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Event-Triggered Control for Linear Descriptor Systems

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Abstract In this paper, event-triggered strategies for linear descriptor systems are presented. By studying the descriptor system in terms of the second equivalent form and employing the Lyapunov theory and linear matrix inequality (LMI) approach, the control law and event-triggered condition guaranteeing the admissible behavior of the descriptor system are proposed. Furthermore, we prove that the inter-sampling times are strictly positive in the proposed event-triggered condition. Finally, a simulation example is given to illustrate the efficiency of the results.

Keywords Event-triggered control · Descriptor systems · Admissibility · Inter-sampling times

1 Introduction

In event-triggered control, the scheduling algorithm is always implemented when a certain error exceeds a threshold. This is in contrast to the commonly used sampled-data control that is implemented in a periodic fashion. As an alternative to periodic control, event-triggered control can reduce the communication among the sensors, the controller and the actuators to some extent, so that the resource utilization of the

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whole system is improved. Some advantages of event-triggered control over periodic control presented in a stochastic setting can be seen in [1].

Recently, event-triggered control has received great interest from the academic community. Related work can be found in the following mentioned papers and the references therein. [12] investigated an event-triggered scheduler that is executed whenever a certain error becomes large enough with respect to the norm of the state. In [13], the authors examined a class of real time control systems and demonstrated that different sampling rates for a control system lead to different acceptable maximum delays. [14] extended the results of [13], in which the assumption that the magnitude of the process noise is bounded was relaxed. Except for a single system, event-triggered control for multi-agent systems was considered, and the related work can be seen in [6, 7] and the references therein. In these references both centralized and decentralized event-triggered cooperative control were treated. Furthermore, in [10] model-based networked control systems and event-triggered control are united under a single framework. The authors provided a strategy that the control laws do not keep constant during the inter-sampling period, which distinguishes from the traditional event-triggered schemes. It is worth to mention that all the above references assumed that the full state is available, but as we all known the assumption is not reasonable in many practical situations, so output-based event-triggered control has attracted some scholars' attention. In [8], event-triggered dynamical output-based controllers guaranteeing L_∞ gain were proposed, see [11] and [4] for more details. Apart from event-triggered control for continuous time systems, event-triggered strategies for control of discrete time systems have been proposed in [9]. Above all, the achievements of event-triggered control have been abundant.

However, the considered plants are normal systems in the previous references. To the best of our knowledge, the event-triggered control problem for descriptor systems has not been addressed. Actually, descriptor systems are more general and they are not only of theoretical interest but also of a great importance in practice. Descriptor systems are frequently encountered in electronic and economic systems, in aerospace and chemical industries. So, there will be a profound meaning applying event-triggered control to descriptor systems. In this paper, event-triggered state feedback control for a linear time-invariant descriptor system is studied. We propose a scheme co-designing the event-triggered condition and controller to guarantee the admissibility of the system. Based on this, a theorem which proves that the inter-sampling times are strictly positive under the control is presented.

The remainder of this paper is organized as follows. Section 2 presents some necessary notations and preliminaries. The problem is stated in Sects. 3 and 4 presents the main results. Simulation is given in Sects. 5 and 6 includes some conclusions.

2 Preliminaries

In what follows, if not explicitly stated, matrices are assumed to have appropriate dimensions. I stands for an identity matrix of any proper size. For a vector $x \in \mathbb{R}^n$, we denote its 2-norm as $\|x\|$. For a symmetric matrix $P \in \mathbb{R}^{n\times n}$, $\lambda_{\max}(P)$, $\lambda_{\min}(P)$ denote its maximum and minimum eigenvalue, respectively. For a matrix $P \in \mathbb{R}^{n\times m}$,

P^T and $\|P\|$, respectively, represent its transposition and induced 2-norm. We denote by $P>0$ the positive definite matrix P and P^{-1} the inverse of an invertible matrix P. $P_\perp$ is denoted the matrix which is perpendicular to P, that is, $P_\perp P=0$. We write a symmetric block matrix

$$M=\begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$$

of the form as

$$M=\begin{bmatrix} A & B \\ * & C \end{bmatrix}.$$

In event-triggered control, an invocation of a control task is called a job. t_k represents the kth release time and also the kth sampling time. $T_k=t_{k+1}-t_k$ denotes the inter-sampling period.

First, we introduce an important result for dealing with linear matrix inequality (LMI) and some basic results about descriptor systems.

Lemma 1 (Schur complement formula [2]) *The symmetric matrix $S\in\mathbb{R}^{n\times n}$, and*

$$S=\begin{bmatrix} S_{11} & S_{12} \\ * & S_{22} \end{bmatrix}<0$$

is equivalent to $S_{11}<0$, $S_{22}-S_{12}^TS_{11}^{-1}S_{12}<0$ or $S_{22}<0$, $S_{11}-S_{12}S_{22}^{-1}S_{12}^T<0$.

Consider a linear time-invariant descriptor systems

$$E\dot{x}(t)=Ax(t), \tag{1}$$

where $x\in\mathbb{R}^n$ is the state vector, the matrices $A,E\in\mathbb{R}^{n\times n}$ and $\operatorname{rank}(E)=q<n$.

Definition 1 [5] (1) The pair (E,A) is said to be regular if $\det(sE-A)\neq 0$ for some complex number s;

(2) The pair (E,A) is called impulse free if $\deg[\det(sE-A)]=\operatorname{rank}(E)$.

Remark 1 The regularity of (E,A) guarantees the existence and uniqueness of solution for system (1) and the impulse immunity avoids impulsive behavior at initial time for inconsistent initial conditions.

For a regular pair (E,A), there always exist two invertible matrices D_1 and D_2 such that

$$D_1ED_2=\begin{bmatrix} I_q & 0 \\ 0 & N \end{bmatrix},$$

$$D_1AD_2=\begin{bmatrix} A_{11} & 0 \\ 0 & I_{n-q} \end{bmatrix},$$

where N is nilpotent whose nilpotent index is denoted by h.

Lemma 2 [5] *If the pair (E, A) is regular, then for any h times piecewise continuously differentiable input function $f(t)$, the system*

$$E\dot{x}(t) = Ax(t) + f(t) \tag{2}$$

is regular, that is, system (2) *has a unique solution. System* (2) *is impulse free if (E, A) is impulse free.*

Definition 2 [5] System (2) is said to be admissible if it is regular, impulse free and asymptotically stable.

3 Problem Statement

In this section, we are going to introduce the event-triggered formulation for a linear time-invariant descriptor system, described by

$$E\dot{x}(t) = Ax(t) + Bu(t), \tag{3}$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^m$ is the control vector, E, A and B are known real constant matrices.

To ensure that system (3) has a unique solution, the pair (E, A) is assumed to be regular.

In this paper, we take into account event-triggered strategies and the state feedback $u(t) = Kx_{t_k}, t \in [t_k, t_{k+1})$, where x_{t_k} stands for the state when the kth job is released, then the state feedback closed loop is given by

$$E\dot{x}(t) = Ax(t) + BKx_{t_k}, \quad t \in [t_k, t_{k+1}).$$

We define $e(t)$ as the error $x_{t_k} - x(t)$, so that system (3) can be written as

$$E\dot{x}(t) = (A + BK)x(t) + BKe(t). \tag{4}$$

The purpose of this paper is to co-design a suitable controller and an event-triggered condition of the form $\gamma(e) < \zeta(x)$, where $\gamma(e)$ and $\zeta(x)$ are functions of e and x, respectively, such that system (4) is admissible.

We denote $A + BK$ by A_c. For any real matrix E, there always exist two invertible matrices U and V such that (E, A_c) is transformed into the second equivalent form $(\bar{E}, W)$ with

$$\bar{E} = UEV = \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix}, \qquad W = UA_cV = \begin{bmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \end{bmatrix},$$

where q is the number of finite dynamic modes. Besides, UB is denoted by T which is partitioned as

$$\begin{bmatrix} T_1 \\ T_2 \end{bmatrix}.$$

Let

$$\tilde{x} = V^{-1}x = \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}$$

and transform system (4) into

$$\bar{E}\dot{\tilde{x}} = W\tilde{x} + TKe,$$

that is,

$$\begin{aligned} \dot{\tilde{x}}_1 &= W_{11}\tilde{x}_1 + W_{12}\tilde{x}_2 + T_1 Ke, \\ 0 &= W_{21}\tilde{x}_1 + W_{22}\tilde{x}_2 + T_2 Ke. \end{aligned} \tag{5}$$

The two systems (4) and (5) are the same with regard to the admissibility.

4 Main Results

In this section, we address a sufficient condition that guarantees performances of system (4) and prove that the inter-sampling times are strictly positive.

First, we give an important lemma for the proof of the main results.

Lemma 3 *If there exists a nonsingular matrix M such that the condition*

$$W_{22}^T M + M^T W_{22} < 0, \tag{6}$$

holds, then system (5) is regular and impulse free.

Proof Inequality (6) implies that the inverse of W_{22} exists.

Define

$$\tilde{e} = \begin{bmatrix} \tilde{e}_1 \\ \tilde{e}_2 \end{bmatrix} = \begin{bmatrix} \tilde{x}_{1tk} - \tilde{x}_1 \\ \tilde{x}_{2tk} - \tilde{x}_2 \end{bmatrix}$$

and partition K and V as $[K_1 \; K_2]$ and

$$\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}.$$

Denote $M_1 = K_1V_{11} + K_2V_{21}$ and $M_2 = K_1V_{12} + K_2V_{22}$. We transform (5) into the following form

$$\begin{aligned} \dot{\tilde{x}}_1 &= (W_{11} - T_1M_1)\tilde{x}_1 + (W_{12} - T_1M_2)\tilde{x}_2 + T_1M_1\tilde{x}_{1tk} + T_1M_2\tilde{x}_{2tk}, \\ 0 &= (W_{21} - T_2M_1)\tilde{x}_1 + (W_{22} - T_2M_2)\tilde{x}_2 + T_2M_1\tilde{x}_{1tk} + T_2M_2\tilde{x}_{2tk}. \end{aligned} \tag{7}$$

Due to the arbitrary piecewise continuous differentiability of $u(t) = Kx_{tk}$, system (3) accepts a unique solution under the assumption that (E, A) is regular. Then obviously we see that $W_{22} - T_2M_2$ is invertible and $\tilde{x}_2$ is a function of $\tilde{x}_1$, $\tilde{x}_{1tk}$ and $\tilde{x}_{2tk}$. Thus $\tilde{x}_1$ is continuously differentiable. We find that $\tilde{x}$, x and e are arbitrary piecewise continuously differentiable.

Furthermore, inequality (6) ensures that the system

$$\bar{E}\dot{\tilde{x}} = W\tilde{x}$$

is regular and impulse free (see [3]).

Above all the regularity and absence of impulse of system (5) can be obtained from Lemma 2. □

Theorem 1 *If there exists a block diagonal matrix* $P \in \mathbb{R}^{n\times n}$ *and* $P > 0$, *an invertible block diagonal matrix* $Q \in \mathbb{R}^{n\times n}$, *a matrix* $R \in \mathbb{R}^{n\times n}$ *and a scalar* $\sigma > 0$ *satisfying the following LMI*

$$\begin{bmatrix} AVPV^TE^T + EVPV^TA^T + BR + R^TB^T \\ + AVQ\bar{E}_\perp U^{-T} + U^{-1}(\bar{E}_\perp)^TQ^TV^TA^T & (\sigma+1)EV \\ * & -(\sigma+1)I \end{bmatrix} < 0, \tag{8}$$

then closed loop system (4) *is admissible with the state feedback* $K = RZ$ *and event-triggered condition*

$$e^T\big(\Phi^T\Phi + \xi I\big)e < \sigma x^T\Psi^T\Psi x, \tag{9}$$

where $\bar{E}_\perp = \mathrm{diag}\{0, I_{n-q}\}$, $Z = XE + E_\perp^TY$ *with* $X = U^TP^{-1}U$, $E_\perp = V\bar{E}_\perp U$ *and* $Y = V^{-T}Q^{-1}V^{-1}$, $\Phi = [I\ -W_{12}W_{22}^{-1}]TK$, $\Psi = V^TE^TXE$ *and* ξ *is chosen to be a positive scalar that renders* $\frac{1}{\lambda_{\min}(\Phi^T\Phi+\xi I)}$ *upper bounded.*

Proof Set

$$\bar{X} = P^{-1} = U^{-T}XU^{-1} = \begin{bmatrix} \bar{X}_{11} & 0 \\ 0 & \bar{X}_{22} \end{bmatrix},$$

and

$$\bar{Y} = Q^{-1} = V^TYV = \begin{bmatrix} \bar{Y}_{11} & 0 \\ 0 & \bar{Y}_{22} \end{bmatrix}.$$

So $\bar{X} > 0$ and $\bar{Y}$ is invertible, which imply $\bar{X}_{11} > 0$ and $\bar{Y}_{22}$ is invertible.

Because

$$\begin{aligned} U^{-T}ZV &= U^{-T}XU^{-1}UEV + U^{-T}U^T\bar{E}_\perp^TV^TYV \\ &= \bar{X}\bar{E} + \bar{E}_\perp^T\bar{Y} \\ &= \begin{bmatrix} \bar{X}_{11} & 0 \\ 0 & \bar{Y}_{22} \end{bmatrix}, \end{aligned}$$

Z is obviously invertible, and its inverse becomes

$$\begin{aligned} Z^{-1} &= V\begin{bmatrix} \bar{X}_{11} & 0 \\ 0 & \bar{Y}_{22} \end{bmatrix}^{-1}U^{-T} \\ &= VUX^{-1}U^TV^TE^T + Y^{-1}V^{-T}\bar{E}_\perp U^{-T} \\ &= VPV^TE^T + VQ\bar{E}_\perp U^{-T}. \end{aligned}$$

Pre-multiplying and post-multiplying the both sides of LMI (8) by

$$\begin{bmatrix} Z^T & 0 \\ 0 & I \end{bmatrix}$$

and

$$\begin{bmatrix} Z & 0 \\ 0 & I \end{bmatrix}$$

respectively, we obtain

$$\begin{bmatrix} (A+BK)^T Z + Z^T(A+BK) & (\sigma+1)\Psi^T \\ * & -(\sigma+1)I \end{bmatrix} < 0. \tag{10}$$

LMIs (8) and (10) are equivalent. So if LMI (8) is feasible, we have

$$(A+BK)^T Z + Z^T(A+BK) < 0, \tag{11}$$

$$(A+BK)^T Z + Z^T(A+BK) + (\sigma+1)\Psi^T\Psi < 0. \tag{12}$$

Pre-multiplying and post-multiplying the both sides of LMI (11) by V^T and V, one has

$$\begin{aligned} &V^T\big((A+BK)^T Z + Z^T(A+BK)\big)V \\ &\quad = W^T\bar{X}\bar{E} + \bar{E}^T\bar{X}W + W^T\bar{E}_\perp^T\bar{Y} + \bar{Y}^T\bar{E}_\perp W \\ &\quad = \begin{bmatrix} W_{11}^T\bar{X}_{11} + \bar{X}_{11}W_{11} & \bar{X}_{11}W_{12} + W_{21}^T\bar{Y}_{22} \\ W_{12}^T\bar{X}_{11} + \bar{Y}_{22}^T W_{21} & W_{22}^T\bar{Y}_{22} + \bar{Y}_{22}^T W_{22} \end{bmatrix} \\ &\quad < 0. \end{aligned} \tag{13}$$

From the above inequality (13), we obtain

$$W_{22}^T\bar{Y}_{22} + \bar{Y}_{22}^T W_{22} < 0,$$

thus W_{22} is invertible and hence system (5) is regular and impulse free.

Based on the conclusion that W_{22}^{-1} exists, system (5) can be rewritten as

$$\begin{aligned} \dot{\tilde{x}}_1 &= \bar{W}_{11}\tilde{x}_1 + \Phi e, \\ \tilde{x}_2 &= -W_{22}^{-1}(W_{21}\tilde{x}_1 + T_2 K e). \end{aligned}$$

Define a Lyapunov function $v(\tilde{x}_1) = \tilde{x}_1^T\bar{X}_{11}\tilde{x}_{11}$. The time derivative of $v(\tilde{x}_1)$ is

$$\begin{aligned} \dot{v}(\tilde{x}_1) &= \tilde{x}_1^T\big(\bar{W}_{11}^T\bar{X}_{11} + \bar{X}_{11}\bar{W}_{11}\big)\tilde{x}_1 + 2\tilde{x}_1^T\bar{X}_{11}\Phi e \\ &\le \tilde{x}_1^T\big(\bar{W}_{11}^T\bar{X}_{11} + \bar{X}_{11}\bar{W}_{11}\big)\tilde{x}_1 + \tilde{x}_1^T\bar{X}_{11}^2\tilde{x}_1 + e^T\Phi^T\Phi e \\ &\le \tilde{x}_1^T\big(\bar{W}_{11}^T\bar{X}_{11} + \bar{X}_{11}\bar{W}_{11} + (\sigma+1)\bar{X}_{11}^2\big)\tilde{x}_1 \\ &< 0. \end{aligned} \tag{14}$$

$\bar{W}_{11}^T\bar{X}_{11} + \bar{X}_{11}\bar{W}_{11} + (\sigma + 1)\bar{X}_{11}^2 < 0$ will be proved in the sequel. In inequality (14), we use $e^T(\Phi^T\Phi + \xi I)e < \sigma\tilde{x}_1^T\bar{X}_{11}^2\tilde{x}_1$ which is equivalent to event-triggered condition (9).

Therefore, $\tilde{x}_1$ is asymptotically stable.

With respect to the stability of $\tilde{x}_2$, the analysis is as follows:

$$\|\tilde{x}_2\| \le \left\|W_{22}^{-1}W_{21}\right\|\|\tilde{x}_1\| + \|T_2K\|\|e\|,$$

while $\|e\|^2 < \sigma\frac{\lambda_{\max}(\bar{X}_{11}^2)\|\tilde{x}_1\|^2}{\lambda_{\min}(\Phi^T\Phi+\xi I)}$.

So

$$\|\tilde{x}_2\| \le \gamma\|\tilde{x_1}\|, \tag{15}$$

where $\gamma = \|W_{22}^{-1}W_{21}\| + \|T_2K\|\sqrt{\frac{\sigma\lambda_{\max}(\bar{X}_{11}^2)}{\lambda_{\min}(\Phi^T\Phi+\xi I)}}$. $\tilde{x}_2$ is bounded and $\lim_{t\to\infty}\|\tilde{x}_2\|=0$, that is, $\tilde{x}_2$ is asymptotically stable.

Thus, $\tilde{x}$ is asymptotically stable, and hence system (5) is admissible, then the admissibility of system (4) is proved.

In the following, $\bar{W}_{11}^T\bar{X}_{11} + \bar{X}_{11}\bar{W}_{11} + (\sigma + 1)\bar{X}_{11}^2 < 0$ is proved.

Consider the two matrices

$$\Sigma = \begin{bmatrix} I & 0 \\ -W_{22}^{-1}W_{21} & I \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} I & -W_{12}W_{22}^{-1} \\ 0 & I \end{bmatrix},$$

and transform W in a diagonal form as

$$\bar{W} = \Gamma W\Sigma = \begin{bmatrix} \bar{W}_{11} & 0 \\ 0 & \bar{W}_{22} \end{bmatrix}$$

with

$$\bar{W}_{11} = W_{11} - W_{12}W_{22}^{-1}W_{21}, \qquad \bar{W}_{22} = W_{22}.$$

Pre-multiplying and post-multiplying the both sides of LMI (12) by Σ^TV^T and $V\Sigma$, we have

$$\begin{aligned}
&\Sigma^TV^T\big((A+BK)^TZ + Z^T(A+BK) + (\sigma+1)\Psi^T\Psi\big)V\Sigma \\
&\quad = \Sigma^T\big(W^T\bar{X}\bar{E} + \bar{E}^T\bar{X}W + W^T\bar{E}_\perp^T\bar{Y} + \bar{Y}^T\bar{E}_\perp W\big)\Sigma + (\sigma+1)\Sigma^TV^T\Psi^T\Psi V\Sigma \\
&\quad = \bar{W}^T\bar{\bar{X}}\bar{\bar{E}} + \bar{\bar{E}}^T\bar{\bar{X}}\bar{W} + \bar{W}^T\bar{\bar{E}}_\perp^T\bar{\bar{Y}} + \bar{\bar{Y}}^T\bar{\bar{E}}_\perp\bar{W} + (\sigma+1)\Sigma^T\big(\bar{E}^T\bar{X}\bar{E}\big)^2\Sigma \\
&\quad = \begin{bmatrix} \bar{W}_{11}^T\bar{\bar{X}}_{11} + \bar{\bar{X}}_{11}\bar{W}_{11} + (\sigma+1)\bar{X}_{11}^2 & \Delta \\ \Delta & \Delta \end{bmatrix} \\
&\quad < 0,
\end{aligned} \tag{16}$$

where

$$\bar{\bar{X}} = \Gamma^{-T}\bar{X}\Gamma^{-1} = \begin{bmatrix} \bar{\bar{X}}_{11} & \Delta \\ \Delta & \Delta \end{bmatrix}$$

with

$$\bar{\bar{X}}_{11} = \bar{X}_{11}, \qquad \bar{\bar{E}} = \Gamma \bar{E} \Sigma = \bar{E}, \qquad \bar{\bar{E}}_{\perp} = \Sigma^{-1} \bar{E}_{\perp} \Gamma^{-1} = \bar{E}_{\perp}, \qquad \bar{\bar{Y}} = \Sigma^T \bar{Y} \Sigma.$$

The "Δ" signs represent some unimportant items.

In light of Lemma 1 and inequality (16), $\bar{W}_{11}^T \bar{X}_{11} + \bar{X}_{11} \bar{W}_{11} + (\sigma + 1)\bar{X}_{11}^2 < 0$ is obtained. □

In order to guarantee the event-triggered control referred above is available, the inter-sampling times should be strictly larger than zero.

Theorem 2 *Given system* (4) *and the controller gain* $K = RZ$, *in which* R, Z *are given in Theorem* 1, *then the inter-sampling times decided by event-triggered condition* (9) *are strictly positive.*

Proof Notice that

$$\frac{\|x\|}{\|Ex\|} = \frac{\|V\tilde{x}\|}{\|EV\tilde{x}\|}, \tag{17}$$

and

$$\frac{\|V\tilde{x}\|}{\|U\|\|EV\tilde{x}\|} \le \frac{\|V\tilde{x}\|}{\|\bar{E}\tilde{x}\|} \le \frac{\|V\tilde{x}\|}{\sqrt{\lambda_{\min}(U^T U)}\|EV\tilde{x}\|},$$

so we have

$$\frac{\sqrt{\lambda_{\min}(U^T U)}\|V\tilde{x}\|}{\|\bar{E}\tilde{x}\|} \le \frac{\|V\tilde{x}\|}{\|EV\tilde{x}\|} \le \frac{\|U\|\|V\tilde{x}\|}{\|\bar{E}\tilde{x}\|}. \tag{18}$$

Combining (17) and (18), one has

$$\sqrt{\lambda_{\min}\big(U^T U\big)\lambda_{\min}\big(V^T V\big)}\frac{\|\tilde{x}\|}{\|\bar{E}\tilde{x}\|} \le \frac{\|x\|}{\|Ex\|} \le \|U\|\|V\|\frac{\|\tilde{x}\|}{\|\bar{E}\tilde{x}\|}, \tag{19}$$

where

$$\frac{\|\tilde{x}\|}{\|\bar{E}\tilde{x}\|} = \sqrt{\frac{(\tilde{x}_1^T \tilde{x}_1 + \tilde{x}_2^T \tilde{x}_2)}{\tilde{x}_1^T \tilde{x}_1}}.$$

Inserting (15) into the above inequality (19), we obtain

$$\begin{aligned}\frac{\|x\|}{\|Ex\|} &\ge \sqrt{\lambda_{\min}\big(U^T U\big)\lambda_{\min}\big(V^T V\big)}\sqrt{1 + \frac{\|\tilde{x}_2\|^2}{\|\tilde{x}_1\|^2}} \\ &\ge \sqrt{\lambda_{\min}\big(U^T U\big)\lambda_{\min}\big(V^T V\big)} \\ &\triangleq b,\end{aligned}$$

and

$$\begin{aligned}\frac{\|x\|}{\|Ex\|} &\le \|U\|\|V\|\sqrt{1+\frac{\|\tilde{x}_2\|^2}{\|\tilde{x}_1\|^2}} \\ &\le \|U\|\|V\|\sqrt{1+\gamma^2} \\ &\triangleq b_0.\end{aligned}$$

Notice that the latter equation in (7) can be rewritten as

$$0 = W_{21}\tilde{x}_{1tk} + W_{22}\tilde{x}_{1tk} + (T_2M_1 - W_{21})\tilde{e}_1 - (W_{22} - T_2M_2)\tilde{e}_2.$$

Therefore, $W_{21}\tilde{x}_{1tk} + W_{22}\tilde{x}_{1tk} = 0$ holds, because $\tilde{e}_1 = 0$ and $\tilde{e}_2 = 0$ at $t = t_k$, and hence

$$\tilde{e}_2 = (W_{22} - T_2M_2)^{-1}(T_2M_1 - W_{21})\tilde{e}_1,$$

and

$$\begin{aligned}\frac{\|e\|}{\|Ex\|} &\le \|V\|\frac{\sqrt{\|\tilde{e}_1\|^2 + \|\tilde{e}_2\|^2}}{\|Ex\|} \\ &\le \|V\|\frac{\sqrt{1+\|(W_{22}-T_2M_2)^{-1}(T_2M_1-W_{21})\|^2}\|\tilde{e}_1\|}{\|Ex\|}.\end{aligned}$$

Besides that,

$$\frac{\|Ee\|}{\|Ex\|} = \frac{\|U^{-1}\bar{E}e\|}{\|Ex\|} \ge \sqrt{\lambda_{\min}\big(U^{-T}U^{-1}\big)}\frac{\|\tilde{e}_1\|}{\|Ex\|}.$$

Thus, there exists a scalar

$$b_1 \ge \|V\|\frac{\sqrt{1+\|(W_{22}-T_2M_2)^{-1}(T_2M_1-W_{21})\|^2}}{\sqrt{\lambda_{\min}(U^{-T}U^{-1})}}$$

such that

$$\frac{\|e\|}{\|Ex\|} \le b_1\frac{\|Ee\|}{\|Ex\|}.$$

Turn event-triggered condition (9) into the form

$$e^T\big(\Phi^T\Phi + \xi I\big)e < \sigma(Ex)^T\Theta^T\Theta(Ex),$$

where $\Theta = V^TE^TX$.

Because

$$\sqrt{\lambda_{\min}\big(V^{-T}V^{-1}\big)}\|x\| \le \|\tilde{x}\| \le \sqrt{1+\gamma^2}\|\tilde{x}_1\|,$$

the following inequality holds:

$$\begin{aligned}\sqrt{\frac{e^T(\Phi^T\Phi+\xi I)e}{(Ex)^T\Theta^T\Theta(Ex)}} &= \sqrt{\frac{e^T(\Phi^T\Phi+\xi I)e}{\tilde{x}_1^T\bar{X}_{11}^2\tilde{x}_1}} \\ &\le \sqrt{\frac{(1+\gamma^2)\lambda_{\max}(\Phi^T\Phi+\xi I)}{\lambda_{\min}(V^{-T}V^{-1})\lambda_{\min}(\bar{X}_{11}^2)}}\frac{\|e\|}{\|x\|} \\ &\le \frac{1}{b}\sqrt{\frac{(1+\gamma^2)\lambda_{\max}(\Phi^T\Phi+\xi I)}{\lambda_{\min}(V^{-T}V^{-1})\lambda_{\min}(\bar{X}_{11}^2)}}\frac{\|e\|}{\|Ex\|} \\ &\le \alpha\frac{\|Ee\|}{\|Ex\|}, \end{aligned} \tag{20}$$

where

$$\alpha = \frac{b_1}{b}\sqrt{\frac{(1+\gamma^2)\lambda_{\max}(\Phi^T\Phi+\xi I)}{\lambda_{\min}(V^{-T}V^{-1})\lambda_{\min}(\bar{X}_{11}^2)}}.$$

The above inequality (20) implies that the inter-sampling times T_k taking for $\frac{e^T(\Phi^T\Phi+\xi I)e}{(Ex)^T\Theta^T\Theta(Ex)}$ to evolve from 0 to σ are lower bounded by the time τ during which $\frac{\|Ee\|}{\|Ex\|}$ evolve from 0 to $\frac{\sqrt{\sigma}}{\alpha}$.

Let us look at the dynamics of $\frac{\|Ee\|}{\|Ex\|}$:

$$\begin{aligned}&\frac{d}{dt}\frac{\|Ee\|}{\|Ex\|} \\ &\quad= \frac{(Ee)^TE\dot{e}}{\|Ex\|\|Ee\|} - \frac{\|Ee\|(Ex)^TE\dot{x}}{\|Ex\|^3} \\ &\quad= \frac{(Ee)^T(-(A+BK)x-BKe)}{\|Ex\|\|Ee\|} - \frac{\|Ee\|(Ex)^T((A+BK)x+BKe)}{\|Ex\|^3} \\ &\quad\le a_0 + a_1\frac{\|Ee\|}{\|Ex\|} + a_2\left(\frac{\|Ee\|}{\|Ex\|}\right)^2,\end{aligned}$$

where $a_0 = b_0\|A+BK\|$, $a_1 = b_0\|A+BK\| + b_1\|BK\|$, $a_2 = b_1\|BK\|$.

Denoting the term $\frac{\|Ee\|}{\|Ex\|}$ by θ, we have $\dot{\theta} \le a_0 + a_1\theta + a_2\theta^2$ and $\theta(t) \le \eta(t,\eta_0)$, where $\eta(t,\eta_0)$ is the solution of

$$\dot{\eta} = a_0 + a_1\eta + a_2\eta^2, \tag{21}$$

the initial value of which is $\eta(0,\eta_0) = \eta_0 = 0$. Denote $\eta(\tau,\eta_0) = \eta_\tau = \frac{\sqrt{\sigma}}{\alpha}$ and by solving equality (21) we have

$$\tau = \frac{\ln\frac{(\eta_\tau+\beta_1-\beta_2)(\eta_0+\beta_1+\beta_2)}{(\eta_\tau+\beta_1+\beta_2)(\eta_0+\beta_1-\beta_2)}}{\varepsilon},$$

where $\varepsilon = \sqrt{a_1^2 - 4a_0a_2}$, $\beta_1 = \frac{a_1}{2a_2}$, $\beta_2 = \frac{\varepsilon}{2a_2}$.

Because

$$\frac{(\eta_\tau+\beta_1-\beta_2)(\eta_0+\beta_1+\beta_2)}{(\eta_\tau+\beta_1+\beta_2)(\eta_0+\beta_1-\beta_2)}>1$$

always holds and $\varepsilon>0$ when $b_0\neq b_1\frac{\|BK\|}{\|A+BK\|}$, we just need to choose a suitable b_1 to avoid $\varepsilon=0$, then $\tau>0$ can be obtained, so that $T_k\geq\tau>0$. □

5 Example

In this section, a simulation example is given to show the feasibility and efficiency of the obtained results. Now we consider the following descriptor system:

$$\begin{bmatrix}1&1&0\\0&1&1\\1&2&1\end{bmatrix}\dot{x}=\begin{bmatrix}-10&-10&1\\1&9&0\\0&-5&-100\end{bmatrix}x+\begin{bmatrix}1\\10\\0.3\end{bmatrix}u. \tag{22}$$

Obviously, E is a singular matrix and $\operatorname{rank}(E)=2$. The two matrices rendering $UEV=\operatorname{diag}\{1,1,0\}$ are

$$U=\begin{bmatrix}-0.1361&-0.1361&-0.2722\\0.7071&-0.7071&0\\-0.5774&-0.5774&0.5774\end{bmatrix},$$

$$V=\begin{bmatrix}-0.4082&0.7071&0.5774\\-0.8165&0.0000&-0.5774\\-0.4082&-0.7071&0.5774\end{bmatrix}.$$

By solving LMI (8), we get $\sigma=58.5327$,

$$P=\begin{bmatrix}2.6730&-1.8028&0\\ *&2.5270&0\\ *&*&58.5327\end{bmatrix},$$

$$Q=\operatorname{diag}\{58.5327;58.5327;1.9398\},$$

$$R=\begin{bmatrix}-0.3194&-5.3396&-5.9279\end{bmatrix}.$$

According to the Theorem 1, we have

$$K=\begin{bmatrix}1.2882&-2.8481&-4.2749\end{bmatrix},$$

$$\Phi=\begin{bmatrix}-6.3394&14.0160&21.0373\\-9.3525&20.6776&31.0361\end{bmatrix},$$

$$\Psi=\begin{bmatrix}0.0694&-0.5888&-0.6581\\0.3293&-0.4200&-0.7479\\0.0000&-0.0000&-0.0000\end{bmatrix}.$$

Besides, ξ is chosen to be 0.0001.

Fig. 1 The state trajectories under the event-triggered control

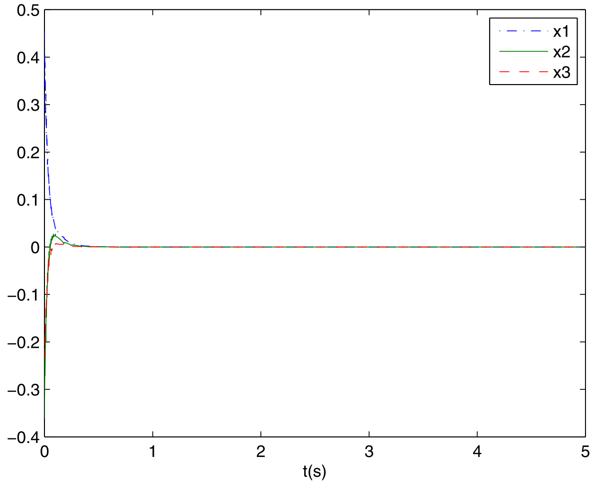
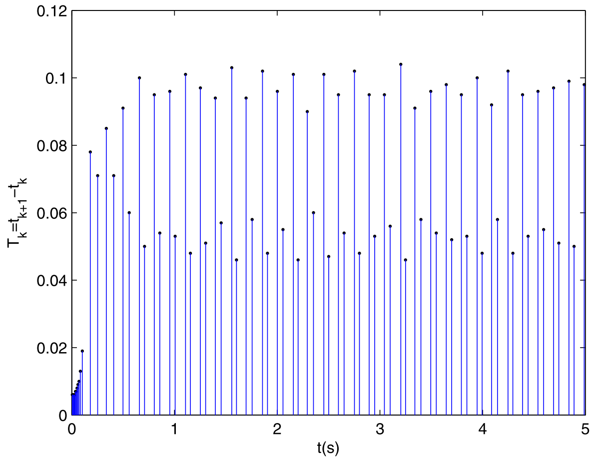


Fig. 2 The inter-sampling times of the event-triggered descriptor system

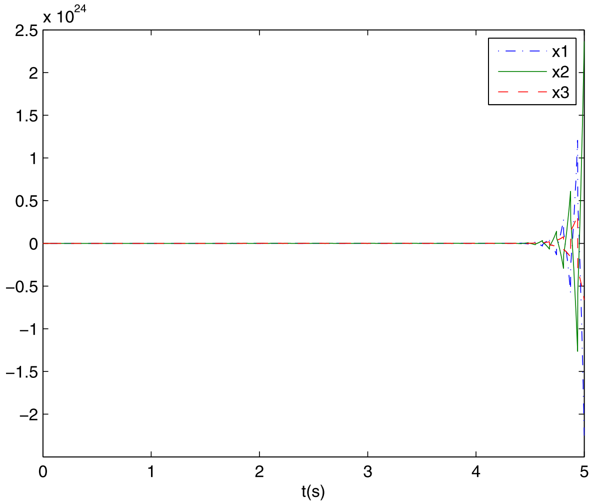


The response of the system under the event-triggered control decided by condition (9) is showed in Fig. 1. It indicates that the system is admissible, because the three states are all impulse free and tend to the zero equilibrium rapidly.

Figure 2 depicts the task periods of the event-triggered descriptor system. It shows that the inter-sampling times are strictly positive. Actually the task periods illustrated in Fig. 2 have an average value of 0.0648, and the maximum and minimum inter-sampling periods are 0.1040 and 0.0060, which illustrate that the inter-sampling times are lower bounded by a positive value.

Furthermore, we show in Fig. 3 the response of system (22) under a periodic control, the sampling period of which is 0.0648. In this case, the system is unstable. Thus we conclude that the proposed event-triggered control can save the use of resource compared with periodic control in terms of system (22).

Fig. 3 The state trajectories under the periodic control when $T_k = 0.0648$



6 Conclusion

This paper studied event-triggered control problem for linear descriptor systems. We proposed an approach to co-design event-triggered condition and controller yielding the admissibility of the system. A sufficient condition was obtained and presented in terms of LMI. Besides, we proved that the inter execution times are strictly positive which is essential to a practical control. The results of this paper were supported through an example. Future work can focus on the extension of previous results obtained in normal systems to descriptor systems, such as L_2 stability, model-based event-triggered control and self-triggered control.

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