Applied Mathematics and Computation 229 (2014) 151–158

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Convergence and stability of Euler method for impulsive stochastic delay differential equations

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ARTICLE INFO

Keywords:
Impulsive stochastic delay differential equations
Numerical method
Euler method
Convergence
Mean square exponential stability

ABSTRACT

This paper deals with the mean square convergence and mean square exponential stability of an Euler scheme for a linear impulsive stochastic delay differential equation (ISDDE). First, a method is presented to take the grid points of the numerical scheme. Based on this method, a fixed stepsize numerical scheme is provided. Based on the method of fixed stepsize grid points, an Euler method is given. The convergence of the Euler method is considered and it is shown the Euler scheme is of mean square convergence with order 1/2. Then the mean square exponential stability is studied. Using Lyapunov-like techniques, the sufficient conditions to guarantee the mean square exponential stability are obtained. The result shows that the mean square exponential stability may be reproduced by the Euler scheme for linear ISDDEs, under the restriction on the stepsize. At last, examples are given to illustrate our results.

1. Introduction

In recent years, impulsive stochastic delay differential equations (ISDDEs) have been studied by many authors and a lot of results have been reported, for example, see [1,2] and the references therein. Generally, the explicit solutions of the ISDDEs are difficult to be obtained, thus it is necessary to develop numerical methods for ISDDEs and study the properties of these numerical methods.

For impulsive differential equations, there are many results on the numerical methods. In 2007, Wu [3] first brought out an Euler method for a random impulsive differential equation using a variable stepsize method. In the same year, in [4], the stability of the Runge–Kutta methods was studied for a linear impulsive differential equation with constant coefficients, where the authors also adopted the variable stepsize numerical scheme. Then these authors considered the nonlinear case in [5]. In [6], the authors considered some numerical methods for impulsive differential equations, where the interval length of the impulsive moments are assumed to be equal. This assumption makes the authors can take a fixed stepsize numerical scheme. In [7] the authors considered the numerical stability and asymptotical stability of the implicit Euler method for a stiff impulsive differential equation in Banach space where the variable stepsize numerical scheme was adopted.

The study of the numerical method for stochastic delay differential equations has been done for many years and a lot of results were reported, see [8–12] and the references therein. For the impulsive delay differential equations, the authors presented a fixed stepsize Euler scheme and considered the convergence of this Euler method in [13]. Moreover, we should point out that, for ISDDEs, in [14], the authors studied the exponential stability of the Euler method with fixed stepsize method. However, just as the case in [6], in that paper, the assumption of the length's equality of each impulsive interval was imposed and the length of the impulsive interval was asked to be equal to the delay. Both the assumptions yield a fixed stepsize numerical scheme for ISDDEs, which also reduced the applications scale of the numerical scheme.

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0096-3003/$ - see front matter
http://dx.doi.org/10.1016/j.amc.2013.12.041

In this paper, we generalize the method of [13] to the case of ISDDEs. First, we present a method to take the grid points and this method yields a fixed stepsize numerical method for ISDDEs. Based on this method, we give an Euler method for the ISDDEs. Then we consider the convergence of the Euler scheme and obtain the order of the convergence. The mean square exponential stability of the Euler method is also studied and the sufficient conditions are obtained to guarantee the stability by using the Lyapunov-like techniques. At last, numerical examples are given to illustrate our results.

For the convenience, we adopt the following standard notations:

$(\Omega, F, \{F_t\}_{t\geqslant 0}, P)$ is a complete probability space with a filtration $\{F_t\}_{t\geqslant slant0}$ satisfying the usual conditions (i.e., the filtration contains all P-null sets and is right continuous). $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^+ = [0, +\infty)$, $\mathbb{N} = \{1, 2, \ldots\}$. Given a positive integer m, define $N_{-m} = \{-m, -m+1, \ldots, 0\}$. $PC(\mathbb{J}; \mathbb{R}) = \{\varphi : \mathbb{J} \to \mathbb{R}, \varphi(s)$ is continuous for all but at most countable points $s \in \mathbb{J}$ and at these points $s \in \mathbb{J}, \varphi(s^+)$ and $\varphi(s^-)$ exist and $\varphi(s^+) = \varphi(s)\}$, where $\mathbb{J}$ is a subset of $\mathbb{R}$, $\varphi(s^+)$ and $\varphi(s^-)$ denote the right-hand and left-hand limits of the function $\varphi(s)$ at time s respectively. $PC^b_{F_0}([-\tau, 0]; \mathbb{R})$ denotes the family of all bounded F_0-measurable, PC-valued random variables. $\|\phi\| = \sup_{-\tau\leqslant\theta\leqslant 0}\{|\phi(\theta)|\}$.

2. Preliminaries

In this paper, we will consider the following system:

$$\begin{cases} dx(t) &= [ax(t) + bx(t-\tau)]dt + cx(t)dW(t), \quad t \geqslant 0, \quad t \neq t_k, \\ x(t_k) &= \beta_k x(t_k^-), \end{cases} \tag{2.1}$$

where a, b, c, β_k are constants, $\tau > 0$ is the delay, $x(t_k^-) = \lim_{t\to t_k^-} x(t)$. The impulsive moments $\{t_k\}$ satisfy: $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$, and $\lim_{k\to\infty} t_k = \infty$. $W(t)$ is an 1-dimension Brownian motion.

We impose the following initial data for system (2.1):

$$x(s) = \phi(s), \quad s \in [-\tau, 0], \tag{2.2}$$

where $\phi \in PC^b_{F_0}([-\tau, 0], \mathbb{R})$.

For the sake of simplicity, we do not consider the more general system:

$$\begin{cases} dx(t) &= [ax(t) + bx(t-\tau)]dt + [cx(t) + dx(t-\tau)]dW(t), \quad t \geqslant 0, \quad t \neq t_k, \\ x(t_k) &= \beta_k x(t_k^-), \end{cases} \tag{2.3}$$

there is no more essential difficulty to study the system (2.1) than to study system (2.3) except the complexity.

Now we state a lemma for system (2.1), it will be used in the sequel.

Lemma 2.1. *For any given positive constant T, there exists an $M > 0$, such that the solution of system (2.1) with initial data (2.2) satisfies:*

$$E|x(t, \phi)|^2 < M, -\tau \leqslant t \leqslant T.$$

Proof. From the property of the impulsive moments, we know that, for a given $T > 0$, there exists a natural number N such that $t_N \leqslant T < t_{N+1}$. By virtue of the exponential estimate theorem in [15] and the induction, we can get the required result easily. □

To end this section, we give a lemma on the mean square exponential stability of system (2.1). It can be obtained directly by the results of [1].

Lemma 2.2. *If the coefficients of system (2.1) satisfy:*

$$2a + 2|b| + c^2 < 0, \quad \beta_k^2 \geqslant 1 \quad \text{and} \quad \sum_{k=1}^{\infty}(\beta_k^2 - 1) < \infty, \tag{2.4}$$

then the trivial solution of system (2.1) is mean square exponentially stable.

3. Convergence of Euler method

In this section, we first present a method to get the grid points for numerical scheme, which is a fixed stepsize method. Based on this method, we derive the fixed stepsize Euler scheme for system (2.1), then we give the convergence result of the Euler scheme.

Given a positive integer m, let $h = \frac{\tau}{m}$, and take the grid points for the numerical scheme as follows:

$$\eta_k = \left[\frac{t_k}{h}\right] + 1 - \delta_{\mathbb{Z}}\left(\frac{t_k}{h}\right),$$

where $[\cdot]$ represents the greatest integer function, and

$$\delta_{\mathbb{Z}}(x) = \begin{cases} 1, & x \in \mathbb{Z}; \\ 0, & \text{otherwise}. \end{cases}$$

Obviously, from the definition of η_k, we can get

$$(\eta_k - 1)h < t_k \leqslant \eta_k h$$

for any $k \in \mathbb{N}$.

For the convenience of the study, we impose the following assumption. At the end of this paper, we will see that this assumption is not necessary.

Assumption 1. $\inf_{n\in\mathbb{N}}(t_{k+1} - t_k)$ is large enough such that $\eta_{k+1} - \eta_k \geqslant m$.

Take $T = K\tau = Kmh$ for some positive integer K. For system (2.1), we present the Euler scheme as follows:

$$\begin{cases} X_{n+1} & = X_n + ahX_n + bhX_{n-m} + cX_n \triangle W_n, \quad n \geqslant 1, \quad n \neq \eta_k - 1, \\ X_{\eta_k} & = \beta_k X_{\eta_k - 1}. \end{cases} \tag{3.1}$$

where X_n is the approximation of the solution of system (2.1) at $t = nh, n \in [-m, Km]$ and $\triangle W_n = W[(n+1)h] - W(nh)$. The coefficients a, b, c and β_k are the same as defined in system (2.1).

Let $X_n = \phi(nh)$ when $n \in N_{-m}$. Take $e_n = E|x(nh) - X_n|^2$. For the convergence of the Euler scheme (3.1), we have the following result:

Theorem 3.1. *There exists a positive constant C, such that*

$$e_n \leqslant Ch, \quad \text{for all } n \in [-m, Km],$$

which means that the Euler scheme (3.1) is mean square convergent with order 1/2.

Proof. Obviously, when $n \in [-m, 0]$, we have $e_n = 0$.

When $n \in [1, \eta_1)$, there is no impulsive effect appears, then in light of the result on numerical method of stochastic delay differential equations [16], we know that there is a positive constant C_1', such that

$$e_n \leqslant C_1' h$$

for $1 \leqslant n < \eta_1$, especially,

$$e_{\eta_1 - 1} \leqslant C_1' h.$$

When $n = \eta_1$,

$$\begin{aligned}
e_{\eta_1} &= E|x(\eta_1 h) - X_{\eta_1}|^2 = E\left|(x(t_1) + \int_{t_1}^{\eta_1 h}[ax(s) + bx(s-\tau)]ds + cx(s)dW(s)) - \beta_1 X_{\eta_1 - 1}\right|^2 \\
&= E\left|\beta_1 x(t_1^-) + \int_{t_1}^{\eta_1 h}[ax(s) + bx(s-\tau)]ds + cx(s)dW(s) - \beta_1 X_{\eta_1 - 1}\right|^2 \\
&= E\left|\beta_1(x((\eta_1 - 1)h) + \int_{(\eta_1-1)h}^{t_1}[ax(s) + bx(s-\tau)]ds + cx(s)dW(t)) + \int_{t_1}^{\eta_1 h}[ax(s) + bx(s-\tau)]ds + cx(s)dW(s) - \beta_1 X_{\eta_1 - 1}\right|^2 \\
&= E\left|(\beta_1 x((\eta_1 - 1)h) - \beta_1 X_{\eta_1 - 1}) + \beta_1 \int_{(\eta_1-1)h}^{t_1}[ax(s) + bx(s-\tau)]ds + cx(s)dW(s) + \int_{t_1}^{\eta_1 h}[ax(s) + bx(s-\tau)]ds + cx(s)dW(s)\right|^2 \\
&\leqslant 3\beta_1^2 E|x((\eta_1 - 1)h) - X_{\eta_1 - 1}|^2 + 3\beta_1^2 E\left|\int_{(\eta_1-1)h}^{t_1}[ax(s) + bx(s-\tau)]ds + cx(s)dW(s)\right|^2 \\
&\quad + 3E\left|\int_{t_1}^{\eta_1 h}[ax(s) + bx(s-\tau)]ds + cx(s)dW(s)\right|^2 \\
&= 3\beta_1^2 e_{\eta_1 - 1} + 3\beta_1^2 E\left|\int_{(\eta_1-1)h}^{t_1}[ax(s) + bx(s-\tau)]ds + cx(s)dW(s)\right|^2 + 3E\left|\int_{t_1}^{\eta_1 h}[ax(s) + bx(s-\tau)]ds + cx(s)dW(s)\right|^2
\end{aligned} \tag{3.2}$$

Using the Hölder inequality, Lemma 2.1 and properties of Itô integral, we have

$$\begin{aligned}
E\left|\int_{(\eta_1-1)h}^{t_1}[ax(t)+bx(t-\tau)]dt+cx(t)dW(t)\right|^2 &= E\left(\int_{(\eta_1-1)h}^{t_1}[ax(t)+bx(t-\tau)]dt+cx(t)dW(t)\right)^2 \\
&= E\left(\int_{(\eta_1-1)h}^{t_1}[ax(t)+bx(t-\tau)]dt\right)^2+E\left(\int_{(\eta_1-1)h}^{t_1}cx(t)dW(t)\right)^2 \\
&\leqslant (t_1-(\eta_1-1)h)E\int_{(\eta_1-1)h}^{t_1}[ax(t)+bx(t-\tau)]^2dt \\
&\quad +\int_{(\eta_1-1)h}^{t_1}E|cx(t)|^2dt \\
&\leqslant h\int_{(\eta_1-1)h}^{t_1}E[ax(t)+bx(t-\tau)]^2dt+\int_{(\eta_1-1)h}^{t_1}E|cx(t)|^2dt \\
&\leqslant h\int_{(\eta_1-1)h}^{t_1}2a^2E|x(t)|^2+2b^2E|x(t-\tau)|^2dt+\int_{(\eta_1-1)h}^{t_1}c^2E|x(t)|^2dt \\
&\leqslant h\int_{(\eta_1-1)h}^{t_1}(2a^2M+2b^2M)dt+\int_{(\eta_1-1)h}^{t_1}c^2Mdt \\
&\leqslant 2(a^2+b^2)Mh^2+c^2Mh=M_1h, \qquad (3.3)
\end{aligned}$$

the last inequality holds for some positive constant M_1 since $h\to 0$.

An analogical calculation gives that

$$E\left|\int_{t_1}^{\eta_1 h}[ax(s)+bx(s-\tau)]ds+cx(s)dW(s)\right|^2\leqslant M_2h \qquad (3.4)$$

for some positive constant M_2.

Substituting (3.3) and (3.4) into (3.2), noting $e_{\eta_1-1}\leqslant C_1'h$, we obtain

$$e_{\eta_1}\leqslant 3\beta_1^2e_{\eta_1-1}+M_1h+M_2h\leqslant 3\beta_1^2C_1'h+(M_1+M_2)h=C_1h, \qquad (3.5)$$

where $C_1=3\beta_1^2C_1'+M_1+M_2$.

In term of the definition of η_k and t_k, we know that there exits a positive integer N such that $\eta_N\leqslant Km<\eta_{N+1}$. Then by the induction and the analogical analysis as $n\in[1,\eta_1]$, for $\eta_1<n\leqslant Km$, we can get the desired result. □

4. Mean square exponential stability of Euler method

In this section, we state a result on the mean square exponential stability of the Euler scheme (3.1) for system (2.1). We can see from this result that the mean square exponential stability of system (2.1) can be reproduced by the Euler scheme (3.1) under the restriction on the stepzie h.

Theorem 4.1. *Suppose condition (2.4) is satisfied and the stepsize h satisfies:*

$$0<h<\min\left\{\frac{1}{|a|},\ \frac{-(2a+2|b|+c^2)}{(a+|b|)^2}\right\}, \qquad (4.1)$$

then there exist positive constants M and γ, such that

$$EX_n^2\leqslant ME\|\phi\|^2e^{-\gamma n},$$

which means that the Euler scheme (3.1) for system (2.1) with the initial condition (2.2) is mean square exponentially stable.

Proof. From the Euler scheme (3.1), using the property of Browinan motion, for $n\neq\eta_k-1$, we have,

$$\begin{aligned}
EX_{n+1}^2 &= E(X_n+ahX_n+bhX_{n-m}+cX_n\triangle W_n)^2=(1+ah)^2EX_n^2+b^2h^2EX_{n-m}^2+c^2hEX_n^2+2(1+ah)bhE(X_nX_{n-m}) \\
&\leqslant [(1+ah)^2+c^2h+|1+ah||b|h]EX_n^2+(b^2h^2+|1+ah||b|h)EX_{n-m}^2=\lambda_1EX_n^2+\lambda_2EX_{n-m}^2 \qquad (4.2)
\end{aligned}$$

where

$$\lambda_1=(1+ah)^2+c^2h+|1+ah||b|h$$

and

$$\lambda_2 = b^2h^2 + |1 + ah||b|h.$$

Obviously, $\lambda_1 + \lambda_2 > 0$. Under the condition (4.1), a direct calculation gives $\lambda_1 + \lambda_2 < 1$. Then, we can take a positive constant $\alpha = \alpha(h)$ for the given stepsize $h = \frac{\tau}{m}$, such that

$$e^{\alpha}(\lambda_1 + \lambda_2 e^{\alpha m}) \leqslant 1. \tag{4.3}$$

Let $U(n) = \max_{\theta\in N_{-m}}\{e^{\alpha(n+\theta)}EX^2_{n+\theta}\}$. For $n \in [\eta_k, \eta_{k+1})$, define

$$\bar{\theta}_n = \max\{\theta : e^{\alpha(n+\theta)}EX^2_{n+\theta} = U(n)\}.$$

Then we get

$$e^{\alpha n}EX^2_n \leqslant e^{\alpha(n+\bar{\theta}_n)}EX^2_{n+\bar{\theta}_n}$$

and

$$e^{\alpha(n-m)}EX^2_{n-m} \leqslant e^{\alpha(n+\bar{\theta}_n)}EX^2_{n+\bar{\theta}_n}.$$

The above inequalities yield that

$$EX^2_n \leqslant e^{\alpha\bar{\theta}_n}EX^2_{n+\bar{\theta}_n}$$

and

$$EX^2_{n-m} \leqslant e^{\alpha(\bar{\theta}_n+m)}EX^2_{n+\bar{\theta}_n}.$$

By virtue of (4.2) and (4.3), keeping the above two inequalities in mind, we have, for $n \in [\eta_k, \eta_{k+1} - 1)$,

$$\begin{aligned} e^{\alpha(n+1)}EX^2_{n+1} &\leqslant e^{\alpha(n+1)}\lambda_1 EX^2_n + e^{\alpha(n+1)}\lambda_2 EX^2_{n-m} \leqslant e^{\alpha(n+1)+\alpha\bar{\theta}_n}\lambda_1 EX^2_{n+\bar{\theta}_n} + e^{\alpha(n+1)+\alpha(\bar{\theta}_n+m)}\lambda_2 EX^2_{n+\bar{\theta}_n} \\ &= e^{\alpha(n+\bar{\theta}_n)}EX^2_{n+\bar{\theta}_n}e^{\alpha}(\lambda_1 + \lambda_2 e^{\alpha m}) \leqslant e^{\alpha(n+\bar{\theta}_n)}EX^2_{n+\bar{\theta}_n} = U(n). \end{aligned} \tag{4.4}$$

From the definition of $U(n)$, we have $U(n+1) \leqslant \max\{e^{\alpha(n+1)}EX^2_{n+1}, U(n)\}$. Then, for $n \in [\eta_k, \eta_{k+1} - 1)$, the inequality (4.4) yields

$$U(n+1) \leqslant U(n).$$

When $n = \eta_{k+1}$,

$$\begin{aligned} U(\eta_{k+1}) &= \max\{e^{\alpha\eta_{(k+1)}}EX^2_{\eta_{(k+1)}}, \max_{\theta\in N_{-m}-\{0\}}\{e^{\alpha(\eta_{(k+1)}+\theta)}EX^2_{\eta_{(k+1)}+\theta}\}\} \\ &= \max\{\beta^2_{k+1}e^{\alpha\eta_{(k+1)}}EX^2_{\eta_{(k+1)}-1}, \max_{\theta\in N_{-m}-\{0\}}\{e^{\alpha(\eta_{(k+1)}+\theta)}EX^2_{\eta_{(k+1)}+\theta}\}\} \leqslant e^{\alpha}\beta^2_{k+1}\max_{\theta\in N_{-m}}\{e^{\alpha(\eta_{(k+1)}-1+\theta)}EX^2_{\eta_{(k+1)}-1+\theta}\} \\ &= e^{\alpha}\beta^2_{k+1}U(\eta_{k+1} - 1). \end{aligned} \tag{4.5}$$

Consequently, for $n \in [\eta_k, \eta_{k+1})$,

$$U(n) \leqslant U(\eta_k) \leqslant \beta_k^2 e^{\alpha}U(\eta_k - 1) \leqslant \beta_k^2 e^{\alpha}U(\eta_{k-1}) \leqslant \prod_{i=1}^{k}(\beta_i^2 e^{\alpha})U(\eta_0).$$

Since $x - 1 \geqslant \ln x$ for $x \geqslant 1$, we have $\sum_{i=1}^{\infty}(\beta_i^2 - 1) \geqslant \sum_{i=1}^{\infty}\ln\beta_i^2 = \ln(\prod_{i=1}^{\infty}\beta_i^2)$, Then $\prod_{i=1}^{\infty}\beta_i^2 < \infty$. Take $\prod_{i=1}^{\infty}\beta_i^2 = M$, from the definition of $U(n)$, we obtain

$$e^{\alpha n}EX^2_n \leqslant e^{k\alpha}ME\|\phi\|^2.$$

Since $\eta_{k+1} - \eta_k \geqslant m$, then $n \geqslant km$, that is $k \leqslant \frac{n}{m}$, by virtue of the above inequality,

$$EX^2_n \leqslant e^{-\alpha\left(1-\frac{1}{m}\right)n}ME\|\phi\|^2.$$

Taking $\gamma = \alpha(1 - \frac{1}{m})$, we get the desired result. □

Table 1
The errors of Euler scheme (3.1) for system (5.1).

stepsize	1/8	1/16	1/32	1/64	1/128	1/256	1/512	1/1024
$e(h)$	0.1722	0.0709	0.0360	0.0169	0.0089	0.0047	0.0025	0.0016

Remark. From the above proof, we can see the Assumption 1 is not necessary, we just need that $\eta_{k+1} - \eta_k \geqslant 2$, which will be satisfied when the setsize h is small enough.

5. Examples

We consider the following equations:

$$\begin{cases} dx = [ax(t) + bx(t-1)]dt + cx(t)dW(t), & t \geqslant 0, \quad t \neq k\sqrt{2}, \\ x(k\sqrt{2}) = \sqrt{1 + \frac{1}{k^2}} x((k\sqrt{2})^-), \\ x(t) = t + 1, & t \in [-1, 0], \end{cases} \tag{5.1}$$

First, we illustrate the theoretical order of convergence for Euler scheme (3.1).

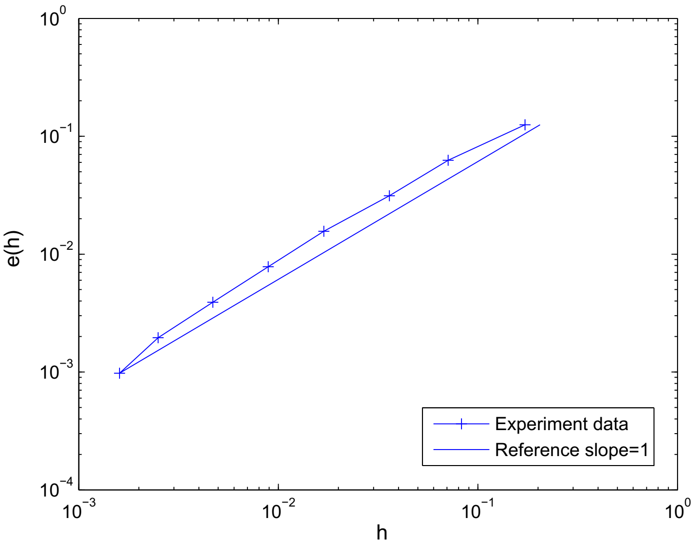

Fig. 1. Linearity of h and $e(h)$.

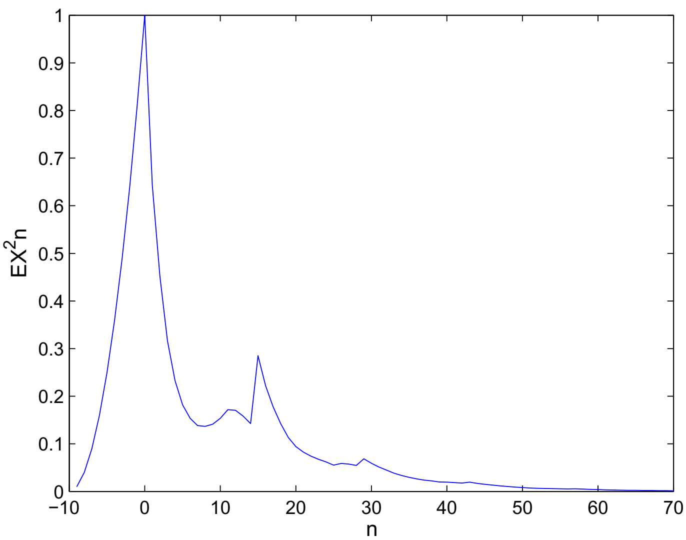

Fig. 2. Mean square stability of Euler scheme (3.1): $h = 1/10$.

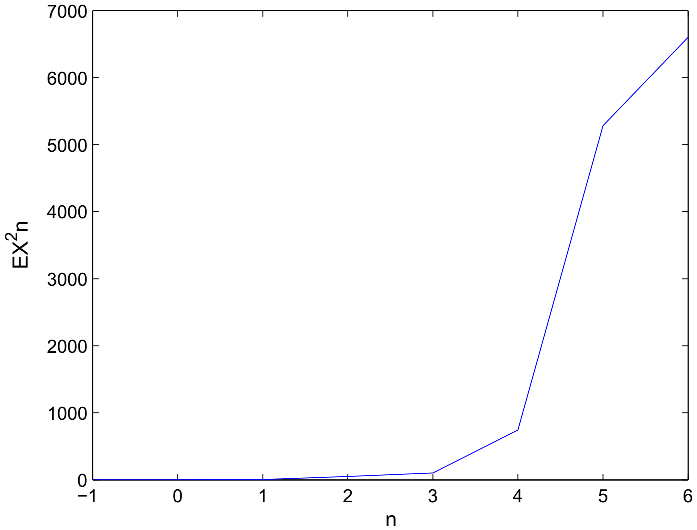

Fig. 3. Instability of Euler scheme (3.1): $h = 1/2$.

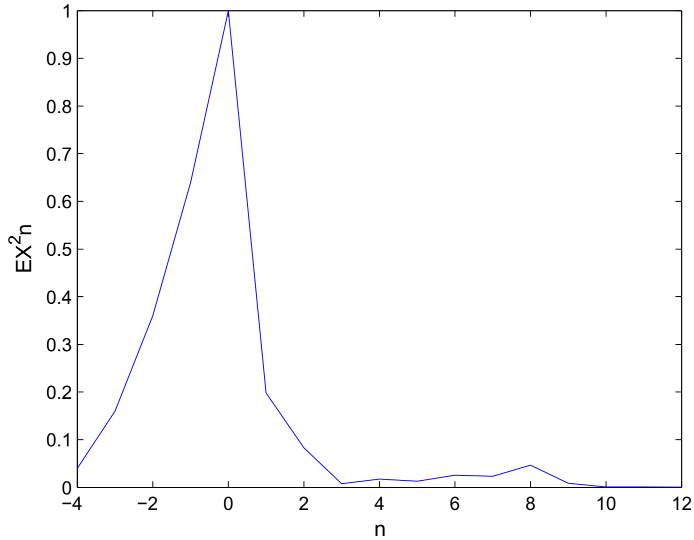

Fig. 4. Stability of Euler scheme (3.1): $h = 1/5$.

Take $a = -2,\ b = 1,\ c = 1$. In our experiment, the mean square error at the time $t = \sqrt{2}$ is estimated in the following way: a set of 20 blocks, each containing 100 outcomes $(\omega_{i,j},\ 1 \leqslant i \leqslant 20,\ 1 \leqslant j \leqslant 100)$ are simulated and for each block, the estimator

$$e_i = \frac{1}{100}\sum_{j=1}^{100} |x(\sqrt{2}, \omega_{i,j}) - X_N(\omega_{i,j})|^2$$

is formed and we take $e(h) = E|x(\sqrt{2}) - X_{\eta_1}|^2 = \frac{1}{20}\sum_{i=1}^{20} e_i$ and the approximate value with $h = 1/2048$ be the exact value at $t = \sqrt{2}$.

The errors are shown in Table 1, and the linearity of stepsize h and $e(h)$ is shown in Fig. 1.

Now we illustrate the stability of Euler scheme (3.1) for system (5.1). First, we take $a = -2,\ b = 1,\ c = 1$. From condition (4.1), we know when $h \in (0, \frac{1}{2})$, Euler scheme (3.1) is mean square exponentially stable. Take $h = \frac{1}{10}$, the stability is shown in Fig. 2.

We should point out that the condition (4.1) is just sufficient, not necessary. To illustrate this point, we give another experiment. For system (5.1), take $a = -7,\ b = 1,\ c = 1$, according condition (4.1), we get if $0 < h < \{\frac{1}{7},\ \frac{25}{36}\} = \frac{1}{7}$, Euler scheme (3.1) is mean square exponentially stable. First, we take $h = 1/2$, obviously, this stepsize is not included in $(0, \frac{1}{7})$. From numerical experiment, we see the Euler method is not stable, and the instability is shown in Fig. 3.

Then we take $h = \frac{1}{5}$, which also is not included in $(0, \frac{1}{7})$, however, through numerical experiment, we get the Euler method is stable. The stability is shown in Fig. 4.

Acknowledgements

This work was supported by the NSFC(11271101) and by the Natural Scientific Research Innovation Foundation in Harbin Institute of Technology (HIT.NSRIF.201014).

Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.amc.2013.12.041.

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