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New insight into reachable set estimation for uncertain singular time-delay systems

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A B S T R A C T

This paper investigates the problem of reachable set estimation for a class of uncertain singular systems with time-varying delays from a new point of view. Our consideration is centered on the design of a proportional-derivative state feedback controller (PDSFC) such that the considered singular system is robustly normalizable and all the states of the closed-loop system can be contained by a bounded set under zero initial conditions. First, a nominal singular time-delay system is considered and sufficient conditions are obtained in terms of matrix inequalities for the existence of a PDSFC and an ellipsoid. In this case, the considered system is guaranteed to be normalizable and the reachable set of the closed-loop systems is contained by the ellipsoid. Then, the result is extended to the case of singular time-delay systems with polytopic uncertainties and relaxed conditions are derived by introducing some weighting matrix variables. Furthermore, based on the obtained results, the reachable set of the considered closed-loop singular system can be contained in a prescribed ellipsoid. Finally, the effectiveness of our results are demonstrated by two numerical examples.

1. Introduction

During the past years, reachable set estimation has been widely applied in control systems with actuator saturation, peak-to-peak gain minimization, and parameter estimation [1,2]. The objective of reachable set estimation is to find a suitable region which can bound all the reachable states of a dynamic system with input disturbances and zero initial conditions. Recently, many researchers have been devoting to studying the problem of reachable set estimation for time-delay systems since time delays often exist in physical systems and may affect the stability and performance of systems. For example, to obtain the existence conditions of an ellipsoid which can bound all the reachable states of linear systems with time-varying delays, Lyapunov–Krasovskii functionals (LKFs) were used in [3–6]; these conditions are less conservative than those in [2] by using Lyapunov–Razumikhin functions. To further improve the conditions of reachable set estimation, a non-uniform delay-partitioning method and a triple integral technique were used in [5]; in [6], a relaxed LKF was constructed where all the involved symmetric matrices are not required to be positive definite. For the problem of reachable set estimation for linear systems with distributed delays; see, e.g., [7–9] and the reference therein.

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On the other hand, singular time-delay systems have also received great attention and lots of results have been developed in the literatures for the analysis and synthesis of continuous- or discrete-time singular systems with time delays, such as admissibility analysis and stabilization [10–13], H_∞ or passive control [14–17], dissipativity analysis [18–20], sliding mode control [21,22], and so on. It is noted that some results on reachable set estimation for singular time-delay systems can also be found; see, e.g. [23,24]. The methods of dealing with the problem of reachable set estimation for normal time-delay systems can only guarantee that the reachable states of the slow subsystem of a singular system are bounded by an ellipsoid. In [23], different bounding techniques were used for the slow subsystem and the fast subsystem of a considered singular time-delay system, respectively. All the reachable states of the considered singular system were contained by an intersection of ellipsoids. However, to decompose the considered singular system into two subsystems with special structures, two nonsingular matrices were required to be found in [23], in which the obtained bounding ellipsoids were related to the two nonsingular matrices. This increases the difficulty of seeking the "smallest" ellipsoid, especially when the structures of the considered systems are complicated. Besides, the applications of the result in [23] are limited since an assumption was introduced. Therefore, the study on the reachable set estimation for singular time-delay systems is still challenging.

In this paper, we present a new insight into the reachable set estimation for a class of uncertain singular time-delay systems, where the considered singular system does not need to be decomposed and a proportional-derivative state feedback controller (PDSFC) is designed to guarantee all the reachable states of the considered closed-loop singular system contained by an ellipsoid. It is worth mentioning that though PDSFCs have been widely used in singular systems; see, e.g. [25–30], we first use them to study the problem of reachable set estimation for uncertain singular systems with time-varying delays. The main contributions are stated as follows.

(i) By using a PDSFC, the considered (uncertain) singular system is (robustly) normalizable and all the reachable states of the closed-loop system are contained by an ellipsoid. By an optimization problem, the "smallest" ellipsoid can be obtained easily. Since the considered (uncertain) singular system does not need to be decomposed and no assumptions are given, our results are less conservative.

(ii) The problem of reachable set estimation for singular time-delay systems with polytopic uncertainties is also studied. The uncertainties exist in both the state matrix and the derivative matrix. By introducing some weighting matrix variables, relaxed conditions are presented.

(iii) Based on the obtained conditions, we can investigate two problems by designing a PDSFC: One is to find the "smallest" ellipsoid to contain all the reachable states of the considered closed-loop singular system; the other is to make the reachable set of the considered closed-loop singular system contained in a prescribed ellipsoid.

The rest part of the paper is organized as follows. Section 2 is the problem formulation which gives the system model and some useful lemmas. Section 3 is the main results. A PDSFC is designed in this section such that the considered nominal singular time-delay system is normalizable and all the reachable states of the closed-loop system are contained by an ellipsoid. Then the result is extended to the case of singular time-delay systems with polytopic uncertainties. Section 4 gives two numerical examples to illustrate the effectiveness of our results and Section 5 is the conclusion of the paper.

Notations: Throughout this paper, $\mathbf{R}^n$ denotes the n-dimensional Euclidean space over the reals and $\mathbf{R}^{n\times m}$ denotes the set of all $n\times m$ real matrices. $|\cdot|$ denotes the Euclidean norm for vectors and $\|\cdot\|$ denotes the spectral norm for matrices. For real symmetric matrices X and Y, the notation $X\geq Y$ (respectively, $X>Y$) means that the matrix $X-Y$ is positive semi-definite (respectively, positive definite). I is an identity matrix with appropriate dimension and 0 is a zero matrix with appropriate dimension. The superscripts 'T' and '-1' represent the matrix transpose and inverse, respectively. $\star$ denotes the symmetric term in a symmetric matrix; $\rho(M)$ denotes the spectral radius of the matrix M; $rank(\cdot)$ stands for the rank of a matrix; $det(\cdot)$ denotes the determinant of a square matrix with appropriate dimension. For any square matrices A and B, define $diag(A,B)=\begin{bmatrix} A & 0 \\ \star & B \end{bmatrix}$. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation

Consider the following uncertain singular system with a time-varying delay:

$$\begin{aligned} &E(\lambda)\dot{x}(t)=A(\lambda)x(t)+A_d(\lambda)x(t-d(t))+B(\lambda)u(t)+D(\lambda)\omega(t),\\ &\qquad x(t)\equiv 0,\quad \forall t\in[-d_2,0], \end{aligned} \tag{1}$$

where $x(t)\in\mathbf{R}^n$ is the state vector; $u(t)\in\mathbf{R}^m$ is the control input; the matrix $E(\lambda)\in\mathbf{R}^{n\times n}$ may be singular and it is assumed that $rank(E(\lambda))=r\leq n$; $d(t)$ is a time-varying continuous function that satisfies

$$0\leq d_1\leq d(t)\leq d_2,\qquad \dot{d}(t)\leq\mu<\infty, \tag{2}$$

where d_1, d_2, μ are constants; $\omega(t)\in\mathbf{R}^l$ is the disturbance satisfying

$$\omega^T(t)\omega(t)\leq\bar{\omega}^2, \tag{3}$$

where $\bar{\omega}$ is a real constant.

In addition, $E(\lambda)$, $A(\lambda)$, $A_d(\lambda)$, $B(\lambda)$, $D(\lambda)$ are appropriately dimensioned matrices with polytopic-type parameter uncertainties. It is assumed that

$$\begin{bmatrix} E(\lambda) & A(\lambda) & A_d(\lambda) & B(\lambda) & D(\lambda) \end{bmatrix} = \sum_{i=1}^{s} \lambda_i \begin{bmatrix} E_i & A_i & A_{di} & B_i & D_i \end{bmatrix},$$

where s is the number of polytope vertices; E_i, A_i, A_{di}, B_i and D_i are known constant real matrices; λ_i are time-invariant scalar uncertainties satisfying $\lambda_i \geq 0$ and $\sum_{i=1}^{s} \lambda_i = 1$.

Similar to [3,6], the reachable set of system (1) is defined as follows:

$$\mathcal{R}_x = \{x(t) |\ x(t)\ and\ \omega(t)\ satisfy\ (1) - (3),\ t \geq 0\}.$$

The purpose of this paper is to design a PDSFC for system (1) of the form

$$u(t) = K_a x(t) - K_e \dot{x}(t), \tag{4}$$

such that for any time-varying delay $d(t)$ satisfying (2), the closed-loop system which is constructed by (1) and (4):

$$\begin{aligned} E_c(\lambda)\dot{x}(t) &= A_c(\lambda)x(t) + A_d(\lambda)x(t-d(t)) + D(\lambda)\omega(t), \\ x(t) &\equiv 0, \quad \forall t \in [-d_2, 0], \end{aligned} \tag{5}$$

with

$$E_c(\lambda) = E(\lambda) + B(\lambda)K_e, \tag{6}$$

$$A_c(\lambda) = A(\lambda) + B(\lambda)K_a, \tag{7}$$

is robustly normal ($E_c(\lambda)$ is nonsingular for all admissible uncertainties [28,29]) and its reachable set can be bounded by an ellipsoid which is defined as $\mathcal{E}(P) = \left\{\xi \in \mathbf{R}^n | \xi^T P \xi \leq 1\right\}$, where $P > 0$ is a real constant matrix.

The following lemmas will be used in the rest of this paper.

Lemma 1 ([23]). *Let $V(x_t)$ be a LKF for system (5) and $V(x_0) = 0$. If $\dot{V} + \alpha V - \frac{\alpha}{\bar{\omega}^2}\omega^T(t)\omega(t) \leq 0$ with a scalar $\alpha > 0$, then $V(x_T) < 1$ for $T \geq 0$.*

Lemma 2 ([32]). *For matrices X, Y and $J > 0$ with appropriate dimensions, the following inequality holds:*

$$XY + Y^T X^T \leq XJX^T + Y^T J^{-1} Y.$$

3. Main results

First, we consider the following nominal singular system which derives from system (1):

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + A_d x(t-d(t)) + Bu(t) + D\omega(t), \\ x(t) &\equiv 0, \quad \forall t \in [-d_2, 0], \end{aligned} \tag{8}$$

where the matrix $E \in \mathbf{R}^{n \times n}$ may be singular and it is assumed that $rank(E) = r \leq n$. A, A_d, B, D are constant matrices. In this case, we provide the existence condition of a PDSFC (4) which can ensure that system (8) is normalizable and the reachable set of the following closed-loop system

$$\begin{aligned} E_c\dot{x}(t) &= A_c x(t) + A_d x(t-d(t)) + D\omega(t), \\ x(t) &\equiv 0, \quad \forall t \in [-d_2, 0], \end{aligned} \tag{9}$$

with $E_c = E + BK_e$, $A_c = A + BK_a$ can be bounded by an ellipsoid.

Theorem 1. *Given scalars d_1, d_2 and μ, if there exist a scalar $\alpha > 0$, matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $R > 0$, M_1, M_2, N_1, N_2, T_1 and T_2 such that the following inequality holds*

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & 0 & \Pi_{14} & -e^{-\alpha d_2}N_1 & T_1^T D & d_2 e^{-\alpha d_2}M_1 & d_{12}e^{-\alpha d_2}N_1 \\ \star & \Pi_{22} & 0 & T_2^T A_d & 0 & T_2^T D & 0 & 0 \\ \star & \star & -e^{-\alpha d_1}Q_1 & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \star & \Pi_{44} & -e^{-\alpha d_2}N_2 & 0 & d_2 e^{-\alpha d_2}M_2 & d_{12}e^{-\alpha d_2}N_2 \\ \star & \star & \star & \star & -e^{-\alpha d_2}Q_2 & 0 & 0 & 0 \\ \star & \star & \star & \star & \star & -\frac{\alpha}{\bar{\omega}^2}I & 0 & 0 \\ \star & \star & \star & \star & \star & \star & -d_2 e^{-\alpha d_2}R & 0 \\ \star & \star & \star & \star & \star & \star & \star & -d_{12}e^{-\alpha d_2}R \end{bmatrix} < 0, \tag{10}$$

with

$$\Pi_{11} = T_1^T A_c + A_c^T T_1 + Q_1 + Q_2 + Q_3 + \alpha P + e^{-\alpha d_2} M_1 + e^{-\alpha d_2} M_1^T,$$
$$\Pi_{12} = P - T_1^T E_c + A_c^T T_2,$$
$$\Pi_{14} = T_1^T A_d - e^{-\alpha d_2} M_1 + e^{-\alpha d_2} M_2^T + e^{-\alpha d_2} N_1,$$
$$\Pi_{22} = -T_2^T E_c - E_c^T T_2 + d_2 R,$$
$$\Pi_{44} = -(1-\mu) e^{-\alpha d_2} Q_3 - e^{-\alpha d_2} M_2 - e^{-\alpha d_2} M_2^T + e^{-\alpha d_2} N_2 + e^{-\alpha d_2} N_2^T$$

and $d_{12} = d_2 - d_1$, then system (9) is normal and its reachable set is contained by the ellipsoid $\mathcal{E}(P)$.

Proof. From (10), we have

$$-T_2^T E_c - E_c^T T_2 < 0,$$

which implies that the derivative matrix E_c is invertible. Choose the following LKF for system (8):

$$\begin{aligned} V(x_t, t) =& x^T(t) P x(t) + \int_{t-d_1}^{t} e^{\alpha(s-t)} x^T(s) Q_1 x(s) ds + \int_{t-d_2}^{t} e^{\alpha(s-t)} x^T(s) Q_2 x(s) ds \\ &+ \int_{t-d(t)}^{t} e^{\alpha(s-t)} x^T(s) Q_3 x(s) ds + \int_{-d_2}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} \dot{x}^T(s) R \dot{x}(s) ds d\theta, \end{aligned} \tag{11}$$

where $x_t(s) = x(t+s)$, $-2d_2 \le s \le 0$. Differentiating $V(x_t)$ with respect to t, we can obtain

$$\begin{aligned} \dot{V}(x_t) + \alpha V(x_t) \le& 2x^T(t) P \dot{x}(t) + \alpha x^T(t) P x(t) + x^T(t)(Q_1 + Q_2 + Q_3) x(t) \\ & - e^{-\alpha d_1} x^T(t-d_1) Q_1 x(t-d_1) - e^{-\alpha d_2} x^T(t-d_2) Q_2 x(t-d_2) \\ & - (1-\mu) e^{-\alpha d_2} x^T(t-d(t)) Q_3 x(t-d(t)) + d_2 \dot{x}^T(t) R \dot{x}(t) \\ & - e^{-\alpha d_2} \int_{t-d(t)}^{t} \dot{x}^T(s) R \dot{x}(s) ds - e^{-\alpha d_2} \int_{t-d_2}^{t-d(t)} \dot{x}^T(s) R \dot{x}(s) ds. \end{aligned} \tag{12}$$

By the Leibniz–Newton formula, for any matrices M_1, M_2, N_1, N_2 with appropriate dimensions, we have

$$2e^{-\alpha d_2}\left[x^T(t) M_1 + x^T(t-d(t)) M_2\right]\left[x(t) - \int_{t-d(t)}^{t} \dot{x}(s) ds - x(t-d(t))\right] = 0, \tag{13}$$

$$2e^{-\alpha d_2}\left[x^T(t) N_1 + x^T(t-d(t)) N_2\right]\left[x(t-d(t)) - \int_{t-d_2}^{t-d(t)} \dot{x}(s) ds - x(t-d_2)\right] = 0. \tag{14}$$

On the other hand, for any matrices T_1, T_2 with appropriate dimensions, we have

$$2\left[x^T(t) T_1 + \dot{x}^T(t) T_2\right]\left[-E_c \dot{x}(t) + A_c x(t) + A_d x(t-d(t)) + D\omega(t)\right] = 0. \tag{15}$$

From (12) to (15), we can verify

$$\begin{aligned} \dot{V}(x_t) + \alpha V(x_t) - \frac{\alpha}{\bar{\omega}^2}\omega^T(t)\omega(t) \le& \xi^T(t)\left(\Phi + d_2 e^{-\alpha d_2} M R^{-1} M^T + d_{12} e^{-\alpha d_2} N R^{-1} N^T\right)\xi(t) \\ & - e^{-\alpha d_2} \int_{t-d(t)}^{t} \left(\xi^T(t) M + \dot{x}^T(s) R\right) R^{-1} \left(M^T \xi(t) + R\dot{x}(s)\right) ds \\ & - \int_{t-d_2}^{t-d(t)} \left(\xi^T(t) N + \dot{x}^T(s) R\right) R^{-1} \left(N^T \xi(t) + R\dot{x}(s)\right) ds, \end{aligned} \tag{16}$$

where

$$\Phi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & 0 & \Pi_{14} & -e^{-\alpha d_2} N_1 & T_1^T D \\ \star & \Pi_{22} & 0 & T_2^T A_d & 0 & T_2^T D \\ \star & \star & -e^{-\alpha d_1} Q_1 & 0 & 0 & 0 \\ \star & \star & \star & \Pi_{44} & -e^{-\alpha d_2} N_2 & 0 \\ \star & \star & \star & \star & -e^{-\alpha d_2} Q_2 & 0 \\ \star & \star & \star & \star & \star & -\frac{\alpha}{\bar{\omega}^2} I \end{bmatrix},$$

$$\xi^T(t) = \begin{bmatrix} x^T(t) & \dot{x}^T(t) & x^T(t-d_1) & x^T(t-d(t)) & x^T(t-d_2) & \omega^T(t) \end{bmatrix},$$
$$M^T = \begin{bmatrix} M_1^T & 0 & 0 & M_2^T & 0 & 0 \end{bmatrix},\ N^T = \begin{bmatrix} N_1^T & 0 & 0 & N_2^T & 0 & 0 \end{bmatrix}.$$

Since $R > 0$, it is easy to find that the last two terms in (16) are less than 0. Therefore, by (10) and the Schur complement, from (16), we can obtain

$$\dot{V}(x_t) + \alpha V(x_t) - \frac{\alpha}{\bar{\omega}^2}\omega^T(t)\omega(t) < 0. \tag{17}$$

By Lemma 1, it can be verified that $x^T(t)Px(t) \le 1$, which implies $x(t) \in \mathcal{E}(P)$. Thus, the reachable set of system (9) is contained by the ellipsoid $\mathcal{E}(P)$. This completes the proof. □

Remark 1. Based on the free-weighting-matrix approach [31], a sufficient delay-dependent condition is presented in Theorem 1 such that system (9) is normal and its reachable set is contained by the ellipsoid $\mathcal{E}(P)$. It can be found that condition (10) cannot be used to obtain the feedback gains K_a and K_e directly. To get the feedback gains of PDSFC (4), we have the following theorem.

Theorem 2. *Given scalars d_1, d_2 and μ, if there exist a scalar $\alpha > 0$, matrices $V_1 > 0$, $L_1 > 0$, $L_2 > 0$, $L_3 > 0$, $R > 0$, V_2, V_3, U_1, U_2, $\tilde{M}_1$, $\tilde{M}_2$, $\tilde{N}_1$, $\tilde{N}_2$ such that the following inequality holds*

$$\begin{bmatrix} \Omega_1 & \Omega_2 & \Omega_3 \\ \star & \Omega_4 & 0 \\ \star & \star & \Omega_5 \end{bmatrix} < 0, \tag{18}$$

with

$$\Omega_1 = \begin{bmatrix} \Phi_{11} & \Phi_{12} & 0 & 0 & 0 & 0 \\ \star & \Phi_{22} & 0 & A_d L_3 & 0 & D \\ \star & \star & -e^{-\alpha d_1} L_1 & 0 & 0 & 0 \\ \star & \star & \star & -(1-\mu)e^{-\alpha d_2} L_3 & 0 & 0 \\ \star & \star & \star & \star & -e^{-\alpha d_2} L_2 & 0 \\ \star & \star & \star & \star & \star & -\frac{\alpha}{\bar{\omega}^2} I \end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix} V_1 & V_1 & V_1 & d_2 V_2^T & e^{-\alpha d_2} V_1 & 0 \\ 0 & 0 & 0 & d_2 V_3^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -e^{-\alpha d_2} L_3 & e^{-\alpha d_2} L_3 \\ 0 & 0 & 0 & 0 & 0 & -e^{-\alpha d_2} L_2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Omega_3 = \begin{bmatrix} d_2 e^{-\alpha d_2} \tilde{M}_1 & d_{12} e^{-\alpha d_2} \tilde{N}_1 & \tilde{M}_1 & \tilde{N}_1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ d_2 e^{-\alpha d_2} \tilde{M}_2 & d_{12} e^{-\alpha d_2} \tilde{N}_2 & \tilde{M}_2 & \tilde{N}_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned} \Omega_4 &= diag\{-L_1, -L_2, -L_3, d_2(R-2I), -R, -R\}, \\ \Omega_5 &= diag\left\{-d_2 e^{-\alpha d_2} R, -d_{12} e^{-\alpha d_2} R, R-2I, R-2I\right\}, \\ \Phi_{11} &= V_2^T + V_2 + \alpha V_1, \quad \Phi_{12} = V_3 + V_1 A^T - V_2^T E^T + U_1^T B^T, \\ \Phi_{22} &= -EV_3 - V_3^T E^T - BU_2 - U_2^T B^T, \quad d_{12} = d_2 - d_1, \end{aligned}$$

then system (8) is normalizable and the reachable set of closed-loop system (9) is contained by the ellipsoid $\mathcal{E}(V_1^{-1})$. Furthermore, the gains of PDSFC (4) are

$$K_a = \left(U_1 + U_2 V_3^{-1} V_2^T\right) V_1^{-1}, \quad K_e = U_2 V_3^{-1}. \tag{19}$$

Proof. By the Schur complement, we can get that (10) is equivalent to

$$\begin{aligned} &\begin{bmatrix} \hat{\Pi}_{11} & \hat{\Pi}_{12} & 0 & T_1^T A_d & 0 & T_1^T D \\ \star & \hat{\Pi}_{22} & 0 & T_2^T A_d & 0 & T_2^T D \\ \star & \star & -e^{-\alpha d_1} Q_1 & 0 & 0 & 0 \\ \star & \star & \star & -(1-\mu)e^{-\alpha d_2} Q_3 & 0 & 0 \\ \star & \star & \star & \star & -e^{-\alpha d_2} Q_2 & 0 \\ \star & \star & \star & \star & \star & -\frac{\alpha}{\bar{\omega}^2} I \end{bmatrix} \\ &+ d_2 e^{-\alpha d_2} M R^{-1} M^T + d_{12} e^{-\alpha d_2} N R^{-1} N^T \\ &+ e^{-\alpha d_2} M e_1 + e^{-\alpha d_2} (M e_1)^T + e^{-\alpha d_2} N e_2 + e^{-\alpha d_2} (N e_2)^T < 0, \end{aligned} \tag{20}$$

where

$$\hat{\Pi}_{11} = T_1^T A_c + A_c^T T_1 + Q_1 + Q_2 + Q_3 + \alpha P,$$
$$\hat{\Pi}_{12} = P - T_1^T E_c + A_c^T T_2,$$
$$\hat{\Pi}_{22} = -T_2^T E_c - E_c^T T_2 + d_2 R,$$
$$M^T = \begin{bmatrix} M_1^T & 0 & 0 & M_2^T & 0 & 0 \end{bmatrix}, \quad N^T = \begin{bmatrix} N_1^T & 0 & 0 & N_2^T & 0 & 0 \end{bmatrix},$$
$$e_1 = \begin{bmatrix} I & 0 & 0 & -I & 0 & 0 \end{bmatrix}, \quad e_2 = \begin{bmatrix} 0 & 0 & 0 & I & -I & 0 \end{bmatrix}.$$

Let

$$X = \begin{bmatrix} P & 0 & 0 & 0 & 0 & 0 \\ T_1 & T_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & Q_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}, \quad X^{-1} = \begin{bmatrix} V_1 & 0 & 0 & 0 & 0 & 0 \\ V_2 & V_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & L_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & L_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & L_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}. \tag{21}$$

It is easy to get $V_1 = P^{-1}$, $V_3 = T_2^{-1}$, $T_1V_1 + T_2V_2 = 0$, $L_1 = Q_1^{-1}$, $L_2 = Q_2^{-1}$ and $L_3 = Q_3^{-1}$. Pre-multiplying and post-multiplying (20) by X^{-T} and its transpose, respectively, we have

$$\begin{aligned} &\begin{bmatrix} \bar{\Pi}_{11} & \bar{\Pi}_{12} & 0 & 0 & 0 & 0 \\ \star & \bar{\Pi}_{22} & 0 & A_d L_3 & 0 & D \\ \star & \star & -e^{-\alpha d_1} L_1 & 0 & 0 & 0 \\ \star & \star & \star & -(1-\mu) e^{-\alpha d_2} L_3 & 0 & 0 \\ \star & \star & \star & \star & -e^{-\alpha d_2} L_2 & 0 \\ \star & \star & \star & \star & \star & -\frac{\alpha}{\bar{\omega}^2} I \end{bmatrix} + d_2 \begin{bmatrix} V_2^T \\ V_3^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} R \begin{bmatrix} V_2^T \\ V_3^T \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \\ &+ d_2 e^{-\alpha d_2} \tilde{M} R^{-1} \tilde{M}^T + d_{12} e^{-\alpha d_2} \tilde{N} R^{-1} \tilde{N}^T + e^{-\alpha d_2} \tilde{M} e_1 X^{-1} + e^{-\alpha d_2} \left(\tilde{M} e_1 X^{-1} \right)^T \\ &+ e^{-\alpha d_2} \tilde{N} e_2 X^{-1} + e^{-\alpha d_2} \left(\tilde{N} e_2 X^{-1} \right)^T < 0. \end{aligned} \tag{22}$$

where

$$\bar{\Pi}_{11} = V_2^T + V_2 + \alpha V_1 + V_1 L_1^{-1} V_1 + V_1 L_2^{-1} V_1 + V_1 L_3^{-1} V_1,$$
$$\bar{\Pi}_{12} = V_3 + V_1 A_c^T - V_2^T E_c^T,$$
$$\bar{\Pi}_{22} = -E_c V_3 - V_3^T E_c^T,$$
$$\tilde{M}^T = M^T X^{-1} = \begin{bmatrix} \tilde{M}_1^T & 0 & 0 & \tilde{M}_2^T & 0 & 0 \end{bmatrix},$$
$$\tilde{N}^T = N^T X^{-1} = \begin{bmatrix} \tilde{N}_1^T & 0 & 0 & \tilde{N}_2^T & 0 & 0 \end{bmatrix}.$$

By Lemma 2, it is easy to get that

$$\begin{aligned} e^{-\alpha d_2} \tilde{M} e_1 X^{-1} + e^{-\alpha d_2} \left(\tilde{M} e_1 X^{-1} \right)^T &\le \tilde{M} R \tilde{M}^T + e^{-\alpha d_2} X^{-T} e_1^T R^{-1} \left(e^{-\alpha d_2} e_1 X^{-1} \right) \\ &= \tilde{M} R \tilde{M}^T + \begin{bmatrix} e^{-\alpha d_2} V_1 \\ 0 \\ 0 \\ -e^{-\alpha d_2} L_3 \\ 0 \\ 0 \end{bmatrix} R^{-1} \begin{bmatrix} e^{-\alpha d_2} V_1 \\ 0 \\ 0 \\ -e^{-\alpha d_2} L_3 \\ 0 \\ 0 \end{bmatrix}^T. \end{aligned} \tag{23}$$

Similarly, we have

$$\begin{aligned} e^{-\alpha d_2} \tilde{N} e_2 X^{-1} + e^{-\alpha d_2} \left(\tilde{N} e_2 X^{-1} \right)^T &\le \tilde{N} R \tilde{N}^T + e^{-\alpha d_2} X^{-T} e_2^T R^{-1} \left(e^{-\alpha d_2} e_2 X^{-1} \right) \\ &= \tilde{N} R \tilde{N}^T + \begin{bmatrix} 0 \\ 0 \\ 0 \\ e^{-\alpha d_2} L_3 \\ -e^{-\alpha d_2} L_2 \\ 0 \end{bmatrix} R^{-1} \begin{bmatrix} 0 \\ 0 \\ 0 \\ e^{-\alpha d_2} L_3 \\ -e^{-\alpha d_2} L_2 \\ 0 \end{bmatrix}^T. \end{aligned} \tag{24}$$

By (19), it is easy to obtain

$$U_1 = K_a V_1 - K_e V_2^T, \quad U_2 = K_e V_3. \tag{25}$$

Since $R > 0$, we also have

$$-R^{-1} < R - 2I. \tag{26}$$

Substituting (25) into (22), by the Schur complement and from (23), (24) and (26), we can deduce that (22) or (10) holds if (18) is true. This completes the proof. □

Remark 2. By Theorem 1 and a matrix transformation technique, Theorem 2 provides a sufficient condition for the reachable set estimation of a class of nominal singular systems with time-varying delays by using PDSFCs. From Theorem 2, we can find that system (8) is normalizable and the reachable set of system (9) can be bounded by the ellipsoid $\mathcal{E}(V_1^{-1})$ and the gains of PDSFC (4) are given in (19).

Remark 3. Since a time-varying delay exists in the considered system, the free-weighting-matrix approach is used in Theorem 1. It should be pointed out that lots of new methods which can improve stability conditions for time-delay systems have been put forward in the literatures recently; see, e.g. [33–38]. However, our main purpose is to design a PDSFC to guarantee all the reachable states of the considered closed-loop singular system contained by an ellipsoid and it is difficult to obtain the feedback gains K_a and K_e by using these new methods.

Now, we provide the existence conditions of a PDSFC (4) such that system (1) is robustly normalizable and the reachable set of closed-loop system (5) can be bounded by an ellipsoid, where we assume that matrix variables L_1, L_2, L_3, R, $\tilde{M}_1$, $\tilde{M}_2$, $\tilde{N}_1$ and $\tilde{N}_2$ are linearly dependent on the uncertain parameters λ_i.

Theorem 3. *Given scalars d_1, d_2 and μ, if there exist a scalar $\alpha > 0$, matrices $V_1 > 0$, $L_{1i} > 0$, $L_{2i} > 0$, $L_{3i} > 0$, $R_i > 0$, V_2, V_3, U_1, U_2, $\tilde{M}_{1i}$, $\tilde{M}_{2i}$, $\tilde{N}_{1i}$, $\tilde{N}_{2i}$, $i = 1, 2, \cdots, s$, and Γ_{ij}, $1 \le i < j \le s$, such that the following inequalities hold*

$$\Pi_{ij} + \Pi_{ji} - \Gamma_{ij} - \Gamma_{ij}^T < 0, \quad (1 \le i < j \le s) \tag{27}$$

$$\begin{bmatrix} \Pi_{11} & \Gamma_{12} & \cdots & \Gamma_{1s} \\ \star & \Pi_{22} & \cdots & \Gamma_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \star & \star & \cdots & \Pi_{ss} \end{bmatrix} < 0, \tag{28}$$

where

$$\Pi_{ij} = \begin{bmatrix} \Omega_{1ij} & \Omega_{2j} & \Omega_{3j} \\ \star & \Omega_{4j} & 0 \\ \star & \star & \Omega_{5j} \end{bmatrix} < 0,$$

with

$$\Omega_{1ij} = \begin{bmatrix} \Phi_{11} & \Phi_{12i} & 0 & 0 & 0 & 0 \\ \star & \Phi_{22i} & 0 & A_{di}L_{3j} & 0 & D_i \\ \star & \star & -e^{-\alpha d_1}L_{1j} & 0 & 0 & 0 \\ \star & \star & \star & -(1-\mu)e^{-\alpha d_2}L_{3j} & 0 & 0 \\ \star & \star & \star & \star & -e^{-\alpha d_2}L_{2j} & 0 \\ \star & \star & \star & \star & \star & -\frac{\alpha}{\bar{\omega}^2}I \end{bmatrix},$$

$$\Omega_{2j} = \begin{bmatrix} V_1 & V_1 & V_1 & d_2V_2^T & e^{-\alpha d_2}V_1 & 0 \\ 0 & 0 & 0 & d_2V_3^T & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -e^{-\alpha d_2}L_{3j} & e^{-\alpha d_2}L_{3j} \\ 0 & 0 & 0 & 0 & 0 & -e^{-\alpha d_2}L_{2j} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Omega_{3j} = \begin{bmatrix} d_2e^{-\alpha d_2}\tilde{M}_{1j} & d_{12}e^{-\alpha d_2}\tilde{N}_{1j} & \tilde{M}_{1j} & \tilde{N}_{1j} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ d_2e^{-\alpha d_2}\tilde{M}_{2j} & d_{12}e^{-\alpha d_2}\tilde{N}_{2j} & \tilde{M}_{2j} & \tilde{N}_{2j} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\begin{aligned} \Omega_{4j} &= diag\left\{-L_{1j}, -L_{2j}, -L_{3j}, d_2(R_j - 2I), -R_j, -R_j\right\}, \\ \Omega_{5j} &= diag\left\{-d_2e^{-\alpha d_2}R_j, -d_{12}e^{-\alpha d_2}R_j, R_j - 2I, R_j - 2I\right\}, \end{aligned}$$

$$\Phi_{11} = V_2^T + V_2 + \alpha V_1, \quad \Phi_{12i} = V_3 + V_1 A_i^T - V_2^T E_i^T + U_1^T B_i^T,$$
$$\Phi_{22i} = -E_i V_3 - V_3^T E_i^T - B_i U_2 - U_2^T B_i^T, \quad d_{12} = d_2 - d_1,$$

then system (1) is robustly normalizable and the reachable set of system (5) is contained by the ellipsoid $\mathcal{E}(V_1^{-1})$. Furthermore, the gains of PDSFC (4) are $K_a = (U_1 + U_2 V_3^{-1} V_2^T) V_1^{-1}$ and $K_e = U_2 V_3^{-1}$.

Proof. From (18) and setting $L_1 = \sum_{i=1}^s L_{1i}$, $L_2 = \sum_{i=1}^s L_{2i}$, $L_3 = \sum_{i=1}^s L_{3i}$, $R = \sum_{i=1}^s R_i$, $\tilde{M}_1 = \sum_{i=1}^s \tilde{M}_{1i}$, $\tilde{M}_2 = \sum_{i=1}^s \tilde{M}_{2i}$, $\tilde{N}_1 = \sum_{i=1}^s \tilde{N}_{1i}$ and $\tilde{N}_2 = \sum_{i=1}^s \tilde{N}_{2i}$, it is easy to complete the proof by following a similar line as the proof in [39] and [40]. □

Remark 4. Theorem 3 gives sufficient conditions for the reachable set estimation of a class of polytopic singular systems with time-varying delays based on PDSFCs. Motivated by [39,40], the weighting matrices Γ_{ij}, $1 \le i < j \le s$, are introduced to relax the conditions.

Remark 5. To get the "smallest" possible bounding ellipsoid for closed-loop system (9) (respectively, system (5)), we need to maximize a positive scalar δ subject to $\delta I \le V_1^{-1}$ and condition (18) (respectively, conditions (27) and (28)), which is equivalent to minimize a positive scalar $\bar{\delta}$ such that

$$V_1 \le \bar{\delta} I, \tag{29}$$

and condition (18) (respectively, conditions (27) and (28)) hold, where $\bar{\delta} = \delta^{-1}$.

Remark 6. It is worth to mention that condition (18) in Theorem 2 (respectively, conditions (27) and (28) in Theorem 3) can also be used to make the reachable set of system (9) (respectively, system (5)) contained in a prescribed ellipsoid, respectively. Since system (9) (respectively, system (5)) is contained in the ellipsoid $\mathcal{E}(V_1^{-1})$, we have $x^T(t)V_1^{-1}x(t) \le 1$. To make the reachable set of system (9) (respectively, system (5)) contained in a given ellipsoid $\mathcal{E}(P_r)$, we need to ensure that $x^T(t)P_r x(t) \le x^T(t)V_1^{-1}x(t) \le 1$. Therefore, we have the following theorems.

Theorem 4. *Consider system (8). Given scalars d_1, d_2 and μ and a matrix P_r, if there exist a scalar $\alpha > 0$, matrices $V_1 > 0$, $L_1 > 0$, $L_2 > 0$, $L_3 > 0$, $R > 0$, V_2, V_3, U_1, U_2, $\tilde{M}_1$, $\tilde{M}_2$, $\tilde{N}_1$, $\tilde{N}_2$ such that $X \le P_r^{-1}$ and condition (18) hold, then system (8) is normalizable and the reachable set of closed-loop system (9) is contained in the given ellipsoid $\mathcal{E}(P_r)$. In this case, the gains of PDSFC (4) are $K_a = (U_1 + U_2 V_3^{-1} V_2^T) V_1^{-1}$, $K_e = U_2 V_3^{-1}$.*

Theorem 5. *Consider system (1). Given scalars d_1, d_2 and μ and a matrix P_r, if there exist a scalar $\alpha > 0$, matrices $V_1 > 0$, $L_{1i} > 0$, $L_{2i} > 0$, $L_{3i} > 0$, $R_i > 0$, V_2, V_3, U_1, U_2, $\tilde{M}_{1i}$, $\tilde{M}_{2i}$, $\tilde{N}_{1i}$, $\tilde{N}_{2i}$, $i = 1, 2, \cdots, s$, and Γ_{ij}, $1 \le i < j \le s$ such that $X \le P_r^{-1}$ and conditions (27) and (28) hold, then system (1) is robustly normalizable and the reachable set of closed-loop system (5) is contained in the given ellipsoid $\mathcal{E}(P_r^{-1})$. In this case, the gains of PDSFC (4) are $K_a = (U_1 + U_2 V_3^{-1} V_2^T) V_1^{-1}$, $K_e = U_2 V_3^{-1}$.*

4. Numerical example

In this section, two numerical examples are provided to show the effectiveness of the proposed methods.

Example 1. Consider singular system (8) with [23]

$$E = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} -1 & -1 \\ -1 & 0 \end{bmatrix}, \quad A_d = \begin{bmatrix} -0.7 & -0.5 \\ -0.7 & -0.7 \end{bmatrix}, \quad B = \begin{bmatrix} -6 \\ -5 \end{bmatrix},$$
$$D = \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix}, \quad d(t) = 0.3 + 0.2\sin(0.5t), \quad \omega(t) = \sin(4t).$$

By (18) and (29), we can get that the maximized value of δ is 16.2826 when $\alpha = 5.22$. In this case, we have

$$V_1^{-1} = 10^4 \times \begin{bmatrix} 1.2899 & 3.1304 \\ 3.1304 & 7.6087 \end{bmatrix}, \quad K_a = 10^6 \times \begin{bmatrix} 1.7466 & 4.2497 \end{bmatrix}, \quad K_e = \begin{bmatrix} 236.3471 & -138.2651 \end{bmatrix}.$$

The bounding ellipsoid $\mathcal{E}(V_1^{-1})$ and the state trajectory of the considered closed-loop system are given in Fig. 1. Fig. 2 shows the comparison of the bounding ellipsoids obtained in this paper and in [23]. From Figs. 1 and 2, we can find that the reachable set of the considered system is contained by the bounding ellipsoid $\mathcal{E}(V_1^{-1})$ and the bounding ellipsoid obtained in this paper is much smaller than those in [23], which show the effectiveness of the result in Theorem 2.

Next, we show the effectiveness of the result in Theorem 4. For a given ellipsoid $\mathcal{E}(P_r)$ with $P_r = \begin{bmatrix} 3.6822 & 3.6822 \\ 3.6822 & 11.6822 \end{bmatrix}$, by Theorem 4, we have

$$V_1^{-1} = \begin{bmatrix} 51.2075 & 137.5387 \\ 137.5387 & 392.5116 \end{bmatrix}, \quad K_a = \begin{bmatrix} 52.1508 & 152.9339 \end{bmatrix}, \quad K_e = \begin{bmatrix} 0.2778 & -1.1599 \end{bmatrix}.$$

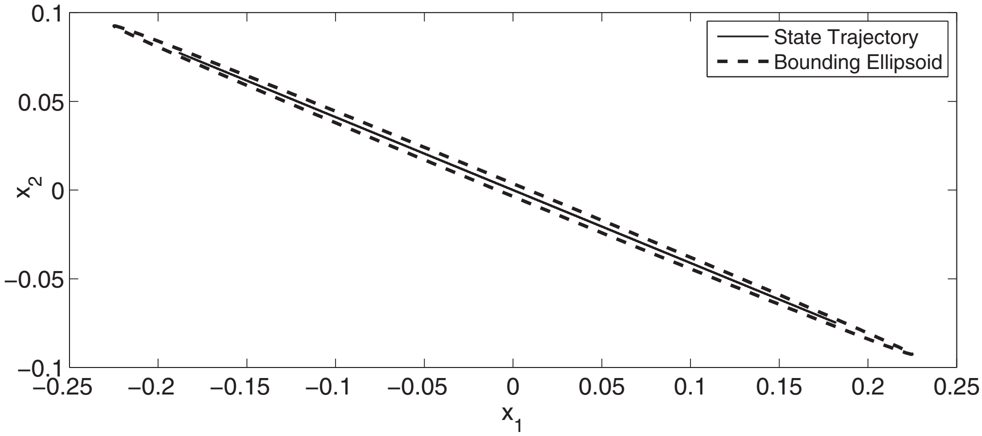

Fig. 1. The bounding ellipsoid and the state trajectory of the closed-loop system for Example 1.

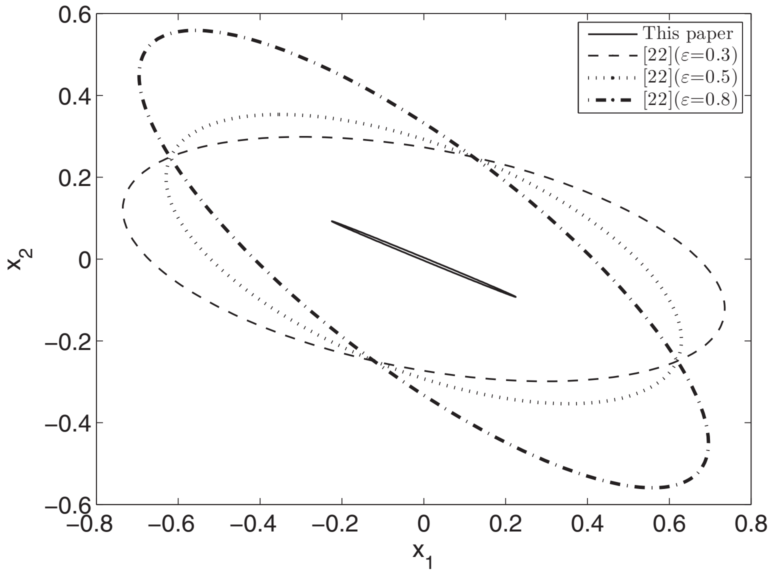

Fig. 2. Comparison of the bounding ellipsoids.

The state trajectory of the considered closed-loop system, the bounding ellipsoid $\mathcal{E}(V_1^{-1})$ and the given ellipsoid $\mathcal{E}(P_r)$ are shown in Fig. 3. From Fig. 3, we can deduce that the reachable set of closed-loop system (9) can be contained in a prescribed ellipsoid.

Example 2. Consider a polytopic singular system in (1) with two vertices, which are given as

$$E_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1 & -1 \\ -1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix},$$
$$A_{d1} = \begin{bmatrix} -0.7 & -0.5 \\ -0.7 & -0.7 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -0.5 & 1 \\ -1 & -0.3 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -6 \\ -5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -4 \\ -4 \end{bmatrix},$$
$$D_1 = \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.1 \\ -0.2 \end{bmatrix}, \quad d(t) = 0.75 + 0.25\sin(0.8t), \quad \omega(t) = \sin(10t).$$

From (27) to (29), it can be obtained that the maximized value of δ is 5.2788 when $\alpha = 1.29$. In this case, we can get

$$V_1^{-1} = 10^4 \times \begin{bmatrix} 1.0645 & 2.6452 \\ 2.6452 & 6.5766 \end{bmatrix}, \quad K_a = 10^5 \times \begin{bmatrix} 0.9492 & 2.3600 \end{bmatrix}, \quad K_e = \begin{bmatrix} 4.1911 & -17.5984 \end{bmatrix}.$$

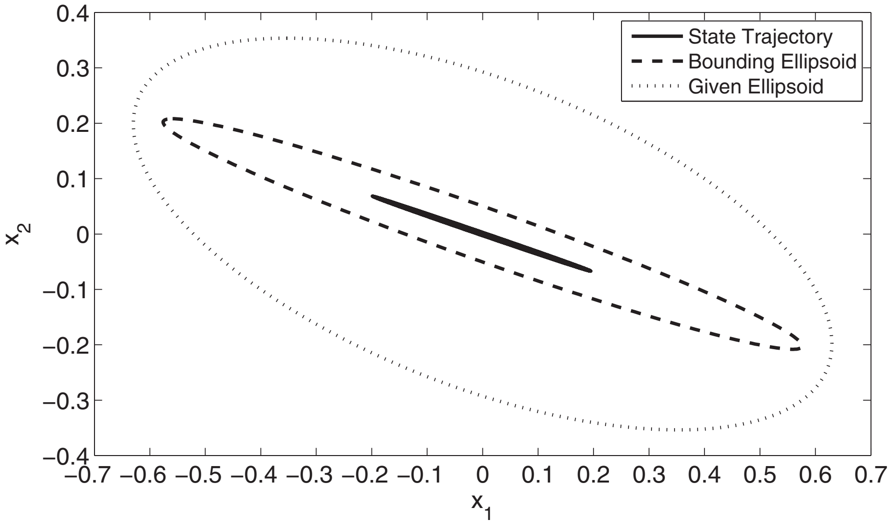

Fig. 3. The given ellipsoid, the bounding ellipsoid and the state trajectory of the closed-loop system for Example 1.

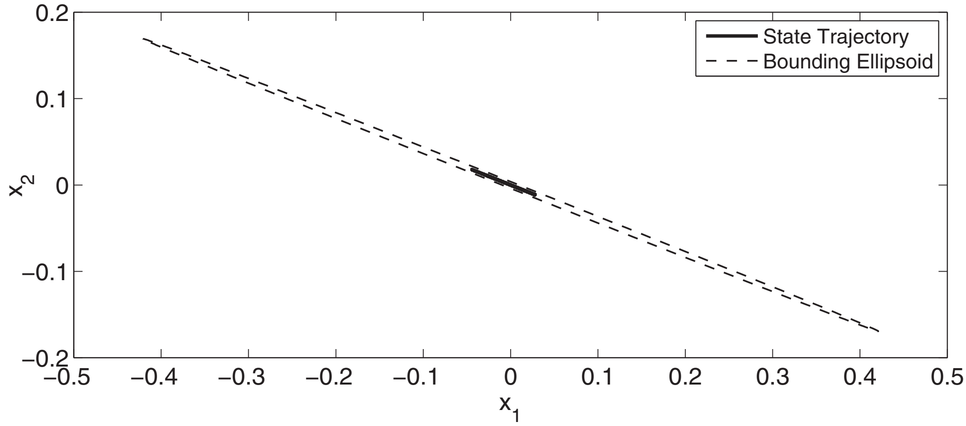

Fig. 4. The bounding ellipsoid and the state trajectory of the closed-loop system for Example 2.

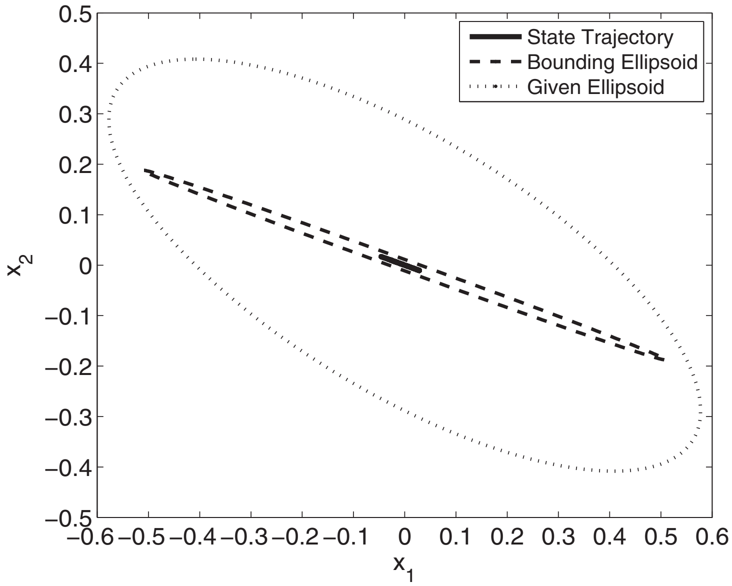

Fig. 5. The given ellipsoid, the bounding ellipsoid and the state trajectory of the closed-loop system for Example 2.

Fig. 4 shows the bounding ellipsoid $\mathcal{E}(V_1^{-1})$ and the state trajectory of the considered closed-loop system. It can be seen from Fig. 4 that the reachable set of the considered system is contained by the bounding ellipsoid $\mathcal{E}(V_1^{-1})$. Thus, the result in Theorem 3 is effective.

Next, for a given ellipsoid $\mathcal{E}(P_r)$ with $P_r = \begin{bmatrix} 6 & 6 \\ 6 & 12 \end{bmatrix}$, we can obtain from Theorem 5 that

$$V_1^{-1} = 10^3 \times \begin{bmatrix} 1.0765 & 2.9145 \\ 2.9145 & 7.9190 \end{bmatrix}, \quad K_a = 10^3 \times \begin{bmatrix} -0.8221 & -2.2337 \end{bmatrix}, \quad K_e = \begin{bmatrix} 0.0537 & 1.0784 \end{bmatrix}.$$

The state trajectory of the considered closed-loop system, the bounding ellipsoid $\mathcal{E}(V_1^{-1})$ and the given ellipsoid $\mathcal{E}(P_r)$ are given in Fig. 5. From Fig. 5, we can find that the reachable set of closed-loop system (5) can be contained in a prescribed ellipsoid, which reveals the effectiveness of the result in Theorem 5.

5. Conclusions

In this paper, the problem of reachable set estimation for a class of uncertain singular systems with time-varying delays has been investigated by using PDSFCs. First, a sufficient condition has been obtained for a nominal singular time-delay system such that there exists a PDSFC to make the considered system normalizable and the reachable set of the considered closed-loop system contained by an ellipsoid. Then, the result has been extended to the case of singular time-delay systems with polytopic uncertainties and relaxed conditions have been obtained by introducing some weighting matrix variables. What is more, sufficient conditions have also been derived to guarantee that the reachable set of the considered closed-loop singular system can be contained in a prescribed ellipsoid. Finally, two numerical examples have been given to show the effectiveness of our results. Recently, much attention has been paid to fuzzy systems and switch systems; see, e.g. [41–46]. Our future work will be centered on reachable set estimation for singular fuzzy systems and singular switch systems.

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