A Simple Success Check for Delay DAEs

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Motivation: Earthquake Engineering

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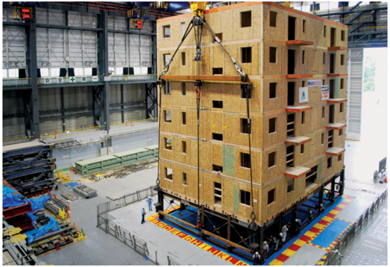
Earthquake Engineering

Main goal: Make structures more resistant to earthquakes.



Problems:

- Actual tests are extremely expensive
- Highly complex system structures with many uncertain parameters

B. Unger. (2020) Delay differential-algebraic equations in real-time dynamic substructuring. ArXiv e-print, 2003.10195.

Delay Differential-Algebraic Equations (DDAEs)

Definition (Delay Differential-Algebraic Equation (DDAE))

Matrices $E, A, B \in \mathbb{R}^{n,n}$, inhomogeneity $f : [0, t_f) \to \mathbb{R}^n$, constant $\tau > 0$

$$E\dot{x}(t) = Ax(t) + Bx(t-\tau) + f(t)$$

Example

$$\begin{aligned} F_1(t) \quad & 0 = x_1(t) - f_1(t), \\ F_2(t) \quad & 0 = x_2(t-\tau) - \dot{x}_1(t) - f_2(t), \end{aligned}$$

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$$\begin{aligned} \dot{F}_1(t) &\quad 0 = \dot{x}_1(t) - \dot{f}_1(t), \\ \hat{F}_2(t) &\quad 0 = x_2(t-\tau) - \dot{f}_1(t) - f_2(t), \\ \hat{F}_2(t+\tau) &\quad 0 = x_2(t+0) - \dot{f}_1(t+\tau) - f_2(t+\tau) \end{aligned}$$

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\end{array}$$

Differentiate and shift equations to solve a DDAE

Structural information: Which variable appears in which equation

Example

$$\begin{aligned} F_1(t) \quad 0 &= x_1(t) - f_1(t) \\ F_2(t) \quad 0 &= x_2(t-\tau) - \dot{x}_1(t) - f_2(t) \end{aligned}$$

I. A. and B. Unger. (2020) The Pantelides algorithm for delay differential-algebraic equations. Trans. Math. Appl., 4, 1–36.

Structural Analysis for DDAEs

Structural information: Which variable appears in which equation

Example

$$\begin{aligned} F_1(t) \quad 0 &= x_1(t) - f_1(t) && F_1(t) \textit{ depends on } x_1(t) \\ F_2(t) \quad 0 &= x_2(t-\tau) - \dot{x}_1(t) - f_2(t) && F_2(t) \textit{ depends on } x_2(t-\tau), \dot{x}_1(t) \end{aligned}$$

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Structural information $\xRightarrow{\textbf{Structural analysis}}$ Number of needed derivatives and shifts

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$F_1(t)$ *depends on* $x_1(t)$
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Example

$$\begin{array}{lll} \tilde{F}_1(t) & 0 & = \sin^2(x_1(t)) + cos^2(x_1(t)) - f_1(t) \\ F_2(t) & 0 & = x_2(t-\tau) - \dot{x}_1(t) - f_2(t) \end{array}$$

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Success check needed

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Restriction to DAEs

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Differentiate ~~and shift~~ equations to solve a $\cancel{\text{D}}$DAE

Reformulation of a DAE

Idea: Reformulate DAE as explicit equation for variables of highest derivative

J. D. Pryce. (2001) A simple structural analysis method for DAEs. BIT Numer. Math., 41(2):364–394.

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$$\begin{array}{ll} G_1 & 0 = y_1 - g_1 \\ G_2 & 0 = y_2 - \dot{y}_1 - g_2 \end{array}$$

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Largest number d_i such that $y_i^{(d_i)}$ appears in any equation

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Example

	y_1	y_2
G_1	0	
G_2	1	0

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Algorithm Signature Method for DAEs

Input: DAE

1: Construct the Signature Matrix
2: Primal assignment problem: Which equation to solve for which variable
3: Dual assignment problem: Number of derivatives

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A Simple Success Check

Question: Can we solve all equations of highest derivative for all variables of highest derivative?

- $\mathcal{G}$: Equations of highest derivative
- $\mathcal{Y}$: Variables of highest derivative
- $\mathcal{G}(t, \mathcal{Y}, \text{lower derivatives}) = 0$
- If $\frac{\partial \mathcal{G}}{\partial \mathcal{Y}}$ nonsingular at consistency point $\overset{\text{IFT}}{\Rightarrow}$ $\mathcal{Y} = \psi(t, \text{lower derivatives})$.

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Theorem

- *Primal and dual assignment problem are solvable*
- $\frac{\partial \mathcal{G}}{\partial \mathcal{Y}}$ *nonsingular at consistency point*

$\Rightarrow$ *Obtain unique solution of DAE*

$\Rightarrow$ *Differentiation index of DAE is bounded above by structural index*

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DDAE

Introduction

DAE

Signature Method

Success Check with Signature Method

Success Check without Signature Method

DDAE

Success Check

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Goal: Success check for arbitrary number of derivatives

Theorem

- *Primal and dual assignment problem are solvable*
- $\frac{\partial \mathcal{G}}{\partial \mathcal{y}}$ *nonsingular at consistency point*

$\Rightarrow$ *Obtain solution of DAE*

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A simple Success Check for DAEs without the Signature Method

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Goal: Success check for arbitrary number of derivatives

Theorem

- ~~*Primal and dual assignment problem are solvable*~~
- $\frac{\partial \mathcal{G}}{\partial \mathcal{Y}}$ *nonsingular at consistency point*

$\overset{?}{\Rightarrow}$ *Obtain solution of DAE*

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Theorem

- $\frac{\partial \mathcal{G}}{\partial \mathcal{Y}}$ *is nonsingular at consistency point*

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Algorithm Success Check for DAEs

Input: DAE, number of derivatives

1: $\mathcal{G}$: Equations of highest derivative
2: $\mathcal{Y}$: Variables of highest derivative
3: **if** $\frac{\partial \mathcal{G}}{\partial \mathcal{Y}}$ nonsingular at consistency point **then**
4: Success
5: **end if**

A simple Success Check for DDAEs: First Ideas

Algorithm Success Check for DDAEs

Input: DDAE, number of derivatives, number of shifts

1: $\mathcal{F}$: Equations of highest derivative and highest shift
2: $\mathcal{X}$: Variables of highest derivative and highest shift
3: **if** $\frac{\partial \mathcal{F}}{\partial \mathcal{X}}$ nonsingular at consistency point **then**
4: Success ?
5: **end if**

The Highest Derivative of the Highest Shift

Definition (Highest derivative and highest shift)

Highest derivative: Largest $d_i \in \mathbb{N}_0$ s.t. $x_i^{(d_i)}(t + k\tau)$ for any $k \geq 0$ appears in any equation

Highest shift: Largest $s_i \in \mathbb{N}_0$ s.t. $x_i^{(\ell)}(t + s_i\tau)$ for any $\ell \geq 0$ appears in any equation

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Example: $0 = \dot{x}_1(t) + x_1(t + \tau)$,

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Highest shift: Largest $s_i \in \mathbb{N}_0$ s.t. $x_i^{(\ell)}(t + s_i\tau)$ for any $\ell \geq 0$ appears in any equation

Example: $0 = \dot{x}_1(t) + x_1(t + \tau)$, highest derivative: 1, highest shift: 1

The Highest Derivative of the Highest Shift

Definition (Highest derivative and highest shift)

Highest derivative: Largest $d_i \in \mathbb{N}_0$ s.t. $x_i^{(d_i)}(t+k\tau)$ for any $k \geq 0$ appears in any equation
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Variable of highest derivative and highest shift: $\dot{x}_1(t+\tau)$

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Definition (Highest derivative of the highest shift)

Highest derivative of the highest shift: Largest $d_i \in \mathbb{N}_0$ such that $x_i^{(d_i)}(t + s_i\tau)$ for the highest shift s_i appears in any equation

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Variable of highest derivative of the highest shift: $x_1(t + \tau)$

A simple Success Check For DDAEs: First Ideas

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Algorithm Success check for DDAEs

Input: DDAE, number of derivatives, number of shifts

1: $\mathcal{F}$: Equations of highest derivative and highest shift
2: $\mathcal{X}$: Variables of highest derivative ~~and~~ of the highest shift
3: **if** $\frac{\partial \mathcal{F}}{\partial \mathcal{X}}$ nonsingular at consistency point **then**
4: Success ?
5: **end if**

A simple Success Check For DDAEs: First Ideas

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Example:

$$
\begin{aligned}
0 &= x_1(t+2\tau) - f_1(t+2\tau)\\
0 &= x_2(t) - \dot{x}_1(t+\tau) - f_2(t+\tau)
\end{aligned}
$$

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$$\begin{aligned} 0 &= x_1(t+2\tau) - f_1(t+2\tau) \\ 0 &= x_2(t) - \dot{x}_1(t+\tau) - f_2(t+\tau) \end{aligned}$$

$$\overset{\text{IFT}}{\Rightarrow} \quad \begin{aligned} x_1(t+2\tau) &= f_1(t+2\tau) \\ x_2(t) &= \dot{x}_1(t+\tau) + f_2(t+\tau) \end{aligned}$$

A simple Success Check For DDAEs: First Ideas

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$$

$$
\overset{\text{shifting}}{\Rightarrow} \quad
\begin{aligned}
x_1(t) &= f_1(t) \\
x_2(t) &= \dot{x}_1(t+\tau) + f_2(t+\tau)
\end{aligned}
$$

Algorithm Success check for DDAEs

Input: DDAE, number of derivatives, number of shifts

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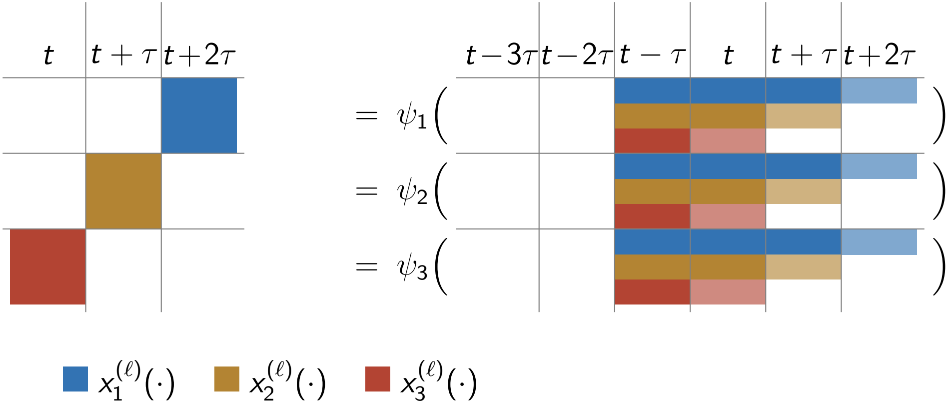
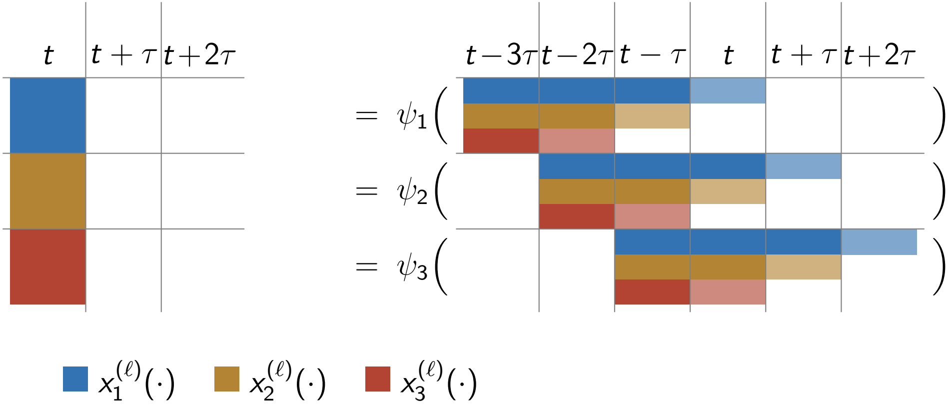
Problems:

- Dependency on future time points
- Dependency on higher or equally high derivatives

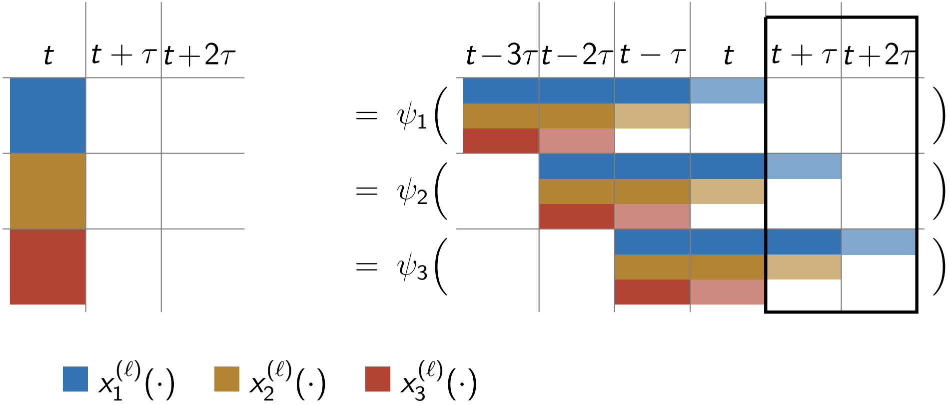


t
$t+\tau$
$t+2\tau$
$t-3\tau$
$t-2\tau$
$t-\tau$
t
$t+\tau$
$t+2\tau$
$= \psi_1($
$)$
$= \psi_2($
$)$
$= \psi_3($
$)$
$x_1^{(\ell)}(\cdot)$
$x_2^{(\ell)}(\cdot)$
$x_3^{(\ell)}(\cdot)$

A simple Success Check For DDAEs: Visualization



A simple Success Check For DDAEs: Visualization



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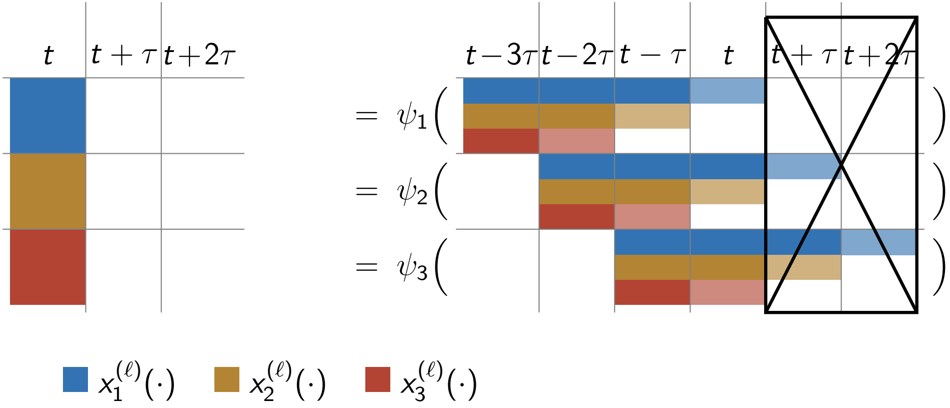

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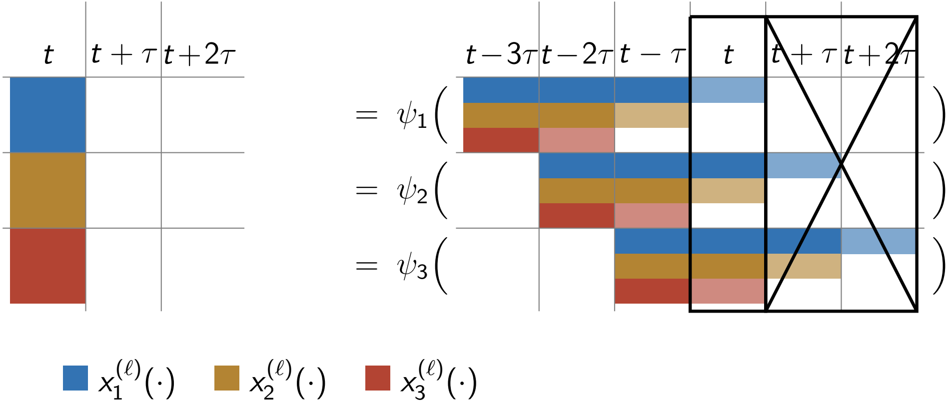

A simple Success Check for DDAEs

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Theorem

- $\mathcal{F}$: *Equations of highest derivative and highest shift*
- $\mathcal{X}$: *Variables of highest derivative of the highest shift*
- $\frac{\partial \mathcal{F}}{\partial \mathcal{X}}$ *nonsingular at consistency point*
- *Shifted equations do not depend on future time points*
- *Variables of highest derivative* $\underset{\textit{of the}}{\overset{\textit{and}}{\parallel}}$ *highest shift*

$\Rightarrow$ *Differentiation index of shifted equation is smaller equal* 1 *and shift index is* 0.

$\Rightarrow$ *If DDAE is LTI: DDAE uniquely solvable*

I. A., B. Unger. (2020) A simple success check for delay differential-algebraic equations. Submitted to PAMM

Summary/Outlook

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Outlook

- Less restrictive success check for delay DAEs
- Finding consistency point
- Signature Method for delay DAEs

Summary

- Success check for DAEs dependent of the Signature Method
- Success check for DAEs independent of the Signature Method
- Success check for delay DAEs