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On exponential stability of linear singular positive delayed systems

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ABSTRACT

In this paper, the problem of positivity and exponential stability for linear singular positive systems with time delay is addressed. By using the singular value decomposition method, necessary and sufficient conditions for the positivity of the system are established. Based on that, a new sufficient condition for exponential stability of the system is derived. All of the criteria obtained in this paper are presented in terms of algebraic matrix inequalities, which make the conditions can be solved directly. A numerical example is given to show the usefulness of the proposed results.

1. Introduction

Singular systems (also called descriptor systems, implicit systems or differential–algebraic systems) arise in a variety of practical systems such as engineering systems, economic systems, network analysis, biological systems, etc. [1,2]. Since singular systems with delays are matrix delay differential equations coupled with matrix difference equations, the study of such systems is much more complicated than that for standard state-space delayed systems. Many significant results based on the theory of regular systems have been extended to the area of singular systems, see e.g. [3–7] and references therein. However, physical systems in the real world involve variables that have nonnegative sign, say, population levels, absolute temperature, and so on. Such systems are referred to as positive systems [8,9], which means that their states and outputs are nonnegative whenever the initial conditions and inputs are nonnegative. Since the states of positive systems are located in the positive orthant rather than in linear spaces, many well-known results for general linear systems cannot be readily applied to positive systems. Moreover, due to the singularity of derivative matrix and the non-negativity of variables in positive singular systems, much of the developed theory for such systems is still not up to a quantitative level. This feature makes the analysis and synthesis of positive systems a challenging and interesting job. Stability analysis of standard positive delayed systems has been studied by many authors (see, e.g. [10–13] and the references therein). Very little is known about positive singular systems with time delay up to now, however, some properties mainly in the undelayed or discrete-time case were studied in [14,15].

In this paper, we investigate the problem of exponential stability of linear singular positive systems with time delay. Firstly, a necessary and sufficient condition for the positivity of linear singular positive delayed systems is provided. Then by using a delay decomposition state-space method, new sufficient conditions for exponential stability are proposed in terms of solutions of some matrix inequalities.

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2. Preliminaries

The following notation will be used in this paper. $R^n_{0,+}$ denotes the space of all nonnegative vectors in R^n; $R^{m\times n}$ denotes the set of all real $(m \times n)$ matrices. $PC([-h, 0], R^n)$ denotes the space of all piecewise continuous functions defined on $[-h, 0]$. A vector $x \in R^n$ is called nonnegative if all entries are nonnegative. I_n is the n-dimensional identity matrix. A matrix $B \in R^{n\times n}$ is called Metzler if all its off diagonal elements are non-negative. A matrix is called a monomial matrix if its every row and its every column contains only one positive entry and the remaining entries are zero. A matrix $B \in R^{m\times n}$ is called nonnegative if all its entries are nonnegative. The nonnegative matrix B will be denoted by $B \succeq 0$. A nonnegative matrix $B \succeq 0$ is called positive if at least one of its entries is positive. The positive matrix B will be denoted by $B \succ 0$. The notation $A \succeq B (A \succ B)$ means that $A - B \succeq 0 (A - B \succ 0)$.

Consider a linear singular system with time delay described by

$$\begin{cases} E\dot{x}(t) = A_0x(t) + A_1x(t-h), & t \geq 0, \\ x(t) = \varphi(t), & t \in [-h, 0], \end{cases} \tag{2.1}$$

where $x(t) \in R^n$ is the state vector, $h > 0, A_0, A_1 \in R^{n\times n}$. The matrix $E \in R^{n\times n}$ is singular and assume that $\operatorname{rank} E = r < n$, $\varphi(t) \in PC([-h, 0], R^n)$.

Definition 2.1 ([2]). (i) The pair (E, A_0) is said to be regular if $\det(sE - A_0)$ is not identically zero. (ii) The pair (E, A_0) is said to be impulse-free if $\deg(\det(sE - A_0)) = \operatorname{rank}(E)$. (iii) The singular delay system (2.1) is said to be regular and impulse-free if the pair (E, A_0) is regular and impulse-free.

Remark 2.1. As in [11,14] we consider the piecewise continuous space of initial conditions $\varphi(t) \in PC([-h, 0], R^n)$. The singular delay system (2.1) may have an impulsive solution, however, the regularity and the absence of impulses of the pair (E, A_0) ensure the existence and uniqueness of an impulse-free solution to system (2.1) with any given initial piecewise continuous vector function (see [16, Theorem 1.2]).

In practice, it is often not enough demanding that the system is asymptotically stable. Then it is desirable that the system can converge quickly that is, has a certain decay rate. To emphasize the decay rate and discuss whether and how the stability changes with the delay's changing, similarly to [3,10], we give the following definition of exponential stability for system (2.1).

Definition 2.2. The singular delay system (2.1) is said to be α-exponentially stable if there exists a positive number $N > 0$ such that, for any initial conditions $\varphi(t)$ the solution $x(t, \varphi)$ satisfies

$$\|x(t, \varphi)\| \leq Ne^{-\alpha t}\|\varphi\|, \quad \forall t \geq 0.$$

Definition 2.3 ([8]). System (2.1) is said to be positive if for any initial positive condition $\varphi : [-h, 0] \to R^n_{0,+}$, the solution $x(t) \succeq 0$ for all $t \geq 0$.

We introduce the following technical propositions, which will be used in the proof of main result.

Lemma 2.1 ([8]). *Let $A \in R^{n\times n}$. Then $e^{At} \succ 0$ for $t \geq 0$ if and only if A is the Metzler matrix. Moreover, the inverse matrix of a positive matrix is positive if and only if it is a monomial matrix.*

Note that the regularity and the absence of impulses of the pair (E, A_0) imply that there exist two invertible matrices P, Q such that ([2])

$$PEQ = \begin{pmatrix} I_r & 0 \\ 0 & 0_{n-r} \end{pmatrix}, \qquad PA_0Q = \begin{pmatrix} A_{01} & A_{02} \\ A_{03} & A_{04} \end{pmatrix}, \qquad PA_1Q = \begin{pmatrix} A_{11} & A_{12} \\ A_{13} & A_{14} \end{pmatrix}.$$

Under coordinate transformation $y(t) = Q^{-1}x(t) = [y_1(t), y_2(t)]$ where $y_1(t) \in R^r$, $y_2(t) \in R^{n-r}$, the system (2.1) is reduced to the system

$$\begin{cases} \dot{y}_1(t) = \bar{A}_{01}y_1(t) + \bar{A}_{11}y_1(t-h) + \bar{A}_{12}y_2(t-h), & y_1(t) = \psi_1(t), \\ y_2(t) = -A_{04}^{-1}\big[A_{03}y_1(t) + A_{13}y_1(t-h) + A_{14}y_2(t-h)\big], & y_2(t) = \psi_2(t), \end{cases} \tag{2.2}$$

where $\bar{A}_{01} = A_{01} - A_{02}A_{04}^{-1}A_{03}$, $\bar{A}_{11} = A_{11} - A_{02}A_{04}^{-1}A_{13}$, $\bar{A}_{12} = A_{12} - A_{02}A_{04}^{-1}A_{14}$.

Lemma 2.2. *Assume that (E, A_0) is regular and impulse-free, Q is a monomial matrix and $\det A_{04} \neq 0$. Then the system (2.1) is positive if and only if the system (2.2) is positive.*

Proof. Suppose that system (2.1) is positive. We have $y(t) = Q^{-1}x(t)$ and from Lemma 2.1, $Q^{-1} \succ 0$ if and only if Q is a monomial matrix, it follows that $y(t) \succeq 0$, $t \geq 0$. Suppose that system (2.2) is positive, i.e. $y(t) \succeq 0$, $t \geq 0$, we have $x(t) = Qy(t) \succeq 0$, $t \geq 0$ because $Q \succ 0$.

3. Positivity of singular delayed systems

In this subsection, we will firstly present a necessary and sufficient condition for the positivity of linear singular positive system with delay.

Theorem 3.1. *Assume the conditions stated in Lemma 2.2. Linear system (2.2) is positive if and only if* $\bar{A}_{01}$ *is Metzler and* $\bar{A}_1 \succeq 0,\ -A_{04}^{-1}A_{03} \succeq 0$, *where*

$$\bar{A}_{01} = A_{01} - A_{02}A_{04}^{-1}A_{03}, \qquad \bar{A}_1 = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ -A_{04}^{-1}A_{13} & -A_{04}^{-1}A_{14} \end{bmatrix}.$$

Proof. *Necessity*. Suppose that system (2.2) is positive. Consider the first equation of system (2.2) on $[0, h]$ with the initial condition:

$$\psi_1(0) = e_j, \qquad \psi_1(t) = 0, \quad t \in [-h, 0); \qquad \psi_2(t) = 0, \quad t \in [-h, 0],$$

where $e_j, j = 1, 2, \ldots, r$ denotes the jth vector of the canonical basis in R^r. We then have the solution on $[0, h]$

$$y_1(t) = e^{\bar{A}_{01}t}e_i, \quad i = 1, 2, \ldots, r,$$

is positive so that the matrix $e^{\bar{A}_{01}t}$, $t \in [0, h]$ is positive. By Lemma 2.1, the matrix $\bar{A}_{01}$ is Metzler. We now prove $\bar{A}_{11}, \bar{A}_{12}$ are non-negative. Suppose that there exist some indices i, j such that $[\bar{A}_{11}]_{ij} \prec 0$. Taking the initial condition

$$\psi_1(0) = 0, \qquad \psi_1(t) = e_j, \quad t \in [-h, 0); \qquad \psi_2(t) = 0, \quad t \in [-h, 0],$$

For $t > 0$ small enough, $t < h$ and $-h < t - h < 0$, from the first equation of (2.2) we have the solution

$$y_1(t) = \int_0^t e^{\bar{A}_{01}(t-s)}\bar{A}_{11}e_j\, ds \succeq 0.$$

Then we have

$$[y_1(t)]_i = \int_0^t \left[\left(I + \bar{A}_{01}(t-s) + O_1((t-s)^2)\right)\bar{A}_{11}e_j\right]_i ds = \int_0^t \left([\bar{A}_{11}]_{ij} + O_2((t-s))\right) ds \prec 0,$$

which contradicts the fact that $y_1(t) \succeq 0$. Therefore, we conclude that $\bar{A}_{11} \succeq 0$. To prove $\bar{A}_{12} \succeq 0$ we choose the initial condition

$$\psi_2(0) = 0, \qquad \psi_2(t) = e_j, \quad t \in [-h, 0); \qquad \psi_1(t) = 0, \quad t \in [-h, 0],$$

where $e_j, j = 1, 2, \ldots, n - r$ denotes the jth vector of the canonical basis in R^{n-r}, and the solution is $y_1(t) = \int_0^t e^{\bar{A}_{01}(t-s)} \bar{A}_{12}e_j\, ds \succeq 0$, then by the same argument we have $\bar{A}_{12} \succeq 0$. We now show that the matrices $-A_{04}^{-1}A_{03}, -A_{04}^{-1}A_{13}, -A_{04}^{-1}A_{14}$ are non-negative. Consider the second equation of (2.2) on $[0, h]$. Taking the initial condition

$$\psi_1(-h) = e_i, \qquad \psi_1(t) = 0, \quad t \in (-h, 0]; \qquad \psi_2(t) = 0, \quad t \in [-h, 0],$$

the solution $y_2(0) = -A_{04}^{-1}A_{13}e_i \succeq 0, i = 1, 2, \ldots, r$, which gives $-A_{04}^{-1}A_{13} \succeq 0$. Taking the initial condition

$$\psi_2(-h) = e_j, \qquad \psi_2(t) = 0, \quad t \in (-h, 0]; \qquad \psi_1(t) = 0, \quad t \in [-h, 0],$$

we have the solution $y_2(0) = -A_{04}^{-1}A_{14}e_j \succeq 0, j = 1, 2, \ldots, n - r$, which gives $-A_{04}^{-1}A_{14} \succeq 0$. Taking the initial condition

$$\psi_1(0) = e_i, \qquad \psi_1(t) = 0, \quad t \in [-h, 0); \qquad \psi_2(t) = 0, \quad t \in [-h, 0],$$

we have the solution $y_2(0) = -A_{04}^{-1}A_{03}e_i \succeq 0, i = 1, 2, \ldots, r$, which gives $-A_{04}^{-1}A_{03} \succeq 0$.

Sufficiency. We have to show that the system (2.2) is positive, i.e. for any initial condition $\psi(\tau) \succeq 0,\ \tau \in [-h, 0]$ we get $y_1(t) \succeq 0$ and $y_2(t) \succeq 0, t \geq 0$. We first prove that the solution $y(t) = (y_1(t), y_2(t))$ is positive on $[0, h]$. From the first equation of (2.2) we have

$$y_1(t) = e^{\bar{A}_{01}t}y_1(0) + \int_0^t e^{\bar{A}_{01}(t-s)}\left[\bar{A}_{11}y_1(s-h) + \bar{A}_{12}y_2(s-h)\right] ds.$$

Since $\bar{A}_{01}$ is a Metzler, by Lemma 2.1, $e^{\bar{A}_{01}t} \succ 0, t \geq 0$. From $y_1(0) \succeq 0$, it follows that $e^{\bar{A}_{01}t}y_1(0) \succeq 0$. By the same way we have $e^{\bar{A}_{01}(t-s)}\bar{A}_{11}y_1(s-h) \succeq 0$ and $e^{\bar{A}_{01}(t-s)}\bar{A}_{12}y_2(s-h) \succeq 0$, for all $0 \leq s \leq t \leq h$. Since integration is monotone then we have $y_1(t) \succeq 0,\ t \in [0, h]$. On the other hand, since $-A_{04}^{-1}A_{03} \succeq 0,\ -A_{04}^{-1}A_{13} \succeq 0,\ -A_{04}^{-1}A_{14} \succeq 0,\ y_1(t) \succeq 0, y_1(t-h) \succeq 0$, $y_2(t-h) \succeq 0,\ t \in [0, h]$ and from the second equation of (2.2) we have

$$y_2(t) = -A_{04}^{-1}\left[A_{03}y_1(t) + A_{13}y_1(t-h) + A_{14}y_2(t-h)\right] \succeq 0.$$

Thus, the solution $y(t)$ is positive on $[0, h]$. Using the step method, we can extend the consideration for the intervals $[h, 2h]$, $[2h, 3h]$, etc.

4. Exponential stability

This section proposes new sufficient conditions for exponential stability of system (2.1).

Theorem 4.1. *Assume that (E, A_0) is regular and impulse-free. Let $\overline{A}_{01},\ \overline{A}_{11}, \overline{A}_{12},\ A_{03},\ A_{04}, A_{13},\ A_{14}$ be defined in Lemma 2.2 and $\|A_{04}^{-1}A_{14}\| < 1$. Then the system (2.1) is α-exponentially stable if there exists a vector $\lambda \succ 0$ such that $\lambda^T[\alpha\tilde{E} + \overline{A}_0 + \overline{A}_1 e^{\alpha h}] \preceq 0$, where*

$$\tilde{E} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \qquad \overline{A}_0 = \begin{bmatrix} \overline{A}_{01} & 0 \\ -A_{04}^{-1}A_{03} & -I_{n-r} \end{bmatrix}, \qquad \overline{A}_1 = \begin{bmatrix} \overline{A}_{11} & \overline{A}_{12} \\ -A_{04}^{-1}A_{13} & -A_{04}^{-1}A_{14} \end{bmatrix}.$$

Proof. By the assumption, the system (2.1) can be rewritten by

$$\begin{cases} \tilde{E}\dot{y}(t) = \overline{A}_0 y(t) + \overline{A}_1 y(t-h), & t \geq 0, \\ y(t) = \psi(t) = [\psi_1(t), \psi_2(t)] & t \in [-h, 0]. \end{cases} \tag{4.1}$$

By Lemma 2.2, system (4.1) is positive and hence by Theorem 3.1, matrix $\overline{A}_1 \succeq 0$. Consider the nonnegative function

$$V(t, y_t) = e^{\alpha t}\lambda^T \tilde{E} y(t) + \int_{t-h}^{t} e^{\alpha(s+h)}\lambda^T \overline{A}_1 y(s)\, ds. \tag{4.2}$$

Taking the derivative along the solution of the system we have

$$\begin{aligned} \dot{V}(t, y_t) &= \alpha\lambda^T e^{\alpha t}\tilde{E}y(t) + \lambda^T e^{\alpha t}\tilde{E}\dot{y}(t) + \lambda^T\overline{A}_1 e^{\alpha(t+h)}y(t) - \lambda^T\overline{A}_1 e^{\alpha t}y(t-h) \\ &= \alpha\lambda^T e^{\alpha t}\tilde{E}y(t) + \lambda^T e^{\alpha t}\overline{A}_0 y(t) + \lambda^T e^{\alpha(t+h)}\overline{A}_1 y(t). \\ &= \lambda^T e^{\alpha t}[\alpha\tilde{E} + \overline{A}_0 + \overline{A}_1 e^{\alpha h}]y(t). \end{aligned}$$

By the assumption of the theorem, we have $\dot{V}(t, y_t) \leq 0,\ \forall t \geq 0$. Integrating both sides of this inequality from 0 to t leads to

$$V(t, y_t) \leq V(0, y_0) = \lambda^T\tilde{E}y(0) + \int_{-h}^{0} e^{\alpha(s+h)}\lambda^T\overline{A}_1 y(s)\, ds \leq \gamma\|\psi\| \tag{4.3}$$

where $\gamma = n\|\lambda\| + nhe^{\alpha h}\|\overline{A}_1^T\lambda\|$. On the other hand, we have

$$V(t, y_t) \geq \lambda^T e^{\alpha t}\tilde{E}y(t) \geq \beta e^{\alpha t}\|y_1(t)\|, \tag{4.4}$$

where $\beta = \min_{i=1,2,\ldots,n} \lambda_i$. Combining (4.3)–(4.4) yields

$$\|y_1(t)\| \leq \frac{\gamma}{\beta}e^{-\alpha t}\|\psi\| := \nu e^{\alpha h}\|\psi\|e^{-\alpha t}, \quad \forall t \geq 0. \tag{4.5}$$

Next, we will prove the second component solution $y_2(t)$ of the system is exponentially stable with the same decay rate α. Let us denote $p(t) = -A_{04}^{-1}A_{03}y_1(t) - A_{04}^{-1}A_{13}y_1(t-h)$. Observe that, if $t > h$ then

$$\|y_1(t-h)\| \leq \frac{\gamma}{\beta}e^{-\alpha(t-h)}\|\psi\| \leq \frac{\gamma}{\beta}e^{\alpha h}\|\psi\|e^{-\alpha t} = \nu e^{\alpha h}\|\psi\|\, e^{-\alpha t}, \quad \forall t > h. \tag{4.6}$$

For $t \in [0, h]$ we have $\|y_1(t-h)\| = \|\psi_1\| \leq \|\psi\| \leq \|\psi\|\, e^{-\alpha(t-h)} \leq e^{\alpha h}\|\psi\|e^{-\alpha t}$ and hence

$$\|y_1(t-h)\| \leq \nu\, e^{\alpha h}\|\psi\|\, e^{-\alpha t}, \quad \forall t \in [0, h]. \tag{4.7}$$

From (4.6) and (4.7) we have

$$\|y_1(t-h)\| \leq \nu e^{\alpha h}\, \|\psi\|\, e^{-\alpha t}, \quad \forall\, t \geq 0. \tag{4.8}$$

By the notation of vector function $p(t)$, we obtain from (4.5) and (4.8) that

$$\|p(t)\| \leq \|A_{04}^{-1}A_{03}\|\|y_1(t)\| + \|A_{04}^{-1}A_{13}\|\|y_1(t-h)\| \leq \nu_1\, \|\psi\|\, e^{-\alpha t}, \quad \forall t \geq 0$$

where $\nu_1 = \nu e^{\alpha h}\left(\|A_{04}^{-1}A_{03}\| + \|A_{04}^{-1}A_{13}\|\right)$. Moreover, from the second equation of (4.1) we have

$$y_2(t) = -A_{04}^{-1}A_{14}y_2(t-h) - A_{04}^{-1}A_{03}y_1(t) - A_{04}^{-1}A_{13}y_1(t-h) = -A_{04}^{-1}A_{14}y_2(t-h) + p(t).$$

Therefore

$$\|y_2(t)\| \leq \|A_{04}^{-1}A_{14}\|\|y_2(t-h)\| + \|p(t)\|, \quad \forall t \geq 0. \tag{4.9}$$

Setting $\sigma := \max\{\nu\, e^{\alpha h}\left(\|A_{04}^{-1}A_{03}\| + \|A_{04}^{-1}A_{13}\|\right);\ e^{\alpha h}\}$. If $t \in [0, h]$ then $t - h \in [-h, 0]$.

So from (4.9) we have

$$\|y_2(t)\| \le \|A_{04}^{-1}A_{14}\|\|\psi\| + \|p(t)\| \le \left(\|A_{04}^{-1}A_{14}\|\sigma + \sigma\right)\|\psi\|e^{-\alpha t}. \tag{4.10}$$

If $t \in [h,\ 2h]$ then $t - h \in [0,\ h]$. From (4.9) and (4.10) we have

$$\|y_2(t)\| \le \left(\|A_{04}^{-1}A_{14}\|^2\sigma + \|A_{04}^{-1}A_{14}\|\sigma + \sigma\right)\|\psi\|e^{-\alpha t}.$$

Suppose that $\forall t \in [(k-1)h, kh]$, then

$$\|y_2(t)\| \le \left(\sigma + \sigma\|A_{04}^{-1}A_{14}\| + \cdots + \sigma(\|A_{04}^{-1}A_{14}\|)^k\right)\|\psi\|e^{-\alpha t}.$$

Thus, when $t \in [kh, (k+1)h]$, $t - h \in [(k-1)h, kh]$, by the inductive supposition and from (4.9) and (4.10) we get that

$$\begin{aligned}\|y_2(t)\| &\le \|A_{04}^{-1}A_{14}\|\left(\sigma + \sigma\|A_{04}^{-1}A_{14}\| + \cdots + \sigma\|A_{04}^{-1}A_{14}\|^k\right)\|\psi\|e^{-\alpha t} + \|p(t)\| \\ &\le \left(\sigma + \sigma\|A_{04}^{-1}A_{14}\| + \cdots + \sigma\|A_{04}^{-1}A_{14}\|^{k+1}\right)\|\psi\|\, e^{-\alpha t}.\end{aligned}$$

If $\|A_{04}^{-1}A_{14}\| < 1$, by induction, we obtain

$$\|y_2(t)\| \le \|\psi\|\, e^{-\alpha t}\left(\sigma + \sigma\|A_{04}^{-1}A_{14}\| + \cdots + \sigma\|A_{04}^{-1}A_{14}\|^k + \cdots\right) \le \frac{\sigma\ \|\psi\|\ e^{-\alpha t}}{1 - \|A_{04}^{-1}A_{14}\|}. \tag{4.11}$$

From (4.5) and (4.11) we finally have $\|y(t)\| < N\|\psi\|\, e^{-\alpha t}$, $t \ge 0$, which completes the proof of the theorem.

Example 4.1. Consider the system (2.1), where

$$E = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \\ 1 & 0 & 0 \end{bmatrix}, \qquad A_0 = \begin{bmatrix} -2 & 4 & -2 \\ 0 & 9 & -34 \\ -26 & 10 & 0 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 0 & -\frac{2}{5} & -2 \\ 0 & \frac{1}{10} & \frac{1}{2} \\ 1 & -\frac{1}{5} & 0 \end{bmatrix}.$$

Direct calculation shows that $\det(sE - A_0) = -2s^2 - 160s - 2388 \neq 0$, for some $s \in R$ and $\deg(\det(sE - A_0)) = \operatorname{rank}(E) = 2$. Then the system is regular and impulse-free. Moreover, there are two nonsingular matrices

$$P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}, \qquad Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix},$$

such that E, A_0, A_1 are partitioned accordingly

$$PEQ = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad PA_0Q = \begin{bmatrix} -26 & 0 & 5 \\ 0 & -68 & 9 \\ -1 & -1 & 1 \end{bmatrix}, \qquad PA_1Q = \begin{bmatrix} 1 & 0 & -\frac{1}{10} \\ 0 & 1 & \frac{1}{10} \\ 0 & -1 & -\frac{1}{10} \end{bmatrix}.$$

By simple computation, we obtain

$$\bar{A}_{01} = \begin{bmatrix} -21 & 5 \\ 9 & -59 \end{bmatrix}, \qquad \bar{A}_{11} = \begin{bmatrix} 1 & 5 \\ 0 & 10 \end{bmatrix}, \qquad \bar{A}_{12} = \begin{bmatrix} \frac{2}{5} \\ 1 \end{bmatrix}, \qquad A_{04}^{-1}A_{03} = \begin{bmatrix} -1 & -1 \end{bmatrix}, \qquad A_{04}^{-1}A_{13} = \begin{bmatrix} 0 & -1 \end{bmatrix},$$

$$A_{04}^{-1}A_{14} = \begin{bmatrix} -\frac{1}{10} \end{bmatrix}, \qquad \bar{A}_0 = \begin{bmatrix} -21 & 5 & 0 \\ 9 & -59 & 0 \\ 1 & 1 & -1 \end{bmatrix}, \qquad \bar{A}_1 = \begin{bmatrix} 1 & 5 & \frac{2}{5} \\ 0 & 10 & 1 \\ 0 & 1 & \frac{1}{10} \end{bmatrix}, \qquad \tilde{E} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Therefore, by Theorem 3.1 the system is positive. Moreover, for $\lambda = (64, 49, 827)$, $\alpha = 0.1$, $h = 0.6$ we can verify that $\|A_{04}^{-1}A_{14}\| = \frac{1}{10} < 1$, and $\lambda^T[\alpha\tilde{E} + \bar{A}_0 + \bar{A}_1 e^{\alpha h}] \preceq 0$, which shows that the system, by Theorem 4.1, is 0.1-exponentially stable.

5. Conclusions

In this paper, we have investigated the problem of positivity and exponential stability of linear singular systems with time delay. By applying delay decomposition state-space method, new sufficient conditions for exponential stability for this class of singular systems are derived in terms of matrix inequalities. One future work is to extend the results in this paper to linear singular positive systems with time-varying delays.

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