Stability Analysis and Synthesis for Linear Positive Systems with Time-Varying Delays

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Abstract. This paper provides necessary and sufficient conditions for the asymptotic stability of linear positive systems subject to time-varying delays. It introduces and initiates an original method for solving directly the proposed stability and stabilization problems without using the well-known Lyapunov theory that is commonly used in the field of stability analysis. In that way and for readers convenience, the paper avoids possible long and tedious superfluous calculus.

1 Introduction

The reaction of real world systems to exogenous signals is never instantaneous and always infected by certain time delays. Differential delay systems known also as hereditary or systems with aftereffects, represent a class of infinite-dimensional systems that can model and take into account the delay influence on wide range of systems such as propagation phenomena, population dynamics and many physical, biological and chemical processes.

The study of the delay effects on the stability and control of dynamical systems (delays in the state and/or in the input) are problems of a great interest in practice. For general linear systems, even nominal stable systems when are affected by delays, may inherit very complex behaviors such as oscillations, instability and bad performances. In addition, it is well-know that small constant delays may destabilize some systems, while large constant delays may stabilize others. Note that the effect of time-varying delays still not well understood for general linear systems.

In contrast, for *positive* linear time-delay systems (systems whose state variables take only nonnegative values are referred to be positive, see [11, 12, 19, 22] for general references), it has been shown in the very beginning of the $80^{\prime s}$ that the

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presence of constant delays does not affect the stability performance of the system [20, 21, 23] (see also recent works [6, 7, 13, 16, 17]). Since then, no one has conjectured that this fact holds true for time-varying delays. In this paper, we go a step further and show that remarkable fact, that is, *the stability performance of* positive *linear time-delay systems is insensitive to any kind of time-varying delays*.

The aim of this paper is to present a new method and techniques for the stability analysis and synthesis of linear positive in presence of time-varying delays. The proposed approach for the stability analysis is quite new and does not use any based Lyapunov technic. This paper develops theoretical results with necessary and sufficient condition for stability and stabilizability of linear positive delayed systems. Specifically, we will show that the stabilization problem can be cast either as an LP problem or as an LMI problem. Since there exist powerful LP softwares (as Cplex) that can solve efficiently very large size problems, we believe that the LP approach is more simple and can have a legitimate numerical advantage in comparison to the LMI approach. We stress out that the proposed LP formulation was introduced by the author in the context of positive observation of delayed systems [6, 7] and interval observers [5]. An old LP formulation has been introduced earlier by the author in [1–4] for dealing with positive observers and positive systems with state and control constraints. It has been adapted for positive system with constant delays [16, 17] and for positive 2D-system [8, 15].

The remainder of the paper is organized as follows. In section 2 some preliminary facts and results are given. Section 3 provides necessary and sufficient conditions for the stability of positive linear systems with time-varying delays. Section 4 solves the stabilization problem for standard state-feedback controls and also for nonnegative state-feedback controls. Finally, section 5 gives some conclusions.

Notations

Re^n_+ denotes the non-negative orthant of the n-dimensional real space Re^n. M^T denotes the transpose of the real matrix M. For a real matrix M, $M > 0$ means that its components are positive: $M_{ij} > 0$, and $M \geq 0$ means that its components are nonnegative: $M_{ij} \geq 0$. $\mathbf{diag}(\lambda)$ is the diagonal matrix whose diagonal is formed by the components of the vector λ. $M \succ 0$, where M is a symmetric real matrix, means that M is definite positive.

2 Statements and Preliminaries

This section provides some necessary preliminary statements and technical keys that are primordial for the characterization and the treatment of positive systems satisfying a differential delayed equation. The introduced facts and results will be essentially used in development and the derivation of our main stability result.

The system under investigation is described by a general forced linear differential delayed equation.

$$\frac{dx}{dt} = Ax + \sum_{i=1}^{m} A_i x(t - \tau_i(t)) + Bu(t), \tag{1}$$

the given matrices $A, A_1, \ldots, A_m \in \mathrm{Re}^{n\times n}$ and $B \in \mathrm{Re}^{n_u \times n}$ are time-invariant and $0 \le \tau_1(\cdot), \cdots, 0 \le \tau_m(\cdot)$ are time-varying delays that are supposed to be Lebesgue measurable. Throughout the paper we use the notation

$$\tau := \max_{1\le i\le m} \sup_{t\ge 0} \tau_i(t).$$

The vector $x(t) \in \mathrm{Re}^n$ is the instantaneous system state at time t and $u(t) \in \mathrm{Re}^{n_u}$ represents an external input. The whole state at time t of system (1) is infinite dimensional which is given by the set $\{x(s)| -\tau \le s \le t\}$.

Following [14], it can be shown that the solution to the system's equation (1) exists, unique and totally determined by any given initial Locally Lebesgue integrable vector function $\phi(\cdot)$ such that

$$x(s) = \phi(s) \text{ for } -\tau \le s \le 0.$$

Throughout this paper, the free system is assumed to satisfy a positivity constraint on its states as follows.

Definition 1. For any nonnegative initial condition $\phi(t) \in \mathrm{Re}^n_+$ such that $x(t) = \phi(t)$ for $-\tau \le t \le 0$, System (1) is said to be positive if the corresponding trajectory is nonnegative, that is $x(t) \in \mathrm{Re}^n_+$ for all $t \ge 0$.

We stress out that that intrinsic properties of the delayed system's positivity behavior are related to Metzlerian matrices and positive matrices.

Definition 2. A real matrix M is called a *Metzler* matrix if its off-diagonal elements are nonnegative: $M_{ij} \ge 0, \ i \ne j$.

Definition 3. A real matrix M is called a *positive* matrix if all its elements are nonnegative: $M_{ij} \ge 0$.

Note that the following result shows how Metzlerian matrices are intrinsically connected to positivity.

Lemma 1. *Let M be a Metzler matrix then the following holds true.*
(a) M Metzler $\Leftrightarrow e^{tM} \ge 0, \ \forall t \ge 0$.
(b) if $v > 0$, *then* $e^{tM} v > 0, \forall t \ge 0$.

Proof. Item (a) is well-know [22]. Item (b) is trivial. □

We emphasize that the following result can be interpreted as an extension of the classical result on positive linear systems (see [22]) and its proof can be obtained in the same spirit of reasoning and then omitted. Also, this result offers an easy test for checking the positivity of the free system.

Proposition 1. *System (1) with* $u = 0$ *is positive if and only if A is a Metzler matrix and* $A_1, \ldots, A_m$ *are positive matrices.*

3 Main Result

In this section, the stability of autonomous linear positive systems subject to time-varying delays is studied. The relevant derived result involves necessary and sufficient conditions.

Previous results and equivalent conditions for a Metzler matrix to be Hurwitz can be found in many places in the literature, see for example [4, 10, 18]. In the following, some well-known facts are presented and will be used in order to derive our main result.

Lemma 2. *Let M be a Metzler matrix. Then, the following conditions are equivalent*

i) M is Hurwitz (has eigenvalues with negative real part).
ii) The inverse of M exists and all its components are negative: $M^{-1} \leq 0$.
iii) There exist a vector $\lambda > 0$ such that $M\lambda < 0$.
iv) There exist a diagonal matrix $D \succ 0$ such that $M^T D + DM \prec 0$.

In the sequel, conditions for the asymptotic stability of the general positive linear time-delay system (1) are derived. But, before we need the following technical lemma.

Lemma 3. *Let $A, A_1, \ldots, A_m \in \mathrm{Re}^{n\times n}$ be constant matrices and assume that the matrix A is Metzler and $A_1, \ldots, A_m$ are nonnegative. Consider the following delayed linear system with constant delay τ.*

$$\begin{aligned} \frac{dz}{dt} &= Az(t) + \sum_{i=1}^{m} A_i z(t-\tau), \\ z(s) &= \lambda, \ \text{for} \ -\tau \leq s \leq 0. \end{aligned} \tag{2}$$

Then, if $(A + \sum_{i=1}^{m} A_i)\lambda < 0$ and $\lambda > 0$, we have that z(t) is strictly decreasing for $t \geq 0$. Moreover, $z(t)$ converges asymptotically to zero. That is $\dot{z}(t) < 0$, $\forall t > 0$ and $\lim_{t\to 0} z(t) = 0$.

Proof. Since A is Metzler and $A_1, \ldots, A_m$ are nonnegative, the minus derivative $-\dot{z}$ of the solution to system (2) satisfies a linear delayed positive system equation and it can be easily shown that $-\dot{z}(t) > 0$, $\forall t \geq 0$. Also, because that $(A + \sum_{i=1}^{m} A_i)\lambda < 0$ and $\lambda > 0$ we have that $z(t)$ converges asymptotically to zero (see for this fact [6, 13] and by this our claim is proved. $\square$

Theorem 1. *Assume that system (1) is positive, or equivalently that the matrix A is Metzler and $A_1, \ldots, A_m$ are positive matrices. Then, the following statements are equivalent.*

i) There exist a constant-time delays $\tau_1^*, \ldots, \tau_m^*$ *and a nonnegative initial functional condition* $\phi^*(\cdot)$ *with* $\phi^*(0) > 0$ *for which the free system (1) (u=0) is asymptotically stable.*
ii) System (1) is asymptotically stable for every nonnegative initial condition $\phi(\cdot) \geq 0$ *and for any bounded arbitrary time-varying delays.*
iii) System (1) is asymptotically stable for every initial condition taking values in Re^n *(*$\phi(\cdot)$ *has indefinite sign) and for any bounded arbitrary time-varying delays.*
iv) There exists $\lambda \in \mathrm{Re}^n$ *such that*

$$(A + \sum_{i=1}^{m} A_i)\lambda < 0, \ \ \lambda > 0. \tag{3}$$

Proof. The implications $(iii) \Rightarrow (ii) \Rightarrow (i)$ are obvious. The rest of the proof will be proceeded in 3 steps. We emphasize that the second step is the more delicate and subtle part of the proposed proof.
$\underline{Step\,1 : (i) \Rightarrow (iv)}$ By integrating System (1) we have

$$x(t) - x(0) = A \int_0^T x(t)dt + \sum_{i=1}^{m} A_i \int_0^T x(t - \tau_i^*)dt,$$

which by change of variable can be expressed as the following identity

$$\begin{aligned}&(A + \sum_{i=1}^{m} A_i) \int_0^T x(t)dt = \\ &x(T) + \sum_{i=1}^{m} A_i \int_T^{T-\tau_i^*} x(t)dt - \sum_{i=1}^{m} A_i \int_{-\tau_i^*}^{0} \phi^*(t)dt - x(0),\end{aligned}$$

since $x(T)$ goes to zeros, then also $\sum_{i=1}^{m} A_i \int_T^{T-\tau_i^*} x(t)dt$. Moreover, since ϕ^* is nonnegative and $x(0)$ is positive, the term $\sum_{i=1}^{m} A_i \int_{-\tau_i^*}^{0} \phi^*(t)dt + x(0)$ is constant and positive. Thus, regarding to these facts, it suffices to select a sufficiently large T to get

$$(A + \sum_{i=1}^{m} A_i)\lambda < 0, \ \ \lambda > 0,$$

where λ is defined as $\lambda = \int_0^T x(t)dt$ which is positive due to the fact that $x(0)$ is positive and the trajectory $x(t)$ is continuous.

$\underline{Step\,2 : (iv) \Rightarrow (ii)}$ Let $\phi(\cdot) \geq 0$ be any initial functional condition and consider its associated trajectory $x(\cdot)$. Now, take any vector $\lambda > 0$ satisfying $(A + \sum_{i=1}^{m} A_i)\lambda < 0$. Of course, there exist a positive constant scalar $\alpha > 0$ such that

$$\alpha\phi(s) < \lambda, \ \ \forall s : -\tau \leq s \leq 0.$$

Note that by linearity the associated trajectory to $\alpha\phi(\cdot)$ is $\alpha x(\cdot)$, so that the scaled trajectory is solution to

$$\alpha x(t) = e^{tA}\alpha x(0) + \int_0^t e^{(t-s)A}\sum_{i=1}^m A_i \alpha x(s-\tau_i(s))ds. \tag{4}$$

Now, consider the following delayed linear system with constant delay τ, such that $\tau \geq \max_{1\leq i\leq m} \sup_{t\geq 0} \tau_i(t)$.

$$\begin{aligned} \frac{dz}{dt} &= Az(t) + \sum_{i=1}^m A_i z(t-\tau), \\ z(s) &= \lambda, \ \text{ for } -\tau \leq s \leq 0. \end{aligned} \tag{5}$$

Next, we claim that

$$\alpha x(t) < z(t), \ \forall t \geq 0.$$

If this fact does not hold, let t^* be the maximal time such that there exist at least a component $x_i(t^*)$ of $x(t^*)$ such that $\alpha x_i(t^*) \geq z_i(t^*)$ and $\alpha x(s) < z(s), \ \forall s : -\tau \leq s < t^*$.

Based on the integral expression (4), we are going to perform a comparison at time t^* by using the fact that $e^{t^*A}(z(0) - \alpha x(0)) > 0$ (since $z(0) > \alpha x(0)$) and also $e^{(t^*-s)A} \geq 0$ if $t^* \geq s$ (apply Lemma 1). Thus, since the matrices $A_1, \ldots, A_m$ are positive (and do not forget that $\alpha x(s) < z(s), \ \forall s : -\tau \leq s < t^*$), we obtain

$$\alpha x(t^*) < e^{t^*A}z(0) + \int_0^{t^*} e^{(t^*-s)A}\sum_{i=1}^m A_i z(s-\tau)ds. \tag{6}$$

At this moment, one can wonder why this holds true? To give a positive answer we use Lemma 3 that asserts that $z(t)$ is strictly decreasing, then from this fact, we can of course see that

$$\alpha x(t-\tau_i) < z(t-\tau_i(t)) \leq z(t-\tau) \ \ \forall t : -\tau \leq t < t^*.$$

Multiplying $\alpha x(t-\tau_i(t)) - z(t-\tau) \leq 0$ by $e^{(t^*-t)A}A_i \geq 0$, integrating and summing from 1 up to m, we obtain

$$\int_0^{t^*} e^{(t^*-s)A}\sum_{i=1}^m A_i x(s-\tau_i(s))ds \leq \int_0^{t^*} e^{(t^*-s)A}\sum_{i=1}^m A_i z(s-\tau)ds,$$

so that by keeping in mind the strict inequality $e^{t^*A}z(0) > \alpha e^{t^*A}x(0)$, now we can be sure that inequality (6) occurs at time t^*.

At this stage, we give the crucial conclusion. The right hand side of the claimed inequality (6) is nothing else than $z(t)$. Consequently, we got $\alpha x(t^*) < z(t^*)$, which turn out to contradict the fact that there is a component such that $\alpha x_i(t^*) \geq z_i(t^*)$, and we are almost done. Because, we have now $0 \leq \alpha x(t) < z(t), \ \forall t$ and $z(t)$ goes to zero (since $A + \sum_{i=1}^m A_i$ is Hurwitz see for this Lemma 3). Henceforth, we have shown

that system (1) is asymptotically stable for every initial functional condition $\phi(\cdot)$ taking values in Re_+^n.
<u>*Step* 3 : $(ii) \Rightarrow (iii)$</u> This implication results from the linearity of the system and the fact that ϕ can be decomposed as $\phi = \phi^+ - \phi^-$ where $\phi^+ \geq 0, \phi^- \geq 0$. So that the proof is complete. □

Corollary 1. *Assume that system (1) is positive, or equivalently that the matrix A is Metzler and $A_1, \ldots, A_m$ are positive matrices. Then, the following statements are equivalent.*

i) There exist a constant-time delays $\tau_1^*, \ldots, \tau_m^*$ and a nonnegative initial functional condition $\phi^*(\cdot)$ with $\phi^*(0) > 0$ for which the free system (1) (u=0) is asymptotically stable.
ii) system (1) is asymptotically stable for every initial condition and for any arbitrary bounded time-varying delays.
iii)The inverse of $A + \sum_{i=1}^{m} A_i$ exists and all its components are negative

$$(A + \sum_{i=1}^{m} A_i)^{-1} \leq 0.$$

iv) There exist a vector λ such that $(A + \sum_{i=1}^{m} A_i)\lambda < 0, \;\; \lambda > 0$.
v) There exist a diagonal matrix D such that

$$(A + \sum_{i=1}^{m} A_i)D + D(A + \sum_{i=1}^{m} A_i)^T \prec 0, \;\; D \succ 0.$$

v) $A + \sum_{i=1}^{m} A_i$ is a Hurwitz matrix.

Proof. It suffices to apply Theorem 1 and Proposition 2. □

4 Controllers Design

The aim of this section is to show how our stability result can be applied in order to compute stabilizing feedback controllers. In particular, those control laws that take only nonnegative values will be considered owing to their importance in practice.

The following result provides necessary and sufficient conditions for the existence of stabilizing control law that preserves the positivity of the system. It also provides an easy and efficient approach for checking the solvability of the stabilization problem and for computing any stabilizing state-feedback control by either using LP or LMI softwares.

Theorem 2. *Assume that $A_1,\dots,A_m$ are positive matrices. Then, the following statements are equivalent*

i) There exists a stabilizing memoryless state-feedback law $u(t) = Kx(t)$ such that the resulting closed-loop system (1) is positive and asymptotically stable for arbitrary bounded time-varying delays.

ii) There exists a matrix $K \in \mathrm{Re}^{n_u \times n}$ such that $A + BK$ is Metzler matrix and $A + BK + \sum_{i=1}^{m} A_i$ is a Hurwitz matrix.

iii) The following LP problem in the variables $\lambda \in \mathrm{Re}^n$ and $Z \in \mathrm{Re}^{n_u \times n}$ is feasible

$$\begin{cases} (A + \sum_{i=1}^{m} A_i)\lambda + BZ \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} < 0, \\ A\mathbf{diag}(\lambda) + BZ + I \geq 0, \\ \lambda > 0, \end{cases} \tag{7}$$

Moreover, a gain matrix K satisfying the conditions (i) and (ii) can be computed as follows

$$K = Z\mathbf{diag}(\lambda)^{-1},$$

where the vector λ and the matrix Z are any feasible solution to the above LP problem.

iv) The following LMI problem in the variables $D \in \mathrm{Re}^{n \times n}$ and $Y \in \mathrm{Re}^{n_u \times n}$ is feasible

$$\begin{cases} (A + \sum_{i=1}^{m} A_i)D + D(A^T + \sum_{i=1}^{m} A_i^T) + BY + Y^T B^T \prec 0, \\ AD + BY + I \geq 0, \\ D \succ 0. \end{cases} \tag{8}$$

Moreover, a gain matrix K satisfying the conditions (i) and (ii) can be computed as follows

$$K = YD^{-1},$$

where the matrices D and Z are any feasible solution to the above LMI problem.

Proof. The equivalence between $i)$ and $ii)$ is straightforward from Theorem 1. Now let us show that $ii)$ and $iii)$ are equivalent. First, consider the implication $ii) \rightarrow iii)$. Note that since $A + BK$ is Metzler and $A_1,\dots,A_m$ are positive matrices then $A + BK + \sum_{i=1}^{m} A_i$ is Metzler. So that by using Corollary 1, we have $A + BK + \sum_{i=1}^{m} A_i$ is Hurwitz if and only if there exists a vector $\lambda > 0$ such that

$$(A + BK + \sum_{i=1}^{m} A_i)\lambda < 0.$$

Now, define $K = Z\mathbf{diag}(\lambda)^{-1}$. Thus, with this change of variable, the above inequality is effectively the first inequality in condition *iii*). The second inequality in the LP constraints, is obtained as follows. Note that $A + BK$ is Metzler if and only if $(A+BK)\mathbf{diag}(\lambda)$ is Metzler, or equivalently (by adding the identity matrix I)

$$(A+BK)\mathbf{diag}(\lambda) + I \geq 0,$$

this holds true by choosing λ with sufficiently small components (since the stability condition is homogeneous in λ). Thus, by recalling that $K = Z\mathbf{diag}(\lambda)^{-1}$, the above inequality is nothing else than the second inequality $A^T\mathbf{diag}(\lambda) + BZ + I \geq 0$, in the LP constraints.
The reverse implication *iii*) $\rightarrow$ *ii*) can be trivially obtained by a simple matrix manipulation as shown above. Also, to show the equivalence between *ii*) and *iv*), it suffices to use the LMI condition given by Corollary 1, make the change of variable $K = YD^{-1}$ and follow the same line of argument as for the LP formulation. Thus, the proof is complete. $\square$

Now, the following result provides necessary and sufficient conditions for the existence of a stabilizing nonnegative control law that preserves the positivity of the system.

Theorem 3. *The following statements are equivalent*

i) There exists a stabilizing nonnegative memoryless state-feedback law $u(t) = Kx(t) \geq 0$ *such that the resulting closed-loop system (1) is positive and asymptotically stable for arbitrary time varying delays.*
ii)There exists a matrix $K \in \mathrm{Re}^{n_u \times n}$ *such that* $K \geq 0$, $A + BK$ *is Metzler matrix and* $A + BK + \sum_{i=1}^{m} A_i$ *is a Hurwitz matrix.*
iii)The following LP problem in the variables $\lambda \in \mathrm{Re}^n$ *and* $Z \in \mathrm{Re}^{n_u \times n}$ *is feasible*

$$\begin{cases} (A + \sum_{i=1}^{m} A_i)\lambda + BZ \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} < 0, \\ A\mathbf{diag}(\lambda) + BZ + I \geq 0, \\ Z \geq 0, \\ \lambda > 0. \end{cases} \tag{9}$$

Moreover, a gain matrix K *satisfying the conditions* (i) *and* (ii) *can be computed as follows*

$$K = \mathrm{Z}\mathbf{diag}(\lambda)^{-1},$$

where the vector λ *and the matrix* Z *are any feasible solution to the above LP problem.*

iv) The following LMI problem in the variables $D \in \mathrm{Re}^{n\times n}$ and $Y \in \mathrm{Re}^{n_u \times n}$ is feasible

$$\begin{cases} (A+\sum_{i=1}^{m} A_i)D + D(A^T + \sum_{i=1}^{m} A_i^T) + BY + Y^T B^T \prec 0, \\ AD + BY + I \geq 0, \\ Y \geq 0 \\ D \succ 0. \end{cases} \tag{10}$$

Moreover, a gain matrix K satisfying the conditions (i) and (ii) can be computed as follows

$$K = YD^{-1},$$

where the matrices D and Z are any feasible solution to the above LMI problem.

Proof. It is easy to see that $u(t) \geq 0$ is equivalent to the positivity of its gain $K \geq 0$. The rest of the proof mimics that one of Theorem 2, so that it is omitted. □

5 Conclusions

We have provided necessary and sufficient conditions for the asymptotic stability of linear positive systems subject to time-varying delays. We have introduced an original method for solving directly the proposed stability and stabilization problems. In addition to developing theoretical results, all the proposed conditions are necessary and sufficient, which turn out to be solvable in terms of LP or LMI.

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