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Stabilization of Rectangular Descriptor Systems*

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Abstract: In this paper a new feedback structure, dynamic output feedback (dynamic compensator) plus state feedback, is utilized to stabilize rectangular descriptor systems. A necessary and sufficient condition is derived such that the resultant closed-loop system is regular, impulse-free and stable (it is called *admissible*). In addition, it is shown that, using this type compensator, the admissible condition is weaker than that using only dynamic compensator. Moreover, a condition for the dynamic order of the proposed compensator being free or not is also derived. Finally, an algorithm for constructing a full order compensator is also presented.
Key Words: Rectangular Descriptor Systems, Dynamic Compensator, State Feedback, Stabilization

1 INTRODUCTION

The rectangular descriptor systems, or named rectangular systems, non-square systems have been investigated by many researchers recently[1-7]. Among them impulse solutions and impulse controllability were discussed in [1,2]. Observer's design was tackled in [3,4]. I-controllability and I-observability and elimination of impulsive modes were solved in [5, 6]. Dynamic compensation is adopted in [7], which can guarantee that the closed-loop systems are square, regular and impulse-free. With regard to dynamic compensation, some new notions on regularizability, controllability and observability have been proposed in [7], and a necessary and sufficient condition for the existence of a dynamic compensator such that the closed-loop system is regular, impulse-free and stable was also derived. For rectangular systems with dynamic compensator applied, the dynamic order of compensator is required to be sufficient large theoretically when stabilizing the system. Undoubtedly, state feedback can improve stability for regular systems, but it can not make system regular for rectangular systems. So this motivates us to simultaneously use the dynamic compensator and state feedback to regularize and stabilize the rectangular descriptor systems.

In this paper, a necessary and sufficient condition is derived such that there exist a dynamic compensator and a state feedback leading to the closed-loop system to be regular, impulse-free, and stable. Also, it is shown that this condition is weaker than that of S-stabilizability and S-detectability. Moreover, the dynamic order of such compensator is proved to be free when a rank condition is satisfied, and an algorithm for constructing a full order compensator is given.

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2 PRELIMINARY

Consider a linear, time-invariant (LTI) rectangular descriptor system

$$E\dot{x} = Ax + Bu\,,\quad y = Cx \tag{1}$$

where $x \in R^n$ is the state vector, $u \in R^q$ and $y \in R^p$ represent the input and output vectors, respectively. $E, A \in R^{m\times n}$, $B \in R^{m\times q}$, and $C \in R^{p\times n}$ are constant matrices. The normal-rank of $(sE - A)$ is defined as the maximum rank of $(sE - A)$. The rank of matrix E is generally assumed as r, i.e., $\mathrm{rank}(E) = r$, with the range of $0 < r \le \min\{m, n\}$. If $m = n$, system (1) is said to be *square*, and more, if $\det(sE - A)$ is not identical zero, system (1) is said to be *regular*, otherwise called *singular*. If $m \ne n$, system (1) is said to be *rectangular.* Let C_+ denote closed right half plane, and $C_{+e} = C_+ \cup \{\infty\}$ denote the extended right half plane. In this paper, the notions about regularizability, R- (I-, S-) controllability and R- (I-, S-) observability, R- (I-, S-) stabilizability and R- (I-, S-) R-detectability for rectangular systems, are used in the sense of dynamic compensation, which are established in [7].

In order to stabilize the system with designed compensator, some new decomposition forms of matrix pencil are required. The following lemmas give us some new decomposition structures and their proofs are based on elementary row or column operations of matrices, which are omitted here.

Lemma 1 For the matrix pencil $(sE - A,\ B,\ C)$ from system (1), there exist nonsingular matrices M and N such that

$$M(sE - A)N = \begin{bmatrix} sI_{n_1} - A_{11} & 0 & -A_{13} \\ 0 & -I_{n_2} & 0 \\ -A_{31} & 0 & 0 \end{bmatrix},$$

$$MB = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} \begin{matrix} \}n_1 \\ \}n_2 \\ \}m_3 \end{matrix}, CN = [\underbrace{C_1}_{n_1} \quad \underbrace{C_2}_{n_2} \quad \underbrace{C_3}_{n_3}] \tag{2}$$

where $A_{13} \in R^{n_1 \times n_3}, A_{31} \in R^{m_3 \times n_1}$, and

$$m = n_1 + n_2 + m_3,\ n = n_1 + n_2 + n_3,\ n_1 = r \tag{3}$$

A more detailed decomposition can be given based on Lemma 1.

Lemma 2 If the rectangular system (1) is regularizable, I-controllable and I-observable, then there exist nonsingular matrices M_1, M_2, N_1, N_2 with compatible dimensions such that

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix} \begin{bmatrix} sE-A & B \\ C & 0 \end{bmatrix} \begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix} = \left[\begin{array}{ccc|cc} sI_{n_1} - A_1 & 0 & -A_2 & 0 & B_{11} \\ 0 & -I_{n_2} & 0 & 0 & B_{21} \\ -A_3 & 0 & 0 & I_{m_3} & 0 \\ \hline 0 & 0 & I_{n_3} & 0 & 0 \\ C_{11} & C_{12} & 0 & 0 & 0 \end{array}\right] \tag{4}$$

Base on the decomposition form in (4), a new controllable and observable condition can be obtained.

Lemma 3 The system (1) is S-controllable and S-observable (S-stabilizable and S-detectable) if and only if the system can be decomposed as (4) and the following two conditions hold:

i) $\mathrm{rank}[sI - A_1 \ \ A_2 \ \ B_{11}] = n_1$ for any $s \in C$ ($s \in C_+$).

ii) $\mathrm{rank}[sI - A_1^{\mathrm{T}} \ \ A_3^{\mathrm{T}} \ \ C_{11}^{\mathrm{T}}] = n_1$, for any $s \in C$ ($s \in C_+$).

According to these lemmas above, a new structure of the compensator can be constructed in following sections.

3 PROBLEM FORMULATION

The compensation of state feedback and dynamic output feedback for system (1) is given, which is named as *united compensator*, with the following form:

$$E_c \dot{x}_c = Sx_c + Ry,$$
$$u = Qx_c + Ky + Hx \tag{5}$$

where $x_c \in R^{n_c}$ is state vector of the compensator, $E_c, S \in R^{m_c \times n_c}$, $R \in R^{m_c \times p}$, $Q \in R^{q \times n_c}$, $K \in R^{q \times p}$ and $H \in R^{q \times n}$ are constant matrices. Let $\mathrm{rank}(E_c) = r_c$, $0 \leqslant r_c \leqslant \min\{m_c, n_c\}$. Feedback (5) also implies that the states of original system (1) can be obtained directly. Without loss of generality, one can assume that $K = 0$. Then the resultant closed-loop system obtained from system (1) and its compensator (5) is:

$$\begin{pmatrix} E & 0 \\ 0 & E_c \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{x}_c \end{pmatrix} = \begin{pmatrix} A+BH & BQ \\ RC & S \end{pmatrix} \begin{pmatrix} x \\ x_c \end{pmatrix} \tag{6}$$

It is desired that the closed-loop system (6) is square, regular and impulse-free, hence the dimension $m_c \times n_c$ of compensator (5) should satisfy.

$$n + n_c = m + m_c \tag{7}$$

The aim of this paper is to derive a condition that the closed-loop system (6) is regular, impulse-free and stable, and further explore a condition under which the dynamic order of compensator (5) has no limitations. In order to solve these problems, the following definition is adopted from [8].

Definition 1 The square system (1) is said to be *admissible*, if it is regular, impulse-free and stable.

4 MAIN RESULTS

In this section, we first give a unified representation for the finite rank and infinite rank of matrix pencil $sE - A$. According to [9] on the notations of infinite rank of matrix pencil, we give the following definition and lemmas.

Definition 2 $\mathrm{rank}[sE-A] := \mathrm{rank}_\infty[sE-A]$, if $s = \infty$.

Lemma 4 $\mathrm{rank}_\infty[sE-A] = \mathrm{rank}\begin{bmatrix} 0 & E \\ E & A \end{bmatrix} - \mathrm{rank}[E]$

Corollary 1 A square system (E, A) is regular and impulse-free if and only if $\mathrm{rank}_\infty[sE - A] = n$.

Corollary 2 A square system (E, A) is admissible if and only if

$$\mathrm{rank}[sE - A] = n, \text{ for any } s \in C_{+e}.$$

Corollary 3 There exists K such that square system $(E, A + BK)$ is admissible if and only if

$$\mathrm{rank}[sE - A \quad B] = n, \text{ for any } s \in C_{+e}.$$

Lemma 5 For almost all $K \in R^{q \times p}$, the following equality holds:

$$\mathrm{rank}_\infty[sE - A - BKC] = \min\left\{\mathrm{rank}_\infty[sE - A \quad B], \mathrm{rank}_\infty\begin{bmatrix} sE - A \\ C \end{bmatrix}\right\} \tag{8}$$

Lemma 6 There exists a dynamic compensator (5) such that the closed-loop system (6) is regular and impulse-free, if and only if

i) $$\mathrm{rank}_\infty[sE - A \quad B] = m \tag{9}$$

ii) $$\mathrm{rank}_\infty\begin{bmatrix} sE - A & B \\ C & 0 \end{bmatrix} \geqslant n \tag{10}$$

Proof From Corollary 1, system (6) is regular and impulse free if and only if for almost all real matrices H, Q, R, S with compatible dimensions, such that

$$\mathrm{rank}_\infty\begin{bmatrix} sE - (A + BH) & -BQ \\ -RC & sE_c - S \end{bmatrix} = m + m_c \tag{11}$$

From Lemma 5, it is easily proved that (11) is equivalent to (9) and (10).

Lemma 7 If $m < n$ and the closed-loop system (6) is regular and impulse-free, then there exists a feedback gain K_0 and nonsingular matrices M_1, M_2, N_1, N_2 with compatible dimensions such that

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{bmatrix} sE-A-BK_0 & B \\ C & 0 \end{bmatrix}\begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix}$$
$$= \left[\begin{array}{ccc|c} sI_{n_1}-A_1 & 0 & -A_2 & B_{11} \\ 0 & -I_{n_2} & 0 & B_{21} \\ \hline 0 & 0 & I_{n_3} & 0 \\ C_{11} & C_{12} & 0 & 0 \end{array}\right] \tag{12}$$

So if (9) and (10) hold, then, without loss of generality, we can assume that the original system has the decomposition form (12), or (4) has the following form:

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{bmatrix} sE-A & B \\ C & 0 \end{bmatrix}\begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix}$$
$$= \left[\begin{array}{ccc|c} sI_{n_1}-A_1 & 0 & -A_2 & B_{11} \\ 0 & -I_{n_2} & 0 & B_{21} \\ \hline 0 & 0 & I_{n_3} & 0 \\ C_{11} & C_{12} & 0 & 0 \end{array}\right] \tag{13}$$

With the above assumption, the following lemma is important to obtain the main results of this paper.

Lemma 8 Assume that $E, A \in R^{n\times n}$, $B \in R^{n\times q}$, $C \in R^{p\times n}$, $D \in R^{n\times q_1}$ are constant matrices, Then there exists a $K \in R^{q_1\times p}$, such that

$\operatorname{rank}\begin{bmatrix} sE-A-DKC & B \end{bmatrix} = n$, for any $s\in C_{+e}$ (14)

if the following conditions hold:

i) $\operatorname{rank}\begin{bmatrix} sE-A & D & B \end{bmatrix} = n$, for any $s\in C_{+e}$, (15)

ii) $\operatorname{rank}\begin{bmatrix} sE-A & B \\ C & 0 \end{bmatrix} \geq n$, for any $s\in C_{+e}$, (16)

iii) $\operatorname{rank}\begin{bmatrix} sE-A & B \\ C & 0 \end{bmatrix} > n$, for some $s\in C_{+e}$. (17)

Proof Without loss of generality, we can assume that (E, A) is regular, then by Theorem 3.1 in [10], we only need to prove that $C(sE-A)^{-1}B \neq 0$, which can be obtained directly from (17).

Theorem 1 1) If $m < n$, there exists a dynamic compensator (5) with dimension $m_c \times n_c$ ($m_c > n_c > 0$) subject to (7) such that closed-loop system (6) is admissible, if and only if

i) $\operatorname{rank}\begin{bmatrix} sE-A & B \end{bmatrix} = m$, for any $s\in C_{+e}$, (18)

ii) $\operatorname{rank}\begin{bmatrix} sE-A & B \\ C & 0 \end{bmatrix} \geq n$, for any $s\in C_{+e}$. (19)

Furthermore, if the inequality in (19) holds for some $s\in C_{+e}$, then the dynamic order of compensator (5) is free.

2) If $m \geqslant n$, there exists a dynamic compensator (5) with dimension $m_c \times n_c$ ($n_c \geqslant m_c > 0$) subject to (7) such that the closed-loop system (6) is admissible, if and only if (18) holds. Furthermore, the dynamic order of compensator (5) is free.

Proof From Corollary 2, the closed-loop system (6) is admissible if and only if there exist real matrices E_c, H, Q, R, S such that

$$\operatorname{rank}\begin{bmatrix} sE-(A+BH) & -BQ \\ -RC & sE_c - S \end{bmatrix} = n + n_c\ (m+m_c),$$

for any $s\in C_{+e}$. (20)

1) *Necessity*. Using Corollary 3 and its duality, we can derive that (18) and (19) hold from (20). So the necessity is proved.

Sufficiency. From (18) and (19), we have

i) $\operatorname{rank}\left\{\begin{bmatrix} sE-A & 0 \\ 0 & sE_c-S \end{bmatrix} \quad \begin{bmatrix} 0 \\ I_{m_c} \end{bmatrix} \quad \begin{bmatrix} B \\ 0 \end{bmatrix}\right\} = m+m_c$,

for any $s\in C_{+e}$, (21)

ii) $\operatorname{rank}\left[\begin{array}{cc|c} sE-A & 0 & B \\ 0 & sE_c-S & 0 \\ \hline C & 0 & 0 \end{array}\right] \geqslant n+n_c$, for any $s\in C_{+e}$, (E_c and S exist). (22)

Condition (22) can be considered in two cases:

(a) $\operatorname{rank}\left[\begin{array}{cc|c} sE-A & 0 & B \\ 0 & sE_c-S & 0 \\ \hline C & 0 & 0 \end{array}\right] > n+n_c$, for some $s\in C_{+e}$. (23)

(b) $\operatorname{rank}\left[\begin{array}{cc|c} sE-A & 0 & B \\ 0 & sE_c-S & 0 \\ \hline C & 0 & 0 \end{array}\right] = n+n_c$, for any $s\in C_{+e}$. (24)

If (23) holds, then from (21)~(23) and Lemma 8, there exist real matrices E_c, R, S, such that

$$\operatorname{rank}\left\{\begin{bmatrix} sE-A & 0 \\ 0 & sE_c-S \end{bmatrix} - \begin{bmatrix} 0 \\ I \end{bmatrix} R\begin{bmatrix} C & 0 \end{bmatrix} \quad \begin{bmatrix} B \\ 0 \end{bmatrix}\right\} = n+n_c,$$

for any $s\in C_{+e}$. (25)

This shows that there exist real matrices E_c, R, S, H, Q, such that (20) holds.

On the other hand, if (24) holds, i.e.,

$$\operatorname{rank}\begin{bmatrix} sE-A & B \\ C & 0 \end{bmatrix} = n, \text{ for any } s\in C_{+e}, \tag{26}$$

then based on decomposition (13), one has

$$[C_{11} \quad C_{12}]\begin{bmatrix} sI_{n_1}-A_1 & 0 \\ 0 & -I_{n_2} \end{bmatrix}^{-1}\begin{bmatrix} B_{11} \\ B_{21} \end{bmatrix}=0 \tag{27}$$

By Kalman canonical form of (27), there exist matrices $\bar{M}_1,\bar{M}_2,\bar{N}_1,\bar{N}_2$, giving a more detailed decomposition to (13) as follows:

$$\begin{bmatrix} \bar{M}_1 & 0 \\ 0 & \bar{M}_2 \end{bmatrix}\begin{bmatrix} sE-A & B \\ C & 0 \end{bmatrix}\begin{bmatrix} \bar{N}_1 & 0 \\ 0 & \bar{N}_2 \end{bmatrix}$$

$$=\left[\begin{array}{ccccc|c} sI_{n_{11}}-A_{11} & A_{12} & 0 & 0 & A_{14} & B_{11} \\ 0 & sI_{n_{12}}-A_{22} & 0 & 0 & A_{24} & 0 \\ 0 & 0 & -I_{n_{21}} & 0 & 0 & B_{23} \\ 0 & 0 & 0 & -I_{n_{22}} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & I_{n_3} & 0 \\ 0 & C_{12} & 0 & C_{14} & 0 & 0 \end{array}\right] \tag{28}$$

where (A_{11},B_{11}) and (A_{22},A_{24}) are both stabilizable and (A_{22},C_{12}) is detectable, with the following constraints.

$n_1=n_{11}+n_{12},\ n_2=n_{21}+n_{22},\ m=n_1+n_2,\ n=n_1+n_2+n_3$.

Then

$$\det\begin{bmatrix} sE-(A+BH) & -BQ \\ -RC & sE_c-S \end{bmatrix}$$

$$=\alpha\cdot\det\begin{bmatrix} sI_{n_{11}}-A_{11}-B_{11}H_1 & * & 0 & 0 & A_{14} & B_{11}Q \\ 0 & sI_{n_{12}}-A_{22} & 0 & 0 & A_{24} & 0 \\ * & * & -I_{n_{21}} & * & * & * \\ 0 & 0 & 0 & -I_{n_{22}} & 0 & 0 \\ 0 & R_1C_{12} & 0 & * & R_2 & sE_c-S \end{bmatrix}$$

$$=\alpha\cdot\det\left(sI_{n_{11}}-A_{11}-B_{11}H_1\right)\cdot\det\begin{bmatrix} sI_{n_{12}}-A_{22} & A_{24} & 0 \\ R_1C_{12} & R_2 & sE_c-S \end{bmatrix}$$

where $\alpha\neq 0$ is a real number, and

$$\begin{bmatrix} sI_{n_{12}}-A_{22} & A_{24} & 0 \\ R_1C_{12} & R_2 & sE_c-S \end{bmatrix}$$

$$=\begin{bmatrix} sI_{n_{12}}-A_{22} & A_{24} & 0 \\ 0 & 0 & sE_c \end{bmatrix}+\begin{bmatrix} 0 \\ I_{m_c} \end{bmatrix}[R_1 \quad R_2 \quad S]\begin{bmatrix} C_{12} & 0 & 0 \\ 0 & I_{n_3} & 0 \\ 0 & 0 & I_{n_c} \end{bmatrix}$$

$$=:\left[s\bar{E}-\bar{A}+\bar{B}\bar{K}\bar{C}\right]$$

Then system $(\bar{E},\bar{A},\bar{B},\bar{C})$ is S-stabilizable and S-detectable because (A_{22},A_{24}) is stabilizable and (A_{22},C_{12}) is detectable. From [7] or [11], there exists $\bar{K}$ such that $(\bar{E},\bar{A}+\bar{B}\bar{K}\bar{C})$ is admissible if $\operatorname{rank}(E_c)=r_c$ satisfies the following equality:

$$\operatorname{rank}\begin{bmatrix} \bar{E} & 0 \\ \bar{A} & \bar{B} \end{bmatrix}+\operatorname{rank}\begin{bmatrix} \bar{E} & \bar{A} \\ 0 & \bar{C} \end{bmatrix}\geqslant 2(n+n_c)+n_{12}+r_c+1 \tag{29}$$

Simplifying (29), one has

$$\operatorname{rank}(A_{24})+\operatorname{rank}(C_{12})+r_c\geqslant n_{12}+1 \tag{30}$$

So system (6) can be stabilized if we select the rank of E_c sufficient large and appropriate matrices H_1,R_1,R_2,S. Thus, 1) has been proved.

2) If $m\geqslant n$, then without loss of generality, one can assume that the original system has the following decomposition form:

$$\begin{bmatrix} M_1 & 0 \\ 0 & M_2 \end{bmatrix}\begin{bmatrix} sE-A & B \\ C & 0 \end{bmatrix}\begin{bmatrix} N_1 & 0 \\ 0 & N_2 \end{bmatrix}$$

$$=\left[\begin{array}{cc|cc} sI_{n_1}-A_1 & 0 & 0 & B_{11} \\ 0 & -I_{n_2} & 0 & B_{21} \\ -A_3 & 0 & I_{m_3} & 0 \\ \hline C_{11} & C_{12} & 0 & 0 \end{array}\right] \tag{31}$$

It is easy to select E_c,R,S,H,Q such that

$$\det\begin{bmatrix} sE-(A+BH) & -BQ \\ -RC & sE_c-S \end{bmatrix}$$
$$=\alpha\cdot\det(sI_{n_1}-A_1-B_{11}H_{11})\cdot\det(sE_{c1}-S_1). \tag{32}$$

The proceeding proof is omitted.

Remark 1 In decomposition (28), if $A_{24}=0$ or $C_{12}=0$, then A_{22} is Hurwitz matrix and it can be proved that r_c is free. If $A_{24}\neq 0$ and $C_{12}\neq 0$, then r_c can be designed smaller than n_{12} (if $n_{12}>0$), actually it can be determined by the subsystem (A_{22},A_{24},C_{12}), the dynamic order can be designed to satisfy

$$r_c\leqslant n_{12}-\operatorname{rank}(C_{12}) \tag{33}$$

From Theorem 1 and its proof, we can give a simple corollary as follows to determine whether the dynamic order of the compensator is free or not.

Corollary 4 If the rectangular system (1) has the decomposition (4), satisfies (18) and (19), and (A_1,B_{11}) is stabilizable, then the dynamic order of the stabilizing compensator (5) is free.

Remark 2 If we only consider the dynamic compensator, i.e., in (5), $H=0$ and $K\neq 0$, then the resultant closed-loop system is

$$\begin{bmatrix} E & 0 \\ 0 & E_c \end{bmatrix}\begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix}=\begin{bmatrix} A+BKC & BQ \\ RC & S \end{bmatrix}\begin{bmatrix} x \\ x_c \end{bmatrix} \tag{34}$$

Then from Theorem 5 of [7], the closed-loop system (34) is admissible if and only if

i) $\operatorname{rank}[sE-A \quad B]=m$, for any $s\in C_{+e}$, (35)

ii) $\operatorname{rank}[sE^T-A^T \quad C^T]=n$, for any $s\in C_{+e}$. (36)

Notice that the condition (36) is apparently stronger than (19), and more, the dynamic order of compensator (5) will not be free for using this type compensator.

In the following we consider the construction of a full order compensator, which is used in many cases. If the rectangular system (1) is S-controllable and S-observable, then on the basis of the decomposition (4), let $E_c=E^{\mathrm{T}}$

and $K = 0, H = 0$, the compensator and the closed-loop system will be in the following form.

$$E^T \dot{x}_c = Sx_c + Ry \text{ , } u = Qx_c \tag{37}$$

$$\begin{bmatrix} E & 0 \\ 0 & E^T \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{x}_c \end{bmatrix} = \begin{bmatrix} A & BQ \\ RC & S \end{bmatrix} \begin{bmatrix} x \\ x_c \end{bmatrix} \tag{38}$$

If let

$$Q = N_2 \begin{bmatrix} 0 & 0 & -I_{m_3} \\ Q_{11} & 0 & 0 \end{bmatrix} M_1^{-\mathrm{T}}, \ R = N_1^{-\mathrm{T}} \begin{bmatrix} 0 & R_{11} \\ 0 & 0 \\ -I_{n_3} & 0 \end{bmatrix} M_2,$$

$$S = N_1^{-\mathrm{T}} \begin{bmatrix} S_{11} & 0 & S_{13} \\ 0 & I_{n_2} & 0 \\ S_{31} & 0 & 0 \end{bmatrix} M_1^{-\mathrm{T}} \tag{39}$$

Then we have

$$\det \begin{bmatrix} sE - A & -BQ \\ -RC & sE^{\mathrm{T}} - S \end{bmatrix}$$

$$= \beta \cdot \det \begin{bmatrix} sI_{n_1} - A_1 & -\tilde{B}_1 Q_1 \\ -R_1 \tilde{C}_1 & sI_{n_1} - S_1 \end{bmatrix} \tag{40}$$

where $\beta \neq 0$ is real number, and

$$\tilde{B}_1 = [A_2 \ \ B_{11}], \ \tilde{C}_1 = \begin{bmatrix} A_3 \\ C_{11} \end{bmatrix}, \ Q_1 = \begin{bmatrix} S_{31} \\ Q_{11} \end{bmatrix},$$

$$R_1 = [S_{13} \ \ R_{11}], \ S_1 = S_{11} - R_{11} C_{12} B_{21} Q_{11} \tag{41}$$

The rest is to construct S_1, Q_1, R_1 in (40). The problem has turned to construct the compensator of normal systems. So we can design the poles in the way of the normal systems [11]. A simple algorithm can be described below:

Step 1: Find nonsingular matrices M_1, M_2, N_1, N_2 satisfying (4).

Step 2: Find Q_1 and R_1 such that the eigenvalues of $A_1 + \tilde{B}_1 Q_1$ and $A_1 + R_1 \tilde{C}_1$ are in desired positions. Thus S_{13}, S_{31}, Q_{11} and R_{11} can be obtained.

Step 3: Let $S_1 = A_1 - \tilde{B}_1 Q_1 + R_1 \tilde{C}_1$ in (40), then

$$S_{11} = A_1 + R_{11} C_{12} B_{21} Q_{11} - \tilde{B}_1 Q_1 + R_1 \tilde{C}_1 .$$

Step 4: We have obtained S, Q, R from (39), then the compensator with the form (38) can be designed.

5 CONCLUSION

In this paper we have considered the problem of stabilization of rectangular systems by using dynamic compensator and state feedback. A necessary and sufficient condition is given such that the closed-loop system is regular, impulse-free and stable. Also it is proved that the proposed conditions are weaker than the existing ones and the dynamic order of the compensators is lower than the existing ones, or free under a rank condition. Although we have used the decomposition forms in the proofs, the main results are still presented with original system parameter matrices except for the condition of the dynamic order of the compensators to be free.

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