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Brief Paper

New result on exponential stability for singular systems with two interval time-varying delays

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Abstract: This article mainly studies the problem of the exponential stability for singular systems with two interval time-varying delays. By constructing a modified Lyapunov–Krasovskii functional (LKF) and utilising a convex polyhedron method to estimate the derivative of LKF, some new delay-dependent criteria can be established in terms of linear matrix inequalities. Compared with some existing literatures, the novelties in this study are that the needed decisive variables are fewer and the obtained delay-dependent stability criteria are less conservative. Finally, two illustrative numerical examples are given to show the effectiveness of the derived results.

Notations: Unless otherwise specified, for a real square matrix A, the notation $A > 0$ ($A \geq 0$, $A < 0$, $A \leq 0$) means that A is a real symmetric and positive definite (positive semi-definite, negative definite, negative semi-definite, respectively); I denotes the unit matrix of appropriate dimension. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum and minimum eigenvalues of the square matrix A, respectively; If A is a vector or a matrix, its transpose is denoted by A^{T}; $|B| = \sqrt{\mathrm{trace}(B^{\mathrm{T}}B)}$ denotes the Euclidean norm of a vector B or its induced norm of a matrix B. The superscript 'T' represents the transpose. '$*$' denotes the symmetric term in a symmetric matrix. Unless explicitly stated, matrices are assumed to have real entries and compatible dimensions. Let $h > 0$ and $C([-h, 0]; R^n)$ be the family of all continuous R^n-valued functions ϕ on the interval $[-h, 0]$ with the norm $\|\phi\| = \sup\{|\phi(\theta)| : -h \leq \theta \leq 0\} < +\infty$.

1 Introduction

Singular time-delay systems have extensively been studied since they have extensive applications in many practical fields, such as electrical circuits, power systems, chemical processes, lossless transmission lines and many other fields [1]. Owing to the fact that these systems can describe the behaviour of engineering systems better than state-space ones, the problem of stability analysis for singular time-delay systems has been a focused topic of theoretical and practical importance; see [2–27] and references therein. Moreover, the sliding-mode control design, the H_∞ control design and the H_∞ filter design for singular time-delay systems have been considered in [28–33]. Current efforts on such problems can be roughly divided into two categories, namely, delay-dependent stability criterion and delay-independent stability criterion. Generally speaking, the latter is more conservative than the former when the delay is small. It should be pointed out that the existence and uniqueness of a solution to a given singular time-delay system is not always guaranteed since this system can have undesirable impulsive behaviour, and the discussion for such systems is much more complicated than that for standard state-space time-delay systems [34–52] and references therein. Thus, for singular time-delay systems, it is necessary to obtain the sufficient conditions, which guarantee that the concerned systems are not only stable but also regular and impulse free.

However, the stability analysis discussed in the above-mentioned works is involved with the asymptotical stability. It should be mentioned that the advantage of the singular systems with exponential stability in comparison with that with asymptotic stability lies in the former can provide fast convergence and desirable accuracy. Besides, in practice, it is also of much importance to derive the estimates of the decay rate of a delay system since the transient process of a system can be described more clearly once the decay rate is determined [45]. Owing to the existence of algebraic constraints, compared with the standard state-space time-delay systems, the problem on the exponential stability analysis for singular time-delay systems is of much interest. For example, by employing the linear matrix inequality (LMI) approach, the delay-dependent exponential stability criteria for uncertain descriptor systems with time-varying delays were obtained in [10], but they are only involved into one case that a time-varying delay appears.

By constructing a modified Lyapunov–Krasovskii functional (LKF), the LMI-based delay-dependent exponential stability criteria for uncertain singular systems with mixed delays were given in [14]. By utilising the properties of matrix measure, the delay-independent exponential stability for singular systems with multiple constant time-delays was studied in [16], which was extended to the case with discrete and distributed delays [17]. The exponential stability for singular time-delay systems was discussed in [23, 33], but it is impossible that the estimate of the convergence rate of the states for such systems can be accomplished. To the best of the author‘s knowledge, the obtained decay rate characterisation is a free parameter, which can be chosen according to different practical conditions. Thus, the exponential stability for singular time-delay systems is worthy of being investigated.

On the other hand, the exponential stability for singular linear systems with time-varying delay mentioned above are only applicable to the case when the lower bound of the time-varying delay is zero. There exists a special type of time-varying delay in practical engineering systems, that is, interval time-varying delay, $h_m \le h(t) \le h_M$ and h_m is not restricted to be zero, which commonly exists in networked control systems, and the delay-dependent stability of such systems was widely studied in [15, 34, 35, 37, 42, 43, 46, 48–50] and references therein. Moreover, in many real applications, signals that are transmitted from one point to another may experience two networks segments, which can possibly produce two delays with different properties because of variable networks transmission conditions. Since the properties of these two delays may not be identical because of the network transmission conditions, it is not reasonable to lump the two delays into one. The methods proposed in [14, 16, 20, 23, 24, 26, 27, 32–34, 37, 42, 43, 48, 49, 51, 52] are inapplicable to obtain the delay-dependent exponential stability criteria for singular systems with two interval time-varying delay components since the two interval time-varying delays make the problem be rather complicated. Although in [38], by using the free-weighting matrices technique combined with the graph theory, the delay-range-dependent exponential stability for singular systems with multiple time-varying delays was considered, the free-weighting matrices approach can bring much more computation burden and produce much conservatism. Besides, in [38], the restrictive condition that the derivative value of time-varying delays is less than one should be imposed, which can narrow the scope of application.

Motivated by the preceding discussions, in this paper, by introducing a new LKF, some new LMIs-based delay-range-dependent sufficient conditions ensuring the exponential stability for singular systems with two interval time-varying delays are obtained with the help of the convex polyhedron method. Unlike some previous results, this work has the following features: (1) compared with the methods in [14, 18, 19, 26, 27, 32–34], some important information neglected in these literatures is fully considered; (2) while maintaining the efficiency of the stability criteria, the proposed method introduces much fewer matrix variables than the existing ones; (3) the restrictive condition that the derivative value of the time-varying delays is less than one is removed. These three distinctive features are the novelty of the work given in this paper.

2 Problem formulation

Consider the following singular systems with two interval time-varying delays (see (1))

$$\begin{cases} E\dot{x}(t) = Ax(t) + A_d x(t-d(t)) + A_h x(t-h(t)), & t \ge 0 \\ x(\theta) = \phi(\theta) \in C([-r,0]; R^n), \quad \theta \in [-r,0], & r = \max\{d_2, h_2\} > 0 \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, the matrix $E \in R^{n\times n}$ may be singular, and it is assumed that rank $E = r \le n$. A, A_d, A_h are known real constant matrices with appropriate dimensions, and $\phi(t)$ is a compatible vector-valued continuous function. The time-varying delays $d(t)$ and $h(t)$ are assumed to satisfy

$$\begin{aligned} &0 \le d_1 \le d(t) \le d_2, \dot{d}(t) \le \mu_1, \\ &0 \le h_1 \le h(t) \le h_2, \dot{h}(t) \le \mu_2 \end{aligned} \tag{2}$$

with d_1, d_2, h_1, h_2 and μ_1, μ_2 are given scalars.

Definition 1 [1, 4]: (1) The singular time-varying systems (1) are said to be regular and impulse free, if the pairs (E, A) is regular and impulse free.
(2) The singular systems (1) are said to be exponential stable if there exist $\sigma > 0$ and $\gamma > 0$ such that, for any compatible initial conditions $\phi(t)$, the solution $x(t)$ of the systems (1) satisfies $|x(t)| \le \gamma e^{-\sigma t}\|\phi\|_C$.
(3) Systems (1) are said to be exponential admissible if it is regular, impulse-free and exponential stable.

Lemma 2 [46, 52]: Suppose that Ω, Ξ_{1i}, Ξ_{2i} $(i = 1, 2)$ are constant matrices of appropriate dimensions, $a, b \in [0, 1]$, then

$$\Omega + [a\Xi_{11} + (1-a)\Xi_{12}] + [b\Xi_{21} + (1-b)\Xi_{22}] < 0$$

holds, if the following inequalities hold simultaneously $\Omega + \Xi_{11} + \Xi_{21} < 0$, $\Omega + \Xi_{11} + \Xi_{22} < 0$, $\Omega + \Xi_{12} + \Xi_{21} < 0$, and $\Omega + \Xi_{12} + \Xi_{22} < 0$.

Similar to the proof of the Jensen's integral inequality provided in [53], we can obtain the following result:

Lemma 3: For any two constant matrices: $M \in R^{n\times n}$, $M = M^{\mathrm{T}} > 0$ and $E \in R^{n\times n}$ is singular with rank $E = r \le n$, a scalar $\gamma > 0$ and a vector function $\omega: [0, \gamma] \to R^n$, such that the integrations in the following are well defined, then

$$\left(\int_0^{\gamma} Ew(s)\,\mathrm{d}s\right)^{\mathrm{T}} M \left(\int_0^{\gamma} Ew(s)\,\mathrm{d}s\right) \le \gamma \int_0^{\gamma} w^{\mathrm{T}}(s)E^{\mathrm{T}}MEw(s)\,\mathrm{d}s$$

Proof: It is easily seen that by using the Schur's complement Lemma [35], we have

$$\begin{bmatrix} (Ew(s))^{\mathrm{T}}M(Ew(s)) & (Ew(s))^{\mathrm{T}} \\ Ew(s) & M^{-1} \end{bmatrix} \ge 0$$

for any $0 \le s \le \gamma$. Integration of the above inequality from 0 to γ, we yield

$$\begin{bmatrix} \int_0^{\gamma} (Ew(s))^{\mathrm{T}}M(Ew(s))\,\mathrm{d}s & \int_0^{\gamma} (Ew(s))^{\mathrm{T}}\,\mathrm{d}s \\ \int_0^{\gamma} (Ew(s))\,\mathrm{d}s & \gamma M^{-1} \end{bmatrix} \ge 0$$

By using the Schur's complement Lemma again, the desired result can obtained. □

3 Main results

In this section, some sufficient LMIs-based conditions of the exponential admissibility for singular systems (1) are obtained by constructing an augmented Lyapunov–Krasovskii functional and using the convex polyhedron method in [46, 52].

Theorem 4: For the time-varying delays: $d(t)$, $h(t)$ satisfying (2) and a positive scalar $\alpha > 0$, if there exists a non-singular matrix P, and some positive-definite matrices: $Q_l, Z_m \in R^{n\times n}$ $(l = 1, 2, 3, 4, 5, 6; m = 1, 2, 3, 4)$, such that the following LMIs hold (see (3)–(6))

with the following constraints

$$E^{\mathrm{T}}P = P^{\mathrm{T}}E \geq 0 \tag{7}$$

and

$$Q_2 \leq Q_5 \tag{8}$$

where (see equation at the bottom of the page)

where other items in Ω are zero, then systems (1) are exponentially admissible.

Proof: Since rank $E = r \leq n$, there exist two singular matrices G and H such that

$$\bar{E} := GEH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \tag{9}$$

and denote

$$\bar{A} := GAH = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \quad \bar{A}_d := GA_dH = \begin{bmatrix} A_{d1} & A_{d2} \\ A_{d3} & A_{d4} \end{bmatrix},$$

$$\bar{A}_h := GA_hH = \begin{bmatrix} A_{h1} & A_{h2} \\ A_{h3} & A_{h4} \end{bmatrix} \tag{10}$$

$$\bar{P} := G^{-T}PH = \begin{bmatrix} P_1 & P_2 \\ P_3 & P_4 \end{bmatrix},$$

$$\bar{Q}_l := H^{\mathrm{T}}Q_lH = \begin{bmatrix} Q_{l1} & Q_{l2} \\ Q_{l3} & Q_{l4} \end{bmatrix}, \quad l = 1, 2, 3, 4, 5, 6 \tag{11}$$

From (3)–(6) and (10)–(11), and some algebraic manipulations, we obtain $A_4^{\mathrm{T}}P_4 + P_4^{\mathrm{T}}A_4 < 0$, which implies that A_4 is nonsingular and thus the pair (E, A) is regular and impulses free. Thus, by Definition 1, systems (1) are regular and impulse free for the time-varying delays $d(t), h(t)$ satisfying (2).

Let $\xi(t) = \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} = H^{-1}x(t)$, where $\xi_1(t) \in R^r$ and $\xi_2(t) \in R^{n-r}$, and substituting (10) and (11) into systems (1), we have (see (12))

Now, choose a new LKF candidate as follows

$$V(t, \xi_t) = V_1(t, \xi_t) + V_2(t, \xi_t) + V_3(t, \xi_t) \tag{13}$$

$$\begin{aligned}
\Omega_1 = \Omega &- e^{-2\alpha d_2}\begin{bmatrix} 0 & -I & 0 & I & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} E^{\mathrm{T}}Z_2E \begin{bmatrix} 0 & -I & 0 & I & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \\
&- e^{-2\alpha h_2}\begin{bmatrix} 0 & 0 & -I & 0 & 0 & I & 0 \end{bmatrix}^{\mathrm{T}} E^{\mathrm{T}}Z_4E \begin{bmatrix} 0 & 0 & -I & 0 & 0 & I & 0 \end{bmatrix}^{\mathrm{T}} < 0
\end{aligned} \tag{3}$$

$$\begin{aligned}
\Omega_2 = \Omega &- e^{-2\alpha d_2}\begin{bmatrix} 0 & -I & 0 & I & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} E^{\mathrm{T}}Z_2E \begin{bmatrix} 0 & -I & 0 & I & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \\
&- e^{-2\alpha h_2}\begin{bmatrix} 0 & 0 & -I & 0 & 0 & 0 & I \end{bmatrix}^{\mathrm{T}} E^{\mathrm{T}}Z_4E \begin{bmatrix} 0 & 0 & -I & 0 & 0 & 0 & I \end{bmatrix}^{\mathrm{T}} < 0
\end{aligned} \tag{4}$$

$$\begin{aligned}
\Omega_3 = \Omega &- e^{-2\alpha d_2}\begin{bmatrix} 0 & -I & 0 & 0 & I & 0 & 0 \end{bmatrix}^{\mathrm{T}} E^{\mathrm{T}}Z_2E \begin{bmatrix} 0 & -I & 0 & 0 & I & 0 & 0 \end{bmatrix}^{\mathrm{T}} \\
&- e^{-2\alpha h_2}\begin{bmatrix} 0 & 0 & -I & 0 & 0 & I & 0 \end{bmatrix} E^{\mathrm{T}}Z_4E \begin{bmatrix} 0 & 0 & -I & 0 & 0 & I & 0 \end{bmatrix}^{\mathrm{T}} < 0
\end{aligned} \tag{5}$$

$$\begin{aligned}
\Omega_4 = \Omega &- e^{-2\alpha d_2}\begin{bmatrix} 0 & -I & 0 & 0 & I & 0 & 0 \end{bmatrix}^{\mathrm{T}} E^{\mathrm{T}}Z_2E \begin{bmatrix} 0 & -I & 0 & 0 & I & 0 & 0 \end{bmatrix}^{\mathrm{T}} \\
&- e^{-2\alpha h_2}\begin{bmatrix} 0 & 0 & -I & 0 & 0 & 0 & I \end{bmatrix}^{\mathrm{T}} E^{\mathrm{T}}Z_4E \begin{bmatrix} 0 & 0 & -I & 0 & 0 & 0 & I \end{bmatrix}^{\mathrm{T}} < 0
\end{aligned} \tag{6}$$

$$\begin{aligned}
&\Omega = (\Omega_{i,j})_{7\times 7} \\
&\Omega_{1,1} = P^{\mathrm{T}}A + A^{\mathrm{T}}P + 2\alpha E^{\mathrm{T}}P + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 - e^{-2\alpha d_1}E^{\mathrm{T}}Z_1E - e^{-2\alpha h_1}E^{\mathrm{T}}Z_3E \\
&\qquad + A^{\mathrm{T}}UA, \Omega_{1,2} = PA_d + A^{\mathrm{T}}UA_d, \Omega_{1,3} = PA_h + A^{\mathrm{T}}UA_h, \Omega_{1,4} = e^{-2\alpha d_1}E^{\mathrm{T}}Z_1E \\
&\Omega_{1,6} = e^{-2\alpha h_1}E^{\mathrm{T}}Z_3E, \Omega_{2,2} = [-e^{-2\alpha d_2}(1-\mu_1)] \vee [-e^{-2\alpha d_1}(1-\mu_1)]Q_2 - 2e^{-2\alpha d_2}E^{\mathrm{T}}Z_2E + A_d^{\mathrm{T}}UA_d \\
&\Omega_{2,3} = A_d^{\mathrm{T}}UA_h, \Omega_{2,4} = \Omega_{2,5} = e^{-2\alpha d_2}E^{\mathrm{T}}Z_2E \\
&\Omega_{3,3} = [-e^{-2\alpha h_2}(1-\mu_2)] \vee [-e^{-2\alpha h_1}(1-\mu_2)]Q_5 - 2e^{-2\alpha h_2}E^{\mathrm{T}}Z_4E + A_h^{\mathrm{T}}UA_h \\
&\Omega_{3,6} = \Omega_{3,7} = e^{-2\alpha h_2}E^{\mathrm{T}}Z_4E, \Omega_{4,4} = -e^{-2\alpha d_1}Q_1 - e^{-2\alpha d_1}E^{\mathrm{T}}Z_1E - e^{-2\alpha d_2}E^{\mathrm{T}}Z_2E \\
&\Omega_{5,5} = -e^{-2\alpha d_2}Q_3 - e^{-2\alpha d_2}E^{\mathrm{T}}Z_2E, \Omega_{6,6} = -e^{-2\alpha h_1}Q_4 - e^{-2\alpha h_1}E^{\mathrm{T}}Z_3E - e^{-2\alpha h_2}E^{\mathrm{T}}Z_4E \\
&\Omega_{7,7} = -e^{-2\alpha h_2}Q_6 - e^{-2\alpha h_2}E^{\mathrm{T}}Z_4E, U = d_1^2Z_1 + d_{12}^2Z_2 + h_1^2Z_3 + h_{12}^2Z_4, d_{12} = d_2 - d_1, h_{12} = h_2 - h_1
\end{aligned}$$

$$\begin{cases} \dot{\xi}_1(t) = A_1\xi_1(t) + A_2\xi_2(t) + A_{d1}\xi_1(t-d(t)) + A_{d2}\xi_2(t-d(t)) + A_{h1}\xi_1(t-h(t)) + A_{h2}\xi_2(t-h(t)), \\ 0 = A_3\xi_1(t) + A_4\xi_2(t) + A_{d3}\xi_1(t-d(t)) + A_{d4}\xi_2(t-d(t)) + A_{h3}\xi_1(t-h(t)) + A_{h4}\xi_2(t-h(t)) \end{cases} \tag{12}$$

where (see equation at the bottom of the page)

for $t \geq r$, where $\bar{Z}_i = G^{-T} Z_i G^{-1}$, $(i = 1, 2, 3, 4)$.

Then, we have the time derivative of $V_i(t, \xi_t)$ $(i = 1, 2, 3)$ along the solution of (12)

$$\dot{V}_1(t, \xi_t) = 2\xi^{\mathrm{T}}(t)\bar{P}^{\mathrm{T}}\bar{E}\dot{\xi}(t) \tag{14}$$

$$\begin{aligned}\dot{V}_2(t, \xi_t) \leq{}& \xi^{\mathrm{T}}(t)[\bar{Q}_1 + \bar{Q}_2 + \bar{Q}_3 + \bar{Q}_4 + \bar{Q}_5 + \bar{Q}_6]\xi(t) \\ &- e^{-2\alpha d_1}\xi^{\mathrm{T}}(t - d_1)\bar{Q}_1\xi(t - d_1) \\ &- (1 - \dot{d}(t))e^{-2\alpha d(t)}\xi^{\mathrm{T}}(t - d(t))\bar{Q}_2\xi(t - d(t)) \\ &- e^{-2\alpha d_2}\xi^{\mathrm{T}}(t - d_2)\bar{Q}_3\xi(t - d_2) \\ &- e^{-2\alpha h_1}\xi^{\mathrm{T}}(t - h_1)\bar{Q}_4\xi(t - h_1) \\ &- (1 - \dot{h}(t))e^{-2\alpha h(t)}\xi^{\mathrm{T}}(t - h(t))\bar{Q}_5\xi(t - h(t)) \\ &- e^{-2\alpha h_2}\xi^{\mathrm{T}}(t - h_2)\bar{Q}_6\xi(t - h_2) - 2\alpha V_2(t, \xi_t)\end{aligned} \tag{15}$$

and

$$\begin{aligned}\dot{V}_3(t, \xi_t) \leq{}& \dot{\xi}(t)\bar{E}^{\mathrm{T}}[d_1^2\bar{Z}_1 + d_{12}^2\bar{Z}_2 + h_1^2\bar{Z}_3 + h_{12}^2\bar{Z}_4]\bar{E}\dot{\xi}(t) \\ &- e^{-2\alpha d_1}d_1\int_{t-d_1}^{t}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_1\bar{E}\dot{\xi}(s)\,\mathrm{d}s \\ &- e^{-2\alpha d_2}d_{12}\int_{t-d_2}^{t-d_1}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_2\bar{E}\dot{\xi}(s)\,\mathrm{d}s \\ &- e^{-2\alpha h_1}h_1\int_{t-h_1}^{t}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_3\bar{E}\dot{\xi}(s)\,\mathrm{d}s \\ &- e^{-2\alpha h_2}h_{12}\int_{t-h_2}^{t-h_1}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_4\bar{E}\dot{\xi}(s)\,\mathrm{d}s - 2\alpha V_3(t, \xi_t)\end{aligned} \tag{16}$$

When $0 \leq \mu_1 < 1$, we have

$$\begin{aligned}&-(1 - \dot{d}(t))e^{-2\alpha d(t)}\xi^{\mathrm{T}}(t - d(t))\bar{Q}_2\xi(t - d(t)) \\ &\quad\leq -(1 - \mu_1)e^{-2\alpha d_2}\xi^{\mathrm{T}}(t - d(t))\bar{Q}_2\xi(t - d(t))\end{aligned} \tag{17}$$

and if $\mu_1 \geq 1$,

$$\begin{aligned}&-(1 - \dot{d}(t))e^{-2\alpha d(t)}\xi^{\mathrm{T}}(t - d(t))\bar{Q}_2\xi(t - d(t)) \\ &\quad\leq -(1 - \mu_1)e^{-2\alpha d_1}\xi^{\mathrm{T}}(t - d(t))\bar{Q}_2\xi(t - d(t))\end{aligned} \tag{18}$$

Similarly, we have

$$\begin{aligned}&-(1 - \dot{h}(t))e^{-2\alpha h(t)}\xi^{\mathrm{T}}(t - h(t))\bar{Q}_5\xi(t - h(t)) \\ &\quad\leq [-(1 - \mu_2)e^{-2\alpha h_2}] \vee [-(1 - \mu_2)e^{-2\alpha h_1}] \\ &\quad\quad\times \xi^{\mathrm{T}}(t - h(t))\bar{Q}_5\xi(t - h(t))\end{aligned} \tag{19}$$

By utilising 2, it obtains

$$\begin{aligned}&-e^{-2\alpha d_1}d_1\int_{t-d_1}^{t}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_1\bar{E}\dot{\xi}(s)\,\mathrm{d}s \\ &\quad\leq -e^{-2\alpha d_1}[\xi(t) - \xi(t - d_1)]^{\mathrm{T}}\bar{E}^{\mathrm{T}}\bar{Z}_1\bar{E}[\xi(t) - \xi(t - d_1)]\end{aligned} \tag{20}$$

and

$$\begin{aligned}&-e^{-2\alpha h_1}h_1\int_{t-h_1}^{t}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_3\bar{E}\dot{\xi}(s)\,\mathrm{d}s \\ &\quad\leq -e^{-2\alpha h_1}[\xi(t) - \xi(t - h_1)]^{\mathrm{T}}\bar{E}^{\mathrm{T}}\bar{Z}_3\bar{E}[\xi(t) - \xi(t - h_1)]\end{aligned} \tag{21}$$

On the other hand, we have

$$\begin{aligned}&-e^{-2\alpha d_2}d_{12}\int_{t-d_2}^{t-d_1}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_2\bar{E}\dot{\xi}(s)\,\mathrm{d}s \\ &\quad= -e^{-2\alpha d_2}[d_2 - d(t)]\int_{t-d_2}^{t-d(t)}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_2\bar{E}\dot{\xi}(s)\,\mathrm{d}s \\ &\quad\quad-e^{-2\alpha d_2}[d_2 - d(t)]\int_{t-d(t)}^{t-d_1}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_2\bar{E}\dot{\xi}(s)\,\mathrm{d}s \\ &\quad\quad-e^{-2\alpha d_2}[d(t) - d_1]\int_{t-d_2}^{t-d(t)}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_2\bar{E}\dot{\xi}(s)\,\mathrm{d}s \\ &\quad\quad-e^{-2\alpha d_2}[d(t) - d_1]\int_{t-d(t)}^{t-d_1}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_2\bar{E}\dot{\xi}(s)\,\mathrm{d}s\end{aligned} \tag{22}$$

Letting $\theta_1 = \{[d_2 - d(t)]/d_{12}\}$, then

$$\begin{aligned}&-e^{-2\alpha d_2}[d_2 - d(t)]\int_{t-d(t)}^{t-d_1}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_2\bar{E}\dot{\xi}(s)\,\mathrm{d}s \\ &-e^{-2\alpha d_2}[d(t) - d_1]\int_{t-d_2}^{t-d(t)}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_2\bar{E}\dot{\xi}(s)\,\mathrm{d}s \\ &\quad\leq -e^{-2\alpha d_2}\theta_1[\xi(t - d_1) - \xi(t - d(t))]^{\mathrm{T}}\bar{E}^{\mathrm{T}}\bar{Z}_2\bar{E}[\xi(t - d_1) \\ &- \xi(t - d(t))] - e^{-2\alpha d_2}(1 - \theta_1)[\xi(t - d(t)) \\ &- \xi(t - d_2)]^{\mathrm{T}}\bar{E}^{\mathrm{T}}\bar{Z}_2\bar{E}[\xi(t - d(t)) - \xi(t - d_2)]\end{aligned} \tag{23}$$

$$\begin{aligned}V_1(t, \xi_t) ={}& \xi^{\mathrm{T}}(t)\bar{E}^{\mathrm{T}}\bar{P}\xi(t) \\ V_2(t, \xi_t) ={}& \int_{t-d_1}^{t}e^{2\alpha(s-t)}\xi^{\mathrm{T}}(s)\bar{Q}_1\xi(s)\,\mathrm{d}s + \int_{t-d(t)}^{t}e^{2\alpha(s-t)}\xi^{\mathrm{T}}(s)\bar{Q}_2\xi(s)\,\mathrm{d}s + \int_{t-d_2}^{t}e^{2\alpha(s-t)}\xi^{\mathrm{T}}(s)\bar{Q}_3\xi(s)\,\mathrm{d}s \\ &+ \int_{t-h_1}^{t}e^{2\alpha(s-t)}\xi^{\mathrm{T}}(s)\bar{Q}_4\xi(s)\,\mathrm{d}s + \int_{t-h(t)}^{t}e^{2\alpha(s-t)}\xi^{\mathrm{T}}(s)\bar{Q}_5\xi(s)\,\mathrm{d}s + \int_{t-h_2}^{t}e^{2\alpha(s-t)}\xi^{\mathrm{T}}(s)\bar{Q}_6\xi(s)\,\mathrm{d}s \\ V_3(t, x) ={}& d_1\int_{-d_1}^{0}\int_{t+\theta}^{t}e^{2\alpha(s-t)}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_1\bar{E}\dot{\xi}(s)\,\mathrm{d}s\,\mathrm{d}\theta + d_{12}\int_{-d_2}^{-d_1}\int_{t+\theta}^{t}e^{2\alpha(s-t)}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_2\bar{E}\dot{\xi}(s)\,\mathrm{d}s\,\mathrm{d}\theta \\ &+ h_1\int_{-h_1}^{0}\int_{t+\theta}^{t}e^{2\alpha(s-t)}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_3\bar{E}\dot{\xi}(s)\,\mathrm{d}s\,\mathrm{d}\theta + h_{12}\int_{-h_2}^{-h_1}\int_{t+\theta}^{t}e^{2\alpha(s-t)}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_4\bar{E}\dot{\xi}(s)\,\mathrm{d}s\,\mathrm{d}\theta\end{aligned}$$

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Similarly

$$-e^{-2\alpha h_2}h_{12}\int_{t-h_2}^{t-h_1}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_4\bar{E}\dot{\xi}(s)\,\mathrm{d}s$$
$$=-e^{-2\alpha h_2}[h_2-h(t)]\int_{t-h_2}^{t-h(t)}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_4\bar{E}\dot{\xi}(s)\,\mathrm{d}s$$
$$-e^{-2\alpha d_2}[h_2-h(t)]\int_{t-h(t)}^{t-h_1}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_4\bar{E}\dot{\xi}(s)\,\mathrm{d}s$$
$$-e^{-2\alpha h_2}[h(t)-h_1]\int_{t-h_2}^{t-h(t)}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_4\bar{E}\dot{\xi}(s)\,\mathrm{d}s$$
$$-e^{-2\alpha h_2}[h(t)-h_1]\int_{t-h(t)}^{t-h_1}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_2\bar{E}\dot{\xi}(s)\,\mathrm{d}s \quad (24)$$

Letting $\theta_2=[h_2-h(t)]/h_{12}$, then

$$-e^{-2\alpha h_2}[h_2-h(t)]\int_{t-h(t)}^{t-h_1}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_4\bar{E}\dot{\xi}(s)\,\mathrm{d}s$$
$$-e^{-2\alpha h_2}[h(t)-h_1]\int_{t-h_2}^{t-h(t)}(\bar{E}\dot{\xi}(s))^{\mathrm{T}}\bar{Z}_4\bar{E}\dot{\xi}(s)\,\mathrm{d}s$$
$$\le -e^{-2\alpha h_2}\theta_2[\xi(t-h_1)-\xi(t-h(t))]^{\mathrm{T}}\bar{E}^{\mathrm{T}}\bar{Z}_4\bar{E}[\xi(t-h_1)$$
$$-\xi(t-h(t))]-e^{-2\alpha h_2}(1-\theta_2)[\xi(t-h(t))$$
$$-\xi(t-h_2)]^{\mathrm{T}}\bar{E}^{\mathrm{T}}\bar{Z}_4\bar{E}[\xi(t-h(t))-\xi(t-h_2)] \quad (25)$$

Combining (14)–(25), it yields that (see (26))

where $\eta(t)=[\xi^{\mathrm{T}}(t)\ \ \xi^{\mathrm{T}}(t-d(t))\ \ \xi^{\mathrm{T}}(t-h(t))\ \ \xi^{\mathrm{T}}(t-d_1)\ \ \xi^{\mathrm{T}}(t-d_2)\ \ \xi^{\mathrm{T}}(t-h_1)\ \ \xi^{\mathrm{T}}(t-h_2)]^{\mathrm{T}}$, $\bar{\Omega}$ is defined in (3)–(6) with $\bar{A}$, $\bar{A}_d$, $\bar{A}_h$, $\bar{Q}_l$, $\bar{Z}_i$ instead of A, A_d, A_h, Q_l, Z_i, $(l=1,2,\ldots,6;\ i=1,2,3,4)$. Pre- and post-multiplying (3)–(6) by $\mathrm{diag}\{H^{\mathrm{T}},H^{\mathrm{T}},H^{\mathrm{T}},H^{\mathrm{T}},H^{\mathrm{T}},H^{\mathrm{T}},H^{\mathrm{T}}\}$ and its transpose, respectively, and then by using the Schur complement Lemma [35] and Lemma 2, it implies that

$$\dot{V}(t,\xi_t)+2\alpha V(t,\xi_t)\le 0$$

which can lead to

$$V(t,\xi_t)\le e^{-2\alpha t}V(t,\phi)$$

Then, the following estimation is derived

$$\lambda_1|\xi_1(t)|^2\le V(t,\xi_t)\le e^{-2\alpha t}V(t,\phi)\le\lambda_2\|\phi\|_c^2e^{-2\alpha t}$$

where $\lambda_1=\lambda_{\min}(P_1)>0$, $\lambda_2>0$ is sufficiently large and can be obtained since $V(t,\phi)$ is bounded quadratic functional of $\phi(t)$. Thus, we obtain

$$|\xi_1(t)|\le\sqrt{\frac{\lambda_2}{\lambda_1}}\|\phi\|_c e^{-\alpha t} \quad (27)$$

Thus, the differential subsystem of (12) is exponentially stable with the decay rate α. The remaining task is to show the exponential stability of the algebraic subsystem.

Define an auxiliary variable $e(t)$ as

$$e(t)=A_3\xi_1(t)+A_{d3}\xi_1(t-d(t))+A_{h3}\xi_1(t-h(t)) \quad (28)$$

From (27) and (28), we obtain that there is a positive constant m such that when $t>0$,

$$|e(t)|\le me^{-\alpha t} \quad (29)$$

To obtain the exponential stability of the algebraic subsystem, a function is constructed as follows

$$J(t)=\xi_2^{\mathrm{T}}(t)[Q_{24}+Q_{54}]\xi_2(t)+[-e^{-2\alpha d_2}(1-\mu_1)]$$
$$\vee[-e^{-2\alpha d_1}(1-\mu_1)]\xi_2^{\mathrm{T}}(t-d(t))Q_{24}\xi_2(t-d(t))$$
$$+[-e^{-2\alpha h_2}(1-\mu_2)]\vee[-e^{-2\alpha h_1}(1-\mu_2)]$$
$$\times\xi_2^{\mathrm{T}}(t-h(t))Q_{54}\xi_2(t-h(t)) \quad (30)$$

By pre-multiplying the second equation of (12) with $\xi_2^{\mathrm{T}}(t)P_4^{\mathrm{T}}$, we obtain that

$$0=\xi_2^{\mathrm{T}}(t)P^{\mathrm{T}}A_4\xi_2(t)+\xi_2^{\mathrm{T}}(t)P_4^{\mathrm{T}}A_{d4}\xi_2(t-d(t))$$
$$+\xi_2^{\mathrm{T}}(t)P_4^{\mathrm{T}}A_{h4}\xi_2(t-h(t))+\xi_2^{\mathrm{T}}(t)P_4^{\mathrm{T}}e(t)$$

Adding it and its transpose to (30) yields that

$$J(t)=\xi_2^{\mathrm{T}}(t)[P_4^{\mathrm{T}}A_4+A_4^{\mathrm{T}}P_4+Q_{24}+Q_{54}]\xi_2(t)$$
$$+2\xi_2^{\mathrm{T}}(t)P_4^{\mathrm{T}}A_{d4}\xi_2(t-d(t))$$
$$+2\xi_2^{\mathrm{T}}(t)P^{\mathrm{T}}A_{h4}\xi_2(t-h(t))$$
$$+[-e^{-2\alpha d_2}(1-\mu_1)]\vee[-e^{-2\alpha d_1}(1-\mu_1)]$$
$$\times\xi_2^{\mathrm{T}}(t-d(t))Q_{24}\xi_2(t-d(t))$$
$$+[-e^{-2\alpha h_2}(1-\mu_2)]\vee[-e^{-2\alpha h_1}(1-\mu_2)]$$
$$\times\xi_2^{\mathrm{T}}(t-h(t))Q_{54}\xi_2(t-h(t))+2\xi_2^{\mathrm{T}}(t)P_4^{\mathrm{T}}e(t)$$
$$\le\eta^{\mathrm{T}}(t)\Lambda\eta(t)+\epsilon_1\xi_2^{\mathrm{T}}(t)\xi_2(t)+\frac{1}{\epsilon_1}e^{\mathrm{T}}(t)P_4P_4^{\mathrm{T}}e(t) \quad (31)$$

where $\eta(t)=[\xi_2^{\mathrm{T}}(t)\ \ \xi_2^{\mathrm{T}}(t-d(t))\ \ \xi_2^{\mathrm{T}}(t-h(t))]^{\mathrm{T}}$, ϵ_1 is a positive constant, and

$$\Lambda=\begin{bmatrix}P_4^{\mathrm{T}}A_4+A_4^{\mathrm{T}}P_4+Q_{24}+Q_{54} & P_4^{\mathrm{T}}A_{d4} & P_4^{\mathrm{T}}A_{h4}\\ * & \Lambda_{22} & 0\\ * & * & \Lambda_{33}\end{bmatrix}$$

where $\Lambda_{22}=[-e^{-2\alpha d_2}(1-\mu_1)]\vee[-e^{-2\alpha d_1}(1-\mu_1)]Q_{24}$ and $\Lambda_{33}=[-e^{-2\alpha h_2}(1-\mu_2)]\vee[-e^{-2\alpha h_1}(1-\mu_2)]Q_{54}$.

$$\dot{V}(t,\xi_t)+2\alpha V(t,\xi_t)$$
$$\le\eta^{\mathrm{T}}(t)\Big\{\bar{\Omega}-e^{-2\alpha d_2}(1-\theta_1)\begin{bmatrix}0 & -I & 0 & I & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}\bar{E}^{\mathrm{T}}\bar{Z}_2\bar{E}\begin{bmatrix}0 & -I & 0 & I & 0 & 0 & 0\end{bmatrix}$$
$$-e^{-2\alpha d_2}\theta_1\begin{bmatrix}0 & -I & 0 & 0 & I & 0 & 0\end{bmatrix}^{\mathrm{T}}\bar{E}^{\mathrm{T}}\bar{Z}_2\bar{E}\begin{bmatrix}0 & -I & 0 & 0 & I & 0 & 0\end{bmatrix}$$
$$-e^{-2\alpha h_2}(1-\theta_2)\begin{bmatrix}0 & 0 & -I & 0 & 0 & I & 0\end{bmatrix}^{\mathrm{T}}\bar{E}^{\mathrm{T}}\bar{Z}_4\bar{E}\begin{bmatrix}0 & 0 & -I & 0 & 0 & I & 0\end{bmatrix}$$
$$-e^{-2\alpha h_2}\theta_2\begin{bmatrix}0 & 0 & -I & 0 & 0 & 0 & I\end{bmatrix}^{\mathrm{T}}\bar{E}^{\mathrm{T}}\bar{Z}_4\bar{E}\begin{bmatrix}0 & 0 & -I & 0 & 0 & 0 & I\end{bmatrix}\Big\}\eta(t) \quad (26)$$

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By using the Schur's complement Lemma on one inequality of (3)–(6), we obtain

$$\begin{bmatrix} \Omega_{1,1} & P^{\mathrm{T}}A_4 & P^{\mathrm{T}}A_h \\ * & \Omega_{2,2} & 0 \\ * & * & \Omega_{3,3} \end{bmatrix} < 0 \tag{32}$$

Pre- and post-multiplying the inequality (32) by $\mathrm{diag}\{H^{\mathrm{T}} \ \ H^{\mathrm{T}}H^{\mathrm{T}}\}$ and its transpose, respectively, and since $U > 0$, by using the Schur's complement Lemma, it yields

$$\begin{bmatrix} \mathrm{Sym}\{P_4^{\mathrm{T}}A_4\} + \sum_{l=1}^{6} Q_{l4} & P^{\mathrm{T}}A_4 & P^{\mathrm{T}}A_h \\ * & \Lambda_{22} & 0 \\ * & * & \Lambda_{33} \end{bmatrix} < 0 \tag{33}$$

where Λ_{22} and Λ_{33} are defined above.

Then, we have $\Lambda \le \mathrm{diag}\{-\Psi \ \ 0 \ \ 0\}$, where $\Psi = -Q_{14} - Q_{34} - Q_{44} - Q_{64}$. From (31) and (33), we have

$$\begin{aligned} &\xi_2^{\mathrm{T}}(t)[Q_{24} + Q_{54}]\xi_2(t) + [-e^{-2\alpha d_2}(1-\mu_1)] \\ &\quad \vee [-e^{-2\alpha d_1}(1-\mu_1)]\xi_2^{\mathrm{T}}(t-d(t))Q_{24}\xi_2(t-d(t)) \\ &\quad + [-e^{-2\alpha h_2}(1-\mu_2)] \vee [-e^{-2\alpha h_1}(1-\mu_2)] \\ &\quad \times \xi_2^{\mathrm{T}}(t-h(t))Q_{54}\xi_2(t-h(t)) \\ &\le \xi_2^{\mathrm{T}}(t)[-\Psi + \epsilon_1 I]\xi_2(t) + \frac{1}{\epsilon_1}e^{\mathrm{T}}(t)P_4^{\mathrm{T}}P_4 e(t) \end{aligned}$$

which can be rewritten as

$$\begin{aligned} &\xi_2^{\mathrm{T}}(t)[Q_{24} + Q_{54} + \Psi - \epsilon_1 I]\xi_2(t) \\ &\quad \le [-e^{-2\alpha d_2}(1-\mu_1)] \vee [-e^{-2\alpha d_1}(1-\mu_1)] \\ &\quad \times \xi_2^{\mathrm{T}}(t-d(t))Q_{24}\xi_2(t-d(t)) \\ &\quad + [-e^{-2\alpha h_2}(1-\mu_2)] \vee [-e^{-2\alpha h_1}(1-\mu_2)] \\ &\quad \times \xi_2^{\mathrm{T}}(t-h(t))Q_{54}\xi_2(t-h(t)) + \frac{1}{\epsilon_1}e^{\mathrm{T}}(t)P_4^{\mathrm{T}}P_4 e(t) \end{aligned} \tag{34}$$

Since ϵ_1 is an arbitrary positive scalar, we can choose a small enough ϵ_1, such that

$$Q_{24} + Q_{54} + \Psi - \epsilon_1 I \ge Q_{54} + \Psi - \epsilon_1 I \ge (1+\upsilon)Q_{54} \tag{35}$$

where $\upsilon > 0$ is a small enough scalar satisfying $\Psi > \upsilon Q_{54}$.

Denote

$$f(t) = \xi_2^{\mathrm{T}}(t)Q_{54}\xi_2(t)$$

and (see equation at the bottom of the page)

Thus, it follows from (34) and (35) that

$$f(t) \le \epsilon_2 \sup_{t-r\le s\le t} f(s) + \epsilon_3 e^{-2\alpha t}$$

By utilising Lemma 1 in [33, 52], the above inequality yields that

$$f(t) \le \sup_{-r\le\theta\le 0} f(\theta)e^{-2\alpha t} + \frac{\epsilon_3}{1-\epsilon_2 e^{2\alpha r}}e^{-2\alpha t}$$

Then, we can find from (27) and the above inequality that system (1) is exponentially stable. □

Remark 1: The Lyapunov–Krasovskii functional (13) employed in this paper has widely been introduced in [38, 49] and references therein, in which it has been proved as an important role in reducing the conservatism of the obtained results while considering the stability of the standard state-space time-delay systems with interval time-varying delays.

Remark 2: In [38], Haidar *et al.* obtained the exponential stability admissibility for singular systems with multiple time-varying delays by using the free-weighting matrices technique [40–42, 50]. Although the Lyapunov–Krasovskii functional (13) is borrowed from ones when $p = 2$ in [38], the convex polyhedron method employed to prove the exponential stability admissibility of systems (1) is different from ones utilised in [38] and the obtained result can weaken the conservatism in [38], which can be shown by Example 1 in the next section. What is more, the restrictive condition that the derivative value of time-varying delay $d(t)$, $h(t)$ is less than one can be removed. And the numbers of the decisive variables needed in Theorem 4 in [38] and Theorem 4 are $17.5n^2 + 5.5n$ and $5.5n^2 + 5.5n$, respectively. Therefore Theorem 1 is obviously better than one given in [38].

Remark 3: From the numerical simulation point of view, it is noticed that the LMIs-conditions in Theorem 4 is not strict. Considering this, (3)–(6) can be changed into some single strict LMIs. Let $\mathcal{P}$ and $S \in R^{n\times(n-r)}$ be any matrix with full column rank, which satisfies $E^{\mathrm{T}}S = 0$. So, replacing P by $\mathcal{P}E + SQ$ in (3)–(6) can produce the strict LMIs.

Remark 4: The sliding-mode control design, the H_∞ control design and H_∞ filter design for singular time-delay systems have widely studied in [28–31, 33, 52]. Inspired by these excellent works, we will discuss these interesting problems for singular linear systems with multiple interval time-varying delays in our later works.

Remark 5: There is an important problem that how to determine an appropriate h_2 (or d_2) with respect to α under the given values d_2 (or h_2), d_1, h_1, μ_1, μ_2. For this optimisation problem, the following algorithm (see Fig. 1) is developed as follows:

When $E = I$, systems (1) can be turned into the following form

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-d(t)) + A_h x(t-h(t)), & t \ge 0 \\ x(\theta) = \phi(\theta), \quad \theta \in [-r, 0], & r = \max\{d_2, h_2\} \end{cases} \tag{36}$$

where the matrices A, A_d, A_h are given in systems (1), and the time-varying delays $d(t)$, $h(t)$ are denoted in (2). Based on the idea proposed in Theorem 4, for systems (36), we can obtain the following result:

Corollary 5: For the time-varying delays: $\tau(t)$, $h(t)$ satisfying (2) and a positive scalar $\alpha > 0$, if there exist some positive-definite matrices: P, Q_l, $Z_m \in R^{n\times n}$ ($l = 1, 2, 3, 4, 5, 6$; $m = 1, 2, 3, 4$), such that the following LMIs hold (see equation at the bottom of next page)

where (see equation at the bottom of next page)

then linear systems (36) are exponentially stable.

$$\epsilon_2 = \frac{[-e^{-2\alpha d_2}(1-\mu_1)] \vee [-e^{-2\alpha d_1}(1-\mu_1)] + [-e^{-2\alpha h_2}(1-\mu_2)] \vee [-e^{-2\alpha h_1}(1-\mu_2)]}{1+\upsilon}, \quad \epsilon_3 = \frac{m\|P_4^{\mathrm{T}}P_4\|}{\epsilon_1(1+\upsilon)}$$

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Algorithm

Step 1: Choose a sufficiently small initial $h_2 > 0$ (or $d_2 > 0$) such that there exists a feasible solution to LMIs in (3)–(8). Set $h_{20} = h_2$(or $d_{20} = d_2$).

Step 2: Search a feasible set $\{P_0, Q_{l0}, Z_{m0} \in R^{n\times n}\ (l = 1,2,3,4,5,6;\ m = 1,2,3,4)\}$ satisfying LMIs in (3)–(8). Set $k = 0$.

Step 3: Solve the following LMIs problem for variables $\{P, Q_l, Z_m \in R^{n\times n}\ (l = 1,2,3,4,5,6;\ m = 1,2,3, 4)\}$ such that (3)–(8). Set $P_{k+1} = P$, $Q_{l(k+1)} = Q_l$, $Z_{m(k+1)} = Z_m \in R^{n\times n}$ $(l = 1,2,3,4,5,6;$ $m = 1,2,3,4)\}$.

Step 4: If (3)–(8) satisfied, then return to step 2 after increasing h_2 (or d_2) to some extent; if (3)–(8) are not satisfied within a specified number of iterations, then exit. Otherwise, set $k = k + 1$, and go to step 3.

Fig. 1 *Finding optimal upper delay bound h_2 (or d_2) with respect to α*

Remark 6: When $E = I$, Corollary 5 provides the exponential stability criterion for standard state-space time-delay systems with two interval time-varying delays. From Example 2 in the next section, it is easily seen that Corollary 5 is regarded as a less conservative one.

4 Two illustrative numerical examples

In this section, two illustrative numerical examples are listed to show the effectiveness of our obtained results.

Example 1: Consider the singular time-delay systems (1) with the following parameters [38]

$$E = \begin{bmatrix} -1 & 2 \\ -2 & 4 \end{bmatrix}, A = \begin{bmatrix} -4.7 & 0.4 \\ -4.9 & 0.8 \end{bmatrix},$$
$$A_d = \begin{bmatrix} 0.7 & -0.95 \\ 1.1 & -1.75 \end{bmatrix}, A_h = \begin{bmatrix} 1 & -0.8 \\ 1.4 & -1.3 \end{bmatrix}$$

Let $d_1 = 0.1$, $d_2 = 0.5$, $h_1 = 0.2$ and $\mu_1 = \mu_2 = 0.3$, for different values α, the allowable upper bounds for h_2 to

$$\begin{aligned}
\Pi_1 = \Pi - e^{-2\alpha d_2}&\begin{bmatrix} 0 & -I & 0 & I & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} Z_2 \begin{bmatrix} 0 & -I & 0 & I & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \\
&- e^{-2\alpha h_2}\begin{bmatrix} 0 & 0 & -I & 0 & 0 & I & 0 & 0 \end{bmatrix}^{\mathrm{T}} Z_4 \begin{bmatrix} 0 & 0 & -I & 0 & 0 & I & 0 & 0 \end{bmatrix}^{\mathrm{T}} < 0 \\
\Pi_2 = \Pi - e^{-2\alpha d_2}&\begin{bmatrix} 0 & -I & 0 & I & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} Z_2 \begin{bmatrix} 0 & -I & 0 & I & 0 & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \\
&- e^{-2\alpha h_2}\begin{bmatrix} 0 & 0 & -I & 0 & 0 & 0 & I & 0 \end{bmatrix}^{\mathrm{T}} Z_4 \begin{bmatrix} 0 & 0 & -I & 0 & 0 & 0 & I & 0 \end{bmatrix}^{\mathrm{T}} < 0 \\
\Pi_3 = \Pi - e^{-2\alpha d_2}&\begin{bmatrix} 0 & -I & 0 & 0 & I & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} Z_2 \begin{bmatrix} 0 & -I & 0 & 0 & I & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \\
&- e^{-2\alpha h_2}\begin{bmatrix} 0 & 0 & -I & 0 & 0 & I & 0 & 0 \end{bmatrix} Z_4 \begin{bmatrix} 0 & 0 & -I & 0 & 0 & I & 0 & 0 \end{bmatrix}^{\mathrm{T}} < 0 \\
\Pi_4 = \Pi - e^{-2\alpha d_2}&\begin{bmatrix} 0 & -I & 0 & 0 & I & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} Z_2 \begin{bmatrix} 0 & -I & 0 & 0 & I & 0 & 0 & 0 \end{bmatrix}^{\mathrm{T}} \\
&- e^{-2\alpha h_2}\begin{bmatrix} 0 & 0 & -I & 0 & 0 & 0 & I & 0 \end{bmatrix}^{\mathrm{T}} Z_4 \begin{bmatrix} 0 & 0 & -I & 0 & 0 & 0 & I & 0 \end{bmatrix}^{\mathrm{T}} < 0
\end{aligned}$$

$$\begin{aligned}
&\Pi = (\Pi_{i,j})_{8\times 8} \\
&\Pi_{1,1} = P^{\mathrm{T}}A + A^{\mathrm{T}}P + 2\alpha P + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 + Q_6 - e^{-2\alpha d_1}Z_1 - e^{-2\alpha h_1}Z_3, \Pi_{1,2} = PA_d \\
&\Omega_{1,3} = PA_h, \Pi_{1,4} = e^{-2\alpha d_1}Z_1, \Pi_{1,5} = 0, \Pi_{1,6} = e^{-2\alpha h_1}Z_3, \Pi_{1,7} = 0, \Pi_{1,8} = A^{\mathrm{T}}U \\
&\Pi_{2,2} = [-e^{-2\alpha d_2}(1-\mu_1)] \vee [-e^{-2\alpha d_1}(1-\mu_1)]Q_2 - 2e^{-2\alpha d_2}Z_2, \Pi_{2,3} = 0, \Pi_{2,4} = \Pi_{2,5} = e^{-2\alpha d_2}Z_2 \\
&\Pi_{2,6} = \Pi_{2,7} = 0, \Pi_{2,8} = A_d^{\mathrm{T}}U, \Pi_{3,3} = [-e^{-2\alpha h_2}(1-\mu_2)] \vee [-e^{-2\alpha h_1}(1-\mu_2)]Q_5 - 2e^{-2\alpha h_2}Z_4 \\
&\Pi_{3,4} = \Pi_{3,5} = 0, \Pi_{3,6} = \Pi_{3,7} = e^{-2\alpha h_2}Z_4 \\
&\Pi_{3,8} = A_h^{\mathrm{T}}U, \Pi_{4,4} = -e^{-2\alpha d_1}Q_1 - e^{-2\alpha d_1}Z_1 - e^{-2\alpha d_2}Z_2, \Pi_{4,5} = \Pi_{4,6} = \Pi_{4,7} = \Pi_{4,8} = 0 \\
&\Pi_{5,5} = -e^{-2\alpha d_2}Q_3 - e^{-2\alpha d_2}Z_2, \Pi_{5,6} = \Pi_{5,7} = \Pi_{5,8} = 0, \Pi_{6,6} = -e^{-2\alpha h_1}Q_4 - e^{-2\alpha h_1}Z_3 - e^{-2\alpha h_2}Z_4 \\
&\Pi_{6,7} = \Pi_{6,8} = 0, \Pi_{7,7} = -e^{-2\alpha h_2}Q_6 - e^{-2\alpha h_2}Z_4, \Pi_{7,8} = 0, \Pi_{8,8} = -U \\
&U = d_1^2Z_1 + d_{12}^2Z_2 + h_1^2Z_3 + h_{12}^2Z_4, d_{12} = d_2 - d_1, h_{12} = h_2 - h_1
\end{aligned}$$

Table 1 Allowable upper bounds of h_2 for different values α

Methods	$\alpha = 0.01$	$\alpha = 0.1$	$\alpha = 0.15$	$\alpha = 0.2$	$\alpha = 0.25$	$\alpha = 0.3$	$\alpha = 0.35$
Hadar *et al.* [38]	5.73	3.30	2.79	2.43	1.84	1.01	0.32
Theorem 4	6.62	5.18	4.87	3.81	2.86	2.35	2.03

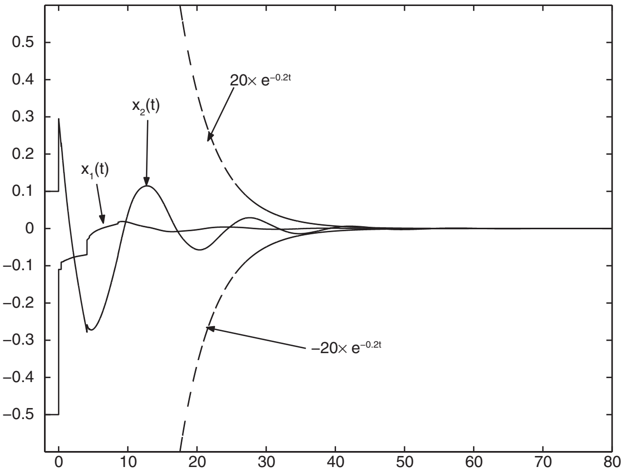

Fig. 2 *Dynamical behaviour of such systems in Example 1*

Table 2 Allowable upper bounds of d_2 for different values α in Example 2

	$\alpha = 0.1$	$\alpha = 0.2$	$\alpha = 0.3$	No. of decisive variables
Amri *et al.* [34] ($\mu_1 = 0.5$)	8.08	3.903	2.582	16
Corollary 5 ($\mu_1 = 0.5$)	11.50	5.19	2.92	12
Amri *et al.* [34] ($\mu_1 = 0.9$)	1.37	0.94	0.74	16
Corollary 5 ($\mu_1 = 0.9$)	3.45	1.91	1.50	12

guarantee that systems (1) are exponentially stable are presented in Table 1. From Table 1, one can easily see that Theorem 4 is much less conservative than ones given in [38]. Fig. 2 provides the simulation results of $x_1(t)$ and $x_2(t)$, when $d_1(t) = 0.3 + 0.1\sin(2t)$ and $d_2(t) = 3.56 + 0.25\sin(t)$ and the initial function is $\phi(t) = [-0.5\ \ 0.1]^{\mathrm{T}}, t \in [-3.5, 0]$.

Example 2: Consider the following linear systems with time-varying delay [34]

$$\dot{x}(t) = Ax(t) + A_{\mathrm{d}}x(t - d(t)) \tag{37}$$

where

$$A = \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}, \quad A_{\mathrm{d}} = \begin{bmatrix} -0.5 & 0.1 \\ 0.3 & 0.0 \end{bmatrix}$$

Letting $d_1 = 0.0$, for different values α, the allowable upper bounds for d_2 when $\mu_1 = 0.5$ and $\mu_1 = 0.9$ to guarantee that systems (37) are exponentially stable are presented in Table 2, respectively. Besides, the number of the decisive variables involved in Corollary 5 is much less than one needed in [34]. From Table 2, it is easily seen that Corollary 1 is much better than ones given in [34].

5 Conclusion

In this paper, the problem on the exponential stability for singular systems with two interval time-varying delays is considered. By introducing a modified LKF and utilising a convex polyhedron method, some LMIs-based sufficient stability criteria for such systems are obtained. The novelties are that the obtained results are less conservative and the involved decisive variables are fewer than ones reported in existing literatures. Finally, two numerical examples are given to illustrate the effectiveness of the obtained results.

The sliding-mode control design, the H_∞ control design and H_∞ filter design for singular time-delay systems have widely studied in [28–31, 33, 52]. Inspired by these excellent works, we will focus our attention to discuss these interesting problems for singular linear systems with multiple interval time-varying delays in our later works.

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