REGULARIZABILITY OF LINEAR DESCRIPTOR SYSTEMS VIA OUTPUT PLUS PARTIAL STATE DERIVATIVE FEEDBACK

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ABSTRACT

Regularizability of a linear descriptor system via output plus partial state derivative feedback is studied. Necessary and sufficient conditions are obtained, which are only dependent upon the open-loop coefficient matrices. It is also shown that under these necessary and sufficient conditions, "almost all" output plus partial state derivative feedback controllers can regularize a regularizable linear descriptor system. The proposed conditions generalize many existing results. The presented example demonstrates the proposed results.

KeyWords: Linear descriptor systems, output plus partial state derivative feedback, regularizability.

I. INTRODUCTION

Regularity is an important property for descriptor linear systems. It guarantees the existence and the uniqueness of the solution of a descriptor linear system [1]. Quite a few reported results for descriptor linear systems have assumed open-loop regularity, see, for example, [2-4] and the bibliographies therein. However, this assumption is unnecessarily strong since it limits the analysis of a number of practical physical systems ([3,5]). Due to this practical reason, the problem of regularizing descriptor systems using various feedbacks has attracted much attention (see [6-21]).

Regularization of descriptor linear systems has been studied by many authors. References [5] and [8] treated the state feedback case and presented various necessary and sufficient conditions for the problem, while [7] and [9] concentrated on the proportional plus state derivative feedback case. In [10] and [11], regularization of descriptor linear systems via proportional output feedback was considered, and again necessary and sufficient conditions were given. In 1999, Chu and Ho [12], considered regularization of a descriptor linear system using proportional plus derivative output feedback and established necessary and sufficient conditions.

Besides giving necessary and sufficient conditions for the regularization problem, many researchers have also developed numerical algorithms for finding the regularizing feedback controllers which guarantee certain desired characteristics of the closed-loop systems (see [6,12-21]). One type of such problems is to design a feedback controller for a descriptor linear system such that the closed-loop system is regular and has index at most one (see [6,12-15,20]). For this type of problems, reference [6] considered the state feedback case, while [14,15] and [20] concentrated on the output feedback case. In references [12] and [14], the more general proportional plus derivative output feedback controllers were adopted. Different from [14], reference [12] considered the problem of designing a feedback controller for a descriptor linear system such that the closed-loop system is regular, has index at most one, and moreover, possesses a desired dynamical order. Extension of this work to the case of output plus partial state derivative feedback control was presented in [13]. Besides the above, the problem of designing a feedback controller for a descriptor linear system such that the closed-loop system is regular and impulse-free ([18,19] and [21]), or regular and strongly controllable and strongly observable ([15] and [20]), have also been investigated. Reference [21] focuses on the existence of a decentralized output

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feedback control law or a P-D decentralized output feedback control law for a descriptor linear system such that the closed-loop system is regular and impulse-free, and at the same time, possesses a maximal dynamical order. It should be pointed out that the regularization problem of linear descriptor systems with variable coefficients has also been studied (see [22] and [23]).

Although regularization of descriptor linear systems has been researched by many authors as stated above, the problem of seeking an output plus partial state derivative feedback for a descriptor linear system such that the closed-loop system is regular has not been investigated. The purpose of this paper is to present necessary and sufficient conditions for this problem, and meanwhile, to consider the solution of such controllers. Several simple necessary and sufficient conditions for the proposed regularizability problem are established, which are dependent only on the open-loop system coefficient matrices. Regarding solutions of the regularizing output plus partial state derivative feedback, it is shown based on the concept of robust set (Zariski open set) that for a descriptor linear system which is regularizable via output plus partial state derivative feedback, "almost all" output plus partial state derivative feedback controllers can regularize the system. This indicates the important fact that such a regularizing controller can be easily obtained by a trial and test procedure.

This paper is divided into 6 sections. The next section formulates the problem to be addressed in the paper. Section 3 states some preliminary results. Some necessary and sufficient conditions for regularizability of descriptor linear systems via output plus partial state derivative feedback are presented in Section 4. Section 5 shows that for a descriptor linear system which is regularizable via output plus partial state derivative feedback, "almost all" output plus partial state derivative feedback controllers can regularize the system. An illustrative example is worked out in Section 6.

II. PROBLEM FORMULATION

Consider the following linear, time invariant descriptor system

$$\begin{cases} E\dot{x} = Ax + Bu \\ y_p = C_p x \\ y_d = C_d \dot{x} \end{cases} \tag{1}$$

where $x \in \mathbf{R}^n$, $u \in \mathbf{R}^r$, $y_p \in \mathbf{R}^m$ and $y_d \in \mathbf{R}^q$ are, respectively, the state-variable vector, the input vector, the proportional output vector and the derivative output vector, and $A, E \in \mathbf{R}^{n\times n}$, $B \in \mathbf{R}^{n\times r}$, $C_p \in \mathbf{R}^{m\times n}$ and $C_d \in \mathbf{R}^{q\times n}$ are known matrices. Without loss of generality, we make the following assumption.

Assumption A1. $\operatorname{rank}B = r$, $\operatorname{rank}C_p = m$, $\operatorname{rank}C_d = q$.

System (1) is said to be regular if (E, A) is a regular pair (see [14]), i.e.,

$$\det(\alpha E - \beta A) \neq 0, \qquad \text{for some } (\alpha, \beta) \in \mathbf{C}^2. \tag{2}$$

When an *output plus partial state derivative feedback controller* of the form

$$u = K_p y_p + K_d y_d = K_p C_p x + K_d C_d \dot{x} \tag{3}$$

is applied to system (1), the closed-loop system is obtained as

$$(E - BK_d C_d)\dot{x} = (A + BK_p C_p)x. \tag{4}$$

Definition 1. The system (1) is said to be regularizable via output plus partial state derivative feedback if the resulted closed-loop system (4) or the matrix pair $(E - BK_dC_d, A + BK_pC_p)$ is regular for some matrices $K_d \in \mathbf{R}^{r\times q}$ and $K_p \in \mathbf{R}^{r\times m}$.

Note that the "regularizability" defined above has the same sense as those in [5,7,9,10] and [17], but is different from those defined in [14] and [24]. In [24], the special case of $C_d = C_p = I$ is treated, and the system is said to be regularizable if $E - BK_d$ is nonsingular for some $K_d \in \mathbf{R}^{r\times q}$. While in [14], the special case of $C_d = C_p = C$ is treated, and the system is said to be regularizable if the matrix pair $(E - BK_dC, A + BK_pC)$ is regular and has index at most one for some matrices $K_d \in \mathbf{R}^{r\times q}$ and $K_p \in \mathbf{R}^{r\times m}$.

Based on the above preparation, the problems to be solved in the paper can be stated as follows.

Problem 1. Determine the necessary and sufficient conditions for regularizability of the descriptor linear system (1) via output plus partial state derivative feedback, that is, to establish necessary and sufficient conditions for the existence of the feedback gain matrices $K_d \in \mathbf{R}^{r\times q}$ and $K_p \in \mathbf{R}^{r\times m}$ such that the matrix pair $(E - BK_dC_d, A + BK_pC_p)$ is regular.

Problem 2. Given the descriptor linear system (1) which is regularizability via output plus partial state derivative feedback, seek a pair of feedback gain matrices $K_d \in \mathbf{R}^{r\times q}$ and $K_p \in \mathbf{R}^{r\times m}$ such that the matrix pair $(E - BK_dC_d, A + BK_pC_p)$ is regular.

III. PRELIMINARY RESULTS

In this section we present a few lemmas which will be used in the next section.

Lemma 1. (see [20, Corollary 5]) Suppose $\Phi \in \mathbf{R}^{n\times n}$, Ψ

$\in \mathbf{R}^{n\times r}$ and $\Omega \in \mathbf{R}^{q\times n}$ are given matrices. Then, there exists $X \in \mathbf{R}^{r\times q}$ such that $\mathrm{rank}(\Phi - \Psi X\Omega) = n$ if and only if

$$\mathrm{rank}\begin{bmatrix}\Phi & \Psi\end{bmatrix} = \mathrm{rank}\begin{bmatrix}\Phi \\ \Omega\end{bmatrix} = n.$$

The following two lemmas can be easily proven (proofs omitted).

Lemma 2. Suppose $M \in \mathbf{R}^{n\times m}$ with $\mathrm{rank}M = n$. Then, for any given matrix $N \in \mathbf{R}^{n\times m}$, there exists a nonzero $k_N \in \mathbf{C}$ with $|k_N|$ sufficiently small such that

$$\mathrm{rank}(M - k_N N) = n.$$

Lemma 3. Suppose $M, N \in \mathbf{R}^{n\times m}$ and $m \geq n$. Then

$$\mathrm{rank}(\gamma M - N) < n, \quad \forall \gamma \in \mathbf{C} \tag{5}$$

if there exists some distinct $\gamma_i \in \mathbf{C}$, $i = 1, \cdots, n + 1$, satisfying

$$\mathrm{rank}(\gamma_i M - N) < n, \quad i = 1, \cdots, n+1. \tag{6}$$

IV. NECESSARY AND SUFFICIENT CONDITIONS

Based on the lemmas stated in Section 3, this section considers solution to the Problem 1 proposed in Section 2.

Theorem 1. Suppose Assumption A1 holds. Then system (1) is regularizable via output plus partial state derivative feedback if and only if

$$\mathrm{rank}\begin{bmatrix}\alpha E - \beta A & B\end{bmatrix} = \mathrm{rank}\begin{bmatrix}\alpha E - \beta A \\ \alpha C_d \\ \beta C_p\end{bmatrix} = n \tag{7}$$

for some $(\alpha, \beta) \in \mathbf{C}^2$.

Proof. It follows from (2) and Definition 1 that the system (1) is regularizable via output plus partial state derivative feedback if and only if there exist $(\alpha, \beta) \in \mathbf{C}^2$, $K_d \in \mathbf{R}^{r\times q}$ and $K_p \in \mathbf{R}^{r\times m}$ such that

$$\det[\alpha(E - BK_dC_d) - \beta(A + BK_pC_p)] \neq 0. \tag{8}$$

Equation (8) can clearly be written as

$$\det\left((\alpha E - \beta A) - B[K_d \quad K_p]\begin{bmatrix}\alpha C_d \\ \beta C_p\end{bmatrix}\right) \neq 0. \tag{9}$$

In view of (9) and applying Lemma 1 to the matrices

$$\Phi = (\alpha E - \beta A), \quad \Psi = B, \quad \Omega = \begin{bmatrix}\alpha C_d \\ \beta C_p\end{bmatrix},$$

it is easily seen that (9) holds for some $K_d \in \mathbf{R}^{r\times q}$ and $K_p \in \mathbf{R}^{r\times m}$ if and only if (7) is met. This completes the proof of the theorem. ■

Theorem 2. Suppose Assumption A1 holds. Then system (1) is regularizable via output plus partial state derivative feedback if and only if there exists nonzero $\gamma \in \mathbf{C}$ such that

$$\mathrm{rank}[\gamma E - A \quad B] = \mathrm{rank}\begin{bmatrix}\gamma E - A \\ C\end{bmatrix} = n \tag{10}$$

or

$$\mathrm{rank}[E - \gamma A \quad B] = \mathrm{rank}\begin{bmatrix}E - \gamma A \\ C\end{bmatrix} = n, \tag{11}$$

where

$$C = \begin{bmatrix}C_d \\ C_p\end{bmatrix}. \tag{12}$$

Proof. The proofs of conditions (10) and (11) are very similar. Therefore, here only (10) is proven.

The "if" part follows directly from Theorem 1 by choosing $\alpha = \gamma$ and $\beta = 1$ in (10). Now we prove the "only if" part.

Suppose that the system (1) is regularizable via output plus partial state derivative feedback, then it follows from Theorem 1 that (7) holds. The proof is dividied into three cases.

Case 1. If $\beta \neq 0$ and $\alpha \neq 0$, by taking $\gamma = \dfrac{\alpha}{\beta}$, condition (7) turns into

$$\mathrm{rank}[\gamma E - A \quad B] = \mathrm{rank}\begin{bmatrix}\gamma E - A \\ \gamma C_d \\ C_p\end{bmatrix} = n \tag{13}$$

through dividing the matrices in (7) by β. Condition (13) is obviously equivalent to (10) in view of $\gamma \neq 0$.

Case 2. When $\beta = 0$, (7) turns into

$$\mathrm{rank}[\alpha E \quad B] = \mathrm{rank}\begin{bmatrix}\alpha E \\ \alpha C_d\end{bmatrix} = n \tag{14}$$

for some nonzero $\alpha \in \mathbf{C}$. Due to (14), by applying

Lemma 2 to matrices $M = [\alpha E \quad B]$ and $N = [A \quad O]$ we have

$$\operatorname{rank}[\alpha E - \beta_1 A \quad B] = n \tag{15}$$

for some nonzero α, $\beta_1 \in \mathbf{C}$ with $|\beta_1|$ being sufficiently small.

Similarly, by applying Lemma 2 to the matrices $M = \begin{bmatrix} \alpha E \\ \alpha C_d \end{bmatrix}$ and $N = \begin{bmatrix} A \\ O \end{bmatrix}$, we have

$$\operatorname{rank}\begin{bmatrix} \alpha E - \beta_2 A \\ \alpha C_d \end{bmatrix} = n \tag{16}$$

for some nonzero α, $\beta_2 \in \mathbf{C}$ with $|\beta_2|$ being sufficiently small. This implies that

$$\operatorname{rank}\begin{bmatrix} \alpha E - \beta_2 A \\ \alpha C_d \\ C_p \end{bmatrix} = n \tag{17}$$

holds for some nonzero α, $\beta_2 \in \mathbf{C}$ with $|\beta_2|$ being sufficiently small. Combining (15) and (17) gives

$$\operatorname{rank}[\alpha E - \beta_0 A \quad B] = \operatorname{rank}\begin{bmatrix} \alpha E - \beta_0 A \\ \alpha C_d \\ C_p \end{bmatrix} = n \tag{18}$$

where

$$\beta_0 = \min\{\beta_1, \beta_2\}$$

Finally, let

$$\gamma = \frac{\alpha}{\beta_0}, \tag{19}$$

by dividing the matrices in (18) by β_0, condition (18) can be converted into (13), which is equivalent to condition (10) since $\gamma \neq 0$.

Case 3. When $\alpha = 0$, the proof is similar to Case 2.

Combining the above three cases, the "only if" part is proven. ■

The above results provide conditions for regularizability of the system (1) via output plus partial state derivative feedback. In the rest of the section, we present conditions to verify the regularizability of the system (1) via output plus partial derivative state feedback.

Theorem 3. Suppose Assumption A1 holds. Then the system (1) is not regularizable via output plus partial derivative state feedback if one of the following two conditions holds:

(1) There exist mutually distinct scalars $\gamma_i \in C$, $i = 1, \cdots, n - r + 1$, such that

$$\operatorname{rank}[\gamma_i E - A \quad B] < n, \quad i = 1, \cdots, n - r + 1. \tag{20}$$

(2) There exist mutually distinct scalars $\gamma_i \in$ C, $i = 1, \cdots, n - t + 1$, $t = \operatorname{rank} C$, such that

$$\operatorname{rank}\begin{bmatrix} \gamma_i E - A \\ C \end{bmatrix} < n, \quad i = 1, \cdots, n - t + 1, \tag{21}$$

where C is defined as in (12).

This theorem can be proven using Theorem 2 and Lemma 3.

The above Theorems 1, 2 and 3 offer a simple test method to determine whether the open-loop system (1) can be regularized via output plus partial state derivative feedback.

V. SOLUTION OF GAIN MATRICES

In this section, we investigate the Problem 2 proposed in Section 2. The following definition and lemmas are used in our treatment.

Definition 2. ([21, Definition 2.1], [25]). A subset of $\mathbf{R}^{m\times n}$ (respectively, $\mathbf{C}^{m\times n}$) is a robust subset (i.e., Zariski open set) of $\mathbf{R}^{m\times n}$ (respectively, $\mathbf{C}^{m\times n}$) if it is nonempty and its complement is the set of solutions in $\mathbf{R}^{m\times n}$ (respectively, $\mathbf{C}^{m\times n}$) to a finite set of polynomial equations.

Lemma 4. ([21, Definition 2.1], [25]) Robust sets admit the following properties:

(1) Robust sets of $\mathbf{R}^{m\times n}$ (respectively, $\mathbf{C}^{m\times n}$) are open and dense in $\mathbf{R}^{m\times n}$ (respectively, $\mathbf{C}^{m\times n}$).

(2) Each robust subset of $\mathbf{C}^{m\times n}$ contains a largest subset which is a robust subset of $\mathbf{R}^{m\times n}$.

(3) The intersection of two robust subsets of $\mathbf{R}^{m\times n}$ (respectively, $\mathbf{C}^{m\times n}$) is also robust in $\mathbf{R}^{m\times n}$ (respectively, $\mathbf{C}^{m\times n}$).

(4) Any union of robust subsets of $\mathbf{R}^{m\times n}$ (respectively, $\mathbf{C}^{m\times n}$) is also robust in $R^{m\times n}$ (respectively, $\mathbf{C}^{m\times n}$).

Lemma 5. ([15], [21, Lemma 2.2]) Let $A_0 \in \mathbf{R}^{m\times n}$, $B_0 \in \mathbf{R}^{m\times h}$, and $C_0 \in \mathbf{R}^{l\times n}$ be fixed real matrices. Further assume

$$R(A_0, B_0, C_0) = \max_{K\in\mathbf{R}^{h\times l}} \operatorname{rank}\,(A_0 + B_0 K C_0). \tag{22}$$

Then

$$R(A_0,B_0,C_0)=\min\left\{\operatorname{rank}[A_0 \quad B_0],\ \operatorname{rank}\begin{bmatrix}A_0\\C_0\end{bmatrix}\right\},$$

and the set

$$S=\left\{K\in\mathbf{R}^{h\times l}\,\middle|\,\operatorname{rank}(A_0+B_0KC_0)=R(A_0,B_0,C_0)\right\}$$

forms a robust set, or equivalently, rank $(A_0 + B_0KC_0)$ reaches its maximum value for "almost all" $K\in\mathbf{R}^{h\times l}$.

Based on the above concept and lemmas, the following theorem can be derived.

Theorem 4. Suppose system (1) satisfies Assumption A1 and is regularizable via output plus partial state derivative feedback. Further, let S be the set of all matrix pairs $(K_d, K_p)\in\mathbf{R}^{r\times q}\times\mathbf{R}^{r\times m}$ such that the closed-loop system (4) is regular. Then the set S is a robust set, or equivalently, "almost all" pairs of matrices $K_d\in\mathbf{R}^{r\times q}$ and $K_p\in\mathbf{R}^{r\times m}$ can regularize the open-loop system (1).

Proof. Since system (1) is regularizable via output plus partial state derivative feedback, there exists $(\alpha,\beta)\in\mathbf{C}^2$ satisfying (7). Denote

$$\Lambda=\{(\alpha,\beta)\in\mathbf{C}^2\,|\,(\alpha,\beta)\text{ satisfies (7)}\} \tag{23}$$

and

$$S_{(\alpha,\beta)}=\{(K_d,K_p)\,|\,(K_d,K_p)\text{ satisfies (8)}\} \tag{24}$$

for any $(\alpha,\beta)\in\Lambda$. Then

$$S=\bigcup_{(\alpha,\beta)\in\Lambda}S_{(\alpha,\beta)}. \tag{25}$$

For arbitrary fixed $(\alpha,\beta)\in\Lambda$, it follows from (7) that

$$\min\left(\operatorname{rank}[\alpha E-\beta A \quad B],\ \operatorname{rank}\begin{bmatrix}\alpha E-\beta A\\ \alpha C_d\\ \beta C_p\end{bmatrix}\right)=n. \tag{26}$$

Using (26) and applying Lemma 5 to the matrices

$$A_0=\alpha E-\beta A,\quad B_0=B,\quad C_0=\begin{bmatrix}\alpha C_d\\ \beta C_p\end{bmatrix} \tag{27}$$

yields

$$R\left(\alpha E-\beta A,\,B,\begin{bmatrix}\alpha C_d\\ \beta C_p\end{bmatrix}\right)=n,$$

and the set

$$\{(K_d,K_p)\,|\,(K_d,K_p)\text{ satisfies (9)}\}$$

is a robust set, or equivalently, the set $S_{(\alpha,\beta)}$ is a robust set. This, together with the fourth condition of Lemma 4, implies that the set S is also a robust set. In other words, "almost all" pairs of matrices $K_d\in\mathbf{R}^{r\times q}$ and $K_p\in\mathbf{R}^{r\times m}$ can regularize the open-loop system (1). ■

It follows from Theorem 4 that the feedback gains $K_d\in\mathbf{R}^{r\times q}$ and $K_p\in\mathbf{R}^{r\times m}$ regularizing the system (1) can be easily sought by a trial and test procedure.

VI. EXAMPLE

Consider a system in the form of (1) with the following parameters

$$E=\begin{bmatrix}0&0&0&1.72&0\\0&0&0&0&0\\-0.82&0&0&0&0\\0&0&0&0&0\\0&0&0&0&1\end{bmatrix},$$

$$A=\begin{bmatrix}0&1.1&0&0&0\\0&0&1.56&0&0\\1.23&0&0&1.98&0\\0&0&0&0&0\\0&0&1.01&0&0\end{bmatrix},$$

$$B=\begin{bmatrix}0&0&0\\1.55&0&0\\0&1.07&0\\0&0&-1.11\\0&-2.5&0\end{bmatrix},$$

$$C_p=\begin{bmatrix}0&1&0&0&0\\1&0&0&-1&0\\0&0&1&1&0\end{bmatrix}, \tag{28}$$

$$C_d=\begin{bmatrix}1&0&0&0&0\\0&0&0&-1&0\end{bmatrix}, \tag{29}$$

where the matrices E, A and B are taken from [7,27,26] and [28]. For this system we have $n = 5$, $r = 3$, $q = 2$ and $m = 3$.

Using MATLAB, it is easy to verify that condition (10) is satisfied with γ =0.5286. Therefore, it follows from Theorem 2 that the above system is regularizable via output plus partial derivative state feedback.

Again according to Theorem 4, "almost all" pairs of matrices $K_d \in \mathbf{R}^{r\times q}$ and $K_p \in \mathbf{R}^{r\times m}$ can regularize the system. Using the Matlab command 0.01*fix(10000*rand(m,n)), we choose randomly 10000 pairs of matrices $K_d \in \mathbf{R}^{3\times 2}$ and $K_p \in \mathbf{R}^{3\times 3}$, and found that every matrix pair regularizes the system.

VII. CONCLUSIONS

Regularizability of descriptor linear systems via output plus partial state derivative feedback is studied. Simple and convenient necessary and sufficient conditions are established. It is also shown that for a descriptor linear system which is regularizable via output plus partial state derivative feedback, the regularizing output plus partial state derivative feedback controller can be easily obtained by a trial and test procedure. The work in this paper generalizes the results of [5] and [7-12].

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