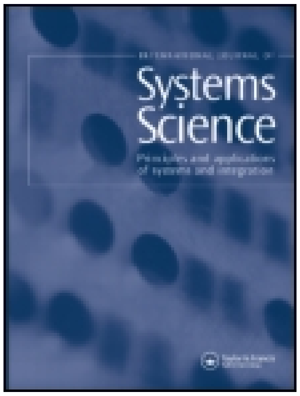


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# A survey of linear matrix inequality techniques in stability analysis of delay systems

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# A survey of linear matrix inequality techniques in stability analysis of delay systems

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Recent years have witnessed a resurgence of research interests in analysing the stability of time-delay systems. Many results have been reported using a variety of approaches and techniques. However, much of the focus has been laid on the use of the Lyapunov–Krasovskii theory to derive sufficient stability conditions in the form of linear matrix inequalities. The purpose of this article is to survey the recent results developed to analyse the asymptotic stability of time-delay systems. Both delay-independent and delay-dependent results are reported in the article. Special emphases are given to the issues of conservatism of the results and computational complexity. Connections of certain delay-dependent stability results are also discussed.

**Keywords:** delay-independent stability; delay-dependent stability; linear matrix inequality; time-delay systems



## 1. Introduction

Time delays are frequently encountered in various engineering systems such as long transmission lines in pneumatic systems, nuclear reactors, rolling mills, hydraulic systems and manufacturing processes (Hale 1977; Gorecki, Fuska, Grabowski, and Korytowski 1989; Hale and Verduyn Lunel 1993; Kolmanovskii and Myshkis 1999; Niculescu 2001a; Gu, Kharitonov, and Chen 2003). Time-delay systems are usually described by functional differential equations; they represent a class of infinite-dimensional systems. Time-delay systems are also referred to as hereditary systems, time-lag systems, dead-time systems, systems with aftereffects, or differential-difference equations (Gu and Niculescu 2003; Richard 2003). It has been shown that time delay is often a source of the generation of oscillation and a source of instability of control systems (Kolmanovskii and Myshkis 1992). Therefore, the problem of stability analysis and control of time-delay systems has attracted much attention during the past years, which is of both practical and theoretical importance. Various types of time-delay systems have been investigated and a great number of results on time-delay systems have been reported in the literature (see, e.g. Chen and Latchman 1995; Chu 1997; Hui and Hu 1997; Dugard and Verriest 1998; Su and Chu 1999; Hmamed 2000; Shi, Agarwal, Boukas, and Shue 2000; Wang and Unbehauen 2000; Liu 2001; Niculescu 2001a; Boukas and Liu 2002; Boukas, Liu, and Shi 2002; Fridman and Shaked 2002a; Xu, Lam, and Yang 2002; Xu, Van Dooren, Stefan, and Lam 2002; Lu, Ho, and Yeung 2003; Niu, Ho, and Lam 2005; Zhou and Li 2005; Chen, Liu, and Tong 2006; Niu and Ho 2006; Shu, Lam, and Xu 2006; Mahmoud, Shi, and Nounou 2007; Shi, Zhang, Qiu, and Xing 2007; Sun, Zhao, and Wang 2007, and the references cited therein).

Usually, stability conditions for time-delay systems can be classified into two types: delay-dependent and delay-independent stability conditions; the former include the information on the size of the delay, while the latter do not. Generally speaking, delay-independent stability conditions are simpler to apply, while delay-dependent stability conditions are less conservative especially in the case when the time delay is small. The objective of the study of the delay-dependent stability problem is twofold:

(1) to develop delay-dependent conditions to provide a maximal allowable delay as large as possible,
(2) to develop delay-dependent conditions by using as few as possible decision variables while keeping the same maximal allowable delay.

In the literature, various approaches have been proposed to obtain delay-dependent stability conditions, among which the linear matrix inequality (LMI) approach is the most popular and has played an important role due to the fact that LMIs can be cast

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into a convex optimisation problem which can be handled efficiently by resorting to recently developed numerical algorithms for solving LMIs (Boyd, El Ghaoui, Feron, and Balakrishnan 1994). Another reason that makes LMI conditions appealing is their frequent readiness to solve the corresponding synthesis problems once the stability (or other performance) conditions have been established, especially when state feedback is employed. The main purpose of the article is to review the recent development on the LMI techniques in deriving both delay-independent and delay-dependent stability results for time-delay systems. Emphases will be given on issues concerned with the conservatism and the computational complexity of the results. Key technical bounding lemmas and slack variable introduction approaches will be discussed. The results will be compared and connections of certain delay-dependent stability results are also discussed.

**Notation:** Throughout this article, for real symmetric matrices $X$ and $Y$, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semi-definite (respectively, positive definite). $I$ is an identity matrix with appropriate dimension. The superscript 'T' represents the transpose. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions.

## 2. Basic stability theories

In the study of stability analysis of time-delay systems, the methods of Lyapunov functions and Lyapunov–Krasovskii functionals play important roles. Before re-stating these methods, we introduce the following notations.

For a given scalar $h > 0$, let $\mathcal{C}_n = \mathcal{C}([-h, 0], \mathbb{R}^n)$ be the Banach space of continuous vector functions mapping the interval $[-h, 0]$ into $\mathbb{R}^n$ with the topology of uniform convergence. For any $\phi \in \mathcal{C}_n$, its norm is defined by

$$\|\phi\|_c = \sup_{-h \leq s \leq 0} \left|\phi(s)\right|,$$

where $|\phi(s)|$ denotes the Euclidean norm of $\phi(s) \in \mathbb{R}^n$. Define a set

$$\mathcal{C}_n^a = \left\{\phi \in \mathcal{C}_n \middle| \|\phi\|_c < a\right\},$$

for some scalar $a > 0$.

Now, consider a time-delay system described by the following differential-difference equation:

$$\dot{x}(t) = f(t, x_t), \quad t \geq t_0, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $x_t$ is defined by

$$x_t = x(t + \theta), \quad -h \leq \theta \leq 0.$$

We assume that the function $f$: $\mathbb{R}^+ \times \mathcal{C}_n \to \mathbb{R}^n$ is continuous and $f(t, 0) = 0$ for all $t \in \mathbb{R}$. The initial condition of the time-delay system in (1) is set by

$$x_{t_0}(\theta) = \phi(\theta), \quad -h \leq \theta \leq 0. \tag{2}$$

It is assumed that for any $\phi \in \mathcal{C}_n$ and for any $t_0 \in \mathbb{R}$, the time-delay system in (1) with the initial condition (2) has a unique solution.

Now, we are in a position to present the method of Lyapunov–Krasovskii functionals.

**Theorem 1** (Krasovskii Stability Theorem): (Hale and Verduyn Lunel 1993) *Suppose that the function $f$ takes bounded sets of $\mathcal{C}_n$ in bounded sets of $\mathbb{R}^n$, and $u, v, w$: $\mathbb{R}^+ \to \mathbb{R}^+$ are continuous, non-decreasing functions satisfying $u(0) = v(0) = 0$ and $u(s)$, $v(s) > 0$ for $s > 0$. If there exists a continuous function $V$: $\mathbb{R} \times \mathcal{C}_n \to \mathbb{R}^+$ such that*

(a) $u(|x|) \leq V(t, x_t) \leq v(|x_t|_c)$.
(b) *The derivative of $V(t, x_t)$ along the solution of* (1) *and* (2), *defined as*

$$\dot{V}(t, x_t) = \lim_{s \to 0^+} \sup \frac{1}{s}[V(t + s, x_{t+s}) - V(t, x_t)],$$

*satisfies*

$$\dot{V}(t, x_t) \leq -w\left(\left|x(t)\right|\right)$$

*then the trivial solution $x = 0$ of the time-delay system in* (1) *and* (2) *is uniformly stable.*

*If $u(s) \to \infty$ as $s \to \infty$, the solutions of the time-delay system in* (1) *and* (2) *are uniformly bounded.*

*If $w(s) > 0$ for $s > 0$, then the solution $x = 0$ is uniformly asymptotically stable.*

**Theorem 2** (Razumikhin Stability Theorem): (Hale and Verduyn Lunel 1993) *Suppose that the function $f$ takes bounded sets of $\mathcal{C}_n$ in bounded sets of $\mathbb{R}^n$ and suppose that $u$, $v$, $w$ : $\mathbb{R}^+ \to \mathbb{R}^+$ are continuous, non-decreasing functions, $u(s)$, $v(s)$, $w(s)$ are positive for $s > 0$, $u(0) = v(0) = 0$. Let $p : \mathbb{R}^+ \to \mathbb{R}^+$ be a continuous non-decreasing function satisfying $p(s) > s$ for $s > 0$. If there exists a continuous function $V : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^+$ such that*

(a) $u(|x|) \leq V(t, x) \leq v(|x|), \quad \forall t \in \mathbb{R},\ x \in \mathbb{R}^n$.
(b) *The derivative of $V(t, x)$ along the solution of* (1) *and* (2), *defined as*

$$\dot{V}(t, x(t)) = \lim_{s \to 0^+} \sup \frac{1}{s}[V(t + s, x(t + s)) - V(t, x(t))],$$

*satisfies*

$$\dot{V}(t, x(t)) \leq -w(|x(t)|)$$

*if*

$$V(t+\theta, x(t+\theta)) \leq p(V(t, x(t))), \quad \forall \theta \in [-h, 0],$$

*then the trivial solution of the time-delay system in* (1) *and* (2) *is uniformly asymptotically stable. Furthermore, if* $u\ (s) \to \infty$ *as* $s \to \infty$, *then the trivial solution is globally uniformly asymptotically stable.*

The following Halanay result also plays an important role in the stability analysis of time-delay systems.

**Theorem 3:** (Halanay 1966) *Suppose that constant scalars* $k_1$ *and* $k_2$ *satisfy* $k_1 > k_2 > 0$, *and* $y(t)$ *is a non-negative continuous function on* $[t_0 - \tau, t_0]$ *satisfying*

$$\frac{\mathrm{d}y(t)}{\mathrm{d}t} \leq -k_1 y(t) + k_2 \bar{y}(t),$$

*for* $t \geq t_0$, *where* $\tau \geq 0$ *and*

$$\bar{y}(t) = \sup_{t-\tau \leq s \leq t} \{y(s)\}.$$

*Then, for* $t \geq t_0$, *we have*

$$y(t) \leq \bar{y}(t_0) \exp(-\kappa(t - t_0)),$$

*where* $\kappa > 0$ *is the unique solution to the following equation*:

$$\kappa = k_1 - k_2 \exp(\kappa\tau).$$

**Remark 1:** Both Theorems 2 and 3 can be used to derive stability conditions for the case when the delay is time-varying, which is continuous but not necessarily differentiable. It is also worth pointing out that Theorem 2 can be used to obtain delay-dependent stability conditions for time-delay systems, which will be shown in the next section.

## 3. LMI stability conditions

For simplicity, we will review the LMI techniques in deriving stability results for the single-delay case. However, the LMI techniques presented in the following can be extended to the multiple-delay case in a straightforward manner. In this section, two classes of time-delay systems will be considered, that is,

$$(\Sigma_1): \quad \dot{x}(t) = Ax(t) + A_1 x(t-h) \tag{3}$$
$$x(t) = \phi(t), \quad \forall t \in [-h, 0] \tag{4}$$

and

$$(\Sigma_2): \quad \dot{x}(t) = Ax(t) + A_1 x(t - h(t)) \tag{5}$$
$$x(t) = \phi(t), \quad \forall t \in [-\bar{h}, 0] \tag{6}$$

where $x(t) \in \mathbb{R}^n$ is the state; $\phi(t)$ is the continuous initial condition. The scalar $h > 0$ is the constant delay of system $(\Sigma_1)$, while $h(t)$ is the time-varying delay of system $(\Sigma_2)$, which is assumed to be continuous and satisfies

$$0 < h(t) \leq \bar{h}. \tag{7}$$

In both the time-delay systems $(\Sigma_1)$ and $(\Sigma_2)$, $A$ and $A_1$ are known real constant matrices.

It is noted that stability results on $(\Sigma_1)$ obtained by the method of Lyapunov–Krasovskii functionals can be easily extended to systems with differentiable time-varying delays. Considering this, time-delay systems with differentiable time-varying delays are not considered, and attention will be focussed on the review of the LMI techniques in deriving both delay-independent and delay-dependent stability conditions for the time-delay systems $(\Sigma_1)$ and $(\Sigma_2)$.

### 3.1. *Delay-independent stability conditions*

For the time-delay system $(\Sigma_1)$, by choosing a Lyapunov–Krasovskii functional as

$$V(t, x_t) = x(t)^T P x(t) + \int_{t-h}^{t} x(\alpha)^T Q x(\alpha) \mathrm{d}\alpha, \tag{8}$$

and resorting to Theorem 1, the following stability condition can be obtained.

**Theorem 4:** (Verriest, Fan, and Kullstam 1993) *The time-delay system* $(\Sigma_1)$ *is asymptotically stable if there exist matrices* $P > 0$ *and* $Q > 0$ *such that*

$$\begin{bmatrix} PA + A^T P + Q & PA_1 \\ A_1^T P & -Q \end{bmatrix} < 0.$$

For the time-delay system $(\Sigma_2)$, since the time-varying delay $h(t)$ may not be differentiable, the Lyapunov–Krasovskii functional similar to (8) as

$$\tilde{V}(t, x_t) = x(t)^T P x(t) + \int_{t-h(t)}^{t} x(\alpha)^T Q x(\alpha) \mathrm{d}\alpha,$$

cannot be used to derive a stability condition. In this case, however, we can use Theorem 2 to give a delay-independent stability condition. Here, we choose a Lyapunov function as

$$V(x(t)) = x(t)^T P x(t), \tag{9}$$

and set

$$p(s) = \delta s, \quad w(s) = \epsilon s^2,$$

where $\delta$ and $\epsilon$ are any scalars satisfying $\delta > 1$ and $\epsilon > 0$. Then, we have the following result.

**Theorem 5:** *The time-delay system* $(\Sigma_2)$ *is asymptotically stable if there exists a matrix* $P > 0$ *such that*

$$\begin{bmatrix} PA + A^TP + P & PA_1 \\ A_1^TP & -P \end{bmatrix} < 0.$$

**Remark 2:** It is easy to see that the LMI condition in Theorem 5 is a special case of that in Theorem 4. Thus, Theorem 5 is more conservative than Theorem 4. However, it is worth pointing out that Theorem 5 can be applied to the case when the delay is time-varying and continuous, which may not be differentiable, while in the time-varying delay case, the use of Theorem 4 usually requires the considered delay being differentiable.

Now, for the time-delay system $(\Sigma_2)$, we note that for any matrices $Y$, $W$ and $S$ with appropriate dimensions, the following equalities hold:

$$\dot{x}(t)^T Y[Ax(t) + A_1x(t - h(t)) - \dot{x}(t)] = 0,$$
$$x(t)^T W[Ax(t) + A_1x(t - h(t)) - \dot{x}(t)] = 0,$$
$$x(t - h(t))^T S[Ax(t) + A_1x(t - h(t)) - \dot{x}(t)] = 0.$$

By noting these and using the Lyapunov function candidate in (9), we can obtain the following delay-independent stability result for the time-delay system $(\Sigma_2)$.

**Theorem 6:** *The time-delay system* $(\Sigma_2)$ *is asymptotically stable if there exist matrices* $P > 0$, $Y$, $W$, *and* $S$ *such that*

$$\begin{bmatrix} WA + A^TW^T & WA_1 + A^TS^T & A^TY^T + P - W \\ A_1^TW^T + SA & SA_1 + A_1^TS^T & A_1^TY^T - S \\ YA + P - W^T & YA_1 - S^T & -Y - Y^T \end{bmatrix} < 0.$$

### 3.2. *Delay-dependent stability conditions*

In this section, LMI techniques in deriving delay-dependent stability conditions will be reviewed.

#### 3.2.1. *Newton–Leibniz formula*

By using the Newton–Leibniz formula and noting (3), we have

$$\begin{aligned} x(t - h) &= x(t) - \int_{t-h}^{t} \dot{x}(\alpha)\mathrm{d}\alpha \\ &= x(t) - \int_{t-h}^{t} [Ax(\alpha) + A_1x(\alpha - h)]\mathrm{d}\alpha. \end{aligned}$$

This together with (3) gives

$$\dot{x}(t) = (A + A_1)x(t) - A_1 \int_{t-h}^{t} [Ax(\alpha) + A_1x(\alpha - h)]\mathrm{d}\alpha. \tag{10}$$

Note that the asymptotic stability of the time-delay system in (10) implies that of the system in (3) and (4). For this reason, we now turn to study the stability of (10). To this end, we choose a Lyapunov–Krasovskii functional candidate as follows:

$$\begin{aligned} V(t, x_t) = x(t)^T P^{-1} x(t) &+ \int_{-h}^{0}\int_{t+\theta}^{t} x(\alpha)^T A_1^T Q_1^{-1} A_1 x(\alpha)\mathrm{d}\alpha\mathrm{d}\theta \\ &+ \int_{-h}^{0}\int_{t-h+\theta}^{t} x(\alpha)^T A_1^T Q_2^{-1} A_1 x(\alpha)\mathrm{d}\alpha\mathrm{d}\theta, \end{aligned} \tag{11}$$

where $P > 0$, $Q_1 > 0$ and $Q_2 > 0$. Then, by Theorem 1, the stability condition for (10) is obtained in the following theorem.

**Theorem 7:** (Cao, Sun, and Lam 1998b) *The time-delay system in* (10) *is asymptotically stable for any delay h satisfying* $0 < h \leq \bar{h}$ *if there exist matrices* $P > 0$, $Q_1 > 0$ *and* $Q_2 > 0$ *such that*

$$\begin{bmatrix} \Omega & \bar{h}PA^T & \bar{h}PA_1^T \\ \bar{h}AP & -Q_1 & 0 \\ \bar{h}A_1P & 0 & -Q_2 \end{bmatrix} < 0, \tag{12}$$

*where*

$$\Omega = (A + A_1)P + P(A + A_1)^T + A_1(Q_1 + Q_2)A_1^T.$$

**Remark 3:** It is easy to see that the LMI stability condition in (12) covers that in (27) of (Li and de Souza 1997b). Therefore, Theorem 7 is less conservative than the stability condition in Corollary 3.1 of (Li and de Souza 1997b).

Similarly, to derive stability conditions for the time-delay system $(\Sigma_2)$, we can use the Newton–Leibniz formula and turn to the study of the stability of the following time-delay system:

$$\dot{x}(t) = (A + A_1)x(t) - A_1 \int_{t-h(t)}^{t} [Ax(\alpha) + A_1x(\alpha - h(\alpha))]\mathrm{d}\alpha. \tag{13}$$

Then, by the Lyapunov function candidate in (9) and the use of Theorem 2, the following stability condition can be obtained.

**Theorem 8:** (Cao, Sun, and Cheng 1998a; Li and de Souza 1997a) *The time-delay system in* (13) *is asymptotically stable for any delay* $h(t)$, *satisfying* $0 < h(t) \leq \bar{h}$ *if there exist matrices* $X > 0$, $X_1 > 0$

*and* $X_2 > 0$ *such that*

$$(A+A_1)X+X(A+A_1)^T+\bar{h}A_1(X_1+X_2)A_1^T+2\bar{h}X<0,$$
$$\begin{bmatrix} X & XA^T \\ AX & X_1 \end{bmatrix} \geq 0,$$
$$\begin{bmatrix} X & XA_1^T \\ A_1X & X_2 \end{bmatrix} \geq 0.$$

**Remark 4:** The technique by using the Newton–Leibniz formula to change the time-delay system $(\Sigma_1)$ and $(\Sigma_2)$ to (10) and (13), respectively, was also adopted by many researchers to deal with various types of time-delay systems to derive delay-dependent stability conditions (see, e.g. Su and Huang 1992; Niculescu, Dion, and Dugard 1996; Liu and Su 1998; Kolmanovskii, Niculescu, and Richard 1999; Liu and Su 1999; Kim 2001; Kharitonov and Melchor-Aguilar 2003a; Tarbouriech, Peres, Garcia, and Queinnec 2002; Zheng and Frank 2002; Ivanescu, Niculescu, Dugard, Dion, and Verriest 2003, and the references therein).

By the Newton–Leibniz formula, we can also change system (3) to

$$\dot{x}(t) = (A+A_1)x(t) - A_1\int_{t-h}^{t}\dot{x}(\alpha)\mathrm{d}\alpha, \tag{14}$$

and

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[x(t)+A_1\int_{t-h}^{t}x(\alpha)\mathrm{d}\alpha\right] = (A+A_1)x(t). \tag{15}$$

Then, attention can be focussed on the study of the stability of time-delay systems in (14) and (15). By appropriate Lyapunov–Krasovskii functionals, delay-dependent stability results for (14) and (15) can be obtained (Kolmanovskii and Richard 1999; Luo and Chung 2002). These techniques were also used to derive delay-dependent results for various types of time-delay systems (see, e.g. Chen, Lien, Fan, and Cheng 2000; Lien, Yu, and Hsieh 2000; Lien 2001; Niculescu 2001b; Lien and Chen 2003; Chen and Liu 2005; Chen, Lam, and Xu 2006, and the references therein).

**Remark 5:** All the time-delay systems in (10), (14), and (15) are transformed from the time-delay system in (3) by using the Newton–Leibniz formula. However, all of them are not equivalent to (3). Compared with (3), additional dynamics are introduced in (10), (14), and (15) (Gu and Niculescu 2000; Kharitonov and Melchor-Aguilar 2000; Gu and Niculescu 2001; Kharitonov and Melchor-Aguilar 2002; Kharitonov and Melchor-Aguilar 2003a; Kharitonov and Melchor-Aguilar 2003b), which may cause conservatism as the delay-dependent conditions are derived based on the transformed systems.

3.2.2. *Bounding techniques*

One of the main purposes in the study of delay-dependent stability for time-delay systems is to find methods to reduce conservatism of existing delay-dependent stability conditions. It is known that the finding of better bounds on some weighted cross products arising in the analysis of the delay-dependent stability problem plays a key role in reducing conservatism. Note that the delay-dependent stability results reported in Li and de Souza (1997a,b) and Cao et al. (1998a,b) were obtained by using the well-known inequality on upper bound for the inner product of two vectors; that is,

$$-2a^Tb \leq a^TXa + b^TX^{-1}b, \tag{16}$$

where $a \in \mathbb{R}^n$, $b \in \mathbb{R}^n$ and $X \in \mathbb{R}^{n\times n}$ with $X>0$. In order to reduce the conservatism in the delay-dependent stability results in Li and de Souza (1997a,b) and Cao et al. (1998a,b), an improved inequality was proposed in Park (1999) which is re-stated as follows:

**Lemma 1** (Park's Inequality)**:** (Park 1999) *Assume that* $a(\alpha) \in \mathbb{R}^{n_a}$, *and* $b(\alpha) \in \mathbb{R}^{n_a}$ *are given for* $\alpha \in \Omega$. *Then, for any* $X \in \mathbb{R}^{n_a\times n_a}$ *with* $X>0$ *and any matrix* $M \in \mathbb{R}^{n_a\times n_a}$, *we have*

$$-2\int_{\Omega} a(\alpha)^T b(\alpha)\mathrm{d}\alpha \leq \int_{\Omega}\begin{bmatrix} a(\alpha) \\ b(\alpha)\end{bmatrix}^T \begin{bmatrix} X & XM \\ M^TX & (M^TX+I)X^{-1}(M^TX+I)^T \end{bmatrix} \times \begin{bmatrix} a(\alpha) \\ b(\alpha)\end{bmatrix}\mathrm{d}\alpha. \tag{17}$$

Then, by using this inequality, an improved delay-dependent stability condition for time-delay system $(\Sigma_1)$ was reported in Park (1999), which is given as follows.

**Theorem 9:** (Park 1999) *The time-delay system* $(\Sigma_1)$ *is asymptotically stable for any delay* $h$ *satisfying* $0<h\leq\bar{h}$ *if there exist matrices* $P>0$, $Q>0$, $V>0$, *and* $W$ *such that*

$$\begin{bmatrix} \Psi & -W^TA_1 & A^TA_1^TV & \bar{h}(W^T+P) \\ -A_1^TW & -Q & A_1^TA_1^TV & 0 \\ VA_1A & VA_1A_1 & -V & 0 \\ \bar{h}(W^T+P)^T & 0 & 0 & -V \end{bmatrix} < 0, \tag{18}$$

*where*

$$\Psi = (A+A_1)^TP + P(A+A_1) + W^TA_1 + A_1^TW + Q.$$

The inequality in Lemma 1 was further improved in Moon, Park, Kwon, and Lee (2001). We re-state this inequality as follows.

**Lemma 2** (Moon's Inequality)**:** (Moon et al. 2001) *Assume that* $a(\alpha) \in \mathbb{R}^{n_a}$, *and* $b(\alpha) \in \mathbb{R}^{n_b}$, *and* $\mathcal{N}(\alpha) \in \mathbb{R}^{n_a \times b_b}$ *are given for* $\alpha \in \Omega$. *Then, for any* $X \in \mathbb{R}^{n_a \times n_a}$ *with* $X > 0$ *and any matrix* $M \in \mathbb{R}^{n_a \times n_a}$, *we have*

$$
\begin{aligned}
&-2\int_{\Omega} a(\alpha)^T \mathcal{N}(\alpha) b(\alpha)\mathrm{d}\alpha \\
&\quad \leq \int_{\Omega} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix}^T \begin{bmatrix} X & Y - \mathcal{N}(\alpha) \\ Y^T - \mathcal{N}(\alpha)^T & Z \end{bmatrix} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix} \mathrm{d}\alpha,
\end{aligned}
$$

*where*

$$
\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0.
$$

By Lemma 2, and the following Lyapunov–Krasovskii functional

$$
\begin{aligned}
V(x_t) = x(t)^T P x(t) + \int_{t-h}^{t} x(\alpha)^T Q x(\alpha)\mathrm{d}\alpha \\
+ \int_{-h}^{0}\int_{t+\beta}^{t} \dot{x}(\alpha)^T Z \dot{x}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta, \qquad (19)
\end{aligned}
$$

a delay-dependent stability result was obtained, which can be used to solve the synthesis problem easily. The delay-dependent stability result in Moon et al. (2001) is re-stated in the following theorem.

**Theorem 10:** (Moon et al. 2001) *The time-delay system* $(\Sigma_1)$ *is asymptotically stable for any delay* $h$ *satisfying* $0 < h < \bar{h}$ *if there exist matrices* $P > 0$, $Q > 0$, $X$, $Y$ *and* $Z$ *such that*

$$
\begin{bmatrix} PA + A^T P + \bar{h}X + Y + Y^T + Q & PA_1 - Y & \bar{h}A^T Z \\ A_1^T P - Y^T & -Q & \bar{h}A_1^T Z \\ -\bar{h}ZA & \bar{h}ZA_1 & -\bar{h}Z \end{bmatrix} < 0, \qquad (20)
$$

$$
\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0. \qquad (21)
$$

It is worth mentioning that by Lemma 2 and the Finsler Lemma (Boyd et al. 1994), a delay-dependent stability condition was also developed in Suplin, Fridman, and Shaked (2004), which is re-written in the following theorem.

**Theorem 11:** (Suplin et al. 2004) *The time-delay system* $(\Sigma_1)$ *is asymptotically stable for any delay* $h$ *satisfying* $0 < h < \bar{h}$ *if there exist matrices* $P_1 > 0$, $S > 0$, $P_2$, $P_3$, $P_4$, $Y_1$, $Y_2$, $Z_1$, $Z_2$, $Z_3$ *and* $R > 0$ *such that the following LMIs hold*:

$$
\begin{bmatrix} P_2^T A + A^T P_2 + Y_1 + Y_1^T + S + \bar{h}Z_1 & P_1 - P_2^T + A^T P_3 + Y_2 + \bar{h}Z_2 & P_2^T A_1 - Y_1^T + A^T P_4 \\ P_1 - P_2 + P_3^T A + Y_2^T + \bar{h}Z_2^T & \bar{h}(R + Z_3) - P_3^T - P_3 & P_3^T A_1 - Y_2^T - P_4 \\ A_1^T P_2 - Y_1 + P_4^T A & A_1^T P_3 - Y_2 - P_4^T & A_1^T P_4 + P_4^T A_1 - S \end{bmatrix} < 0, \qquad (22)
$$

$$
\begin{bmatrix} R & Y_1 & Y_2 \\ Y_1^T & Z_1 & Z_2 \\ Y_2^T & Z_2^T & Z_3 \end{bmatrix} \geq 0. \qquad (23)
$$

**Remark 6:** Since the inequality in Lemma 2 is more general than both the inequalities in (16) and (17), Lemma 2 is now extensively used in dealing with various issues related to time-delay systems to obtain delay-dependent results (see Park 2001; L. Yu, Han, Yu, and Gao 2003; Gao, Lam, Wang, and Wang 2004; Li, Wang, and Liao 2004; Chen, Feng, and Guan 2005; Guan, Chen, and Xu 2005; Liu, Zhang, and Zhang 2005; Palhares, Campos, Ekel, Leles, and D'Angelo 2005a,b; Yue and Han 2005b; Zhang, Wu, She, He 2005; Huang, Li, and Zhong 2006; Lam and Leung 2006; Lin, Wang, and Lee 2006c; Liu, Martin, Wu, and Tang 2006; Suplin, Fridman, and Shaked 2006; Wu 2006; Xie and Tang 2006; Chen and Zheng 2007; Cho and Park 2007; Lou and Cui 2007; Yoneyama 2007). However, it has been theoretically established in Xu and Lam (2005) that the results in Theorem 10 is more conservative than Theorem 1 in Xu and Lam (2005), which was derived by introducing some slack variables. Therefore, without using Theorem 10, one still can obtain a less conservative delay-dependent stability result. It is also worth mentioning that Theorem 1 in Xu and Lam (2005) can also be easily applied to deal with synthesis problems.

Now, we present another important inequality, which is also effective in the derivation of delay-dependent stability conditions.

**Lemma 3** (Jensen's Inequality)**:** (Gu 2000) *For any constant matrix* $\mathcal{M} \in \mathbb{R}^{m \times m}$ *with* $\mathcal{M} > 0$, *scalars* $b > a$, *vector function* $\omega : [a,b] \to \mathbb{R}^m$ *such that the integrations in the following are welldefined, then*

$$(b-a)\int_a^b \omega(\beta)^T M\omega(\beta)\mathrm{d}\beta \geq \left[\int_a^b \omega(\beta)\mathrm{d}\beta\right]^T M\left[\int_a^b \omega(\beta)\mathrm{d}\beta\right].$$

By using this inequality, and choosing the following Lyapunov–Krasovskii functional for $(\Sigma_1)$:

$$V(x_t) = x(t)^T Px(t) + \int_{t-h}^{t} x(\alpha)^T Qx(\alpha)\mathrm{d}\alpha + h\int_{-h}^{0}\int_{t+\beta}^{t} \dot{x}(\alpha)^T Z\dot{x}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta, \tag{24}$$

the following result can be obtained.

**Theorem 12:** (Gouaisbaut and Peaucelle 2006a) *The time-delay system* $(\Sigma_1)$ *is asymptotically stable for any delay h satisfying* $0 < h < \bar{h}$ *if there exist matrices* $P > 0$, $Q > 0$, *and* $Z > 0$ *such that*

$$\begin{bmatrix} PA + A^T P + Q - Z & PA_1 + Z & \bar{h}A^T Z \\ A_1^T P + Z & -Q - Z & \bar{h}A_1^T Z \\ \bar{h}ZA & \bar{h}ZA_1 & -Z \end{bmatrix} < 0. \tag{25}$$

Similarly, by Lemma 3 and the Lyapunov–Krasovskii functional for $(\Sigma_2)$ as

$$V(x_t) = x(t)^T Px(t) + \bar{h}\int_{-\bar{h}}^{0}\int_{t+\beta}^{t} \dot{x}(\alpha)^T Z\dot{x}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta, \tag{26}$$

we have the following result.

**Theorem 13:** *The time-delay system* $(\Sigma_2)$ *is asymptotically stable for all continuous* $h(t)$ *satisfying* $0 < h(t) \leq \bar{h}$ *if there exist matrices* $P > 0$, *and* $Z > 0$ *such that*

$$\begin{bmatrix} PA + A^T P - Z & PA_1 + Z & \bar{h}A^T Z \\ A_1^T P + Z & -Z & \bar{h}A_1^T Z \\ -\bar{h}ZA & \bar{h}ZA_1 & -Z \end{bmatrix} < 0.$$

Note that Theorem 12 proves that the time-delay system $(\Sigma_1)$ is asymptotically stable for any delay $h$ satisfying $0 < h < \bar{h}$ when the LMI in (25) is feasible, which implies that for $h$ satisfying $0 < h < \bar{h}/2$ the time-delay system $(\Sigma_1)$ is asymptotically stable too. Then, introducing the half delay into the time-delay system $(\Sigma_1)$ will take more information on the system and thus may reduce the conservatism in Theorem 12. Consequently, one can consider the following artificially augmented system (Gouaisbaut and Peaucelle 2006):

$$(\tilde{\Sigma}):\quad \dot{x}(t) = Ax(t) + A_1 x(t-h),$$
$$\dot{x}\left(t+\frac{h}{2}\right) = Ax\left(t+\frac{h}{2}\right) + A_1 x\left(t-\frac{h}{2}\right).$$

Set

$$x_2(t) = \left[x\left(t+\frac{h}{2}\right)^T \quad x(t)^T\right]^T.$$

Then, the time-delay system $(\tilde{\Sigma})$ can be re-written as

$$\dot{x}_2(t) = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} x_2(t) + \begin{bmatrix} A_1 & 0 \\ 0 & A_1 \end{bmatrix} x_2(t-h). \tag{27}$$

Now, by Lemma 3 and the following Lyapunov–Krasovskii functional for the augmented system (27):

$$V(x_{2t}) = x_2(t)^T Px_2(t) + \sum_{i=1}^{2}\int_{t-(ih/2)}^{t} x_2(\alpha)^T Q_{2i}x_2(\alpha)\mathrm{d}\alpha + h\sum_{i=1}^{2}\int_{-(ih/2)}^{0}\int_{t+\beta}^{t} \dot{x}(\alpha)^T Z_{2i}\dot{x}(\alpha)\mathrm{d}\alpha\mathrm{d}\beta,$$

the following result can be obtained.

**Theorem 14:** (Gouaisbaut and Peaucelle 2006) *The time-delay system* $(\Sigma_1)$ *is asymptotically stable for any delay h satisfying* $0 < h < \bar{h}$ *if there exist matrices* $P > 0$, $Q_{21} > 0$, $Q_{22} > 0$, $Z_{21} > 0$, *and* $Z_{22} > 0$ *such that*

$$B_2^{\perp T}\mathcal{M}_2(\bar{h})B_2^{\perp} < 0,$$

*where* $B_2^{\perp}$ *is an orthogonal complement of* $B_2$ *given by*

$$B_2 = \begin{bmatrix} I & \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} & 0 & \begin{bmatrix} A_1 & 0 \\ 0 & A_1 \end{bmatrix} & 0 & 0 \\ 0 & -I & I & 0 & I & 0 \\ 0 & -I & 0 & I & 0 & I \\ 0 & \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} & \begin{bmatrix} -I & 0 \\ 0 & I \end{bmatrix} & \begin{bmatrix} 0 & 0 \\ -I & 0 \end{bmatrix} & 0 & 0 \end{bmatrix},$$

$$\mathcal{M}_2(\bar{h}) = \begin{bmatrix} \frac{h^2}{2}Z_{21} + h^2 Z_{22} & P & 0 & 0 \\ P & Q_{21} + Q_{22} & 0 & 0 \\ 0 & 0 & -\mathcal{Q}_2 & 0 \\ 0 & 0 & 0 & -\mathcal{Z}_2 \end{bmatrix},$$

$$\mathcal{Q}_2 = \mathrm{diag}(Q_{21}, \quad Q_{22}), \quad \mathcal{Z}_2 = \mathrm{diag}(2Z_{21}, \quad Z_{22}).$$

**Remark 7:** Theorem 14 can be further improved by discretising $r > 2$ times of the interval $[-h, 0]$; see, Theorem 6 in Gouaisbaut and Peaucelle (2006).

**Remark 8:** The inequality in Lemma 3 has been used to deal with different kinds of time-delay systems in order to obtain delay-dependent results (see Gu et al. 2003; Han 2003a,b; Han 2004; Han 2005a,c; Jiang and Han 2005; Lien 2005; Park 2005; Park and Kwon 2005; Kwon and Park 2006; Lien 2006; Wu, Shi, Wang, and Gao 2006; Chen and Zheng 2007; Han 2007).

### 3.2.3. *Descriptor system approach*

It is known that delay-dependent stability results obtained via a transformed model, which is not equivalent to the original time-delay system, may have conservatism to some extent (see Remark 5). In order to reduce such potential conservatism, a method based on descriptor systems has been introduced in the literature (Fridman 2001; Fridman and Shaked 2002a; Fridman and Shaked 2003). This method uses a descriptor system model, which is equivalent to the original time-delay system, to derive delay-dependent stability conditions.

To introduce the descriptor system approach, we first consider the time-delay system $(\Sigma_1)$ and represent (3) in the following form:

$$\dot{x}(t) = y(t),$$
$$0 = -y(t) + (A + A_1)x(t) - A_1 \int_{t-h}^{t} y(\alpha)\mathrm{d}\alpha,$$

which can be re-written as

$$E\dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{A}_1 \int_{t-h}^{t} y(\alpha)\mathrm{d}\alpha, \tag{28}$$

where

$$\bar{x}(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \quad E = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \tag{29}$$

$$\bar{A} = \begin{bmatrix} 0 & I \\ A + A_1 & -I \end{bmatrix}, \quad \bar{A}_1 = \begin{bmatrix} 0 \\ A_1 \end{bmatrix}. \tag{30}$$

Note that the descriptor time-delay system in (28) is equivalent to the original time-delay system in (3). Then, attention will be focussed on the derivation of delay-dependent stability conditions for (28). By the Lyapunov–Krasovskii functional chosen as

$$V(x_t) = \bar{x}(t)^T EP\bar{x}(t) + \int_{t-h}^{t} x(\alpha)^T S_1 x(\alpha)\mathrm{d}\alpha + \int_{-h}^{0}\int_{t+\theta}^{t} y(\alpha)^T R_1 y(\alpha)\mathrm{d}\alpha,$$

together with the bounding technique in Lemma 2, a delay-dependent stability condition for system $(\Sigma_1)$ is obtained, which is given in the following theorem.

**Theorem 15:** (Fridman and Shaked 2002b) *The time-delay system* $(\Sigma_1)$ *is asymptotically stable for any delay h satisfying* $0 < h \le \bar{h}$ *if there exist matrices* $P_1 > 0$, $P_2$, $P_3$, $R_1 > 0$, $S_1$, $Y_{11}$, $Y_{12}$, $Z_{11}$, $Z_{12}$ *and* $Z_{13}$, *such that*

$$\begin{bmatrix} \Omega + \bar{h}Z_1 & P^T\begin{bmatrix} 0 \\ A_1 \end{bmatrix} - Y_1^T \\ \begin{bmatrix} 0 & A_1^T \end{bmatrix}P - Y_1 & -S_1 \end{bmatrix} < 0, \tag{31}$$

*and*

$$\begin{bmatrix} R_1 & Y_1 \\ Y_1^T & Z_1 \end{bmatrix} \ge 0, \tag{32}$$

*where*

$$\Omega = P^T\begin{bmatrix} 0 & I \\ A & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A & -I \end{bmatrix}^T P + \begin{bmatrix} S_1 & 0 \\ 0 & \bar{h}R_1 \end{bmatrix} + \begin{bmatrix} Y_1 \\ 0 \end{bmatrix} + \begin{bmatrix} Y_1 \\ 0 \end{bmatrix}^T, \tag{33}$$

$$P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \tag{34}$$

$$Y_1 = \begin{bmatrix} Y_{11} & Y_{12} \end{bmatrix}, \tag{35}$$

$$Z_1 = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{12}^T & Z_{13} \end{bmatrix}. \tag{36}$$

An equivalent form of Theorem 15 with fewer variables is reported in Xu, Lam, and Zou (2005), which is re-stated in the following theorem.

**Theorem 16:** (Xu, Lam, and Zou 2005c) *The time-delay system* $(\Sigma_1)$ *is asymptotically stable for any delay h satisfying* $0 < h \le \bar{h}$ *if there exist matrices* $P_1 > 0$, $P_2$, $P_3$, $R_1 > 0$, $S_1$, $Y_{11}$ *and* $Y_{12}$ *such that the following LMI holds*:

$$\begin{bmatrix} \Omega & P^T\begin{bmatrix} 0 \\ A_1 \end{bmatrix} - Y_1^T & -\bar{h}Y_1^T \\ \begin{bmatrix} 0 & A_1^T \end{bmatrix}P - Y_1 & -S_1 & 0 \\ -\bar{h}Y_1 & 0 & -\bar{h}R_1 \end{bmatrix} < 0, \tag{37}$$

*where* $\Omega$, $P$ *and* $Y_1$ *are given in* (33)–(35), *respectively*.

Similar to the derivation of Theorem 15, by applying the descriptor system approach to the time-delay system $(\Sigma_2)$, a delay-dependent stability condition can be obtained as follows.

**Theorem 17:** (Fridman and Shaked 2002b) *The time-delay system* $(\Sigma_2)$ *is asymptotically stable for all continuous* $h(t)$ *satisfying* $0 < h(t) \le \bar{h}$ *if there exist matrices* $P_1 > 0$, $P_2$, $P_3$, $R_1 > 0$, $Z_{11}$, $Z_{12}$ *and* $Z_{13}$ *such that*

$$\Omega_1 + \bar{h}Z_1 < 0, \tag{38}$$

*and*

$$\begin{bmatrix} R_1 & \begin{bmatrix} 0 & A_1^T \end{bmatrix} P \\ P^T \begin{bmatrix} 0 \\ A_1 \end{bmatrix} & -Z_1 \end{bmatrix} \geq 0, \tag{39}$$

*where* $P$ *and* $Z_1$ *are given in* (34) *and* (36), *respectively, and*

$$\Omega_1 = P^T \begin{bmatrix} 0 & I \\ A + A_1 & -I \end{bmatrix} + \begin{bmatrix} 0 & I \\ A + A_1 & -I \end{bmatrix}^T P + \begin{bmatrix} 0 & 0 \\ 0 & \bar{h} R_1 \end{bmatrix}. \tag{40}$$

An equivalent form of Theorem 17 with fewer variables is also reported in Xu, Lam, and Zou (2005), which is re-stated in the following theorem.

**Theorem 18:** (Xu et al. 2005c) *The time-delay system* $(\Sigma_2)$ *is asymptotically stable for all continuous* $h(t)$ *satisfying* $0 < h(t) \leq \bar{h}$ *if there exist matrices* $P_1 > 0$, $P_2$, $P_3$ *and* $R_1 > 0$ *such that*

$$\begin{bmatrix} \Omega_1 & -\bar{h} P^T \begin{bmatrix} 0 \\ A_1 \end{bmatrix} \\ -\bar{h} \begin{bmatrix} 0 & A_1^T \end{bmatrix} P & -\bar{h} R_1 \end{bmatrix} < 0, \tag{41}$$

*where* $P$ *and* $\Omega_1$ *are given in* (34) *and* (40), *respectively*.

By the descriptor system approach together with the following functional

$$V(x_t) = x(t)^T P x(t) + \int_0^h (h - \sigma) \dot{x}(t - \sigma)^T X_{33} \dot{x}(t - \sigma) \mathrm{d}\sigma + \int_0^t \int_{\sigma - h}^{\sigma} e^T X e \, \mathrm{d}s \mathrm{d}\sigma,$$

with

$$e = \begin{bmatrix} x(\sigma) \\ x(\sigma - h) \\ \dot{x}(s) \end{bmatrix}, \quad X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} > 0,$$

a delay-dependent stability condition was reported in Jing, Tan, and Wang (2004), which is re-stated as follows.

**Theorem 19:** (Jing et al. 2004) *The time-delay system* $(\Sigma_1)$ *is asymptotically stable for any delay h satisfying* $0 < h \leq \bar{h}$ *if there exist matrices* $P > 0$, $Q > 0$, $P_1$, $P_2$, $X_{ij}$ $(i \leq j,\ i, j = 1, 2, 3)$ *such that*

$$\begin{bmatrix} X_{11} & X_{12} & X_{13} \\ X_{12}^T & X_{22} & X_{23} \\ X_{13}^T & X_{23}^T & X_{33} \end{bmatrix} > 0,$$

$$\begin{bmatrix} A^T P_1 + P_1^T A + \bar{h} X_{11} + X_{13} + X_{13}^T + Q & P - P_1^T + A^T P_2 & P_1^T A_1 + \bar{h} X_{12} - X_{13} + X_{23}^T \\ P - P_1 + P_2^T A & \bar{h} X_{33} - P_2 - P_2^T & P_2^T A_1 \\ A_1^T P_1 + \bar{h} X_{12}^T - X_{13}^T + X_{23} & A_1^T P_2 & \bar{h} X_{22} - X_{23} - X_{23}^T - Q \end{bmatrix} < 0.$$

By an alternative functional defined as

$$V(x_t) = \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T EP \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + \int_{t-h}^t x(\alpha)^T Q x(\alpha) \mathrm{d}\alpha + \int_0^t \int_{\beta - h}^{\beta} \begin{bmatrix} x(\beta) \\ \dot{x}(\beta) \\ \dot{x}(\alpha) \end{bmatrix}^T \begin{bmatrix} X_{11} & X_{12} & Y_1 \\ X_{12}^T & X_{22} & Y_2 \\ Y_1^T & Y_2^T & Z \end{bmatrix} \begin{bmatrix} x(\beta) \\ \dot{x}(\beta) \\ \dot{x}(\alpha) \end{bmatrix} \mathrm{d}\alpha \mathrm{d}\beta,$$

with $E$ and $P$ given in (29) and (34), respectively, a delay-dependent stability condition can be obtained as follows:

**Theorem 20:** (Lee, Moon, Kwon, and Park 2004) *The time-delay system* $(\Sigma_1)$ *is asymptotically stable for any delay h satisfying* $0 < h \leq \bar{h}$ *if there exist matrices* $P_1 > 0$, $P_2$, $P_3$, $Q$, $X_{11}$, $X_{12}$, $X_{22}$, $Y_1$, $Y_2$ *and* $Z > 0$ *such that the following LMIs hold*:

$$\begin{bmatrix} X_{11} & X_{12} & Y_1 \\ X_{12}^T & X_{22} & Y_2 \\ Y_1^T & Y_2^T & Z \end{bmatrix} \geq 0, \tag{42}$$

$$\begin{bmatrix} P_2^T A + A^T P_2 + \bar{h} X_{11} + Q + Y_1 + Y_1^T & P_1 - P_2^T + A^T P_3 + \bar{h} X_{12} + Y_2^T & P_2^T A_1 - Y_1 \\ P_1 - P_2 + P_3^T A + \bar{h} X_{12}^T + Y_2 & -P_3^T - P_3 + \bar{h} X_{22} + \bar{h} Z & P_3^T A_1 - Y_2 \\ A_1^T P_2 - Y_1^T & A_1^T P_3 - Y_2^T & -Q \end{bmatrix} < 0. \tag{43}$$

**Remark 9:** The descriptor system approach has been widely used to deal with various problems of time-delay systems in order to provide delay-dependent results (see Fridman and Shaked 2001a,b; Fridman and Shaked 2002c; Fridman, Pila, and Shaked 2003; Fridman, Shaked, and Xie 2003; Gao and Wang 2003; Han 2003a,b; Chen, Guan, and Lu 2004; Fridman, Seuret, and Richard 2004; Ismail and Mahmoud 2004; Liu and Zhang 2005; Fridman 2006; Parlakci 2006; Sun, Wang, and Xie 2006; Han 2007, and the references therein).

### 3.2.4. *Discretised Lyapunov–Krasovskii functional approach*

The discretised Lyapunov–Krasovskii functional approach has been proposed in Gu (1997) in order to reduce conservatism in delay-dependent stability conditions. One feature of this method is that for a given time-delay system without uncertainty, analytical stability limit can be approached as grid size approaches zero. To show this method, we choose a quadratic Lyapunov–Krasovskii functional for the time-delay system $(\Sigma_1)$ as given in Gu (1997):

$$\begin{aligned} V(\phi) = {}& \frac{1}{2}\phi(0)^T P\phi(0) + \phi(0)^T \int_{-h}^{0} Q(\xi)\phi(\xi)\mathrm{d}\xi \\ &+ \frac{1}{2}\int_{-h}^{0}\int_{-h}^{0} \phi(\xi)^T R(\xi-\eta)\phi(\eta)\mathrm{d}\eta\mathrm{d}\xi \\ &+ \frac{1}{2}\int_{-h}^{0} \phi(\xi)^T S(\xi)\phi(\xi)\mathrm{d}\xi, \end{aligned} \tag{44}$$

where $P>0$, and

$$\begin{aligned} S(\xi)^T &= S(\xi) > 0, \quad \xi \in [-h, 0], \\ R(-\xi) &= R(\xi)^T, \quad \xi \in [-h, h]. \end{aligned}$$

The following result is well known.

**Theorem 21:** (Hale and Verduyn Lunel 1993) *The time-delay system* $(\Sigma_1)$ *is asymptotically stable if there exists a quadratic Lyapunov function* $V$ *in* (44) *such that for some* $\epsilon>0$, *it satisfies*

$$V(\phi) \geq \epsilon\phi(0)^T\phi(0),$$

*and its derivative along the solution of* (3) *satisfies*

$$\dot{V}(\phi) \leq -\epsilon\phi(0)^T\phi(0),$$

*where*

$$\dot{V}(\phi)\Delta{=}\frac{\mathrm{d}}{\mathrm{d}t}V(x_t)\Big|_{x_t=\phi}.$$

Now, let the delay interval $[-h, 0]$ be divided into $N$ segments $[\theta_{i-1}, \theta_i]$ of equal length $r$, where

$$\theta_i = -h + ir, \quad i = 0, 1, 2, \ldots, N. \tag{45}$$

Set

$$Q_i = Q(\theta_i), \quad S_i = S(\theta_i), \quad T_i = T(\theta_i), \quad T_{kj,i} = T_{kj}(\theta_i). \tag{46}$$

Write

$$\tau_i = ir, \quad i = 0, \pm1, \pm2, \ldots, \pm N, \tag{47}$$

$$R_i = R_{-i}^T = R(\tau_i). \tag{48}$$

For $0 \leq \alpha \leq r$, let

$$Q^i(\alpha) \triangleq Q(\theta_{i-1}+\alpha) = (1-\alpha/r)Q_{i-1} + (\alpha/r)Q_i, \tag{49}$$

$$S^i(\alpha) \triangleq S(\theta_{i-1}+\alpha) = (1-\alpha/r)S_{i-1} + (\alpha/r)S_i, \tag{50}$$

$$T^i(\alpha) \triangleq T(\theta_{i-1}+\alpha) = (1-\alpha/r)T_{i-1} + (\alpha/r)T_i, \tag{51}$$

$$T_{kj}^i(\alpha) \triangleq T_{kj}(\theta_{i-1}+\alpha) = (1-\alpha/r)T_{kj,i-1} + (\alpha/r)T_{kj,i}, \tag{52}$$

and

$$R^i(\alpha) = R(\tau_i+\alpha) = (1-\alpha/r)R_{i-1} + (\alpha/r)R_i. \tag{53}$$

Then, by Theorem 21, the following result can be obtained.

**Theorem 22:** (Gu 1997) *The time-delay system* $(\Sigma_1)$ *is asymptotically stable if there exist matrices* $P>0$, $Q_i$, $S_i$, $R_i$, *and* $T_{kj,i}$, $i=0, 1, 2, \ldots, N$, $k$, $j=1,2$ *as defined in* (45)–(53) *such that*

$$S_i \geq 0, \quad i = 0, 1, 2, \ldots, N, \tag{54}$$

$$\begin{bmatrix} \hat{R} & \hat{Q}^T \\ \hat{Q} & P \end{bmatrix} > 0, \tag{55}$$

$$\begin{bmatrix} \Delta_1 + T_{11,i+k-1} & T_{12,i+k-1} - \Delta_2 & \Delta_{3ik} \\ T_{12,i+k-1}^T - \Delta_2^T & T_{22,i+k-1} + S_0 & \Delta_{4ik} \\ \Delta_{3ik}^T & \Delta_{4ik}^T & \frac{1}{N}(S_i - S_{i-1}) \end{bmatrix} > 0,$$
$$i = 1, 2, \ldots, N, \quad k = 0, 1, \tag{56}$$

*and*

$$T_0 + T_N + 2\sum_{i=1}^{N-1} T_i = 0, \tag{57}$$

*where*

$$\hat{R} = \begin{bmatrix} R_0 & R_{-1} & \cdots & R_{-N} \\ R_1 & R_0 & \cdots & R_{-N+1} \\ \vdots & \vdots & \ddots & \vdots \\ R_N & R_{N-1} & \cdots & R_0 \end{bmatrix},$$

$$\hat{Q} = \begin{bmatrix} Q_0 & Q_1 & \cdots & Q_N \end{bmatrix},$$

$$\Delta_1 = -\left(PA + A^T P + S_N + Q_N + Q_N^T\right),$$

$$\Delta_2 = PA_1 - Q_0,$$

$$\Delta_{3i0} = rA^T Q_{i-1} - (Q_i - Q_{i-1}) + rR_{i-N-1}^T,$$

$$\Delta_{4i0} = rA_1^T Q_{i-1} - rR_{i-1}^T,$$

$$\Delta_{3i1} = rA^T Q_i - (Q_i - Q_{i-1}) + rR_{i-N}^T,$$

$$\Delta_{4i1} = rA_1^T Q_i - rR_i^T,$$

$$T_i = \begin{bmatrix} T_{11,i} & T_{12,i} \\ T_{12,i}^T & T_{22,i} \end{bmatrix}.$$

**Remark 10:** It is pointed out in Gu (1997) that the equality constraint in (57) can be relaxed to

$$T_0 + T_N + 2\sum_{i=1}^{N-1} T_i \leq 0.$$

Together with (54)–(56), it can be seen that Theorem 22 provides an LMI condition for delay-dependent stability of the time-delay system $(\Sigma_1)$.

**Remark 11:** Theorem 22 is effective in the analysis of delay-dependent stability for time-delay systems. This theorem was generalised and used to deal with stability problems of different kinds of time-delay systems (see Gu 1999a,b,c; Gu 2001a,b; Gu, Han, Luo, and Niculescu 2001; Han and Gu 2001; Han 2002; Gu 2003a,b; Han, Gu, and Yu 2003; Han, Yu, and Gu 2004; Han 2005b). However, it is not easy to apply this to study synthesis problems due to the cross terms of AP and $AQ_i$.

### 3.2.5. *Slack matrix variables*

By the Newton–Leibniz formula

$$x(t-h) = x(t) - \int_{t-h}^{t} \dot{x}(\alpha)\mathrm{d}\alpha,$$

it is easy to see that for any matrices $Y$ and $W$ with appropriate dimensions,

$$2x(t)^T Y\left[x(t) - x(t-h) - \int_{t-h}^{t} \dot{x}(\alpha)\mathrm{d}\alpha\right] = 0, \tag{58}$$

$$2x(t-h)^T W\left[x(t) - x(t-h) - \int_{t-h}^{t} \dot{x}(\alpha)\mathrm{d}\alpha\right] = 0. \tag{59}$$

Furthermore, it can be seen that

$$h\zeta(t)^T X\zeta(t) - \int_{t-h}^{t} \zeta(t)^T X\zeta(t)\mathrm{d}s = 0, \tag{60}$$

where

$$\zeta(t) = \begin{bmatrix} x(t)^T & x(t-h)^T \end{bmatrix}^T,$$
$$X = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix}.$$

Then, by (58)–(60) and the Lyapunov–Krasovskii functional in (24), the following delay-dependent stability result was obtained in Wu et al. (2004).

**Theorem 23:** (Wu, He, She, and Liu 2004) *The time-delay system* $(\Sigma_1)$ *is asymptotically stable for any delay* $h$ *satisfying* $0 < h \leq \bar{h}$ *if there exist matrices* $P > 0$, $Q > 0$, $Z > 0$, $X_{11}$, $X_{12}$, $X_{22}$, *Y and W such that*

$$\begin{bmatrix} PA + A^T P + Y + Y^T + Q + \bar{h}X_{11} & PA_1 - Y + W^T + \bar{h}X_{12} & \bar{h}A^T Z \\ A_1^T P - Y^T + W + \bar{h}X_{12}^T & -Q - W - W^T + \bar{h}X_{22} & \bar{h}A_1^T Z \\ \bar{h}ZA & \bar{h}ZA_1 & -\bar{h}Z \end{bmatrix} < 0, \tag{61}$$

$$\begin{bmatrix} X_{11} & X_{12} & Y \\ X_{12}^T & X_{22} & W \\ Y^T & W^T & Z \end{bmatrix} \geq 0 \tag{62}$$

Without resorting to the equality in (60), we can have the following result.

**Theorem 24:** (Xu and Lam 2005) *The time-delay system* $(\Sigma_1)$ *is asymptotically stable for any delay* $h$ *satisfying* $0 < h \le \bar{h}$ *if there exist matrices* $P>0$, $Q>0$, $Z>0$, $Y$ *and* $W$ *such that*

$$\begin{bmatrix} PA+A^TP+Y+Y^T+Q & PA_1-Y+W^T & -\bar{h}Y & \bar{h}A^TZ \\ A_1^TP-Y^T+W & -Q-W-W^T & -\bar{h}W & \bar{h}A_1^TZ \\ -\bar{h}Y^T & -\bar{h}W^T & -\bar{h}Z & 0 \\ \bar{h}ZA & \bar{h}ZA_1 & 0 & -\bar{h}Z \end{bmatrix} < 0. \tag{63}$$

Similarly, by the equalities in (58) and (59) and the Lyapunov–Krasovskii functional in (26), we have the following result.

**Theorem 25:** (Xu and Lam 2007) *The time-delay system* $(\Sigma_2)$ *is asymptotically stable for all continuous* $h\,(t)$ *satisfying* $0 < h(t) \le \bar{h}$ *if there exist matrices* $P>0$, $Z>0$, $Y$ *and* $W$ *such that*

$$\begin{bmatrix} PA+A^TP+Y+Y^T & PA_1-Y+W^T & -\bar{h}Y & \bar{h}A^TZ \\ A_1^TP-Y^T+W & -W-W^T & -\bar{h}W & \bar{h}A_1^TZ \\ -\bar{h}Y^T & -\bar{h}W^T & -\bar{h}Z & 0 \\ \bar{h}ZA & \bar{h}ZA_1 & 0 & -\bar{h}Z \end{bmatrix} < 0. \tag{64}$$

**Remark 12:** The method based on introducing slack variables has been extensively used in the derivation of delay-dependent results for time-delay systems, which is also effective in reducing conservatism in existing delay-dependent results; delay-dependent results on various kinds of time-delay systems obtained by using such a method can be found in He, Wu, She, and Liu (2004a,b); Wu, He, and She (2004, 2006); Yue and Han (2004); He, Wang, Lin, and Wu (2005, 2007); Lam, Gao, and Wang (2005); Xu, Lam, Chen, and Zou (2005a,b); Xu, Lam, Ho, and Zou (2005); Xu, Lam, Mao, and Zou (2005); Xu, Lam, and Zou (2005a,b); Yue and Han (2005a); Lin, Wang, and Lee (2006a,b); Wu, Chen, and Wang (2006); Xu, Lam, and Zou (2006a,b); Gao and Chen (2007).

3.2.6. *Delay-partitioning projection approach*

To show the idea of the delay-partition projection approach, we consider

$$\dot{x}(t) = Ax(t) + A_1x(t - \sum_{i=1}^{r} h_i) \tag{65}$$

$$x(t) = \phi(t), \quad \forall t \in [-\bar{h}, 0] \tag{66}$$

where the scalars $h_i>0$, $i=1,\ldots,r$, and

$$\sum_{i=1}^{r} h_i \le \bar{h}.$$

Then, it can be seen that $h_i$, $i=1,\ldots,r$, represents a partition of the lumped delay $\bar{h}$. Let

$$\sigma_0 = 0, \quad \sigma_j = \sum_{i=1}^{j} h_i.$$

By making such a partition and using the Lyapunov–Krasovskii functional:

$$\begin{aligned} V(x_t) &= x(t)^TPx(t) + \int_{t-h}^{t} x(s)^TRx(s)\mathrm{d}s \\ &\quad + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{x}(s)^TX\dot{x}(s)\mathrm{d}s\mathrm{d}\theta \\ &\quad + \sum_{j=1}^{r}\int_{t-\sigma_j}^{t-\sigma_{j-1}} x(s)^TQ_jx(s)\mathrm{d}s, \end{aligned}$$

the following result can be obtained.

**Theorem 26:** (Du, Lam, Shu, and Wang, in press) *The time-delay system in* (65) *and* (66) *is asymptotically stable if there exist matrices* $P>0$, $X>0$, *and* $Q_i$, $i=1,\ldots,r$, *such that*

$$B^{\perp T}\begin{pmatrix} \Omega_1+\Omega_2 & 0 \\ 0 & \Omega_3 \end{pmatrix}B^{\perp} < 0,$$

*where* $B^{\perp} \in \mathbb{R}^{(2r+1)n\times(r+1)n}$ *is the orthogonal complement of*

$$B_{rn\times(2r+1)n} = \left(\begin{array}{cccccc|cccc} I & -I & 0 & \cdots & 0 & & -I & 0 & \cdots & 0 \\ 0 & I & -I & \cdots & 0 & & 0 & -I & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots & & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I & -I & & 0 & 0 & \cdots & -I \end{array}\right),$$

*and*

$$\Omega_1 = \begin{pmatrix} PA+A^TP+Q_1 & 0 & \cdots & 0 & PA_d \\ * & Q_2-Q_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & \cdots & Q_r-Q_{r-1} & 0 \\ * & * & \cdots & * & -Q_r \end{pmatrix},$$

$$\Omega_2 = \left(\sum_{i=1}^{r}\bar{h}_i\right)\begin{pmatrix} A & 0 & \cdots & 0 & A_d \end{pmatrix}^TX\begin{pmatrix} A & 0 & \cdots & 0 & A_d \end{pmatrix},$$

$$\Omega_3 = \mathrm{diag}\{-\bar{h}_1^{-1}X,\ldots,-\bar{h}_r^{-1}X\}.$$

**Remark 13:** By the delay-partition projection approach, it can be seen that the variables to be determined are fewer than those in Peaucelle, Arzelier, Henrion, and Gouaisbaut (2007). Numerical examples in (Du et al. in press) show that Theorem 26 can achieve the same maximal allowable delay while using fewer variables compared with that in Peaucelle et al. (2007). The approach in (Du et al. in press) is also related to systems with multiple delay components investigated in Lam, Gao, and Wang (2007). The delay-partitioning approach has also been applied to neutral delay systems (Du, Lam, and Shu 2008) and T-S fuzzy systems (Zhao, Gao, Lam, and Du 2008).

### 3.3. *Connections of certain LMI stability conditions*

In the above section, various techniques in the derivation of delay-dependent stability conditions are reviewed, and corresponding delay-dependent stability conditions in terms of LMIs are presented. These conditions are in different appearances. However, some of these conditions are equivalent although they are obtained via different approaches, which is theoretically established in Xu and Lam (2007). We re-state the equivalence of these conditions in the following lemma.

**Lemma 4:** (Xu and Lam 2007) *The delay-dependent conditions in Theorems* 11, 15, 16, 19, 20, 23, *and* 24 *are equivalent*.

In Gouaisbaut and Peaucelle (2006b), the following equivalence result is also established.

**Lemma 5:** (Gouaisbaut and Peaucelle 2006b) *The delay-dependent conditions in Theorems* 12 *and* 24 *are equivalent*.

**Remark 14:** In the proof of Lemma 5, the Finsler Lemma is used in Gouaisbaut and Peaucelle (2006b) to establish the fact that the feasibility of LMI (63) in Theorem 24 implies that of LMI (25) in Theorem 12.

Now, by Lemmas 4 and 5, it is easy to have the following result.

**Theorem 27:** *The delay-dependent conditions in Theorems* 11, 12, 15, 16, 19, 20, 23, *and* 24 *are equivalent*.

**Remark 15:** By Theorem 27, it is now clear that the essence in the use of 'descriptor system approach' and the bounding techniques in Lemma 2 is the introduction of slack variables, which can also be seen in the derivation of LMI conditions in Theorems 11, 15, 16, 19, and 20.

**Remark 16:** Note that the number of variables involved in LMI (25) is 1.5 $(n^2+n)$. Therefore, by Xu and Lam (2007), it can be seen that Theorem 12 involves the least number of variables to be determined compared with Theorems 11, 15, 16, 19, 20, 23, and 24.

**Remark 17:** In the case when norm-bounded parameter uncertainties appear in the delay system $(\Sigma)$, Theorems 12, 23, and 24 will provide equivalent robust stability results if the techniques in eliminating uncertainties in Khargonekar, Petersen, and Zhou (1990) are resorted to.

## 4. Conclusion

This article has reviewed certain existing LMI techniques in stability analysis of time-delay systems. LMI techniques in deriving both delay-independent and delay-dependent stability conditions have been reviewed. It is worth pointing out that by these LMI techniques, various synthesis problems such as stabilisation and $H_\infty$ control can also be solved. However, the conditions on the solvability of these synthesis problems may not be expressed in LMIs; see, e.g. Moon et al. (2001); Xu, Lam, and Zou (2006), and the references therein.

As the stability results obtained through the use of various Lyapunov-type functions or functionals are of sufficiency nature, they invariably have conservatism to some extent. Much of the research has been focussed on reducing the conservatism of the stability conditions. Earlier effort in this area has been relied mainly on improving the bounding techniques (Park's inequality, Moon's inequality, and Jensen's Inequality) for use in guaranteeing the negative definiteness of the derivative of the Lyapunov–Krasovskii functionals. More recently, the reduction of conservatism is achieved by introducing slack variables; this is often realised by discretising of the Lyapunov–Krasovskii functionals and the use of identities such as the Newton–Leibniz formula incorporated by delay partitioning. On the other hand, the increase of slack variables can also significantly increase the computational complexity, and there has been some effort in reducing the redundancy of some of the slack variables. In this respect, the projection approach appears to be effective. Therefore, how to develop new methods in order to further reduce the conservatism in existing stability results while keeping a reasonably low computational complexity is an important issue to be investigated in the future.

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