar{\lambda}_1 \|y(\alpha - h_2)\|^2 d\alpha + \varepsilon \lambda_2 \bar{h} e^{\varepsilon \bar{h}} \int_{\bar{h}}^{t} e^{\varepsilon \alpha} \|y(\alpha)\|^2 d\alpha$$

$$- \int_{\bar{h}}^{t} e^{\varepsilon \alpha} \lambda_0 \|y(\alpha)\|^2 d\alpha - \int_{\bar{h}}^{t} e^{\varepsilon \alpha} \lambda_0 \|y(\alpha - h_2)\|^2 d\alpha + \bar{h}^2 e^{2\varepsilon \bar{h}} \|\varphi(t)\|_{\bar{h}}^2. \tag{25}$$

It is easy to see that ε is small enough such that $\max\{\varepsilon \bar{\lambda}_1, \varepsilon \lambda_1 + \varepsilon \lambda_2 \bar{h} e^{\varepsilon \bar{h}}\} \leq \lambda_0$. Therefore, by (25), for any $t \geq \bar{h}$, there exists a scalar $\kappa > 0$ such that

$$V(y_t, t) \leq \kappa e^{-\varepsilon t} \|\varphi(t)\|_{\bar{h}}^2. \tag{26}$$

From (25) and considering $\tau_1 \|y_2(t - h_2)\|^2 \leq (\mathfrak{D} y)^T(t) \tilde{P} (\mathfrak{D} y)(t) \leq V(y_t, t) \leq \kappa e^{-\varepsilon t} \|\varphi(t)\|_{\bar{h}}^2$, we have for any $t \geq \bar{h} - h_2$

$$\|y_2(t)\| \leq \|\varphi(t)\|_{\bar{h}}^2 \frac{\kappa}{\tau_1} e^{-\frac{\varepsilon}{2} t}. \tag{27}$$

Integrating both sides of (27) from $\bar{h} - h_2$ to t, we get that

$$\int_{\bar{h} - h_2}^{t} \|y_2(s)\| ds < \infty. \tag{28}$$

According to (22) and (28), yields

$$\int_{0}^{t} \|y_2(s)\| ds = \int_{0}^{\bar{h} - h_2} \|y_2(s)\| ds + \int_{\bar{h} - h_2}^{t} \|y_2(s)\| ds < \infty. \tag{29}$$

Since (29), we obtain

$$\int_{0}^{t} \|y_2(s - h_1)\| ds < \infty. \tag{30}$$

Applying (22), (29), (30), $\lambda(\tilde{B}_3^T \tilde{B}_3) \neq 0$ and integrating the sides of the second equation of (23) from 0 to t, we have

$$\left\| \int_{0}^{t} y_1(s - h_1) ds \right\| < \infty. \tag{31}$$

Utilizing (31), we can deduce that

$$\left\| \int_{0}^{t} y_1(s) ds \right\| < \infty. \tag{32}$$

Similarly, integrating the sides of the first equation of (23) from 0 to t, there exists a scalar $L_0 > 0$ such that

$$\|y_1(t)\| \leq L_0 + \|\tilde{C}_1\| \|y_1(t - h_2)\|. \tag{33}$$

Since $\|\tilde{C}_1\| < 1$, it easy to get

$$\|y_1(t)\| < \infty. \tag{34}$$

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By (22), (34) and det($\tilde{C}_2$) ≠ 0, it can be found from the second equation of (23) that

$$\|\dot{y}_2(t)\| < \infty. \tag{35}$$

Next, we will show $\|\dot{y}_1(t)\| < \infty$. From (22)and (34) and considering the expression of the first equation in (23), there exists a scalar $L_1 > 0$ such that

$$\|\dot{y}_1(t)\| \leq L_1 + \|\tilde{C}_1\| \|\dot{y}_1(t - h_2)\|. \tag{36}$$

Since $\|\dot{y}_1(t - h_2)\| \leq L_1 + \|\tilde{C}_1\| \|\dot{y}_1(t - 2h_2)\|$, then (36) can reduce into

$$\|\dot{y}_1(t)\| \leq L_1 + L_1\|\tilde{C}_1\| + \|\tilde{C}_1\|^2 \|\dot{y}_1(t - 2h_2)\|. \tag{37}$$

From (36)-(37) and using the recursive method, we can get

$$\|\dot{y}_1(t)\| \leq L_1 + L_1\|\tilde{C}_1\| + L_1\|\tilde{C}_1\|^2 + ... + L_1\|\tilde{C}_1\|^k + \|\tilde{C}_1\|^{k+1}\|\dot{y}_1(t - (k+1)h_2)\|, k \in \mathbb{N}^+. \tag{38}$$

Taking into account (38), there exists an integer k satisfying $t - kh_2 > 0$ and $t - (k+1)h_2 < 0$. Noting that initial condition $\Theta(t)$ is a continuous in $[-\bar{h}, 0]$, we can conclude that

$$\|\dot{y}_1(t)\| < \infty. \tag{39}$$

By (35), (39) and Lemma 1, we know that $y(t)$ is uniformly continuous. Therefore, noting (29), (32) and Lemma 2, we obtain

$$\lim_{t\to\infty} \|y(t)\| = 0.$$

Thus, the system (12) is stable. That is, (1) is stable. By Definition 1-2, we can deduce that the system (1) is admissible. This completes the proof.

Remark 2. *The conditions, $\|\tilde{C}_1\| < 1, \det(\tilde{C}_2) \neq 0$, are indispensable. In fact, by Lemma 3, the operator $\mathcal{D}$ is stable, which ensures the existence and uniqueness of the solution for the neutral singular systems (1)*

Remark 3. *In this paper, the neutral singular systems are with mixed delays, which means the stability criteria in [30, 33] is not suitable for the case with mixed delays.*

Remark 4. *In [29], Li et al. have been studied stability of neutral type descriptor system with mixed delays. Compared with the Theorem 5 in [29], the superiority of the Theorem 1 in this paper can be shown from the following three aspects. First, the stability criterion of Theorem 1 in this paper is less conservative and more general in form. It is not difficult to find out that Theorem 1 can reduce to Theorem 5 in [29] when free matrices $M_i = 0, i = 1, 2, 3, 4$. Second, the regularity and non-impulsiveness can be guaranteed in Theorem 1 according to the given sufficient conditions, while they can not be obtained from Theorem 5 in [29], which implies the results presented in [29] may be invalid. Third, the proof of asymptotic stability is not strictly given due to the existence of singular matrix E and state derivative term $\dot{x}(t - h_2)$, which may cause the solution $x(t) \to 0$ can not be obtained. Nevertheless, in this paper, these issues are addressed effectively.*

Remark 5. *In [31], Zhao et al. studied the stability of neutral-type descriptor systems with multiple time-varying delays. However, it has come to our notice that the principal diagonal block matrix terms $-(1 - \tau_i)E^T S_k E$ ($S_k > 0, \tau_i < 1$) appear in linear matrix inequalities, which may cause the stability criterion infeasibility owing to the existence of singular matrix E. In [32], the stability criterion may be not acquired since the inequality $-(1 - v)\dot{y}(t - h(t))\bar{E}^T \bar{Q}_2 \bar{E}\dot{y}(t - h(t)) \leq -(1 - v)\dot{y}(t - h(t))(\bar{E}^T + \bar{E} + 2mI + m^2\bar{Q}_2)\dot{y}(t - h(t))$ can not be established. In this paper, the zero equation (14) on the basis of Leibniz-Newton formula plays an important role in ensuring the stability criterion to be feasible by utilizing MATLAB LMI control toolbox.*

7

Consider the following neutral singular system:

$$\begin{cases} E\dot{x}(t) - C\dot{x}(t - h_1) = Ax(t) + Bx(t - h_1), \\ x(t) = \phi(t), \dot{x}(t) = \theta(t), t \in [-h_1, 0]. \end{cases} \tag{40}$$

The stability analysis of the neutral singular system (40) have been discussed in [30, 33]. However, we can see that the singular neutral system (40) is a special case of the system (1). Assuming $Q_2 = 0, M_4 = 0$ in Theorem 1 and similar to the proof of Theorem 1, the following Corollary 1 can easily be acquired.

Corollary 1. *For given matrices C, B in system (1) satisfying $\|\tilde{C}_1\| < 1, \det(\tilde{C}_2) \neq 0, \lambda(\tilde{B}_3^T \tilde{B}_3) \neq 0$ (see (10)), system (40) is asymptotically stable if there exist matrices $P \in \mathbb{S}_+^n$, $Q_1, X_i \in \mathbb{S}_+^n, i = 1, 2, 3$ and any matrices $M_j \in \mathbb{R}^{n \times n}$ with $\bar{A}_2^T \bar{M}_{12} = 0$ (see (6)), $j = 1, 2, 3$ such that the following LMIs hold:*

$$\begin{bmatrix} -I & X_i \\ \star & -I \end{bmatrix} < 0, i = 1, 2, 3,$$

$$\begin{bmatrix} \Delta_{11} & \Delta_{12} \\ \star & \Delta_{22} \end{bmatrix} < 0,$$

where

$$\Delta_{11} = \begin{bmatrix} A^T PE + E^T PA + Q_1 + A^T M_1 + M_1^T A & E^T PB - A^T PC + M_2^T & M_3 \\ \star & -B^T PC - C^T PB - Q_1 - M_2 - M_2^T & -M_3^T \\ \star & \star & -M_3 - M_3^T \end{bmatrix},$$

$$\Delta_{12} = \begin{bmatrix} A^T M_1 & 0 & 0 & 0 & A^T M_1 & 0 \\ 0 & M_2 & 0 & 0 & 0 & X_3 \\ 0 & 0 & X_1 & X_2 & 0 & 0 \end{bmatrix},$$

$$\Delta_{22} = diag\{-X_1, -X_2, -I, -I, -X_3, -I\}.$$

Remark 6. *The stability criteria obtained in [30] have focused mainly on cases with identical delays in discrete and neutral terms. Nevertheless, the regularity and non-impulsiveness problems were not considered, which may lead to a certain degree of conservatism. Moreover, the asymptotic stability can not be acquired directly from inequality $\dot{V}(x(t)) < -\rho\|x(t)\|^2, \rho > 0$ due to the singularity of E. To overcome these shortcomings, new stability criteria in Corollary 1 in this paper are proposed. In addition, in [33], Long et al. also studied similar situations. But the stability criterion in Corollary 2 may be with some errors because the conditions $ES = C$ is not established wherein C satisfying the equation (77), which makes the zero equation (31) with free-weighting matrices untenable. In short, our results are more effective and less conservative than the ones obtained in previous literatures.*
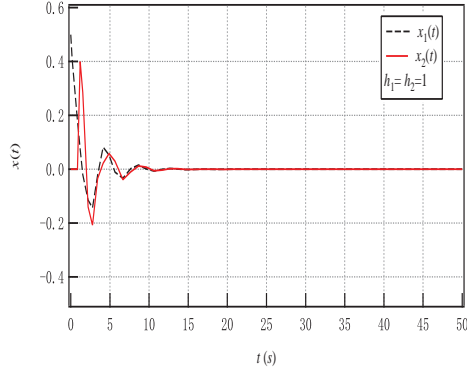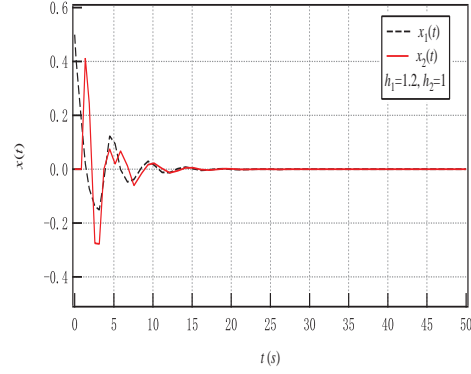
4. Numerical examples

In this section, three numerical examples and simulation studies are given to show the application and admissibility for the neutral singular system (1) with mixed delays.

Table 1: Feasibility comparisons for different stability criterion in Example 1

Cases	Theo 5 in [29]	Theo 1 in [31, 32]	Coro 2 in [33]	Theo 3.2 in [30]	Coro 1	Theo 1
$h_1 = h_2$	NO	NO	NO	NO	YES	YES
$h_1 \neq h_2$	NO	NO	NO	NO	NO	YES

Example 1. *Consider the neutral singular system (1) with parameters as*

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -1.2 & 0 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} -0.6 & 0.7 \\ -1 & -0.8 \end{bmatrix}, C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

Fig. 1: Trajectories of the state vector $x(t)$ in Example 1.

Fig. 2: Trajectories of the state vector $x(t)$ in Example 1.

117   We can easily verify that the system (1) has regularity and non-impulsiveness. Therefore, there exist two in-

118   vertible matrices $\tilde{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tilde{N} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ such that $\tilde{M}E\tilde{N} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \tilde{M}A\tilde{N} = \begin{bmatrix} -1.2 & 0 \\ 0 & 1 \end{bmatrix}, \tilde{M}B\tilde{N} =$

119    $\begin{bmatrix} -0.6 & -0.7 \\ -1 & 0.8 \end{bmatrix}, \tilde{M}C\tilde{N} = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix}$. Table 1 lists the feasibility comparisons for different stability criterion in

120   [29–33], respectively. 'YES' means the corresponding stability criterion is feasible and 'NO' means the correspond-

121   ing stability criterion is not feasible. Meanwhile, 'Theo' and 'Coro' denote 'Theorem' and 'Corollary', respectively.

122   Thus, we can see that the stability criterion in this paper are feasible based on $h_1 = h_2$ and $h_1 \neq h_2$, while those

123   in [29–33] are not. Without loss of generally, suppose that the initial function is $\phi(t) = \begin{bmatrix} 0.5 & 0 \end{bmatrix}^T$. The results

124   of numerical simulations under $h_1 = h_2$ and $h_1 \neq h_2$ are presented in Fig.1 and Fig.2, respectively, which show the

125   effectiveness and feasible of the Theorem 1 and Corollary 1 .
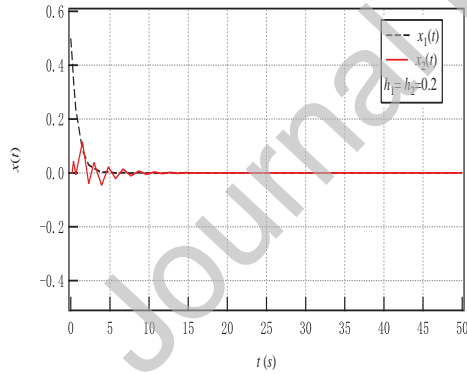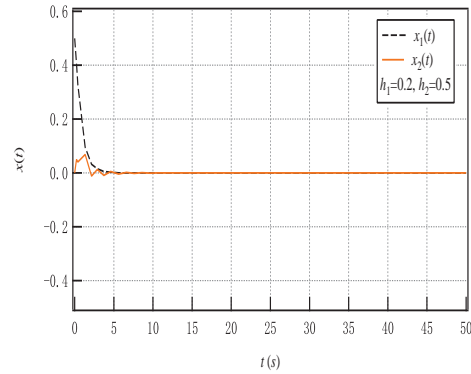

Fig. 3: Trajectories of the state vector $x(t)$ in Example 2.

Fig. 4: Trajectories of the state vector $x(t)$ in Example 2.

Table 2: Feasibility comparisons for different stability criterion in Example 2

Cases	Theo 5 in [29]	Theo 1 in [31, 32]	Coro 2 in [33]	Theo 3.2 in [30]	Coro 1	Theo 1
$h_1 = h_2$	NO	NO	NO	YES	YES	YES
$h_1 \neq h_2$	NO	NO	NO	NO	NO	YES

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Example 2. *System (1) with the following parameters was investigated*

$$E = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} -0.9 & -1.8 \\ -1 & -1 \end{bmatrix}, B = \begin{bmatrix} -0.1 & -0.3 \\ -0.1 & -0.2 \end{bmatrix}, C = \begin{bmatrix} 0.1 & 0.2 \\ -0.1 & -0.1 \end{bmatrix}.$$

It is not hard to see that the pair (E, A) is regular and impulse-free. Let invertible matrices $\tilde{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\tilde{N} = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$, we can get from (10) that $\tilde{M}E\tilde{N} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \tilde{M}A\tilde{N} = \begin{bmatrix} -0.9 & 0 \\ 0 & 1 \end{bmatrix}, \tilde{M}B\tilde{N} = \begin{bmatrix} -0.2 & -0.1 \\ -0.1 & 0 \end{bmatrix}, \tilde{M}C\tilde{N} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$. Then, in order to check the admissibility of this singular system, the feasibility comparisons for different stability criteria in this neutral singular system are listed in Table 2. It is easy to see that the stability criterion in Theorem 1 and Corollary 1 are feasible under $h_1 \neq h_2$ and $h_1 = h_2$, respectively; and infeasible in [29, 31–33]. Although the stability criterion in Theorem 3.2 in [30] is feasible, it is invalid if $h_1 \neq h_2$. Thus, our results are less conservative than those in [29–33]. To this end, in view of the simulation studies, suppose that the initial function is $\phi(t) = \begin{bmatrix} 0.5 & 0 \end{bmatrix}^T$. The trajectories of the states for this system are confirmed via simulations in Matlab/Simulink environment, which are shown in Fig.3 and Fig.4 based on $h_1 \neq h_2$ and $h_1 = h_2$, respectively.
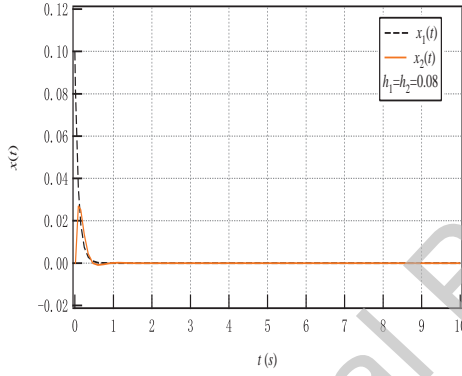

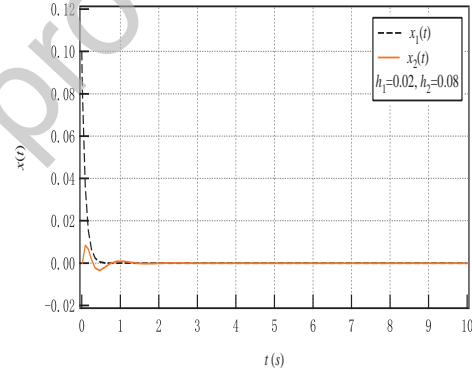Fig. 5: Trajectories of the state vector $x(t)$ in Example 3.

Fig. 6: Trajectories of the state vector $x(t)$ in Example 3.

Table 3: Feasibility comparisons for different stability criterion in Example 3

Cases	Theo 5 in [29]	Theo 1 in [31, 32]	Coro 2 in [33]	Theo 3.2 in [30]	Coro 1	Theo 1
$h_1 = h_2$	NO	NO	NO	NO	YES	YES
$h_1 \neq h_2$	NO	NO	NO	NO	NO	YES

Example 3. *Let us consider the System (1) with the following parameters*

$$E = \begin{bmatrix} 9 & 3 \\ 6 & 2 \end{bmatrix}, A = \begin{bmatrix} -104.6 & -35.2 \\ -70.4 & -23.8 \end{bmatrix}, B = \begin{bmatrix} -0.1 & -0.2 \\ -0.1 & -0.1 \end{bmatrix}, C = \begin{bmatrix} -0.16 & -0.05 \\ -0.1 & -0.03 \end{bmatrix}.$$

Obviously, the pair (E, A) is regular and impulse free. There exist two invertible matrices $\tilde{M} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix}$ and $\tilde{N} = \begin{bmatrix} 1 & 1 \\ -2 & -3 \end{bmatrix}$ such that $\tilde{M}E\tilde{N} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \tilde{M}A\tilde{N} = \begin{bmatrix} -11.4 & 0 \\ 0 & 1 \end{bmatrix}, \tilde{M}B\tilde{N} = \begin{bmatrix} 0.2 & 0.3 \\ -0.3 & -0.4 \end{bmatrix}, \tilde{M}C\tilde{N} = \begin{bmatrix} -0.02 & 0 \\ 0 & -0.01 \end{bmatrix}$. In Table 3, it is clear that the stability conditions in Theorems 1 and Corollary 1 are effective

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138 for this neutral singular system ensuring the admissibility of the neutral singular system. Nevertheless, the stability
139 criterion in [29–33] are infeasible, which indicates the validity and superiority of our results in this paper. Suppose that
140 the initial function is $\phi(t) = \begin{bmatrix} 0.1 & 0 \end{bmatrix}^T$. The trajectories of states for the system (1) in example 3 are demonstrated
141 in Fig.5 and Fig.6 in two cases $h_1 = h_2 = 0.08$ and $h_1 = 0.02, h_2 = 0.08$, respectively , which show the asymptotical
142 stability of the system.

5. Conclusion

144 This paper has discussed the stability problem for neutral singular systems with mixed delays. According to the
145 singular system characteristics and Lyapunov-Krasovskill functional methodology, new stability criteria are obtained
146 ensuring the systems to be regular, impulse free and asymptotically stable. To this end, the zero equation with free-
147 weighting matrices is indispensable in order to obtain feasible stability criteria and better use MATLAB LMI control
148 toolbox. Furthermore, the asymptotically stability for neutral singular systems are further explored. Finally, the
149 advantages and admissibility of the proposed stability criteria are demonstrated by three numerical examples. In
150 addition, the research will not be stopped at any time. It will be our future work to be further and deeper research
151 related to the stability criterion with dependent time-varying delays.

Acknowledgment

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Conflict of Interest Statement

We declare that we have no financial and personal relationships with other people or organizations that can inappropriately influence our work. There is no professional or other personal interest of any nature or kind in any product, service and/or company that could be construed as influencing the position presented in, or the review of, the manuscript entitled, " New results on stability analysis for a kind of neutral singular systems with mixed delays".

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