Automatica 44 (2008) 3222–3226

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Technical communique

Stability and stabilization of nonuniform sampling systems[☆]

Young Soo Suh[*]

Department of Electrical Eng., University of Ulsan, Namgu, Ulsan 680-749, Republic of Korea

ARTICLE INFO

Article history:
Received 11 December 2007
Received in revised form
30 June 2008
Accepted 12 October 2008
Available online 4 November 2008

Keywords:
Sampled-data control
Stability
Linear matrix inequalities
Networked control systems

ABSTRACT

This paper is concerned with nonuniform sampling systems, where the sampling interval is time-varying within a certain known bound. The system is transformed into a time-varying discrete time system, where time-varying parts due to the sampling interval variation are treated as norm bounded uncertainties using robust control techniques. To reduce conservatism arising from modeling time-varying parts as a single uncertainty, the time-varying parts are modeled as N uncertainties. With larger N, a less conservative stability condition is derived at sacrifice of more computation. It is shown through a numerical example that the proposed stability condition is better than existing stability conditions.

1. Introduction

Recent interest in sampled-data systems with uncertain sampling intervals is mainly from networked control systems (Yang, 2006). In networked control systems with a limited bandwidth, a nonuniform sampling strategy could achieve better performance than a uniform sampling strategy. For example, the maximum error algorithm was first used in Walsh and Ye (2001), where fast changing signals are transmitted more often than slowly changing signals. In this case, the transmission interval of each node is not known in advance and time-varying. From the control viewpoint, the node transmission interval is the same as the sampling interval; thus the sampling interval is time-varying.

Sampled-data systems with a known sampling interval have been studied extensively over the past decades and the systems can be analyzed in the discrete time (Chen & Francis, 1995; Franklin, David Powell, & Workman, 1997). When the sampling interval is unknown, the situation is more complicated and there are three main approaches for stability analysis.

The first approach is to view a system as a continuous time system with a delayed control input. If the sampling instant is t_k and the next sampling instant is t_{k+1}, the control input between the sampling instants is $u(t) = u(t_k)$, $t \in [t_k, t_{k+1})$. From $u(t) = u(t-(t-t_k))$, we can model the system as a time-varying input delay system with the input delay $t-t_k$. This input delay approach is proposed in Fridman (2004) and improved in Mirkin (2007) and Hu, Bai, Shi, and Wu (2007).

The second approach is a hybrid modeling of sampled data systems (Naghshtabrizi, Hespanha, & Teel, 2006), where a Lyapunov function with jumps is used. Stability conditions are derived both for time-varying uncertain sampling intervals and time-invariant uncertain sampling intervals (i.e., the sampling interval is constant but unknown).

The third approach is to model a sampled data system as a discrete time system. In Zhang, Branicky, and Phillips (2001), a sampled data system with a delay is modeled as a discrete time system and a stability condition is derived. A delay is treated as a parameter variation in the transformed discrete time system. Our approach is based on the third approach in the sense that we first model the nonuniform sampling system as a time-varying discrete system. The problem with this approach is that it is not easy to derive a stability condition for the time-varying discrete system. To overcome the difficulty, we model the time-varying system as an uncertain time-invariant system, where the time-varying parts are modeled as a norm-bounded uncertainty. We also provide a method to divide time-varying parts into N norm-bounded uncertainties. Using this method, we can derive a less conservative stability condition while the computational burden is increasing. We note that a similar approach was taken in Fujioka (2008) and our approach uses less conservative bounds of the time-varying parts.

Throughout the paper, $\|M\|_2$ for a matrix M denotes the 2-norm, which is the maximum singular value of M. For a matrix M, M' denotes the transpose of M.

[☆] This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor Mayuresh V. Kothare under the direction of Editor André L. Tits.

[*] Tel.: +82 52 259 2196; fax: +82 52 259 1686.
E-mail address: suh@ieee.org.

0005-1098/$ – see front matter © 2008 Elsevier Ltd. All rights reserved.
doi:10.1016/j.automatica.2008.10.002

2. Problem formulation

Consider continuous time linear systems of the form

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x \in R^n$ is the state and $u \in R^m$ is the control input.

We assume that the state $x(t)$ is sampled at the discrete time instances

$$0 = t_0 < t_1 < \cdots < t_k < \cdots$$

and the control input $u(t)$ is piecewise constant between the discrete time instances:

$$u(t) = Kx(t_k), \quad \forall t \in [t_k, t_{k+1}). \tag{2}$$

The sampling interval T_k is defined as

$$T_k \triangleq t_{k+1} - t_k.$$

If T_k is constant, it is just a standard sampled-data control problem (Chen & Francis, 1995). In this paper, it is assumed that T_k is time-varying and its lower bound and upper bound are known:

$$0 < T_{\min} \le T_k \le T_{\max} < \infty, \quad \forall k. \tag{3}$$

In networked control systems, $T_{\min}$ and $T_{\max}$ depend on network types and scheduling methods. In the case of $T_k = 0$, the sampled-data system is not well-defined: (4) is not well-defined if $T_k = 0$. Thus we assume $T_{\min} > 0$, which is not restricted since sampling periods of digital systems cannot be zero.

The system (1) and (2) can be written as a time-varying discrete time system:

$$x(t_{k+1}) = G(T_k)x(t_k) \tag{4}$$

where

$$G(T_k) \triangleq \exp(AT_k) + \int_0^{T_k} \exp(Ar)B\mathrm{d}r\, K.$$

The standard Lyapunov stability condition for (4) is given in the next lemma (Khalil, 1996):

Lemma 1. *The system (4) is stable if there exists a matrix $P = P' > 0$ satisfying*

$$G(T)'PG(T) - P < 0, \quad \textit{for all } T_{\min} \le T \le T_{\max}. \tag{5}$$

If T is fixed, it is easy to find P satisfying (5). It is, however, not easy to find P satisfying (5) for all $T_{\min} \le T \le T_{\max}$. In the next section, we will derive a sufficient condition of (5) using the robust control techniques.

3. Sufficient stability condition with one nominal point

In this section, we will model $G(T)$, $T_{\min} \le T \le T_{\max}$ as $G(T_{\text{nom}}) + \Delta Q(T_{\text{nom}})$, where $\|\Delta\|_2$ is a norm bounded uncertainty and T_{nom} is a constant to be chosen. We define Δ so that for $T_{\min} \le T \le T_{\max}$

$$\{G(T)\} \subset \{G(T_{\text{nom}}) + \Delta Q(T_{\text{nom}}) \mid \|\Delta\|_2 \le \beta\}.$$

Thus a stability condition for $G(T_{\text{nom}}) + \Delta Q(T_{\text{nom}})$ is a sufficient stability condition for $G(T)$, $T_{\min} \le T \le T_{\max}$.

Let H and $F(T)$ be defined by

$$H \triangleq \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}, \qquad F(T) \triangleq \exp(HT).$$

From the relationship (Van Loan, 1978)

$$\exp\left(\begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix} T\right) = \begin{bmatrix} \exp(AT) & \int_0^T \exp(Ar)B\,\mathrm{d}r \\ 0 & I \end{bmatrix}, \tag{6}$$

we have

$$G(T) \triangleq \begin{bmatrix} I & 0 \end{bmatrix} F(T) \begin{bmatrix} I \\ K \end{bmatrix}. \tag{7}$$

A nominal point T_{nom} satisfies the following

$$T_{\min} \le T_{\text{nom}} \le T_{\max}, \tag{8}$$

and T_{nom} is chosen so that β, a norm bound of Δ, is small. The numerical process to choose T_{nom} will be given in (19).

Given T_{nom}, $\tau(T, T_{\text{nom}})$ is defined as

$$\tau(T, T_{\text{nom}}) \triangleq T - T_{\text{nom}}. \tag{9}$$

Note that for a given T_{nom} satisfying (8), we have

$$T_{\min} - T_{\text{nom}} \le \tau(T, T_{\text{nom}}) \le T_{\max} - T_{\text{nom}}. \tag{10}$$

For simplicity, $\tau(T, T_{\text{nom}})$ is sometimes written as just τ.

In the following lemma, we express $G(T)$ in the form of $G(T_{\text{nom}}) + \Delta(\tau)Q(T_{\text{nom}})$, where $\Delta(\tau)$ is defined by

$$\Delta(\tau) \triangleq \int_0^\tau \exp(Ar)\,\mathrm{d}r. \tag{11}$$

Lemma 2. *$G(T)$ can be expressed as follows:*

$$G(T) = G(T_{\text{nom}}) + \Delta(\tau)\begin{bmatrix} A & B \end{bmatrix} F(T_{\text{nom}}) \begin{bmatrix} I \\ K \end{bmatrix}. \tag{12}$$

Proof. Note that

$$\begin{aligned} F(T) &= F(\tau + T_{\text{nom}}) = F(\tau)F(T_{\text{nom}}) \\ &= F(T_{\text{nom}}) + (F(\tau) - I)F(T_{\text{nom}}). \end{aligned} \tag{13}$$

Invoking the relationship (Bernstein, 2005):

$$\exp(H\tau) - I_{2n} = \int_0^\tau \exp(Hr)H\,\mathrm{d}r,$$

we have

$$\begin{aligned} F(\tau) - I &= \int_0^\tau \exp(Hr)H\,\mathrm{d}r \\ &= \begin{bmatrix} I \\ 0 \end{bmatrix} \int_0^\tau \exp(Ar)\mathrm{d}r \begin{bmatrix} A & B \end{bmatrix}. \end{aligned} \tag{14}$$

Inserting (14) into (13) and again inserting (13) into (7), we obtain (12). □

Note that the only time-varying part in (12) is $\Delta(\tau)$. In the following, $\Delta(\tau)$ will be treated as a norm bounded uncertainty. To do that, a norm bound of $\Delta(\tau)$ is given in the next lemma.

Lemma 3. *Let α_1 be the maximum real part of the eigenvalues of A and α_2 be the maximum real part of the eigenvalues of $-A$. The Schur decomposition of A is given by*

$$U'AU = D + N$$

where U is an orthogonal matrix, D is a diagonal matrix, and N is a strictly upper triangular matrix. The following is satisfied:

$$\left\| \int_0^\tau \exp(At)\,\mathrm{d}t \right\|_2 \le \beta(\tau) \tag{15}$$

where

$$\beta(\tau) \triangleq \begin{cases} \displaystyle\sum_{k=0}^{n-1} \|N\|_2^k \left(-\frac{(-1)^k}{\alpha_1^{k+1}} + \frac{\exp(\alpha_1\tau)}{\alpha_1} \sum_{i=0}^{k} \frac{(-1)^i \tau^{k-i}}{\alpha_1^i (k-i)!} \right), \\ \quad \text{if } \tau \ge 0, \alpha_1 \ne 0 \\ \displaystyle\sum_{k=0}^{n-1} \|N\|_2^k \left(-\frac{(-1)^k}{\alpha_2^{k+1}} + \frac{\exp(\alpha_2|\tau|)}{\alpha_2} \sum_{i=0}^{k} \frac{(-1)^i |\tau|^{k-i}}{\alpha_2^i (k-i)!} \right), \\ \quad \text{if } \tau < 0, \alpha_2 \ne 0 \\ \displaystyle\sum_{k=0}^{n-1} \frac{\|N\|_2^k}{(k+1)!} |\tau|^{k+1}, \quad \text{otherwise.} \end{cases}$$

Proof. It is proved in Van Loan (1977) that for $t \geq 0$

$$\|\exp(At)\|_2 \leq \exp(\alpha_1 t) \sum_{k=0}^{n-1} \frac{\|Nt\|_2^k}{k!}. \tag{16}$$

Using (16), we have

$$\begin{aligned}\left\| \int_0^\tau \exp(At)\,dt \right\|_2 &\leq \int_0^\tau \|\exp(At)\|_2 \,dt \\ &\leq \int_0^\tau \exp(\alpha_1 t) \sum_{k=0}^{n-1} \frac{\|Nt\|_2^k}{k!}\,dt \\ &= \sum_{k=0}^{n-1} \frac{\|N\|_2^k}{k!} \int_0^\tau \exp(\alpha_1 t) t^k \,dt. \end{aligned} \tag{17}$$

Integrating the last equation of (17), we obtain (15) in the case of $\tau \geq 0$. When $\tau < 0$, by changing the integral variable $r = -t$, we have

$$\left\| \int_0^\tau \exp(At)\,dt \right\|_2 = \left\| \int_0^{|\tau|} \exp(-Ar)\,dr \right\|_2.$$

Repeating the processes (16) and (17), we can obtain (15) in the case of $\tau < 0$. $\square$

We treat $\Delta(\tau)$ as a norm bounded uncertainty and recall from (10) that the range of time-varying τ is as follows:

$$T_{\min} - T_{\text{nom}} \leq \tau(T, T_{\text{nom}}) \leq T_{\max} - T_{\text{nom}}.$$

When T_{nom} is given, we have from Lemma 3

$$\|\Delta(\tau)\|_2 \leq \max_{T_{\min}-T_{\text{nom}} \leq r \leq T_{\max}-T_{\text{nom}}} \beta(r).$$

It is desirable the right hand side bound is as small as possible and thus we choose T_{nom} from the following minimization problem:

$$\min_{T_{\min} \leq T \leq T_{\max}} \; \max_{T_{\min}-T \leq r \leq T_{\max}-T} \beta(r). \tag{18}$$

Since $\beta(r)$ is a monotonically increasing function as $|r|$ increases, (18) can be simplified as follows:

$$\bar{\beta} = \min_{T_{\min} \leq T \leq T_{\max}} \max \left\{ \beta(T_{\min} - T), \beta(T_{\max} - T) \right\}. \tag{19}$$

Moreover let T_{nom} be the sample period corresponding to which β attains its minimum, i.e.,

$$\bar{\beta} = \beta(T_{\text{nom}}). \tag{20}$$

Note that $\bar{\beta}$ satisfies that

$$\|\Delta(\tau)\|_2 \leq \bar{\beta}, \quad T_{\min} - T_{\text{nom}} \leq \tau \leq T_{\max} - T_{\text{nom}}. \tag{21}$$

Now the stability condition is given in the next theorem.

Theorem 4. *Let $\bar{\beta}$ be defined as in (19) and (20). If there exist $P = P' \in R^{n \times n} > 0$ and $\epsilon \in R > 0$ satisfying the following*

$$\begin{bmatrix} -P & \star & \star \\ G(T_{\text{nom}})P & -P + \epsilon I & \star \\ \begin{bmatrix} A & B \end{bmatrix} F(T_{\text{nom}}) \begin{bmatrix} I \\ K \end{bmatrix} P & 0 & -\dfrac{\epsilon}{\bar{\beta}^2} I \end{bmatrix} < 0, \tag{22}$$

then the system (1) with the feedback control (2) is stable for any sampling intervals satisfying (3). In (22), $\star$ denotes symmetric elements of the matrix and $\bar{\beta}$ satisfies (21) for all τ satisfying (10).

Proof. The system is stable if there exist $P = P' > 0$ and $\epsilon > 0$ satisfying (5). Using the Schur complement (Boyd, El Ghaoui, Feron, & Balakrishnan, 1994), (5) is equivalent to the following

$$\begin{bmatrix} -P & PG(T)' \\ G(T)P & -P \end{bmatrix} < 0, \quad T_{\min} \leq T \leq T_{\max}. \tag{23}$$

Using (12), we can rewrite (23) as follows:

$$\begin{bmatrix} -P & \star \\ G(T_{\text{nom}})P + \Delta(\tau) \begin{bmatrix} A & B \end{bmatrix} F(T_{\text{nom}}) \begin{bmatrix} I \\ K \end{bmatrix} P & -P \end{bmatrix} < 0. \tag{24}$$

We use the following inequality: if $\|\Delta\|_2 \leq \bar{\beta}$, then

$$R\Delta S + S'\Delta' R' \leq \epsilon RR' + \frac{\bar{\beta}^2}{\epsilon} S'S \tag{25}$$

for any $\epsilon > 0$. The proof of (25) is given in Wang, Xie, and de Souza (1992). Note that Lemma 2.2 in Wang et al. (1992), it is assumed that $\|\Delta\|_2 \leq 1$. Setting $\frac{\Delta}{\bar{\beta}}$ and $\bar{\beta} S$, we can use Lemma 2.2.

Invoking (25) in (23) and (24), we have for any $\epsilon > 0$

$$\begin{bmatrix} -P & PG(T)' \\ G(T)P & -P \end{bmatrix} < \begin{bmatrix} -P & \star \\ G(T_{\text{nom}})P & -P \end{bmatrix} + \epsilon RR' + \frac{\bar{\beta}^2}{\epsilon} S'S \tag{26}$$

where

$$R \triangleq \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad S \triangleq \begin{bmatrix} \begin{bmatrix} A & B \end{bmatrix} F(T_{\text{nom}}) \begin{bmatrix} I \\ K \end{bmatrix} P & 0 \end{bmatrix}.$$

Applying the Schur complement to the right hand side of (26), we obtain (22). Thus if there exist $P = P' > 0$ and $\epsilon > 0$ satisfying (22), there exists $P = P' > 0$ satisfying (23). $\square$

Using Theorem 4, we can check the stability of (1) (the state feedback K is given) without checking infinitely many linear matrix inequalities in (5).

In the next theorem, we consider the stabilization problem.

Theorem 5. *If there exist $P = P' \in R^{n \times n} > 0$, $\epsilon \in R > 0$, and $Z \in R^{m \times n}$ satisfying*

$$\begin{bmatrix} -P & \star & \star \\ \Xi & -P + \epsilon I & 0 \\ A\Xi + BZ & 0 & -\dfrac{\epsilon}{\bar{\beta}^2} I \end{bmatrix} < 0 \tag{27}$$

with

$$\Xi \triangleq \exp(AT_{\text{nom}})P + \int_0^{T_{\text{nom}}} \exp(Ar)B\,dr\,Z,$$

then the system (1) with a state feedback controller $K = ZP^{-1}$ is stable for any sampling intervals satisfying (3).

Proof. The proof is straightforward by setting $Z = KP$ in (22). $\square$

4. Sufficient stability condition with multiple nominal points

Using Theorem 4, we can check the stability of (1) from one linear matrix inequality instead of infinitely many matrix inequalities in (5). The price for this convenience is that the condition in Theorem 4 is now a sufficient condition, which could be conservative. Conservativeness of Theorem 4 depends on the tightness of the bound (15), which is usually tight bound for small τ (see Table 2 and numerical verification in Section 5). One way to reduce conservativeness of Theorem 4 is to reduce the maximum τ value in (10) by introducing multiple nominal points.

Table 1
Largest stable $T_{\max}$ value by Theorem 6.

	$N = 1$	$N = 2$	$N = 3$
$T_{\min} = 0.3$	1.335	1.651	1.688
$T_{\min} = 0.5$	1.619	1.725	1.729

Divide $[T_{\min}, T_{\max}]$ into N partitions: each partition $[T_{\min,i}, T_{\max,i}]$ $(i = 1, \ldots, N)$ satisfies

$$T_{\min} = T_{\min,1} < T_{\max_1} = T_{\min,2} < T_{\max,2} \\ = T_{\min,2} < \cdots T_{\max,N} = T_{\max}.$$

It is not required that the length of each partition be the same: that is, in general, $T_{\max,i} - T_{\min,i} \neq T_{\max,j} - T_{\min,j}$ if $i \neq j$. For each partition, we can find $\bar{\beta}_i$ and $T_{nom,i}$ ($T_{nom,i} \in [T_{\min,i}, T_{\max,i}]$) from the optimization problem (19), where $\bar{\beta}_i$ satisfies

$$\|\Delta(\tau)\|_2 \leq \bar{\beta}_i, \quad T_{\min,i} - T_{nom,i} \leq \tau \leq T_{\max,i} - T_{nom,i}. \tag{28}$$

Using the multiple nominal points, we can derive a less conservative stability condition in the next theorem.

Theorem 6. *If there exist $P = P' \in R^{n\times n} > 0$ and $\epsilon_i \in R > 0$ satisfying*

$$\begin{bmatrix} -P & \star & \star \\ G(T_{nom,i})P & -P + \epsilon_i I & 0 \\ \begin{bmatrix} A & B \end{bmatrix} F(T_{nom,i}) \begin{bmatrix} I \\ K \end{bmatrix} P & 0 & -\dfrac{\epsilon_i}{\bar{\beta}_i^2} I \end{bmatrix} < 0, \tag{29}$$

for $i = 1, \ldots, N$, where $\bar{\beta}_i$ satisfies (28), then the system (4) is stable.

Proof. We can rewrite the stability condition (5) as follows:

$$\begin{aligned} &G(T)'PG(T) - P < 0, \quad \text{for all } T_{\min,1} \leq T \leq T_{\max,1} \\ &\qquad \vdots \\ &G(T)'PG(T) - P < 0, \quad \text{for all } T_{\min,N} \leq T \leq T_{\max,N}. \end{aligned} \tag{30}$$

Applying the same arguments with the proof of Theorem 4 to each inequality in (30), we can show that if there exist $P = P' > 0$ and $\epsilon_i > 0$ satisfying (29), there exists $P = P' > 0$ satisfying (30). Thus the system is stable if (29) are satisfied from Lemma 1. □

For $N = 1$, Theorem 6 is the same as Theorem 4. If we use large N (that is, each partition is small), the stability condition in (29) becomes less conservative at the sacrifice of more computation.

A stabilization problem based on Theorem 6 can also be derived exactly the same way as in Theorem 5 and is omitted.

5. Example

To illustrate the proposed stability condition, we consider the following system

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}, \qquad K = \begin{bmatrix} -3.75 & -11.5 \end{bmatrix}.$$

This system was considered in Zhang et al. (2001) and also considered in Naghshtabrizi et al. (2006) and Mirkin (2007).

We used the stability condition in Theorem 6 for $N = 1, 2,$ and 3. For $N = 1$, we increased $T_{\max}$ value until the stability condition in Theorem 6 is satisfied. For $N = 2$, we chose $T_{\max,1} = T_{\min,2}$ as the value $T_{\max}$ for the case $N = 1$. Then we increased $T_{\max,2}$ value until the stability condition is satisfied. For $N = 3$, we chose $T_{\max,1}$ and $T_{\max,2}$ as the same value for $N = 2$ and we increased $T_{\max,3}$ value until the stability condition is satisfied.

The largest $T_{\max}$ values by Theorem 6 are given in Table 1. As N increases, we can see the stability condition of Theorem 6 becomes less conservative. Conservativeness of the stability condition depends mainly on the tightness of the bound (15). To see how tight the norm bound (15) is, the actual norm and its bound values in (15) are given in Table 2 for different τ values. We can see that as τ becomes larger, the bound becomes less tight. This trend was also verified numerically from 10,000 randomly generated 2×2 matrices: for all 10,000 matrices, $\beta(\tau) - \left\| \int_0^\tau \exp(At)\,dt \right\|_2$ were monotonically nonincreasing with respect to τ.

Table 2
$\| \int_0^\tau \exp(At)\,dt \|_2$ and $\beta(\tau)$ in (15).

τ	$\| \int_0^\tau \exp(At)\,dt \|_2$	$\beta(\tau)$	Percentage error
0.100	0.100	0.105	5.0
0.300	0.297	0.345	16.2
0.500	0.490	0.625	27.6
0.700	0.679	0.945	39.2
0.900	0.865	1.305	50.9

Table 3
Largest stable $T_{\max}$ by existing stability conditions.

	Largest stable $T_{\max}$
Result 1 (Fridman, 2004)	0.869
Result 2 (Naghshtabrizi et al., 2006)	1.113
Result 3 (Fujioka, 2008) ($T_{\min} = 0.3$)	1.283
Result 4[a] (Naghshtabrizi et al., 2006)	1.327
Result 5 (Mirkin, 2007)	1.365
Result 3 (Fujioka, 2008) ($T_{\min} = 0.5$)	1.607
Result 6[b]	1.729

[a] Constant sampling case.
[b] Analytic result for the constant sampling case.

Comparison with existing stability conditions is given in Table 3.

Result 4 is derived with the assumption that the sampling interval is constant while Result 1, 2, 3, and 5 allow the sampling interval to be time-varying. Result 6 is the analytic result when the sampling interval is constant: that is, we computed the largest T satisfying $\rho(G(T)) < 1$, $T > 0$, where $\rho(\cdot)$ is the spectral radius. In Result 1, 2, 4, and 5, $T_{\min} = 0$ is assumed. In Result 3, the time-varying interval is not partitioned ($N = 1$) and less conservative result can be obtained following the algorithm in Fujioka (2008).

Comparing Tables 1 and 3, we can see that the proposed stability condition is less conservative. One difference between the proposed stability condition and the existing stability conditions is $T_{\min}$. In existing stability conditions except for Result 3, $T_{\min} = 0$ while in the proposed condition we must assume $T_{\min} > 0$. However, we can explicitly assign $T_{\min}$ value. In networked control systems, $T_{\min}$ cannot be zero and can be determined by network types and scheduling methods. Exploiting $T_{\min}$ information, we can derive a less conservative stability condition.

6. Conclusion

We have derived new stability conditions and stabilization methods for nonuniform sampling systems. The main technical contribution is that the system is transformed into a time-varying discrete system and then modeled as a norm bounded uncertainty systems. The stability condition is given in linear matrix inequalities. Furthermore, we provide a method to reduce conservativeness using multiple matrix inequalities. It is shown that the proposed stability condition is less conservative than existing conditions. How to extend the proposed method for uncertain systems is a topic of future studies.

Acknowledgments

This work was supported by the Korea Science and Engineering Foundation (KOSEF) grant (No. R01-2006-000-11334-0) and the Network-based Automation Research Center (NARC) grant funded by Ministry of Knowledge Economy and Ulsan Metropolitan City.

References

Bernstein, Dennis S. (2005). *Matrix mathematics: Thoery, facts, and formulas with application to linear system theory*. Princeton, NJ: Princeton University Press.

Boyd, Stephen, El Ghaoui, Laurent, Feron, Eric, & Balakrishnan, Venkataramanan (1994). *Linear matrix inequalities in system and control theory*. Philadelphia: Society for Industrial and Applied Mathematics.

Chen, Tongwen, & Francis, Bruce (1995). *Optimal sampled-data control systems*. Tokyo: Springer-Verlag.

Franklin, Gene F., David Powell, J., & Workman, Michael (1997). *Digital control of dynamic systems* (3rd ed.). Menlo Park: Addison Wesley Longman.

Fridman, E., Seuret, Alexandre, & Richard, Jean-Pierre (2004). Robust sampled-data stabilization of linear systems: An input delay approach. *Automatica*, *40*, 1441–1446.

Fujioka, Hisaya (2008). Stability analysis for a class of networked/embedded control systems: A discrete-time approach. In *Proceedings of American control conference* (pp. 4997–5002).

Hu, Li-Sheng, Bai, Tao, Shi, Peng, & Wu, Ziming (2007). Sampled-data control of networked linear control systems. *Automatica*, *43*, 903–911.

Khalil, Hassan K. (1996). *Nonlinear systems* (2nd ed.). New Jersey: Prentice-Hall.

Mirkin, Leonid (2007). Some remarks on the use of time-varying delay to model sample-and-hold circuits. *IEEE Transactions on Automatic Control*, *52*(6), 1109–1112.

Naghshtabrizi, Payam, Hespanha, Joao P., & Teel, Andrew R. (2006). On the robust stability and stabilization of sampled-data systems: A hybrid system approach. In *Proc. of the 45th IEEE conference on decision and control* (pp. 4873–4878).

Van Loan, Charles F. (1978). Computing integrals involving the matrix exponential. *IEEE Transactions on Automatic Control*, *AC-23*(3), 395–404.

Van Loan, Charles (1977). The sensitivity of the matrix exponential. *SIAM Journal on Numerical Analysis*, *14*(6), 971–981.

Walsh, G. C., & Ye, Hong (2001). Scheduling of networked control systems. *IEEE Control Systems Magazine*, *21*(1), 57–65.

Wang, Y., Xie, L., & de Souza, C. E. (1992). Robust control of a class of uncertain nonlinear systems. *Systems & Control Letters*, *19*(2), 139–149.

Yang, T. C. (2006). Networked control system: A brief survey. *IEE Proceedings on - Control Theory and Applications*, *153*(4), 403–412.

Zhang, Wei, Branicky, M. S., & Phillips, S. M. (2001). Stability of networked control systems. *IEEE Control Systems Magazine*, *21*(1), 84–99.