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A Riemannian Framework for $\mathcal{H}_2$-Optimal Model Reduction of Port-Hamiltonian Systems

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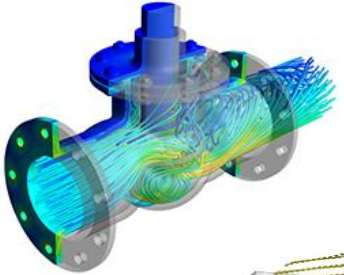
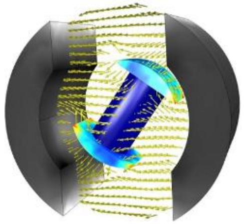
Modeling and Reduction of Multi-Physics Systems

- Interplay of phenomena from different domains
- Energy serves as the *lingua franca*

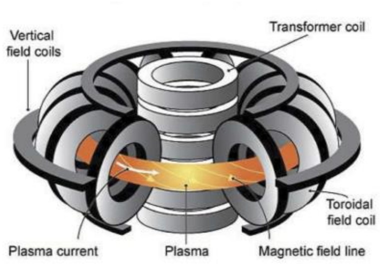
→ Energy-based modeling



[ANSYS '19]



[CAMNET '19]



[Vu '16]

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Port-Hamiltonian Systems (PHS)

We consider linear, single-input/single-output PHS in input-state-output representation

$$\dot{\mathbf{x}} = (\mathbf{J} - \mathbf{R})\nabla_{\mathbf{x}}\mathcal{H}(\mathbf{x}) + \mathbf{b}u$$
$$y = \mathbf{b}^T\nabla_{\mathbf{x}}\mathcal{H}(\mathbf{x})$$

with Hamiltonian $\mathcal{H}(\mathbf{x})$, $\mathbf{J}, \mathbf{R} \in \mathbb{R}^{n \times n}$ and $\mathbf{b}, \mathbf{x} \in \mathbb{R}^n$.

Structural properties:

- *Interconnection* matrix $\mathbf{J} = -\mathbf{J}^T$
- *Dissipation* matrix $\mathbf{R} = \mathbf{R}^T \geq 0$

Assumptions:

- Hamiltonian $\mathcal{H}(\mathbf{x}) = \frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x}$
- *Energy* matrix $\mathbf{Q} = \mathbf{Q}^T > 0$

$\rightarrow$ Inherent Passivity: $\dot{\mathcal{H}}(\mathbf{x}) = \frac{d}{dt}\frac{1}{2}\mathbf{x}^T(t)\mathbf{Q}\mathbf{x}(t) \leq u(t)y(t)$

$\rightarrow$ Interconnectivity and Controller Design

$\rightarrow$ Structure-preserving reduction

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$\mathcal{H}_2$-Optimal Model Reduction of PHS

Consider a port-Hamiltonian system

$$\dot{\mathbf{x}} = (\mathbf{J} - \mathbf{R})\mathbf{Q}\mathbf{x} + \mathbf{b}u$$
$$y = \mathbf{b}^T\mathbf{Q}\mathbf{x}$$

with transfer function $G(s) = \sum_{j=1}^{n} \frac{\phi_j}{s-\lambda_j}$.

(1)

The reduced model is port-Hamiltonian

$$\dot{\hat{\mathbf{x}}} = (\hat{\mathbf{J}} - \hat{\mathbf{R}})\hat{\mathbf{Q}}\hat{\mathbf{x}} + \hat{\mathbf{b}}u$$
$$\hat{y} = \hat{\mathbf{b}}^T\hat{\mathbf{Q}}\hat{\mathbf{x}}$$

with $\mathbf{x} \in \mathbb{R}^r (r \ll n)$ and $\hat{G}(s) = \sum_{k=1}^{r} \frac{\hat{\phi}_k}{s-\hat{\lambda}_k}$.

(2)

The reduced model is a (local) minimizer of

$$\min_{\dim(\hat{G})=r} \quad \mathcal{F} := \|G - \hat{G}\|_{\mathcal{H}_2}^2,$$

where

$$\|G\|_{\mathcal{H}_2} = \left(\frac{1}{2\pi}\int_{-\infty}^{\infty} |G(i\omega)|^2 d\omega\right)^{\frac{1}{2}}.$$

$\mathcal{H}_2$-Optimal Model Reduction of LTI systems

- *Interpolatory* framework
  - Cost function [Beattie '09]

$$\mathcal{F} = \|G\|_{\mathcal{H}_2}^2 - 2\sum_{j=1}^{r} \hat{\phi}_j G(-\hat{\lambda}_j) + \sum_{k,l=1}^{r} \frac{\hat{\phi}_k \hat{\phi}_l}{-\hat{\lambda}_k - \hat{\lambda}_l}$$

  - First-order optimality conditions [Meier '67]

$$\frac{\partial \mathcal{F}}{\partial \hat{\phi}_k} = 2(\hat{G}(-\hat{\lambda}_k) - G(-\hat{\lambda}_k)) \qquad = 0$$

$$\frac{\partial \mathcal{F}}{\partial \hat{\lambda}_k} = 2\hat{\phi}_k(G'(-\hat{\lambda}_k) - \hat{G}'(-\hat{\lambda}_k)) = 0$$

$$\text{for all } k = 1, \dots, r$$

- *Lyapunov* framework: Computation of cost function and gradient by solving (coupled) Lyapunov equations [Wilson '70], [Van Dooren '08]

$\mathcal{H}_2$-Inspired Tangential Interpolation of PHS [Gugercin '12]

- Structure-preserving Petrov-Galerkin projection

$$\begin{aligned} \dot{\mathbf{x}} &= (\mathbf{J} - \mathbf{R})\mathbf{Q}\mathbf{x} + \mathbf{b}u \\ y &= \mathbf{b}^T\mathbf{Q}\mathbf{x} \end{aligned} \xrightarrow[\mathbf{W} = \mathbf{Q}\mathbf{V}(\mathbf{V}^T\mathbf{Q}\mathbf{V})^{-1}]{\mathbf{V} \in \mathbb{R}^{n \times r}}$$

$$\begin{aligned} \dot{\hat{\mathbf{x}}} &= (\hat{\mathbf{J}} - \hat{\mathbf{R}})\hat{\mathbf{Q}}\hat{\mathbf{x}} + \hat{\mathbf{b}}u \\ \hat{y} &= \hat{\mathbf{b}}^T\hat{\mathbf{Q}}\hat{\mathbf{x}} \end{aligned}$$

$$\text{with } \hat{\mathbf{J}} = \mathbf{W}^T\mathbf{J}\mathbf{W}, \hat{\mathbf{R}} = \mathbf{W}^T\mathbf{R}\mathbf{W}$$
$$\hat{\mathbf{Q}} = \mathbf{V}^T\mathbf{Q}\mathbf{V}, \hat{\mathbf{b}} = \mathbf{W}^T\mathbf{b}$$

$\mathcal{H}_2$-Inspired Tangential Interpolation of PHS [Gugercin '12]

- Structure-preserving Petrov-Galerkin projection
- Iterative Rational Krylov Algorithm for PHS (IRKA-PH)

Algorithm IRKA-PH

Input: System $\mathbf{J}, \mathbf{R}, \mathbf{Q}, \mathbf{b}$, initial values $\{s_1, ..., s_r\}$, tolerance ε

Output: Reduced pH system $\hat{\mathbf{J}}, \hat{\mathbf{R}}, \hat{\mathbf{Q}}, \hat{\mathbf{b}}$

1: **repeat:**
2: $\quad \text{colsp}\{\mathbf{V}\} = \text{span}\left\{(s_1\mathbf{I} - (\mathbf{J}-\mathbf{R})\mathbf{Q})^{-1}\mathbf{b}, ..., (s_r\mathbf{I} - (\mathbf{J}-\mathbf{R})\mathbf{Q})^{-1}\mathbf{b}\right\}$
3: $\quad \mathbf{W} = \mathbf{Q}\mathbf{V}(\mathbf{V}^{\mathbf{T}}\mathbf{Q}\mathbf{V})^{-1}$
4: $\quad \hat{\mathbf{J}} = \mathbf{W}^T\mathbf{J}\mathbf{W}, \hat{\mathbf{R}} = \mathbf{W}^T\mathbf{R}\mathbf{W}, \hat{\mathbf{Q}} = \mathbf{V}^T\mathbf{Q}\mathbf{V}, \hat{\mathbf{b}} = \mathbf{W}^T\mathbf{b}, \hat{\mathbf{A}} = (\hat{\mathbf{J}} - \hat{\mathbf{R}})\hat{\mathbf{Q}}$
5: $\quad s_k \leftarrow -\hat{\lambda}_k(\hat{\mathbf{A}})$
6: **until:** $|s_k + \hat{\lambda}_k(\hat{\mathbf{A}})| < \varepsilon, \quad \forall k = 1, \dots, r$

$\mathcal{H}_2$-Optimality

(1) $\hat{G}(-\hat{\lambda}_k) = G(-\hat{\lambda}_k)$ ✓

(2) $\hat{G}'(-\hat{\lambda}_k) = G'(-\hat{\lambda}_k)$ ✗

for all $k = 1, \dots, r$

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$\mathcal{H}_2$-Optimal Model Reduction of PHS [Sato '18]

- Reduce number of optimization parameters

$$\begin{aligned} \dot{\tilde{\mathbf{x}}} &= (\tilde{\mathbf{J}} - \tilde{\mathbf{R}})\tilde{\mathbf{Q}}\tilde{\mathbf{x}} + \tilde{\mathbf{b}}u \\ y &= \tilde{\mathbf{b}}^T\tilde{\mathbf{Q}}\tilde{\mathbf{x}} \qquad \tilde{\mathbf{x}} \in \mathbb{R}^r \end{aligned} \quad \xrightarrow[\hat{\mathbf{x}} = \mathbf{L}^T\tilde{\mathbf{x}}]{\tilde{\mathbf{Q}} = \mathbf{L}\mathbf{L}^T} \quad \begin{aligned} \dot{\hat{\mathbf{x}}} &= (\hat{\mathbf{J}} - \hat{\mathbf{R}})\hat{\mathbf{x}} + \hat{\mathbf{b}}u \\ y &= \hat{\mathbf{b}}^T\hat{\mathbf{x}} \qquad \hat{\mathbf{x}} \in \mathbb{R}^r \end{aligned}$$

- New optimization problem

$$\boxed{\begin{aligned} &\min_{(\hat{\mathbf{J}},\hat{\mathbf{R}},\hat{\mathbf{b}})\in\mathcal{M}} \quad \mathcal{F} := \|G - \hat{G}\|_{\mathcal{H}_2}^2 \\ &\mathcal{M} := \mathrm{Skew}_r \times \mathrm{Sym}_r^+ \times \mathbb{R}^r \end{aligned}}$$

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Optimization on Matrix Manifolds [Absil '07]

- In Euclidean space

$$\boldsymbol{\chi}_{k+1} = \boldsymbol{\chi}_k + \alpha \boldsymbol{\eta}$$

- Riemannian metric

$$\langle \cdot, \cdot \rangle_{\boldsymbol{\chi}} : T_{\boldsymbol{\chi}}\mathcal{M} \times T_{\boldsymbol{\chi}}\mathcal{M} \to \mathbb{R} \qquad \|\boldsymbol{\xi}\|_{\boldsymbol{\chi}} = \sqrt{\langle \boldsymbol{\xi}, \boldsymbol{\xi} \rangle_{\boldsymbol{\chi}}}$$

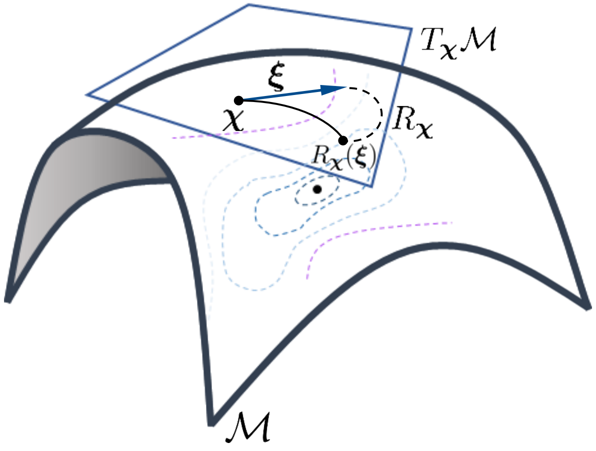
- Retraction and pullback of $\mathcal{F}$

$$R_{\boldsymbol{\chi}} : T_{\boldsymbol{\chi}}\mathcal{M} \to \mathcal{M} \qquad \hat{\mathcal{F}} := \mathcal{F} \circ R_{\boldsymbol{\chi}} \, : \, T_{\boldsymbol{\chi}}\mathcal{M} \to \mathbb{R}$$

- Second-order approximation of $\hat{\mathcal{F}}$ on $T_{\boldsymbol{\chi}}\mathcal{M}$

$$m_{\boldsymbol{\chi}}(\boldsymbol{\xi}) = \mathcal{F}(\boldsymbol{\chi}) + \langle \mathrm{grad}\mathcal{F}(\boldsymbol{\chi}), (\boldsymbol{\xi}) \rangle_{\boldsymbol{\chi}} + \frac{1}{2} \langle \mathrm{Hess}\mathcal{F}(\boldsymbol{\chi})[\boldsymbol{\xi}], (\boldsymbol{\xi}) \rangle_{\boldsymbol{\chi}}$$

$$\min_{\boldsymbol{\chi} \in \mathcal{M}} \mathcal{F}(\boldsymbol{\chi})$$



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Optimization on Manifold $\mathcal{M}$

$$\min_{(\hat{\mathbf{J}},\hat{\mathbf{R}},\hat{\mathbf{b}})\in\mathcal{M}} \quad \mathcal{F} := \|G - \hat{G}\|^2_{\mathcal{H}_2}$$

$$\mathcal{M} := \mathrm{Skew}_r \times \mathrm{Sym}_r^+ \times \mathbb{R}^r$$

- Riemannian metric [Lang '99]

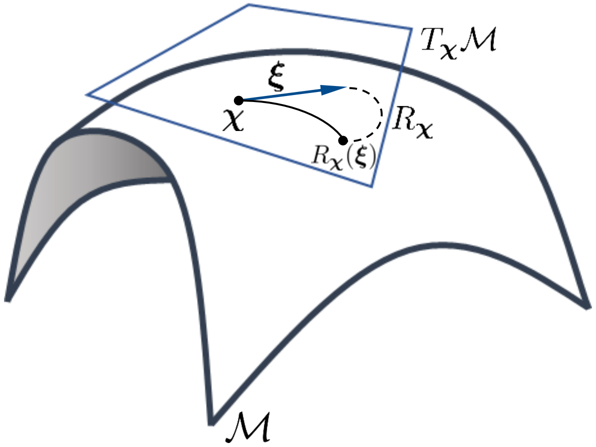
$$\langle(\boldsymbol{\xi}_1,\boldsymbol{\eta}_1,\zeta_1),(\boldsymbol{\xi}_2,\boldsymbol{\eta}_2,\zeta_2)\rangle_{(\hat{\mathbf{J}},\hat{\mathbf{R}},\hat{\mathbf{b}})} := \mathrm{tr}(\boldsymbol{\xi}_1^T\boldsymbol{\xi}_2) + \mathrm{tr}(\hat{\mathbf{R}}^{-1}\boldsymbol{\eta}_1\hat{\mathbf{R}}^{-1}\boldsymbol{\eta}_2) + \mathrm{tr}(\zeta_1^T\zeta_2)$$

- Retraction [Jeuris '12]

$$\mathrm{R}_{(\hat{\mathbf{J}},\hat{\mathbf{R}},\hat{\mathbf{b}})}(\boldsymbol{\xi},\boldsymbol{\eta},\zeta) := (\hat{\mathbf{J}}+\boldsymbol{\xi}, \hat{\mathbf{R}}+\boldsymbol{\eta}+\frac{1}{2}\boldsymbol{\eta}\hat{\mathbf{R}}^{-1}\boldsymbol{\eta}, \hat{\mathbf{b}}+\zeta)$$

- Second-order approximation of $\hat{\mathcal{F}}$ on $T_{\chi}\mathcal{M}$

$$m_{\chi}(\boldsymbol{\xi},\boldsymbol{\eta},\zeta) = \mathcal{F}(\hat{\mathbf{J}},\hat{\mathbf{R}},\hat{\mathbf{b}}) + \langle \mathrm{grad}\mathcal{F}(\hat{\mathbf{J}},\hat{\mathbf{R}},\hat{\mathbf{b}}), (\boldsymbol{\xi},\boldsymbol{\eta},\zeta)\rangle_{\chi} + \frac{1}{2}\langle \mathrm{Hess}\mathcal{F}(\hat{\mathbf{J}},\hat{\mathbf{R}},\hat{\mathbf{b}})[\boldsymbol{\xi},\boldsymbol{\eta},\zeta], (\boldsymbol{\xi},\boldsymbol{\eta},\zeta)\rangle_{\chi}$$



Derivation of the Euclidean Gradient

Optimization parameters $\mathbf{q} = \mathrm{vec}\left(\left[\hat{\mathbf{J}}, \hat{\mathbf{R}}, \hat{\mathbf{b}}\right]\right) \in \mathbb{R}^{r(2r+1)}$

$$\frac{\partial \mathcal{F}}{\partial \hat{\phi}_k} = 2(\hat{G}(-\hat{\lambda}_k) - \boxed{G(-\hat{\lambda}_k)})$$

e.g. $$\frac{\partial \hat{\phi}_j}{\partial \hat{J}_{k,l}} = \mathbf{e}_j^T \mathbf{Z}^{-1} \hat{\mathbf{b}} \hat{\mathbf{b}}^T \frac{\partial \mathbf{Z}}{\partial \hat{J}_{k,l}} \mathbf{e}_j - \mathbf{e}_j^T \mathbf{Z}^{-1} \frac{\partial \mathbf{Z}}{\partial \hat{J}_{k,l}} \mathbf{Z}^{-1} \hat{\mathbf{b}} \hat{\mathbf{b}}^T \mathbf{Z} \mathbf{e}_j$$

$$\nabla \mathcal{F}_{\mathbf{q}}^T = \left[\mathrm{D}_{\hat{\boldsymbol{\phi}}} \mathcal{F}, \mathrm{D}_{\hat{\boldsymbol{\lambda}}} \mathcal{F}\right] \begin{bmatrix} \mathrm{D}\hat{\boldsymbol{\phi}}(\mathbf{q}) \\ \mathrm{D}\hat{\boldsymbol{\lambda}}(\mathbf{q}) \end{bmatrix}$$

$$\frac{\partial \mathcal{F}}{\partial \hat{\lambda}_k} = 2\hat{\phi}_k (G'(-\hat{\lambda}_k) - \hat{G}'(-\hat{\lambda}_k))$$

$$(\hat{\mathbf{J}} - \hat{\mathbf{R}})\mathbf{z}_j = \hat{\lambda}_j \mathbf{z}_j$$
$$(\hat{\mathbf{J}} - \hat{\mathbf{R}})^* \mathbf{w}_j = \overline{\hat{\lambda}}_j \mathbf{w}_j$$

e.g. $$\frac{\partial \hat{\lambda}_j}{\partial \hat{J}_{k,l}} = \frac{\mathbf{w}_j^* \mathbf{E}_{k,l} \mathbf{z}_j}{\mathbf{w}_j^* \mathbf{z}_j}$$

→ Major computational cost: r evaluations of $G(s)$

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Riemannian Gradient and Hessian

- Riemannian gradient

$$\nabla \mathcal{F}_{(\hat{\mathbf{J}},\hat{\mathbf{R}},\hat{\mathbf{b}})} = \left[\frac{\partial \mathcal{F}}{\delta \hat{\mathbf{J}}}, \frac{\partial \mathcal{F}}{\partial \hat{\mathbf{R}}}, \frac{\partial \mathcal{F}}{\partial \hat{\mathbf{b}}}\right] \in \mathbb{R}^{r\times(2r+1)} \xrightarrow{\langle\cdot,\cdot\rangle_{\mathcal{X}}} \mathrm{grad}\mathcal{F}(\hat{\mathbf{J}},\hat{\mathbf{R}},\hat{\mathbf{b}}) = \left[\mathrm{sk}\left(\frac{\partial \mathcal{F}}{\partial \hat{\mathbf{J}}}\right), \hat{\mathbf{R}}\mathrm{sym}\left(\frac{\partial \mathcal{F}}{\partial \hat{\mathbf{R}}}\right)\hat{\mathbf{R}}, \frac{\partial \mathcal{F}}{\partial \hat{\mathbf{b}}}\right]$$

- FD approximation of Riemannian Hessian [Boumal ‘15]

$\rightarrow$ Requires another evaluation of $\mathrm{grad}\mathcal{F}$

Riemannian Trust-Region Method

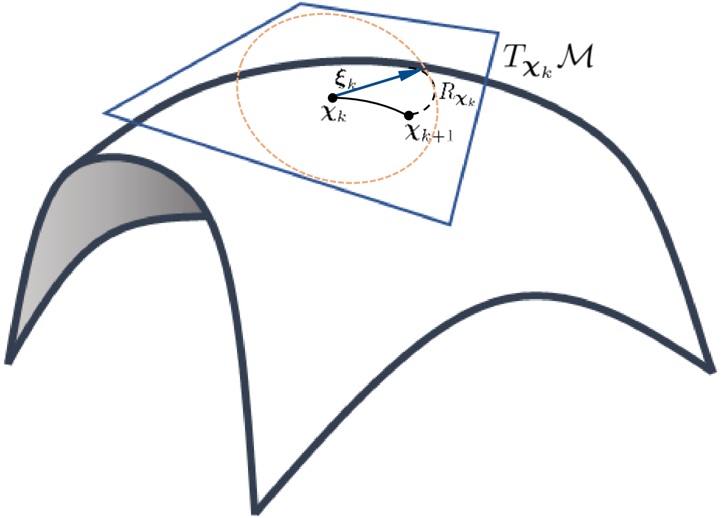
Algorithm $\mathcal{H}_2$-Optimal RTR Method for PHS

Input: Initial iterate $(\hat{\mathbf{J}}_0, \hat{\mathbf{R}}_0, \hat{\mathbf{b}}_0) \in \mathcal{M}$
parameters $r \in [2, n), \bar{\Delta} > 0, \Delta_0 \in (0, \bar{\Delta}), \rho' \in [0, \frac{1}{4})$
Output: $\mathcal{H}_2$-optimal model $(\hat{\mathbf{J}}, \hat{\mathbf{R}}, \hat{\mathbf{b}})$

for $k = 0, 1, 2, \ldots$ **do**
  Solve trust-region subproblem
    $\min_{(\boldsymbol{\xi}, \boldsymbol{\eta}, \boldsymbol{\zeta}) \in T_{\boldsymbol{\chi}_k}\mathcal{M}} m_{\boldsymbol{\chi}_k}(\boldsymbol{\xi}, \boldsymbol{\eta}, \boldsymbol{\zeta})$
    subject to $\|(\boldsymbol{\xi}, \boldsymbol{\eta}, \boldsymbol{\zeta})\|_{\boldsymbol{\chi}_k} \leq \Delta_k$
  Evaluate ρ_k
  if $\rho_k < \frac{1}{4}$ **then**
    $\Delta_{k+1} = \frac{1}{4}\Delta_k$;
  else if $\rho_k > \frac{3}{4}$ **and** $\|(\boldsymbol{\xi}_k, \boldsymbol{\eta}_k, \boldsymbol{\zeta}_k)\|_{\boldsymbol{\chi}_k} = \Delta_k$ **then**
    $\Delta_{k+1} = \min(2\Delta_k, \bar{\Delta})$;
  else
    $\Delta_{k+1} = \Delta_k$;
  end if
  if $\rho_k > \rho'$ **then**
    $(\hat{\mathbf{J}}_{k+1}, \hat{\mathbf{R}}_{k+1}, \hat{\mathbf{b}}_{k+1}) = \mathrm{R}_{\boldsymbol{\chi}_k}(\boldsymbol{\xi}_k, \boldsymbol{\eta}_k, \boldsymbol{\zeta}_k)$
  else
    $(\hat{\mathbf{J}}_{k+1}, \hat{\mathbf{R}}_{k+1}, \hat{\mathbf{b}}_{k+1}) = (\hat{\mathbf{J}}_k, \hat{\mathbf{R}}_k, \hat{\mathbf{b}}_k)$
  end if
end for

In which region can we trust our model?

$$\rightarrow \rho_k = \frac{\mathcal{F}(\hat{\mathbf{J}}_k, \hat{\mathbf{R}}_k, \hat{\mathbf{b}}_k) - \mathcal{F}(\mathrm{R}_{\boldsymbol{\chi}_k}(\boldsymbol{\xi}_k, \boldsymbol{\eta}_k, \boldsymbol{\zeta}_k))}{m_{\boldsymbol{\chi}_k}(\mathbf{0}, \mathbf{0}, \mathbf{0}) - m_{\boldsymbol{\chi}_k}(\boldsymbol{\xi}_k, \boldsymbol{\eta}_k, \boldsymbol{\zeta}_k)}$$



Riemannian Trust-Region Method

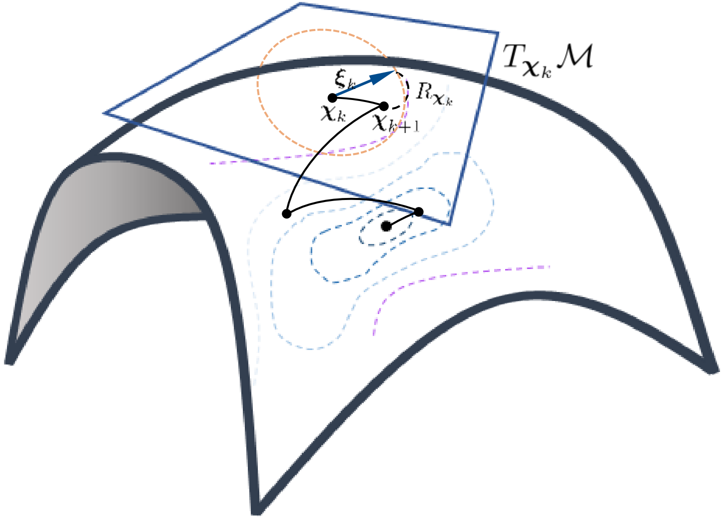
Algorithm $\mathcal{H}_2$-Optimal RTR Method for PHS

Input: Initial iterate $(\hat{\mathbf{J}}_0, \hat{\mathbf{R}}_0, \hat{\mathbf{b}}_0) \in \mathcal{M}$
parameters $r \in [2, n), \bar{\Delta} > 0, \Delta_0 \in (0, \bar{\Delta}), \rho' \in [0, \frac{1}{4})$
Output: $\mathcal{H}_2$-optimal model $(\hat{\mathbf{J}}, \hat{\mathbf{R}}, \hat{\mathbf{b}})$

```
for k = 0, 1, 2, ... do
    Solve trust-region subproblem
        min_{(ξ,η,ζ) ∈ T_{χ_k} M}  m_{χ_k}(ξ, η, ζ)
        subject to ‖(ξ, η, ζ)‖_{χ_k} ≤ Δ_k
    Evaluate ρ_k
    if ρ_k < 1/4 then
        Δ_{k+1} = 1/4 Δ_k;
    else if ρ_k > 3/4 and ‖(ξ_k, η_k, ζ_k)‖_{χ_k} = Δ_k then
        Δ_{k+1} = min(2Δ_k, Δ̄);
    else
        Δ_{k+1} = Δ_k;
    end if
    if ρ_k > ρ' then
        (Ĵ_{k+1}, R̂_{k+1}, b̂_{k+1}) = R_{χ_k}(ξ_k, η_k, ζ_k)
    else
        (Ĵ_{k+1}, R̂_{k+1}, b̂_{k+1}) = (Ĵ_k, R̂_k, b̂_k)
    end if
end for
```

In which region can we trust our model?

$$\rightarrow \rho_k = \frac{\mathcal{F}(\hat{\mathbf{J}}_k, \hat{\mathbf{R}}_k, \hat{\mathbf{b}}_k) - \mathcal{F}(\mathrm{R}_{\chi_k}(\boldsymbol{\xi}_k, \boldsymbol{\eta}_k, \boldsymbol{\zeta}_k))}{m_{\chi_k}(\mathbf{0}, \mathbf{0}, \mathbf{0}) - m_{\chi_k}(\boldsymbol{\xi}_k, \boldsymbol{\eta}_k, \boldsymbol{\zeta}_k)}$$



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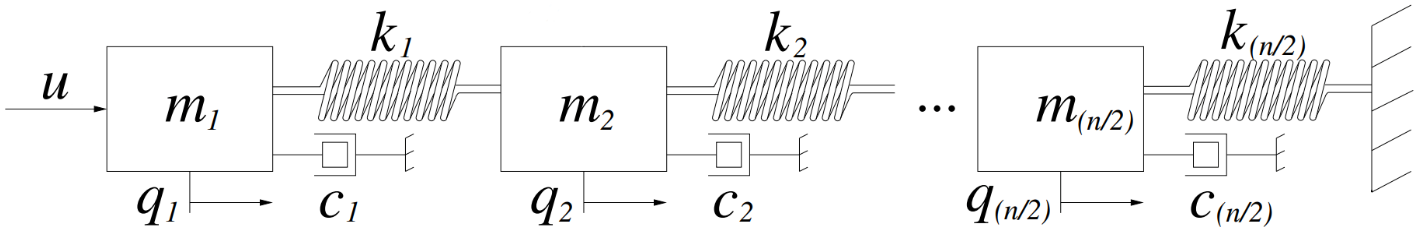
Numerical Example: Mass-Spring-Damper Model



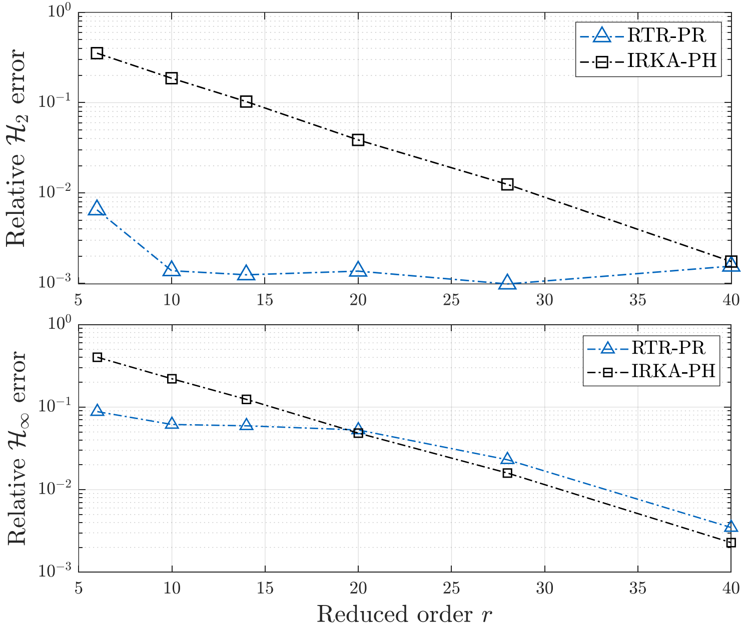
with $m_i = 4, \quad k_i = 4, \quad c_i = 1$ [Gugercin '12]

e.g. $n = 4$:

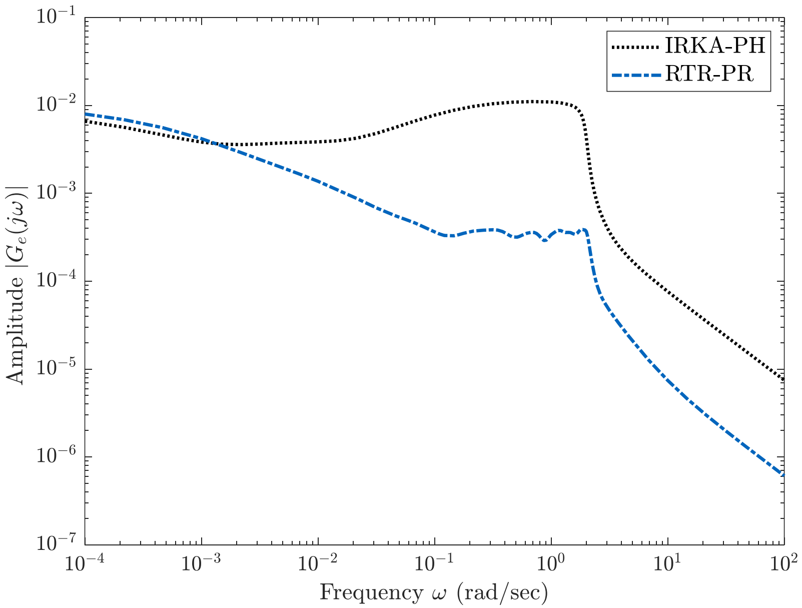
$$\mathbf{J} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & c_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & c_2 \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} k_1 & 0 & -k_1 & 0 \\ 0 & \frac{1}{m_1} & 0 & 0 \\ -k_1 & 0 & k_1 + k_2 & 0 \\ 0 & 0 & 0 & \frac{1}{m_2} \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

Numerical Example: Mass-Spring-Damper Model

$(n = 500)$



$(r = 20, n = 10000)$



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Conclusion and Future Work

- Derivation of the Riemannian gradient in the *interpolation* framework
- Can be used to solve the problem of structure-preserving MOR of PHS
- An RTR method illustrates the potential compared to projective methods

Future Work

- Proof of global convergence
- Impact of different initialization strategies, solvers etc.
- Application to real-world systems

Thank you for your attention!

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Thank you for your attention!

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