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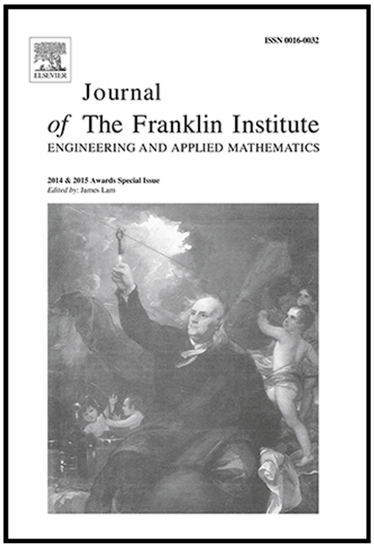

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Sampled-data Stabilization of a Class of Nonlinear Differential Algebraic Systems via Partial-State and Output Feedback

Qixun Lan[1*], Liying Sun[2], Jiarong Liang[3], Yajie Li[1], Yujie Cai[1], Zhe Wang[1], Xiaofeng Wang[1]

1. School of Mathematics and Physics, Henan Provincial Engineering Laboratory of Building-Photovoltaics, Henan University of Urban Construction, Pingdingshan 467036, China,
2. School of Mathematics Science, University of Jinan, Jinan 250022, China
3. School of Computer, Electronics and Information, Guangxi University, Nanning 530004, China

Abstract: In this paper, the problem of sampled-data stabilization for a class of nonlinear differential-algebraic systems is considered by using partial-state and output feedback. First, based on the output feedback domination approach, a systematic design procedure for sampled-data output feedback controller is proposed without using the states of the algebraic subsystem. It is shown that the proposed sampled-data controller can ensure the whole closed-loop nonlinear differential-algebraic system is asymptotically stable by choosing appropriate scaling gains and sampling period. In addition, due to the domination nature of the proposed method, the obtained results can be extended to more general class of nonlinear differential-algebraic systems easily. Finally, simulation examples are provided to illustrate the effectiveness of the proposed control method.

Keywords: Nonlinear differential algebraic system; Sampled-data stabilization; Output feedback domination; Partial-state feedback; Stability

1. Introduction

Systems described by differential-algebraic equations (DAEs) are also referred to as singular systems [1–8], descriptor systems [9–14], or implicit systems [15] widely exist in many practical systems, such as power systems, robot systems, economic systems and so on. Systems modeled by DAEs represent the more general systems than normal systems (described by ordinary equations), the study of DAE systems has attracted a lot of attentions of researchers. Compared to normal systems without algebraic constraints, the stability analysis and stabilization problem for DAE systems are more complicated. Many significant results have been obtained in the literature, however, most of them are concentrated on linear DAE systems, such as, [2–4, 6–8, 10] and the reference therein.

Although the stability analysis and stabilization problems for systems described by nonlinear DAEs (NDAEs) are more difficult than that of linear DAEs, there still exist some interesting results. The work [17] considered the stability problems for a class of NDAE systems, and proposed the stability criteria for NDAE systems with special forms. For Lipschitz NDAE systems, the actuator fault estimation and fault-tolerant control problems were considered in [12, 18]. And the existence and uniqueness of a solution and the stability problems for a class of Lipschitz discrete-time NDAE systems were addressed in [32]. H_∞ control problem and sufficient condition of locally asymptotically stability for a class of NDAE systems were considered in [11]. The Lyapunov stability and strong passivity for a class of NDAE systems were studied in [13], and a strongly absolute stability criterion was derived at the same time. For a class of affine NDAE systems, the necessary and sufficient condition for the solvability of the regulation problem was derived in [5]. By using the Hamiltonian function method, the stabilization and H_∞ control problems of NDAE systems were investigated in [20]. Motivated by the Hamiltonian function method, state undecomposed approach was proposed in [14] to study the finite-time stabilization and control problems for a class of NDAE systems. For a class of NDAE systems, it was shown that under suitable diffeomorphism transformation NDAE systems can be transformed

*Corresponding Author. Tel.: +(375)2089151; E-mail address: q.lan@hncj.edu.cn (Q. Lan).

into systems with triangular structure on the constrained manifold [29]. Based on backstepping technique, a robust controller design method for a class of NDAE systems was proposed [19].

Due to the inherent nonlinear characteristics of NDAE systems, the output feedback stabilization problems are more challenging. Inspired by [6], the output regulation problem for a class of nonlinear singular systems was addressed in [24], and the generalized version of the centre manifold theorem was given at the same time. The normalizability assumption [24] was removed in [25], and an output dynamic feedback controller for a class of nonlinear singular systems was constructed. The work [26] extended the results of [24], and considered the robust output regulation problem nonlinear singular systems with uncertain parameters. For a general class of nonlinear singular systems, a complete solution of output regulation problem was considered in [27] without the normalizability assumption [24]. By using internal model approach, the robust output regulation problem for a class of nonlinear singular systems was further investigated by [28], which significantly improved the previous work [26]. For a class of singular systems with input saturation, an output feedback composite nonlinear feedback controller design method was proposed in [33].

Note that all the aforementioned results on the stabilization problems of NDAE systems were achieved by using continuous time controllers. In the actual engineering systems, more and more systems are being operated by using digital computers [36, 49]. Up to now, the existing results on the sampled-data stabilization problems for nonlinear systems mainly focused on normal nonlinear systems, that is, the nonlinear systems without algebraic constraints [34, 37–49]. Therefore, it is of great significance to study the stabilization problems of NDAE systems by using sampled-data controllers. However, to the best of our knowledge, there is few results on the sampled-data stabilization problem of NDAE systems in the literature. In this paper, we will investigate the sampled-data output feedback stabilization problem for a class of NDAE systems. Based on the output feedback domination approach [23], a tunable scaling gain is introduced to the differential subsystems through an appropriate coordinate transformation firstly. Then, a discrete-time linear observer and an emulated linear output feedback controller is constructed by using partial sates only and without considering the unknown nonlinearities. Finally, it is shown that the proposed discretized output feedback controller can stabilize the considered NDAE systems by choosing appropriate scaling gain and sampling period. In the simulation part, both the numerical and practical examples are employed to show the effectiveness of the sampled-data output feedback controllers proposed in this paper.

The remainder of the paper is organized as follows. In the next Section, the preliminaries and problem description are given. In Section 3, we focus on the sampled-data output feedback controller design problem for NDAE systems. Some discussions and extension are given in Section 4. In Section 5, the effectiveness of the proposed control algorithm are testified by two simulation examples. Section 6 contains some concluding remarks.

2. Preliminaries and problem description

Consider a class of uncertain NDAE systems in the following form

$$
\begin{aligned}
\dot{x}_i(t) &= x_{i+1}(t) + f_i(t, x(t), z(t), u(t)), i = 1, \cdots, n, &\quad (1a)\\
0 &= g_i(\underline{x}_i(t), \underline{z}_i(t)), i = 1, \cdots, n, &\quad (1b)\\
y(t) &= x_1(t)
\end{aligned}
$$

where $\underline{x}_i(t) := (x_1(t), \cdots, x_i(t))^T \in \mathbb{R}^i(x(t) = \underline{x}_n(t))$ are the vectors of differential variables, $\underline{z}_i(t) := (z_1(t), \cdots, z_i(t))^T \in \mathbb{R}^i(z(t) = \underline{z}_n(t))$ are the vectors of algebraic variables, $x_{n+1}(t) = u(t) \in \mathbb{R}$ is the control input, $y(t) \in \mathbb{R}$ is the output of the system, respectively; $f_i, i = 1, \cdots, n$ are unknown continuous nonlinear functions with $f_i(t, 0, \cdots, 0) = 0$, and nonlinear functions $g_i, i = 1, \cdots, n$ are $\mathbb{C}^1$ and vanish at the origin.

To address the sampled-data stabilization of the NDAE systems (1), the following two assumptions are needed.

Assumption 2.1. *For $i = 1, \cdots, n$, there is a constant $c_1 > 0$ such that*

$$|f_i(x(t), z(t), u(t))| \le c_1 (|x_1(t)| + |x_2(t)| + \cdots + |x_i(t)| + |z_1(t)| + \cdots + |z_i(t)|). \quad (2)$$

Remark 2.1. *In Assumption 2.1, the nonlinear term in system (1a) should be satisfy linear growth conditions, which means that (1a) may be a nonsmooth system [14, 22, 30]. In [29], the nonlinear terms $f_i(\cdot), i = 1, \cdots, n$ are assumed to be precisely known and global Lipschitz with respect to $(z(t), x(t))$. The assumptions on the nonlinear terms $f_i(\cdot), i = 1, \cdots, n$ were relaxed in [30], but the nonlinear terms should satisfy the condition: $|f_i(\cdot)| \le$*

$c_1\left(|x_1(t)| + |x_2(t)| + \cdots + |x_i(t)|\right), i = 1, \cdots, n$. *Obviously, compared to [29, 30], the system considered in this paper includes more general class of NDAE systems.*

Assumption 2.2. *There exist two constants* $c_{i1} > 0$ *and* $c_{i2} > 0$*, such that* $\frac{\partial g_i(\cdot)}{\partial z_i} \geq c_{i1}$ *and*

$$|g_i(z_1(t), \cdots, z_{i-1}(t), 0, \underline{x}_i(t))| \leq c_{i2}(|x_1(t)| + \cdots + |x_i(t)| + |z_1(t)| + \cdots + |z_{i-1}(t)|). \tag{3}$$

Remark 2.2. *If we define* $G(x(t), z(t)) = [g_1(\cdot), \cdots, g_n(\cdot)]^T$*, and* $\Omega = \{(x(t), z(t)) \in \mathbb{R}^{2n} : 0 = G(x(t), z(t))\}$*, then it follows from Assumption 2.2 that the Jacobian of* G *with respect to* z *has full rank on* Ω*, that is,* $\left[\frac{\partial G}{\partial z}\right] = n$ *which means that NDAE system (1) is index one. It should be pointed out that when a NDAE system has a higher index, by using index reduction techniques [1, 5], the NDAE system with a higher index can often reduced to a NDAE system with index one[16, 17, 19, 20]. Therefore, we merely concentrated on the NDAE systems with index one.*

Remark 2.3. *Based on Assumption 2.2, we have*

$$\begin{aligned}
0 &= g_i(\underline{z}_i, \underline{x}_i) = g_i(z_1, \cdots, z_{i-1}, z_i, \underline{x}_i) \\
&= g_i(z_1, \cdots, z_{i-1}, z_i, \underline{x}_i) - g_i(z_1, \cdots, z_{i-1}, 0, \underline{x}_i) + g_i(z_1, \cdots, z_{i-1}, 0, \underline{x}_i) \\
&= \int_0^1 \frac{\partial g_i(z_1, \cdots, z_{i-1}, \lambda z_i, \underline{x}_i)}{\partial z_i} d\lambda z_i + g_i(z_1, \cdots, z_{i-1}, 0, \underline{x}_i).
\end{aligned} \tag{4}$$

It follows from (4) that

$$\int_0^1 \frac{\partial g_i(z_1, \cdots, z_{i-1}, \lambda z_i, \underline{x}_i)}{\partial z_i} d\lambda z_i = -g_i(z_1, \cdots, z_{i-1}, 0, \underline{x}_i). \tag{5}$$

According to (4) and Assumption 2.2, it can be shown that

$$\begin{aligned}
c_{i1}|z_i| &\leq \left| \int_0^1 \frac{\partial g_i(z_1, \cdots, z_{i-1}, \lambda z_i, \underline{x}_i)}{\partial z_i} d\lambda z_i \right| = |-g_i(z_1, \cdots, z_{i-1}, 0, \underline{x}_i)| \\
&\leq c_{i2}\left(|x_1| + \cdots + |x_i| + |z_1| + \cdots + |z_{i-1}|\right).
\end{aligned} \tag{6}$$

This implies that the following inequality holds

$$\begin{aligned}
|z_i| &\leq \frac{c_{i2}}{c_{i1}}\left(|x_1| + \cdots + |x_i| + |z_1| + \cdots + |z_{i-1}|\right) \\
&\leq c_2\left(|x_1| + \cdots + |x_i| + |z_1| + \cdots + |z_{i-1}|\right)
\end{aligned} \tag{7}$$

with $c_2 = \max_{1\leq i\leq n}\left\{\frac{c_{i2}}{c_{i1}}\right\}$.

That is, the algebraic variable z_i *could be bounded by a homogeneous polynomial.*

It should be pointed out that in Assumption 2.2, if $c_{i1} < 0$ *and* $\frac{\partial g_i}{\partial z_i} \leq c_{i1}$*, it could be easily proved that the inequality (7) still holds with* $c_2 = \max_{1\leq i\leq n}\left\{\frac{c_{i2}}{|c_{i1}|}\right\}$.

The next lemma is a consequence of the Young's inequality [22], which plays a key role in the proof of the main results.

Lemma 2.1. *Let* c *and* d *be positive constants. Given any positive number* $\gamma > 0$*, the following inequality holds:*

$$|x|^c|y|^d \leq \frac{c}{c+d}\gamma|x|^{c+d} + \frac{d}{c+d}\gamma^{-c/d}|y|^{c+d}$$

for $x \in \mathbb{R}, y \in \mathbb{R}$.

The control objective of this paper is to design a dynamic sampled-data output feedback controller for NDAE system (1), such that the closed-loop system is asymptotically stable.

3. Sampled-data output feedback control law design

In this section, based on output feedback domination approach [23], we will present a step by step procedure to design a sampled-data output feedback controller to solve the stabilization problem of NDAE system (1).

Theorem 3.1. *For NDAE system (1), if Assumptions 2.1 and 2.2 hold, then there exist an appropriate sampling period T and a scaling gain $L > 1$ such that under the sampled-data output feedback controller*

$$u(t) = -L^n\left(k_1\hat{x}_1(t_k) + \frac{1}{L}k_2\hat{x}_2(t_k) + \cdots + \frac{1}{L^{n-1}}k_n\hat{x}_n(t_k)\right), \tag{8}$$

$$\hat{x}(t_{k+1}) = M\hat{x}(t_k) + Ny(t_k), \forall t \in [t_k, t_{k+1}), t_k = kT, k = 0, 1, 2, \cdots, \tag{9}$$

the closed-loop system composed by (1), (8) and (9) is asymptotically stable, where k_j, $j = 1, \cdots, n$ are coefficients of the Hurwitz polynomial $p_1(s) = s^n + k_n s^{n-1} + \cdots + k_2 s + k_1$, $M \in \mathbb{R}^{n\times n}$ and $N \in \mathbb{R}^{n\times 1}$ are appropriate matrices to be determined.

Proof. The proof of the theorem can be divided into three steps. In the first step, a tunable scaling gain L is introduced to the subsystem (1a) by using a coordinate transformation. In the second step, a linear discrete-time observer is designed to estimate the unmeasurable states of the subsystem (1a), and then a linear discrete-time controller is constructed by using the estimated states of subsystem (1a) only by using emulation technique. In the last step, based on the output feedback domination approach, it is shown that the proposed sampled-data output feedback controller will render the closed-loop system of (1) be asymptotically stable by choosing appropriate scaling gain L and sampling period T.

Step 1: The coordinate transformation of subsystem (1a).

For subsystem (1a), first, we consider the following coordinate transformation

$$\eta_i(t) = \frac{x_i(t)}{L^{i-1}}, i = 1, \cdots, n, \upsilon(t) = \frac{u(t)}{L^n}. \tag{10}$$

Under coordination transformation (10), subsystem (1a) is transformed into the following system

$$\begin{cases} \dot{\eta}_1(t) = L\eta_2(t) + \overline{f_1}(\eta(t), z(t), \upsilon(t) \\ \dot{\eta}_i(t) = L\eta_{i+1}(t) + \overline{f_i}(\eta(t), z(t), \upsilon(t)), i = 2, \cdots, n-1, \\ \dot{\eta}_n(t) = L\upsilon(t) + \overline{f_n}(\eta(t), z(t), \upsilon(t)), \\ y(t) = \eta_1(t), \end{cases} \tag{11}$$

where $\overline{f_i}(\eta(t), z(t), \upsilon(t)) = f_i(x(t), z(t), u(t))/L^{i-1}, i = 1, \cdots, n$.

Define

$$F(\eta(t), z(t), \upsilon(t)) = \begin{bmatrix} \overline{f_1}(\eta(t), z(t), \upsilon(t)) \\ \overline{f_2}(\eta(t), z(t), \upsilon(t)) \\ \vdots \\ \overline{f_n}(\eta(t), z(t), \upsilon(t)) \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}, C = [1\ 0\ \cdots\ 0].$$

Then system (11) can be written in the following compact form

$$\begin{aligned} \dot{\eta}(t) &= LA\eta(t) + LB\upsilon(t) + F(\eta(t), z(t), \upsilon(t)), \\ y(t) &= C\eta(t). \end{aligned} \tag{12}$$

Noticed that only the output $y(t) = \eta_1(t)$ is measurable at sampling points $t_k, k = 0, 1, 2, \cdots$, and the states $\eta_2(t), \cdots, \eta_n(t)$ are not available. In the next step, we will focus on the estimation problem for the unmeasured states of system (12), and then use the estimations to construct sampled-data output feedback controller.

Step 2: Construction of sampled-data output feedback controller.

To estimate the unmeasured states of system (12), inspired by [23, 34], we design the following observer over $[t_k, t_{k+1})$ by utilising $\eta_1(t_k)$ and $\upsilon(t_k)$

$$\begin{cases} \dot{\hat{\eta}}_1(t) = L\hat{\eta}_2(t) + La_1(\eta_1(t_k) - \hat{\eta}_1(t)) \\ \dot{\hat{\eta}}_i(t) = L\hat{\eta}_{i+1}(t) + La_i(\eta_1(t_k) - \hat{\eta}_1(t)), i = 2, \cdots, n-1, \\ \dot{\hat{\eta}}_n(t) = L\upsilon(t_k) + La_n(\eta_1(t_k) - \hat{\eta}_1(t)), \forall t \in [t_k, t_{k+1}), \end{cases} \tag{13}$$

where a_j, $j = 1, \cdots, n$ are coefficients of the Hurwitz polynomial $p_2(s) = s^n + a_n s^{n-1} + \cdots + a_2 s + a_1$.

Let $\hat{\eta}(t) = [\hat{\eta}_1(t), \cdots, \hat{\eta}_n(t)]^T$, $\Pi = [a_1, \cdots, a_n]^T$ and $\hat{A} = A - \Pi C$, then the observer (13) can be written as

$$\dot{\hat{\eta}}(t) = L\hat{A}\hat{\eta}(t) + LB\upsilon(t_k) + L\Pi\eta_1(t_k), \forall t \in [t_k, t_{k+1}). \tag{14}$$

Integrating on the both sides of equation (14) from t_k to t_{k+1}, it can be shown that system (14) is equivalent to the following discrete-time system

$$\begin{aligned} \hat{\eta}(t_{k+1}) &= e^{L\hat{A}T}\hat{\eta}(t_k) + \int_0^T e^{L\hat{A}s}(LB\upsilon(t_k) + L\Pi\eta_1(t_k))ds \\ &= \Sigma\hat{\eta}(t_k) + \Lambda\upsilon(t_k) + N_0\eta_1(t_k) \end{aligned} \tag{15}$$

where $\Sigma = e^{L\hat{A}T}$, $\Lambda = \int_0^T e^{L\hat{A}s}dsLB$, and $N_0 = \int_0^T e^{L\hat{A}s}dsL\Pi$.

If we choose the following controller for system (13) or system (14)

$$\upsilon(t_k) = -K\hat{\eta}(t_k) = -k_1\hat{\eta}_1(t_k) - k_2\hat{\eta}_2(t_k) - \cdots - k_n\hat{\eta}_n(t_k) \tag{16}$$

with $K = [k_1, \cdots, k_n]$, where $k_1, \cdots, k_n$ are the same as that defined in (8).

By (15) and (16), we have

$$\hat{\eta}(t_{k+1}) = M_0\hat{\eta}(t_k) + N_0\eta_1(t_k), \tag{17}$$

where $M_0 = \Sigma - \Lambda K$ and N_0 are depend on the sampling period T.

According to (10), we define $\Theta = diag\left(1, L, \cdots, L^{n-1}\right)$. This combining with (17), the equation (9) could be easily derived with $M = \Theta M_0 \Theta^{-1}$ and $N = \Theta N_0$.

By the definitions of a_i's and k_i's, it is not difficult to prove that both $\hat{A} = A - \Pi C$ and $A - BK$ are Hurwitz matrices. In what follows, we will show that by choosing appropriate scaling gain L and T, the closed-loop system composed by (11),(14) and $\upsilon(t) = -K\hat{\eta}(t_k)$ is asymptotically stable.

Substituting the control law $\upsilon(t) = -K\hat{\eta}(t_k)$ into systems (11) and (14) yields

$$\begin{aligned} \begin{bmatrix} \dot{\eta}(t) \\ \dot{\hat{\eta}}(t) \end{bmatrix} &= L\begin{bmatrix} A & 0 \\ 0 & \hat{A} \end{bmatrix}\begin{bmatrix} \eta(t) \\ \hat{\eta}(t) \end{bmatrix} - L\begin{bmatrix} B \\ B \end{bmatrix}K\hat{\eta}(t_k) + L\begin{bmatrix} 0 \\ \Pi \end{bmatrix}\eta_1(t_k) + \begin{bmatrix} F(\cdot) \\ 0 \end{bmatrix} \\ &= L\begin{bmatrix} A & -BK \\ \Pi C & \hat{A} - BK \end{bmatrix}\begin{bmatrix} \eta(t) \\ \hat{\eta}(t) \end{bmatrix} - L\begin{bmatrix} B \\ B \end{bmatrix}K(\hat{\eta}(t_k) - \hat{\eta}(t)) + L\begin{bmatrix} 0 \\ \Pi C \end{bmatrix}(\eta(t_k) - \eta(t)) + \begin{bmatrix} F(\cdot) \\ 0 \end{bmatrix}. \end{aligned} \tag{18}$$

Denote $\overline{A} = \begin{bmatrix} A & -BK \\ \Pi C & \hat{A} - BK \end{bmatrix}$, it could be easily proved that

$$\overline{A} = \begin{bmatrix} A & -BK \\ \Pi C & \hat{A} - BK \end{bmatrix} = \begin{bmatrix} I & 0 \\ -I & I \end{bmatrix}^{-1}\begin{bmatrix} A - BK & -BK \\ 0 & \hat{A} \end{bmatrix}\begin{bmatrix} I & 0 \\ -I & I \end{bmatrix}.$$

Since both $\hat{A} = A - \Pi C$ and $A - BK$ are Hurwitz matrices. It is easy to show that $\overline{A}$ is a Hurwitz matrix also, thus there exists a positive definite matrix $P = P^T \in \mathbb{R}^{2n\times 2n} > 0$ such that $P^T\overline{A} + \overline{A}^T P = -I$.

Consider the Lyapunov function $V(\overline{\eta}(t)) = \overline{\eta}^T(t)P\overline{\eta}(t)$ with $\overline{\eta}(t) = \left[\eta^T(t), \hat{\eta}^T(t)\right]^T$ for system (18). Take the derivative of $V(\overline{\eta}(t))$ along system (18) yields

$$\dot{V}(\overline{\eta}(t)) = -L\|\overline{\eta}(t)\|^2 + 2L\overline{\eta}^T(t)P\begin{bmatrix} B \\ B \end{bmatrix}K(\hat{\eta}(t) - \hat{\eta}(t_k)) + 2L\overline{\eta}^T(t)\begin{bmatrix} 0 \\ \Pi C \end{bmatrix}(\eta(t_k) - \eta(t)) + 2\overline{\eta}^T(t)P\begin{bmatrix} F(\cdot) \\ 0 \end{bmatrix}. \tag{19}$$

In what follows, we will show that by choosing appropriate scaling gain L and sampled-data period T, the closed-loop system (18) is asymptotically stable.

Step 3: Stability analysis of the closed-loop system.

To analysis the stability of the closed-loop system (18) and to determine L and T, the last three nonlinear terms in righthand side of (19) need to be estimated.

To estimate the last term in righthand side of (19), we introduce the following proposition, the proof of which is collected in the Appendix.

Proposition 3.1. *If Assumption 2.1 and Assumption 2.2 hold, then there exist a constant $c > 0$ such that*

$$|f_i(x(t), z(t), u(t)| \leq c(|x_1(t)| + |x_2(t)| + \cdots + |x_i(t)|), i = 1, 2, \cdots, n. \tag{20}$$

It follows from (20) that

$$|\overline{f}_i(\eta(t), z(t), \upsilon(t))| = \frac{c}{L^{i-1}}(|\eta_1(t)| + \cdots + |L^{i-1}\eta_i(t)|) \leq c(|\eta_1(t)| + \cdots + |\eta_i(t)|). \tag{21}$$

With this in mind, it is not difficult to deduce that

$$||F(\eta(t), z(t), \upsilon(t)|| \leq c\sqrt{\eta_1^2(t) + \cdots + (|\eta_1(t)| + \cdots + |\eta_n(t)|)^2} \leq c_0||\eta(t)|| \leq c_0||\overline{\eta}(t)||, \tag{22}$$

where $c_0 = c\sqrt{n(n+1)/2} > 0$.

With the help of (22) and Lemma 2.1, we have

$$\left|2\hat{\eta}^T(t)P\begin{bmatrix}\overline{F}(\cdot)\\0\end{bmatrix}\right| \leq 2c_0||P||||\overline{\eta}(t)||^2. \tag{23}$$

Combining (19) and (23), we obtain

$$\dot{V}(\overline{\eta}(t)) \leq -L||\overline{\eta}(t)||^2 + 2L||\overline{\eta}(t)||||P||(\sqrt{2}||K|| + ||\Pi||)||\overline{\eta}(t) - \overline{\eta}(t_k)|| + 2c_0||\overline{\eta}(t)||||P||||\overline{\eta}(t)||. \tag{24}$$

So far, we only need to estimate the second term in righthand side of (24). To estimate the term $||\overline{\eta}(t) - \overline{\eta}(t_k)||$, the following proposition is needed. The proof of which is included in the Appendix.

Proposition 3.2. *There exists a function $\theta(t) > 0$, such that the following inequality holds*

$$||\overline{\eta}(t) - \overline{\eta}(t_k)|| \leq \frac{\theta(t)}{1-\theta(t)}||\overline{\eta}(t)||, \forall t \in [t_k, t_{k+1}).$$

In conjunction with (24) and Proposition 3.2, we have

$$\begin{aligned}\dot{V}(\overline{\eta}(t)) &\leq -L||\overline{\eta}(t)||^2 + 2L||P||\left(\sqrt{2}||K|| + ||\Pi||\right)\frac{\theta(t)}{1-\theta(t)}||\overline{\eta}(t)||^2 + 2c_0||P||||\overline{\eta}(t)||^2\\ &\leq -\left(\frac{L}{\lambda_{max}(P)} - \frac{2L||P||\left(\sqrt{2}||K|| + ||\Pi||\right)}{\lambda_{min}(P)}\frac{\theta(t)}{1-\theta(t)} - \frac{2c_0||P||}{\lambda_{min}(P)}\right)V(\overline{\eta}(t)), \forall t \in [t_k, t_{k+1}),\end{aligned} \tag{25}$$

where $\lambda_{max}(P)$ and $\lambda_{min}(P)$ represent the maximum eigenvalue and minimal eigenvalue of the matrix P, respectively.

Therefore, if we choose a small enough constant T and a large enough constant L such that

$$\theta(t) = \left(e^{\mu_1(t-t_k)} - 1\right)\frac{\mu_2}{\mu_1} < 1, \text{and } L > \max\left\{\frac{\lambda_{max}(P)(2c_0||P|| + 0.5\lambda_{min}(P))(1-\theta(t))}{\lambda_{min}(P)(1-\theta(t)) - 2\lambda_{max}(P)||P||\left(\sqrt{2}||K|| + ||\Pi||\right)\theta(t)}, 1\right\} \tag{26}$$

where μ_1 and μ_2 are defined in (A.4), then we have

$$\dot{V}(\overline{\eta}(t)) < -0.5V(\overline{\eta}(t)), \forall t \in [t_k, t_{k+1}), \tag{27}$$

which means that the closed-loop system (18) is globally exponentially stable. Note that the coordinate transformation does not change the properties of the system, the under the sampled-data output feedback controller (8)-(9) states of subsystem (1a) will converge to the origin asymptotically, that is, $\lim_{t\to+\infty} x_i(t) = 0, i = 1, \cdots, n$.

Based on the fact $\lim_{t\to+\infty} x_i(t) = 0, i = 1, \cdots, n$, Assumption 2.2 and inequality (7), let $t \to +\infty$, it follows from $|z_1| \leq c_2|x_1|$ that $\lim_{t\to+\infty} z_1 = 0$. And then based on the fact $x_1 \to 0$, $x_2 \to 0$, $z_1 \to 0$ as $t \to +\infty$, and $|z_2| \leq c_2 (|x_1| + |x_2| + |z_1|)$ that we have $\lim_{t\to+\infty} z_2 = 0$.

Proceed in the same manner, it can be shown that $\lim_{t\to+\infty} z_1 = 0, \cdots, \lim_{t\to+\infty} z_n = 0$ step by step.

Therefore, it can be concluded that NDAE system (1) can be asymptotically stabilized by the sampled-data output feedback controller (8)-(9). □

Remark 3.1. *In order to facilitate the analysis and design, we assume that every subsystem of system (1a) has a corresponding algebraic equation for NDAE system (1). In fact, part of the subsystems of system (1a) are allowed to have no algebraic constraint equations, e.g., without of loss generality, if we assume that the first $n - m$ subsystems of system (1a) have no corresponding algebraic equations, then (1b) becomes*

$$0 = g_i(\underline{x}_{n-m+i}(t), \underline{z}_i(t)), i = 1, \cdots, m.$$

In this condition, to solve the sampled-data output feedback stabilization problem of system (1), the Assumptions 2.1 and 2.2 should be modified respectively as:

Assumption 2.1***.** *There is a constant $c_1 > 0$ such that*

$$|f_i(x(t), z(t), u(t))| \leq c_1 (|x_1(t)| + |x_2(t)| + \cdots + |x_i(t)|), i = 1, \cdots, n - m,$$
$$|f_{n-m+i}(x(t), z(t), u(t))| \leq c_1 (|x_1(t)| + |x_2(t)| + \cdots + |x_{n-m}(t)| + \cdots + |x_{n-m+i}| + |z_1(t)| + \cdots + |z_i(t)|), i = 1, \cdots, m.$$

Assumption 2.2***.** *For $i = 1, \cdots, m$, there exist two constants $c_{i1} > 0$ and $c_{i2} > 0$ such that $\frac{\partial g_i(\cdot)}{\partial z_i} \geq c_{i1}$, and*

$$|g_i(0, z_{i-1}(t), \cdots, z_1(t), \underline{x}_{n-m+i}(t))| \leq c_{i2}(|x_1(t)| + \cdots + |x_{n-m}(t)| + \cdots + |x_{n-m+i}(t)| + |z_1(t)| + \cdots + |z_{i-1}(t)|).$$

Assumption 2.2* means that the Jacobian of G with respect to z has full rank on Ω, that is, NDAE system is still a index one system.

Note that Assumptions 2.1* and 2.2* are only special cases of Assumptions 2.1 and 2.2, respectively. Based on Assumptions 2.1* and 2.2*, the corresponding result for NDAE system (1) follows.

Corollary 3.1. *For NDAE system (1), if Assumptions 2.1* and 2.2* hold, then NDAE system (1) can be asymptotic stabilized by the dynamic sampled-data output feedback controller (8)-(9).*

Remark 3.2. *For stabilization problems of NDAE systems by using discrete-time feedback control, as far as we know that there few results besides [35]. However, [35] only considered the sampled-data state feedback stabilization problem of a class of linear singular systems. When partial states of the considered system are not measured or the DAE system is nonlinear, the proposed method in [35] is invalid. In Theorem 3.1, motivated by [23, 34] the sampled-data controller is constructed by discretizing continuous time controller approach (that is, by emulation approach [34, 37, 38]) rather than by the discrete time approximation of the nonlinear systems. The proposed dynamic sampled-data output feedback controller (8)-(9) in Theorem 3.1 is linear and only partial states are involved. Therefore, from a practical perspective, the proposed dynamic output feedback controller can be easily implemented by digital computers. Furthermore, it is known to all that many practical systems are modelled by NDAE systems. Hence, the considered NDAE systems have widely applications scopes, and the proposed control algorithm in this paper is more interesting.*

4. Discussions and extension

It should be pointed out that Theorem 3.1 is obtained based on the assumptions that the nonlinear terms $f_i(\cdot)$ should satisfy linear growth condition about $x_1, \cdots, x_i$ and $z_1, \cdots, z_i$, and in the algebraic equations g_i is only a function of

$x_1, \cdots, x_i$ and $z_1, \cdots, z_i$. In this subsection, we will show that the result obtained in the preceding section can be further extended to a more general class of NDAE systems described by

$$\dot{x}_i(t) = x_{i+1}(t) + f_i(x(t), z(t), u(t)), i = 1, \cdots, n, \tag{28a}$$
$$0 = g_i(x(t), z(t)), i = 1, \cdots, n. \tag{28b}$$
$$y(t) = x_1(t).$$

Due to the domination nature of the output feedback domination approach, the result obtained can be extended under the following more general assumptions.

Assumption 4.1. *(i) There exist two constants $c_{i1} > 0$ and $c_{i2} > 0$, such that $\frac{\partial g_i(\cdot)}{\partial z_i} \geq c_{i1}$ and*

$$|g_i(z_n(t), \cdots, z_{i+1}(t), 0, z_{i-1}(t), \cdots, z_1(t), \underline{x}_i(t))| \leq c_{i2}\left(\sum_{j=1}^{n} |x_j(t)| + \sum_{j\neq i} |z_j(t)|\right). \tag{29}$$

(ii) The Jacobian matrix $\left[\frac{\partial G}{\partial z}\right]$ is a strictly diagonally dominant matrix on Ω.

Assumption 4.2. $c_2 := \max\{\frac{c_{i2}}{c_{i1}}\} \in (\frac{1}{2n-1}, \frac{1}{n-1})$.

According to Assumption 4.1, it is obvious that NDAE system (28) is index one. Furthermore, by using a similar method adopted in Remark 2.3, it can be proved that

$$|z_i(t)| \leq \frac{c_{i2}}{c_{i1}}\left(\sum_{j=1}^{n} |x_j(t)| + \sum_{j\neq i} |z_j(t)|\right) \leq c_2\left(\sum_{j=1}^{n} |x_j(t)| + \sum_{j\neq i} |z_j(t)|\right), i = 1, \cdots, n. \tag{30}$$

Assumption 4.3. *There are constants $c > 0$ and $\alpha \in [0, 1)$ such that for $i = 1, \cdots, n$*

$$\left|\frac{f_i(\eta_1(t), L\eta_2(t), \cdots, L^{n-1}\eta_n(t), z(t), L^n\upsilon(t))}{L^{i-1}}\right| \leq L^\alpha c\left(|\eta_1(t)| + \cdots + |\eta_n(t)|\right), \tag{31}$$

for any $L > 1$.

Based on Assumptions 4.1-4.3, the following result can be established for the NDAE system (28).

Theorem 4.1. *Consider the NDAE system (28). If Assumptions 4.1, 4.2 and 4.3 hold, then the sampled-data stabilization problem of system (28) is solvable by using a partial-state and output feedback controller.*

Proof. Due to the domination nature of the proposed design scheme, we can show this theorem easily similar to that of Theorem 3.1. In fact, Assumption 4.1 in conjunction with the same discrete-time observer (16) and controller (17), the relations (22), (23), (24), and (25) are consequently need to be revised. Similar to the proof of Theorem 3.1, it can be shown that if the parameters L and T are selected according to the following formulas

$$\theta(t) = \left(e^{\mu_1(t-t_k)} - 1\right)\frac{\mu_2}{\mu_1} < 1, \text{and } L > \max\left\{\frac{\lambda_{max}(P)(2c_0L^\alpha\|P\| + 0.5\lambda_{min}(P))(1-\theta(t)))}{\lambda_{min}(P)(1-\theta(t)) - 2\lambda_{max}(P)\|P\|(\sqrt{\|K\|} + \|\Pi\|)\theta(t)}, 1\right\}, \tag{32}$$

then under the dynamic sampled-data output feedback controller (8),(9) the state of the system (28a) will converge to the origin asymptotically, that is, $\lim_{t\to+\infty} x_i(t) = 0, i = 1, \cdots, n$.

By (30), it can be shown that

$$\sum_{j=1}^{n} |z_j(t)| \leq \frac{nc_2}{1-(n-1)c_2}\sum_{j=1} |x_j(t)|. \tag{33}$$

Based on Assumption 4.2 and the fact $\lim_{t\to+\infty} x_i(t) = 0, i = 1, \cdots, n$, we have $\lim_{t\to+\infty} \sum_{j=1} |x_j(t)| \to 0$. By inequality (33), it is obviously that $\lim_{t\to+\infty} \sum_{j=1} |z_j(t)| \to 0$, that is, $\lim_{t\to+\infty} z_i(t) = 0, i = 1, \cdots, n$.

Therefore, it can be concluded that NDAE system (28) can be asymptotically stabilized by the dynamic sampled-data output feedback controller (8),(9). □

5. Simulation examples

In this section, we use two examples to show the effectiveness of the proposed results in this paper.

Example 5.1. *Consider the following NDAE system*

$$\begin{cases} \dot{x}_1(t) = x_2(t) + 0.5cos(z_1(t))(x_1(t) + z_1(t)) \\ \dot{x}_2(t) = u(t) + 0.2(x_1^{2/3}(t)z_1^{1/3}(t) + sin(x_2(t))z_2(t)) \\ y(t) = x_1(t), \end{cases} \tag{34a}$$

$$\begin{cases} 0 = z_1(t) + \frac{1}{3}z_1^3(t) + x_1(t) \\ 0 = z_2(t) + x_1(t) + \frac{2}{3}z_1(t) + x_2(t). \end{cases} \tag{34b}$$

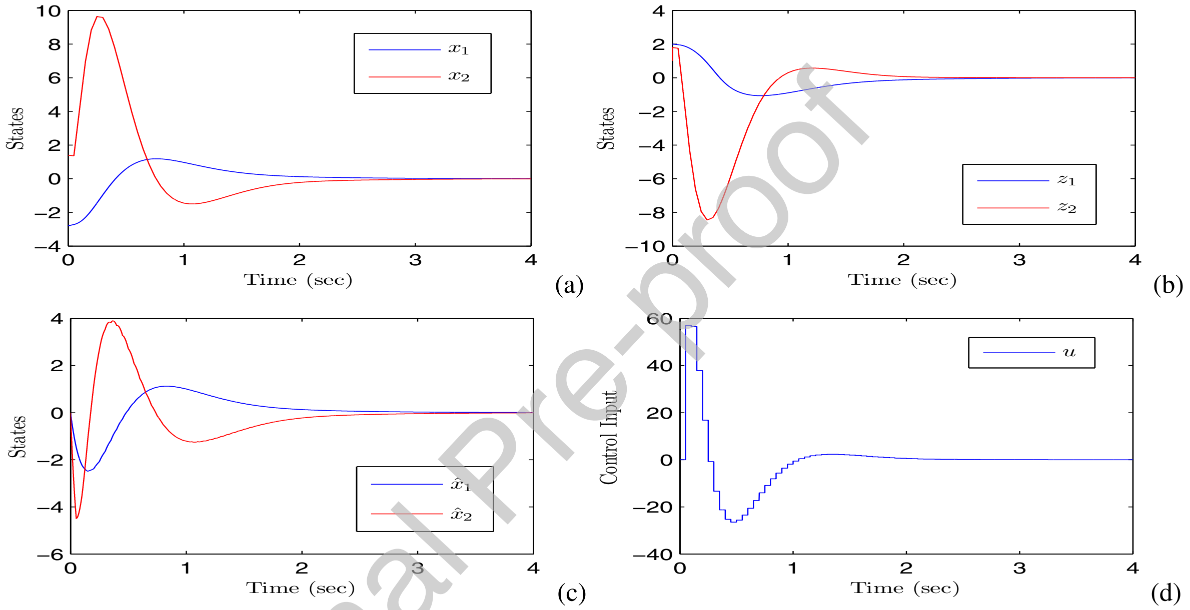


Fig.1. Response curves of the closed-loop system (34),(38) and(39).

For subsystem (34a), by Lemma2.1, it can be verified that

$$|f_1(\cdot)| \leq 0.5(|x_1(t)| + |z_1(t)|), \ \textit{and}\ |f_2(\cdot)| \leq 0.2(|x_1(t)| + |z_1(t)| + |z_2(t)|). \tag{35}$$

Furthermore, for the algebraic equations $g_1(\cdot) = 0$ and $g_2(\cdot) = 0$, it is can be easily obtained that

$$\frac{\partial g_1(z_1, x_1)}{\partial z_1} = 1 + z_1^2(t) > 0, \ \frac{\partial g_2(z_2, z_1, x_2, x_1)}{\partial z_2} = 1 > 0, \tag{36}$$

and

$$|g_1(0, x_1)| \leq |x_1(t)|, \ \textit{and}\ |g_2(0, z_1, x_2, x_1,)| \leq (|x_1(t)| + |x_2(t)| + |z_1(t)|). \tag{37}$$

Obviously, Assumptions 2.2 and 2.1 hold. Because the nonlinear term $f_2(t, x(t)) = 0.2(x_1^{2/3}(t)z_1^{1/3}(t)+sin(x_2(t))z_2(t))$ is a non-Lipschitz continuous function, and the righthand sides of the inequality in (37) contain not only $|x_1(t)|, |x_2(t)|$ but also $|z_1(t)|$ and $|z_2(t)|$, the output feedback stabilization problem can not be solved by using the existing results. However, according to Theorem 3.1, it can be shown that NDAE system (34) can be stabilized by the following sampled-data output feedback controller

$$\hat{x}(t_{k+1}) = M\hat{x}(t_k) + Ny(t_k), M \in \mathbb{R}^{2\times2}, N \in \mathbb{R}^{2\times1} \tag{38}$$

$$u(t) = u(t_k) = -L^2\left(k_1\hat{x}_1(t_k) + k_2\hat{x}_2(t_k)/L\right), \forall t \in [t_k, t_{k+1}), \tag{39}$$

with proper choice of the parameters L, k_1, k_2, a_1, a_2 and sampling period T.

The simulation is carried out by choosing

$$M = \begin{bmatrix} 0.1197 & 0.4071 \\ -0.2278 & 0.5500 \end{bmatrix}, N = \begin{bmatrix} 2.0485 \\ 2.7744 \end{bmatrix},$$

$L = 1.5, k_1 = 5, k_2 = 6, a_1 = 9, a_2 = 20$, *and the sampling period* $T = 0.05$, *and the admissible initial conditions* $(x_1(0), x_2(0), z_1(0), z_2(0), \hat{x}_1(0), \hat{x}_2(0)) = (-2.8, 1.4, 2, 1, 0, 0)$, *and the simulation results are shown in Fig.1.*

The response curves differential state variables and algebraic state variables of the closed-loop NDAE system (34), (38) and (39) are shown in Fig.1 (a) and (b), respectively. Fig.1 (c) shows the estimations of the differential state variables that generated by the observer (38). Fig.1 (d) illustrates the time history of the sampled-data control signal.

Example 5.2. *Consider nonlinear circuit system in Fig.2, where a dc source with voltage μ is connected in series to a linear resistor, a linear inductor and a nonlinear capacitor with a $q - v$ characteristic $q = z(v) = v + v^3$. Similar nonlinear capacitors are considered in [13, 21].*

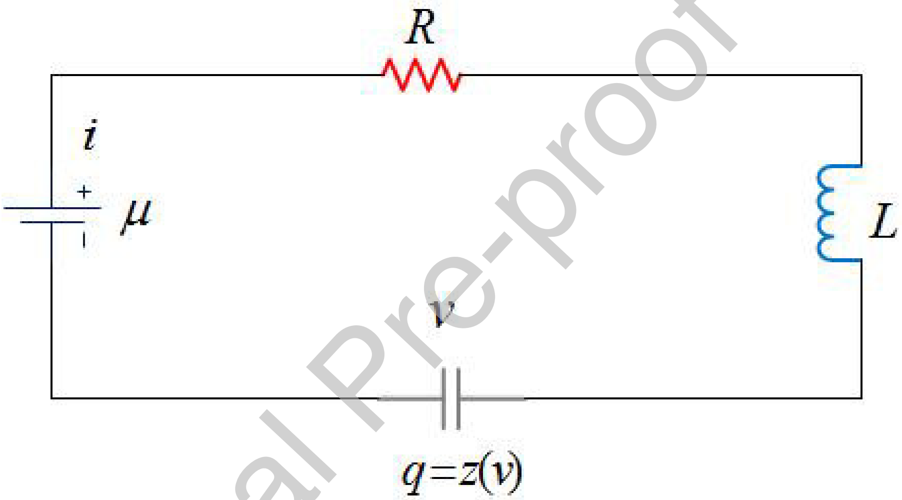


Fig.2. Nonlinear RLC circuit.

This circuit may be easily shown to be admit the charge-flux description

$$\begin{cases} \dot{q} = \frac{\phi}{L} \\ \dot{\phi} = \mu - \frac{\phi R}{L} - v \end{cases} \tag{40a}$$

$$0 = -(q - q_0) + (v - v_0) + (v - v_0)^3. \tag{40b}$$

where ϕ is the magnetic flux in the inductor, q_0 and v_0 are known constants. Obviously, system (40) is a NDAE system with differential variables (q, ϕ) and algebraic variable v.

Define $x = [x_1, x_2]^T = [q - q_0, \phi]^T$, $z_1 = v - v_0$, and $u = \mu - v_0$, let $\boldsymbol{L} = 1$ and $\boldsymbol{R} = 1.5$, and assume that q could be the measured, then the regulation problem system (40) is converted to the output feedback stabilization problem of the following system

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = u(t) - 1.5x_2(t) - z_1(t) \end{cases} \tag{41a}$$

$$0 = -x_1(t) + z_1(t) + z_1^3(t). \tag{41b}$$

$$y(t) = x_1(t) \tag{41c}$$

NDAE system (41) only has one algebraic equation, thus Assumptions 2.1 and 2.2 are not satisfied. However, it can be verified easily that Assumptions 2.1^* *and* 2.2^* *hold. Thus, the sampled-data output feedback stabilization* *problem of NDAE system (41) can also be solved by the obtained results in this paper. According to Corollary 3.1, NDAE system (41) can be stabilized by the following sampled-data output feedback controller*

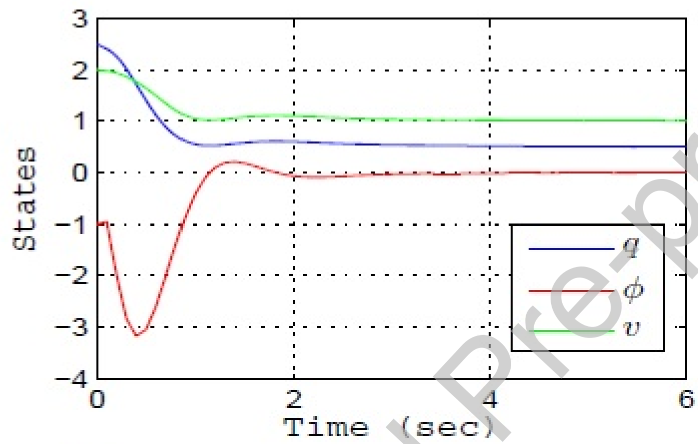
$$\begin{aligned} &\hat{x}(t_{k+1}) = M\hat{x}(t_k) + Ny(t_k), M \in \mathbb{R}^{2\times 2}, N \in \mathbb{R}^{2\times 1} \\ &\mu(t) = \mu(t_k) = u(t_k) + v_0 = -L^2(k_1\hat{x}_1(t_k) + k_2\hat{x}_2(t_k)/L) + v_0, \forall t \in [t_k, t_{k+1}), \end{aligned} \tag{42}$$

by choosing appropriate parameters L, k_1, k_2, a_1, a_2 and sampling period T.

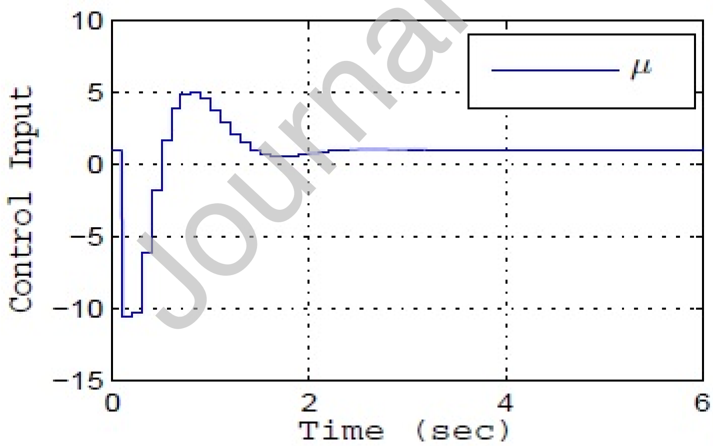
The simulation is conducted by choosing

$$M = \begin{bmatrix} 0.2447 & 0.5588 \\ -0.1503 & 0.7000 \end{bmatrix}, N = \begin{bmatrix} 3.8301 \\ 4.9276 \end{bmatrix},$$

$L = 1.5, a_1 = 6, a_2 = 10, k_1 = 1, k_2 = 2$, *$(q(0), \phi(0), v(0), \hat{x}_1(0), \hat{x}_2(0)) = (2.5, -1, 2, 0, 0)$, and the sampling period $T = 0.1$. The simulation results are shown in Fig.3.*



(a)



(b)

Fig.3. Response curves of system (41) under the dynamic sampled-data output feedback controller (42).

6. Conclusions

In this paper, we have proposed a sampled-data stabilization approach for a class of NDAE systems by partial-state and output feedback. Combining the output feedback domination approach [23] and emulation technique [34, 37, 38], a sampled-data output feedback controller is explicitly designed with a tunable scaling gain and tunable sampling period based on the nominal linear system of the differential subsystem firstly. Then, the scaling gain is chosen to dominate the unknown nonlinear terms of the differential subsystem and the sampling period is selected to ensure the asymptotic stability of the closed-loop system. The proposed dynamic sampled-data output feedback controller is composed by a linear observer and a linear controller, and no information of the algebraic subsystem is involved, thus it can be easily implemented.

Appendix A

The proof of the Propositions 3.1 and 3.2 is given in this section.

The proof the Proposition 3.1:

Based on subsystem (1b), Assumption 2.2 and the inequality (7), we have for $j = 1, \cdots, i$

$$|z_j| \le c_2 \left(|x_1| + \cdots + |x_j| + |z_1| + \cdots + |z_{j-1}|\right). \tag{A.1}$$

By using (A.1) and Assumptions 2.1, it follows from (1b) that

$$\begin{aligned}
|f_i| &\le c_1 \left(|x_1| + \cdots + |x_i| + |z_1| + |z_2| + \cdots + |z_{i-1}| + c_2 \left(|x_1| + \cdots + |x_i| + |z_1| + \cdots + |z_{i-1}|\right)\right) \\
&= c_1(1 + c_2)\left(|x_1| + \cdots + |x_i|\right) + c_1(1 + c_2)\left(|z_1| + |z_2| + \cdots + |z_{i-1}|\right) \\
&\le c_1(1 + c_2)\left(|x_1| + \cdots + |x_i|\right) + c_1(1 + c_2)\left(|z_1| + |z_2| + \cdots + |z_{i-2}| + c_2\left(|x_1| + \cdots + |x_{i-1}| + |z_1| + \cdots + |z_{i-2}|\right)\right) \\
&\le c_1(1 + c_2)^2\left(|x_1| + \cdots + |x_i|\right) + c_1(1 + c_2)^2\left(|z_1| + |z_2| + \cdots + |z_{i-2}|\right) \\
&\vdots \\
&\le c_1(1 + c_2)^i\left(|x_1| + \cdots + |x_i|\right) \\
&\le c\left(|x_1| + \cdots + |x_i|\right)
\end{aligned} \tag{A.2}$$

where $c = c_1(1 + c_2)^n$. This complete the proof of Proposition 3.1.

The proof the Proposition 3.2:

Based on system (18) and equation (22), we have

$$\|\dot{\overline{\eta}}(t)\| \le L(\|A\| + \|\hat{A}\| + c_0)\|\overline{\eta}(t)\| + L\left(\sqrt{2}\|K\| + \|\Pi\|\right)\|\overline{\eta}(t_k)\|, \forall t \in [t_k, t_{k+1}). \tag{A.3}$$

Integrating (18) on both sides from t_k to t leads to

$$\|\overline{\eta}(t) - \overline{\eta}(t_k)\| \le \int_{t_k}^{t} \|\dot{\overline{\eta}}(s)\| ds \le \int_{t_k}^{t} (\mu_1\|\overline{\eta}(s) - \overline{\eta}(t_k)\| + \mu_2\|\overline{\eta}(t_k)\|) ds, \forall t \ge t_k, \tag{A.4}$$

where $\mu_1 = L(\|A\| + \|\hat{A}\| + c_0)$ and $\mu_2 = L(\|A\| + \|\hat{A}\| + c_0 + \sqrt{2}\|K\| + \|\Pi\|)$.

Let $\beta(t) = \int_{t_k}^{t}(\mu_1\|\overline{\eta}(s) - \overline{\eta}(t_k)\| + \mu_2\|\overline{\eta}(t_k)\|)ds$. According to (A.4), it can be shown that

$$\begin{aligned}
\dot{\beta}(t) &\le \mu_1\|\overline{\eta}(t) - \overline{\eta}(t_k)\| + \mu_2\|\overline{\eta}(t_k)\|) \\
&\le \mu_1\beta(t) + \mu_2\|\overline{\eta}(t_k)\|, \forall t \ge t_k.
\end{aligned} \tag{A.5}$$

It can be proved that

$$\begin{aligned}
\beta(t) &\le e^{\mu_1(t-t_k)} \int_{t_k}^{t} e^{-\mu_1(s-t_k)}\mu_2\|\overline{\eta}(t_k)\| ds \\
&\le \left(e^{\mu_1(t-t_k)} - 1\right)\frac{\mu_2}{\mu_1}\|\overline{\eta}(t_k)\| \\
&\le \theta(t)\left(\|\overline{\eta}(t) - \overline{\eta}(t_k)\| + \|\overline{\eta}(t)\|\right)
\end{aligned} \tag{A.6}$$

where $\theta(t) = \left(e^{\mu_1(t-t_k)} - 1\right)\frac{\mu_2}{\mu_1}$.

If $0 < \theta(t) < 1$, then it follows from (A.4) and (A.6) that

$$\|\overline{\eta}(t) - \overline{\eta}(t_k)\| \le \frac{\theta(t)}{1 - \theta(t)}\|\overline{\eta}(t)\|, \forall t \in [t_k, t_{k+1}). \tag{A.7}$$

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Conflict of interest statement

The authors declared that they have no conflicts of interest to this work.

We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

Qixun Lan

Henan University of Urban Construction

Liying Sun

Shanghai Dianji University

Jiarong Liang

Guangxi University