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EIGENSTRUCTURE ASSIGNMENT FOR LINEAR DESCRIPTOR SYSTEMS VIA OUTPUT FEEDBACK

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ABSTRACT

Parametric approaches for eigenstructure assignment in linear descriptor systems via output feedback are proposed. General parametric expressions for both the left and right closed-loop finite and infinite eigenvectors and the output feedback gain matrix are established in terms of certain parameter vectors. The approaches do not impose any restriction on the finite closed-loop eigenvalues, assign the whole sets of the left and right closed-loop finite and infinite eigenvectors, and need less computational work.

Key Words: Descriptor systems, eigenstructure assignment, output feedback.

I. INTRODUCTION

Descriptor systems are encountered in many fields such as network theory, robotics, economics, large scale systems, *etc.*, and have attracted much attention in the past three decades (see, for example, [1,2] and references therein).

For a linear descriptor system, the stability and the speed of the response are mainly governed by the eigenvalues of the system, while the transient response and sensitivities of the eigenvalues are basically determined by the eigenvectors of the system. As one of the important problems in descriptor system theory, the problem of eigenstructure assignment (simultaneous assignment of eigenvalues and eigenvectors) in linear descriptor systems has been studied by many researchers (see, for example, [3–6] and references therein).

In particular, eigenstructure assignment for linear descriptor systems via output feedback has been studied by several researchers [5–10]. However, these reported results are subject to the following limitations.

(i) The approaches in [5,8–10] only deal with the simple case of distinct finite closed-loop eigenvalues, and assign part, not all, of the left and right closed-loop finite eigenvectors.
(ii) The approaches in [5–7,9,10] only assign part, not all, of the left and right closed-loop infinite eigenvectors. It will be seen that the assignment of the whole sets of the left and right closed-loop infinite eigenvectors may have applications in sensitivity analysis of regularity and impulse elimination of the closed-loop system.
(iii) In [7], conditions for eigenstructure assignment contain n^2 algebraic equations (*i.e.*, the equations in Constraints C2–C4 in [7]). It will be seen that the number of algebraic equations can be reduced.
(iv) The approaches in [5,6] adopt the inner inverses of matrices in the solutions of the feedback gain matrix. This not only leads to a complicated expression of the feedback gain matrix but also results in a difficult condition for the closed-loop regularity (see [7]).

This paper also deals with the problem of eigenstructure assignment in linear descriptor systems via output feedback. Parametric approaches for the problem are proposed. General parametric expressions for both the left and right closed-loop finite and infinite eigenvectors and the output feedback gain matrix are given. The approaches overcome the drawbacks of the previously published results [5–10].

II. FORMULATION OF THE PROBLEM

Let

$$E\dot{x} = Ax + Bu, \quad y = Cx \tag{1}$$

be a linear descriptor system, where $x \in \mathbf{R}^n, u \in \mathbf{R}^m, y \in \mathbf{R}^p$ are, respectively, the state vector, the input vector and the output vector; $E, A \in \mathbf{R}^{n\times n},\ B \in \mathbf{R}^{n\times m}$ and $C \in \mathbf{R}^{p\times n}$ are known matrices with $\operatorname{rank} E = n_0 \le n, \operatorname{rank} B = m$

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and rank $C = p$. When an output feedback control law

$$u = Ky,\ K \in \mathbf{R}^{m\times p} \tag{2}$$

is applied to the system (1), a closed-loop system is obtained as

$$E\dot{x} = (A + BKC)x \tag{3}$$

It is assumed that the system (1) is strongly controllable (S-controllable), *i.e.*, both R-controllable and impulse controllable, and strongly observable (S-observable), *i.e.*, both R-observable and impulse observable. The system (1) is both R-controllable and R-observable if and only if

$$\begin{aligned} &\operatorname{rank}\left[\, A - sE \;\; B \,\right] \\ &= \operatorname{rank}\left[\, A^T - sE^T \;\; C^T \,\right] = n,\ \forall s \in \mathbf{C} \end{aligned} \tag{4}$$

The system (1) is both impulse controllable and impulse observable if and only if

$$\begin{aligned} &\operatorname{rank}\left[\, E \;\; AN_\infty \;\; B \,\right] \\ &= \operatorname{rank}\left[\, E^T \;\; A^T H_\infty \;\; C^T \,\right] = n \end{aligned} \tag{5}$$

where N_∞ and H_∞ are $n \times (n - n_0)$ matrices defined by

$$EN_\infty = 0,\ H_\infty^T E = 0,\ \operatorname{rank} N_\infty = \operatorname{rank} H_\infty = n - n_0 \tag{6}$$

See [1,11] for details.

Let $\Lambda = \{\lambda_1, \lambda_2, \cdots, \lambda_{n_0'}\}$ $(1 \le n_0' \le n_0)$, which is symmetric about the real axis, be the set of distinct finite eigenvalues of the matrix pair $(E, A + BKC)$, and denote the algebraic and geometric multiplicities of λ_i by q_i and r_i respectively. Then in the Jordan form J determined by the finite eigenvalues of the matrix pair $(E, A + BKC)$, there are r_i Jordan blocks associated with λ_i. Denote the orders of the r_i Jordan blocks associated with λ_i by $p_{ij}, j = 1, 2, \cdots, r_i$. Then the following relation holds:

$$p_{i1} + p_{i2} + \cdots + p_{ir_i} = q_i \tag{7}$$

It is aimed to assign n_0 (= rank E) finite closed-loop eigenvalues and thus

$$q_1 + q_2 + \cdots + q_{n_0'} = n_0 \tag{8}$$

is required. It follows from the above description that the Jordan form J, determined by the finite closed-loop eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_{n_0'}$, is in the following form

$$J = \operatorname{diag}(J_1, J_2, \cdots, J_{n_0'})$$

$$J_i = \operatorname{diag}(J_{i1}, J_{i2}, \cdots, J_{ir_i})$$

$$J_{ij} = \begin{bmatrix} \lambda_i & 1 & & & \\ & \lambda_i & 1 & & \\ & & \ddots & \ddots & \\ & & & \ddots & 1 \\ & & & & \lambda_i \end{bmatrix}_{p_{ij}\times p_{ij}}$$

Further, let the right and left eigenvector chains associated with λ_i be denoted, respectively, by $v_{ij}^k \in \mathbf{C}^n$ and $t_{ij}^k \in \mathbf{C}^n, k = 1, 2, \cdots, p_{ij}, j = 1, 2, \cdots, r_i$. Then we have the following equations by definition

$$(A + BKC - \lambda_i E)v_{ij}^k = Ev_{ij}^{k-1},\ v_{ij}^0 = 0 \tag{9}$$

$$(t_{ij}^k)^T(A + BKC - \lambda_i E) = (t_{ij}^{k+1})^T E,\ t_{ij}^{p_{ij}+1} = 0 \tag{10}$$

$$k = 1, 2, \cdots, p_{ij}, j = 1, 2, \cdots, r_i, i = 1, 2, \cdots, n_0'$$

For convenience, we make the following convention throughout the rest of this paper.

Convention 1. For any group of vectors $x_{ij}^k, k = 1, 2, \cdots, p_{ij}, j = 1, 2, \cdots, r_i, i = 1, 2, \cdots, n_0'$, a matrix X_f is defined by

$$X_f = \left[\, X_1 \;\; X_2 \;\; \cdots \;\; X_{n_0'} \,\right]$$

$$X_i = \left[\, X_{i1} \;\; X_{i2} \;\; \cdots \;\; X_{ir_i} \,\right]$$

$$X_{ij} = \left[\, x_{ij}^1 \;\; x_{ij}^2 \;\; \cdots \;\; x_{ij}^{p_{ij}} \,\right]$$

With Convention 1, the equations in (9) and (10) can be equivalently written in the following unified matrix forms:

$$(A + BKC)V_f = EV_f J \tag{11}$$

$$T_f^T(A + BKC) = JT_f^T E \tag{12}$$

The matrices V_f and T_f satisfying (11) and (12) are called respectively the right and left finite closed-loop eigenvector matrices. They are said to be a normalized pair if they satisfy [12,13]

$$T_f^T E V_f = I_{n_0} \tag{13}$$

Let the infinite eigenvalue of the matrix pair $(E, A + BKC)$ be denoted by $\lambda_\infty = \infty$. Then $s_\infty = 1/\lambda_\infty = 0$ is the eigenvalue of the matrix pair $(A + BKC, E)$. Because of (8), s_∞ is a multiple eigenvalue with both algebraic and geometric multiplicities being equal to $n - n_0$. Thus the Jordan form N determined by the zero eigenvalue $s_\infty = 0$ of the matrix pair $(A + BKC, E)$ is $N = 0_{(n-n_0)\times(n-n_0)}$.

Further, let the right and left eigenvectors associated with s_∞ be denoted, respectively, by $v_{\infty j} \in \mathbf{R}^n$ and $t_{\infty j} \in \mathbf{R}^n, j = 1, 2, \cdots, n - n_0$. Then we have the following equations by definition

$$(E - s_\infty(A + BKC))v_{\infty j} = 0 \tag{14}$$

$$t_{\infty j}^T(E - s_\infty(A + BKC)) = 0 \tag{15}$$

$$j = 1, 2, \cdots, n - n_0$$

For convenience, we also make the following convention throughout the rest of this paper.

Convention 2. For any group of vectors $x_{\infty j}, j = 1, 2, \cdots, n - n_0$, a matrix X_∞ is defined by

$$X_\infty = \begin{bmatrix} x_{\infty 1} & x_{\infty 2} & \cdots & x_{\infty, n-n_0} \end{bmatrix}$$

With Convention 2, the equations in (14) and (15) can be equivalently written in the following unified matrix forms:

$$EV_\infty = (A + BKC)V_\infty N = 0_{n\times(n-n_0)} \tag{16}$$

$$T_\infty^T E = NT_\infty^T(A + BKC) = 0_{(n-n_0)\times n} \tag{17}$$

We call the matrices V_∞ and T_∞ satisfying (16) and (17) respectively the right and left infinite closed-loop eigenvector matrices. They are said to be a normalized pair if they satisfy

$$T_\infty^T(A + BKC)V_\infty = I_{n-n_0} \tag{18}$$

Let

$$V = \begin{bmatrix} V_f & V_\infty \end{bmatrix}, \ T = \begin{bmatrix} T_f & T_\infty \end{bmatrix} \tag{19}$$

Then the matrices V and T are respectively the right and left entire closed-loop eigenvector matrices. They are said to be a normalized pair if both (13) and (18) hold.

If the matrices V and T are a normalized pair, *i.e.*, both (13) and (18) hold, then, from (11), (12), (16), (17) and (19), we can obtain

$$T^T EV = \begin{bmatrix} I_{n_0} & 0 \\ 0 & 0 \end{bmatrix}, \ T^T(A + BKC)V = \begin{bmatrix} J & 0 \\ 0 & I_{n-n_0} \end{bmatrix} \tag{20}$$

With the above descriptions, the following eigenstructure assignment problem in the linear descriptor system (1) via the output feedback control law (2) can be proposed.

Problem EA. Given the S-controllable S-observable linear descriptor system (1), the distinct finite closed-loop eigenvalues set Λ as described previously, and integers $q_i, r_i, p_{ij}, j = 1, 2, \cdots, r_i, i = 1, 2, \cdots, n_0'$ satisfying (7) and (8), find a matrix $K \in \mathbf{R}^{m\times p}$ and two groups of vectors $t_{ij}^k \in \mathbf{C}^n, k = 1, 2, \cdots, p_{ij}, j = 1, 2, \cdots, r_i, i = 1, 2, \cdots, n_0', t_{\infty j} \in \mathbf{R}^n, j = 1, 2, \cdots, n - n_0$ and $v_{ij}^k \in \mathbf{C}^n, k = 1, 2, \cdots, p_{ij}, j = 1, 2, \cdots, r_i, i = 1, 2, \cdots, n_0', v_{\infty j} \in \mathbf{R}^n, j = 1, 2, \cdots, n - n_0$ (or, equivalently, a matrix triple (K, V, T)), such that the following four requirements are simultaneously satisfied:

(a) equations (11), (12), (16) and (17) hold;
(b) $v_{ij}^k, t_{ij}^k \in \mathbf{R}^n$ for $\lambda_i \in \mathbf{R}$ and $v_{ij}^k = \bar{v}_{lj}^k, t_{ij}^k = \bar{t}_{lj}^k \in \mathbf{C}^n$ for $\lambda_i = \bar{\lambda}_l$;
(c) equation (13) holds;
(d) equation (18) holds.

The requirement (b) in Problem EA is essential since it is required that K is real. The requirements (c) and (d) in Problem EA are the conditions for the matrices V and T to form a normalized pair. It is easily seen from (20) that these conditions guarantee that $\det V \neq 0$, $\det T \neq 0$ and the closed-loop system is regular and impulse-free.

The above Problem EA differs from that treated in [7] mainly in the aspect of infinite eigenstructure assignment: The whole sets of the left and right closed-loop infinite eigenvector matrices are assigned to form normalized pairs in the above Problem EA, while in that of [7] only a single pair of the left and right closed-loop infinite eigenvector matrices are assigned.

Proposition 1. Let (K, V, T) be a solution of Problem EA and Δ be a perturbation of the matrix $A + BKC$. Then the perturbed closed-loop matrix pair $(E, A + BKC + \Delta)$ remains regular and impulse-free for all disturbances Δ satisfying

$$\|\Delta\|_2 < \frac{1}{\|T_\infty\|_2 \|V_\infty\|_2} \tag{21}$$

where $\| \ \|_2$ represents the spectral norm.

Proof. Using (20), we have

$$\begin{aligned} &T^T(sE - A - BKC - \Delta)V \\ &= \begin{bmatrix} sI_{n_0} - J - T_f^T\Delta V_f & -T_f^T\Delta V_\infty \\ -T_\infty^T\Delta V_f & -I_{n-n_0} - T_\infty^T\Delta V_\infty \end{bmatrix} \end{aligned}$$

It is easy to show that the matrix pair $(E, A + BKC + \Delta)$ remains regular and impulse-free if and only if

$$\det(I_{n-n_0} + T_\infty^T\Delta V_\infty) \neq 0 \tag{22}$$

It follows from matrix analysis theory that (22) holds if

$$\|T_\infty^T \Delta V_\infty\|_2 < 1 \tag{23}$$

Using inequality $\|T_\infty^T \Delta V_\infty\|_2 \leq \|T_\infty^T\|_2\|\Delta\|_2\|V_\infty\|_2$, we can show that the inequality (23) holds for all disturbances Δ satisfying the condition (21). The proof has been completed.

It can be seen from Proposition 1 that normalized pairs of right and left infinite eigenvector matrices of the closed-loop system play an important role in sensitivity analysis of regularity and impulse elimination of the closed-loop system. In fact, Proposition 1 gives an upper bound on the spectral norm of all disturbances Δ which make the disturbed closed-loop system remain regular and impulse-free. This bound can be maximized through selecting the left and right infinite eigenvector matrices.

Remark 1. It is known from [14] that, when $mp > n_0$, Problem EA is almost always solvable provided that a slight modification of the eigenvalues to be assigned are allowed.

III. SOLUTION TO PROBLEM EA

In this section, we consider the solution to Problem EA proposed in Section II.

Let

$$w_{ij}^k = KCv_{ij}^k,\ \left(z_{ij}^k\right)^T = \left(t_{ij}^k\right)^T BK \tag{24}$$

$$k = 1, 2, \cdots, p_{ij}, j = 1, 2, \cdots, r_i, i = 1, 2, \cdots, n_0'$$

With Convention 1, we have

$$W_f = KCV_f,\ Z_f^T = T_f^T BK \tag{25}$$

Then (11) and (12) can be changed equivalently into the following generalized Sylvester matrix equations:

$$AV_f + BW_f = EV_f J \tag{26}$$

$$T_f^T A + Z_f^T C = JT_f^T E \tag{27}$$

When $\mathrm{rank}[sE - A \quad B] = n$ for $\forall s \in \mathbf{C}$, we apply matrix elementary transformations to the matrices $[A - \lambda_i E\ B]$, obtaining

$$P(\lambda_i)\left[A - \lambda_i E\ \ B\right] Q(\lambda_i) = \left[I_n\ \ 0\right] \tag{28}$$

$$i = 1, 2, \cdots, n_0'$$

where $P(\lambda_i) \in \mathbf{C}^{n\times n}$ and $Q(\lambda_i) \in \mathbf{C}^{(n+m)\times(n+m)}$ are nonsingular matrices. Partition $Q(\lambda_i)$ as

$$Q(\lambda_i) = \begin{bmatrix} Q_{11}(\lambda_i) & Q_{12}(\lambda_i) \\ Q_{21}(\lambda_i) & Q_{22}(\lambda_i) \end{bmatrix},\ Q_{22}(\lambda_i) \in \mathbf{C}^{m\times m} \tag{29}$$

$$i = 1, 2, \cdots, n_0'$$

Let

$$N_k(\lambda_i) = (Q_{11}(\lambda_i)P(\lambda_i)E)^{k-1}Q_{12}(\lambda_i) \tag{30}$$

$$D_k(\lambda_i) = \begin{cases} Q_{22}(\lambda_i), & k = 1 \\ (Q_{21}(\lambda_i)P(\lambda_i)E)(Q_{11}(\lambda_i)P(\lambda_i)E)^{k-2}Q_{12}(\lambda_i), & k \neq 1 \end{cases} \tag{31}$$

$$k = 1, 2, \cdots, d_i, i = 1, 2, \cdots, n_0'$$

where $d_i = \max_{1\leq j\leq r_i}\{p_{ij}\}$.

Lemma 1. [15]. Let $\mathrm{rank}[sE - A\ \ B] = n, \forall s \in \mathbf{C}$. Then the general solution (V_f, W_f) to the generalized Sylvester matrix equation (26) is given by

$$\begin{aligned} \begin{bmatrix} v_{ij}^k \\ w_{ij}^k \end{bmatrix} = \begin{bmatrix} N_1(\lambda_i) \\ D_1(\lambda_i) \end{bmatrix} f_{ij}^k + \begin{bmatrix} N_2(\lambda_i) \\ D_2(\lambda_i) \end{bmatrix} f_{ij}^{k-1} + \\ \cdots + \begin{bmatrix} N_k(\lambda_i) \\ D_k(\lambda_i) \end{bmatrix} f_{ij}^1 \end{aligned} \tag{32}$$

$$k = 1, 2, \cdots, p_{ij}, j = 1, 2, \cdots, r_i, i = 1, 2, \cdots, n_0'$$

where $f_{ij}^k \in \mathbb{C}^m, k = 1, 2, \cdots, p_{ij}, j = 1, 2, \cdots, r_i, i = 1, 2, \cdots, n_0'$ are a group of arbitrary parameter vectors, and the matrices $N_k(\lambda_i)$ and $D_k(\lambda_i), k = 1, 2, \cdots, d_i, i = 1, 2, \cdots, n_0'$ are given by (28)– (31).

Taking the transpose over (27) yields

$$A^T T_f + C^T Z_f = E^T T_f J^T \tag{33}$$

which is in the form of (26). When $\mathrm{rank}[sE^T - A^T\ \ C^T] = n$ for $\forall s \in \mathbf{C}$, we apply matrix elementary transformations to the matrices $[A^T - \lambda_i E^T\ \ C^T]$, obtaining

$$F(\lambda_i)\left[A^T - \lambda_i E^T\ \ C^T\right] G(\lambda_i) = \left[I_n\ \ 0\right], i = 1, 2, \cdots, n_0' \tag{34}$$

where $F(\lambda_i) \in \mathbf{C}^{n\times n}$ and $G(\lambda_i) \in \mathbf{C}^{(n+p)\times(n+p)}$ are nonsingular matrices. Partition $G(\lambda_i)$ as

$$G(\lambda_i) = \begin{bmatrix} G_{11}(\lambda_i) & G_{12}(\lambda_i) \\ G_{21}(\lambda_i) & G_{22}(\lambda_i) \end{bmatrix},\ G_{22}(\lambda_i) \in \mathbf{C}^{p\times p} \quad i = 1, 2, \cdots, n_0' \tag{35}$$

Let

$$H_k(\lambda_i) = (G_{11}(\lambda_i)F(\lambda_i)E^T)^{k-1}G_{12}(\lambda_i) \tag{36}$$

$$L_k(\lambda_i) = \begin{cases} G_{22}(\lambda_i), & k = 1 \\ (G_{21}(\lambda_i)F(\lambda_i)E^T)(G_{11}(\lambda_i)F(\lambda_i)E^T)^{k-2}G_{12}(\lambda_i), \\ k \neq 1 \end{cases} \tag{37}$$

$$k = 1, 2, \cdots, d_i, i = 1, 2, \cdots, n_0'$$

Applying Lemma 1 to equation (33), we obtain the general solution (T_f, Z_f) to the generalized Sylvester matrix equation (27) as follows:

$$\begin{bmatrix} t_{ij}^{p_{ij}-k+1} \\ z_{ij}^{p_{ij}-k+1} \end{bmatrix} = \begin{bmatrix} H_1(\lambda_i) \\ L_1(\lambda_i) \end{bmatrix} g_{ij}^k + \begin{bmatrix} H_2(\lambda_i) \\ L_2(\lambda_i) \end{bmatrix} g_{ij}^{k-1} + \cdots + \begin{bmatrix} H_k(\lambda_i) \\ L_k(\lambda_i) \end{bmatrix} g_{ij}^1 \tag{38}$$

$$k = 1, 2, \cdots, p_{ij}, j = 1, 2, \cdots, r_i, i = 1, 2, \cdots, n_0'$$

where $g_{ij}^k \in \mathbf{C}^p, k = 1, 2, \cdots, p_{ij}, j = 1, 2, \cdots, r_i, i = 1, 2, \cdots, n_0'$ are a group of arbitrary parameter vectors, and the matrices $H_k(\lambda_i)$ and $L_k(\lambda_i), k = 1, 2, \cdots, d_i, i = 1, 2, \cdots, n_0'$ are given by (34)- (37).

Let

$$w_{\infty j} = KCv_{\infty j}, z_{\infty j}^T = t_{\infty j}^T BK, j = 1, 2, \cdots, n - n_0 \tag{39}$$

With Convention 2, we have

$$W_\infty = KCV_\infty,\ Z_\infty^T = T_\infty^T BK \tag{40}$$

Then equations (16) and (17) can be changed equivalently into the following matrix equations:

$$EV_\infty = AV_\infty N + BW_\infty N \tag{41}$$

$$T_\infty^T E = NT_\infty^T A + NZ_\infty^T C \tag{42}$$

Since $N = 0_{(n-n_0)\times(n-n_0)}$, it is easily shown that the general solution (V_∞, W_∞) to the matrix equation (41) is given by

$$v_{\infty j} = N_\infty f_{\infty j},\ w_{\infty j} \in \mathbf{R}^m \text{ arbitrary} \tag{43}$$

$$j = 1, 2, \cdots, n - n_0$$

and the general solution (T_∞, Z_∞) to the matrix equation (42) is given by

$$t_{\infty j} = H_\infty g_{\infty j},\ z_{\infty j} \in \mathbf{R}^p \text{ arbitrary} \tag{44}$$

$$j = 1, 2, \cdots, n - n_0$$

where N_∞ and H_∞ are determined by (6), and $f_{\infty j}, g_{\infty j} \in \mathbf{R}^{n-n_0}, j = 1, 2, \cdots, n - n_0$ are two groups of arbitrary parameter vectors.

Denote

$$W = \begin{bmatrix} W_f & W_\infty \end{bmatrix},\ Z = \begin{bmatrix} Z_f & Z_\infty \end{bmatrix} \tag{45}$$

From (25) and (40), we obtain

$$W = KCV,\ Z^T = T^T BK \tag{46}$$

Lemma 2. Let V_f, W_f, T_f and Z_f be matrices satisfying the generalized Sylvester matrix equations (26) and (27) and condition (13), and let V_∞, T_∞ and W_∞ be matrices satisfying the equations (41) and (42). Then the two equations in (46) have a common solution K for some Z_∞ if and only if the following equation holds:

$$T_f^T BW_\infty = Z_f^T CV_\infty \tag{47}$$

Proof. It follows from the matrix analysis theory that the following two equations

$$W = KCV,\ Z_f^T = T_f^T BK \tag{48}$$

have a common solution K if and only if each of the equations in (48) has solutions and the following relation holds:

$$T_f^T BW = Z_f^T CV \tag{49}$$

Clearly, equation (49) can be decomposed into equation (47) and the following equation:

$$T_f^T BW_f = Z_f^T CV_f \tag{50}$$

It is shown in [7] that under the condition (13) the equation (50) holds automatically and there always exists a matrix K satisfying the two equations in (25). With this matrix K and the matrices W_∞ and Z_∞ defined by the two equations in (40), both the equations in (46) hold. The proof has been completed.

From the fist equation in (40), we obtain

$$T_\infty^T(A+BKC)V_\infty = T_\infty^T AV_\infty + T_\infty^T BW_\infty$$

Then the requirement (d) in Problem EA can be turned equivalently into the following equation

$$T_\infty^T AV_\infty + T_\infty^T BW_\infty = I_{n_0} \tag{51}$$

Lemma 3. If the conditions (13), (47) and (51) are satisfied, then the two equations in (46) have a unique common solution K for some Z_∞, and the matrix K is given by

$$K = W(CV)^T[(CV)(CV)^T]^{-1} \tag{52}$$

Proof. It follows from Lemma 2 that the two equations in (46) have a common solution K for some Z_∞. Note that the term CV in the first equation in (46) is of full column rank due to the full rank assumption of the matrix C. Therefore, the matrix K satisfying both the first equation and the second equation in (46) is unique, which can be given by (52). The proof is done.

By using Lemmas 2 and 3, we can prove the following theorem for the general solution to Problem EA.

Theorem 1. Let the S-controllable S-observable linear descriptor system (1) be given. Then Problem EA has solutions if and only if there exist a group of parameter vectors $f_{ij}^k \in \mathbf{C}^m, g_{ij}^k \in \mathbf{C}^p, k = 1,2,\cdots,p_{ij}, j = 1,2,\cdots,r_i, i = 1,2,\cdots,n_0', f_{\infty j}, g_{\infty j} \in \mathbf{R}^{n-n_0}, w_{\infty j} \in \mathbf{R}^m, j = 1,2,\cdots,n-n_0$ satisfying the following constraints.

Constraint 1. $f_{ij}^k \in \mathbf{R}^m, g_{ij}^k \in \mathbf{R}^p$ for $\lambda_i \in \mathbf{R}$, and $f_{ij}^k = \bar{f}_{lj}^k \in \mathbf{C}^m, g_{ij}^k = \bar{g}_{lj}^k \in \mathbf{C}^p$ for $\lambda_i = \bar{\lambda}_l \in \mathbf{C}$.

Constraint 2. $T_f^T EV_f = I_{n_0}$.

Constraint 3. $T_f^T BW_\infty = Z_f^T CV_\infty$.

Constraint 4. $T_\infty^T AV_\infty + T_\infty^T BW_\infty = I_{n_0}$.

Furthermore, if this condition is satisfied, then the general solution (K, V, T) to Problem EA is given by (19), (32), (38), (43), (44) and (52) with Constraints 1–4 satisfied.

Proof. Note that, with the general expressions (32), (38), (43) and (44), the requirements (b) and (c) in Problem EA, the condition (51) and the requirement (d) in Problem EA can be turned equivalently in turn into Constraints 1-4 on the group of parameter vectors $f_{ij}^k, g_{ij}^k, k = 1,2,\cdots,p_{ij}, j = 1,2,\cdots,r_i, i = 1,2,\cdots, n_0', f_{\infty j}, g_{\infty j}, w_{\infty j}, j = 1,2,\cdots,n-n_0$. Then the theorem follows directly from the results of Lemmas 2 and 3.

For the case where $1 \le p < n_0$, we introduce the following additional constraint on the group of parameter vectors $f_{ij}^k, k = 1,2,\cdots,p_{ij}, j = 1,2,\cdots,r_i, i = 1,2,\cdots,n_0'$ as follows:

Constraint 5. $\mathrm{rank}(CV_f) = p$.

It is easy see that, when Constraint 5 is met, there exists a unique K satisfying the first equation in (25), which can be given by

$$K = W_f(CV_f)^T[(CV_f)(CV_f)^T]^{-1} \tag{53}$$

With this K, Constraint 3 can easily be shown to hold automatically and Constraint 4 can be turned equivalently into the following constraint on the group of parameter vectors $f_{ij}^k, k = 1,2,\cdots,p_{ij}, j = 1,2,\cdots,r_i, i = 1,2,\cdots,n_0', f_{\infty j}, g_{\infty j}, j = 1,2,\cdots,n-n_0$ as follows:

Constraint 6. $T_\infty^T(A+BW_f(CV_f)^T[(CV_f)(CV_f)^T]^{-1}C)V_\infty = I_{n_0}$.

Now applying Theorem 1 to this special case gives the following corollary.

Corollary 1. Let the S-controllable S-observable linear descriptor system (1) with $1 \le p < n_0$ be given. Then Problem EA has solutions if there exist a group of parameter vectors $f_{ij}^k \in \mathbf{C}^m, g_{ij}^k \in \mathbf{C}^p, k = 1,2,\cdots,p_{ij}, j = 1,2,\cdots,r_i, i = 1,2,\cdots,n_0', f_{\infty j}, g_{\infty j} \in \mathbf{R}^{n-n_0}, j = 1,2,\cdots,n-n_0$ satisfying Constraints 1, 2, 5 and 6. Furthermore, if this condition is satisfied, then the general solution (K, V, T) to Problem EA is given by (19), (32), (38), (43), (44) and (53) with Constraints 1, 2, 5 and 6 satisfied.

Remark 2. The general solution to Problem EA for the case where $1 \le p < n_0$ given by Corollary 1 can be simplified further. For this purpose, we partition the matrices V_f and W_f into

$$V_f = \left[\, \hat{V}_f \;\; \hat{V}_f^c \,\right], \quad W_f = \left[\, \hat{W}_f \;\; \hat{W}_f^c \,\right] \tag{54}$$

where $\hat{V}_f \in \mathbf{R}^{n\times p}$ and $\hat{W}_f \in \mathbf{R}^{m\times p}$. Then the first equation in (25) can be decomposed into the following two equations:

$$\hat{W}_f = KC\hat{V}_f, \quad \hat{W}_f^c = KC\hat{V}_f^c \tag{55}$$

We introduce, instead of Constraint 5, the following constraint on the group of parameter vectors $f_{ij}^k, k = 1,2,\cdots,p_{ij}, j = 1,2,\cdots,r_i, i = 1,2,\cdots,n_0'$ as follows.

Constraint 7. $\det(C\hat{V}_f) \neq 0$.

We can see that, when Constraint 7 is met, Constraint 5 holds automatically and there exists a unique K satisfying the first equation in (55), which can be given by

$$K = \hat{W}_f (C\hat{V}_f)^{-1} \tag{56}$$

With this K, Constraint 4 becomes

Constraint 8. $T_\infty^T (A + B\hat{W}_f (C\hat{V}_f)^{-1} C) V_\infty = I_{n_0}$.

It follows from Corollary 1 that the general solution (K, V, T) to Problem EA is given by (19), (32), (38), (43), (44), and (56) with Constraints 1, 2, 7 and 8 satisfied.

Turn now to the case where $n_0 \leq p \leq n$. Suppose that the parametric forms of the matrices V and W are already obtained through formulas (32) and (43) with the group of parameter vectors $f_{ij}^k, k = 1,2,\cdots,p_{ij}, j = 1,2,\cdots,r_i, i = 1,2,\cdots,n_0', f_{\infty j}, j = 1,2,\cdots,n-n_0$ satisfying the following constraints:

Constraint 9. $f_{ij}^k \in \mathbf{R}^m$ for $\lambda_i \in \mathbf{R}$, and $f_{ij}^k = \bar{f}_{lj}^k \in \mathbf{C}^m$ for $\lambda_i = \bar{\lambda}_l \in \mathbf{C}$.

Constraint 10. $\det V \neq 0$.

We partition the matrices V and W into

$$V = \begin{bmatrix} \hat{V} & \hat{V}^c \end{bmatrix}, \quad W = \begin{bmatrix} \hat{W} & \hat{W}^c \end{bmatrix} \tag{57}$$

where $\hat{V} \in \mathbf{R}^{n \times p}$ and $\hat{W} \in \mathbf{R}^{m \times p}$. Then the first equation in (46) can be decomposed into the following two equations:

$$\hat{W} = KC\hat{V}, \quad \hat{W}^c = KC\hat{V}^c \tag{58}$$

We introduce the following additional constraint on the group of parameter vectors $f_{ij}^k, k = 1,2,\cdots,p_{ij}, j = 1,2,\cdots,r_i, i = 1,2,\cdots,n_0', f_{\infty j}, j = 1,2,\cdots,n-n_0$ as follows:

Constraint 11. $\det(C\hat{V}) \neq 0$.

It is easy to show that, when Constraint 11 is met, there exists a unique K satisfying the first equation in (58), which can be given by

$$K = \hat{W}(C\hat{V})^{-1} \tag{59}$$

This K, together with V, satisfies the equations (11) and (16).

For the case where $n_0 \leq p \leq n$, we have the following corollary.

Corollary 2. Let the S-controllable S-observable linear descriptor system (1) with $n_0 \leq p \leq n$ be given. Then Problem EA has solutions if there exist a group of parameter vectors $f_{ij}^k \in \mathbf{C}^m, k = 1,2,\cdots,p_{ij}, j = 1,2,\cdots,r_i, i = 1,2,\cdots,n_0', f_{\infty j} \in \mathbf{R}^{n-n_0}, j = 1,2,\cdots,n-n_0, w_{\infty j} \in \mathbf{R}^m, j = 1,2,\cdots,p-n_0$ satisfying Constraints 9, 11 and the following constraint:

Constraint 12. $\det[EV_f \ (A + B\hat{W}(C\hat{V})^{-1}C)V_\infty] \neq 0$.

Furthermore, if this condition is satisfied, then the general solution (K, V, T) to Problem EA is given by (19), (32), (43), (59) and

$$T^T = \begin{bmatrix} EV_f & (A + B\hat{W}(C\hat{V})^{-1}C)V_\infty \end{bmatrix}^{-1} \tag{60}$$

with Constraints 9, 11 and 12 satisfied.

Proof. We have shown in the above that, when Constraints 9-11 are met, there always exists a matrix pair (K, V) satisfying equations (11) and (16), and the general solution for (K, V) is given by (19), (32), (43), and (59). Now suppose that the general solution (K, V) is already obtained. Then there always exists a matrix T satisfying equations (12) and (17) and Constraints1-4. From equations (12), (16), (17), (19), (40) and (59) and Constraints 2 and 4, we have

$$\begin{aligned}
&T^T \begin{bmatrix} EV_f & (A + B\hat{W}(C\hat{V})^{-1}C)V_\infty \end{bmatrix} \\
&= \begin{bmatrix} T_f^T \\ T_\infty^T \end{bmatrix} \begin{bmatrix} EV_f & (A + BKC)V_\infty \end{bmatrix} \\
&= \begin{bmatrix} T_f^T EV_f & T_f^T (A + BKC)V_\infty \\ T_\infty^T EV_f & T_\infty^T (A + BKC)V_\infty \end{bmatrix} \\
&= \begin{bmatrix} T_f^T EV_f & JT_f^T EV_\infty \\ T_\infty^T EV_f & T_\infty^T AV_\infty + T_\infty^T BW_\infty \end{bmatrix} \\
&= \begin{bmatrix} I_{n_0} & 0 \\ 0 & I_{n-n_0} \end{bmatrix}
\end{aligned}$$

It follows from the above equation and Constraint 12 that the general solution for T is given by formula (60). Note that, when Constraint 12 is met, Constraint 10 holds automatically. We have completed the proof.

Similarly, we can prove the following theorem and its corollaries, which are dual versions of Theorem 1 and Corollaries 1 and 2.

Theorem 2. Let the S-controllable S-observable linear descriptor system (1) be given. Then Problem EA has solutions if and only if there exist a group of parameter vectors $f_{ij}^k \in \mathbf{C}^m, g_{ij}^k \in \mathbf{C}^p, k = 1, 2, \cdots, p_{ij}, j = 1, 2, \cdots, r_i, i = 1, 2, \cdots, n_0', f_{\infty j}, g_{\infty j} \in \mathbf{R}^{n-n_0}, z_{\infty j} \in \mathbf{R}^p, j = 1, 2, \cdots, n - n_0$ satisfying Constraints 1 and 2 and the following constraints

Constraint 13. $T_\infty^T B W_f = Z_\infty^T C V_f$.

Constraint 14. $T_\infty^T A V_\infty + Z_\infty^T C V_\infty = I_{n-n_0}$.

Furthermore, if this condition is satisfied, then the general solution (K, V, T) to Problem EA is given by (19), (32), (38), (43), (44) and

$$K = [(T^T B)^T (T^T B)]^{-1} (T^T B)^T Z^T \tag{61}$$

with Constraints 1, 2, 13 and 14 satisfied.

Corollary 3. Let the S-controllable S-observable linear descriptor system (1) with $1 \leq m < n_0$ be given. Then Problem EA has solutions if there exist a group of parameter vectors $f_{ij}^k \in \mathbf{C}^m, g_{ij}^k \in \mathbf{C}^p, k = 1, 2, \cdots, p_{ij}, j = 1, 2, \cdots, r_i, i = 1, 2, \cdots, n_0', f_{\infty j}, g_{\infty j} \in \mathbf{R}^{n-n_0}, j = 1, 2, \cdots, n - n_0$ satisfying Constraints 1 and 2 and the following constraints

Constraint 15. $\operatorname{rank}(T_f^T B) = m$.

Constraint 16. $T_\infty^T (A + B[(T_f^T B)^T (T_f^T B)]^{-1} (T_f^T B)^T Z_f^T C) V_\infty = I_{n_0}$.

Furthermore, if this condition is satisfied, then the general solution (K, V, T) to Problem EA is given by (19), (32), (38), (43), (44) and

$$K = [(T_f^T B)^T (T_f^T B)]^{-1} (T_f^T B)^T Z_f^T \tag{62}$$

with Constraints 1, 2, 15 and 16 satisfied.

Remark 3. The general solution to Problem EA for the case where $1 \leq m < n_0$ given by Corollary 3 can also be simplified further. We partition the matrices T_f and Z_f into

$$T_f = \left[\hat{T}_f \ \hat{T}_f^c \right], \ \ Z_f = \left[\hat{Z}_f \ \hat{Z}_f^c \right] \tag{63}$$

where $\hat{T}_f \in \mathbf{R}^{n \times m}$ and $\hat{Z}_f \in \mathbf{R}^{p \times m}$. Then Problem EA for the case where $1 \leq m < n_0$ has solutions if there exist a group of parameter vectors $f_{ij}^k \in \mathbf{C}^m, g_{ij}^k \in \mathbf{C}^p, k = 1, 2, \cdots, p_{ij}, j = 1, 2, \cdots, r_i, i = 1, 2, \cdots, n_0', f_{\infty j}, g_{\infty j} \in \mathbf{R}^{n-n_0}, j = 1, 2, \cdots, n - n_0$ satisfying Constraints 1 and 2 and the following constraints

Constraint 17. $\det(\hat{T}_f^T B) \neq 0$.

Constraint 18. $T_\infty^T (A + B(\hat{T}_f^T B)^{-1} \hat{Z}_f^T C) V_\infty = I_{n_0}$.

Furthermore, if this condition is satisfied, then the general solution (K, V, T) to Problem EA is given by (19), (32), (38), (43), (44), and

$$K = (\hat{T}_f^T B)^{-1} \hat{Z}_f^T \tag{64}$$

with Constraints 1, 2, 17 and 18 satisfied.

When $n_0 \leq m \leq n$, we partition the matrices T and Z into

$$T = \left[\hat{T} \ \hat{T}^c \right], \ \ Z = \left[\hat{Z} \ \hat{Z}^c \right] \tag{65}$$

where $\hat{T} \in \mathbf{R}^{n \times m}$ and $\hat{Z} \in \mathbf{R}^{p \times m}$. Then we have the following corollary.

Corollary 4. Let the S-controllable S-observable linear descriptor system (1) with $n_0 \leq m \leq n$ be given. Then Problem EA has solutions if there exist a group of parameter vectors $g_{ij}^k \in \mathbf{C}^p, k = 1, 2, \cdots, p_{ij}, j = 1, 2, \cdots, r_i, i = 1, 2, \cdots, n_0', g_{\infty j} \in \mathbf{R}^{n-n_0}, j = 1, 2, \cdots, n - n_0, z_{\infty j} \in \mathbf{R}^p, j = 1, 2, \cdots, m - n_0$ satisfying the following constraints

Constraint 19. $g_{ij}^k \in \mathbf{R}^p$ for $\lambda_i \in \mathbf{R}$, and $g_{ij}^k = \bar{g}_{lj}^k \in \mathbf{C}^p$ for $\lambda_i = \bar{\lambda}_l \in \mathbf{C}$.

Constraint 20. $\det(\hat{T}^T B) \neq 0$.

Constraint 21. $\det \begin{bmatrix} T_f^T E \\ T_\infty^T (A + B(\hat{T}^T B)^{-1} \hat{Z}^T C) \end{bmatrix} \neq 0$.

Furthermore, if this condition is satisfied, then the general solution (K, V, T) to Problem EA is given by (19), (38), (44), and

$$K = (\hat{T}^T B)^{-1} \hat{Z}^T \tag{66}$$

$$V = \begin{bmatrix} T_f^T E \\ T_\infty^T (A + B(\hat{T}^T B)^{-1} \hat{Z}^T C) \end{bmatrix}^{-1} \tag{67}$$

with Constraints 19-21 satisfied.

Remark 4. By Lemma 6 in [14], we can easily show that each introduced additional condition (Constraint 5 or Constraint 7 or Constraint 11 or Constraint 15 or Constraint 17 or Constraint 20) holds for almost all matrices C or B. This means that the case that the additional condition cannot be satisfied is rare.

Remark 5. The approaches given by Theorems 1 and 2 and their corollaries have several advantages over the approaches in [5–10]:

(i) Our approaches eliminate the conditions required in [5,8–10] on the closed-loop eigenvalues.
(ii) Unlike the approaches in [5–10], our approaches assign the whole sets of the left and right closed-loop finite and infinite eigenvectors.
(iii) Our approaches reduce the numbers of algebraic equations in conditions for eigenstructure assignment in [7].
(iv) Unlike the approaches in [5,6], our approaches do not adopt the inner inverses of matrices in the solutions of the feedback gain matrices.

IV. AN EXAMPLE

Consider a system in the form of (1) with the following matrix parameters

$$E = \begin{bmatrix} 1&0&0&0&0&0\\ 0&1&0&0&0&0\\ 0&0&1&0&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&0\\ 1&0&0&0&0&0 \end{bmatrix}, A = \begin{bmatrix} 0&0&1&0&0&0\\ 1&0&0&0&0&0\\ 0&1&0&1&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&1&0\\ 1&0&0&0&0&1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1&0\\ 0&0\\ 0&0\\ 0&0\\ 1&1\\ 0&0 \end{bmatrix}, \; C = \begin{bmatrix} 0&1&0&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&0&1 \end{bmatrix}$$

where E, A and B are taken from [16] (see also [13,17]). For this system we have $n = 6, n_0 = 4, m = 2, p = 3$ and the matrices N_∞ and H_∞ defined by (6) are easily obtained as

$$N_\infty = \begin{bmatrix} 0&0\\ 0&0\\ 0&0\\ 1&0\\ 0&0\\ 0&1 \end{bmatrix}, \; H_\infty = \begin{bmatrix} 0&-1\\ 0&0\\ 0&0\\ 0&0\\ 1&0\\ 0&1 \end{bmatrix}$$

It is easy to verify that the system is S-controllable and S-observable. The finite eigenvalues of the system are $1, \frac{-1\pm\sqrt{3}i}{2}$ and so the system is unstable and possesses one impulsive mode. We consider the assignment of the following closed-loop eigenstructure:

$$\Lambda = \{\lambda_i \in \mathbf{R}, i = 1,2,3,4\},$$

$$n_0' = 4, q_i = r_i = 1, i = 1,2,3,4$$

Since $1 \le \max\{m,p\} = p < n_0$, we can use the approach given in Remark 2.

By our approach, we obtain

$$N_1(\lambda_i) = \begin{bmatrix} 0 & \lambda_i^2\\ 0 & \lambda_i\\ 1 & 0\\ \lambda_i & -\lambda_i\\ 1 & -1\\ 0 & \lambda_i^2(\lambda_i - 1) \end{bmatrix},$$

$$D_1(\lambda_i) = \begin{bmatrix} -1 & \lambda_i^3\\ 0 & -\lambda_i^3 + 1 \end{bmatrix},$$

$$H_1(\lambda_i) = \begin{bmatrix} \lambda_i & 0 & 0\\ \lambda_i^2 & 0 & \lambda_i - 1\\ 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & \lambda_i & 0\\ 0 & 0 & 1 \end{bmatrix},$$

$$L_1(\lambda_i) = \begin{bmatrix} \lambda_i^3 - 1 & 0 & \lambda_i(\lambda_i - 1)\\ -1 & -1 & 0\\ 0 & 0 & -1 \end{bmatrix},$$

$i = 1,2,3,4$

Then the right and left finite closed-loop eigenvectors are given by

$$v_{i1}^1 = N_1(\lambda_i)f_{i1}^1, \; t_{i1}^1 = H_1(\lambda_i)g_{i1}^1, \; i = 1,2,3,4$$

and the corresponding vectors are given by

$$w_{i1}^1 = D_1(\lambda_i)f_{i1}^1, \; z_{i1}^1 = L_1(\lambda_i)g_{i1}^1, \; i = 1,2,3,4$$

where $f_{i1}^1 \in \mathbf{R}^2, \; g_{i1}^1 \in \mathbf{R}^3, \; i = 1,2,3,4.$

Also, the right and left infinite closed-loop eigenvectors are given by

$$v_{\infty j} = N_\infty f_{\infty j}, \; t_{\infty j} = H_\infty g_{\infty j}, \; j = 1,2$$

where $f_{\infty i}, g_{\infty i} \in \mathbf{R}^2, \; j = 1,2.$

Thus Constraint 1 holds automatically and Constraints 2, 7 and 8 can be easily obtained as

$$\left(t_{j1}^1\right)^T E v_{i1}^1 = \delta_{ij},\ i,j = 1,2,3,4 \tag{68}$$

$$\det\left(C\left[\,v_{11}^1\ \ v_{21}^1\ \ v_{31}^1\,\right]\right) \neq 0 \tag{69}$$

$$t_{\infty j}^T\left(A + B\left[w_{11}^1\ w_{21}^1\ w_{31}^1\right]\left(C\left[v_{11}^1\ v_{21}^1\ v_{31}^1\right]\right)^{-1}\right) v_{\infty i} = \delta_{ij}, \tag{70}$$

$$i,j = 1,2$$

where, in (68) and (70), δ_{ij} represents the Kronecker function. When (68)- (70) are satisfied, according to (19) and (56), the general solution (K, V, T) to Problem EA is given by

$$T = \left[\,t_{11}^1\ t_{21}^1\ t_{31}^1\ t_{41}^1\ t_{\infty 1}\ t_{\infty 2}\,\right]$$

$$V = \left[\,v_{11}^1\ v_{21}^1\ v_{31}^1\ v_{41}^1\ v_{\infty 1}\ v_{\infty 2}\,\right]$$

$$K = \left[\,w_{11}^1\ w_{21}^1\ w_{31}^1\,\right]\left(C\left[\,v_{11}^1\ v_{21}^1\ v_{31}^1\,\right]\right)^{-1}$$

In the following, we choose

$$\lambda_i = -i,\ i = 1,2,3,4$$

Through some simple investigations and preselecting some parameters, we can find a special solution of the system of equations in (68), (69) and (70) to be

$$f_{11}^1 = \begin{bmatrix} 1 \\ \frac{5}{2} \end{bmatrix},\ f_{21}^1 = \begin{bmatrix} 1 \\ -\frac{5}{2} \end{bmatrix},\ f_{31}^1 = \begin{bmatrix} 1 \\ -\frac{5}{7} \end{bmatrix},$$

$$f_{41}^1 = \begin{bmatrix} \frac{86}{15} \\ -\frac{13}{6} \end{bmatrix},\ g_{11}^1 = \begin{bmatrix} \frac{6}{5} \\ \frac{112}{15} \\ \frac{34}{15} \end{bmatrix},\ g_{21}^1 = \begin{bmatrix} \frac{6}{5} \\ \frac{124}{5} \\ \frac{27}{5} \end{bmatrix},$$

$$g_{31}^1 = \begin{bmatrix} -\frac{7}{5} \\ -\frac{448}{5} \\ -14 \end{bmatrix},\ g_{41}^1 = \begin{bmatrix} 0 \\ 10 \\ 1 \end{bmatrix},\ f_{\infty 1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$

$$f_{\infty 2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ g_{\infty 1} = \begin{bmatrix} \frac{88}{5} \\ -136 \end{bmatrix},\ g_{\infty 2} = \begin{bmatrix} \frac{3}{5} \\ -6 \end{bmatrix}$$

With these parameters, we obtain the following special solution

$$T = \begin{bmatrix} -\frac{6}{5} & -\frac{12}{5} & \frac{21}{5} & 0 & -\frac{3}{5} & 6 \\ -\frac{10}{3} & -\frac{57}{5} & \frac{217}{5} & -5 & 0 & 0 \\ \frac{6}{5} & \frac{6}{5} & -\frac{7}{5} & 0 & 0 & 0 \\ \frac{112}{15} & \frac{124}{5} & -\frac{448}{5} & 10 & 0 & 0 \\ -\frac{112}{15} & -\frac{248}{5} & \frac{1344}{5} & -40 & \frac{88}{5} & -136 \\ \frac{34}{15} & \frac{27}{5} & -14 & 1 & \frac{3}{5} & -6 \end{bmatrix}$$

$$V = \begin{bmatrix} \frac{5}{2} & -10 & -\frac{45}{7} & -\frac{104}{3} & 0 & 0 \\ -\frac{5}{2} & 5 & \frac{15}{7} & \frac{26}{3} & 0 & 0 \\ 1 & 1 & 1 & \frac{86}{15} & 0 & 0 \\ \frac{3}{2} & -7 & -\frac{36}{7} & -\frac{158}{5} & 1 & 0 \\ -\frac{3}{2} & \frac{7}{2} & \frac{12}{7} & \frac{79}{10} & 0 & 0 \\ -5 & 30 & \frac{180}{7} & \frac{520}{3} & 0 & 1 \end{bmatrix}$$

$$K = \begin{bmatrix} \frac{4}{3} & \frac{17}{3} & \frac{26}{15} \\ -\frac{11}{6} & -\frac{65}{12} & -\frac{41}{24} \end{bmatrix}$$

V. CONCLUSION

Based on a general parametric solution to a type of generalized Sylvester matrix equations, simple parametric approaches for eigenstructure assignment in linear descriptor systems via output feedback are proposed. General parametric expressions for both the left and right closed-loop finite and infinite eigenvectors and the output feedback gain matrix are given. The approaches overcome the drawbacks of some previously published results.

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