Mathematical and Computer Modelling of Dynamical Systems 1387-3954/01/0702-189$16.00
2001, Vol. 7, No. 2, pp. 189–203

A PDAE Model for Interconnected Linear RLC Networks

M. GÜNTHER*

ABSTRACT

In electrical circuit simulation, a refined generalized network approach is used to describe secondary and parasitic effects of interconnected networks. Restricting our investigations to linear RLC circuits, this ansatz yields linear initial-boundary value problems of mixed partial-differential and differential-algebraic equations, so-called PDAE systems. If the network fulfils some topological conditions, this system is well-posed and has perturbation index 1 only: the solution of a slightly perturbed system does not depend on derivatives of the perturbations. As method-of-lines applications are often used to embed PDAE models into time-domain network analysis packages, it is reasonable to demand that the analytical properties of the approximate DAE system obtained after semidiscretization are consistent with the original PDAE system. Especially, both should show the same sensitivity with respect to initial and boundary data. We will learn, however, that semidiscretization may act like a deregularization of an index-1 PDAE model, if an inappropriate type of semidiscretization is used.

Keywords: refined modeling, generalized network models, differential-algebraic equations (DAEs), partial differential equations (PDEs), partial differential-algebraic equations (PDAEs), a-priori estimates, perturbation index, method-of-lines (MOL), approximate DAE systems (ADAEs).

1 INTRODUCTION

In network simulation packages, real circuit elements and interconnections are commonly replaced by companion models of ideal and compact network elements, whose properties are determined uniquely by fixing electrical para-

*Fakultät für Mathematik und Wirtschaftswissenschaften, Universität Ulm, D-89081 Ulm, Germany. – Permanent address: Universität Karlsruhe (TH), Institut für Wissenschaftliches Rechnen und Mathmatische Modellbildung (IWRMM), Engesserstr.6, D-76128 Karlsruhe, Germany.

meters like capacitances or inductances. This yields a unique modeling approach, which allows for including parasitic and second order effects into the differential-algebraic (DAE) network approach. Examples are transistor models which approximate the physical behavior of semiconductor devices by companion models of different modeling levels, or transmission line models, which consist of RLC elements and controlled sources. Mathematically, this approach corresponds to a spatial discretization of the governing partial differential equations (PDEs) already at the modeling level. Another short-coming is the frequent use of arbitrary coupled sources that may destroy the structure of the network equations and thus lead to high-index systems [5, 9].

As an alternative, the co-simulation approach makes use of already existing simulation software for single parts of the system: different parts of the systems are modeled independently of each other and simulated by two simulation packages for electrical networks and electromagnetic fields; coupling is ensured by coupling the simulators. In addition to convergence problems, difficulties may arise, since coupled systems are often characterized by very different time constants.

A third approach is the use of generalized network models [6]. Refined models are allowed for interconnects and semiconductor devices, whose characteristic equations define PDE models. Hence numerical methods can be tailored exactly to the resulting mathematical models – the spatial discretization is not yet made at modeling level. Mathematically spoken, this approach leads to a coupled system of DAEs and PDEs, with the boundary conditions for the PDEs linked to the DAEs at the boundary nodes. Such systems are called partial differential-algebraic equations, shortly PDAE systems.

In this paper, we will concentrate on the last approach for linear RLC networks. This ansatz yields linear initial-boundary value problems of mixed partial-differential and differential-algebraic equations. The analysis of such PDAE systems and their numerical discretization is in the focus of actual research: one aim is to generalize the DAE index concept to PDAE systems to get some knowledge on structural properties before discretization [4, 7, 8, 10]: for example, the sensitivity of the solution to small perturbations in the initial data and/or input signals. On the other hand, estimates are required for the impact of semidiscretization on the index of the resulting approximate DAE (ADAE) system: does the ADAE system properly reflect the behavior of the original PDAE system? Or does one detect an artificial smoothing effect? Or even a coarsening one?

It is already known that the last but one question has to be answered in the affirmative. Semidiscretization may act like a regularization [1, 3]: the ADAE system is less sensitive w.r.t. input data than the PDAE model, and may yield physically incorrect solutions. In this contribution we will affirm the last question, too.

The paper is organized as follows. We first derive the generalized network approach for interconnected linear RLC networks and discuss the impact of topological index-1 conditions on the structure of the DAE part. In the analysis of the PDAE model in Section 3 we first prove uniqueness and derive a-priori estimates of the solution to establish well-posedness of the system. We see that the solution of a slightly perturbed system does not depend on derivatives of the perturbations – the PDAE has perturbation index 1. These properties should be reflected by the approximate DAE system obtained after semidiscretization. We will learn, however, in Section 4 that semidiscretization may act like a deregularization of an index-1 PDAE model, if an inappropriate type of semidiscretization is used.

2 GENERALIZED NETWORK APPROACH

In the following, we consider two electrical networks which are coupled by a system of d uniform lossy transmission lines. To derive a mathematical model, we use a generalized network approach: the electrical circuits are described by DAE models, whereas the transmission lines shown in Figure 1 are governed by a PDE model. Both models are linked via boundary node voltages and currents.

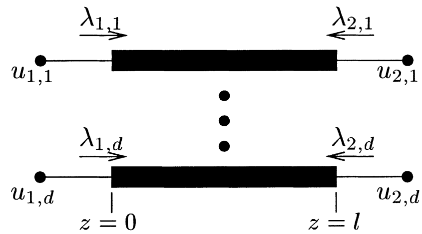

Fig. 1. PDE-network model for a system of d uniform lossy transmission lines.

2.1 PDE Network Model for Transmission Lines

Assuming quasi stationary behavior transverse to the wave propagation, the signal propagation in the transmission lines can be characterized by the telegrapher's equation

$$0 = \boldsymbol{V}_z(z,t) + \boldsymbol{L}\boldsymbol{J}_t(z,t) + \boldsymbol{R}\boldsymbol{J}(z,t), \tag{1a}$$

$$0 = \boldsymbol{J}_z(z,t) + \boldsymbol{C}\boldsymbol{V}_t(z,t) + \boldsymbol{G}\boldsymbol{V}(z,t), \tag{1b}$$

where $\boldsymbol{R}, \boldsymbol{L}, \boldsymbol{G}$ and $\boldsymbol{C} \in \mathbf{R}^{d\times d}$ are the positive-definite symmetric resistance, inductance, conductance and capacitance matrices per unit length. $\boldsymbol{V}(z,t)$ is a d-dimensional vector of line voltages with respect to ground, and $\boldsymbol{J}(z,t)$ is a d-dimensional vector of line currents. For $\boldsymbol{V}, \boldsymbol{J} \in \mathcal{H} := H^1(0,l)^d$ the telegrapher's equation (1) holds in the sense of distributions in $\mathcal{V} := L^2(0,l)^d$.

This first order hyperbolic system of partial differential equations is initialized by a set of initial values

$$\boldsymbol{V}(z,t_0) = \boldsymbol{V}^0(z), \tag{2a}$$

$$\boldsymbol{J}(z,t_0) = \boldsymbol{J}^0(z), \tag{2b}$$

for all $z \in I := [0,l]$ at initial time point t_0.

After introducing $2d$ virtual current sources $\boldsymbol{\lambda}_1 := (\lambda_{1,1}, \ldots, \lambda_{1,d})^\top$ and $\boldsymbol{\lambda}_2 := (\lambda_{2,1}, \ldots, \lambda_{2,d})^\top$ at the boundaries, the characteristic equation for the PDE model of a lossy transmission line system in admittance form reads

$$\boldsymbol{\lambda} = \begin{pmatrix} \boldsymbol{J}(0,t) \\ -\boldsymbol{J}(l,t) \end{pmatrix} \quad \text{with} \quad \boldsymbol{\lambda} = \begin{pmatrix} \boldsymbol{\lambda}_1 \\ \boldsymbol{\lambda}_2 \end{pmatrix}, \tag{3}$$

where the line currents $\boldsymbol{J}$ are defined by the telegrapher's equation (1). The PDE model is completed by the boundary conditions

$$\begin{pmatrix} \boldsymbol{V}(0,t) \\ \boldsymbol{V}(l,t) \end{pmatrix} = \begin{pmatrix} \boldsymbol{u}_1 \\ \boldsymbol{u}_2 \end{pmatrix} =: \boldsymbol{A}_\lambda^{\mathrm{T}} \boldsymbol{u}, \tag{4}$$

which couples the PDE network model with the node potentials $\boldsymbol{u}$ of the DAE model for both electrical networks via an incidence matrix $\boldsymbol{A}_\lambda$.

2.2 DAE Network Model for Linear Electrical Circuits

A network model is used to describe the electrical behavior of the circuit: network equations for node potentials are derived using Kirchhoff's laws and characteristic equations for the elements. This results in a system of differential-algebraic equations since only topology and no spatial dimension is considered. Using classical Modified Nodal Analysis (MNA), only node potentials $\boldsymbol{u}$, currents through inductive and resistive branches $\boldsymbol{\jmath}_L$ and $\boldsymbol{\jmath}_V$, and currents $\boldsymbol{\lambda}$ at the boundaries of interconnects are unknowns. The DAE network equations with $x := (\boldsymbol{u}, \boldsymbol{\jmath}_L, \boldsymbol{\jmath}_V)^\top$ read for a linear RLC network consisting of only linear capacitors, inductors and resistors, as well as independent voltage and current sources,

$$\begin{pmatrix} \boldsymbol{A}_C\widetilde{\boldsymbol{C}}\boldsymbol{A}_C^{\mathrm{T}} & 0 & 0 \\ 0 & \widetilde{\boldsymbol{L}} & 0 \\ 0 & 0 & 0 \end{pmatrix} \dot{\boldsymbol{x}} + \begin{pmatrix} \boldsymbol{A}_R\widetilde{\boldsymbol{G}}\boldsymbol{A}_R^\top & \boldsymbol{A}_L & \boldsymbol{A}_V \\ -\boldsymbol{A}_L^\top & 0 & 0 \\ -\boldsymbol{A}_V^\top & 0 & 0 \end{pmatrix} \boldsymbol{x} + \begin{pmatrix} \boldsymbol{A}_\lambda \\ 0 \\ 0 \end{pmatrix} \boldsymbol{\lambda} + \begin{pmatrix} \boldsymbol{A}_I\, \boldsymbol{\imath}(t) \\ 0 \\ \boldsymbol{v}(t) \end{pmatrix} = 0 \tag{5}$$

with consistent initial values

$$\boldsymbol{x}(t_0) = \boldsymbol{x}_0. \tag{6}$$

The element-related incidence matrices $\boldsymbol{A}_C$, $\boldsymbol{A}_L$, $\boldsymbol{A}_R$, $\boldsymbol{A}_V$, $\boldsymbol{A}_I$ and $\boldsymbol{A}_\lambda$ describe the branch-current relations for capacitive, inductive, resistive branches and branches for voltage sources, current sources and transmission line elements. The capacitance, inductance and conductance matrices $\widetilde{\boldsymbol{C}}$, $\widetilde{\boldsymbol{L}}$ and $\widetilde{\boldsymbol{G}}$ are assumed to be positive-definite and symmetric [9].

2.3 Topological Index-1 Conditions

With $\boldsymbol{Q}_c$ projecting onto $\boldsymbol{A}_C^\top$ and $\boldsymbol{P}_c := \boldsymbol{I} - \boldsymbol{Q}_c$, the network variables $\boldsymbol{x}$ can be split into the differential part $\boldsymbol{y} := (\boldsymbol{P}_c\, \boldsymbol{u}, \boldsymbol{\jmath}_L)^\top$ and the algebraic components $\boldsymbol{z} := (\boldsymbol{Q}_c\, \boldsymbol{u}, \boldsymbol{\jmath}_V)^\top$. Correspondingly, the network equations read after pure algebraic transformations

$$\begin{aligned}
0 &= \widetilde{\boldsymbol{H}}_1\dot{\boldsymbol{y}}_1 + \boldsymbol{P}_c^\top\Big(\boldsymbol{A}_R\widetilde{\boldsymbol{G}}\boldsymbol{A}_R^\top(\boldsymbol{y}_1 + \boldsymbol{z}_1) + \boldsymbol{A}_L\boldsymbol{y}_2 + \boldsymbol{A}_V\boldsymbol{z}_2 + \boldsymbol{A}_I\, \boldsymbol{\imath}(t) + \boldsymbol{A}_\lambda\boldsymbol{\lambda}\Big) \\
0 &= \widetilde{\boldsymbol{L}}\, \dot{\boldsymbol{\jmath}}_L - \boldsymbol{A}_L^\top(\boldsymbol{y}_1 + \boldsymbol{z}_1) \\
0 &= \begin{pmatrix} \boldsymbol{Q}_c^\top\boldsymbol{A}_R\widetilde{\boldsymbol{G}}\boldsymbol{A}_R^\top\boldsymbol{Q}_c & \boldsymbol{Q}_c^\top\boldsymbol{A}_V \\ \boldsymbol{A}_V^\top\boldsymbol{Q}_c & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{z}_1 \\ \boldsymbol{z}_2 \end{pmatrix} + \begin{pmatrix} \boldsymbol{Q}_c^\top\boldsymbol{A}_R\widetilde{\boldsymbol{G}}\boldsymbol{A}_R^\top & \boldsymbol{Q}_c^\top\boldsymbol{A}_L \\ \boldsymbol{A}_V^\top & 0 \end{pmatrix}\begin{pmatrix} \boldsymbol{y}_1 \\ \boldsymbol{y}_2 \end{pmatrix} \\
&\quad + \begin{pmatrix} \boldsymbol{Q}_c^\top(\boldsymbol{A}_I\, \boldsymbol{\imath}(t) + \boldsymbol{A}_\lambda\boldsymbol{\lambda}) \\ -\boldsymbol{v}(t) \end{pmatrix}
\end{aligned}$$

with $\widetilde{\boldsymbol{H}}_1 := \boldsymbol{A}_C\widetilde{\boldsymbol{C}}\boldsymbol{A}_C^\top + \boldsymbol{Q}_C^\top \boldsymbol{Q}_c$ positive-definite and symmetric. The topological index-1 conditions

$$\begin{array}{ll} \mathsf{T1} & \ker(\boldsymbol{A}_{\mathrm{C}}, \boldsymbol{A}_{\mathrm{R}}, \boldsymbol{A}_{\mathrm{V}})^\top = \{0\}, \\ \mathsf{T2} & \ker \boldsymbol{Q}_c^\top \boldsymbol{A}_V = \{0\} \end{array}$$

guarantee

$$\ker \begin{pmatrix} \boldsymbol{Q}_c^\top \boldsymbol{A}_R \widetilde{\boldsymbol{G}} \boldsymbol{A}_R^\top \boldsymbol{Q}_c & \boldsymbol{Q}_c^\top \boldsymbol{A}_V \\ \boldsymbol{A}_V^\top \boldsymbol{Q}_c & 0 \end{pmatrix} = \ker \boldsymbol{Q}_c \times \{0\}.$$

Thus $\boldsymbol{z}(t)$ is given as a linear function in $\boldsymbol{y}(t)$, $\boldsymbol{\imath}(t)$, $\boldsymbol{v}(t)$ and $\boldsymbol{\lambda}(t)$. If there is a capacitive path to ground for all coupling nodes, the condition

$$\mathsf{T3} \qquad \boldsymbol{Q}_c^\top \boldsymbol{A}_\lambda = 0 \qquad (\ker \boldsymbol{P}_c^\top \boldsymbol{A}_\lambda = \{0\})$$

holds, and $\boldsymbol{z}(t)$ is given as a linear function in $\boldsymbol{y}(t)$, $\boldsymbol{\imath}(t)$ and $\boldsymbol{u}(t)$ alone.

Assuming that T1–T3 holds, the equations (1–6) define a uniquely solvable mixed initial-boundary value problem of PDEs and DAEs, or partial differential-algebraic equations (PDAEs), of *perturbation index 1*. This will be shown in the next section.

3 ANALYTICAL PROPERTIES OF THE PDAE MODEL

To derive the analytical properties of the PDAE (1–6), we proceed in three steps: First, we prove uniqueness in an L^2-sense independent of the network topology by using energy estimates. Based on these technique, we are able to derive a-priori estimates for the solution depending only on the model data, i.e., initial values, input sources and matrices of the linear PDE and DAE models. If the topological index-1 conditions hold, these two ingredients establish well-posedness of the PDAE system. In a last step, we investigate the sensitivity of the model with respect to small perturbations.

3.1 Uniqueness

Let $\Delta\boldsymbol{V}, \Delta\boldsymbol{J}, \Delta\boldsymbol{u}, \Delta\boldsymbol{\jmath}_L, \Delta\boldsymbol{\jmath}_V$ be the difference of two (sufficiently smooth) solutions of the PDAE (1–6) for the same data, i.e., initial values $\boldsymbol{V}^0, \boldsymbol{J}^0, \boldsymbol{x}^0$, input sources $\boldsymbol{\imath}(t)$, $\boldsymbol{v}(t)$, and matrices $\boldsymbol{L}, \boldsymbol{C}, \boldsymbol{R}, \boldsymbol{G}$ of the linear PDE and $\widetilde{\boldsymbol{L}}, \widetilde{\boldsymbol{C}}, \widetilde{\boldsymbol{G}}$

of the DAE model. Considering the inner product of (1a) and (1b) with $\boldsymbol{\Delta J}$ and $\boldsymbol{\Delta V}$, resp., in $\mathcal{V}$, one obtains after integration by parts due to $\boldsymbol{C}, \boldsymbol{L}$ symmetrical and positive-definite

$$0 = \frac{1}{2}\frac{d}{dt}\Big\{\|\boldsymbol{C}^{1/2}\boldsymbol{\Delta V}(\cdot,t)\|_{\mathcal{V}}^2 + \|\boldsymbol{L}^{1/2}\boldsymbol{\Delta J}(\cdot,t)\|_{\mathcal{V}}^2\Big\}$$
$$+ \ \langle \boldsymbol{G\Delta V}(\cdot,t), \boldsymbol{\Delta V}(\cdot,t)\rangle + \langle \boldsymbol{R\Delta J}(\cdot,t), \boldsymbol{\Delta J}(\cdot,t)\rangle - \ (\boldsymbol{A}_\lambda^\top \boldsymbol{\Delta u})^\top \boldsymbol{\Delta\lambda}, \quad (7)$$

where $\langle\cdot,\cdot\rangle$ denotes the inner product in $\mathcal{V}$ and $\|\cdot\|_{\mathcal{V}}$ the associated norm. The network equations (5) yield for the last term

$$-(\boldsymbol{A}_\lambda^\top \boldsymbol{\Delta u})^\top \boldsymbol{\Delta\lambda} = \boldsymbol{\Delta u}^\top\Big(\boldsymbol{A}_C\widetilde{\boldsymbol{C}}\boldsymbol{A}_C^\top\boldsymbol{\Delta}\,\dot{\boldsymbol{u}} + \boldsymbol{A}_R\widetilde{\boldsymbol{G}}\boldsymbol{A}_R^\top\boldsymbol{\Delta u}\Big) + \boldsymbol{\Delta\jmath}_L^\top\widetilde{\boldsymbol{L}}\boldsymbol{\Delta j}_L\,. \quad (8)$$

As the matrices $\widetilde{\boldsymbol{C}}, \widetilde{\boldsymbol{L}}$ and $\widetilde{\boldsymbol{G}}$ are symmetrical and positive-definite, we get after inserting (8) into (7) and integration with respect to time due to $\boldsymbol{R}, \boldsymbol{G}$ positive-semidefinite the estimate

$$\|\boldsymbol{C}^{1/2}\boldsymbol{\Delta V}(\cdot,t_e)\|_{\mathcal{V}}^2 + \|\boldsymbol{L}^{1/2}\boldsymbol{\Delta J}(\cdot,t_e)\|_{\mathcal{V}}^2 + \|\widetilde{\boldsymbol{L}}^{1/2}\boldsymbol{\Delta\jmath}_L(t_e)\|_2^2$$
$$+ \|\widetilde{\boldsymbol{C}}^{1/2}\boldsymbol{A}_C^\top\boldsymbol{\Delta u}(t_e)\|_2^2 + \int_{t_0}^{t_e} \|\widetilde{\boldsymbol{G}}^{1/2}\boldsymbol{A}_R^\top\boldsymbol{\Delta u}(t)\|_2^2\,dt \le 0, \quad (9)$$

which is equivalent to

$$\boldsymbol{\Delta V} \equiv \boldsymbol{\Delta J} \equiv 0, \quad \boldsymbol{\Delta\jmath}_L \equiv 0, \quad \boldsymbol{A}_C^\top\boldsymbol{\Delta u} \equiv 0, \quad \boldsymbol{A}_R^\top\boldsymbol{\Delta u} \equiv 0.$$

in the L^2-sense. Since $\boldsymbol{A}_V^\top\boldsymbol{\Delta u} \equiv \boldsymbol{\Delta v}(t) \equiv 0$ and $\boldsymbol{A}_L^\top\boldsymbol{\Delta u} \equiv \widetilde{\boldsymbol{L}}\,\boldsymbol{\Delta}\,\boldsymbol{\jmath}_L \equiv 0$ hold, $\boldsymbol{\Delta u} \equiv 0$ follows from the fact that a physically reasonable network must not contain cut sets of current sources: $(\boldsymbol{A}_C, \boldsymbol{A}_R, \boldsymbol{A}_V, \boldsymbol{A}_L)^\top = \{0\}$. Finally, the identity $\boldsymbol{A}_V\boldsymbol{\Delta\jmath}_V \equiv 0$ in (5) together with the topological condition $\boldsymbol{A}_V = \{0\}$ (no loops of only voltage sources) yields $\boldsymbol{\Delta\jmath}_V = 0$.

Thus uniqueness is established independent of the network topology whether the topological conditions T1–T3 hold or not.

3.2 A-Priori Estimates

Analogous to (7–8) we have for a solution of the PDAE (1–6)

$$0 = \frac{1}{2}\frac{d}{dt}\Big\{\langle \boldsymbol{CV}(\cdot,t), \boldsymbol{V}(\cdot,t)\rangle + \langle \boldsymbol{LJ}(\cdot,t), \boldsymbol{J}(\cdot,t)\rangle + \boldsymbol{u}^\top\boldsymbol{A}_C\widetilde{\boldsymbol{C}}\boldsymbol{A}_C^\top\boldsymbol{u} + \boldsymbol{\jmath}_L^\top\widetilde{\boldsymbol{L}}\,\boldsymbol{\jmath}_L\Big\}$$
$$+ \langle \boldsymbol{GV}(\cdot,t), \boldsymbol{V}(\cdot,t)\rangle + \langle \boldsymbol{RJ}(\cdot,t), \boldsymbol{J}(\cdot,t)\rangle + \boldsymbol{u}^\top\boldsymbol{A}_R\widetilde{\boldsymbol{G}}\boldsymbol{A}_R^\top\boldsymbol{u}$$
$$+ \boldsymbol{u}^\top\boldsymbol{A}_I\boldsymbol{\imath}(t) + \boldsymbol{v}^\top(t)\boldsymbol{\jmath}_V.$$

The symmetry of $\widetilde{\boldsymbol{C}}$, $\widetilde{\boldsymbol{L}}$, $\widetilde{\boldsymbol{G}}$, $\boldsymbol{C}$ and $\boldsymbol{L}$ yields the energy estimate

$$\rho(t) \leq \rho(t_0) + c_1 \left(\int_{t_0}^{t} \left(\rho(\tau) + \| \boldsymbol{z}(\tau) \|_2^2 \right) d\tau + \| \boldsymbol{\imath} \|_{L^2(t_0,t)}^2 + \| \boldsymbol{v} \|_{L^2(t_0,t)}^2 \right) \tag{10}$$

in the differential components $\boldsymbol{V}$, $\boldsymbol{J}$ and $\boldsymbol{y}$ with

$$\rho(t) := \|\boldsymbol{V}(\cdot,t)\|_{\mathcal{V}}^2 + \|\boldsymbol{J}(\cdot,t)\|_{\mathcal{V}}^2 + \|\boldsymbol{y}(t)\|_2^2,$$
$$\|\boldsymbol{\imath}\|_{L^2(t_0,t)} := \sqrt{\int_{t_0}^{t} \|\boldsymbol{\imath}(\tau)\|_2^2 \, d\tau}, \quad \|\boldsymbol{v}\|_{L^2(t_0,t)} := \sqrt{\int_{t_0}^{t} \|\boldsymbol{v}(\tau)\|_2^2 \, d\tau}$$

and the constant

$$c_1 = 2 \max \left\{ \|\boldsymbol{C}^{-1}\|_2, \|\boldsymbol{L}^{-1}\|_2, \left\|\widetilde{\boldsymbol{H}}_1^{-1}\right\|_2, \left\|\widetilde{\boldsymbol{L}}^{-1}\right\|_2 \right\}$$
$$\cdot \max \left\{ \|\boldsymbol{G}\|_2, \|\boldsymbol{R}\|_2, \|\widetilde{\boldsymbol{G}}\boldsymbol{A}_R^\top\|_2, \frac{1}{2}\|\boldsymbol{A}_I\|_2, \frac{1}{2} \right\},$$

depending on the matrices of the linear models. Using the topological conditions T1–T3, the algebraic components $\boldsymbol{z}$ are given as a linear function in $\boldsymbol{y}$, $\boldsymbol{\imath}$ and $\boldsymbol{v}$. Thus we have with $\|\boldsymbol{z}(t)\|_2^2 \leq c_2(\|\boldsymbol{y}(t)\|_2^2 + \|\boldsymbol{\imath}(t)\|_2^2 + \|\boldsymbol{v}(t)\|_2^2)$ and $c_3 := c_1(1 + c_2)$

$$\rho(t) \leq \rho(t_0) + c_3 \left(\int_{t_0}^{t} \rho(\tau) \, d\tau + \|\boldsymbol{\imath}\|_{L^2(t_0,t)}^2 + \|\boldsymbol{v}\|_{L^2(t_0,t)}^2 \right).$$

By Gronwall's lemma this yields the a-priori estimate

$$\rho(t) \leq \exp(c_3 t) \cdot \left(\rho(t_0) + c_3 \left(\|\boldsymbol{\imath}\|_{L^2(t_0,t)}^2 + \|\boldsymbol{v}\|_{L^2(t_0,t)}^2 \right) \right), \tag{11}$$

and finally for the algebraic components

$$\|\boldsymbol{z}(t)\|_2^2 \leq c_2 \left(\rho(t) + \|\boldsymbol{\imath}(t)\|_2^2 + \|\boldsymbol{v}(t)\|_2^2 \right). \tag{12}$$

If the topological conditions T1–T3 hold, the PDAE initial-boundary value problem (1–6) is well posed. The solution depends continuously on the data, i.e., on the initial values, on the input signals and via the constants c_1, c_2 on the matrices of the linear models. In the DAE sense it is well conditioned, too, as only data enter the solution, but not its derivatives.

Based on the a-priori estimates, one can show the existence of a unique solution for the PDAE (1–6): Under the assumptions of T1–T3 the PDAE system (1–6) has a unique solution $(\boldsymbol{V}, \boldsymbol{J}, \boldsymbol{u}, \boldsymbol{\jmath}_L, \boldsymbol{\jmath}_V,)$, if

$$\boldsymbol{V}^0, \boldsymbol{J}^0 \in \mathcal{H}, \quad \boldsymbol{\imath} \in H^1(t_0, t_e)^{n_\imath}, \quad \boldsymbol{v} \in H^1(t_0, t_e)^{n_v}$$

holds for the initial values and input signals. Both $\boldsymbol{V}, \boldsymbol{J}$ and $\boldsymbol{V}_t, \boldsymbol{J}_t$ are bounded on the finite time interval $[t_0, t_e]$ with

$$\boldsymbol{V}, \boldsymbol{J} \in \mathcal{H}, \quad \boldsymbol{V}_t, \boldsymbol{J}_t \in \mathcal{V}.$$

For the network variables one gets

$$\boldsymbol{u} \in H^1(t_0, t_e)^{n_u}, \quad \boldsymbol{\jmath}_L \in H^1(t_0, t_e)^{n_{\jmath L}}, \quad \boldsymbol{\jmath}_V \in H^1(t_0, t_e)^{n_{\jmath V}}.$$

3.3 Perturbation Analysis

Let $(\boldsymbol{V}, \boldsymbol{J}, \boldsymbol{x})$ be the (reference) solution of the PDAE system (1–6) with consistent initial values $\boldsymbol{V}^0$, $\boldsymbol{J}^0$ and $\boldsymbol{x}_0$. To investigate the sensitivity of this solution, we apply perturbations $\boldsymbol{\delta}(z, t)$ and $\boldsymbol{\mu}(t)$ to the right-hand sides of (1) and (5). The corresponding perturbed solution is denoted by $(\widehat{\boldsymbol{V}}, \widehat{\boldsymbol{J}}, \widehat{\boldsymbol{x}})$ with consistent perturbed initial values $\widehat{\boldsymbol{V}}^0, \widehat{\boldsymbol{J}}^0$ and $\widehat{\boldsymbol{x}}_0$. The aim is to obtain estimates for the sensitivity of the solution, i.e., for the difference

$$(\boldsymbol{\Delta V}, \boldsymbol{\Delta J}, \boldsymbol{\Delta x}) := (\boldsymbol{V}, \boldsymbol{J}, \boldsymbol{x}) - (\widehat{\boldsymbol{V}}, \widehat{\boldsymbol{J}}, \widehat{\boldsymbol{x}})$$

between both solutions.

Due to the linearity of the systems, we get an a-priori energy estimate for

$$\widetilde{\rho}(t) := \|\boldsymbol{\Delta V}(\cdot, t)\|_{\mathcal{V}}^2 + \|\boldsymbol{\Delta J}(\cdot, t)\|_{\mathcal{V}}^2 + \|\boldsymbol{\Delta y}(t)\|_2^2,$$

and $\|\boldsymbol{\Delta} \widetilde{z}(t)\|_2^2$ of the type (11, 12):

$$\widetilde{\rho}(t) \leq \exp(\widetilde{c}_3 t) \cdot \Big(\widetilde{\rho}(t_0) + \widetilde{c}_3 \Big(\|\delta\|_{L^2([t_0, t], \mathcal{V})} + \|\mu\|_{L^2(t_0, t)}^2\Big)\Big), \tag{13a}$$

$$\|\boldsymbol{\Delta} \widetilde{z}(t)\|_2^2 \leq c_2(\widetilde{\rho}(t) + \|\delta(\cdot, t)\|_{\mathcal{V}}^2 + \|\mu(t)\|_2^2) \tag{13b}$$

with the norm

$$\|\delta\|_{L^2([t_0, t], \mathcal{V})} := \int_{t_0}^{t} \|\boldsymbol{\delta}(\cdot, \tau)\|_{\mathcal{V}}^2 \, d\tau$$

and the constants

$$\widetilde{c}_1 = \max\Big\{ \|\boldsymbol{C}^{-1}\|_2, \|\boldsymbol{L}^{-1}\|_2, \Big\|\widetilde{\boldsymbol{H}}_1^{-1}\Big\|_2, \Big\|\widetilde{\boldsymbol{L}}^{-1}\Big\|_2 \Big\}$$
$$\cdot \max\Big\{ 2\|\boldsymbol{G}\|_2 + 1, 2\|\boldsymbol{R}\|_2 + 1, 2\|\widetilde{\boldsymbol{G}}\boldsymbol{A}_R^\top\|_2 + 1, 1 \Big\}$$

and $\widetilde{c}_3 := \widetilde{c}_1(1 + c_2)$.

Thus the norm of the difference of reference and perturbed solution can be estimated by the norm of the perturbations itself, and does not depend on derivatives of the perturbations, neither with respect to space nor time. Hence the PDAE system (1–6) has perturbation index 1. For a precise definition of the perturbation index of linear PDAE systems of hyperbolic-type (1–6) we refer to [7, 10].

4 IMPACT OF SEMIDISCRETIZATION: PERTURBATION INDEX OF PDAE AND ADAE

Summing up the results of Section 3, the PDAE model (1–6) meeting the topological conditions T1–T3 is well-posed and has perturbation index 1. This analytical behavior should be reflected by any type of numerical discretization. As the numerical simulation of interconnected electrical networks has to be embedded into time-domain network analysis packages that are based on a DAE description of systems in time alone, the method-of-lines (MOL) approach is a natural candidate for numerical discretization of the PDAE model: first semidiscretization of the PDAE model with respect to space, and secondly numerical integration of the originating approximate system of differential-algebraic equations (ADAEs) in time. To reflect the analytical properties of the PDAE properly, the equality of PDAE and ADAE perturbation index defines an inalienable demand on the respective type of semidiscretization. However, as we will see in the following, this demand will not always be met. As an example, we will use linear finite elements that are based on two analytically equivalent mixed weak / strong formulation.

4.1 Mixed Weak / Strong Formulation

Under the regularity assumptions

$$\boldsymbol{V}, \boldsymbol{J} \in \mathcal{H}, \quad \boldsymbol{V}_t, \boldsymbol{J}_t \in \mathcal{V}, \quad \boldsymbol{x} \in H^1(t_0, t_e)^{n_x}$$

of Section 3.2 the strong formulation (1) is equivalent to the weak formulation

$$\left\langle \begin{pmatrix} \boldsymbol{L}\boldsymbol{J}_t(\cdot,t) + \boldsymbol{V}_z(\cdot,t) + \boldsymbol{R}\boldsymbol{J}(\cdot,t) \\ \boldsymbol{C}\boldsymbol{V}_t(\cdot,t) + \boldsymbol{J}_z(\cdot,t) + \boldsymbol{G}\boldsymbol{V}(\cdot,t) \end{pmatrix}, \boldsymbol{w} \right\rangle = 0 \quad \forall\, \boldsymbol{w} = \begin{pmatrix} \boldsymbol{w}_1 \\ \boldsymbol{w}_2 \end{pmatrix} \in \mathcal{H} \times \mathcal{H}\,.$$

By integrating the term

$$\left\langle \begin{pmatrix} \boldsymbol{V}_z(\cdot,t) \\ \boldsymbol{J}_z(\cdot,t) \end{pmatrix}, \boldsymbol{w} \right\rangle$$

twice by parts and formulating the boundary conditions (4) weakly for the node voltages, the PDAE system (1–6) can be transformed into the mixed weak / strong formulation

$$\begin{aligned} &\left\langle \begin{pmatrix} \boldsymbol{L}\boldsymbol{J}_t(\cdot,t) + \boldsymbol{V}_z(\cdot,t) + \boldsymbol{R}\boldsymbol{J}(\cdot,t) \\ \boldsymbol{C}\boldsymbol{V}_t(\cdot,t) + \boldsymbol{J}_z(\cdot,t) + \boldsymbol{G}\boldsymbol{V}(\cdot,t) \end{pmatrix}, \boldsymbol{w} \right\rangle \\ &\quad + \left\{ \boldsymbol{A}_\lambda^\top \boldsymbol{u} - \begin{pmatrix} \boldsymbol{V}(0,t) \\ \boldsymbol{V}(l,t) \end{pmatrix} \right\}^\top \begin{pmatrix} -\boldsymbol{w}_1(0) \\ \boldsymbol{w}_1(l) \end{pmatrix} = 0 \quad \forall \boldsymbol{w} \in \mathcal{H} \times \mathcal{H}, \end{aligned} \tag{14a}$$

$$\boldsymbol{\lambda} - \begin{pmatrix} \boldsymbol{J}(0,t) \\ -\boldsymbol{J}(l,t) \end{pmatrix} = 0 \tag{14b}$$

$$\begin{aligned} &\begin{pmatrix} \boldsymbol{A}_C \widetilde{\boldsymbol{C}} \boldsymbol{A}_C^\top & 0 & 0 \\ 0 & \widetilde{\boldsymbol{L}} & 0 \\ 0 & 0 & 0 \end{pmatrix} \dot{\boldsymbol{x}} + \begin{pmatrix} \boldsymbol{A}_R \widetilde{\boldsymbol{G}} \boldsymbol{A}_R^\top & \boldsymbol{A}_L & \boldsymbol{A}_V \\ -\boldsymbol{A}_L^\top & 0 & 0 \\ -\boldsymbol{A}_V^\top & 0 & 0 \end{pmatrix} \boldsymbol{x} \\ &\quad + \begin{pmatrix} \boldsymbol{A}_\lambda \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{J}(0,t) \\ -\boldsymbol{J}(l,t) \end{pmatrix} + \begin{pmatrix} \boldsymbol{A}_I\, \boldsymbol{\imath}(t) \\ 0 \\ \boldsymbol{v}\,(t) \end{pmatrix} = 0. \end{aligned} \tag{14c}$$

One notes that the boundary condition (3) for the *algebraic* component $\boldsymbol{\lambda}$ is formulated in a strong sense. Correspondingly, if the coupling condition (3) for the coupling currents is formulated weakly, we get the system

$$\begin{aligned} &\left\langle \begin{pmatrix} \boldsymbol{L}\boldsymbol{J}_t(\cdot,t) + \boldsymbol{V}_z(\cdot,t) + \boldsymbol{R}\boldsymbol{J}(\cdot,t) \\ \boldsymbol{C}\boldsymbol{V}_t(\cdot,t) + \boldsymbol{J}_z(\cdot,t) + \boldsymbol{G}\boldsymbol{V}(\cdot,t) \end{pmatrix}, \boldsymbol{w} \right\rangle \\ &\quad - \left\{ \boldsymbol{\lambda} - \begin{pmatrix} \boldsymbol{J}(0,t) \\ -\boldsymbol{J}(l,t) \end{pmatrix} \right\}^\top \begin{pmatrix} \boldsymbol{w}_2(0) \\ \boldsymbol{w}_2(l) \end{pmatrix} = 0 \quad \forall \boldsymbol{w} \in \mathcal{H} \times \mathcal{H}, \end{aligned} \tag{15a}$$

$$A_\lambda^\top \boldsymbol{u} - \begin{pmatrix} \boldsymbol{V}(0,t) \\ \boldsymbol{V}(l,t) \end{pmatrix} = 0 \tag{15b}$$

$$\begin{aligned} &\begin{pmatrix} A_C \widetilde{C} A_C^\top & 0 & 0 \\ 0 & \widetilde{L} & 0 \\ 0 & 0 & 0 \end{pmatrix} \dot{\boldsymbol{x}} + \begin{pmatrix} A_R \widetilde{G} A_R^\top & A_L & A_V \\ -A_L^\top & 0 & 0 \\ -A_V^\top & 0 & 0 \end{pmatrix} \boldsymbol{x} \\ &\qquad + \begin{pmatrix} A_\lambda \\ 0 \\ 0 \end{pmatrix} \boldsymbol{\lambda} + \begin{pmatrix} A_I\, \boldsymbol{\imath}(t) \\ 0 \\ v(t) \end{pmatrix} = 0. \end{aligned} \tag{15c}$$

Now the boundary condition (4) for the *differential* components $A_\lambda^\top \boldsymbol{u}$ is formulated in a strong sense.

These two analytically equivalent formulations may form the basis for a Ritz–Galerkin approach with linear finite elements.

4.2 Semidiscretization

If we apply semidiscretization with respect to space, we seek for an approximate solution $\boldsymbol{V}_m, \boldsymbol{J}_m, \boldsymbol{x}_m, \boldsymbol{\lambda}_m$ of (14) and (15) resp., with

$$\begin{pmatrix} \boldsymbol{J}_m(z,t) \\ \boldsymbol{V}_m(z,t) \end{pmatrix} = \sum_{i=1}^{m} \xi_i(t) \cdot \boldsymbol{w}^i(z)$$

and $\boldsymbol{w}^1, \ldots, \boldsymbol{w}^m$ elements of a finite subspace of $\mathcal{H} \times \mathcal{H}$. Using Galerkin's principle, this approach defines the approximated index-1 DAE (ADAE) system

$$0 = \boldsymbol{M}_1 \dot{\boldsymbol{\xi}} + (\boldsymbol{K}_1 + \boldsymbol{K}_2 - \boldsymbol{K}_3)\boldsymbol{\xi} + \boldsymbol{b}_1 A_\lambda^\top \boldsymbol{y}_{1,m} \tag{16a}$$

$$0 = \boldsymbol{\lambda} + \boldsymbol{b}_1^\top \boldsymbol{\xi} \tag{16b}$$

$$\begin{aligned} 0 = \widetilde{\boldsymbol{H}}_1 \dot{\boldsymbol{y}}_{1,m} + \boldsymbol{P}_c^\top \Big(& A_R \widetilde{G} A_R^\top (\boldsymbol{y}_{1,m} + \boldsymbol{z}_{1,m}) + A_L \boldsymbol{y}_{2,m} \\ & + A_V \boldsymbol{z}_{2,m} + A_I\, \boldsymbol{\imath}(t) \Big) - (\boldsymbol{b}_1 A_\lambda^\top)^\top \boldsymbol{\xi} \end{aligned} \tag{16c}$$

$$0 = \widetilde{\boldsymbol{L}} \dot{\boldsymbol{y}}_{2,m} - A_L^\top (\boldsymbol{y}_1 + \boldsymbol{z}_1) \tag{16d}$$

$$0 = \begin{pmatrix} \boldsymbol{Q}_C^\top \boldsymbol{A}_R \widetilde{\boldsymbol{G}} \boldsymbol{A}_R^\top \boldsymbol{Q}_C & \boldsymbol{Q}_C^\top \boldsymbol{A}_V \\ \boldsymbol{A}_V^\top \boldsymbol{Q}_C & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{z}_{1,m} \\ \boldsymbol{z}_{2,m} \end{pmatrix} + \begin{pmatrix} \boldsymbol{Q}_C^\top \boldsymbol{A}_R \widetilde{\boldsymbol{G}} \boldsymbol{A}_R^\top & \boldsymbol{Q}_C^\top \boldsymbol{A}_L \\ \boldsymbol{A}_V^\top & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{y}_{1,m} \\ \boldsymbol{y}_{2,m} \end{pmatrix} + \begin{pmatrix} \boldsymbol{A}_I \boldsymbol{\imath}(t) \\ -\boldsymbol{v}(t) \end{pmatrix} \tag{16e}$$

with $\boldsymbol{\xi} := (\xi_1, \ldots, \xi_m)^\top$ and the element matrices

$$\boldsymbol{M}_1 = \left\langle \begin{pmatrix} \boldsymbol{C} & 0 \\ 0 & \boldsymbol{L} \end{pmatrix} \boldsymbol{w}^i, \boldsymbol{w}^j \right\rangle_{i,j},$$

$$\boldsymbol{K}_1 = \left\langle \begin{pmatrix} 0 & \boldsymbol{I} \\ \boldsymbol{I} & 0 \end{pmatrix} \boldsymbol{w}_z^i, \boldsymbol{w}^j \right\rangle_{i,j},$$

$$\boldsymbol{K}_2 = \left\langle \begin{pmatrix} \boldsymbol{R} & 0 \\ 0 & \boldsymbol{G} \end{pmatrix} \boldsymbol{w}^i, \boldsymbol{w}^j \right\rangle_{i,j},$$

$$\boldsymbol{K}_3 = \left([\boldsymbol{w}_1^i(l)]^\top \boldsymbol{w}_2^j(l) \right)_{i,j} - \left([\boldsymbol{w}_1^i(0)]^\top \boldsymbol{w}_2^j(0) \right)_{i,j},$$

$$\boldsymbol{b}_1 = \begin{pmatrix} -\boldsymbol{w}_1^1(0) & \boldsymbol{w}_1^1(l) \\ \vdots & \vdots \\ -\boldsymbol{w}_1^m(0) & \boldsymbol{w}_1^m(l) \end{pmatrix}$$

for (14). Thus the index of the ADAE systems fits to the perturbation index for the original PDAE that has to be solved numerically.

For (15) we get the ADAE system

$$0 = \boldsymbol{M}_1 \dot{\boldsymbol{\xi}} + (\boldsymbol{K}_1 + \boldsymbol{K}_2 + \boldsymbol{K}_4)\boldsymbol{\xi} - \boldsymbol{b}_2 \boldsymbol{\lambda} \tag{17a}$$

$$0 = \boldsymbol{A}_\lambda^\top \boldsymbol{y}_{1,m} - \boldsymbol{b}_2^\top \boldsymbol{\xi} \tag{17b}$$

$$0 = \widetilde{\boldsymbol{H}}_1 \dot{\boldsymbol{y}}_{1,m} + \boldsymbol{P}_c^\top \Big(\boldsymbol{A}_R \widetilde{\boldsymbol{G}} \boldsymbol{A}_R^\top (\boldsymbol{y}_{1,m} + \boldsymbol{z}_{1,m}) + \boldsymbol{A}_L \boldsymbol{y}_{2,m} + \boldsymbol{A}_V \boldsymbol{z}_{2,m} + \boldsymbol{A}_I \boldsymbol{\imath}(t) \Big) + \boldsymbol{A}_\lambda \boldsymbol{\lambda}_m \tag{17c}$$

$$0 = \widetilde{\boldsymbol{L}} \dot{\boldsymbol{y}}_{2,m} - \boldsymbol{A}_L^\top (\boldsymbol{y}_1 + \boldsymbol{z}_1) \tag{17d}$$

$$0 = \begin{pmatrix} \boldsymbol{Q}_C^\top \boldsymbol{A}_{R\tilde{G}} \boldsymbol{A}_R^\top \boldsymbol{Q}_C & \boldsymbol{Q}_C^\top \boldsymbol{A}_V \\ \boldsymbol{A}_V^\top \boldsymbol{Q}_C & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{z}_{1,m} \\ \boldsymbol{y}_{2,m} \end{pmatrix} + \begin{pmatrix} \boldsymbol{Q}_C^\top \boldsymbol{A}_R \widetilde{\boldsymbol{G}} \boldsymbol{A}_R^\top & \boldsymbol{Q}_C^\top \boldsymbol{A}_L \\ \boldsymbol{A}_V^\top & 0 \end{pmatrix} \begin{pmatrix} \boldsymbol{y}_{1,m} \\ \boldsymbol{y}_{2,m} \end{pmatrix} + \begin{pmatrix} \boldsymbol{A}_I \boldsymbol{\imath}(t) \\ -\boldsymbol{v}(t) \end{pmatrix} \tag{17e}$$

with the element matrices

$$\boldsymbol{K}_4 = \Big([\boldsymbol{w}_2^i(l)]^\top \boldsymbol{w}_1^j(l)\Big)_{i,j} + \Big([\boldsymbol{w}_2^i(0)]^\top \boldsymbol{w}_1^j(0)\Big)_{i,j},$$

$$\boldsymbol{b}_2 = \begin{pmatrix} \boldsymbol{w}_2^1(0) & \boldsymbol{w}_2^1(l) \\ \vdots & \vdots \\ \boldsymbol{w}_2^m(0) & \boldsymbol{w}_2^m(l) \end{pmatrix}.$$

One notes that the index is larger than 1: the algebraic equation (19) does only depend on the differential components $\boldsymbol{y}_{1,m}$ and ξ. After differentiation of (17b), the equations (17a–17c) can be solved for $\dot{\xi}, \dot{\mathbf{y}}_1$ and λ as a linear function in $\xi, \boldsymbol{x}$: the matrix

$$\left(\begin{array}{cc|c} \boldsymbol{M}_1 & 0 & -\boldsymbol{b}_2 \\ 0 & \widetilde{\boldsymbol{H}}_1 & \boldsymbol{A}_\lambda \\ \hline -\boldsymbol{b}_2^\top & \boldsymbol{A}_\lambda^\top & 0 \end{array}\right)$$

is regular, since $\boldsymbol{M_1}$ and $\widetilde{\boldsymbol{H}}_1$ are symmetric positive-definite and $\boldsymbol{A}_\lambda$ has full column rank. Thus the index is two, independent of the particular Galerkin ansatz.

How can one explain these different results for system (14) and (15)? In both cases, semidiscretization yields coupled systems of two DAEs [2]. The structure of the coupling is given by the way the boundary conditions are formulated in a weak or strong sense. In (14) the algebraic component is defined in a strong sense. Thus we get two index-1 DAE systems

$$\left.\begin{aligned} \dot{y}_i(t) &= f_i(y, z_i), \\ 0 &= h_i(y, z_i) \end{aligned}\right\} \quad (i = 1, 2) \tag{18}$$

with $y_1 := \xi$, $z_1 := \lambda_m$, and $y_2 := (\boldsymbol{y}_{1,m}, \boldsymbol{y}_{2,m})^\top$, $z_2 := (\boldsymbol{z}_{1,m}, \boldsymbol{z}_{2,m})^\top$, which are coupled only via the right-hand side – thus the whole system (14) has index 1.

In (15), however, the differential components $\boldsymbol{A}_\lambda^\top \boldsymbol{u}$ are defined in a strong manner. Correspondingly, we get

$$\left.\begin{aligned} \dot{y}_i(t) &= f_i(y, z_i, u), \\ 0 &= h_i(y, z_i) \end{aligned}\right\} \quad (i = 1, 2) \tag{19a}$$

$$0 = g(y, z) \tag{19b}$$

with $y_1 := \xi$, $z_1 := \{\}, h_1 := \{\}, y_2 := (\boldsymbol{y}_{1,m}, \boldsymbol{y}_{2,m})^\top$, $z_2 := (\boldsymbol{z}_{1,m}, \boldsymbol{z}_{2,m})^\top$ and $u := \lambda_m$, i.e., one ODE and one index-1 DAE coupled via right-hand sides and

n_u algebraic equaitons. One obtains index 2 for the whole system (19) due to the algebraic coupling, since $\partial g(y,z)/\partial z = 0$.

5 CONCLUSION

Using generalized network models for interconnects, the DAE network equations for linear RLC circuits are generalized to a linear PDAE model. If the network fulfils the topological index-1 conditions, the system is well-posed and has perturbation index 1. Using semidiscretization with respect to space, the method-of-lines approach converts this PDAE model into an approximate DAE system in time only. To reflect the physical properties of the original PDAE model, the ADAE system should neither be more nor less sensitive. Although the formulation of PDAE boundary conditions in a weak or strong sense does not affect the analytical solution, it may have an impact on the index of the ADAE systems, if one applies a semidiscretization scheme such as linear finite elements that does not reflect the hyperbolic type of the PDE system.

REFERENCES

1. Arnold, M.: *A Note on the Uniform Perturbation index*. Rostock. Math. Kolloq. 52 (1998), pp. 33–46.
2. Arnold, M. and Günther, M.: Preconditioned Dynamic Iteration for Coupled Differential-Algebraic System. *BIT* 41(1) (2001), pp. 1–25.
3. Campbell, S.L. and Marszalek, W.: *ODE/DAE Integrators and MOL Problems*. *Z.f. Angew. Math*. 76 (Suppl. 1) (1996), pp. 251–254.
4. Campbell, S.L. and Marszalek, W.: The Index of an Infinite Dimensional Implicit System. *Math. Comput. Modeling Dyn. Syst*. 5 (1999), pp. 18–42.
5. Günther, M. and Feldmann, U.: CAD Based Electric Circuit Modeling in Industry. I: Mathematical Structure and Index of Network Equations. II: Impact of Circuit Configurations and Parameters. *Surv. Math. Ind.* 8 (1999), pp. 97–157.
6. Günther, M. and Rentrop, P.: PDAE-Netzwerkmodelle in der Elektrischen Schaltungssimulation. In: W. John (Ed.): *Analog '99: 5.ITG/GMM-Diskussionssitzung*. 'Entwicklung von Analogschaltungen mit CAE-Methoden mit dem Schwerpunkt Entwurfsmethodik und Parasitäre Effekte.' Frankfurt, 2001, pp. 31–38.
7. Günther, M. and Wagner, Y.: Index Concepts for Linear Mixed Systems of Differential-Algebraic and Hyperbolic-Type Equations. *SIAM J. Sci. Comp.* 22 (5) (2000), pp. 1610–1629.
8. Lucht, W., Strehmel, K. and Eichler-Liebenow, C.: Indexes and Special Discretization Methods for Linear Partial Differential Algebraic Equations. *BIT* 39 (1999), pp. 484–512.
9. Tischendorf, C.: Topological Index Calculation of Differential-Algebraic Equations in Circuit Simulation. *Surv. Math. Ind.* 8 (1999), pp. 187–199.
10. Wagner, Y.: A further Index Concept for PDAEs of Hyperbolic Type. *Mathematics and Computers in Simulation* 53 (2000), pp. 287–291.