Qualitative Properties of Equilibria in MNA Models of Electrical Circuits

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Abstract We present in this communication some tools for the qualitative analysis of lumped circuits directed to differential-algebraic MNA models. The attention is focused on equilibria, which describe operating points of the circuit. Specifically, hyperbolicity and asymptotic stability of linearized models are analyzed in terms of the circuit topology and device characteristics. The topological conditions arising in this qualitative study are proved independent of those supporting the index of the differential-algebraic circuit model. An example containing a Josephson junction circuit illustrates the discussion.

1 Introduction

Qualitative properties of nonlinear circuits have been often discussed assuming that a state-space model describing network dynamics is available [Chu80, GW92]. However, such a state model does not always exist or is difficult to obtain in practice; this has led to semistate formalisms based on differential-algebraic equations (DAEs), which currently frame approaches such as Modified Nodal Analysis (MNA) or Tableau Analysis [ET00, GF99, Tis99]. In this differential-algebraic context, we address in the present communication several qualitative properties of equilibria in MNA-modeled nonlinear circuits, using and extending previous results from [Ria04, Tis99].

Qualitative features of circuits have been also addressed in the last decades within a geometric framework. This stems from the work [BM64]; see also [DW72, HB84, HB86, Mat87, Sma72, WM97, WMT98]. This approach provides a coordinate-free point of view for the analysis of several intrinsic properties of circuit dynamics. Our approach, in contrast, uses the natural coordinates arising in the widely-used MNA models of electrical circuits.

We work with nonlinear RLC circuits assuming that capacitors, resistors and inductors are respectively controlled through C^1 relations of the form $q = \psi(v_c)$, $i_r = \gamma(v_r)$, $\phi = \varphi(i_l)$. Denote the capacitance, inductance, and conductance matrices as $C(v_c) = \psi'(v_c),\ L(i_l) = \varphi'(i_l),\ G(v_r) = \gamma'(v_r)$. In circuit-theoretic terms, symmetric capacitance or inductance matrices will be said to describe *reciprocal* devices, whereas positive definite capacitance, inductance or conductance matrices will be said to yield *strictly locally passive* elements [Chu80]; positive definiteness of an $n \times n$ matrix B means in this work that $x^{\mathrm{T}}Bx > 0$ for any $x \in \mathbb{R}^n - \{0\}$, not implying that B is symmetric.

Conventional MNA equations for circuits without controlled sources read

$$A_C C(A_C^{\mathrm{T}} e) A_C^{\mathrm{T}} e' + A_R \gamma(A_R^{\mathrm{T}} e) + A_L i_l + A_V i_v = -A_I i_s(t) \tag{1a}$$

$$L(i_l) i_l' - A_L^{\mathrm{T}} e = 0 \tag{1b}$$

$$-A_V^{\mathrm{T}} e = -v_s(t). \tag{1c}$$

Here, e stands for node voltages; i_l, i_v represent currents in inductors and voltage sources, respectively, and $i_s(t)$, $v_s(t)$ denote currents and voltages in the (independent) sources. A_R (resp. $A_L,\ A_C,\ A_V,\ A_I$)

describes the *incidence* between resistive (resp. inductive, capacitive, voltage source, current source) branches and nodes in the circuit, once a reference node has been chosen. Specifically, the incidence matrix $(a_{ij}) \in \mathbb{R}^{(n-1)\times b}$ (n and b being the number of nodes and branches in the circuit, respectively) is given by

$$a_{ij} = \begin{cases} 1 & \text{if branch } j \text{ leaves node } i \\ -1 & \text{if branch } j \text{ enters node } i \\ 0 & \text{if branch } j \text{ is not incident with node } i. \end{cases}$$

Note that (1) is a quasilinear DAE of the form

$$A(x)x' + f(x) = s(t), \tag{2}$$

where $x = (e,\ i_l,\ i_v)^{\mathrm{T}}$, s is the excitation term $(-A_I i_s,\ 0,\ -v_s)^{\mathrm{T}}$, and

$$A = \begin{pmatrix} A_C C(A_C^{\mathrm{T}} e) A_C^{\mathrm{T}} & 0 & 0 \\ 0 & L(i_l) & 0 \\ 0 & 0 & 0 \end{pmatrix},\ f = \begin{pmatrix} A_R \gamma(A_R^{\mathrm{T}} e) + A_L i_l + A_V i_v \\ -A_L^{\mathrm{T}} e \\ -A_V^{\mathrm{T}} e \end{pmatrix}.$$

Many analytical and numerical features of a semistate circuit model rely upon its *index* (see [ET00, GF99, Ria04, Tis99] and references therein). We compile in Proposition 1 below Theorems 4 and 5 of [Tis99], replacing in the first claim positive definiteness by just non-singularity on L:

Proposition 1. *Assume that the capacitance and conductance matrices are positive definite, and that the inductance matrix is non-singular.*

1. *If the network contains neither I-L cutsets nor V-C loops (except for C-loops), then the MNA system (1) has index ≤ 1.*
2. *Assume additionally that the inductance matrix is positive definite. If the network contains an I-L cutset or a V-C loop (with at least one voltage source), then the MNA system (1) has index 2.*

Assume that a given circuit has only independent DC sources, so that s in (2) is a constant vector. We may hence rewrite this equations as the quasilinear autonomous DAE

$$A(x)x' + g(x) = 0, \tag{3}$$

with $g(x) = f(x) - s$.

Equilibrium points of (3) are defined by the condition $g(x^*) = 0$, and the linearization of the DAE at equilibrium leads to the *matrix pencil* $\lambda A(x^*) + g'(x^*)$, i.e.,

$$\lambda \begin{pmatrix} A_C C(A_C^{\mathrm{T}} e^*) A_C^{\mathrm{T}} & 0 & 0 \\ 0 & L(i_l^*) & 0 \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} A_R G(A_R^{\mathrm{T}} e^*) A_R^{\mathrm{T}} & A_L & A_V \\ -A_L^{\mathrm{T}} & 0 & 0 \\ -A_V^{\mathrm{T}} & 0 & 0 \end{pmatrix}. \tag{4}$$

Several qualitative properties of equilibria can be characterized in terms of the spectrum $\sigma\{A(x^*), g'(x^*)\} = \{\lambda \in \mathbb{C} \ / \ \det(\lambda A(x^*) + g'(x^*)) = 0\}$ of the matrix pencil depicted in (4). The reader is referred to [Ria04] and references therein for background in this regard. The purpose of the present work is to characterize the spectrum of (4) in terms of the circuit topology.

We compile below some results coming from graph theory which will be useful in this regard. $\mathcal{K}$ will represent a subset of the set of branches of a connected graph $\mathcal{G}$. We denote as $A_{\mathcal{K}}$ (resp. $A_{\mathcal{G}-\mathcal{K}}$) the submatrix of A formed by the columns corresponding to the branches in $\mathcal{K}$ (resp. not in $\mathcal{K}$).

Lemma 1. *$\mathcal{K}$ does not contain loops if and only if $A_{\mathcal{K}} y = 0 \Rightarrow y = 0$, that is,* $\operatorname{Ker} A_{\mathcal{K}} = \{0\}$.

The subset $\mathcal{K}$ is a *cutset* if the deletion of $\mathcal{K}$ results in a disconnected graph, and it is minimal with respect to this property (i.e., removing any proper subset of $\mathcal{K}$ does not disconnect the graph).

Lemma 2. *$\mathcal{K}$ does not contain cutsets if and only if $x^{\mathrm{T}} A_{\mathcal{G}-\mathcal{K}} = 0 \Rightarrow x = 0$ or, equivalently, $A^{\mathrm{T}}_{\mathcal{G}-\mathcal{K}} x = 0 \Rightarrow x = 0$, that is,* $\mathrm{Ker}\ A^{\mathrm{T}}_{\mathcal{G}-\mathcal{K}} = \{0\}$.

The following two properties will be useful later on.

Lemma 3. *Let $\mathcal{J}_1,\ \mathcal{J}_2$ be two sets of branches of a connected graph $\mathcal{G}$, $\mathcal{J}_1 \subseteq \mathcal{J}_2$. If all loops within $\mathcal{J}_2$ are contained in $\mathcal{J}_1$, then $A_{\mathcal{J}_1} w_1 + A_{\mathcal{J}_2-\mathcal{J}_1} w_2 = 0 \Rightarrow w_2 = 0$. Equivalently, letting the first columns of $A_{\mathcal{J}_2}$ be those of $A_{\mathcal{J}_1}$,* $\mathrm{Ker}\ A_{\mathcal{J}_2} = \mathrm{Ker}\ A_{\mathcal{J}_1} \times \{0\}$.

Lemma 4. *Let $\mathcal{K}_1,\ \mathcal{K}_2$ be two sets of branches of a connected graph $\mathcal{G}$, $\mathcal{K}_1 \subseteq \mathcal{K}_2$. If all cutsets within $\mathcal{K}_2$ are contained in $\mathcal{K}_1$, then $w^{\mathrm{T}} A_{\mathcal{G}-\mathcal{K}_2} = 0 \Rightarrow w^{\mathrm{T}} A_{\mathcal{K}_2-\mathcal{K}_1} = 0$. Equivalently,* $\mathrm{Ker}\ A^{\mathrm{T}}_{\mathcal{G}-\mathcal{K}_2} = \mathrm{Ker}\ A^{\mathrm{T}}_{\mathcal{G}-\mathcal{K}_1}$.

2 Hyperbolicity

Equilibrium points of (3) are defined by the vanishing of $g(x)$. An equilibrium x^* is said to be *hyperbolic* if the spectrum of the linearization has no purely imaginary eigenvalues. Null eigenvalues are depicted if and only if $g'(x^*)$ is singular; non-singularity of $g'(x^*)$ guarantees the isolation of this equilibrium and follows, in circuits with definite conductance, from the topological conditions of Theorem 1 below. We skip the proof of this result for the sake of brevity; note that it is a restatement, in a matrix pencil setting, of a known result [HB86, MCM79]. Non-vanishing, purely imaginary eigenvalues will be ruled out by the conditions in Theorem 2.

Theorem 1. *Let $x^* = (e^*, i_l^*, i_v^*)$ be an equilibrium point of (3). Denote $G = G(A_R^{\mathrm{T}} e^*)$, and assume that G is (positive or negative) definite. Then x^* is non-singular (equivalently, $0 \notin \sigma\{A(x^*), g'(x^*)\}$) if and only if there are neither V-L loops nor I-C cutsets in the circuit.*

Theorem 2. *If G is (positive or negative) definite, both $C = C(A_C^{\mathrm{T}} e^*)$ and $L = L(i_l^*)$ are symmetric and non-singular, and any one of the conditions*

a) there are no I-C-L cutsets; or
b) there are no V-C-L loops;

holds, then there are no purely imaginary eigenvalues $\lambda = \alpha j$ with $\alpha \in \mathbb{R} - \{0\}$.

Proof: $\lambda \in \mathbb{C}$ is an eigenvalue if and only if there exists a nonvanishing vector $w = (w_e, w_l, w_v)$ such that $(\lambda A(x^*) + g'(x^*))w = 0$, what yields

$$\lambda A_C C A_C^{\mathrm{T}} w_e + A_R G A_R^{\mathrm{T}} w_e + A_L w_l + A_V w_v = 0 \tag{5a}$$

$$-A_L^{\mathrm{T}} w_e + \lambda L w_l = 0 \tag{5b}$$

$$-A_V^{\mathrm{T}} w_e = 0. \tag{5c}$$

Multiplying (5a) by the conjugate transpose $w_e^\star$, we get

$$\lambda w_e^\star A_C C A_C^{\mathrm{T}} w_e + w_e^\star A_R G A_R^{\mathrm{T}} w_e + w_e^\star A_L w_l + w_e^\star A_V w_v = 0. \tag{6}$$

Note that (5b) yields $w_e^\star A_L = \overline{\lambda} w_l^\star L$, where we have used the symmetry of L. On the other hand, from (5c), it follows that $w_e^\star A_V = 0$. Some simple computations lead to

$$(\mathrm{Re}\lambda) w_e^\star A_C C A_C^{\mathrm{T}} w_e + w_e^\star A_R \frac{G + G^{\mathrm{T}}}{2} A_R^{\mathrm{T}} w_e + (\mathrm{Re}\lambda) w_l^\star L w_l = 0. \tag{7}$$

Let λ be a non-vanishing eigenvalue with $\mathrm{Re}\lambda = 0$. Equation (7) then leads to $A_R^{\mathrm{T}} w_e = 0$, due to the definiteness of G. Now, assume first that condition a) is satisfied. The exclusion of I-C-L cutsets, together with $A_R^{\mathrm{T}} w_e = 0$ and $A_V^{\mathrm{T}} w_e = 0$ (from (5c)), implies that $w_e = 0$. From (5b), the assumption $\lambda \neq 0$, and the non-singularity of L, we get $w_l = 0$. Then, from (5a), we get $A_V w_v = 0$, and the exclusion of V-loops in well-posed circuits would yield $w_v = 0$.

Assume now that condition b) is satisfied, and write (5a) as

$$A_C(\lambda C A_C^{\mathrm{T}} w_e) + A_L w_l + A_V w_v = 0,$$

since $A_R^{\mathrm{T}} w_e = 0$. From the V-C-L loop exclusion property, it follows that $\lambda C A_C^{\mathrm{T}} w_e = 0$, $w_l = 0$, $w_v = 0$. From the first identity, the non-vanishing of λ, and the non-singularity of C, we get $A_C^{\mathrm{T}} w_e = 0$. On the other hand, $w_l = 0$ yields, in the light of (5b), $A_L^{\mathrm{T}} w_e = 0$. Together with the conditions $A_C^{\mathrm{T}} w_e = 0$, $A_R^{\mathrm{T}} w_e = 0$, $A_V^{\mathrm{T}} w_e = 0$, and the exclusion of I cutsets in well-posed circuits, we would get $w_e = 0$. □

Theorems 1 and 2 together provide a sufficient condition for the hyperbolicity of the matrix pencil. Merging the topological conditions and using Lemmas 3 and 4, we may assert hyperbolicity allowing for the existence of V-C loops and I-L cutsets, so that the resulting topological conditions be entirely independent of the index conditions appearing in Proposition 1. Therefore, Theorem 3 will naturally apply to both index-1 and index-2 problems.

Theorem 3. *If G is (positive or negative) definite, both C and L are symmetric and non-singular, and any one of the two pairs of conditions*

a) there are neither V-L loops nor I-C-L cutsets (except maybe I-L cutsets); or
b) there are neither I-C cutsets nor V-C-L loops (except maybe V-C loops);

is satisfied, then $\mathrm{Re}\lambda \neq 0$, $\forall \lambda \in \sigma\{A(x^*), g'(x^*)\}$.

Proof: Since I-C-L cutsets include in particular I-C cutsets, and so do V-C-L loops with regard to V-L loops, the only cases which do not follow automatically from Theorem 1 and Theorem 2 are those in which either I-L cutsets or V-C loops are present. We have to show that purely imaginary non-vanishing eigenvalues may not exist in this situation.

Let us first consider case a). Proceeding as in the proof of Theorem 2, we get $A_R^{\mathrm{T}} w_e = 0$ and $A_V^{\mathrm{T}} w_e = 0$. Denote as $\mathcal{K}_1$ the set of branches corresponding to inductors and current sources, and as $\mathcal{K}_2$ the ones corresponding to capacitors, inductors and current sources. If $\mathcal{G}$ stands for the graph of the circuit, the branches in $\mathcal{G} - \mathcal{K}_2$ correspond to resistors and voltage sources, whereas those in $\mathcal{K}_2 - \mathcal{K}_1$ are the capacitive ones. With this notation, and in the light of Lemma 4, we get that $w_e^{\mathrm{T}}(A_R \ A_V) = 0 \Rightarrow w_e^{\mathrm{T}} A_C = 0$, that is, $A_C^{\mathrm{T}} w_e = 0$. From this property, (5a) reads $A_L w_l + A_V w_v = 0$, and the exclusion of V-L loops in a) yields $w_l = 0$, $w_v = 0$. Additionally, (5b) implies $A_L^{\mathrm{T}} w_e = 0$, and the absence of I cutsets in well-posed circuits implies $w_e = 0$.

Now consider case b). Again, $A_R^{\mathrm{T}} w_e = 0$ and $A_V^{\mathrm{T}} w_e = 0$ hold. Using $A_R^{\mathrm{T}} w_e = 0$, equation (5a) reads $\lambda A_C C A_C^{\mathrm{T}} w_e + A_L w_l + A_V w_v = 0$. Let $\mathcal{J}_1$ stand for the capacitor and voltage source branches, and assume that $\mathcal{J}_2$ includes these and, additionally, the inductive branches. Based upon the absence of V-C-L loops except for V-C loops, application of Lemma 3 yields $w_l = 0$. In virtue of (5b), it is $A_L^{\mathrm{T}} w_e = 0$, and the properties $A_R^{\mathrm{T}} w_e = 0$, $A_V^{\mathrm{T}} w_e = 0$, together with the exclusion of I-C cutsets, lead to $w_e = 0$. Finally, $w_v = 0$ from (5a) and the absence of V-loops in well-posed circuits. □

3 Asymptotic stability

Proposition 2. *If G is positive definite, and both C and L are symmetric positive definite, then* $\mathrm{Re}\lambda \leq 0$, $\forall \lambda \in \sigma\{A(x^*), g'(x^*)\}$.

Proof: The derivation of (7) in Theorem 2 is still valid under the current working assumptions. Let λ be an eigenvalue with $\mathrm{Re}\lambda > 0$. From the assumption of symmetry and positive definiteness on C and L, it follows that

$$w_e^{\star} A_C C A_C^{\mathrm{T}} w_e = w_e^{\star} A_R \frac{G + G^{\mathrm{T}}}{2} A_R^{\mathrm{T}} w_e = w_l^{\star} L w_l = 0, \tag{8}$$

so that $A_C^{\mathrm{T}} w_e = 0$, $A_R^{\mathrm{T}} w_e = 0$, $w_l = 0$ and (using (5b)) $A_L^{\mathrm{T}} w_e = 0$. Additionally, $A_V^{\mathrm{T}} w_e = 0$ as displayed in (5c). Since current source cutsets are forbidden in well-posed circuits, it follows that $w_e = 0$. From (5a), we get $A_V w_v = 0$ and, since voltage source loops are also excluded in well-posed circuits, it follows that $w_v = 0$. This would yield the contradiction $w = 0$, meaning that it must be $\mathrm{Re}\lambda \leq 0$. □

Adding to Proposition 2 the topological conditions of Theorem 3, we get the following asymptotic stability criterion, where the topological conditions are again independent of those characterizing the index in Proposition 1.

Theorem 4. *Assume that:*

1) *G is positive definite, and both C and L are symmetric positive definite.*
2) *At least one of the two pairs of topological conditions holds:*
   2a) *There are neither V-L loops nor I-C-L cutsets (except maybe I-L cutsets); or*
   2b) *There are neither I-C cutsets nor V-C-L loops (except maybe V-C loops).*

Then, all eigenvalues in the spectrum $\sigma\{A(x^*), g'(x^*)\}$ verify $\mathrm{Re}\lambda < 0$. □

4 Example

Consider the nonlinear circuit depicted in Fig. 1. The device labeled as L_2 is a *Josephson junction*, which can be treated as a nonlinear inductor with a current-flux characteristic $i_2 = I_0 \sin k\phi_2$, where $I_0 > 0$ is a device parameter, and k is a positive physical constant. The incremental inductance of this device is $L_2 = (I_0 k \cos k\phi_2)^{-1}$.

The two resistors are linear with conductances $G_1 > 0$, $G_2 \geq 0$, and the inductor is linear with inductance $L_1 > 0$. MNA equations read

$$L_1 i_1' = e_1 \tag{9a}$$
$$L_2 i_2' = e_2 \tag{9b}$$
$$0 = i_1 + G_1(e_1 - e_2) - I \tag{9c}$$
$$0 = i_2 - G_1(e_1 - e_2) + G_2 e_2. \tag{9d}$$

Equilibrium points are given by $e_1 = e_2 = 0,\ i_1 = I,\ i_2 = 0$. The latter yields $\sin k\phi_2 = 0$, i.e., $\phi_2 = n\pi/k,\ n \in \mathbb{Z}$, so that the incremental inductance L_2 at equilibrium is $\pm(I_0 k)^{-1}$, the sign depending on the parity of n.

Stability properties have been analyzed in [Ria04] via a DAE model of the circuit. Our present goal is to illustrate that this qualitative analysis can be performed checking only device characteristics and circuit topology, without making explicit use of any model. We will distinguish the two cases $G_2 > 0$ and $G_2 = 0$. Note that, in both cases, the (symmetric) inductance matrix $L = \mathrm{diag}(L_1, L_2)$ is positive definite (resp. indefinite) at equilibria for which $L_2 = (I_0 k)^{-1}$ (resp. $L_2 = -(I_0 k)^{-1}$).

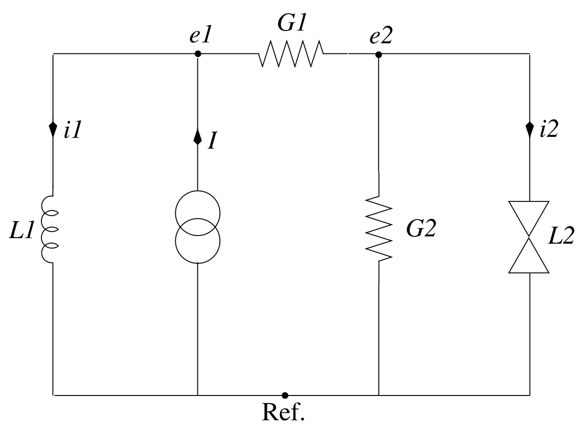

Fig. 1. A Josephson junction circuit

Index. In the absence of capacitors and voltage sources, according to Proposition 1 it suffices to check for I-L cutsets in order to compute the index of (9). This yields index-1 regardless of the sign of L_2 when $G_2 > 0$. In contrast, the case $G_2 = 0$ yields an I-L cutset defined by the linear inductor, the current source and the Josephson junction. In this situation, Proposition 1 only allows one to conclude that the index is 2 if L is positive definite, that is, around equilibria in which $L_2 = (I_0k)^{-1} > 0$. Using (9), it is not difficult to check that, at the remaining equilibria (for which $L_2 = -(I_0k)^{-1} < 0$), the index is 2 if and only if the additional condition $L_1 \neq -L_2$ is satisfied.

Hyperbolicity. The absence of capacitors and voltage sources make the topological conditions in Theorem 3 amount to the absence of L-loops, which is verified for all equilibria independently of the value of G_2, making all of them hyperbolic regardless of the sign of L_2.

Asymptotic stability. Theorem 4 guarantees that equilibria with $L_2 = (I_0k)^{-1} > 0$ are asymptotically stable, since for them the inductance matrix is symmetric positive definite. The case $L_2 = -(I_0k)^{-1} < 0$ cannot be assessed in these terms. It can be checked that, actually, when $G_2 > 0$, these equilibria are unstable; in contrast, if $G_2 = 0$, these equilibria are asymptotically stable if $-L_2 = (I_0k)^{-1} < L_1$, and unstable if $-L_2 = (I_0k)^{-1} > L_1$.

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