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ROBUST STABILITY AND STABILIZATION OF A CLASS OF SINGULAR SYSTEMS WITH MULTIPLE TIME-VARYING DELAYS

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Abstract: This paper deals with the problem of robust stability and stabilization for uncertain continuous singular systems with multiple time-varying delays. The parametric uncertainty is assumed to be norm bounded. The purpose of the robust stability problem is to give conditions such that the uncertain singular system is regular, impulse free, and stable for all admissible uncertainties. The purpose of the robust stabilization problem is to design a feedback control law such that the resulting closed-loop system is robustly stable. This problem is solved via generalized quadratic stability approach. A strict linear matrix inequality (LMI) design approach is developed.

Keywords: singular systems, multiple time-varying delays, robust stability, robust stabilization, linear matrix inequality.

1. INTRODUCTION

In recent years, considerable efforts have been devoted to the analysis and synthesis of singular systems (known also as descriptor systems, semi–state systems, differential algebraic systems, generalized state-space systems, (Dai, 1989), (Lewis, 2002)). These systems arise naturally in various fields including electrical networks, robotics, social, biological, and automatic control. Alike the case of uncertain systems without delay, methods based on the concepts of quadratic stability and quadratic stabilizability have been shown to be effective in dealing with these problems in both continuous and discrete contexts (Mahmoud and Al-Muthairi, 1994), (S. Xu and Yang, 2001).
On the other hand, control of singular systems has been extensively studied in the past years due to the fact that singular system better describe physical systems than regular ones. Recently, robust stability and robust stabilization for uncertain singular systems have been considered. The notions of quadratic stability and quadratic stabilization of regular systems have been extended (S. Xu, 2000), (S. Xu and Lam, 2001). It should be pointed out that the robust stability problem for singular systems is much more complicated than that for regular systems because it requires to consider not only stability robustness, but also regularity and absence of impulses (for continuous singular systems) or causality (for discrete singular systems) at the same time (Fang and Chang, 1993), (C. H. Fang and Chang, 1994), and the latter two properties need not be considered in regular systems. Very recently, much attention has been paid to singular systems with time delay. For the continuous case, numerical methods for such systems were discussed in (Campbell, 1980) and (S. Xu and Yang, 1994). To the best of our knowledge, there is not much results on the problems of robust stability or robust stabilization for singular systems with multiple time-varying delays in

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the literature.
In this note, we address the problems of stability/stabilization and robust stability/stabilization for uncertain singular systems with multiple time-varying delays. The parameter uncertainties are time invariant and unknown, but norm bounded.

The paper is organized as follows. In section 2, the problem is stated and the required assumptions are formulated. Section 3 deals with the stability problem. In section 4 we address the robust stability problem and in section 5 we address the stabilization problem. Section 6 deals with the robust stabilization. In section 7 we present a numerical example to show the usefulness of the proposed results.

Notation
In the sequel $\mathsf{Sym}\{.\}$ is defined as

$$\mathsf{Sym}\{X\} = X + X^\top$$

for any matrix X

2. PROBLEM STATEMENT AND PRELIMINARIES

Consider the following uncertain singular systems with multiple delays :

$$\begin{aligned} E\dot{x}(t) &= A_0(t)x(t) + \sum_{j=1}^{p} A_j(t)x(t-h_j(t)) + B(t)u(t) \\ y(t) &= Cx(t) \\ x(t) &= \phi(t), \ -\bar{h} < t < 0 \end{aligned}$$

where $x(t)$ is the state vector, in $\mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ is the control, $y(t) \in \mathbb{R}^r$ is the output vector, $h_j(t)$; $j = 1, 2, \ldots, p$, are the time-varying delays of the system and the matrices $A_j(t)$; $j = 0, 1, 2, \ldots, p$ and $B(t)$ are given by :

$$A_j(t) = A_j + D_jF_j(t)N_j \qquad B(t) = B + D_bF_b(t)N_b \quad (3)$$

with A_j, $j = 0, 1, 2, \ldots, p$, B, D_j, N_j; $j = 0, 1, 2, \ldots, p$, D_b and N_b are given matrices with appropriate dimensions and $F_b(t)$ and $F_j(t)$; $j = 0, 1, 2, \ldots, p$ represent the system uncertainties satisfying the following assumption.

Assumption 2.1. Assume that the uncertainty terms satisfy what follows

$$F_0^\top(t)R_0F_0(t) \le R_0 \qquad F_d^\top(t)R_dF_d(t) \le R_d, \qquad F_b^\top(t)R_bF_b(t) \le R_b \quad (4)$$

where R_d and $F_d(t)$ are diagonal matrices given by

$$F_d(t) = \mathsf{diag}\,(\,F_1(t) \ \ \ldots \ \ F_p(t)\,) \qquad R_d = \mathsf{diag}\,(\,R_1 \ \ \ldots \ \ R_p\,)$$

Definition 2.1. (Dai, 1989)

(1) The pair (E, A) is said to be regular if $det(sE - A)$ is not identically zero.
(2) The pair (E, A) is said to be impulse free if $deg(det(sE - A)) = rankE$.

The singular system (2) may have an impulsive solution, however the regularity and the absence of impulses of the pair (E, A_0) ensure the existence and uniqueness of an impulse free solution to this system, which is stated by Lemma 2.1.

Lemma 2.1. (S. Xu and Lam, 2002) Suppose the pair (E, A_0) is regular and impulse free, then the solution to (2) exists and is impulse free and unique on $[0, \infty)$

In view of this, we introduce the following definition for singular delay system (2).

Definition 2.2.

- The singular delay system (2) is said to be regular and impulse free if the pair (E, A_0) is regular and impulse free.
- The singular delay system (2) is said to be stable if for any $\varepsilon > 0$ there exists a scalar $\delta(\varepsilon) > 0$ such that, for any compatible initial conditions $\phi(t)$ satisfying $\sup_{-\tau \le t \le 0} \parallel \phi(t) \parallel \le \delta(\varepsilon)$, the solution $x(t)$ of system (2) satisfies $\parallel x(t) \parallel \le \varepsilon$ for $t \ge 0$. Furthermore

$$\lim_{t\to\infty} x(t) = 0$$

The following three lemmas are very useful for our development in this paper.

Lemma 2.2. (Xie, 1996) Let Z, E, F, R and Δ be matrices of appropriate dimensions. Assume that Z is symmetric, R is symmetric and positive definite and $\Delta^\top R\Delta \le R$, then

$$Z + E\Delta F + F^\top\Delta^\top E^\top < 0$$

if and only if there exists a scalar $\lambda > 0$ satisfying

$$Z + E\Big(\lambda R\Big)E^\top + F^\top\Big(\lambda R\Big)^{-1}F < 0$$

Lemma 2.3. (M.S. Saadni, 2003) Let Φ, a and b, then the two statement are equivalent

a) the LMI

$$\begin{bmatrix} \Phi & a \\ a^\top & 0 \end{bmatrix} + \mathsf{Sym}\left\{\begin{bmatrix} f \\ g \end{bmatrix}\begin{bmatrix} b^\top & -I \end{bmatrix}\right\} < 0$$

is feasible in the variable f and g.

b) Φ, a and b satisfy : $\Phi + ab^\top + ba^\top < 0$

Lemma 2.4. (M.S. Saadni, 2003) Let Φ, a and b be given matrices of appropriate dimension, then the two statements are equivalent

a) the following LMI

$$\begin{aligned} \begin{bmatrix} \Phi & a + bG^\top \\ a^\top + Gb^\top & -G - G^\top \end{bmatrix} &= \begin{bmatrix} \Phi & a \\ a^\top & 0 \end{bmatrix} \\ &+ \mathsf{Sym}\left\{\begin{bmatrix} 0 \\ I \end{bmatrix} G \begin{bmatrix} b^\top & -I \end{bmatrix}\right\} < 0 \end{aligned}$$

is feasible in the variable G.

b) Φ, a and b satisfies $\Phi < 0$ and $\Phi + ab^\top + ba^\top < 0$

In our subsequent developments we need the following lemma :

Lemma 2.5. (S. Xu and Lam, 2002) Consider the function $\varphi : \mathbb{R}^+ \to \mathbb{R}$. if $\dot{\varphi}$ is bounded on $[0, \infty)$, that is, there exists a scalar $\alpha > 0$ such that $|\dot{\varphi}(t)| \leq \alpha$ for all $t \in [0, \infty)$, then φ is uniformly continuous on $[0, \infty)$.

Lemma 2.6. Barbalat's Lemma: Consider the function $\varphi : \mathbb{R}^+ \to \mathbb{R}$.if φ is uniformly continuous and $\int_0^\infty \varphi(s)ds < \infty$, then

$$\lim_{t\to\infty} \varphi(t) = 0$$

In the rest of the paper the notation is standard unless it is specified otherwise. $L > 0$ ($L < 0$) means that the matrix L is symmetric and positive-definite (symmetric and negative-definite).

Assumption 2.2. The delays $h_j(t)$, $j = 1, 2, \ldots, p$ are assumed to satisfy the following constraint:

$$0 \leq h_j(t) \leq \bar{h}_j \text{ and } 0 \leq \dot{h}_j(t) \leq \bar{l}_j < 1,$$

where $\bar{h}_j$, are given positive constants.

Let us define $\bar{h}$ as $\bar{h} = max\left(\bar{h}_1, \ldots, \bar{h}_p\right)$.

3. STABILITY ANALYSIS

The goal of this section consists of establishing what will be the sufficient conditions that can be used to check whether or not the class of systems under study is stable. We consider the system given by the following dynamics:

$$E\dot{x}_t = A_0(t)x(t) + \sum_{j=1}^{p} A_j(t)x(t - h_j(t)) \quad (5)$$

or in a compact form as

$$E\dot{x}_t = A_0(t)x(t) + A_d(t)x_h(t) \quad (6)$$

with

$$A_d(t) = [\, A_1(t) \quad A_2(t) \quad \cdots \quad A_p(t) \,]$$
$$x_h(t) = \left[\, x(t-h_1)^\top \quad x(t-h_2)^\top \cdots \quad x(t-h_p)^\top \,\right]^\top$$

The goal of this subsection consists of developing some conditions that can be used to check whether the class of systems under study is stable or not. The conditions we are looking for should depend on the upper bound of the delay as given in Assumption 2.2. The following theorem states such a result.

Theorem 3.1. Assume that the assumption 2.2 is satisfied. If there exist F_i, $i = 1, \ldots, 4$, $P > 0$, $Q_j > 0$, $W_j > 0$, Y_j and Z_j for $j = 1, 2, \ldots, p$ such that the following hold:

$$E^\top P = P^\top E \geq 0 \quad (7)$$

$$\begin{bmatrix} Z_j & Y_j \\ Y_j^\top & E^\top W_j E \end{bmatrix} \geq 0 \quad (8)$$

$$\begin{bmatrix} \Psi_1 & -\Psi_3 & 0 & P \\ -\Psi_3^\top & -\Psi_2 & 0 & 0 \\ 0 & 0 & -\mathcal{W} & \mathcal{W} \\ P^\top & 0 & \mathcal{W} & 0 \end{bmatrix} + \text{Sym}\left\{\mathcal{F}_1^\top \mathcal{F}_2\right\} < 0 \quad (9)$$

are feasible with

$$\mathcal{F}_1 = \left[\, F_1^\top \quad F_2^\top \quad F_3^\top \quad F_4 \,\right]^\top$$
$$\mathcal{F}_2 = [\, A_0 \quad A_d \quad 0 \quad -I \,]$$
$$A_d = [\, A_1 \quad A_2 \quad \cdots \quad A_p \,] \qquad \mathcal{W} = \sum_{j=1}^{p} \bar{h}_j W_j$$
$$\Psi_1 = \sum_{j=1}^{p} \left(Q_j + (1 - \bar{l}_j)\left(\bar{h}_j Z_j + Y_j + Y_j^\top\right)\right)$$
$$\Psi_2 = diag\left((1-\bar{l}_1)Q_1, \ldots, (1-\bar{l}_p)Q_p\right)$$
$$\Psi_3 = [\, (1-\bar{l}_1)Y_1 \quad (1-\bar{l}_2)Y_2 \quad \cdots \quad (1-\bar{l}_p)Y_p \,]$$

then, system (6) is asymptotically stable.

Proof of Theorem 3.1 Note that the regularity and the absence of impulses of the pair (E, A_0) implies that there exist two invertible matrices G and $H \in \mathbb{R}^{n\times n}$ such that (Dai, 1989)

$$\bar{E} = GEH = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \quad \bar{A}_0 = GA_0H = \begin{bmatrix} \bar{A}_1 & 0 \\ 0 & I_{n-r} \end{bmatrix}$$
$$\bar{A}_j = GA_jH = \begin{bmatrix} A_{11j} & A_{12j} \\ A_{21j} & A_{22j} \end{bmatrix} \quad (10)$$

where $I_r \in \mathbb{R}^{r\times r}$ and $I_{n-r} \in \mathbb{R}^{n-r\times n-r}$ are identity matrices. Using the same transformation as in (10), let

$$\bar{P} = H^\top P G^{-1}, \quad \bar{W}_j = G^{-\top} W_j G^{-1} \quad (11)$$
$$\bar{Q}_j = H^\top Q_j H, \quad \bar{Z}_j = H^\top Z_j H, \quad \bar{Y}_j = H^\top Y_j H$$

Taking account of (10) and using (7), we deduce that $\bar{P}_{11} = \bar{P}_{11}^\top \geq 0$ and $\bar{P}_{12} = 0$. Now, let

$$\zeta(t) = \left[\, \zeta_1^\top(t) \quad \zeta_2^\top(t) \,\right]^\top = H^{-1}x(t) \quad (12)$$

where $\zeta_1 \in \mathbb{R}^r$ and $\zeta_2 \in \mathbb{R}^{n-r}$. Using the expression in (6), the singular delay system (2) can be decomposed as

$$\dot{\zeta}_1(t) = \bar{A}_1\zeta_1(t) + \sum_{j=1}^{p} [A_{11j}\zeta_1(t-h_j(t)) + A_{12j}\zeta_2(t-h_j(t))]$$
$$0 = \zeta_2(t) + \sum_{j=1}^{p} [A_{21j}\zeta_1(t-h_j(t)) + A_{22j}\zeta_2(t-h_j(t))] \quad (13)$$
$$\phi(\zeta_t) = \zeta(t+\beta) \qquad \beta \in [-\bar{h}, 0]$$

It is easy to see that the stability of the singular delay system (2) is equivalent to that of the system (13). In view of this, we shall prove next that the system (13) is stable.

We consider the Lyapunov function candidate :

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) \quad (14)$$

with

$$V_1(\zeta_t) = \zeta_t^\top \bar{P}^\top \bar{E}\zeta_t$$
$$V_2(\zeta_t) = \sum_{j=1}^{p} \int_{t-h_j(t)}^{t} \int_{s}^{t} \dot{\zeta}_z^\top \bar{E}^\top \bar{W}_j \bar{E}\dot{\zeta}_z \, dz ds$$

$$V_3(\zeta_t) = \sum_{j=1}^{p} \int_{t-h_j(t)}^{t} \zeta_s^\top \bar{Q}_j \zeta_s ds$$

$$V_4(\zeta_t) = \sum_{j=1}^{p} \int_0^t \left(1 - \bar{l}_j\right) \int_{z-h_j(z)}^{z} \left[\zeta_z^\top \ \ \dot{\zeta}(s)^\top \right] \left[\begin{array}{cc} \bar{Z}_j & \bar{Y}_j \\ \bar{Y}_j^\top & \bar{E}^\top \bar{W}_j \bar{E} \end{array} \right] \left[\begin{array}{c} \zeta_z \\ \dot{\zeta}(s) \end{array} \right] dsdz$$

Recall that for any matrices F_1, F_2 and F_3 of appropriate dimensions with $F_2 > 0$

$$F_1^\top F_3 + F_3^\top F_1 \le F_1^\top F_2 F_1 + F_3^\top F_2^{-1} F_3$$

After taking the derivatives of these functionals and performing some algebraic manipulations and assuming that

$$\bar{A}_d^\top \bar{\mathcal{W}} \bar{A}_d - \bar{\Psi}_2 < 0 \tag{15}$$

we get :

$$\dot{V}(\zeta_t) \le \zeta^\top(t)(M_{11} - M_{12} M_{22}^{-1} M_{12}^\top)\zeta(t)$$

with

$$M_{11} = \bar{A}_0^\top \bar{P} + \bar{P}^\top \bar{A}_0 + \bar{A}_0^\top \bar{\mathcal{W}} \bar{A}_0 + \bar{\Psi}_1 \tag{16}$$

$$M_{12} = \bar{P} \bar{A}_d - \bar{\Psi}_3 + \bar{A}_0^\top \bar{\mathcal{W}} \bar{A}_d \tag{17}$$

$$M_{22} = \bar{A}_d^\top \bar{\mathcal{W}} \bar{A}_d - \bar{\Psi}_2 \tag{18}$$

It comes then that $\dot{V}(\zeta_t)$ is definite negative if $\left(M_{11} - M_{12} M_{22}^{-1} M_{12}^\top\right) < 0$ which associated with (15) and using the Schur complement and rewriting the result we get

$$\left[\begin{array}{ccc} \bar{\Psi}_1 & -\bar{\Psi}_3 & 0 \\ -\bar{\Psi}_3^\top & -\bar{\Psi}_2 & 0 \\ 0 & 0 & -\bar{\mathcal{W}} \end{array} \right] + \mathsf{Sym} \left\{ \left[\begin{array}{c} \bar{P} \\ 0 \\ \bar{\mathcal{W}} \end{array} \right] [\, \bar{A}_0 \quad \bar{A}_d \quad 0 \,] \right\} < 0$$

According to lemma 2.3, the condition above holds if there exist F_m, $m = 1, ..., 4$, such that (9) is satisfied.

It follows from inequality (**??**) that $\dot{V}(\zeta_t) < 0$ and

$$\lambda_1 \parallel \zeta_1(t) \parallel^2 - V(\zeta_0) \le \int_0^t \dot{V}(\zeta(s))ds \le -\lambda_2 \int_0^t \parallel \zeta(s) \parallel^2 ds$$

$$\le -\lambda_2 \int_0^t \parallel \zeta_1(s) \parallel^2 ds$$

with

$$\lambda_1 = \lambda_{min}(P_{11}) > 0$$

$$\lambda_2 = -\lambda_{max} \left\{ M_{11} - M_{12} M_{22}^{-1} M_{12}^\top \right\} > 0$$

Taking into account (19), we deduce that

$$\lambda_1 \parallel \zeta_1(t) \parallel^2 + \lambda_2 \int_0^t \parallel \zeta_1(s) \parallel^2 ds \le V(\zeta_0) \tag{19}$$

Therefore

$$\parallel \zeta_1(t) \parallel^2 \le c_1 \text{ and } \int_0^t \parallel \zeta_1(s) \parallel^2 \le c_2 \tag{20}$$

where

$$c_1 = \frac{1}{\lambda_1} V(\zeta_0) \qquad c_2 = \frac{1}{\lambda_2} V(\zeta_0) \tag{21}$$

Thus, $\parallel \zeta_1(t) \parallel$ is bounded and from system (13) we note that $\frac{d}{dt} \parallel \zeta_1(t) \parallel^2$ is bounded too. By Lemma 2.5, we have that $\parallel \zeta_1(t) \parallel^2$ is uniformly continuous. Therefore, with (20) in mind and using Lemma 2.5, we obtain

$$\lim_{t \to \infty} \parallel \zeta_1(t) \parallel = 0 \tag{22}$$

Now, note that for any $t > 0$, there exists a positive integer k such that $k\bar{h} - \bar{h} \le t < k\bar{h}$, we have $\zeta_2(t) = -\sum_{i=1}^{k} \sum_{j=1}^{p} ((-A_{21j})^{i-1} \zeta_1(t - ih_j(t)) + \sum_{j=1}^{p} (-A_{22j})^k \zeta_2(t - kh_j(t)))$ with $\bar{h} = max(h_1 ... h_p)$. Since $\parallel \zeta_1(t) \parallel$ is bounded and if

$$\rho(A_{22j}) < 1 \qquad \text{for} \qquad j = 1 \dots p \tag{23}$$

which implies that

$$\lim_{t \to \infty} \parallel \zeta_2(t) \parallel = 0 \tag{24}$$

Thus, the system (13) is stable.

Remark 3.1. The results of Theorem (3.1) are only sufficient and therefore if these conditions are not verified we can't claim that the system under study is not stable.

4. ROBUST STABILITY

We assume that the system has uncertainties on all the matrices, *i.e* : $E\dot{x}_t = \tilde{A}_0(t)x_t + \tilde{A}_d(t)x_h(t)$ with $\tilde{A}_0(t) = A_0 + D_0 F_0(t) N_0$, $\tilde{A}_d(t) = A_d + D_d F_d(t) N_d$ and D_d, F_d, N_d are given by $D_d = [\, D_1 \dots D_p \,]$, $N_d = diag\left(N_1, \ \dots N_p\right)$, $F_d(t) = diag\left(F_1(t) \quad \dots \quad F_p(t) \right)$.

Note that conditions (7) and (8) do not depend on the system matrices so they do not need to be adapted to the uncertain case. Besides, we have to replace A_0 and A_d respectively by $\tilde{A}_0(t)$ and $\tilde{A}_d(t)$ in condition (9). Separating the nominal and the uncertain part and applying Lemma (2.2) and using the Schur complement we get a condition for the robust case which is stated by Theorem 4.1.

Theorem 4.1. Assume that assumptions (2.1) and (2.2) are satisfied. If there exist F_i for $i = 1, \dots, 4$ and $P > 0$, $Q_j > 0$, $W_j > 0$, Y_j, Z_j for $j = 1, 2, \dots, p$ and λ such that conditions (7), (8) and

$$\left[\begin{array}{cccccc} f_{11} & -\Psi_3 & 0 & P & 0 & 0 \\ -\Psi_3^\top & f_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & -\mathcal{W} & \mathcal{W} & 0 & 0 \\ P & 0 & \mathcal{W} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\lambda R & 0 \\ 0 & 0 & 0 & 0 & 0 & -\lambda R_d \end{array} \right] + \mathsf{Sym} \left\{ \mathcal{F}_3^\top \mathcal{F}_4 \right\} < 0 \tag{25}$$

are feasible, with

$$\mathcal{F}_3 = [\, \mathcal{F}_1^\top \quad 0 \quad 0 \,]^\top \tag{26}$$

$$\mathcal{F}_4 = [\, \mathcal{F}_2 \quad D_0 \quad D_d \,] \tag{27}$$

$$f_{11} = \Psi_1 + \lambda N_0^\top R_0 N_0 \qquad f_{22} = -\Psi_2 + \lambda N_d^\top R_d N_d \tag{28}$$

then the uncertain system under study is asymptotically stable for all admissible uncertainties.

Proof of Theorem 4.1 We have to replace A_0 and A_d respectively by $\tilde{A}_0(t)$ and $\tilde{A}_d(t)$ in condition (9) ,which gives after separating the uncertain terms

$$\underbrace{\mathsf{Sym}\left\{\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}\begin{bmatrix} D_0^\top \\ D_d^\top \end{bmatrix}^\top \begin{bmatrix} F_0(t) & \\ & F_d(t) \end{bmatrix}\begin{bmatrix} N_0^\top & 0 \\ 0 & N_d^\top \\ 0 & 0 \\ 0 & 0 \end{bmatrix}^\top\right\}}_{X} +$$

$$\begin{bmatrix} \Psi_1 & -\Psi_3 & 0 & P \\ -\Psi_3^\top & -\Psi_2 & 0 & 0 \\ 0 & 0 & -\mathcal{W} & \mathcal{W} \\ P & 0 & \mathcal{W} & 0 \end{bmatrix} + \mathsf{Sym}\left\{\mathcal{F}_1^\top \mathcal{F}_2\right\} < 0$$

Applying Lemma 2.2 for expression (X) above, and using of the Schur complement, we get a condition (25)holds with f_{11} and f_{22} given, respectively, by (28) and (28). Hence the uncertain system under study is asymptotically stable for all admissible uncertainties.

5. STABILIZABILITY ANALYSIS

We consider the nominal system (2) given as

$$\begin{aligned} E\dot{x}(t) &= A_0 x(t) + A_d x_h(t) + Bu(t) \\ y(t) &= Cx(t) \\ x(t) &= \phi(t),\ -\bar{h} < t < 0 \end{aligned} \tag{29}$$

We propose to synthesize a stabilizing output feedback controller. Notice that a dynamical output feedback could be obtained as a statical output feedback for an augmented system. In addition state feedback could be obtained by adopting $C = I$, where I indicates the unit matrix. The controller is thus given by

$$u_t = Ky_t \tag{30}$$

Substituting (30) in the plant model (29) and with $A^{cl} = A_0 + BKC$ we get the closed-loop dynamic

$$\begin{aligned} E\dot{x}(t) &= (A_0 + BKC)x(t) + A_d x_h(t) \\ x(t) &= \phi(t),\ -\bar{h} < t < 0 \end{aligned}$$

with

$$\begin{aligned} A^{cl} &= A_0 + BKC = A_0 + BK_s + B(KC - K_s) \\ A_d^{cl} &= A_d + BK_{d_s} - BK_{d_s} \\ A_s &= A_0 + BK_s \quad A_{d_s} = A_d + BK_{d_s} \end{aligned} \tag{31}$$

The objective of this study is to develop a new delay-dependent stabilization method that provides an output feedback controller $u(t) = Ky(t)$ for of class of dynamical singular systems. The following theorem states such a result:

Theorem 5.1. Assume that assumption 2.2 is satisfied and there exist F_m pour $m = 1, 2, ..., 4$, $P > 0$, $W_j > 0$, $Q_j > 0$, Y_j, Z_j for $j = 1, 2, ..., p$, L et G such that the conditions (7), (8) and

$$\begin{bmatrix} \Psi_1 & -\Psi_3 & 0 & P & 0 \\ -\Psi_3^\top & -\Psi_2 & 0 & 0 & 0 \\ 0 & 0 & -\mathcal{W} & \mathcal{W} & 0 \\ P & 0 & \mathcal{W} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} + \mathrm{Sym}\left\{\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ 0 \end{bmatrix}\left[A_s \quad A_{d_s} \quad 0 \quad -I \quad B \right]\right\} \tag{32}$$

$$+\mathrm{Sym}\left\{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ I \end{bmatrix}\left[LC - GK_s) \quad -GK_{d_s} \quad 0 \quad 0 \quad -G \right]\right\} < 0$$

are feasible. Then, the system (29) is asymptotically stable and the output feedback control law is given by

$$K = G^{-1}L$$

The proof of Theorem 5.1 can be carried out with similar arguments as for Theorem 4.1.

6. ROBUST STABILIZATION

In this section, we are concerned by robust stabilizability of the uncertain system under the control law (30). Introducing the uncertainty terms in (4), the closed loop system equation becomes

$$\begin{aligned} E\dot{x}(t) &= \tilde{A}^{cl}(t)x(t) + \tilde{A}_d(t)x_h(t) \\ x(t) &= \phi(t),\ -\bar{h} < t < 0 \end{aligned} \tag{33}$$

with

$$\tilde{A}^{cl}(t) = A(t) + B(t)KC \quad \tilde{A}_d(t) = A_d + D_d F_d(t) N_d$$

where N_d and D_d are given by

$$D_d = [\, D_1 \quad \dots \quad D_p \,] \quad N_d = diag\big(N_1,\ \dots N_p\big)$$

There exist any matrices K_s and K_{d_s}, such as

$$\begin{aligned} \tilde{A}^{cl}(t) &= A(t) + B(t)KC + B(t)K_s - B(t)K_s \\ \tilde{A}^{cl}(t) &= A(t) + B(t)K_s + B(t)(KC - K_s) \\ \tilde{A}_d(t) &= A_d(t) + B(t)K_{d_s} - B(t)K_{d_s} \\ A_s(t) &= A(t) + B(t)K_s \quad A_{d_s}(t) = A_d(t) + B(t)K_{d_s} \end{aligned}$$

Note that conditions (7) and (8) do not depend on the system matrices so they do not need to be adapted to the uncertain case. Besides, we have to replace A^{cl} and A_d respectively by $\tilde{A}^{cl}(t)$ and $\tilde{A}_d(t)$ in condition (9) to get a condition for the robust case which is stated by Theorem 6.1.

Theorem 6.1. Assume that assumptions 2.1-2.2 are satisfied. If there exist F_i for $i = 1, \dots, 4$ and $P > 0$, $Q_j > 0$, $W_j > 0$, Y_j, Z_j for $j = 1, 2, \dots, p$, L, G and λ such that conditions (7), (8) and

$$\begin{bmatrix} f_{11} & f_{12} & 0 & P & f_{15}^\top & 0 & 0 & 0 \\ f_{12}^\top & f_{22} & 0 & 0 & f_{25}^\top & 0 & 0 & 0 \\ 0 & 0 & -\mathcal{W} & \mathcal{W} & 0 & 0 & 0 & 0 \\ P & 0 & \mathcal{W} & 0 & 0 & 0 & 0 & 0 \\ f_{15} & f_{25} & 0 & 0 & f_{55} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\lambda R_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\lambda R_b & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\lambda R_d \end{bmatrix} \tag{34}$$

$$+\mathsf{Sym}\left\{\begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}\left[A_s \quad A_{d_s} \quad 0 \quad -I \quad B \quad D_0 \quad D_b \quad D_d \right]\right\} < 0$$

hold with

$$f_{11} = \Psi_1 + \lambda N_0^\top R_0 N_0 + \lambda K_s^\top N_b^\top R_b N_b K_s$$
$$f_{22} = -\Psi_2 + \lambda N_d^\top R_d N_d + \lambda K_{d_s}^\top N_b^\top R_b N_b K_{d_s}$$
$$f_{12} = -\Psi_3 + \lambda K_s^\top N_b^\top R_b N_b K_{d_s}$$
$$f_{15} = (LC - GK_s) + \lambda K_s^\top N_b^\top R_b N_b$$
$$f_{25} = -GK_{d_s} + \lambda K_{d_s}^\top N_b^\top R_b N_b$$
$$f_{55} = -(G+G)^\top + N_b^\top R_b N_b$$

then system (33) is robustly asymptotically stabilizable by the output feedback controller

$$K = G^{-1}L$$

The proof of Theorem 6.1 can be carried out with similar arguments as for Theorem 4.1.

Example 6.1. In this example, we consider the problem of state feedback robust stabilization for the system whose data are given as

$$A_0 = \begin{bmatrix} 1.5 & 0.5 & 1 \\ -1 & 0 & 1 \\ 0.5 & 0 & 1 \end{bmatrix}, A_1 = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 0.5 \\ 0.3 & 0.5 & -1 \end{bmatrix}$$
$$E = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 1 \\ 2 & 0 & 1 \end{bmatrix} B = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (35)$$

The admissible uncertainties are given by

$$D_0 = D_1 = 0.2I_3, \; N_0 = N_1 = I_3, \; N_b = 0, \; D_b = 0.2I_3$$

We apply theorem 6.1 for the overall system and for

$$K_s = \begin{bmatrix} 4.8976 & 10.1848 & 21.3709 \\ -9.6414 & -35.0424 & -41.3699 \end{bmatrix}$$
$$K_{d_s} = \begin{bmatrix} 1.8620 & 1.3457 & 1.0246 \\ 0.6899 & -1.2852 & -0.3757 \end{bmatrix}$$

we find out that this system is asymptotically stabilizable with the state feedback gain

$$K = \begin{bmatrix} 177.5158 & 420.7779 & 663.7208 \\ -47.0935 & -198.5964 & -210.2011 \end{bmatrix}$$

for $\bar{h} = 2s$ and $0 \leq \dot{h}(t) \leq 0.8$.

Figure 1 shows the behaviour of system (35) for a maximum delay.

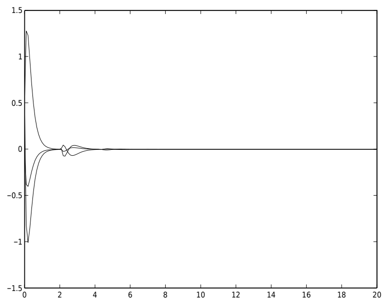

Fig. 1. Evolution of states x_1, x_2 and x_3 of system (35)

7. CONCLUSION

This paper deals with a class of dynamical uncertain singular systems with multiple time-varying states delays. Delay-dependent sufficient conditions have been developed to check whether a system of this class is stable or unstable, an output feedback controller with consequent parameters has been used to stabilize the system. The LMI technique is used in all the development. A numerical example is given to illustrate the obtained results.

REFERENCES

C. H. Fang, L. Lee and F-R. Chang (1994). Robust control analysis design for discrete-time singular systems. *Automatica* **30**, 1741–1750.

Campbell, S. L. (1980). Singular linear systems of diferential equations with delays. *Appl. Anal* **2**, 129–136.

Dai, L. (1989). *Singular control systems*. Springer-Verlag. New-York.

Fang, C. H. and F. R. Chang (1993). Analysis of stability robustness for generalized state-space systems with structured perturbations. *Sys. Control. Lett* **21**, 109–114.

Lewis, F. L. (2002). A survey of linear singular systems. *Circiuts, Syst. Signal processing* **5**, 03–36.

Mahmoud, M. S and N. F. Al-Muthairi (1994). Quadratic stabilization of continous time systems with state-delay and norm-bounded time-varying uncertainties. *IEEE Transactions on Automatic Control* **39**, 2135–2139.

M.S. Saadni, D. Mehdi (2003). Ostabilisation des systèmes dynamiques à temps discret et à retards multiples variant dans le temps. *JDA-JNA, Valencienne*.

S. Xu, C. Yang (2000). An algebraic approach to the robust stability analysis and robust stabilization of uncertain singular systems. *Int.J. Syst. Science* **31**, 55–61.

S. Xu, C. Yang, Y . Niu and J. Lam (2001). Robust stabilization for uncertain discrete singular systems. *Automatica* **37**, 769–774.

S. Xu, J. Lam and C. Yang (1994). Robust h_∞ control for discrete singular systems with state delay and parameter uncertainty. *Dyna .continous, discrete, Impul. syst.*

S. Xu, J. Lam and C. Yang (2001). Quadratic stability and stabilization of uncertain linear discrete-time systems with state delay. *Syst. control. Lett* **43**, 77–84.

S. Xu, P. V . Dooren, R. Stefan and J. Lam (2002). Robust stability and stabilization for singular systems with state delay and parameter uncertainty. *IEEE transaction on automatic control* **47**, 1122–1128.

Xie, L. (1996). Output feedback H_∞ control of systems with parameter uncertainty. *Int. J. Control* **63**, 741– 750.