Event-triggered control for singular linear positive systems

Huy Hoàng Lê[a], Văn Hiện Lê[b], Phi Hà[c]

[a] *Department of Mathematics, National University of Civil Engineering, 55 Giai Phong road, Dong Tam, Hai Ba Trung, Hanoi, Vietnam*
[b] *Department of Mathematics, Hanoi National University of Education, 136 Xuan Thuy road, Cau Giay, Hanoi, Viet Nam*
[c] *Faculty of Mathematics, Mechanics and Informatics, VNU University of Science, 334 Nguyen Trai, Thanh Xuan, Hanoi, Vietnam*

Abstract

This paper consider the event-triggered strategy for singular linear positive systems ...

Keywords: singular linear positive systems; event-triggered control; linear programming;

Nomenclature

$\mathbb{N}$, $\mathbb{N}_0$	set of natural numbers, $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$
$\mathbb{C}_-$	$= \{ \lambda \in \mathbb{C} \mid \operatorname{Re}\lambda < 0 \}$

1. Introduction

(adding later).....

2. Preliminaries

Consider the linear systems

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}Ex(t) &= Ax(t) + Bu(t), \quad t \geq 0, \\ x(0) &= x_0, \end{aligned} \tag{1}$$

where E, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control vector, $\operatorname{rank} E = r < n$.

Definition 2.1. see [1]

(i) The matrix pencil (E,A) is regular if $\det(sE - A) \in \mathbb{R}[s] \setminus \{0\}$.

(ii) The matrix pencil (E,A) impulse-free if $\deg(\det(sE - A)) = \operatorname{rank} E$.

(iii) The systems (1) ($u(t) = 0$) is regular and impulse-free if the matrix pencil (E,A) is regular and impulse-free.

Let us introduce some basics of positive systems and event-triggered control.

Definition 2.2. see [4] The regular and impulse-free system (1) is positive if for all $t \geq 0$ we have $x(t) \succeq 0$ for any input function $u(\tau) \succeq 0$ with $0 \leq \tau \leq t$ and any consistent initial value $x_0 \succeq 0$.

Definition 2.3. see [2] Given $\alpha > 0$, the regular and impulse-free system (1) with $u(t) = 0$ is α-stable if there exist a positive number $N > 0$ such that the solution $x(t,x_0)$ satisfies

$$\|x(t,x_0)\| \leq Ne^{-\alpha t}\|x_0\|, \quad \text{for all } t \geq 0.$$

Email addresses: hoanglh[at]nuce.edu.vn (Huy Hoàng Lê), hienlv[at]hnue.edu.vn (Văn Hiện Lê), haphi.hus[at]vnu.edu.vn (Phi Hà)

Definition 2.4. see [2] Given $\alpha > 0$, the regular and impulse-free system (1) is α-stabilizable if there exists a feedback control $u(t) = Kx(t)$, $K \in \mathbb{R}^{m\times n}$ such that the closed-loop system

$$\begin{aligned} E\dot{x}(t) &= (A+BK)x(t), \\ x(0) &= x_0, \end{aligned}$$

is positive and α-stable.

Since $\operatorname{rank} E = r < n$, it is known that there exist regular matrix $P, Q \in \mathbb{R}^{n\times n}$ such that $PEQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$. Now, we denote

$$\tilde{E} := PEQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \ \tilde{A} := PAQ = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \ \tilde{B} := PB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}.$$

Remark 2.5. The regular system (1) is impulse-free if and only if $\det(A_4) \neq 0$.

Using coordinate transformation $y(t) = \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} := Q^{-1}x(t)$, where $y_1(t) \in \mathbb{R}^r$, $y_2(t) \in \mathbb{R}^{n-r}$, the system (1) is reduced to the system

$$\begin{aligned} \dot{y}_1(t) &= A_1y_1(t) + A_2y_2(t) + B_1u(t) \\ 0 &= A_3y_1(t) + A_4y_2(t) + B_2u(t) \\ y_1(0) &= y_{10} \\ y_2(0) &= y_{20} \end{aligned} \tag{2}$$

where $Q^{-1}x_0 = y_0 = \begin{pmatrix} y_{10} \\ y_{20} \end{pmatrix}$.

Lemma 2.6. *The system* (2) *with* $\det(A_4) \neq 0$ *is positive if and only if* A_4 *is Hurwitz,* $\tilde{A}$ *is Metzler, and* $\tilde{B} \succeq 0$.

Lemma 2.7. *Let A be a Metzler matrix. Then the following statements are equivalent.*

(i) A is Hurwitz.

(ii) There exists $\gamma \in \mathbb{R}^n$ *such that* $\gamma \succ 0$ *and* $A\gamma \prec 0$.

(iii) There exists $\lambda \in \mathbb{R}^n$ *such that* $\lambda \succ 0$ *and* $\lambda^\top A \prec 0$.

(iv) The matrix is nonsingular and satisfies $A^{-1} \preceq 0$.

Lemma 2.8. *Suppose that* $Q \succeq 0$. *If the system* (2) *is* α*-stabilizable by the feedback control* $u(t) = Ky(t)$*, then the system* (1) *is* α*-stabilizable by the feedback control* $u(t) = KQ^{-1}x(t)$.

The goal of this paper is to develop an event-triggered feedback control law, which has been introduced in [3], for positive descriptor systems. We use linear feedback control law

$$u(t) = K\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} = \begin{bmatrix} K_1 & K_2 \end{bmatrix}\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} = K_1y_1(t) + K_2y_2(t), \tag{3}$$

where $K_1 \in \mathbb{R}^{m\times r}$, $K_2 \in \mathbb{R}^{m\times(n-r)}$. We assume the inputs to be held constant in between the successive recomputations of (3).

$$u(t) = u(t_k) \text{ for } t \in [t_k, t_{k+1}) \tag{4}$$

where the sequence $\{t_k\}_{k\in\mathbb{N}}$ represents the instants at which (3) is re-computed and the actuator signals are updated. We refer to these instants as the *triggering* times. The state measurement error is defined by

$$e(t) = \begin{pmatrix} e_1(t) \\ e_2(t) \end{pmatrix} = \begin{pmatrix} y_1(t_k) \\ y_2(t_k) \end{pmatrix} - \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} = y(t_k) - y(t) \text{ for } t \in [t_k, t_{k+1}) \tag{5}$$

Using this error we express the evolution of the closed-loop system during the interval $[t_k, t_{k+1})$ by

$$\begin{aligned} \dot{y}_1(t) &= (A_1 + B_1K_1)y_1(t) + (A_2 + B_1K_2)y_2(t) + B_1K_1e_1(t) + B_1K_2e_2(t), \\ 0 &= (A_3 + B_2K_1)y_1(t) + (A_4 + B_2K_2)y_2(t) + B_2K_1e_1(t) + B_2K_2e_2(t). \end{aligned} \tag{6}$$

We rewrite (6) in matrix form

$$\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \end{pmatrix} = \left(\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \begin{bmatrix} K_1 & K_2 \end{bmatrix} \right) \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{pmatrix} e_1(t) \\ e_2(t) \end{pmatrix} \tag{7}$$

or

$$\tilde{E}\dot{y}(t) = (\tilde{A} + \tilde{B}K)y(t) + \tilde{B}Ke(t).$$

Suppose that the system (2) is positive. We want to find a matrix K, and a event-triggered condition which generate the *triggering times* $\{t_k\}_{k\in\mathbb{N}}$ such that the system (6) or (7) is positive and α-stable.

3. Main Results

The event-triggered condition is generated by

$$\|e\| \geq \sigma \|y\|, \tag{8}$$

where σ is a constant satisfying $\sigma > 0$. The event-triggered mechanism means that the control input $u(t)$ is updated when the condition (8) holds. We note that the *triggering times* $\{t_k\}_{k\in\mathbb{N}}$ is implicitly defined by the (8) as follow

$$t_0 = 0,\ t_{k+1} = \inf\{t > t_k \mid \|e(t)\| \geq \sigma\|y(t)\|\}, \tag{9}$$

where $0 < \sigma < 1$. In case of regular, impulse-free system (2), we prove that the Zeno behavior does not happen, which means that there exists a time $\tau > 0$ such that $t_{k+1} - t_k \geq \tau$ for any $k \in \mathbb{N}$.

Proposition 3.1. *Suppose that (2) is regular and impulse-free system. There exists a time $\tau > 0$ such that for any consistent initial value y_0 the inter-execution times $\{t_{k+1} - t_k\}_{k\in\mathbb{N}}$ implicitly defined by the execution rule (9) are lower bounded by τ.*

Proof. Using feedback control (3), it is obtained the closed-loop system (6),

$$\begin{aligned} \dot{y}_1(t) &= (A_1 + B_1K_1)y_1(t) + (A_2 + B_1K_2)y_2(t) + B_1K_1e_1(t) + B_1K_2e_2(t), \\ 0 &= (A_3 + B_2K_1)y_1(t) + (A_4 + B_2K_2)y_2(t) + B_2K_1e_1(t) + B_2K_2e_2(t). \end{aligned}$$

Differentiate the second equation of the closed-loop system (6) with the note that $\dot{y}_1(t) = -\dot{e}_1(t)$, and $\dot{y}_2(t) = -\dot{e}_2(t)$, we have

$$0 = A_3\dot{y}_1(t) + A_4\dot{y}_2(t).$$

Since the system (2) is regular and impulse-free, the matrix A_4 is non-singular. Hence,

$$\dot{y}_2(t) = -A_4^{-1}A_3\dot{y}_1(t) = -A_4^{-1}A_3\left[(A_1 + B_1K_1)y_1(t) + (A_2 + B_1K_2)y_2(t) + B_1K_1e_1(t) + B_1K_2e_2(t)\right].$$

Recall the first equation of the closed-loop system (6), we obtain the differential equation

$$\begin{aligned} \begin{pmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \end{pmatrix} = &\begin{bmatrix} A_1 + B_1K_1 & A_2 + B_1K_2 \\ -A_4^{-1}A_3(A_1 + B_1K_1) & -A_4^{-1}A_3(A_2 + B_1K_2) \end{bmatrix} \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} \\ &+ \begin{bmatrix} B_1K_1 & B_1K_2 \\ -A_4^{-1}A_3B_1K_1 & -A_4^{-1}A_3B_1K_2 \end{bmatrix} \begin{pmatrix} e_1(t) \\ e_2(t) \end{pmatrix}. \end{aligned}$$

Thereafter,

$$\begin{aligned}\|\dot{y}\| &\leq \left\| \begin{bmatrix} A_1+B_1K_1 & A_2+B_1K_2 \\ -A_4^{-1}A_3(A_1+B_1K_1) & -A_4^{-1}A_3(A_2+B_1K_2) \end{bmatrix} \right\| \|y\| + \left\| \begin{bmatrix} B_1K_1 & B_1K_2 \\ -A_4^{-1}A_3B_1K_1 & -A_4^{-1}A_3B_1K_2 \end{bmatrix} \right\| \|e\| \\ &= a\|y\| + b\|e\|, \end{aligned} \tag{10}$$

where $a = \left\| \begin{bmatrix} A_1+B_1K_1 & A_2+B_1K_2 \\ -A_4^{-1}A_3(A_1+B_1K_1) & -A_4^{-1}A_3(A_2+B_1K_2) \end{bmatrix} \right\|$, and $b = \left\| \begin{bmatrix} B_1K_1 & B_1K_2 \\ -A_4^{-1}A_3B_1K_1 & -A_4^{-1}A_3B_1K_2 \end{bmatrix} \right\|$.

We can now bound the inter-event times by looking at the dynamics of $\dfrac{\|e\|}{\|y\|}$

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\frac{\|e\|}{\|y\|} &= \frac{\mathrm{d}}{\mathrm{d}t}\frac{(e^\top e)^{1/2}}{(y^\top y)^{1/2}} \\ &= -\frac{e^\top \dot{y}}{\|e\|\|y\|} - \frac{y^\top \dot{y}}{\|y\|^2}\frac{\|e\|}{\|y\|} \quad (\text{ by } \dot{y} = -\dot{e}) \\ &\leq \frac{\|e\|\|\dot{y}\|}{\|e\|\|y\|} + \frac{\|y\|\|\dot{y}\|}{\|y\|^2}\frac{\|e\|}{\|y\|} \\ &= \left(1+\frac{\|e\|}{\|y\|}\right)\frac{\|\dot{y}\|}{\|y\|} \\ &\leq a + (a+b)\frac{\|e\|}{\|y\|} + b\left(\frac{\|e\|}{\|y\|}\right)^2 \quad (\text{ by } (10)). \end{aligned}$$

Consequently, the inter-event times are lower bounded by time τ satisfying

$$\phi(\tau, 0) = \sigma,$$

where $\phi(t,\phi_0)$ is the solution of

$$\dot{\phi} = a + (a+b)\phi + b\phi^2$$

satisfying $\phi(0,\phi_0) = \phi_0$. As a result, $\tau = \dfrac{1}{a-b}\ln\dfrac{a+a\sigma}{a+b\sigma} > 0$, and $t_{k+1} - t_k \geq \tau$, for all $k \in \mathbb{N}$. □

Seeking of simplicity, let us denote $\tilde{A} = [a_{ij}]_{n\times n}$, and $b_i^\top$ is the ith row of $\tilde{B}$.

Theorem 3.2. *Suppose that* (2) *is regular and impulse-free system. Given* $\alpha > 0$*, if there exist constant* $0 < \sigma < 1$*, and vectors* $\beta = \begin{pmatrix}\beta_1 & \beta_2 & \ldots & \beta_n\end{pmatrix}^\top \in \mathbb{R}^n_+$*,* $k_j \in \mathbb{R}^m$*,* $j = 1,\ldots,n$*, such that*

$$\begin{gathered} a_{ij}\beta_j + b_i^\top k_j \geq 0, \quad i,j = 1,\ldots,n,\ i \neq j, \\ b_i^\top k_j \geq 0, \quad i,j = 1,\ldots,n, \end{gathered} \tag{11}$$

and

$$(\alpha\tilde{E} + \tilde{A})\beta + (\sigma+1)\tilde{B}\sum_{j=1}^{n} k_j \prec 0, \tag{12}$$

then under the event-triggered control law (4) *with*

$$K = \begin{bmatrix} \frac{k_1}{\beta_1} & \frac{k_2}{\beta_2} & \cdots & \frac{k_n}{\beta_n}, \end{bmatrix} \tag{13}$$

the resulting closed-loop system of system (2) *is positive and* α*-stable.*

Proof. We proceed in several steps.

Step 1: We show that the resulting closed-loop system (7) is positive, regular and impulse-free.

$$\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \dot{y}_1(t) \\ \dot{y}_2(t) \end{pmatrix} = \left(\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \begin{bmatrix} K_1 & K_2 \end{bmatrix} \right) \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{pmatrix} e_1(t) \\ e_2(t) \end{pmatrix}$$

Since (11), $a_{ij}\beta_j + b_i^\top k_j \geq 0, \quad i,j = 1,\ldots,n,\ i \neq j$ which means that $a_{ij} + b_i^\top \dfrac{k_j}{\beta_j} \geq 0, \quad i,j = 1,\ldots,n,\ i \neq j$. These implies that matrix $\tilde{A} + \tilde{B}K$ or matrix

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \begin{bmatrix} K_1 & K_2 \end{bmatrix} \text{ is Metzler.}$$

On the other hand, we have

$$\sum_{j=1}^{n} k_j = \begin{bmatrix} \frac{k_1}{\beta_1} & \frac{k_2}{\beta_2} & \cdots & \frac{k_n}{\beta_n} \end{bmatrix} \beta = K\beta$$

Invoking condition (12), we have

$$(\alpha\tilde{E} + \tilde{A})\beta + (\sigma+1)\tilde{B}K\beta \prec 0.$$

In conjunction with the fact that $(\alpha\tilde{E} + \sigma\tilde{B}K)\beta \succeq 0$, we obtain

$$(\tilde{A} + \tilde{B}K)\beta \prec 0.$$

Decomposing $\beta := \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$, with $v_1 \in \mathbb{R}^r_+$, and $v_1 \in \mathbb{R}^{n-r}_+$, we derive that

$$(\tilde{A} + \tilde{B}K)\beta = \begin{bmatrix} A_1 + B_1K_1 & A_2 + B_1K_2 \\ A_3 + B_2K_1 & A_4 + B_2K_2 \end{bmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \prec 0,$$

which gives

$$(A_3 + B_2K_1)v_1 + (A_4 + B_2K_2)v_2 \prec 0.$$

Since $\tilde{A} + \tilde{B}K$ is Metzler matrix, and $v_1 \in \mathbb{R}^r_+$, we have $(A_3 + B_2K_1)v_1 \succeq 0$. Therefore,

$$(A_4 + B_2K_2)v_2 \prec 0$$

We note that matrix $A_4 + B_2K_2$ is Metzler. Hence, according to Lemma 2.7,the matrix $A_4 + B_2K_2$ is Hurwitz, and $\det(A_4 + B_2K_2) \neq 0$, which implies that the system (7) is regular, impulse-free. Furthermore, since $\tilde{A} + \tilde{B}K$, $A_4 + B_2K_2$ is Hurwitz, $\det(A_4 + B_2K_2) \neq 0$, and $\tilde{B}K \succeq 0$, we obtain that the system (7) is positive by using Lemma 2.6.

Step 2: We prove that the resulting closed-loop system (7) is α-stable.

Step 2a: We show that $y_1(t)$ is α-stable.

According to **Step 1**, we have already known that the matrix $\alpha\tilde{E} + \tilde{A} + (\sigma+1)\tilde{B}K$ is Metzler. Recall condition (12), and using Lemma 2.7, there exists $\lambda = \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} \in \mathbb{R}^n_+$, $\lambda_1 \in \mathbb{R}^r$, $\lambda_2 \in \mathbb{R}^{n-r}$ such that

$$\lambda^\top \left(\alpha\tilde{E} + \tilde{A} + (\sigma+1)\tilde{B}K \right) \prec 0, \tag{14}$$

We now construct the Lyapunov functional candidate as follows

$$V(t, y(t)) = e^{\alpha t} \lambda^\top \tilde{E} y(t). \tag{15}$$

Taking the derivative in t along the solution we have

$$\begin{aligned}\frac{d}{dt}V(t,y(t)) &= \alpha e^{\alpha t}\lambda^\top \tilde{E}y(t) + e^{\alpha t}\lambda^\top \tilde{E}\dot{y}(t)\\ &= \alpha e^{\alpha t}\lambda^\top \tilde{E}y(t) + e^{\alpha t}\lambda^\top \left[\left(\tilde{A}+\tilde{B}K\right)y(t) + \tilde{B}Ke(t)\right]\\ &= e^{\alpha t}\lambda^\top \left[\left(\alpha\tilde{E}+\tilde{A}+\tilde{B}K\right)y(t) + \tilde{B}Ke(t)\right].\end{aligned}$$

By event-triggered condition (8), the event-triggered controller is not updated. We have

$$\|e(t)\| < \sigma y(t).$$

Therefore,

$$\frac{d}{dt}V(t,y(t)) < e^{\alpha t}\lambda^\top \left(\alpha\tilde{E}+\tilde{A}+(\sigma+1)\tilde{B}K\right)y(t) \le 0, \quad t\ge 0. \tag{16}$$

Hence,

$$V(t,y(t)) \le V(0,y(0)) = \lambda^\top \tilde{E}y_0 = \lambda^\top \begin{pmatrix} y_{01}\\ 0\end{pmatrix} = \lambda_1^\top y_{01} \le \|\lambda_1\|\|y_{01}\|.$$

On the other hand, we have

$$V(t,y(t)) = e^{\alpha t}\lambda^\top \tilde{E}y(t) \ge \Lambda e^{\alpha t}\|y_1(t)\|,$$

where $\Lambda = \min\limits_{1\le i\le r}\lambda_i$. As a result,

$$\|y_1(t)\| \le \frac{\|\lambda\|\|y_0\|}{\Lambda}e^{-\alpha t}, \quad \text{for all } t\ge 0. \tag{17}$$

Step 2b: We show that $y_2(t)$ is α-stable.
Invoking condition (12), we have

$$(\alpha\tilde{E}+\tilde{A})\beta + (\sigma+1)\tilde{B}K\beta \prec 0.$$

In conjunction with the fact that $(\alpha\tilde{E})\beta \succeq 0$, we obtain

$$(\tilde{A}+(\sigma+1)\tilde{B}K)\beta \prec 0.$$

Imitating the process in **Step 1**, we have

$$(\tilde{A}+(\sigma+1)\tilde{B}K)\beta = \begin{bmatrix} A_1+(\sigma+1)B_1K_1 & A_2+(\sigma+1)B_1K_2\\ A_3+(\sigma+1)B_2K_1 & A_4+(\sigma+1)B_2K_2\end{bmatrix}\begin{pmatrix} v_1\\ v_2\end{pmatrix} \prec 0,$$

with $\beta := \begin{pmatrix} v_1\\ v_2\end{pmatrix}$, $v_1\in\mathbb{R}^r_+$, and $v_1\in\mathbb{R}^{n-r}_+$. Hence,

$$[A_3+(\sigma+1)B_2K_1]\,v_1 + [A_4+(\sigma+1)B_2K_2]\,v_2 \prec 0.$$

Since $\tilde{A}+(\sigma+1)\tilde{B}K$ is Metzler matrix, and $v_1\in\mathbb{R}^r_+$, we have $[A_3+(\sigma+1)B_2K_1]\,v_1 \succeq 0$. Therefore,

$$[A_4+(\sigma+1)B_2K_2]\,v_2 \prec 0$$

We note that matrix $A_4+(\sigma+1)B_2K_2$ is Metzler. Hence, according to Lemma 2.7,the matrix $A_4+(\sigma+1)B_2K_2$ is Hurwitz, and $\det[A_4+(\sigma+1)B_2K_2]\neq 0$.
On the other hand, recall (6), we have

$$0 = (A_3+B_2K_1)y_1(t) + (A_4+B_2K_2)y_2(t) + B_2\begin{bmatrix} K_1 & K_2\end{bmatrix}\begin{pmatrix} e_1(t)\\ e_2(t)\end{pmatrix}.$$

By event-triggered condition (8), the event-triggered controller is not updated. We have

$$\|e(t)\| < \sigma y(t).$$

Moreover, $BK \succeq 0$. Therefore,

$$[A_3 + (\sigma+1)B_2K_1]\, y_1(t) + [A_4 + (\sigma+1)B_2K_2]\, y_2 \succ 0. \tag{18}$$

Since, the matrix $A_4 + (\sigma + 1)B_2K_2$ is both Metzler and Hurwitz, using Lemma 2.7, we obtain $-[A_4+(\sigma+1)B_2K_2]^{-1} \succeq 0$. Hence, pre-multiplying both sides of equation (18) with the non-singular matrix $-[A_4+(\sigma+1)B_2K_2]^{-1} \succeq 0$, we have

$$-[A_4+(\sigma+1)B_2K_2]^{-1}[A_3+(\sigma+1)B_2K_1]\, y_1(t) - y_2(t) \succeq 0.$$

Thereafter,

$$y_2(t) \preceq -[A_4+(\sigma+1)B_2K_2]^{-1}[A_3+(\sigma+1)B_2K_1]\, y_1(t). \tag{19}$$

Consequently,

$$\|y_2(t)\| \leq \|[A_4+(\sigma+1)B_2K_2]^{-1}\|\|[A_3+(\sigma+1)B_2K_1]\|\|y_1(t)\|.$$

In corporate with (17), we have

$$\|y_2(t)\| \leq \|[A_4+(\sigma+1)B_2K_2]^{-1}\|\|[A_3+(\sigma+1)B_2K_1]\|\frac{\|\lambda\|\|y_0\|}{\Lambda}e^{-\alpha t}, \quad \text{for all } t \geq 0. \tag{20}$$

Finally, from the both (17) and (20), we conclude that

$$\|y(t)\| \leq M\|y_0\|e^{-\alpha t}, \quad \text{for all } t \geq 0.$$

□

4. Example

In current section, to demonstrate the application of our controller, we consider the following academic example.

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix}\begin{pmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\end{pmatrix} = \begin{bmatrix}-5&0&0\\-2&-3&-2\\-1&-3&-4\end{bmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix} + \begin{bmatrix}0&0\\1&0\\0&1\end{bmatrix}\begin{pmatrix}u_1\\u_2\end{pmatrix}. \tag{21}$$

System (21) is as in the form (2) with

$$A_1 = \begin{bmatrix}-5&0\\-2&-3\end{bmatrix},\ A_2 = \begin{bmatrix}0\\-2\end{bmatrix},\ A_3 = \begin{bmatrix}-1&-3\end{bmatrix},\ A_4 = \begin{bmatrix}-4\end{bmatrix},\ B_1 = \begin{bmatrix}0&0\\1&0\end{bmatrix},\ B_2 = \begin{bmatrix}0&1\end{bmatrix}.$$

It can be seen that (21) is a impulse-free singular systems but not positive systems. To show the feasibility of our control strategy, we choose parameters $\alpha = 0.5$ and $\sigma = 0.5$. By using linear programming to solve the conditions (11), (12), and (13), we obtain

$$K = \begin{bmatrix}2.0299&0.0400&0.0538\\1.3404&3.1254&0.2711\end{bmatrix}.$$

With gain matrix K, the simulation of the controller (3) and (4) apply to the system (21) over the time interval $0-10\ s$ has been performed in MATLAB and is depicted in Fig. 1. The Fig. 1a shows that the inter-execution time strictly positive and the Fig. 1b indicates that all three state variables tend to zero which mean that the closed-loop systems is stable.

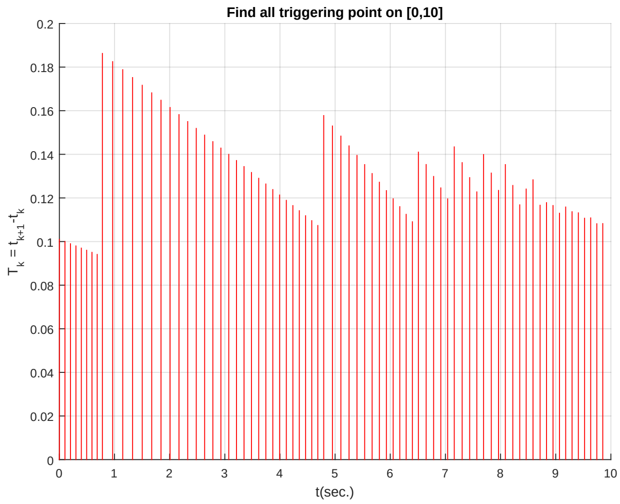

Fig. 1a: Inter-execution times

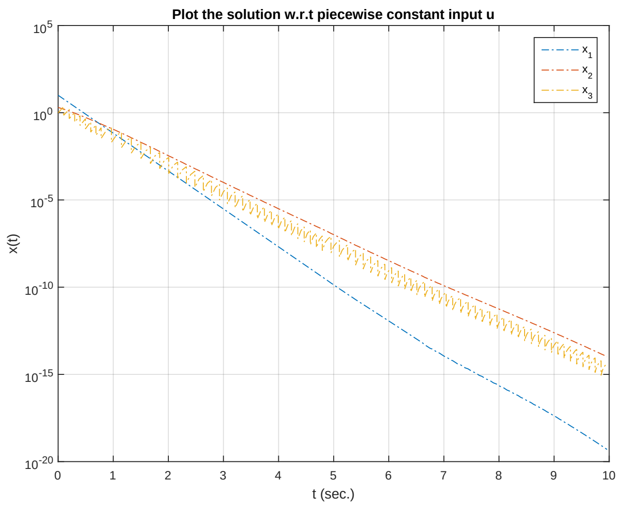

Fig. 1b: State trajectories

Figure 1: Simulation of the controller (3) and (4) for the system (21).

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