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MR3606589 93B07 05E10 20C40 37C80 93B05 93C05

Dellnitz, Michael (D-PDRB); **Klus, Stefan** (D-FUB-C)

Sensing and control in symmetric networks. (English. English summary)

Dyn. Syst. **32** (2017), *no.* 1, 61–79.

In this paper the authors focus on the relationship between controllability (and observability) of complex networked systems and symmetries characterizing the underlying network structure. The analysis is carried out by exploiting results from group representation theory and, more specifically, the concepts of irreducible representations and isotypic decomposition.

In this framework, network symmetries can be expressed as higher-dimensional irreducible representations of the matrix describing the network structure. It is well known that this matrix's structural property leads to the existence of repeated eigenvalues, which generally decrease the controllability of the system; the authors' goal is to study how these equivariance properties affect the controllability and observability of linear systems.

First of all, it is proved that if the pair (A, B) is controllable and A is Γ-equivariant, where Γ is a finite group of orthogonal matrices, then the rank of the input-to-state matrix B cannot be smaller than the maximal dimension of the irreducible representation of Γ having nontrivial projection onto the corresponding isotypic component. Secondly, the problem of computing a sparse input matrix B is addressed. By exploiting representation of A with respect to the symmetry-adapted basis and projections onto the isotypic components, the authors develop an algorithm that allows them to compute the matrix B with the minimum number of columns and the minimum number of nonzero entries.

Due to duality of controllability and observability properties, results presented in the paper can also be used to study observability of a given pair (A, C) and compute the sparse output matrix C. *Irene Zorzan*

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Pang, Shaopeng (PRC-BUAA-7R); **Hao, Fei** (PRC-BUAA-STA)

Optimizing controllability of edge dynamics in complex networks by perturbing network structure. (English. English summary)

Phys. A **470** (2017), 217–227.

The tools developed in complex network theory and engineering control theory suggest new methods to help us understand the complex system better since Y.-Y. Liu, J.-J. Slotine and A.-L. Barabási published "Controllability of complex networks" in [Nature **473** (2011), no. 7346, 167–173, doi:10.1038/nature10011]. For a large-scale complex network, how can we find the least number of control nodes to control the whole network to achieve some desired state? Is it better to control the node or to control the edge of complex network? This is one of the hot topics in the research of complex network controllability. In this paper, the authors investigate a complex network system whose dynamics is defined on its edges and optimize its controllability by minimum structural perturbations using the proposed adding-edge strategy and turning-edge strategies. This approach is very different from the conventional method of controlling nodes in complex networks, and provides a new idea for the controllability of networks.

The authors first select the switchboard dynamics as a dynamical process on the edge of a directed network. Then they propose the adding-edge strategy and turning-edge strategy to optimize network controllability by minimum structural perturbations and design a detailed algorithm for them, respectively. The adding-edge strategy is implemented by adding the minimum number of additional links to the directed network. The turning-edge strategy is also used to optimize network controllability by turning a minimum number of edges in the directed network. An interesting result in this paper is that the minimum number of turning edges required for the optimal controllability of edge dynamics in complex networks is equal to the minimum number of adding edges. Numerical and theoretical results show the effectiveness of the conclusion.

To the best of my knowledge, there is not much research on the dynamical process defined on the edges of complex networks. This seems to be a very fruitful approach to reveal the intrinsic nature of the structure and controllability of a complex network via dynamic properties on the edges. *Lixiang Li*

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MR3588197 93B05 05C82

Li, Xiang (PRC-FUDAN-ET); **Yao, Peng** (PRC-FUDAN-ET); **Pan, Yujian** (PRC-FUDAN-ET)

★**Towards structural controllability of temporal complex networks. (English. English summary)**

Complex systems and networks, 341–371, *Underst. Complex Syst.*, *Springer*, *Heidelberg*, 2016.

Summary: "Temporal complex networks are ubiquitous in human daily life whose topologies evolve with time, such as communication networks and transportation networks. Investigations on the structural controllability of temporal complex networks show the properties and performances of controllability when the weights of edges in temporal networks are arbitrary values rather than exact ones. There are two frameworks proposed in this chapter to analyze the structural controllability of temporal networks. In the first framework, a temporal network is treated as a sequence of characteristic subgraphs with different characteristic time stamps. After finding the maximum characteristic subgraph set from these subgraphs, priority maximum methods are applied to improve the controlling efficiency by which temporal information of the network remains. On the other hand, in the later framework, a temporal network is represented by time-ordered graph (TOG). Instead of calculating the rank of controllability matrix directly, finding and classifying temporal trees of the time-ordered graph provides an effective way to estimate the controlling centrality of a node in the network."

Alexey N. Zhirabok

MR3579692 93B05 60J05 90C40 93E03

Diallo, Tidiane (F-PEMV-LAM); **Goreac, Dan** (F-PEMV-LAM)

Controllability metrics on networks with linear decision process-type interactions and multiplicative noise. (English. English summary)

SIAM J. Control Optim. **54** (2016), *no.* 6, 3126–3151.

This article by Diallo and Goreac is a valuable contribution to stochastic optimal control at its interface with both (i) economic and cultural sciences and (ii) computational biology, bioinformatics and medicine. This work is rigorous, and based on it, more future research may be raised and real-life applications made, e.g., portfolio optimization in finance under regime switches and investments in culture and education in paradigm shifting environments.

In fact, the authors aim at a deep understanding of complex networks and possible controllability properties and induced controllability metrics on them, governed by a class of time-discrete linear decision processes equipped with multiplicative noise. These underlying dynamics are given by a pair consisting of a Markovian trend and a linear decision process for which both the "deterministic" and the noise components are based on trend-dependent matrices. The authors discuss approximate and exact (null-)controllability.

A number of examples are provided to illustrate the links between those concepts, and to compare the concepts with their time-continuous counterparts presented by Goreac and M. Martínez [Math. Control Signals Systems **27** (2015), no. 4, 551–578; MR3412378]. These examples especially show that qualitative properties of models in gene networks strongly depend on the time framework. The authors introduce a class of backward stochastic Riccati difference schemes (BSRDSs) and study the solvability of these BSRDSs for special frameworks. These BSRDSs allow for the introduction of Gramian-like controllability metrics. The authors suggest a minimal intervention-targeted reduction in the investigation of gene networks as a possible application of these metrics.

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This excellent work is well embedded into the research landscape, well structured, deep, exemplified and illustrated numerically, and well written.

The four sections of this rich work are: 1. Introduction, 2. The main concepts and results, 3. A minimal intervention-targeted application in biological networks, 4. Proofs of Theorems 2 and 8, solvability Propositions 10 and 12, and necessary condition (Proposition 17).

Further refinements and generalizations in theory and numerical methods, results and codes could be expected in the research community, motivated by this work and its continuous-time predecessor [D. Goreac and M. Martínez, op. cit.]. Those might be made in terms of stochastic optimal control with jumps, game theory, etc. Such future progress could foster achievements in science and engineering, finance and economics, bio-, environmental and earth sciences, chemical engineering, neuroscience and medicine, social and developmental sciences, video reconstruction and processing.

Gerhard-Wilhelm Weber

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Babazadeh, Maryam (IR-SHAR-EE); **Nobakhti, Amin** (IR-SHAR-EE)

Robust controllability assessment via semi-definite programming. (English. English summary)

Systems Control Lett. **98** (2016), 1–7.

In this paper the authors study the controllability of uncertain linear time-invariant (LTI) network systems. Two types of networks are considered: continuous-time LTI network systems described by the equations

$$\begin{aligned}\dot{x}(t) &= Ax(t) + Bu(t),\\ y(t) &= Cx(t)\end{aligned}$$

and discrete-time LTI network systems described by the equations

$$\begin{aligned}x(t+1) &= Ax(t) + Bu(t),\\ y(t) &= Cx(t).\end{aligned}$$

Uncertainties, present in (A, B), cause the evaluation of known controllability conditions to require sweeping a certain parameter over the entire complex plane, which is numerically intractable. To avoid this, a new necessary and sufficient condition is derived which permits effective evaluation of the controllability of both directed and undirected network systems. These conditions are reformulated as two (resp. four) Lyapunov matrix-based linear matrix inequalities (LMIs) for undirected (resp. directed) networks. To solve these matrix inequalities, the authors use a reformulation as a generalized eigenvalue problem (which is a quasi-convex optimization problem). The method is implemented using LMI tools and can be used for the assessment of robust controllability. Using duality, similar results can be established for assessing robust observability of network systems. *Radek Cibulka*

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Wang, Wen-Xu (PRC-BJN-SSY); **Lai, Ying-Cheng** (1-AZS-ECE);
Grebogi, Celso (4-ABERK-CXB)

Data based identification and prediction of nonlinear and complex dynamical systems. (English. English summary)

Phys. Rep. **644** (2016), 1–76.

Summary: "The problem of reconstructing nonlinear and complex dynamical systems from measured data or time series is central to many scientific disciplines including physical, biological, computer, and social sciences, as well as engineering and economics. The classic approach to phase-space reconstruction through the methodology of delay-coordinate embedding has been practiced for more than three decades, but the paradigm is effective mostly for low-dimensional dynamical systems. Often, the methodology yields only a topological correspondence of the original system. There are situations in various fields of science and engineering where the systems of interest are complex and high dimensional with many interacting components. A complex system typically exhibits a rich variety of collective dynamics, and it is of great interest to be able to detect, classify, understand, predict, and control the dynamics using data that are becoming increasingly accessible due to the advances of modern information technology. To accomplish these goals, especially prediction and control, an accurate reconstruction of the original system is required.

"Nonlinear and complex systems identification aims at inferring, from data, the mathematical equations that govern the dynamical evolution and the complex interaction patterns, or topology, among the various components of the system. With successful reconstruction of the system equations and the connecting topology, it may be possible to address challenging and significant problems such as identification of causal relations among the interacting components and detection of hidden nodes. The 'inverse' problem thus presents a grand challenge, requiring new paradigms beyond the traditional delay-coordinate embedding methodology.

"The past fifteen years have witnessed rapid development of contemporary complex graph theory with broad applications in interdisciplinary science and engineering. The combination of graph, information, and nonlinear dynamical systems theories with tools from statistical physics, optimization, engineering control, applied mathematics, and scientific computing enables the development of a number of paradigms to address the problem of nonlinear and complex systems reconstruction. In this Review, we describe the recent advances in this forefront and rapidly evolving field, with a focus on compressive sensing based methods. In particular, compressive sensing is a paradigm developed in recent years in applied mathematics, electrical engineering, and nonlinear physics to reconstruct sparse signals using only limited data. It has broad applications ranging from image compression/reconstruction to the analysis of large-scale sensor networks, and it has become a powerful technique to obtain high-fidelity signals for applications where sufficient observations are not available. We will describe in detail how compressive sensing can be exploited to address a diverse array of problems in data based reconstruction of nonlinear and complex networked systems. The problems include identification of chaotic systems and prediction of catastrophic bifurcations, forecasting future attractors of time-varying nonlinear systems, reconstruction of complex networks with oscillatory and evolutionary game dynamics, detection of hidden nodes, identification of chaotic elements in neuronal networks, reconstruction of complex geospatial networks and nodal positioning, and reconstruction of complex spreading networks with binary data. A number of alternative methods, such as those based on system response to external driving, synchronization, and noise-induced dynamical correlation, will also be discussed. Due to the high relevance of network reconstruction to

biological sciences, a special section is devoted to a brief survey of the current methods to infer biological networks. Finally, a number of open problems including control and controllability of complex nonlinear dynamical networks are discussed.

"The methods outlined in this Review are principled on various concepts in complexity science and engineering such as phase transitions, bifurcations, stabilities, and robustness. The methodologies have the potential to significantly improve our ability to understand a variety of complex dynamical systems ranging from gene regulatory systems to social networks toward the ultimate goal of controlling such systems."

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MR3531218 93B05 93A15 93C55

Liu, Bo (PRC-NCHUT-CSC); **Han, Yue** (PRC-NCHUT-CSC);
Jiang, Fangcui (PRC-SHDWH-SMS); **Su, Housheng** (PRC-HUST-SAU);
Zou, Jietao (PRC-NCHUT-CSC)

Group controllability of discrete-time multi-agent systems. (English. English summary)

J. Franklin Inst. **353** (2016), *no.* 14, 3524–3559.

The interest in studying complex systems arising from ecology, biology, sociology, engineering, etc., existed since the last decades of the 20th century, but the stage of the evolution of personal computers at that time couldn't sufficiently back up the research efforts. The tremendous progress in computer science and communications and the huge amount of processing power that came in handy since the beginning of the 21st century have made possible new approaches in the study of such systems and also the emergence of some new ones, like distributed networks, which brought a more practical flavor to the field.

In recent years the study of multi-agent systems has crystallized as a new branch in this area as the classical problems (controllability, observability) became more challenging due to the many factors involved: the dynamic evolution of the subsystems/agents, communication topology and protocols, feedback gains, external interference, dimension of the system, etc.

This paper is motivated by and follows up previous work performed by the authors and some other researchers [Z. Ji, H. Lin and H. Yu, Systems Control Lett. **61** (2012), no. 9, 918–925; MR2958715; J. Yu and L. Wang, Systems Control Lett. **59** (2010), no. 6, 340–348; MR2674834]. It addresses the controllability of multi-agent systems (containing multiple subgroups) extended at the group level. The communication protocol is different from the leader-follower one proposed in [B. Liu et al., IEEE Trans. Automat. Control **53** (2008), no. 4, 1009–1013; MR2419448] and is based upon a *consensus protocol*, in which communication delays occur in a discrete-time framework. Various communication topologies are considered.

More precisely, a multi-agent system $(\mathcal{G}, x)$ is considered, where the graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ with its set of vertices $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$, set of edges $\mathcal{E}$ and weight matrix $A = (a_{ij})$ defines the topology of the network, and the states $x_1, x_2, \dots, x_N$ corresponding to the nodes follow a *group-consensus communication protocol*

$$x_i(k+1) = u_i(k).$$

The group of agents is divided, without loss of generality, into two subnetworks $(\mathcal{G}_1, x^1)$ and $(\mathcal{G}_2, x^2)$, $x^1 = (x_1, \dots, x_n)^\mathsf{T}$, $x^2 = (x_{n+1}, \dots, x_{n+m})^\mathsf{T}$, and the protocol is described as

$$u_i(k) =$$
$$\begin{cases} x_i(k) + \sum\limits_{j \in N_{1i}} a_{ij}[x_j(k-h) + x_i(k-h)] + \sum\limits_{j \in N_{2i}} a_{ij} x_j(k), & i = 1, \dots, n, \\ x_i(k) + \sum\limits_{j \in N_{2i}} a_{ij}[x_j(k-h) + x_i(k-h)] + \sum\limits_{j \in N_{1i}} a_{ij} x_j(k), & i = n+1, \dots, n+m, \end{cases}$$

where N_{1i}, N_{2i} are the separate neighbor sets of agent i and h is the integer time delay.

In order to be able to study the controllability of the above system via the classical theory, the new states $X^1(k) = (x^1(k)^\mathsf{T}, x^1(k-1)^\mathsf{T}, \dots, x^1(k-h)^\mathsf{T})^\mathsf{T}$ and $X^2(k) = (x^2(k)^\mathsf{T}, x^2(k-1)^\mathsf{T}, \dots, x^2(k-h)^\mathsf{T})^\mathsf{T}$ are considered and an augmented system without time delays is obtained:

$$\begin{cases} X^1(k+1) = \mathcal{L}_{11} X^1(k) + \mathcal{L}_{12} X^2(k), \\ X^2(k+1) = \mathcal{L}_{21} X^1(k) + \mathcal{L}_{22} X^2(k); \end{cases}$$

this system's controllability is equivalent to the controllability of the original one.

Two main general cases are considered in the sequel from a topological point of view:

(a) the switching topology, where some group-controllability criteria that involve cyclic-invariant subspaces related to the matrices $\mathcal{L}_{ij}$ are obtained;

(b) fixed topology, where some Popov-Belevitch-Hautus criteria are proven (e.g $\operatorname{rank}(sI - \mathcal{L}_{11}, \mathcal{L}_{12}) = (h+1)n$ and $\operatorname{rank}(sI - \mathcal{L}_{22}, \mathcal{L}_{21}) = (h+1)m$, $\forall s, t \in \mathbf{C}$).

Then some particular cases (featuring $\mathcal{L}_{21}^\mathsf{T} = \mathcal{L}_{12}$, the symmetry: $\mathcal{L}_{ii}^\mathsf{T} = \mathcal{L}_{ii}$, $i = 1, 2$, as well balanced couple, star graph, isolated agent) are used to illustrate the power of the theoretical general results.

The final section brings more boots-on-the-ground experience by including some examples and simulations that on the one hand provide more insight on the multi-

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agent systems and on the other hand illustrate the effectiveness of the approach and the importance of the theoretical results. *Tiberiu Vasilache*

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★**Complex systems and networks.**
Dynamics, controls and applications.
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MR3508713 93B05 93C55 94C30

Lu, Jianquan (PRC-SEU); **Zhong, Jie** (PRC-SEU); **Huang, Chi** (PRC-TYUT-CM); **Cao, Jinde** (PRC-SEU)

On pinning controllability of Boolean control networks. (English. English summary)

IEEE Trans. Automat. Control **61** (2016), *no.* 6, 1658–1663.

In this paper, the authors study the pinning controllability of Boolean control networks (BCNs). They present some necessary and sufficient conditions for the reachability and controllability of BCNs with pinning controllers. The main tool used in the paper is an algebraic state space representation approach.

It is known that controllability is a basic concept for dynamical systems. Once a criterion is obtained for the controllability, it becomes a powerful tool for solving various control design issues. Pinning control comes from the field of complex networks, which aims to control as few nodes as possible to reach some desirable objectives. Hence, the concept of pinning controllability proposed in the paper is very important for the control design of gene regulatory networks.

It is believed that the criterion of pinning controllability provides a novel way to stabilize and track BCNs. *Haitao Li*

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Chen, Yu-Zhong (1-AZS-ECE); **Wang, Le-Zhi** (1-AZS-ECE);
Wang, Wen-Xu (PRC-BJN-SYM); **Lai, Ying-Cheng** (1-AZS-ECE)

Energy scaling and reduction in controlling complex networks. (English. English summary)

R. Soc. Open Sci. **3** (2016), *April*, 160064, 10 *pp.*

Summary: "Recent works revealed that the energy required to control a complex network depends on the number of driving signals and the energy distribution follows an algebraic scaling law. If one implements control using a small number of drivers, e.g. as determined by the structural controllability theory, there is a high probability that the energy will diverge. We develop a physical theory to explain the scaling behaviour through identification of the fundamental structural elements, the longest control chains (LCCs), that dominate the control energy. Based on the LCCs, we articulate a strategy to drastically reduce the control energy (e.g. in a large number of real-world networks). Owing to their *structural* nature, the LCCs may shed light on energy issues associated with control of nonlinear dynamical networks."

MR3481732 93C85 70Q05 93A14

Ma, Mihua (PRC-SGH-AMM); **Zhou, Jin** (PRC-SGH-AMM);
Cai, Jianping (PRC-MNNU-SMS)

Pinning synchronization in networked Lagrangian systems. (English. English summary)

Asian J. Control **18** (2016), *no.* 2, 569–580.

Summary: "This paper investigates the synchronization problem of networked Lagrangian systems based on pinning control framework on complex networks. We propose a pinning algorithm to guarantee the controlled synchronization of networked identical Lagrangian systems by applying local linear feedback injections to a small fraction of nodes. We also present some simple yet generic criteria on pinning synchronization for such an algorithm over undirected connected graphs, where all the agents are regulated to follow a synchronization state. Furthermore, the pinning controllability in networked Lagrangian systems is also discussed. Compared with some existing works on networked Lagrangian systems, the distinctive advantages of the proposed pinning algorithm include: (i) independence on the knowledge of system models; (ii) explicit consideration of agent's intrinsic complex dynamics; and (iii) simplicity of implement procedure in practice. Subsequently, the results are illustrated by a typical Lagrangian network composing of eight two-link revolute manipulators. Numerical simulations with different kinds of pinning schemes are finally given to demonstrate the effectiveness of the proposed control methodology."

MR3477965 93B05 90C27 93B07 93B51 93C05

Summers, Tyler H. (1-TXD-ME); **Cortesi, Fabrizio L.** (CH-ETHZ-AC);
Lygeros, John (CH-ETHZ-AC)

On submodularity and controllability in complex dynamical networks. (English. English summary)

IEEE Trans. Control Netw. Syst. **3** (2016), *no.* 1, 91–101.

In this paper, the authors address the problem of actuator (or input) selection in a complex network system represented by the state dynamic matrix A. The input selection is modeled as the choice of several input columns from a set of potential column vectors: $V = \{b_1, \ldots, b_M\}$; the chosen input columns together comprise the input matrix B. The design objective is to choose the columns such that some linear functionals of the controllability Gramian are optimized, using as few input columns as possible.

The controllability Gramian associated with the state and input matrix pair (A, B) is given by

$$W_c = \int_0^\infty e^{A\tau} B B^\mathsf{T} e^{A^\mathsf{T}\tau} d\tau.$$

The authors investigate several functionals of the controllability Gramian matrix, including $\mathrm{trace}(W_c)$, $-\mathrm{trace}(W_c^{-1})$, $\log\det(W_c)$, $\lambda_{\min}(W_c)$, $\mathrm{rank}(W_c)$. These functionals measure on average, or in the worst case for $\lambda_{\min}(W_c)$, how much control energy is required to steer the complex dynamical system in its state space.

The authors show that all but $\lambda_{\min}(W_c)$ are submodular and monotone increasing set functions in the choice of the input columns, while $\mathrm{trace}(W_c)$ satisfies the stronger property of modularity. Subsequently, $\mathrm{trace}(W_c)$ can be efficiently maximized using a greedy algorithm; likewise for the remaining submodular functionals, one can use a greedy selection algorithm to obtain an input set whose value is no worse than 63% of an optimal actuator set with the same number of actors. This is in spite of the combinatorial nature of the input selection problem, which makes a brute-force search prohibitively expensive in terms of the required capacity for computations.

A closely related actuator placement problem was also considered in [IEEE Trans. Control Netw. Syst. **3** (2016), no. 1, 67–78; MR3477963], where V. Tzoumas et al. also adopted an approach based on greedy algorithms, and exploited the submodularity of the control energy functional to approximate the optimal actuator sets.

M. Amin Rahimian

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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MR3476088 93B05 05C82 90B10 93C05 94C15 94C30

Li, Jian (PRC-CSU-DPS); **Dueñas-Osorio, Leonardo** (1-RICE-CEV); **Chen, Changkun** (PRC-CSU-DPS); **Berryhill, Benjamin** (1-ONESUB); **Yazdani, Alireza**

Characterizing the topological and controllability features of U.S. power transmission networks. (English. English summary)

Phys. A **453** (2016), 84–98.

Controllability is a key system property studied in modern control theory. A linear system is said to be controllable if it can be driven from any initial state to any desired final state within finite time with a suitable choice of control inputs. Understanding this property in the case of complex networks is a relevant field of study and in this paper the authors focus on the electrical power grids class of networks. Specifically, they study the controllability features of an ensemble of 58 U.S. city-level power transmission networks in seven U.S. states.

In order to do structural controllability analyses, the topological characteristics of the ensemble of networks are first quantified, including degree, shortest path length, clustering coefficient, meshedness and betweenness centrality, as well as the uncertainty associated with these and related properties. After that, the authors focus on the controllability features of complex networks so as to detect the minimal sets of driver nodes to possibly control the networks given system linearity assumptions. Accordingly, a node is critical, intermittent or redundant if it acts as a driver node in all, some, or none of the potentially controllable system configurations. It is worth mentioning to put the study into context that, in power grids, linear dynamics could be associated with slow dynamics, such as those related to power supply/demand balance, and not with fast dynamics related to network stability.

The paper presents a new methodology to quantify the probability of driver nodes being among the intermittent nodes, and reveals the controllability importance of system components. Results show that a small proportion of driver nodes can provide the conditions for controlling the slow dynamics of entire power transmission networks from a topological perspective, despite variations in network sizes and configurations. Besides, the results also reveal that the driver nodes tend to avoid high degree nodes and high triangulation sub-graph nodes as well as high betweenness centrality nodes.

It should be noted that, as the selection of networks in the paper is one of the largest ensembles of real power transmission networks published to date, insights could be attributed to the class of electrical power transmission systems. *Robert Griñó*

MR3459647 93C23 34D06

Li, Shaolin (PRC-HHU); **Cao, Jinde** (PRC-SEU-SNW); **He, Yinghui** (PRC-HHU)

Pinning controllability scheme of directed complex delayed dynamical networks via periodically intermittent control. (English. English summary)

Discrete Dyn. Nat. Soc. **2016,** *Art. ID* 1585928, 10 *pp.*

Summary: "This paper studies the pinning controllability of directed complex delayed dynamical networks by using periodic intermittent control scheme. The general and low-dimensional pinning synchronization criteria are derived to illustrate the design of periodic intermittent control scheme. According to our low-dimensional pinning criterion, especially, the constraint condition of coupling strength is obtained when the network structure and amounts of pinned nodes are fixed. An algorithm is presented to determine the amounts of periodically intermittent controllers and locate these intermittent controllers in a directed network, in which the significance of nodes out-(in-) degree in pinning control of complex network is also illustrated. Finally, a directed network consisting of 12 coupled delayed Chua oscillators is designed as numerical

example to verify the effectiveness of the theoretical analysis."

MR3451312 93B51 93A14 93B05

Li, Xin-Feng (PRC-ZHJ-SAE); **Lu, Zhe-Ming** (PRC-ZHJ-SAE)

Optimizing the controllability of arbitrary networks with genetic algorithm. (English. English summary)

Phys. A **447** (2016), 422–433.

Summary: "Recently, as the controllability of complex networks attracts much attention, how to optimize networks' controllability has become a common and urgent problem. In this paper, we develop an efficient genetic algorithm oriented optimization tool to optimize the controllability of arbitrary networks consisting of both state nodes and control nodes under Popov-Belevitch-Hautus rank condition. The experimental results on a number of benchmark networks show the effectiveness of this method and the evolution of network topology is captured. Furthermore, we explore how network structure affects its controllability and find that the sparser a network is, the more control nodes are needed to control it and the larger the differences between node degrees, the more control nodes are needed to achieve the full control. Our framework provides an alternative to controllability optimization and can be applied to arbitrary networks without any limitations."

MR3443701 93-02 37N40 90B18 90C27 91A43 93A14 93B51 94C30

Clark, Andrew (1-WPI-CP); **Alomair, Basel** (SAR-KACST);
Bushnell, Linda (1-WA-E); **Poovendran, Radha** (1-WA-E)

★**Submodularity in dynamics and control of networked systems.**

With a foreword by John Baillieul.

Communications and Control Engineering Series.

Springer, *Cham*, 2016. *xvii*+210 *pp. ISBN* 978-3-319-26975-7; 978-3-319-26977-1

This book aims to unify techniques for the analysis and design of control algorithms. It contains applications for modeling the spread of influence in social networks, the optimal placement of nodes in sensor networks and the contagion theories in epidemiology. The common tool is a branch of combinatorial optimization: submodular function optimization. The book also shows how submodular optimization can help to improve dynamics and control of networked systems. There are two main parts:

(1) submodular functions and optimization, and
(2) submodularity in dynamics and control.

The first part is divided into three chapters. The first one gives definitions and properties of submodular (supermodular) functions. The concept of matroid is also introduced. It generalizes properties, including linear independence in vector spaces. This structure reduces the complexity of algorithms and improves their performance guaranties. The second chapter presents centralized algorithms for solving problems of the following form on the reference set V:

$$\max_{S\in\mathcal{C}} f(S),$$

where $f\colon 2^V \to \mathbb{R}$ is a submodular function and $\mathcal{C}$ is a collection of subsets of V. The authors propose three applications to motivate the use of this model. Cardinality-constrained submodular maximization is studied and greedy algorithms, optimality bounds and worst-case complexity results are given.

The third chapter is dedicated to distributed submodular maximization, as large networks' datasets require multiple processors to perform graph computation. The authors first describe an online greedy algorithm which achieves the same optimal bounds as the centralized algorithm but requires time synchronization and periodic broadcast.

Then they consider an exchange-based approach which removes these requirements but reduces the optimal bounds. The two approaches are compared via a case study. At the end of this chapter, they examine the influence of the number of processors and the number of elements in each processor in parallel computation on submodularity optimization.

The fourth chapter presents background on control of networked systems. Basic concepts in graph theory are recalled, including algebraic graph theory and properties of the Laplacian spectrum. Consensus is the process by which a set of nodes come to agreement on a common parameter or state. Consensus in network systems is described in both the static and dynamic cases. Distributed estimation algorithms for sensor networks are enunciated. Each node must sense its environment in a distributed manner. The authors employ consensus dynamics to agree on a global parameter based on the sensor measurements and employ opinion dynamics in social networks. The latter ones are formed by the interactions between individual users and include mechanisms for sharing opinions and innovation. They discuss a stubborn agent interaction model, which is seen as a noncooperative game of the users of the social network, and its relationship to the consensus dynamics. The objective is to reach a convergence of the users' opinions to a consensus state.

Chapter 5 develops a submodular optimization paradigm for smooth convergence via input selection. A networked system achieves smooth convergence if the nodes reach their desired states with a given time bound and with minimal error in the intermediate states prior to convergence. Lower bounds on the convergence error are derived. They can be calculated by any input selection algorithm using the knowledge of network topologies from previous time periods. An efficient algorithm for selecting input nodes to minimize the convergence error is presented for the case of unknown time-varying topologies. Furthermore, expert algorithms based on the submodular optimization approach are used to predict the best input node to be selected at each iteration.

The subject of chapter 6 concerns the synchronization of complex systems. The approach presented is to pin a subset of nodes, the catalysts, to a desired state. Then these pinned nodes influence their neighbors and progressively synchronize the whole network. A submodular optimization approach guarantees the existence of stable synchronized states and the synchronization, starting from any initial state. Chapter 7 deals with minimization of the impact of link noise (errors in data exchange and interference from neighboring nodes). The presented submodular optimization algorithm selects an optimal subset of input nodes to ensure noise robustness for static and dynamic systems or networks with random walks.

In chapter 8, the authors study input selection under link noise injection attacks by an intelligent adversary. A game theory approach is introduced to model the network and the adversary, as well as develop input selection algorithms. The framework is developed in two cases:

- When an input set is fixed and the error due to noise injection is minimized, the problem is formulated as a Stackelberg game, and the adversary's optimization problem is shown convex.
- When the input nodes set varies, a simultaneous Nash game is used.

In both cases the submodularity of the error due to link noise introduced in chapter 7 helps to develop efficient algorithms for approximating the equilibrium strategies for both the network and the adversary.

Chapter 9 presents an input selection model which relates performance optimization of the network and its controllability, while the last chapter is devoted to emerging applications of submodular optimization to control energy systems.

This book gives a very precise theoretical presentation of submodular optimization.

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It also contains many examples, which facilitate the understanding of this subject and guide the reader along the book. *Fatiha Bendali*

MR3437807 93B05

Chao, Luo (PRC-SHDN-IFE); **Hong, Liu** (PRC-SHDN-IFE)

Controllability of Boolean control networks under asynchronous stochastic update with time delay. (English. English summary)

J. Vib. Control **22** (2016), *no.* 1, 235–246.

In this paper, the authors consider the controllability properties of asynchronous Boolean control networks with time delay. The approach considered consists in obtaining a linear representation for the problem and general formulas for control-dependent network transition matrices using semi-tensor products.

In the introduction, the authors recall the notion of Boolean networks and give the state of the art of control theory of complex systems.

In section 2, the definition of the semi-tensor product is recalled and some notations are presented.

In section 3, the main important results are presented: First the algebraic expression of asynchronous control network with time delay is given. Then the authors consider the following:

- Deterministic controllability of asynchronous Boolean control networks with time delay.
- Controllability of asynchronous Boolean control networks with time delay via free Boolean sequences.
- Controllability of asynchronous Boolean control networks with time delay via input control network and closed-loop control.

Some examples are presented in the fourth section.

The paper is well written and well presented. *Abdes-Samed Bernoussi*

MR3437669 93B05 94C30

Zhou, Ming-Yang (PRC-SZU-CSF); **He, Xingsheng** (PRC-HEF-ELT);
Fu, Zhong-Qian (PRC-HEF-ELT); **Liao, Hao** (PRC-SZU-CSF);
Cai, Shi-min (PRC-EST-DTR); **Zhuo, Zhao** (PRC-HEF-ELT)

Diffusion inspires selection of pinning nodes in pinning control. (English. English summary)

Phys. A **446** (2016), 120–128.

The outstanding problem of controlling a complex network via pinning is related to network dynamics and has the potential to master large-scale real-world systems as well.

In this paper the authors address the issue of how to choose pinning nodes for pinning control, where pinning control aims to control a network to an identical state by injecting feedback control signals to a small fraction of nodes.

For a network with only previously known edge connections, how to select the controlled nodes is still a challenge and it has piqued the interest of many researchers [see X. Wang, X. Li and J. Lu, IEEE Circuits Syst. Mag. **10** (2010), no. 3, 83–91, doi:10.1109/MCAS.2010.937887; Y.-Y. Liu, J.-J. Slotine and A.-L. Barabási, Nature **473** (2011), no. 7346, 167–173, doi:10.1038/nature10011; T. Nepusz and T. Vicsek, Nat. Phys. **8** (2012), no. 7, 568–573, doi:10.1038/nphys2327; M. Pósfai et al., Sci. Rep. **3** (2013), 1067, doi:10.1038/srep01067]. Among those, Wang et al. sstudied pinning control of networks and the results showed that controlling high degree nodes was better than random selection in scale-free artificial networks [X. Wang and G. Chen, Phys. A **310** (2002), no. 3-4, 521–531; MR1946327; R. Olfati-Saber, IEEE Trans. Automat. Control **51** (2006), no. 3,

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401–420; MR2205679]. M. Jalili-Kharaajoo, O. A. Sichani and X. Yu explored the optimal pinning nodes, which was only appropriate for small networks due to calculation complexity [Phys. Rev. E (3) **91** (2015), no. 1, 012803; MR3416662]. Apart from pinning control (of non-linear systems), many researchers aim to control each node to any arbitrary final state in linear systems (i.e., full control). Since pinning control is a particular case of full control, some achievement in full control may also contribute to pinning control.

Unlike previous work, in this paper the authors firstly find the poor controllability of conventional large-degree selection in practical networks by showing an abnormal phenomenon — the large-degree selection method performs worse than random selection in real-world networks. They explore network controllability from not only mathematical analysis, but also the aspects of network topology and information diffusion. Then, the relationship between pinning control and information diffusion is introduced, by which the problem of pinning node selection is transferred into multi-spreaders in information diffusion, which this has been widely investigated [see A. Rahmani et al., SIAM J. Control Optim. **48** (2009), no. 1, 162–186; MR2480130; K. Lu et al., Sci. China Inf. Sci. **53** (2010), no. 11, 2332–2342, doi:10.1007/s11432-010-4092-8; K. Lu et al., IET Wirel. Sens. Syst. **1** (2011), no. 1, 1–6, doi:10.1049/iet-wss.2010.0004]. Then the authors use multi-spreader theory to explain why extra high degree nodes do not benefit to enhancing controllability in real-world networks. If you want to enhance controllability, periphery nodes should also be taken into account and the distances of information diffusion from sources to destinations should be reduced.

Based on information diffusion, a novel and simple method is proposed in this work to select pinning nodes by optimizing the spreading ability of multiple spreaders. This method selects pinning nodes according to the overall influences of pinning nodes. It should be noted that the overall influences are not the sum of each node's influence due to the overlapping influence between nodes. The proposed method enhances the controllability a lot compared to that of traditional large-degree selection. What is more, the method provides a new perspective (information diffusion) to study pinning nodes selection and could inspire many other effective heuristic algorithms in the future.

Cristina Jordán

MR3416819 93B05 05C82 90B10

Yin, Hongli (PRC-QGD-CX); **Zhang, Siying** (PRC-QGD-CX)

Minimum structural controllability problems of complex networks. (English. English summary)

Phys. A **443** (2016), 467–476.

In this paper the authors study the structural controllability of complex networks and demonstrate that the network cannot be fully controlled by the minimum driver vertices when the associated graph contains inaccessible strongly connected components which have perfect matching. Then they show that the minimum controlled vertex set required to achieve the full control of the network with arbitrary structure is the union of the minimum unmatched vertex set (all unmatched vertices of a maximum edge set matching M) and the minimum input inaccessible vertex set (the set of vertices which are selected from inaccessible strongly connected components, with only one vertex selected from each component) which have the biggest intersection. Finally, the authors propose a mathematical model to find the pair of the minimum unmatched vertex set and the minimum input inaccessible vertex set which have the biggest intersection.

Ma. Isabel García-Planas

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★**Rough sets, fuzzy sets, data mining, and granular computing.**
Proceedings of the 15th International Conference (RSFDGrC 2015) held in Tianjin, November 20–23, 2015.
Edited by Yiyu Yao, Qinghua Hu, Hong Yu and Jerzy W. Grzymala-Busse.
Lecture Notes in Computer Science, 9437.
Lecture Notes in Artificial Intelligence.
Springer, *Cham*, 2015. *xvi*+502 *pp.* *ISBN* 978-3-319-25783-9; 978-3-319-25782-2
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Caihui Liu, Meizhi Wang, Yanfei Dai and Yueli Luo, "A new subsystem-based definition of generalized rough set model", 82–89. MR3487420

Yanfang Liu and William Zhu, "A comparison of two types of covering-based rough sets through the complement of coverings", 90–101. MR3487421

Keyun Qin and Bo Li, "On the nearness measures of near sets", 102–111. MR3487422

Mauricio Restrepo and Jonatan Gómez, "Topological properties for approximation operators in covering based rough sets", 112–123. MR3487423

Giampiero Chiaselotti, Davide Ciucci, Tommaso Gentile and Federico Infusino, "Preclusively and simple graphs", 127–137. MR3487424

Giampiero Chiaselotti, Davide Ciucci, Tommaso Gentile and Federico Infusino, "Preclusively and simple graphs: the n-cycle and n-path cases", 138–148. MR3487425

Hui Li and William Zhu, "Connectedness of graph and matroid by covering-based rough sets", 149–160. MR3487426

Yunyun Yang, Gang Xie and Zehua Chen, "Controllability in directed complex networks: granular computing perspective", 161–169. MR3487427

Yanyong Huang, Tianrui Li and Shi-jinn Horng, "Dynamic maintenance of rough fuzzy approximations with the variation of objects and attributes", 173–184. MR3487428

Richard Jensen, Sarah Vluymans, Neil Mac Parthaláin, Chris Cornelis and Yvan Saeys, "Semi-supervised fuzzy-rough feature selection", 185–195. MR3487429

Zbigniew Suraj, "Modified generalised fuzzy Petri nets for rule-based systems", 196–206. MR3487430

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Marek Grzegorowski and Sebastian Stawicki, "Window-based feature engineering for prediction of methane threats in coal mines", 452–463. MR3487452

Petre Lameski, Eftim Zdravevski, Riste Mingov and Andrea Kulakov, "SVM parameter tuning with grid search and its impact on reduction of model over-fitting", 464–474. MR3487453

Krzysztof Pawłowski and Karol Kurach, "Detecting methane outbreaks from time series data with deep neural networks", 475–484. MR3487454

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