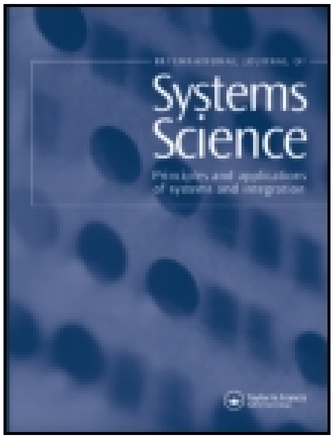

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Positive real control for descriptor systems with uncertainties in the derivative matrix via a proportional plus derivative feedback

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Positive real control for descriptor systems with uncertainties in the derivative matrix via a proportional plus derivative feedback

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This article deals with the problem of normalisation and positive real control synthesis for a class of descriptor systems with uncertainties in the derivative matrix. Attention is focused on the design of a proportional plus derivative (PD) state feedback, which guarantees that the closed-loop system is quadratically normal and quadratically stable (QNQS) and the closed-loop transfer function is extended strictly positive real (ESPR). We first present a new linear matrix inequality (LMI) characterisation of positive realness for normal descriptor systems. This new characterisation provides a kind of decoupling among the Lyapunov matrix, the system matrix and the derivative matrix. Based on this, a necessary and sufficient condition for the desired PD controllers is given and a PD controller is constructed by solving LMIs. Finally, a numerical example is given to demonstrate that the proposed method is effective.

Keywords: descriptor systems; positive real control; proportional plus derivative feedback; parameter uncertainty; singular systems

1. Introduction

It is well known that descriptor system models provide a more general mathematical description than state space models due to their capacity in preserving the structure of physical systems, and involving the dynamic and algebraic relationships between state variables simultaneously (Dai 1989; Xu and Lam 2006). Descriptor systems are of great utility for the modelling of many practical systems such as electrical networks, mechanical systems, economical systems, robotics and other areas (Luenberger 1977; Hemami and Wyman 1979; Mills and Goldenberg 1989; Newcomb and Dziurla 1989). In the past years, many notions and results developed for state-space systems have been successfully generalised to descriptor systems (Lewis 1986; Dai 1989; Xu and Lam 2006; Ma, Zhang, and Wu 2008; Xu, Lam, and Zou 2008).

On the other hand, the notion of positive realness is one of the oldest, and has played an important role in control and system theory, see, e.g. Anderson and Vongpanitlerd (1973), Molander and Willems (1980, 2000), Haddad and Bernstein (1991), Vidyasagar (1993), Bernstein and Haddad (1994), Bernussou, Geromel, and De Oliveira (1999) and Shim (1996). Since the notion of positive realness is introduced, many researchers have considered the positive real analysis and control problems for state space systems. Various approaches have been developed and a great number of results for both continuous state space systems and discrete contexts have been reported in the literature, see, e.g. Haddad and Bernstein (1994), Sun, Khargonekar, and Shim (1994), Xie and Soh (1995), Turan, Safonov, and Huang (1997), Zhou, Lam, and Feng (2005), Du and Yang (2009) and references therein. Applications of positive realness have been found in many areas such as the analysis of the properties of immittance or hybrid matrices of various classes of networks, the inverse problem of linear optimal control, the stability analysis for linear systems, and so on (Anderson and Vongpanitlerd 1973; Vidyasagar 1993; Xu and Lam 2004; Papageorgiou and Smith 2006). In parallel, these results of positive real theory for state-space systems have been perfectly extended to descriptor systems (Zhang, Lam, and Xu 2002; Li and Chen 2003; Masubuchi 2007; Yang, Zhang, Lin, and Zhou 2007; Chen 2008; Chu and Tan 2008; Camlibel and Frasca 2009). With respect to the positive real control problems of both state space systems and descriptor systems, all the works mentioned above are concerned with using conventional state or output feedbacks. The main objective of this article is to present a construction for an alternative class of feedbacks for uncertain descriptor systems with perturbations in the derivative matrix, i.e.

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derivative feedbacks or proportional plus derivative (PD) state feedbacks employing both states and derivative states.

As we know from classical control theory, the use of derivative feedback has a well engineering motivation, which is sometimes essential for achieving the desired control objectives such as reducing sensitivity to parameter variations and improving performance index of the considered systems compared to a state feedback (Haraldsdottir, Kabamba, and Ulsoy 1988, 1990). In industrial practice, derivative feedbacks are usually employed to provide anticipatory action for overshoot reduction in the responses (Kuo 1980). The other stimulus for using a derivative feedback instead of the conventional state feedback is the fact that in mechanical applications where accelerometer and velocity sensors are used for measuring the system motion, thus, the accelerations and velocities are the sensed variables as opposed to the displacements, see, e.g. Reithmeier and Leitmann (2003), Assunção, Teixeira, Faria, Da Silva, and Cardim (2007), Faria et al. (2009) and Vyhldal, Michiels, Ztek, Nijmeijer, and McGahan (2009) and the references therein. Derivative controls for descriptor systems with perturbations in derivative matrix are more meaningful than state space systems. Consider the following descriptor system with perturbations in the derivative matrix

$$E\dot{x}(t) = Ax(t) + B_1u(t), \tag{1}$$

where

$$E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

We can know that the system (1)($u(t)=0$) is regular, impulse-free and stable (Dai 1989; Xu and Lam 2006). Suppose that the derivative matrix E has an additive perturbation given by the following matrix

$$\Delta E = \begin{bmatrix} 0 & 0 \\ \delta & 0 \end{bmatrix}. \tag{2}$$

We can verify that the system (1) becomes unstable for any arbitrary small $\delta > 0$. It has been shown that one can always choose a perturbation in the derivative matrix E with an arbitrary small norm to destroy the stability of the system (1) even though the system (1) is regular, impulse-free and stable. However, if we can design the derivative state feedback

$$u(t) = K_d\dot{x}(t), \quad K_d = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

for the system (1) and suppose that the derivative matrix E still has an additive perturbation given in (2), for any $\delta \neq 0$, we can prove that the resulting closed-loop system

$$(E + B_1K_d)\dot{x}(t) = Ax(t) \tag{3}$$

is stable with the two same eigenvalues -1. It can be concluded that the stability of a descriptor system may be *fragile* to arbitrarily small perturbations in the derivative matrix, and hence it is impossible to consider unstructured uncertainties in the derivative matrix E when studying robust stabilisation via a static state feedback alone if some assumptions and constraints on the perturbations in the derivative matrix are not given because of singularity and possible rank change of the derivative matrix (Lin, Wang, Yang, and Lam 2000; Xu and Lam 2006). The derivative control may provide a choice way to deal with the control problem of descriptor systems with perturbations in the derivative matrix due to the fact that under the very weak condition $\text{rank}\begin{bmatrix} E & B_1 \end{bmatrix} = n$ (the system dimension) there *almost always* exists a matrix K_d such that $E + B_1\text{K}_d$ is nonsingular and non-defective matrix pair possesses relative eigenvalues with less sensitivities to the matrix parameter perturbations (Duan and Patton 1997, 1999). But this fact does not mean that the derivative gain K_d can be designed by a test-and-try procedure and systematic methods are not needed when dealing with the control problems of descriptor systems with perturbations in all the system coefficient matrices (Remark 4). In addition, for descriptor systems which are not I-controllable, any state feedback cannot eliminate impulse term but a properly selected derivative feedback can work it out. Still consider the descriptor system (1) with the following parameters

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

It can be verified that the system (1) is not I-controllable, and hence the impulse term cannot be eliminated by any state feedback. If we choose the derivative state feedback alone

$$u(t) = K_d\dot{x}(t), \quad K_d = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix},$$

the resulting closed-loop system

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}x(t) \tag{4}$$

can be easily verified to be impulse-free. Due to the above mentioned advantages, derivative controls in descriptor systems have been considered by a group of researchers and many results have been obtained, see, e.g. Shayman and Zhou (1987), Chang, Fang, and Wang (1991), Le (1992), Duan and Patton (1997, 1999) and Wu, Duan, and Zhao (2007) and the references therein. Recently, the problem of quadratic normalisation and stabilisation via PD state and PD output feedbacks for uncertain descriptor systems with norm-bounded perturbations in the derivative matrix has been investigated and such problem is solved by converting it into a corresponding stabilisation problem of an augmented uncertain system (Lin, Wang, and Lee 2005). Up to date, however, no results on positive real control problem for descriptor systems with perturbations in the derivative matrix is available in the literature by using a PD feedback, this problem is still open.

In this article, we study the problem of positive real control for uncertain descriptor systems via a PD feedback. The parameter uncertainties are assumed to be time-invariant and unknown but norm-bounded. The problem to be addressed is the design of a PD feedback controller such that the resulting closed-loop system is quadratically normal and quadratically stable (QNQS) and the closed-loop transfer function from the disturbance to the controlled output is extended strictly positive real (ESPR) for all admissible uncertainties. To solve this problem, we first establish a new linear matrix inequality (LMI) characterisation of positive realness for normal descriptor systems by introducing new slack matrix variables. The new result provides a kind of decoupling among the Lyapunov matrix, the system matrix and the derivative matrix, which is crucial for the design of positive real PD controllers in an LMI frame. Based on the new result, a necessary and sufficient condition for positive real control problem is obtained for the considered descriptor systems. Finally, an example is presented to demonstrate the validity of the proposed approach.

Notation: Throughout this article, the notation $X \geq Y$ (respectively, $X > Y$) with X and Y being symmetric matrices, means that the matrix $X - Y$ is positive semidefinite (respectively, positive definite). I is the identity matrix with appropriate dimension. The superscripts 'T', '*' and '-1' stand for the matrix transpose, the complex matrix conjugate transpose and inverse, respectively.

2. Main results

Consider the following unforced linear descriptor system

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bw(t), \\ z(t) = Cx(t) + Dw(t), \end{cases} \tag{5}$$

where $x(t) \in R^n$ is the state; $w(t) \in R^l$ is the disturbance input; $z(t) \in R^p$ is the control output. The matrix $E \in R^{n\times n}$ may be singular. We shall assume that $p = l$. E, A, B, C and D are known real constant matrices with appropriate dimensions.

The transfer function from the exogenous input $w(t)$ to the controlled output $z(t)$ of the system (5) is given by

$$G(s) = C(sE - A)^{-1}B + D \tag{6}$$

Throughout this article, we shall use the following concepts.

Definition 1:

(1) The system (5) is said to be normal (N) if E is invertible.
(2) The system (5) is said to be stable (S) if all roots of $\det(\mathrm{sE} - A) = 0$ have negative real parts.

Definition 2 (Zhang et al. 2002; Xu and Lam 2006)**:**

(1) The system (5) is said to be positive real (PR) if its transfer function $G(s)$ is analytic in $\mathrm{Re}(s) > 0$ and satisfies
$$G(s) + G^*(s) \geq 0$$
for $\mathrm{Re}(s) > 0$.
(2) The system (5) is said to be strictly positive real (SPR) if its transfer function $G(s)$ is analytic in $\mathrm{Re}(s) \geq 0$ and satisfies
$$G(j\theta) + G^T(-j\theta) > 0$$
for $\theta \in [0, \infty)$.
(3) The system (5) is said to be ESPR if the system (5) is SPR and
$$G(j\infty) + G^T(-j\infty) > 0.$$

Lemma 1: *The following statements are equivalent*:

(1) *The system* (5) *is NS and ESPR.*
(2) *There exists a matrix* $P_1 > 0$ *such that*
$$\begin{bmatrix} AP_1E^T + EPA^T & B - EP_1C^T \\ B^T - CP_1E^T & -(D + D^T) \end{bmatrix} < 0. \tag{7}$$

Proof: If the system (5) is normal, obviously, it is simple to transform the system (5), by pre-multiplying the inverse of E. This yields the standard state space system

$$\begin{cases} \dot{x}(t) = E^{-1}Ax(t) + E^{-1}Bw(t), \\ z(t) = Cx(t) + Dw(t). \end{cases} \tag{8}$$

By Lemma 2.3 in Sun et al. (1994), we have

$$\begin{bmatrix} (E^{-1}A)P + P(E^{-1}A)^T & (E^{-1}B) - PC^T \\ (E^{-1}B)^T - CP & -(D + D^T) \end{bmatrix} < 0. \quad (9)$$

Multiplying (9) the left-handside and the right-handside, respectively, by

$$\begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}$$

and its transpose, results in (7).

Theorem 1: *The following statements are equivalent*:

(1) *The system* (5) *is NS and ESPR.*
(2) *There exist matrices* $P_1 > 0$, P_2, P_3 *such that*

$$\begin{bmatrix} P_2 + P_2^T & \left\{\begin{matrix} P_1A^T - P_2^T \\ \times E^T + P_3 \end{matrix}\right\} & -P_1C^T \\ \left\{\begin{matrix} AP_1 - E \\ \times P_2 + P_3^T \end{matrix}\right\} & -EP_3 - P_3^TE^T & B \\ -CP_1 & B^T & -(D + D^T) \end{bmatrix} < 0. \quad (10)$$

Proof: Based on Lemma 1, we only need to show that the feasibility of (10) for decision variables P_1, P_2 and P_3 is equivalent to the feasibility of (7) for decision variable P_1.

$(7) \Rightarrow (10)$. If (7) is true, we always can choose an appropriately dimensioned matrix P_2 such that

$$P_2 + P_2^T < 0, \ -(D + D^T) - CP_1(P_2 + P_2^T)^{-1}P_1C^T < 0.$$

This together with (7), by Schur complement, gives

$$\begin{bmatrix} P_2 + P_2^T & 0 & -P_1C^T \\ 0 & AP_1E^T + EPA^T & B - EP_1C^T \\ -CP_1 & B^T - CP_1E^T & -(D + D^T) \end{bmatrix} < 0. \quad (11)$$

Let matrix P_3 be such that

$$P_1A^T + P_2E^T + P_3 = 0.$$

By (11), we have

$$\begin{bmatrix} P_2 + P_2^T & \left\{\begin{matrix} P_1A^T + P_2 \\ \times E^T + P_3 \end{matrix}\right\} & -P_1C^T \\ \left\{\begin{matrix} AP_1 + E \\ \times P_2^T + P_3^T \end{matrix}\right\} & AP_1E^T + EPA^T & B - EP_1C^T \\ -CP_1 & B^T - CP_1E^T & -(D + D^T) \end{bmatrix} < 0. \quad (12)$$

Now, multiplying (12) the left-hand side and the right-hand side, respectively, by

$$\begin{bmatrix} I & 0 & 0 \\ -E & I & 0 \\ 0 & 0 & I \end{bmatrix}$$

and its transpose, we can conclude that (10) holds.

$(10) \Rightarrow (7)$. From the above proof, it is obvious.

This completes the proof.

Remark 1: The importance of (10) compared to (7), is to separate the Lyapunov matrix P_1 from the system matrix A and the derivative matrix E by introducing slack matrix variables. This technique is often used to get parameter-dependent Lyapunov functions instead of using single Lyapunov quadratic functions in order to reduce the conservatism due to the extra degrees of freedom when dealing with robust control systems with polytopic uncertainties (De Oliveira, Bernussou, and Geromel 1999; He, Wu, and She 2005; Zhou et al. 2005). Here, Theorem 1 enables a PD controller synthesis in LMI frame to be possible, which will be demonstrated in the sequel. It is noted that the matrices P_2 and P_3 introduced are not necessarily constrained to be symmetric.

Consider an uncertain descriptor system described by

$$\begin{cases} E_\Delta \dot{x}(t) = A_\Delta x(t) + B_{1\Delta}u(t) + Bw(t), \\ z(t) = Cx(t) + Dw(t), \end{cases} \quad (13)$$

where

$$E_\Delta = E + \Delta E(\sigma), \ A_\Delta = A + \Delta A(\sigma), \ B_{1\Delta} = B_1 + \Delta B_1(\sigma).$$

where $u(t) \in R^{n\times m}$ the control input. B_1 the known real constant matrices with appropriate dimensions. $\Delta E(\sigma)$, $\Delta A(\sigma)$ and $\Delta B_1(\sigma)$ are time-invariant matrices representing norm-bounded parameter uncertainties, and are assumed to be of the form

$$\begin{bmatrix} \Delta E(\sigma) & \Delta A(\sigma) & \Delta B_1(\sigma) \end{bmatrix} = MF(\sigma)\begin{bmatrix} N_e & N_a & N_b \end{bmatrix}, \quad (14)$$

where M, N_e, N_a and N_b are known real constant matrices with appropriate dimensions. The uncertain matrix $F(\sigma) \in R^{g\times k}$ satisfies

$$F^T(\sigma)F(\sigma) \le I \quad (15)$$

and $\sigma \in \Xi$, where Ξ is a compact set in R. The matrices $\Delta E(\sigma)$, $\Delta A(\sigma)$ and $\Delta B_1(\sigma)$ are said to be admissible if both (14) and (15) hold. The matrix $E_\Delta \in R^{n\times n}$ may be singular. We shall assume that

$$\mathrm{Rank}(E_\Delta) \le n.$$

for all admissible uncertainties.

Remark 2: The structure of the parameter uncertainty with the form (14) and (15) has been widely used when dealing with the problem of robust stabilisation for uncertain systems in both continuous and discrete time contexts; see, e.g. Xu and Lam (2004) and Lin et al. (2005) and the references therein.

Definition 3: The unforced uncertain descriptor system (13) is said to be QNQS and ESPR, if there exist matrices P_1, P_2 and P_3 such that

$$\begin{bmatrix} P_2+P_2^T & \begin{Bmatrix} P_1A_\Delta^T-P_2^T \\ \times E_\Delta^T+P_3 \end{Bmatrix} & -P_1C^T \\ \begin{Bmatrix} A_\Delta P_1-E_\Delta \\ \times P_2+P_3^T \end{Bmatrix} & -E_\Delta P_3-P_3^TE_\Delta^T & B \\ -CP_1 & B^T & -(D+D^T) \end{bmatrix} < 0. \tag{16}$$

In this article, we will construct a proportional PD state feedback controller for the system(13) in the form of

$$u(t) = K_p x(t) - K_d\dot{x}(t), \tag{17}$$

where K_p and K_d are to be designed matrices with appropriate dimensions. Substituting (17) into the system (13) leads to the closed-loop system

$$\begin{cases} E_{\Delta K}\dot{x}(t) = A_{\Delta K}x(t) + Bw(t), \\ z(t) = Cx(t) + Dw(t), \end{cases} \tag{18}$$

where

$$\begin{aligned} E_{\Delta K} &= E_K + \Delta E_K, & A_{\Delta K} &= A_K + \Delta A_K, \\ E_K &= E + B_1K_d, & A_K &= A + B_1K_p, \\ \Delta E_K &= MF(\sigma)N_{eK}, & \Delta A_K &= MF(\sigma)N_{aK}, \\ N_{eK} &= N_e + N_bK_d, & N_{aK} &= N_a + N_bK_p. \end{aligned}$$

The transfer function from the exogenous input $w(t)$ to the controlled output $z(t)$ of the closed-loop system (18) is given by

$$G_{\Delta K}(s) = C(sE_{\Delta K} - A_{\Delta K})^{-1}B + D \tag{19}$$

In this article, our aim is to determine K_p and K_d such that the closed-loop system (18) is QNQS and ESPR. In the following, attention will be focused on the development of conditions for the desired matrices K_p and K_d. To this end, we need the following result.

Lemma 2 (Petersen 1987)**:** *Given matrices $\mathcal{Z}$, $\mathcal{X}$ and $\mathcal{Y}$ of appropriate dimensions and with symmetric $\mathcal{Z}$, then*

$$\mathcal{Z} + \mathcal{X}\mathcal{F}(\sigma)\mathcal{Y} + (\mathcal{X}\mathcal{F}(\sigma)\mathcal{Y})^T < 0$$

for all $\mathcal{F}(\sigma)$ satisfying $\mathcal{F}^T(\sigma)\ \mathcal{F}(\sigma) \le I$, if and only if there exists a scalar $\varepsilon > 0$ such that

$$\mathcal{Z} + \varepsilon\mathcal{X}\mathcal{X}^T + \varepsilon^{-1}\mathcal{Y}^T\mathcal{Y} < 0.$$

Theorem 2: *Consider the uncertain descriptor system* (13). *There exists a PD feedback controller* (17) *such that the closed-loop system* (18) *is QNQS and ESPR, if and only if there exist a scalar $\varepsilon > 0$, matrices $P_1 > 0$, P_2, P_3, Q_1 and Q_2 such that*

$$\begin{bmatrix} \Theta_{11} & \Theta_{12} & -P_1C^T & \Lambda_{15} \\ \Theta_{12}^T & \Theta_{22} & B & \Lambda_{25} \\ -CP_1 & B^T & -(D+D^T) & 0 \\ \Lambda_{15}^T & \Lambda_{25}^T & 0 & -\varepsilon I \end{bmatrix} < 0. \tag{20}$$

where

$$\begin{aligned} \Theta_{11} &= P_2 + P_2^T, \\ \Theta_{12} &= P_1A^T - P_2^TE^T + P_3 + Q_1^TB_1^T, \\ \Theta_{22} &= -EP_3 - P_3^TE^T + B_1Q_2 + Q_2^TB_1^T + \varepsilon MM^T, \\ \Lambda_{15} &= P_1^TN_a^T - P_2^TN_e^T + Q_1^TN_b^T, \\ \Lambda_{25} &= -P_3^TN_e^T + Q_2^TN_b^T. \end{aligned}$$

In this case, a desired PD feedback controller is given by

$$u(t) = \Upsilon P_1^{-1}x(t) + Q_2P_3^{-1}\dot{x}(t), \tag{21}$$

where

$$\Upsilon = Q_1 - Q_2P_3^{-1}P_2. \tag{22}$$

Proof (Sufficiency)**:** Suppose that there exist a scalar $\varepsilon > 0$, matrices $P_1 > 0$, P_2, P_3, Q_1 and Q_2 such that (20) holds. Now, applying the PD feedback (21) to (13), we have the closed-loop system as

$$\begin{cases} E_{\Delta c}\dot{x}(t) = A_{\Delta c}x(t) + Bw(t), \\ z(t) = Cx(t) + Dw(t), \end{cases} \tag{23}$$

where

$$\begin{aligned} E_{\Delta c} &= E_c + \Delta E_c, & A_{\Delta c} &= A_c + \Delta A_c, \\ E_c &= E - B_1Q_2P_3^{-1}, & A_c &= A + B_1\Upsilon P_1^{-1}, \\ \Delta E_c &= MF(\sigma)N_{ec}, & \Delta A_c &= MF(\sigma)N_{ac}, \\ N_{ec} &= N_e - N_bQ_2P_3^{-1}, & N_{ac} &= N_a + N_b\Upsilon P_1^{-1}. \end{aligned}$$

Let

$$\mathcal{G} = \begin{bmatrix} P_2 + P_2^T & P_1A^T - P_2^TP^T + P_3 \\ AP_1 - EP_2 + P_3^T & -EP_3 - P_3^TE^T \end{bmatrix},$$

$$\mathcal{H} = \begin{bmatrix} -P_1C^T \\ B \end{bmatrix}(D+D^T)^{-1}\begin{bmatrix} -P_1C^T \\ B \end{bmatrix}^T,$$

$$\mathcal{J}=\begin{bmatrix} P_2+P_2^T & \left\{\begin{matrix} P_1A^T-P_2^TE^T \\ +P_3+Q_1^TB_1^T \end{matrix}\right\} \\ \left\{\begin{matrix} AP_1-EP_2 \\ +P_3^T+B_1Q_1 \end{matrix}\right\} & \left\{\begin{matrix} -EP_3-P_3^TE^T \\ +B_1Q_2+Q_2^TB_1^T \end{matrix}\right\} \end{bmatrix}.$$

Then, we have

$$\begin{aligned}
&\begin{bmatrix} P_2+P_2^T & P_1A_c^T-P_2^TE_c^T+P_3 \\ A_cP_1-E_cP_2+P_3^T & -E_cP_3-P_3^TE_c^T \end{bmatrix} \\
&=\mathcal{G}+\begin{bmatrix} 0 \\ B_1 \end{bmatrix}\begin{bmatrix} \Upsilon P_1^{-1} & Q_2P_3^{-1} \end{bmatrix}\begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix} \\
&\quad+\begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}^T\begin{bmatrix} \Upsilon P_1^{-1} & Q_2P_3^{-1} \end{bmatrix}^T\begin{bmatrix} 0 \\ B_1 \end{bmatrix}^T \\
&=\mathcal{G}+\begin{bmatrix} 0 \\ B_1 \end{bmatrix}\begin{bmatrix} Q_1 & Q_2 \end{bmatrix}+\begin{bmatrix} Q_1 & Q_2 \end{bmatrix}^T\begin{bmatrix} 0 \\ B_1 \end{bmatrix}^T \\
&=\mathcal{J}.
\end{aligned} \tag{24}$$

Note that

$$\begin{aligned}
&\begin{bmatrix} 0 & P_1\Delta A_c^T-P_2^T\Delta E_c^T \\ \Delta A_cP_1-\Delta E_cP_2 & -\Delta E_cP_3-P_3^T\Delta E_c^T \end{bmatrix} \\
&=\begin{bmatrix} 0 \\ M \end{bmatrix}F(\sigma)\begin{bmatrix} \Lambda_{15}^T & \Lambda_{25}^T \end{bmatrix}+\begin{bmatrix} \Lambda_{15}^T & \Lambda_{25}^T \end{bmatrix}^TF^T(\sigma)\begin{bmatrix} 0 \\ M \end{bmatrix}^T.
\end{aligned} \tag{25}$$

Then, by (24), (25), (20) and Lemma 2, it follows that

$$\begin{aligned}
&\mathcal{H}+\begin{bmatrix} P_2+P_2^T & P_1A_{\Delta c}^T-P_2^TE_{\Delta c}^T+P_3 \\ A_{\Delta c}P_1^T-E_{\Delta c}P_2+P_3^T & -E_{\Delta c}P_3-P_3^TE_{\Delta c}^T \end{bmatrix} \\
&=\mathcal{H}+\begin{bmatrix} P_2+P_2^T & P_1A_c^T-P_2^TE_c^T+P_3 \\ A_cP_1^T-E_cP_2+P_3^T & -E_cP_3-P_3^TE_c^T \end{bmatrix} \\
&\quad+\begin{bmatrix} 0 & P_1\Delta A_c^T-P_2^T\Delta E_c^T \\ \Delta A_cP_1^T-\Delta E_cP_2 & -\Delta E_cP_3-P_3^T\Delta E_c^T \end{bmatrix} \\
&\le\mathcal{H}+\mathcal{J}+\varepsilon\begin{bmatrix} 0 \\ M \end{bmatrix}\begin{bmatrix} 0 \\ M \end{bmatrix}^T+\varepsilon^{-1}\begin{bmatrix} \Lambda_{15} \\ \Lambda_{25} \end{bmatrix}\begin{bmatrix} \Lambda_{15} \\ \Lambda_{25} \end{bmatrix}^T<0;
\end{aligned} \tag{26}$$

equivalently,

$$\begin{bmatrix} P_2+P_2^T & P_1A_{\Delta c}^T-P_2^TE_{\Delta c}^T+P_3 & -P_1C^T \\ A_{\Delta c}P_1^T-E_{\Delta c}P_2+P_3^T & -E_{\Delta c}P_3-P_3^TE_{\Delta c}^T & B \\ -CP_1 & B^T & -(D+D^T) \end{bmatrix}<0, \tag{27}$$

Thus, by Definition 3, we know that the closed-loop system (23) is QNQS and ESPR.

(Necessity): Suppose that there exists a PD state feedback control law (17) such that the system (18) is QNQS and ESPR. Then, by Definition 3, we have that there exist matrices $P_1>0$, P_2, P_3 such that

$$\begin{bmatrix} P_2+P_2^T & P_1A_{\Delta K}^T-P_2^TE_{\Delta K}^T+P_3 & -P_1C^T \\ A_{\Delta K}P_1^T-E_{\Delta K}P_2+P_3^T & -E_{\Delta K}P_3-P_3^TE_{\Delta K}^T & B \\ -CP_1 & B^T & -(D+D^T) \end{bmatrix}<0. \tag{28}$$

First, let

$$\begin{cases} Q_1=K_pP_1-K_dP_2, \\ Q_2=-K_dP_3. \end{cases} \tag{29}$$

Then, by (28) and considering Schur complement, we have

$$\begin{aligned}
0&>\mathcal{H}+\begin{bmatrix} P_2+P_2^T & P_1A_{\Delta K}^T-P_2^TE_{\Delta K}^T+P_3 \\ A_{\Delta K}P_1^T-E_{\Delta K}P_2+P_3^T & -E_{\Delta K}P_3-P_3^TE_{\Delta K}^T \end{bmatrix} \\
&=\mathcal{H}+\mathcal{G}+\begin{bmatrix} 0 \\ B_1 \end{bmatrix}\begin{bmatrix} K_pP_1-K_dP_2 & -K_dP_3 \end{bmatrix} \\
&\quad+\begin{bmatrix} K_pP_1-K_dP_2 & -K_dP_3 \end{bmatrix}^T\begin{bmatrix} 0 \\ B_1 \end{bmatrix}^T+\begin{bmatrix} 0 \\ M \end{bmatrix}F(\sigma) \\
&\quad\times\begin{bmatrix} \left\{\begin{matrix} N_aP_1-N_eP_2 \\ +N_b(K_pP_1-K_dP_3) \end{matrix}\right\} & -N_eP_3+N_b(-K_dP_3) \end{bmatrix} \\
&\quad+\begin{bmatrix} \left\{\begin{matrix} N_aP_1-N_eP_2 \\ +N_b(K_pP_1-K_dP_3) \end{matrix}\right\} & -N_eP_3+N_b(-K_dP_3) \end{bmatrix}^T \\
&\quad\times F^T(\sigma)\begin{bmatrix} 0 \\ M \end{bmatrix}^T \\
&=\mathcal{H}+\mathcal{J}+\begin{bmatrix} 0 \\ M \end{bmatrix}F(\sigma)\begin{bmatrix} \Lambda_{15}^T & \Lambda_{25}^T \end{bmatrix} \\
&\quad+\begin{bmatrix} \Lambda_{15}^T & \Lambda_{25}^T \end{bmatrix}^TF^T(\sigma)\begin{bmatrix} 0 \\ M \end{bmatrix}^T.
\end{aligned} \tag{30}$$

By considering Lemma 2, (30) holds if and only if there exists a scalar $\varepsilon>0$ such that

$$\mathcal{H}+\mathcal{J}+\varepsilon\begin{bmatrix} 0 \\ M \end{bmatrix}\begin{bmatrix} 0 \\ M \end{bmatrix}^T+\varepsilon^{-1}\begin{bmatrix} \Lambda_{15} \\ \Lambda_{25} \end{bmatrix}\begin{bmatrix} \Lambda_{15} \\ \Lambda_{25} \end{bmatrix}^T<0 \tag{31}$$

which, by Schur complement, implies that (20) holds. Finally, noting (29), it is easily showed that

$$K_p=\Upsilon P_1^{-1}, \quad K_d=-Q_2P_3^{-1}. \tag{32}$$

This completes the proof.

Remark 3: Theorem 3 provides an LMI condition for designing a PD feedback controller which makes the resulting closed-loop system be QNQS while achieving the ESPR property. It is noted that the LMI in (20) is directly expressed in terms of the coefficient matrices of the original descriptor system without resorting to the conversion of the considered system. Based on this result, the prevailing LMI-toolbox in Matlab can be directly applicable. This will make the design procedure relatively simple.

Remark 4: An insight into robust normalisation and robust stabilisation via a PD feedback in an LMI frame was given in Lin et al. (2005). But the approach proposed in Lin et al. (2005) seems difficult to deal with the control problem considered this article due to the fact that it is not an easy task to show the equivalence of ESPR property between the origin system and the corresponding augmented system. In addition, it is worth pointing out that a simple two-step approach, which is first to find the gain matrix K_d such that the matrix $E+B_1K_d$ is nonsingular, and then to seek a stabilising gain matrix K_p for the normalisation system, is not feasible when dealing with the control problem considered in this article because we can not exactly know coefficient matrices E, A and B_1 with perturbations.

In (21), If we set

$$Q_1 = Q_2(\triangleq Q) \tag{33}$$

and

$$P_2 = P_3(\triangleq V), \tag{34}$$

we have

$$K_p = (Q_1 - Q_2P_3^{-1}P_2)P_1^{-1} = 0.$$

Therefore, we have the following result.

Corollary 1: *Consider the uncertain descriptor system* (13). *There exists a derivative state feedback control law*

$$u(t) = -K_d\dot{x}(t)$$

such that the closed-loop system (18) *is QNQS and ESPR, if there exist a scalar* $\varepsilon > 0$, *matrices* $P_1 > 0$, V *and* Q *such that*

$$\begin{bmatrix} \bar{\Theta}_{11} & \bar{\Theta}_{12} & -P_1C^T & \bar{\Lambda}_{15} \\ \bar{\Theta}_{12}^T & \bar{\Theta}_{22} & B & \bar{\Lambda}_{25} \\ -CP_1 & B^T & -(D+D^T) & 0 \\ \bar{\Lambda}_{15}^T & \bar{\Lambda}_{25}^T & 0 & -\varepsilon I \end{bmatrix} < 0. \tag{35}$$

where

$$\begin{aligned}
\bar{\Theta}_{11} &= V + V^T,\\
\bar{\Theta}_{12} &= P_1A^T - V^TE^T + V + Q^TB_1^T,\\
\bar{\Theta}_{22} &= -EV - V^TE^T + B_1Q + Q^TB_1^T + \varepsilon MM^T,\\
\bar{\Lambda}_{15} &= P_1^TN_a^T - V^TN_e^T + Q^TN_b^T,\\
\bar{\Lambda}_{25} &= -V^TN_e^T + Q^TN_b^T.
\end{aligned}$$

In this case, a desired derivative state feedback controller is given by

$$u(t) = QP_1^{-1}\dot{x}(t).$$

Remark 5: Applications of derivative state feedbacks can be found in Reithmeier and Leitmann (2003), Abdelaziz and Valášek (2004), Assunção et al. (2007), Faria et al. (2009) and Vyhldal et al. (2009) and the references therein. The geometric theory for descriptor systems under derivative state feedbacks was provided in Lewis and Syrmos (1991).

In (21), if we set

$$Q_2 = 0,$$

we have

$$K_d = 0,$$

or vice versa. Thus, in the case of $\mathrm{rank}(E+\Delta E)=n$, we have the following result.

Corollary 2: *Consider the uncertain system* (13) *with the constraint of* $\mathrm{rank}(E+\Delta E)=n$. *There exists a state feedback control law*

$$u(t) = K_px(t)$$

such that the closed-loop system (18) *is quadratically stable and ESPR, if and only if there exist a scalar* $\varepsilon > 0$, *matrices* $P_1 > 0$, P_2, P_3 *and* Q_1 *such that*

$$\begin{bmatrix} \tilde{\Theta}_{11} & \tilde{\Theta}_{12} & -P_1C^T & \tilde{\Lambda}_{15} \\ \tilde{\Theta}_{12}^T & \tilde{\Theta}_{22} & B & \tilde{\Lambda}_{25} \\ -CP_1 & B^T & -(D+D^T) & 0 \\ \tilde{\Lambda}_{15}^T & \tilde{\Lambda}_{25}^T & 0 & -\varepsilon I \end{bmatrix} < 0 \tag{36}$$

where

$$\begin{aligned}
\tilde{\Theta}_{11} &= P_2 + P_2^T,\\
\tilde{\Theta}_{12} &= P_1A^T - P_2^TE^T + P_3 + Q_1^TB_1^T,\\
\tilde{\Theta}_{22} &= -EP_3 - P_3^TE^T + \varepsilon MM^T,\\
\tilde{\Lambda}_{15} &= P_1^TN_a^T - P_2^TN_e^T + Q_1^TN_b^T,\\
\tilde{\Lambda}_{25} &= -P_3^TN_e^T.
\end{aligned}$$

In this case, a desired state feedback controller is given by

$$u(t) = Q_1P_1^{-1}x(t).$$

Remark 6: When $E=I$ and $\Delta E=0$ (i.e. $N_e=0$), obviously, the constraint of $\mathrm{rank}(E+\Delta E)=n$ is satisfied and Corollary 2 is the conclusion for a state space system.

3. Numerical example

Consider descriptor systems (13) with the following parameters:

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.5 & 0.8 \\ 0.2 & 0.4 \\ 0 & 1 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.5 & 1 \\ 2 & 1 \\ 1 & 1.5 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.5 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}, \quad D = \begin{bmatrix} 1.5 & 2 \\ 0.5 & 2 \end{bmatrix}, \quad M = \begin{bmatrix} 0.5 \\ 0.2 \\ 0.8 \end{bmatrix},$$

$$N_e = \begin{bmatrix} 1 & 0.8 & 0.2 \end{bmatrix}, \quad N_a = \begin{bmatrix} 1.5 & 1.2 & 0.6 \end{bmatrix},$$
$$N_b = \begin{bmatrix} 1.5 & 0.4 \end{bmatrix}.$$

It is supposed that the uncertain matrix is given as $F(\sigma) = \sin(\sigma)$, $\sigma \in [0, 2\pi]$. Since $\det(E) = 0$, we can know that the system is a singular one. Noting that

$$\deg(\det(sE - A)) = \deg(s - 1) = 1 < \operatorname{rank}(E) = 2,$$

we can know that the nominal descriptor system is not impulsive free (Xu and Lam 2006; Chen 2008). We also can see that the nominal descriptor system is not stable with a finite eigenvalue $s = 1$. The transfer function matrix of open-loop nominal descriptor system is

$$G(s) = \begin{bmatrix} 2 & \dfrac{10s + 13}{10} \\ \dfrac{15s - 13}{10s - 10} & \dfrac{10s^2 - 2s - 6}{5s - 5} \end{bmatrix},$$

which is not ESPR because the minimal eigenvalue of $G(j\theta) + G^T(-j\theta)$ at $\theta = 20$ is -16.6098. According to Theorem 2 and with the aid of the LMI toolbox, then, we can show that a set of solutions to (20) is as follows:

$$P_1 = \begin{bmatrix} 0.1949 & -0.0386 & 0.2032 \\ -0.0386 & 0.3648 & -0.1849 \\ 0.2032 & -0.1849 & 1.0479 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} -0.9961 & -0.2265 & 2.4272 \\ 0.6717 & -1.3834 & -1.7872 \\ -2.2868 & 1.9492 & -3.0161 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 1.0817 & 0.3017 & -0.0320 \\ -1.1689 & 0.3826 & -0.5089 \\ 3.2421 & -1.8325 & 0.1485 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} -0.0952 & -0.4305 & 0.0338 \\ -0.1456 & 0.4321 & 0.0889 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} -0.1068 & -0.5260 & -0.1039 \\ -0.2243 & 0.1336 & -0.7805 \end{bmatrix}, \quad \varepsilon = 2.2254.$$

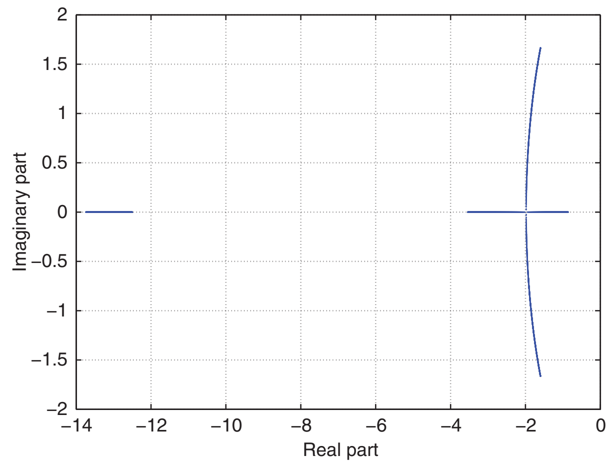

Figure 1. The eigenvalue distribution of the closed-loop system.

The desired PD controller parameters can be calculated as follows:

$$K_p = \begin{bmatrix} -5.0043 & -0.4974 & 3.4746 \\ -2.9744 & 7.7544 & 4.6038 \end{bmatrix},$$

$$K_d = \begin{bmatrix} 0.5462 & -0.3152 & -0.2629 \\ -0.4956 & -1.6027 & -0.3433 \end{bmatrix}.$$

For all $\sigma \in [0,\ 2\pi]$ and with the above designed controller, the determinant of derivative matrix of the corresponding closed-loop system is

$$\det(E_{\Delta K}) = 0.0288 \sin(\sigma) - 1.3249 \neq 0,$$

which implies that the closed-loop system is normal. The eigenvalues of the closed-loop system for all $\sigma \in [0, 2\pi]$ are shown in Figure 1. This figure illustrates the stability of the closed-loop system for all $\sigma \in [0, 2\pi]$. The minimal eigenvalues of $G_{\Delta K}(j\theta) + G^T_{\Delta K}(-j\theta)$ for θ ranging from 0 to 50 and σ ranging from 0 to 2π are displayed in Figure 2. It is clear to see that $G_{\Delta K}(s)$ is ESPR for all $\sigma \in [0, 2\pi]$. From the above analysis, we can conclude that the closed-loop system is QNQS and ESPR.

4. Conclusions

This article has investigated the normalisation and positive real control problem for descriptor systems with parameter uncertainties in the state, input and derivative matrices. A new LMI characterisation of positive realness for normal descriptor systems is given. Based on this, a necessary and sufficient condition for robust normalisation and robust stabilisation with ESPR has been obtained in terms of LMIs. An explicit desired PD feedback controllers has also been given. An numerical example has been provided to illustrate the effectiveness of our methods.

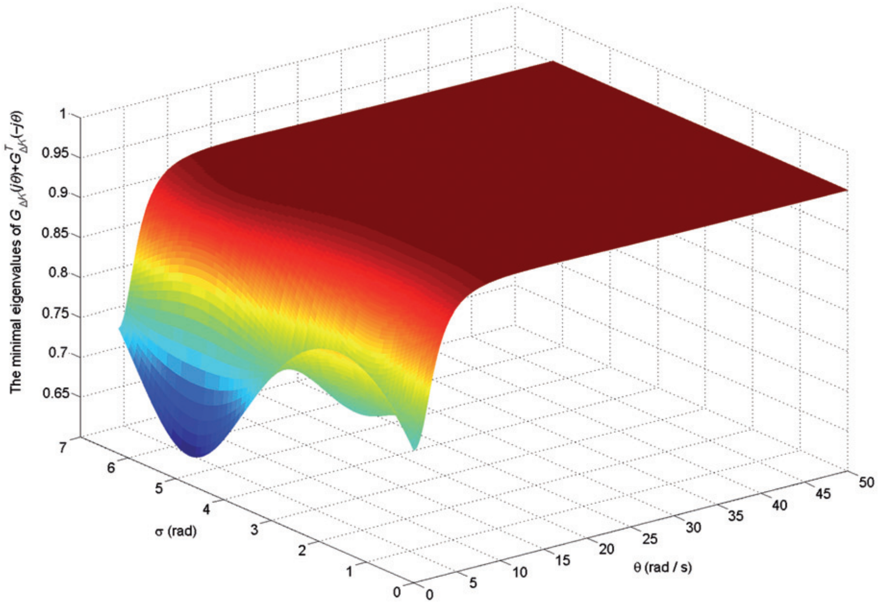

Figure 2. The minimal eigenvalues of $G_{\Delta K}(j\theta) + G^T_{\Delta K}(-j\theta)$.

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