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Eigenstructure assignment and impulse elimination for singular second-order system via feedback control

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Abstract: This study presents the eigenstructure assignment approach for singular second-order system via feedback control. The solvability for the problem of normalisation is considered by analysing characteristic polynomial of closed-loop system. All the parametric expressions of controller gains for normalisation are derived. Combining infinite eigenstructure and finite eigenstructure of closed-loop system, impulse elimination approach via feedback control is presented. More precisely, based on desired eigenstructure, the parametric expressions of the gains controller making the closed-loop system impulse-free and assigning the finite eigenstructure are formulated. Simulation results are provided to verify the effectiveness of the proposed method.

1 Introduction

Second-order linear systems have found wide applications in many scientific and engineering fields, such as the control of large flexible space structures, earthquake engineering, robotics control, control of mechanical multibody systems, and vibration control in structural dynamics. The model of second-order system has aroused great interest and a wide variety of practical applications over the last few decades [1, 2]. There are many mathematical methods for solving model and designing controllers [3–9]. It is well known that the eigenstructure assignment (ESA) and pole assignment are effective approaches to improve the dynamic characteristics of singular second-order system, including stabilisation, impulse elimination and decoupling [5–12]. The ESA problem has attracted growing interest in the last few years [13–17].

The ESA based on state and derivative feedback control for second-order systems has many valuable results [5–9, 18–20]. Robust ESA based on state feedback in quadratic matrix polynomials is studied in [20]. The ESA problem is investigated by solving the so-called second-order Sylvester matrix equations in [5, 6]. Several algorithms and complete parametric approach are presented for second-order system via proportional and derivative feedbacks [7, 8, 18, 19, 21]. The output regulation for second-order system via feedback is presented in [13, 14]. The eigenvalue assignment with minimum sensitivity for second-order systems via proportional–derivative state feedback is proposed in [21]. The parametric approach for ESA in second-order system via velocity-plus-acceleration feedback is presented in [15–17], and then, the parametric expressions of gain controllers assigning the eigenstructure via velocity-plus-acceleration feedback are formulated. The partial pole assignment [22, 23] and partial eigenstructure assignment [24, 25] are concerned by various approaches.

The impulsive behaviour is an important characteristic of descriptor systems. Impulse terms may destroy the system and hence are expected to be eliminated in descriptor systems. There are many approaches and results of impulse elimination for descriptor systems [9, 26–28]. The impulsive modes can be eliminated in descriptor systems via state feedback and output feedback [9, 28]. The impulse controllability and impulse observability are necessary conditions for impulse elimination in descriptor systems [6, 29–31]. The impulsive mode controllablisability is proposed for descriptor systems in [30], and the criteria of impulsive mode controllablisability are established. In [26], a structured output proportional and derivative feedback approach is presented for problem of impulsive modes elimination in descriptor systems. Disturbance impulse controllability for descriptor system is introduced in [31]. The controllability and observability conditions of second-order linear systems are analysed in [32]. The impulse elimination problems for second-order systems have not been investigated in the literatures.

In this work, the normalisation problem and impulse elimination problem are investigated, respectively, via a class of feedback controllers by ESA approach for singular second-order system. The solvability of proposed problem is given by analysing desired eigenstructure. Under the conditions of solvability, the complete parametric expressions for controller gains of normalisation and impulse elimination are derived, respectively. Finally, illustrating examples are given.

2 Problem formulation and preliminaries

Consider a following singular second-order dynamical system

$$M\ddot{x} + D\dot{x} + Kx = Bu \tag{1}$$

where $x \in \mathbb{R}^n$ is system state, M, D, $K \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times r}$ are constant matrices and $u \in \mathbb{R}^r$ is controlled input. M, K are singular with $0 < \operatorname{rank}M < n$, $0 < \operatorname{rank}K < n$, and $\operatorname{rank}B = r$. M, D and K represent, respectively, the mass, damping and stiffness matrices when considering a mechanical system. $x(t)$, $\dot{x}(t)$ and $\ddot{x}(t)$ are position, velocity and acceleration, respectively [8, 15–17]. We know that singular system may contain impulse terms in its state solution. Impulse elimination is to design a controller such that the state solution of resulted closed-loop system has no impulse terms. This paper is concerned with ESA and impulse elimination of system (1) via the following feedback controller

$$u = -F_0x - F_1\dot{x} - F_2\ddot{x} \tag{2}$$

where $F_i \in \mathbb{R}^{r\times n}$, $i = 0, 1, 2$. (2) is called position–velocity–acceleration feedback.

The system (1) is singular and distinct from [15, 16] one. The feedback controllers given in [5–8, 15–17] are different from (2). M needs to be non-singular when considering proportional–derivative feedback [8] and K needs to be non-singular when considering velocity-plus-acceleration feedback [15–17]. For feedback (2), these conditions will no longer be needed. Besides, the impulse elimination problem that we will study in Section 4 for singular second-order system is not considered in [5–8, 15–17].

By feedback (2), the system (1) can be transformed into

$$(M + BF_2)\ddot{x} + (D + BF_1)\dot{x} + (K + BF_0)x = 0 \tag{3}$$

The corresponding quadratic polynomial matrix is

$$P(s) = s^2(M + BF_2) + s(D + BF_1) + K + BF_0$$

For analysis and design purpose, we usually transform system (1) to the first-order form

$$\begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix}\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -K & -D \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} u \tag{4}$$

Simultaneously, rewrite (2) as

$$u = -[F_0, F_{11}]\begin{bmatrix} x \\ \dot{x} \end{bmatrix} - [F_{12}, F_2]\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix},$$

where $F_{11} + F_{12} = F_1$. Substituting this into (4), we have

$$\begin{bmatrix} I & 0 \\ BF_{12} & M + BF_2 \end{bmatrix}\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -(K + BF_0) & -(D + BF_{11}) \end{bmatrix}\begin{bmatrix} x \\ \dot{x} \end{bmatrix} \tag{5}$$

There are similar statements about regularisation and normalisation, but terminology is not uniform in the literatures. To avoid confusion, we first show the following definition.

Definition 1: The system (1) is called normalisable via feedback controller (2), if $\operatorname{degdet} P(s) = 2n$.

Before starting main results, we introduce invariant property of system (1) under feedback controller (2). It is necessary for solvability of normalisation.

Lemma 1 [32]: The system (1) is

(i) $\mathcal{R}2$-controllable, if and only if

$$\operatorname{rank}[s^2M + sD + K, B] = n, \quad \forall s \in \mathbb{C};$$

(ii) $\mathcal{C}2$-controllable, if and only if it is $\mathcal{R}2$-controllable and

$$\operatorname{rank}[M, D, B] = n;$$

(iii) strongly $\mathcal{C}2$-controllable, if and only if it is $\mathcal{R}2$-controllable and

$$\operatorname{rank}[M, B] = n.$$

Lemma 2: The feedback controller (2) does not change controllability of system (1). That is to say, if the open-loop system is controllable, then the closed-loop system is controllable, and vice versa.

Proof:

$$\begin{aligned} &[s^2(M + BF_2) + s(D + BF_1) + K + BF_0, B] \\ &\quad = [s^2M + sD + K, B]\begin{bmatrix} I & 0 \\ s^2F_2 + sF_1 + F_0 & I \end{bmatrix} \end{aligned}$$

$$[M + BF_2, D + BF_1, B] = [M, D, B]\begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ F_2 & F_1 & I \end{bmatrix}$$

$$[M + BF_2, B] = [M, B]\begin{bmatrix} I & 0 \\ F_2 & I \end{bmatrix}$$

Then one can have that

$$\begin{aligned} &\operatorname{rank}[s^2(M + BF_2) + s(D + BF_1) + K + BF_0, B] \\ &\quad = \operatorname{rank}[s^2M + sD + K, B], \\ &\operatorname{rank}[M + BF_2, D + BF_1, B] = \operatorname{rank}[M, D, B], \\ &\operatorname{rank}[M + BF_2, B] = \operatorname{rank}[M, B]. \end{aligned}$$

Therefore, $\mathcal{R}2$-controllability, $\mathcal{C}2$-controllability and strong $\mathcal{C}2$-controllability do not change. □

3 Eigenstructure assignment

In this section, the normalisation can be realised by means of the method of ESA. Based on desired eigenstructure, the parametric expressions for the gain matrices of feedback controller are established. Thus, the normalisation problem of system (1) via feedback controller (2) is solved.

Generally, if $\operatorname{rank} M < n$, then the number of finite eigenvalues of system (1) is less than $2n$. The objective of this section is to design the feedback controller (2) such that the closed-loop system has $2n$ eigenvalues. This implies

$$\operatorname{degdet}[s^2(M + BF_2) + s(D + BF_1) + K + BF_0] = 2n.$$

The following theorem gives the conditions to guarantee that the system (1) is normalisable via controller (2).

Theorem 1: The system (1) is normalisable via feedback controller (2) if and only if $\operatorname{rank}[M, B] = n$ and the normalising gain F_2 can be given by

$$F_2 = \left[0_{r,q}, B_2^T\right] T_2.$$

Moreover, $\operatorname{rank}[K, B] = n$ if and only if there exists gain F_0 with

$$F_0 = \left[0_{r,\bar{q}}, B_0^T\right] S_2$$

such that the eigenvalues of closed-loop system are non-zero, where T_2, $S_2 \in \mathbb{R}^{n\times n}$ are orthogonal matrices, q, $\bar{q}$ are the rank of M, K, respectively, and $B_2 \in \mathbb{R}^{(n-q)\times r}$, $B_0 \in \mathbb{R}^{(n-\bar{q})\times r}$ are row full rank matrices.

Proof: Let

$$\begin{aligned} &\det(s^2(M + BF_2) + s(D + BF_1) + K + BF_0) \\ &\quad = a_{2n}s^{2n} + \cdots + a_1 s + a_0 \end{aligned}$$

It is seen that

$$a_{2n} = \det(M + BF_2), \quad a_0 = \det(K + BF_0), \quad \prod_{i=1}^{2n} s_i = \frac{a_0}{a_{2n}},$$

where s_i is the eigenvalue of closed-loop system. By Definition 1, the system (1) is normalisable if and only if $a_{2n} \neq 0$, that is,

$\det(M + BF_2) \neq 0$. Besides, $s_i \neq 0$ for $i = 1{:}2n$, if and only if $a_0 \neq 0$, i.e. $\det(K + BF_0) \neq 0$. It is obtained directly by Lemma 1 of [26] (or Theorem 2.1 of [33]) that there exist F_2, F_0 such that

$$\det(M + BF_2) \neq 0, \quad \det(K + BF_0) \neq 0$$

if and only if

$$\operatorname{rank}[M, B] = n, \quad \operatorname{rank}[K, B] = n.$$

Next, we give an approach to construct gains F_2, F_0. Let $\operatorname{rank} M = q < n$, then the singular value decomposition of M is

$$M = T_1 \begin{bmatrix} \Sigma_M & 0_{q,n-q} \\ 0_{n-q,q} & 0_{n-q,n-q} \end{bmatrix} T_2.$$

where T_1, T_2 are orthogonal matrices, and Σ_M is the singular value matrix of M. Note that

$$\begin{aligned} M + BF_2 &= T_1 \begin{bmatrix} \Sigma_M & 0_{q,n-q} \\ 0_{n-q,q} & 0_{n-q,n-q} \end{bmatrix} T_2 + BF_2 \\ &= T_1 \left(\begin{bmatrix} \Sigma_M & 0_{q,n-q} \\ 0_{n-q,q} & 0_{n-q,n-q} \end{bmatrix} + T_1^{-1} BF_2 T_2^{-1} \right) T_2 \end{aligned}$$

Let

$$T_1^{-1} B := \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad F_2 := [F_{21}, F_{22}]\, T_2$$

where $B_1 \in \mathbb{R}^{q \times r}$, $B_2 \in \mathbb{R}^{(n-q) \times r}$, $F_{21} \in \mathbb{R}^{r \times q}, F_{22} \in \mathbb{R}^{r \times (n-q)}$. Then we have that

$$M + BF_2 = T_1 \begin{bmatrix} \Sigma_M + B_1 F_{21} & B_1 F_{22} \\ B_2 F_{21} & B_2 F_{22} \end{bmatrix} T_2$$

Note that $\operatorname{rank}[M, B] = n$, and

$$\begin{aligned} \operatorname{rank}[M, B] &= \operatorname{rank} \left(T_1^{-1} [M, B] \begin{bmatrix} T_2^{-1} & 0_{n,r} \\ 0_{r,n} & I_r \end{bmatrix} \right) \\ &= \operatorname{rank} \begin{bmatrix} \Sigma_M & 0_{q,n-q} & B_1 \\ 0_{n-q,q} & 0_{n-q,n-q} & B_2 \end{bmatrix}. \end{aligned}$$

These imply that B_2 is a row full rank matrix. If taking $F_{21} = 0$, $F_{22} = B_2^T$, then $\det(M + BF_2) \neq 0$. Thus, $F_2 = [0_{r,q}, B_2^T] T_2$ is a gain matrix such that $\det(M + BF_2) \neq 0$. Similarly, there exist $S_2 \in \mathbb{R}^{n \times n}$, $B_0 \in \mathbb{R}^{(n-\bar{q}) \times r}$ with $F_0 = [0_{r,\bar{q}}, B_0^T] S_2$ such that $\det(K + BF_0) \neq 0$.

In fact, if the system (1) is $\mathcal{R}2$-controllable, then $\operatorname{rank}[K, B] = n$ always holds. In what follows, without loss of generality, we assume that s_i is non-zero. Next, we are to derive complete parametric expressions of gains controller under the conditions of Theorem 1. Assume that desired eigenstructure for (5) is as follows

$$\begin{cases} J = \operatorname{diag}(J_1, J_2, \ldots, J_p) \\ J_i = \operatorname{diag}(J_{i1}, J_{i2}, \ldots, J_{iq_i}) \\ J_{ij} = \begin{bmatrix} s_i & 1 & & \\ & s_i & \ddots & \\ & & \ddots & 1 \\ & & & s_i \end{bmatrix}_{p_{ij} \times p_{ij}} \end{cases} \tag{6}$$

where $J_{ij}, j = 1, 2, \ldots, q_i$, are the q_i Jordan blocks associated with the eigenvalue s_i. J is the Jordan matrix associated with all the finite eigenvalues of closed-loop system. It is seen that the closed-loop system has $2n$ finite eigenvalues, i.e.

$$\sum_{i=1}^{p} \sum_{j=1}^{q_i} p_{ij} = 2n.$$

Define that

$$\begin{aligned} V &= [V_1, V_2, \ldots, V_p] \\ V_i &= [V_{i1}, V_{i2}, \ldots, V_{iq_i}] \\ V_{ij} &= [v_{ij}^1, v_{ij}^2, \ldots, v_{ij}^{p_{ij}}] \end{aligned} \tag{7}$$

are eigenvector matrices corresponding to Jordan matrices in (6). V_i is an eigenvector matrix associated with the eigenvalue s_i, where $v_{ij}^k \in \mathbb{C}^{2n}$, $k = 1, 2, \ldots, p_{ij}$. To derive parametric expressions of feedback gains, we first give following important lemma. □

Lemma 3: Let $L \in \mathbb{C}^{n \times n}$, $N \in \mathbb{C}^{n \times m}$, $X \in \mathbb{C}^{m+n}$, $Y \in \mathbb{C}^n$, rank $[L, N] = n$. Then all the parametric solutions of equation

$$[L, N] X = Y \tag{8}$$

can be expressed by

$$X = S \begin{bmatrix} \Sigma^{-1} U Y \\ G \end{bmatrix},$$

where $S \in \mathbb{C}^{(m+n) \times (m+n)}$, $U \in \mathbb{C}^{n \times n}$ are unitary matrices. Σ is the singular value matrix of $[L, N]$. $G \in \mathbb{C}^m$ is arbitrary column vector.

Proof: Since $\operatorname{rank}[L, N] = n$, by the singular value decomposition, there exist $U \in \mathbb{C}^{n \times n}$, $S \in \mathbb{C}^{(m+n) \times (m+n)}$, such that

$$U[L, N] S = [\Sigma, 0_{n,m}]. \tag{9}$$

Substituting (9) into (8), we have

$$[\Sigma, 0_{n,m}] S^{-1} X = UY. \tag{10}$$

Premultiplying both sides of (10) by Σ^{-1}, we obtain

$$[I, 0_{n,m}] S^{-1} X = \Sigma^{-1} UY.$$

Thus

$$S^{-1} X = \begin{bmatrix} \Sigma^{-1} UY \\ G \end{bmatrix},$$

where $G \in \mathbb{C}^m$ is arbitrary column vector. This shows that

$$X = S \begin{bmatrix} \Sigma^{-1} UY \\ G \end{bmatrix}.$$

□

Theorem 2: Assume that the system (1) is $\mathcal{R}2$-controllable and $\operatorname{rank}[M, B] = n$. Then based on the prescribed eigenstructure in (6), all the parametric expressions of the gain controllers for normalisation are expressed by

$$[F_0, F_{11}] = RV^{-1}, \quad [F_{12}, F_2] = HJ^{-1} V^{-1} \tag{11}$$

where $V \in \mathbb{C}^{2n \times 2n}$ is given as (7) and $R \in \mathbb{C}^{r \times 2n}$, $H \in \mathbb{C}^{r \times 2n}$ are defined by

$$\begin{aligned} R &= [R_1, R_2, \ldots, R_p], \quad R_i = [R_{i1}, R_{i2}, \ldots, R_{iq_i}], \\ R_{ij} &= [r_{ij}^1, r_{ij}^2, \ldots, r_{ij}^{p_{ij}}] \end{aligned} \tag{12}$$

$$\begin{aligned} H &= [H_1, H_2, \ldots, H_p], \quad H_i = [H_{i1}, H_{i2}, \ldots, H_{iq_i}], \\ H_{ij} &= [h_{ij}^1, h_{ij}^2, \ldots, h_{ij}^{p_{ij}}] \end{aligned} \tag{13}$$

$$v_{ij}^k = \begin{bmatrix} S_{i,12}G_{ij}^k - S_{i,11}\Sigma_i^{-1}U_i[s_iM+D,M]v_{ij}^{k-1} \\ s_iS_{i,12}G_{ij}^k - s_iS_{i,11}\Sigma_i^{-1}U_i[s_iM+D,M]v_{ij}^{k-1} + \tilde{v}_{ij}^{k-1} \end{bmatrix},$$
$$r_{ij}^k = S_{i,22}G_{ij}^k - S_{i,21}\Sigma_i^{-1}U_i[s_iM+D,M]v_{ij}^{k-1}, \tag{14}$$
$$h_{ij}^k = S_{i,32}G_{ij}^k - S_{i,31}\Sigma_i^{-1}U_i[s_iM+D,M]v_{ij}^{k-1},$$
$$v_{ij}^0 = 0, i=1,2,\dots,p; j=1,2,\dots,q_i; k=1,2,\dots,p_{ij},$$

with $r_{ij}^k, h_{ij}^k \in \mathbb{C}^r$. $G_{ij}^k \in \mathbb{C}^{2r}, \forall i,j$ are parameter vectors satisfying

(i) $(G_{ij}^k)^* = G_{ij}^k$, when $s_l^* = s_i, \forall i, l$;
(ii) $\det V \neq 0$.

Proof: Combining (6), (7) and the definition of closed-loop jth eigenvector chain associated with eigenvalue s_i, the following relation holds for system (5)

$$\begin{bmatrix} 0 & I \\ -(K+BF_0) & -(D+BF_{11}) \end{bmatrix} v_{ij}^k = \begin{bmatrix} I & 0 \\ BF_{12} & M+BF_2 \end{bmatrix} (s_iv_{ij}^k + v_{ij}^{k-1})$$

That is

$$\begin{bmatrix} 0 & I \\ -(K+BF_0) & -(D+BF_{11}) \end{bmatrix} \begin{bmatrix} \tilde{v}_{ij}^k \\ \hat{v}_{ij}^k \end{bmatrix} = \begin{bmatrix} I & 0 \\ BF_{12} & M+BF_2 \end{bmatrix} \begin{bmatrix} s_i\tilde{v}_{ij}^k + \tilde{v}_{ij}^{k-1} \\ s_i\hat{v}_{ij}^k + \hat{v}_{ij}^{k-1} \end{bmatrix}$$

where

$$v_{ij}^k \equiv \begin{bmatrix} \tilde{v}_{ij}^k \\ \hat{v}_{ij}^k \end{bmatrix}, \quad \tilde{v}_{ij}^k, \hat{v}_{ij}^k \in \mathbb{C}^n, \quad v_{ij}^0 = 0.$$

Then

$$\hat{v}_{ij}^k = s_i\tilde{v}_{ij}^k + \tilde{v}_{ij}^{k-1},$$
$$-(K+BF_0)\tilde{v}_{ij}^k - (D+BF_{11})\hat{v}_{ij}^k = BF_{12}(s_i\tilde{v}_{ij}^k + \tilde{v}_{ij}^{k-1}) + (M+BF_2)(s_i\hat{v}_{ij}^k + \hat{v}_{ij}^{k-1}),$$

Rewrite it as

$$(s_i^2M + s_iD + K)\tilde{v}_{ij}^k + B[F_0, F_{11}] \begin{bmatrix} \tilde{v}_{ij}^k \\ \hat{v}_{ij}^k \end{bmatrix} + B[F_{12}, F_2] \begin{bmatrix} s_i\tilde{v}_{ij}^k + \tilde{v}_{ij}^{k-1} \\ s_i\hat{v}_{ij}^k + \hat{v}_{ij}^{k-1} \end{bmatrix} = -(s_iM+D)\tilde{v}_{ij}^{k-1} - M\hat{v}_{ij}^{k-1}, \tag{15}$$

Let

$$[F_0, F_{11}]v_{ij}^k = r_{ij}^k, \quad [F_{12}, F_2](s_iv_{ij}^k + v_{ij}^{k-1}) = h_{ij}^k.$$

Then the parametric expressions for the gains can be, respectively, arranged into the forms of

$$[F_0, F_{11}]V = R, \quad [F_{12}, F_2]VJ = H. \tag{16}$$

That is

$$[F_0, F_{11}] = RV^{-1}, \quad [F_{12}, F_2] = HJ^{-1}V^{-1}.$$

It follows from (15) that

$$[s_i^2M + s_iD + K, B, B] \begin{bmatrix} \tilde{v}_{ij}^k \\ r_{ij}^k \\ h_{ij}^k \end{bmatrix} = -[s_iM+D, M]v_{ij}^{k-1}.$$

$\mathcal{R}2$-controllable of system (1) implies

$$\operatorname{rank}[s^2M + sD + K, B, B] = \operatorname{rank}[s^2M + sD + K, B] = n.$$

By Lemma 3, we have

$$\begin{bmatrix} \tilde{v}_{ij}^k \\ r_{ij}^k \\ h_{ij}^k \end{bmatrix} = S_i \begin{bmatrix} -\Sigma_i^{-1}U_i[s_iM+D,M]v_{ij}^{k-1} \\ G_{ij}^k \end{bmatrix} \tag{17}$$

where $S_i \in \mathbb{C}^{(n+2r)\times(n+2r)}$, $U_i \in \mathbb{C}^{n\times n}$ are unitary matrices and $G_{ij}^k \in \mathbb{C}^{2r}$. Note that conjugated eigenvalues correspond to conjugated eigenvectors. Hence, G_{ij}^k satisfies (i). For the solvability of gains in (16), condition (ii) is necessary and easy to be met. Partition the matrix S_i as follows

$$S_i := \begin{bmatrix} S_{i,11} & S_{i,12} \\ S_{i,21} & S_{i,22} \\ S_{i,31} & S_{i,32} \end{bmatrix}.$$

Substituting it into (17) produces (14). Thus, (11) holds and F_0, F_{11}, F_{12}, F_2 are obtainable. The proof is then completed. □

4 Impulse elimination

We know that impulsive behaviour is an important issue in descriptor system, and impulse terms are not expected to exist. Analogous to descriptor system, we say that second-order system has impulsive modes if its state solution contains an impulse term. A system is called impulse-free, or equivalently, having no impulsive modes if there is no impulse term in the state solution. Similarly, the response of singular second-order system (1) may contain impulse terms. In this section, we consider the problem of impulse elimination for system (1) via feedback controller (2). By using ESA method, we present the parametric expressions of gain controllers ensuring the closed-loop system impulse-free.

The following lemma, which is obtained directly by the results of [34], gives a simple rank criterion for a second-order system to be impulse-free.

Lemma 4: The system (1) is impulse-free if and only if

$$\operatorname{rank}\begin{bmatrix} M & D & K \\ 0 & M & D \\ 0 & 0 & M \end{bmatrix} - \operatorname{rank}\begin{bmatrix} M & D \\ 0 & M \end{bmatrix} = n$$

This result provides an approach to detect impulsive modes of system (1). In this section, the impulse elimination problem is investigated based on the following lemma instead of Lemma 4.

Lemma 5: If the system (4) is impulse-free, then the system (1) is impulse-free.

Lemma 5 is easy to prove by matrix decomposition. By Lemma 5, in this section, we consider impulse elimination in the system (4) via feedback (2). Our objective of this section is to design the controller (2) such that system (5) is impulse-free. For the system (5), we know that $\text{rank}E \geq \text{degdet}(sE - A)$ and closed-loop system is impulse-free if and only if $\text{rank}E = \text{degdet}(sE - A)$, where

$$E = \begin{bmatrix} I & 0 \\ BF_{12} & M + BF_2 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ -(K + BF_0) & -(D + BF_{11}) \end{bmatrix}.$$

In what follows, we assume that $n_1 = n + \text{rank}[M, B]$ is the number of desired finite eigenvalues, and $n_2 = 2n - n_1$. Then

$$\begin{aligned} n_1 = \text{degdet}(sE - A) \leq \text{rank}E = n \\ + \text{rank}(M + BF_2) \leq n + \text{rank}[M, B] \end{aligned}$$

Therefore, $\text{rank}E = \text{degdet}(sE - A)$, and the closed-loop system is impulse-free. Based on ESA approach and Lemma 5, parametric expressions of gains controller making the closed-loop system impulse-free and assigning the finite eigenstructure (i.e. Jordan structure of finite eigenvalues) of the closed-loop system are derived. The results are presented by the following theorem.

Theorem 3: Assume that system (1) is $\mathcal{R}2$-controllable. Then based on the prescribed finite eigenstructure with the form of (6), the controller gains for impulse elimination and arbitrary finite ESA via feedback (2) are parameterised as

$$[F_{12}, F_2] = [H_f J^{-1}, H_\infty] V^{-1}, \quad [F_0, F_{11}] = [R_f, R_\infty] V^{-1} \quad (18)$$

where $V \equiv [V_f, V_\infty]$, $V_f \in \mathbb{C}^{2n \times n_1}$, H_f, $R_f \in \mathbb{C}^{r \times n_1}$ with the forms of (7), (12) and (13), respectively,

$$V_\infty = \begin{bmatrix} 0 \\ \hat{V}_\infty \end{bmatrix} \in \mathbb{C}^{2n \times n_2},$$

$H_\infty \in \mathbb{C}^{r \times n_2}$ and

$$\begin{bmatrix} \hat{V}_\infty \\ H_\infty \end{bmatrix} \in \ker[M, B].$$

$R_\infty \in \mathbb{C}^{r \times n_2}$ is any given matrix.

Proof: Based on the finite ESA results in Theorem 2, we can get a result similar to (16). Write this as

$$[F_0, F_{11}] V_f = R_f, \quad [F_{12}, F_2] V_f J = H_f, \quad (19)$$

where $V_f \in \mathbb{C}^{2n \times n_1}$ with $\text{rank}V_f = n_1$, R_f, $H_f \in \mathbb{C}^{r \times n_1}$, respectively, have the same definition with (7), (12) and (13) except number of columns. Since the closed-loop system (5) is impulse-free. The infinite Jordan matrix $J_\infty = 0$ and $\text{rank}V_\infty = n_2$, where J_∞ is infinite Jordan matrix of the matrix pair (E, A). V_∞ is infinite eigenvector matrix associated with J_∞. It follows immediately that

$$\begin{bmatrix} I & 0 \\ BF_{12} & M + BF_2 \end{bmatrix} \begin{bmatrix} \tilde{V}_\infty \\ \hat{V}_\infty \end{bmatrix} = 0, \quad \text{rank} \begin{bmatrix} \tilde{V}_\infty \\ \hat{V}_\infty \end{bmatrix} = n_2, \quad (20)$$

where

$$V_\infty = \begin{bmatrix} \tilde{V}_\infty \\ \hat{V}_\infty \end{bmatrix},$$

that is

$$\tilde{V}_\infty = 0, \quad M\hat{V}_\infty + B[F_{12}, F_2] V_\infty = 0, \quad \text{rank}\hat{V}_\infty = n_2.$$

Let

$$[F_{12}, F_2] V_\infty = H_\infty, \quad (21)$$

then $H_\infty \in \mathbb{C}^{r \times n_2}$ and

$$[M, B] \begin{bmatrix} \hat{V}_\infty \\ H_\infty \end{bmatrix} = 0. \quad (22)$$

Hence

$$\begin{bmatrix} \hat{V}_\infty \\ H_\infty \end{bmatrix} \in \ker[M, B].$$

For given R_∞, let

$$[F_0, F_{11}] V_\infty = R_\infty, \quad (23)$$

and then $R_\infty \in \mathbb{C}^{r \times n_2}$. Combining (19), (21) and (23), we obtain

$$\begin{aligned} [F_{12}, F_2][V_f, V_\infty] &= [H_f J^{-1}, H_\infty], \\ [F_0, F_{11}][V_f, V_\infty] &= [R_f, R_\infty] \end{aligned}$$

Thus

$$[F_{12}, F_2] = [H_f J^{-1}, H_\infty] V^{-1}, \quad [F_0, F_{11}] = [R_f, R_\infty] V^{-1},$$

where

$$V \equiv [V_f, V_\infty] = \begin{bmatrix} \tilde{V}_f & 0 \\ \hat{V}_f & \hat{V}_\infty \end{bmatrix}.$$

The proof is therefore completed. □

Remark 1: In fact, $\text{rank}[M, B] < n$ in Theorem 3. There are different selection methods for

$$\begin{bmatrix} \hat{V}_\infty \\ H_\infty \end{bmatrix}$$

with

$$\begin{bmatrix} \hat{V}_\infty \\ H_\infty \end{bmatrix} \in \ker[M, B].$$

Let $\tau = \text{rank}[M, B]$, then $\tau = n_1 - n$. By singular value decomposition and Lemma 3, there exist $\mathcal{T}_1 \in \mathbb{C}^{n \times n}$, $\mathcal{T}_2 \in \mathbb{C}^{(n+r) \times (n+r)}$ such that

$$[M, B] = \mathcal{T}_1 \begin{bmatrix} \sum_{\tau,\tau} & 0_{\tau, n+r-\tau} \\ 0_{n-\tau,\tau} & 0_{n-\tau, n+r-\tau} \end{bmatrix} \mathcal{T}_2^H.$$

Thus

$$\begin{bmatrix} \hat{V}_\infty \\ H_\infty \end{bmatrix}$$

can be taken as

$$\begin{bmatrix} \hat{V}_\infty \\ H_\infty \end{bmatrix} = \mathcal{T}_2 \begin{bmatrix} 0_{\tau,\tau} & * \\ * & * \end{bmatrix} \text{with rank} \begin{bmatrix} 0_{\tau,\tau} & * \\ * & * \end{bmatrix} = n_2$$

where $*$ represents possible non-zero terms. Especially, since $n + r - n_2 > \tau$, taking

$$\begin{bmatrix} \hat{V}_\infty \\ H_\infty \end{bmatrix} = \mathcal{T}_2 \begin{bmatrix} 0_{n+r-n_2, n_2} \\ I_{n_2} \end{bmatrix}.$$

5 Simulation examples

As mentioned above, the controller (2) is suitable for system with non-singular matrices M or K (e.g. the system models of [8, 15–17]). However, proportional–derivative feedback controller and velocity–acceleration feedback controller are not feasible for normalisation of system (1). To verify the results, we give three numerical examples. Examples 1 and 3 verify the effectiveness of the results of Theorems 2 and 3, respectively. Example 2 shows that the results of [17] can also be obtained by feedback (2), and therefore feedback (2) is more general than proportional–derivative feedback or velocity–acceleration feedback.

Example 1: Consider system (1) with

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 1 & -1 \\ -2 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

$$K = \begin{bmatrix} 3 & 0 & -1 \\ -1 & 2 & 2 \\ 1 & 4 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ -1 & 2 \end{bmatrix}$$

It can be obtained that $\det(s^2M + sD + K) = 4s^3 + 21s^2 + 29s$. One can observe that the system has three finite eigenvalues and three infinite eigenvalues. The following simulation will be shown for choosing desired finite eigenvalues $s_1 = -2$, $s_2 = -3$, $s_{3,4} = -5$, $s_{5,6} = -4 \pm 4i$, where -5 is a multiple root with partial multiplicity 2. The parameter vectors are chosen as

$$G_1 = G_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \\ 2 \end{bmatrix}, \quad G_3 = G_4 = \begin{bmatrix} 3 \\ -2 \\ 0 \\ 1 \end{bmatrix}, \quad G_5 = G_6 = \begin{bmatrix} -5 \\ 4 \\ 3 \\ -1 \end{bmatrix}.$$

According to Theorem 2 and (12)–(14), V, R and H can be constructed. Then the feedback gains F_0, F_{11}, F_{12} and F_2 are obtained, respectively, as follows

$$F_0 = \begin{bmatrix} 5.5171 & -0.9031 & 5.0034 \\ -3.2550 & 0.7231 & -11.0170 \end{bmatrix},$$

$$F_{11} = \begin{bmatrix} 0.0318 & -0.5166 & 0.3458 \\ 4.8832 & -0.0180 & -0.4788 \end{bmatrix},$$

$$F_{12} = \begin{bmatrix} -0.6355 & 0.5957 & 0.3828 \\ -1.6796 & 1.2575 & -2.5888 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} -0.0293 & 0.1136 & 0.0343 \\ 1.2234 & 0.1633 & -0.2006 \end{bmatrix}.$$

Furthermore

$$F_1 = \begin{bmatrix} -0.6038 & 0.0792 & 0.7286 \\ 3.2035 & 1.2395 & -3.0676 \end{bmatrix}.$$

The eigenvalues of closed-loop system are obtained by solving the roots of polynomial $\det[(M + BF_2)s^2 + (D + BF_1)s^2 + K + BF_0]$. The errors of eigenvalues for closed-loop pencil are, respectively,

$$-5.3291 \times 10^{-15}, 3.4195 \times 10^{-14}, \pm 1.0728 \times 10^{-8},$$
$$(1.4655 - 1.3323i) \times 10^{-14}, (0.9326 + 1.5987i) \times 10^{-14}$$

Finally, the simulation results of closed-loop system are presented in Fig. 1, where the initial state conditions are given by

$$x_0 = [-0.1, -0.1, 0.2]^{\mathrm{T}}, \quad \dot{x}_0 = [0.2, 0.1, -0.3]^{\mathrm{T}}.$$

In this example, above results are missing if proportional–derivative feedback or velocity–acceleration feedback is used instead of feedback (2).

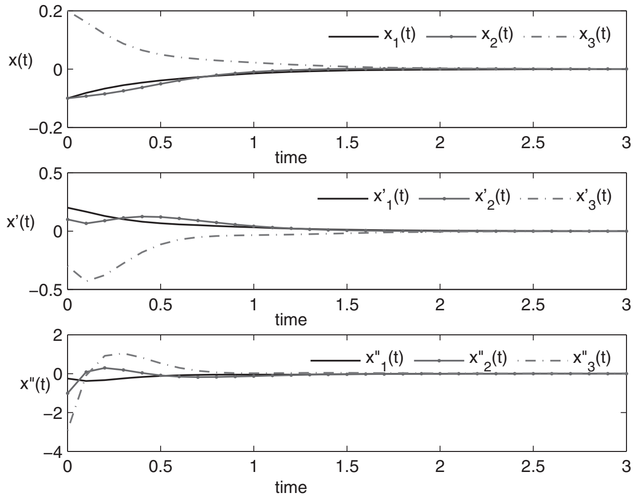

Fig. 1 *System's response for normalisation*

Example 2: To illustrate that controller (2) is suitable for systems given in [8, 15–17], without loss of generality, we give the simulation results for numerical example of [17] via feedback (2).

$$M = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 25 & -15 & 0 & 0 \\ -15 & 35 & -20 & 0 \\ 0 & -20 & 60 & -40 \\ 0 & 0 & -40 & 40 \end{bmatrix},$$

$$K = \begin{bmatrix} 15 & -10 & 0 & 0 \\ -10 & 25 & -15 & 0 \\ 0 & -15 & 35 & -20 \\ 0 & 0 & -20 & 20 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

As given in simulation of [17], the desired eigenvalues and parameter vectors are chosen as follows

$$s_{1,2} = -1 \pm 0.5i, \quad s_{3,4} = -2 \pm i,$$
$$s_{5,6} = -3 \pm i, \quad s_{7,8} = -4 \pm 4i$$
$$G_1 = G_2 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}, \quad G_3 = G_4 = \begin{bmatrix} 5 \\ -1 \end{bmatrix},$$
$$G_5 = G_6 = \begin{bmatrix} -3 \\ 4 \end{bmatrix}, \quad G_7 = G_8 = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

According to Theorem 2, the feedback gains are

$$F_0 = \begin{bmatrix} 25.7516 & -233.8821 & 729.8007 & -522.7613 \\ -8.9919 & 161.7217 & -532.2418 & 386.3812 \end{bmatrix},$$

$$F_1 = \begin{bmatrix} -0.1877 & -0.1000 & 1.8427 & -1.3790 \\ -0.0170 & 0.5231 & -1.9471 & 1.3031 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} 3.5183 & -11.9068 & 33.0550 & -19.3303 \\ -1.3063 & 3.4149 & -21.5258 & 12.9428 \end{bmatrix}$$

The errors of closed-loop system are

$$10^{-13} \times \{-0.1532 \pm 0.0177i, -0.2531 \pm 0.0999i,$$
$$-1.5055 \pm 0.4463i, -0.0888 \pm 0.0266i\}$$

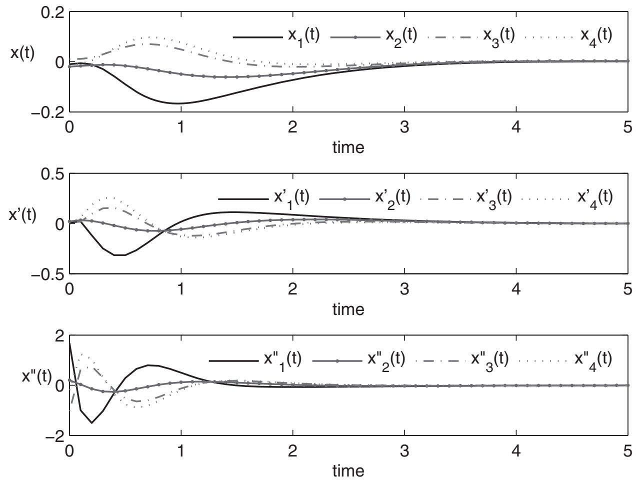

Fig. 2 *System's response via feedback (2)*

The simulation results are displayed in Fig. 2 using the following identical initial state conditions

$$x_0 = [-0.01, -0.02, 0.01, -0.01]^{\mathrm{T}};$$
$$\dot{x}_0 = [0.01, 0.02, 0.02, -0.01]^{\mathrm{T}}.$$

One can see that the simulation results are nearly identical with [17] ones, and therefore the simulation diagrams of [17] are not presented here.

Example 3: Consider singular second-order system

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \ddot{x} + \begin{bmatrix} 0 & 2 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{x} + \begin{bmatrix} 1 & -1 & 0 \\ 0 & -3 & 0 \\ 1 & 0 & 0 \end{bmatrix} x = \begin{bmatrix} 1 & -2 \\ 1 & 0 \\ -1 & 0 \end{bmatrix} u \tag{24}$$

It is clearly seen that there exist impulsive modes for system (24). Since $\mathrm{rank}[M, B] = 2$. Selecting

$$s_{1,2} = -2 \pm 3i, \quad s_{3,4} = -1 \pm 2i, \quad s_5 = -1.$$

The parameter vectors G_1, G_2, G_3, G_4, G_5 are same as ones of Example 1, and $R_\infty = [1, 2]^{\mathrm{T}}$. $\hat{V}_\infty$, H_∞ are chosen as Remark 1 with $[\hat{V}_\infty^{\mathrm{T}}, H_\infty^{\mathrm{T}}]^{\mathrm{T}} = \mathcal{T}_2[0, 0, 1, 1, 1]^{\mathrm{T}}$. Then, the impulsive modes can be eliminated and finite eigenstructure can be assigned arbitrarily via feedback controller (2). By (18), the feedback gains are

$$F_0 = \begin{bmatrix} -3.5423 & 3.4586 & 1.2981 \\ -0.5613 & 6.3949 & -1.0831 \end{bmatrix},$$
$$F_{11} = \begin{bmatrix} -1.0330 & 1.9016 & 0.2374 \\ 3.6470 & -1.1901 & -0.1945 \end{bmatrix},$$
$$F_{12} = \begin{bmatrix} -3.0996 & -0.6291 & 1.3825 \\ 0.7852 & 2.4568 & -0.8232 \end{bmatrix},$$
$$F_1 = \begin{bmatrix} -4.1326 & 1.2726 & 1.6200 \\ 4.4322 & 1.2667 & -1.0177 \end{bmatrix},$$
$$F_2 = \begin{bmatrix} -2.0639 & 1.6599 & 0.3541 \\ 1.7419 & -0.9229 & -0.2724 \end{bmatrix}.$$

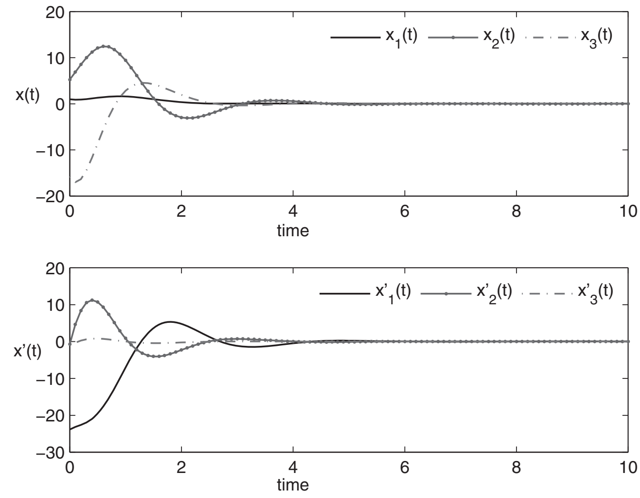

Fig. 3 *System's response for impulse elimination*

It is calculated that

$$\begin{bmatrix} I & 0 \\ BF_{12} & M + BF_2 \end{bmatrix} V_\infty = 0.$$

The error grade is 10^{-15}. This implies that $J_\infty = 0$. One may conclude that the closed-loop system is impulse-free. The simulation results for impulse responses of the closed loop are presented in Fig. 3. According to the results of Theorem 3, closed-loop system is still singular, therefore the state's second derivatives are not contained in Fig. 3.

6 Conclusions

This paper has addressed normalisation and impulse elimination problems via a class of feedback controllers for second-order system by using ESA approach. The parametric expressions of controller gains for normalisation are derived by using ESA method. A condition for impulse elimination and arbitrary finite ESA are given. Under this condition, parametric forms for controller gains are established. The controller is suitable for practical application because the position, velocity and acceleration signals are obtainable. The controller is feasible for mechanical system with the singular mass and stiffness matrices.

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