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Finite spectrum assignment of linear systems with a class of non-commensurate delays

KEIJI WATANABE†

This paper is concerned with finite spectrum assignment of linear systems with non-commensurate delays. It is proved that if a feedforward pass between states variables does not have multi non-commensurate delays in parallel but only one or no delay in series, then the system is finite spectrum assignable even though the feedback pass has multi non-commensurate delays in parallel.

1. Introduction

In general, a linear system with delays in state variables has an infinite number of eigenvalues. A finite spectrum assignment is a static feedback control such that delay terms are eliminated from the characteristic equation of the corresponding closed-loop system and a finite number of eigenvalues are located at an arbitrarily preassigned set of points in the left-half part of the complex plane. The number of the assigned eigenvalues is equal to the dimension of the differential equation describing the system. The other eigenvalues are automatically moved to infinity of the left side of the complex plane.

The finite spectrum assignment problem of systems with delays in only controls has been completely solved by Manitius and Olbrot (1979). The problem of systems with commensurate delays in only the state variables was first studied by Maeda and Yamada (1975), Morse (1976) and Kamen (1978) on the basis of algebraic methods. Purely algebraic methods give simple solutions whenever quite restrictive controllability conditions are met. Manitius and Olbrot (1979) have broken through the restrictive condition by enlarging the class of feedback operator to include finite integrals over past value of state trajectory. They also showed that the necessary condition for finite spectrum assignment is for the system to be spectrally controllable and presented a design method of feedback law on the basis of finite Laplace transforms. Watanabe *et al.* (1983 a, b) and Watanabe and Ito (1984) proved that spectral controllability is also sufficient for finite spectrum assignment and presented a systematic design method of control law. Furthermore, Watanabe *et al.* (1984) have solved the finite spectrum assignment problem of systems with commensurate delays in both states and control. Hyun *et al.* (1987 a) presented a different finite spectrum assignment procedure of systems with commensurate delays in states and control. Manitius and Manousiouthakis (1985) have a sufficient condition for finite spectrum assignment of multivariable systems. Watanabe (1986) has proved that the necessary and sufficient condition for finite spectrum assignment of multivariable systems with commensurate delays in states and controls.

With these developments of finite spectrum assignment, our interests now lie in the finite spectrum assignment problem of systems with non-commensurate delays. Shin

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and Kitamori (1986) and Hyun *et al.* (1987 b) have presented a finite spectrum assignment procedure of systems with different delays in states and control.

In this paper, we study the finite spectrum assignment problem of systems with a class of non-commensurate delays in states and control. It is proved that if a feedforward pass between state variables does not have multi non-commensurate delays in parallel but only one or no delay in series, the system is finite spectrum assignable even though a feedback pass between states has multi non-commensurate delays in parallel. A systematic design procedure of the control law is suggested and a numerical example is given.

2. Preliminary

Consider the system

$$\left.\begin{aligned} \dot{x}(t) &= A_0 x(t) + A_1 x(t-h) + bu(t-L) \\ y(t) &= cx(t) \end{aligned}\right\} \tag{1}$$

where $x \in R^n$, $A_0, A_1 \in R^{n \times n}$, $b \in R^n$, $c \in R^{1 \times n}$, $h \geqq 0$ and $L \geqq 0$. There is the unique solution $x(t)$ of (1) with the initial conditions $x(0) = x_0 \in R^n$, $x(\tau) = 0$, $\tau < 0$, and $u(t) = 0$, $t \geqq -L$. The solution $x(t)$ is represented by using the state transition matrix $K(t)$ as follows.

$$x(t) = K(t)x_0 \tag{2}$$

Lemma 1

The state transition matrix $K(t)$ has the following properties.

(i) $$K(t) = 0, \quad t < 0 \tag{3}$$

(ii) $$K(t) = I, \quad t = 0 \tag{4}$$

(iii) $$\dot{K}(t) = A_0 K(t) = K(t) A_0, \quad 0 \leqq t \leqq h \tag{5}$$

(iv) $$\begin{aligned} \dot{K}(t) &= A_0 K(t) + A_1 K(t-h) \\ &= K(t) A_0 + K(t-h) A_1, \quad h \leqq t \end{aligned} \tag{6}$$

Proof

(i), (ii) and the first equations of (iii) and (iv) are self-evident. The second equations of (iii) and (iv) are proved as follows. Let $u(t) = 0$, $t \geqq -L$, and replace t in (1) by τ. The first equation of (1) yields

$$\dot{x}(\tau) = A_0 x(\tau) + A_1 x(\tau - h), \quad (\tau \geqq 0) \tag{7}$$

Premultiplying (7) by $K(t-\tau)$ and integrating (7) yield

$$\int_0^t K(t-\tau)\dot{x}(\tau)\, d\tau = \int_0^t K(t-\tau) A_0 x(\tau)\, d\tau + \int_0^t K(t-\tau) A_1 x(\tau - h)\, d\tau \tag{8}$$

The integral term on the left-hand side of (8) is expanded as

$$\begin{aligned} \int_0^t K(t-\tau)\dot{x}(\tau)\, d\tau &= [K(t-\tau)x(\tau)]_0^t - \int_0^t \left[\frac{d}{d\tau} K(t-\tau)\right] x(\tau)\, d\tau \\ &= -\int_0^t \left[\frac{d}{d\tau} K(t-\tau)\right] x(\tau)\, d\tau \end{aligned} \tag{9}$$

Since $x(\tau)=0$ for $\tau<0$, the second term on the right-hand side of (8) is rewritten as follows:

$$\int_0^t K(t-\tau)A_1 x(\tau-h)\,d\tau = \int_h^t K(t-\tau)A_1 x(\tau-h)\,d\tau = \int_0^{t-h} K(t-h-\tau)A_1 x(\tau)\,d\tau \tag{10}$$

Substituting (9) and (10) into (8) and with some manipulations, (8) can be written as

$$\int_0^{t-h}\left[\frac{d}{d\tau}K(t-\tau)+K(t-\tau)A_0+K(t-h-\tau)A_1\right]x(\tau)\,d\tau + \int_{t-h}^{t}\left[\frac{d}{d\tau}K(t-\tau)+K(t-\tau)A_0\right]x(\tau)\,d\tau=0 \tag{11}$$

Since (11) holds for any $x(\tau)$ satisfying (7), we have

$$\frac{d}{d\tau}K(t-\tau)+K(t-\tau)A_0+K(t-h-\tau)A_1=0, \quad 0\leqq\tau\leqq t-h \tag{12}$$

$$\frac{d}{d\tau}K(t-\tau)+K(t-\tau)A_0=0, \quad t-h\leqq\tau\leqq t \tag{13}$$

Equations (12) and (13) imply

$$\frac{d}{dt}K(t)=K(t)A_0+K(t-h)A_1, \quad h\leqq t \tag{14}$$

$$\frac{d}{dt}K(t)=K(t)A_0, \quad 0\leqq t\leqq h \tag{15}$$

This completes the proof. □

Let $x(t)$ be the state variable of (1) with arbitrary initial conditions $x(\tau)=\phi(\tau)$ $(-h\leqq\tau\leqq 0)$ and $u(\tau)=\psi(\tau)$ $(-L\leqq\tau<0)$. Define $z(t+\theta)$ $(\theta\leqq L)$ as follows:

$$z(t+\theta)=x(t+\theta) \quad (\theta\leqq 0) \tag{16 a}$$

$$z(t+\theta)=K(\theta)x(t)+\int_{t-h}^{t-h+\theta}K(t-h+\theta-\tau)A_1x(\tau)\,d\tau + \int_{t-L}^{t-L+\theta}K(t-L+\theta-\tau)bu(\tau)\,d\tau \quad (0\leqq\theta\leqq h) \tag{16 b}$$

$$z(t+\theta)=K(\theta)x(t)+\int_{t-h}^{t}K(t-h+\theta-\tau)A_1x(\tau)\,d\tau + \int_{t-L}^{t-L+\theta}K(t-L+\theta-\tau)bu(\tau)\,d\tau \quad (h\leqq\theta) \tag{16 c}$$

Differentiating (16) and using Lemma 1, (1) is transformed to

$$\left.\begin{aligned}\dot z(t+\theta)&=A_0z(t+\theta)+A_1z(t+\theta-h)+bu(t+\theta-L)\\ y(t)&=cz(t)\end{aligned}\right\} \tag{17}$$

Let $\theta = L$ and then we have

$$\left.\begin{aligned} \dot{z}(t+L) &= A_0 z(t+L) + A_1 z(t+L-h) + bu(t) \\ y(t) &= cz(t) \end{aligned}\right\} \tag{18}$$

This transformation can be extended to the system

$$\left.\begin{aligned} \dot{x}(t) &= \sum_{i=0}^{p} A_i x(t-h_i) + bu(t-L) \\ y(t) &= cx(t) \end{aligned}\right\} \tag{19}$$

where $0 = h_0 = h_1 < h_2 < \dots < h_p$. Denote the state transition matrix of (19) by $K(t)$ $(t \geqq 0)$. Define $z(t+\theta)$ $(\theta \leqq h_p)$ by

$$z(t+\theta) = x(t+\theta) \quad (\theta \leqq 0) \tag{20 a}$$

$$\begin{aligned} z(t+\theta) = K(\theta)x(t) &+ \sum_{i=0}^{j} \int_{t-h_i}^{t} K(t-h_i+\theta-\tau)A_i x(\tau)\, d\tau \\ &+ \sum_{i=j+1}^{p} \int_{t-h_i}^{t-h_i+\theta} K(t-h_i+\theta-\tau)A_i x(\tau)\, d\tau \\ &+ \int_{t-L}^{t-L+\theta} K(t-L+\theta-\tau)bu(\tau)\, d\tau \quad (h_j \leqq \theta < h_{j+1}) \end{aligned} \tag{20 b}$$

Differentiating (20), (19) is transformed to

$$\left.\begin{aligned} \dot{z}(t+L) &= \sum_{i=0}^{p} A_i z(t+L-h_i) + bu(t) \\ y(t) &= cz(t) \end{aligned}\right\} \tag{21}$$

3. Main result

Consider a system

$$\dot{x}(t) = \sum_{i=0}^{p} A_i x(t-h_i) + bu(t-L) \tag{22}$$

where $x \in R^n$, $u \in R$, $0 = h_0 < h_1 < \dots < h_p$ and $L \geqq 0$. For this system, consider a linear feedback control

$$\begin{aligned} u(t) = \sum_i \sum_j f_{ij} x(t-jh_i) &+ \int_{-Mh_p}^{0} \xi(\tau)x(t+\tau)\, d\tau \\ &+ \int_{-NL}^{0} \zeta(\tau)u(t+\tau)\, d\tau \end{aligned} \tag{23}$$

where M and N are positive integers, $\xi \in L_2([-Mh_p, 0], R^{1\times n})$ and $\zeta \in L_2[-NL, 0], R)$. The characteristic function of the closed-loop system is given by

$$\Delta_f(s) = \begin{vmatrix} sI - A(s) & -b(s) \\ -f_1(s) & 1 - f_2(s) \end{vmatrix} \tag{24}$$

where

$$\left.\begin{aligned} A(s) &= \sum_{i=0}^{p} A_i \exp(-s_j h_i), \quad b(s) = b \exp(-sL) \\ f_1(s) &= \sum_i \sum_j f_{ij} \exp(-s_j h_i) + \int_{-Mh_p}^{0} \xi(\tau) \exp(s\tau)\, d\tau \\ f_2(s) &= \int_{-NL}^{0} \zeta(\tau) \exp(s\tau)\, d\tau \end{aligned}\right\} \tag{25}$$

Definition 1

The system (22) is said to be finite spectrum assignable if there are $f_1(s)$ and $f_2(s)$ such that

$$\Delta_f(s) = s^n + \beta_1 s^{n-1} + \ldots + \beta_n \tag{26}$$

for arbitrarily preassigned real numbers $\beta_1, \beta_2, \ldots, \beta_n$.

Theorem 1

Let $U_c(s)$ be denoted by

$$U_c(s) = [b(s) \quad A(s)b(s) \quad \ldots \quad A^{n-1}(s)b(s)] \tag{27}$$

If the system satisfies

$$\det U_c(s) = \alpha \exp(-s\lambda) \tag{28}$$

where α is a non-zero real number and $\lambda \geqq 0$, then the system (22) is finite spectrum assignable.

Proof

Applying the transformations similar to the elementary row operations (Wolovich 1974) to $U_c(s)$, we have

$$T(s)U_c(s) = \begin{bmatrix} 0 & & * \\ & \diagup & \vdots \\ * & \cdots & * \end{bmatrix} \tag{29}$$

where $T(s)$ is the matrix with real numbers and exponential terms $\exp(-s\rho)$ $(\rho \geqq 0)$ and

$$\det T(s) = (\text{non-zero real number}) \tag{30}$$

Define $z_0(s)$ by

$$z_0(s) = T(s)x(s) \tag{31}$$

From (28), (29) and (31), (22) is transformed to

$$sz_0(s) = F_0(s)z_0(s) + g(s)u(s) \tag{32}$$

where

$$F_0(s) = T(s)A(s)T^{-1}(s)$$
$$= \begin{bmatrix} * & \exp(-s\rho_1) & & 0 \\ \vdots & & \ddots & \exp(-s\rho_{n-1}) \\ * & \cdots & & * \end{bmatrix} \tag{33}$$

$$g(s) = T(s)b(s) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \exp(-sL) \end{bmatrix} \tag{34}$$

From (30) and (33), we have

$$|sI - F_0(s)| = |T(s)(sI - A(s))T^{-1}(s)| = |sI - A(s)| \tag{35}$$

We restrict attention to the first row of (32). $F_0(s)$ is rewritten as

$$F_0(s) = \begin{array}{c} \begin{bmatrix} F_{01}(s) & F_{02}\exp(-s\rho_1) \\ F_{03}(s) & F_{04}(s) \end{bmatrix} \begin{array}{l} 1 \\ n-1 \end{array} \\ \begin{array}{cc} \leftarrow 1 \rightarrow & \qquad \leftarrow n-1 \rightarrow \end{array} \end{array} \tag{36}$$

Applying the transformation of (20) to the first row of (32), there is a transformation

$$z_1(s, \theta) = K_1(s, \theta)z_0(s)$$
$$= \begin{bmatrix} k_{11}(s,\theta) & k_{12}(s,\theta) & 0 & \dots & 0 \\ 0 & 1 & & \ddots & 0 \\ 0 & 0 & & & 1 \end{bmatrix} z_0(s) \tag{37}$$

such that

$$sz_1(s, \rho_1) = F_1(s)z_1(s, \rho_1) + g(s)u(s) \tag{38}$$

$$F_1(s) = \begin{bmatrix} F_{01}(s) & F_{01} \\ F_{02}(s)\exp(-s\rho_1) & F_{04}(s) \end{bmatrix}$$
$$= \begin{bmatrix} * & 1 & & 0 \\ \vdots & \exp(-s\rho_2) & \ddots & \\ \vdots & & \ddots & \exp(-s\rho_{n-1}) \\ * & \cdots & & * \end{bmatrix}$$
$$= \begin{array}{c} \begin{bmatrix} F_{11}(s) & F_{12}\exp(-s\rho_2) \\ F_{13}(s) & F_{14}(s) \end{bmatrix} \begin{array}{l} 2 \\ n-2 \end{array} \\ \begin{array}{cc} \leftarrow 2 \rightarrow & \qquad \leftarrow n-2 \rightarrow \end{array} \end{array} \tag{39}$$

where $k_{11}(s, \theta)$ and $k_{12}(s, \theta)$ are the finite Laplace transforms. From (36) and the first equation of (39), we obtain

$$|sI - F_1(s)| = |sI - F_0(s)| = |sI - A(s)| \tag{40}$$

Applying the transformation of (20) to the first two rows of (38), we have

$$z_2(s, \theta) = K_2(s, \theta) z_1(s, \rho_1) \tag{41}$$

and

$$s z_2(s, \rho_2) = F_2(s) z_2(s, \rho_2) + g(s) u(s) \tag{42}$$

$$F_2(s) = \begin{bmatrix} F_{11}(s) & F_{12} \\ F_{13}(s) \exp(-s\rho_2) & F_{14}(s) \end{bmatrix}$$

$$= \begin{bmatrix} * & 1 & & & 0 \\ \vdots & & 1 & & \\ \vdots & & \ddots & \exp(-s\rho_3) & \\ \vdots & & & \ddots & \exp(-s\rho_{n-1}) \\ * & \cdots & \cdots & \cdots & * \end{bmatrix} \tag{43}$$

From (39), (43) and (40), the following equation holds.

$$|sI - F_2(s)| = |sI - F_1(s)| = |sI - A(s)| \tag{44}$$

Continuing this transformation, we have

$$z_{n-1}(s, \theta) = K_{n-1}(s, \theta) z_{n-2}(s, \rho_{n-2}) \tag{45}$$

and

$$s z_{n-1}(s, \rho_{n-1}) = F_{n-1}(s) z_{n-1}(s, \rho_{n-1}) + g(s) u(s) \tag{46}$$

$$F_{n-1}(s) = \begin{bmatrix} * & 1 & & 0 \\ \vdots & \ddots & 1 & \\ \vdots & & \ddots & 1 \\ * & \cdots & \cdots & * \end{bmatrix} \tag{47}$$

In the same manner as (44), we have

$$|sI - F_{n-1}(s)| = |sI - A(s)| \tag{48}$$

From (20) and (21), there is a transformation

$$z_n(s, \theta) = K_n(s, \theta) z_{n-1}(s, \rho_{n-1}) + K_{n+1}(s, \theta) u(s) \tag{49}$$

such that

$$sz_n(s, L) = F_{n-1}(s)z_n(s, L) + g_0 u(s) \tag{50}$$

$$g_0 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{51}$$

As seen from (47), there is a transformation

$$z(s) = K_m(s)z_n(s, L) \tag{52}$$

such that

$$sz(s) = F(s)z(s) + g_0 u(s) \tag{53}$$

$$F(s) = \begin{bmatrix} 0 & 1 & & 0 \\ \vdots & \ddots & 1 & \\ \vdots & & \ddots & \ddots \\ 0 & \cdots & 0 & 1 \\ * & & & * \end{bmatrix} \tag{54}$$

where $K_m(s)$ is a matrix with real numbers and exponential terms, det $K_m(s) =$ (non-zero real number) and $*$ denotes quasi-polynomials of exponentials with real coefficients. It follows from (48) and (52) that

$$|sI - F(s)| = |sI - F_{n-1}(s)| = |sI - A(s)| \tag{55}$$

Summarizing these transformations, we have

$$z(s) = K_a(s)x(s) + K_b(s)u(s) \tag{56}$$

$$k_a(s) = K_m(s)K_n(s, L)K_{n-1}(s, \rho_{n-1}) \dots K_1(s, \rho_1)T(s) \tag{57}$$

$$K_b(s) = K_m(s)K_{n+1}(s, L) \tag{58}$$

and (53) is obtained.

Since $(F(s), g_0)$ is reachable as seen from (54), there is a $f(s)$ with quasi-quasi-polynomials of exponentials with real coefficients such that

$$|sI - F(s) - g_0 f(s)| = s^n + \beta_1 s^{n-1} + \dots + \beta_n \tag{59}$$

Using $f(s)$, the control for (1) is given by

$$u(s) = f(s)z(s) = f(s)K_a(s)x(s) + f(s)K_b u(s) \tag{60}$$

The characteristic function of the closed-loop system constructed from (22) and (60) is given by

$$\Delta_f(s) = \begin{vmatrix} sI - A(s) & -b(s) \\ -f(s)K_a(s) & 1 - f(s)K_b(s) \end{vmatrix}$$

$$= |sI - A(s)|\,|1 - f(s)K_b(s) - f(s)K_a(s)(sI - A(s))^{-1}b(s)| \tag{61}$$

Substituting (22) and (53) into (56), we have

$$(sI - F(s))^{-1}g_0 = K_a(s)(sI - A(s))^{-1}b(s) + K_b(s) \tag{62}$$

Substituting (55) and (62) into (61) yields

$$\begin{aligned}\Delta_f(s) &= |sI - F(s)|\,|I - f(s)(sI - F(s))^{-1}g_0| \\ &= |sI - F(s)|\,|I - (sI - F(s))^{-1}g_0 f(s)| \\ &= |sI - F(s) - g_0 f(s)|\end{aligned} \tag{63}$$

Equations (59) and (63) yield

$$\Delta_f(s) = s^n + \beta_1 s^{n-1} + \ldots + \beta_n \tag{64}$$

The system (22) which satisfies (28) is finite spectrum assignable. This completes the proof. □

The control law can be computed according to the procedure of the proof. This is shown in the following numerical example.

4. Numerical example

Consider the system

$$\begin{aligned}\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} &= \begin{bmatrix}1 & 0\\ 1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1(t-0\cdot 8)\\ x_2(t-0\cdot 8)\end{bmatrix} \\ &\quad + \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1(t-1)\\ x_2(t-1)\end{bmatrix} + \begin{bmatrix}0\\ 1\end{bmatrix}u(t-0\cdot 5)\end{aligned} \tag{65}$$

In this system, $A(s)$ and $b(s)$ are given by

$$A(s) = \begin{bmatrix}1 + \exp(-0\cdot 8s) & \exp(-s)\\ 1 & 0\end{bmatrix}, \quad b(s) = \begin{bmatrix}0\\ \exp(-0\cdot 5s)\end{bmatrix} \tag{66}$$

The matrix $U_c(s)$ is given by

$$U_c(s) = [b(s) \quad A(s)b(s)] = \begin{bmatrix}0 & \exp(-1\cdot 5s)\\ \exp(-0\cdot 5s) & 0\end{bmatrix} \tag{67}$$

Since $\det U_c(s) = -\exp(-2s)$, the system is finite spectrum assignable by Theorem 1. The control can be determined as follows.

We first consider the transformation (29). In this case, $T(s) = I$. It follows that $z_0(s) = x(s)$ in (31), $F_0(s) = A(s)$ and $g(s) = b(s)$ in (32).

Then we turn to the transformation (37). Since the first row of (65) is written as

$$\dot{x}_1(t) = x_1(t) + x_1(t - 0\cdot 8) + x_2(t-1) \tag{68}$$

denote the state $z_1(s, \theta)$ in (37) as $z_1(t, \theta) = [z_{11}(t+\theta) \quad z_{12}(t)]^{\mathrm{T}}$ in the time domain.

They are given by

$$z_{11}(t+\theta) = x_1(t+\theta) \quad (-0\cdot 1 \leqq \theta \leqq 0) \tag{69 a}$$

$$z_{11}(t+\theta) = k_1(\theta)x_1(t) + \int_{t-0\cdot 8}^{t-0\cdot 8+\theta} k_1(t-0\cdot 8+\theta-\tau)x_1(\tau)\,d\tau + \int_{t-1}^{t-1+\theta} K_1(t-1+\theta-\tau)x_2(\tau)\,d\tau \quad (0 \leqq \theta \leqq 0\cdot 8) \tag{69 b}$$

$$z_{11}(t+\theta) = k_1(\theta)x_1(t) + \int_{t-0\cdot 8}^{t} k_1(t-0\cdot 8+\theta-\tau)x_1(\tau)\,d\tau + \int_{t-1}^{t-1+\theta} k_1(t-1+\theta-\tau)x_2(\tau)\,d\tau \quad (0\cdot 8 \leqq \theta \leqq 1) \tag{69 c}$$

and

$$z_{12}(t) = x_2(t) \tag{69 d}$$

where $k_1(t)$ is the function satisfying

$$\dot{k}_1(t) = k_1(t) + k_1(t-0\cdot 8) \tag{70}$$

and $k_1(t)$ and $z_{11}(t+\theta)$ can be computed numerically. This transformation yields

$$s\begin{bmatrix} z_{11}(s)\exp(s) \\ z_{12}(s) \end{bmatrix} = \begin{bmatrix} 1+\exp(-0\cdot 8s) & 1 \\ \exp(-s) & 0 \end{bmatrix}\begin{bmatrix} z_{11}(s)\exp(s) \\ z_{12}(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(s)\exp(-0\cdot 5s) \tag{71}$$

Let $z_2(s,\theta)$ in (41) be represented by $z_2(t,\theta) = [z_{21}(t+\theta) \quad z_{22}(t+\theta)]^{\mathrm{T}}$ in the time domain. The transformation (41) can be accomplished as follows.

$$\begin{bmatrix} z_{21}(t+\theta) \\ z_{22}(t+\theta) \end{bmatrix} = \begin{bmatrix} z_{11}(t+1+\theta) \\ z_{12}(t+\theta) \end{bmatrix} \quad (-1\cdot 1 \leqq \theta \leqq 0)$$

$$\begin{bmatrix} z_{21}(t+\theta) \\ z_{22}(t+\theta) \end{bmatrix} = K_2(\theta)\begin{bmatrix} z_{11}(t+1) \\ z_{12}(t) \end{bmatrix} + \int_{t-0\cdot 8}^{t-0\cdot 8+\theta} K_2(t-0\cdot 8+\theta-\tau)\begin{bmatrix} z_{11}(\tau+1) \\ z_{12}(\tau) \end{bmatrix} d\tau + \int_{t-1}^{t-1+\theta} K_2(t-1+\theta-\tau)\begin{bmatrix} z_{11}(\tau+1) \\ z_{12}(\tau) \end{bmatrix} d\tau + \int_{t-0\cdot 5}^{t-0\cdot 5+\theta} K_2(t-0\cdot 5+\theta-\tau)\begin{bmatrix} 0 \\ 1 \end{bmatrix} u(\tau)\,d\tau \quad (0 \leqq \theta \leqq 0\cdot 5)$$

where $K_2(t)$ is the function matrix satisfying

$$\dot{K}_2(t) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} K_2(t) + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} K_2(t-0\cdot 8) + \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} K_2(t-1) \tag{73}$$

The transformation (72) yields

$$s\begin{bmatrix} z_{21}(s)\exp(0\cdot5s) \\ z_{22}(s)\exp(0\cdot5s) \end{bmatrix} = \begin{bmatrix} 1+\exp(-s0\cdot8) & 1 \\ \exp(-s) & 0 \end{bmatrix}\begin{bmatrix} z_{21}(s) \\ z_{22}(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(s) \tag{74}$$

We can consider the transform (52). Let

$$\begin{bmatrix} z_{31}(s) \\ z_{32}(s) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1+\exp(-0\cdot8s) & 1 \end{bmatrix}\begin{bmatrix} z_{21}(s)\exp(0\cdot5s) \\ z_{22}(s)\exp(0\cdot5s) \end{bmatrix} \tag{75}$$

then we have

$$s\begin{bmatrix} z_{31}(s) \\ z_{32}(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \exp(-s) & 1+\exp(-0\cdot8s) \end{bmatrix}\begin{bmatrix} z_{31}(s) \\ z_{32}(s) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(s) \tag{76}$$

If the eigenvalues of the closed-loop system are required to be -2 and -2, then $f(s)$ is given by

$$f(s) = [-\exp(-s) - 4 \quad -(1+\exp(-0\cdot8s) - 4] \tag{77}$$

and the control $u(s)$ is given by

$$u(s) = f(s)\begin{bmatrix} z_{31}(s) \\ z_{32}(s) \end{bmatrix} \tag{78}$$

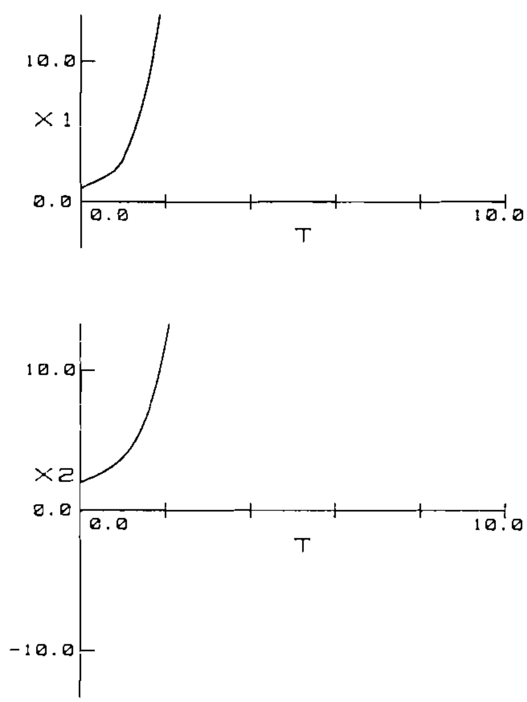

Figure 1. Free response of the controlled system.

The free response of the system (65) with the initial conditions $x_1(0) = 1$, $x_2(0) = 1$, $x_1(\tau) = x_2(\tau) = 0$ ($\tau < 0$) is shown in Fig. 1. This system is unstable. The response of the closed-loop system consisting of (65) and (78) is shown in Fig. 2. It is seen that the theory established here works well.

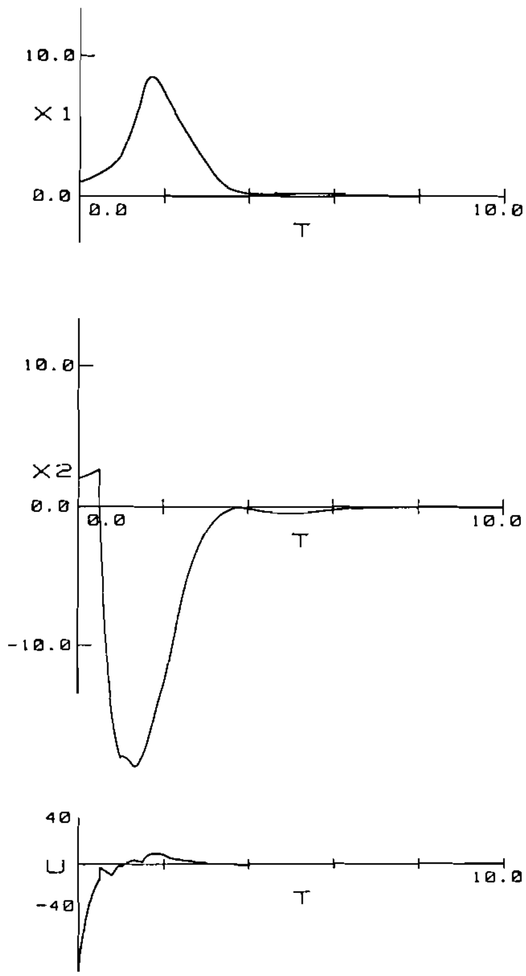

Figure 2. Response of the closed-loop system.

5. Conclusion

In this paper, we prove that if the feedforward pass does not have multi non-commensurate delays in parallel but only one or no delay in series, then the system is finite spectrum assignable even though a feedback pass has multi non-commensurate delays in parallel.

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