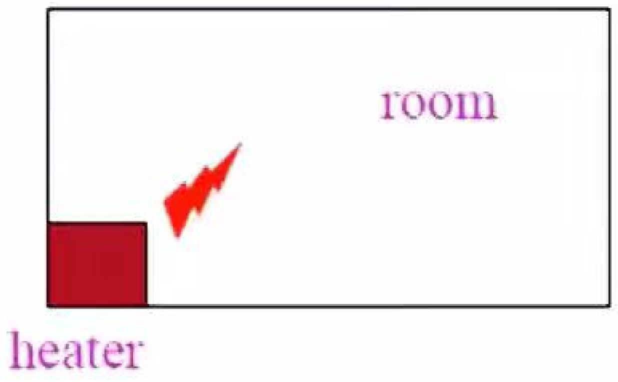
EXAMPLE#1-THERMOSTAT



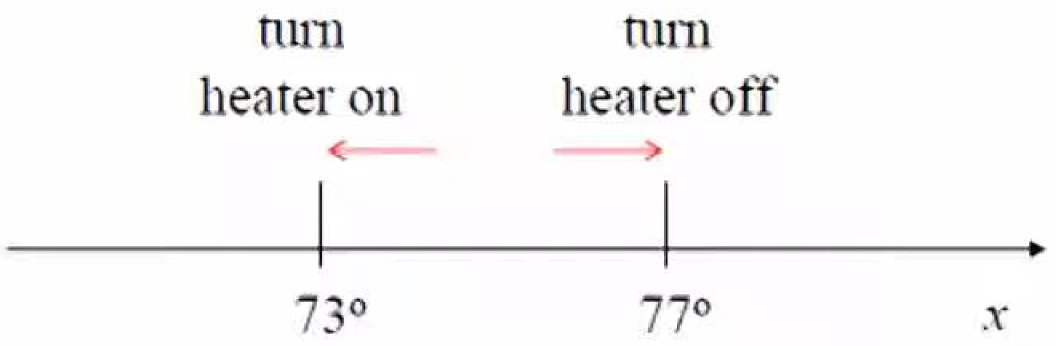
goal $\equiv$ regulate temperature around 75°

$x \equiv$ mean temperature

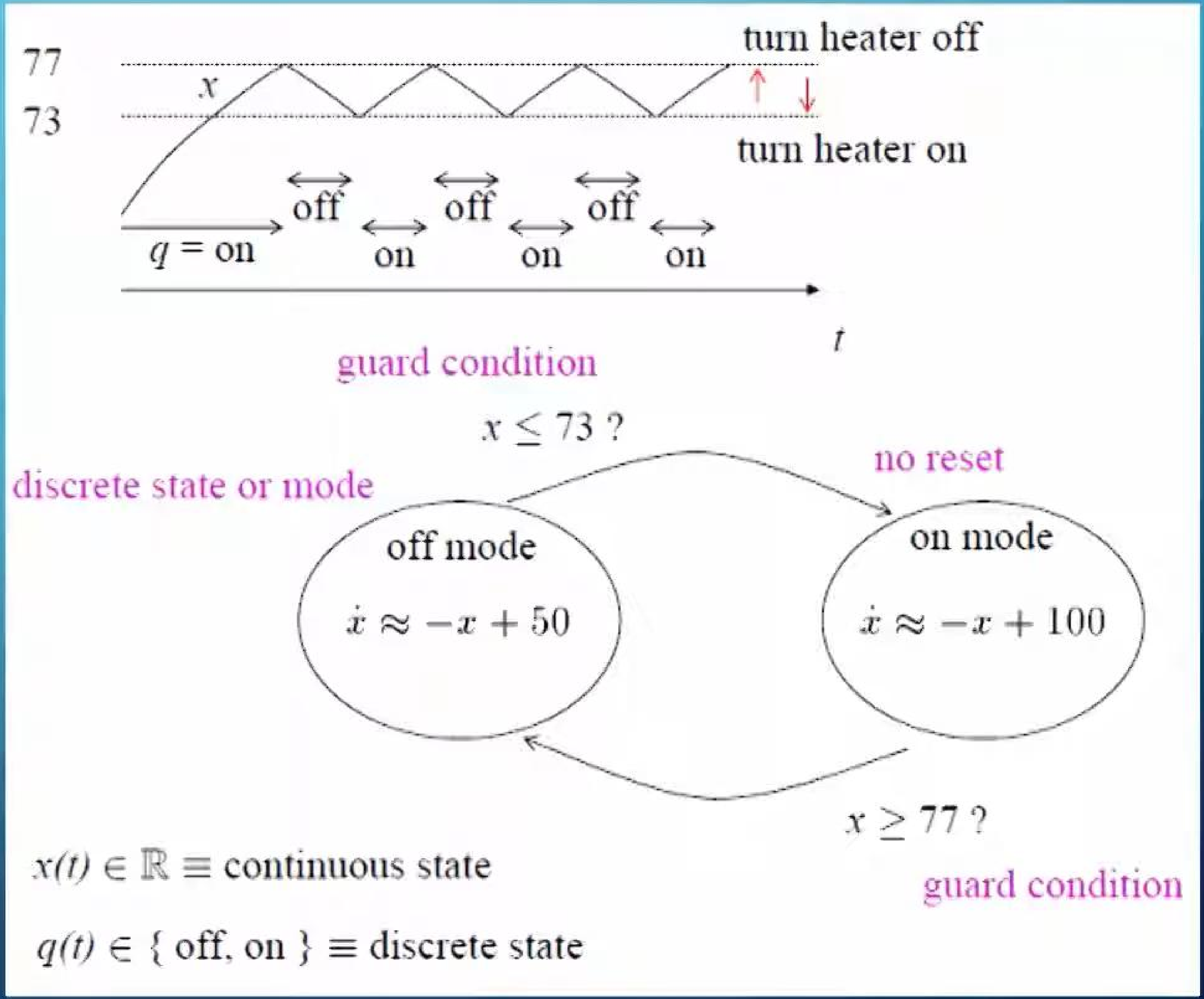
when heater is off: $\dot{x} \approx -x + 50$ $\quad (x \to 50°)$

when heater is on: $\dot{x} \approx -x + 100$ $\quad (x \to 100°)$

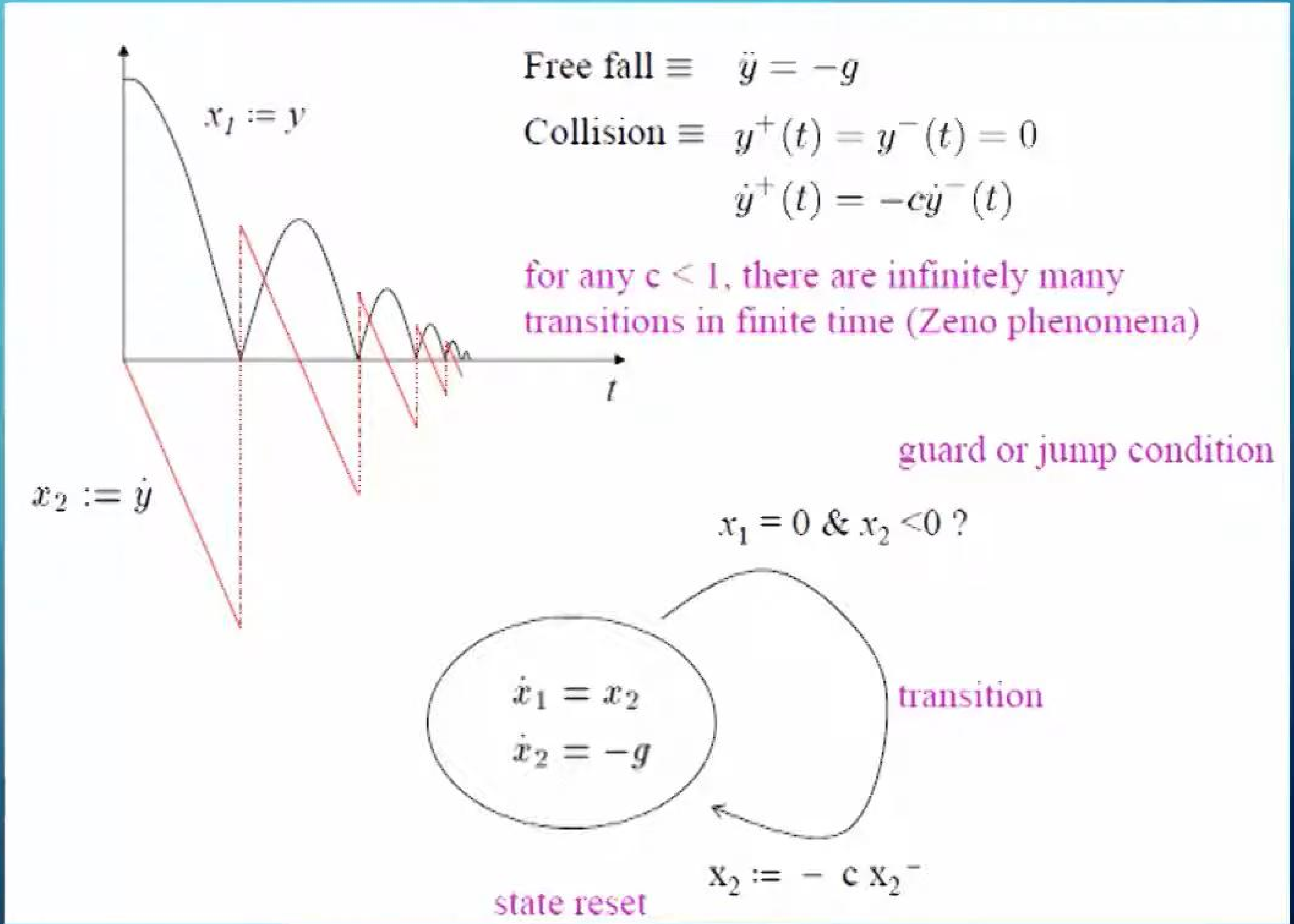
event-based control



EXAMPLE#1-THERMOSTAT

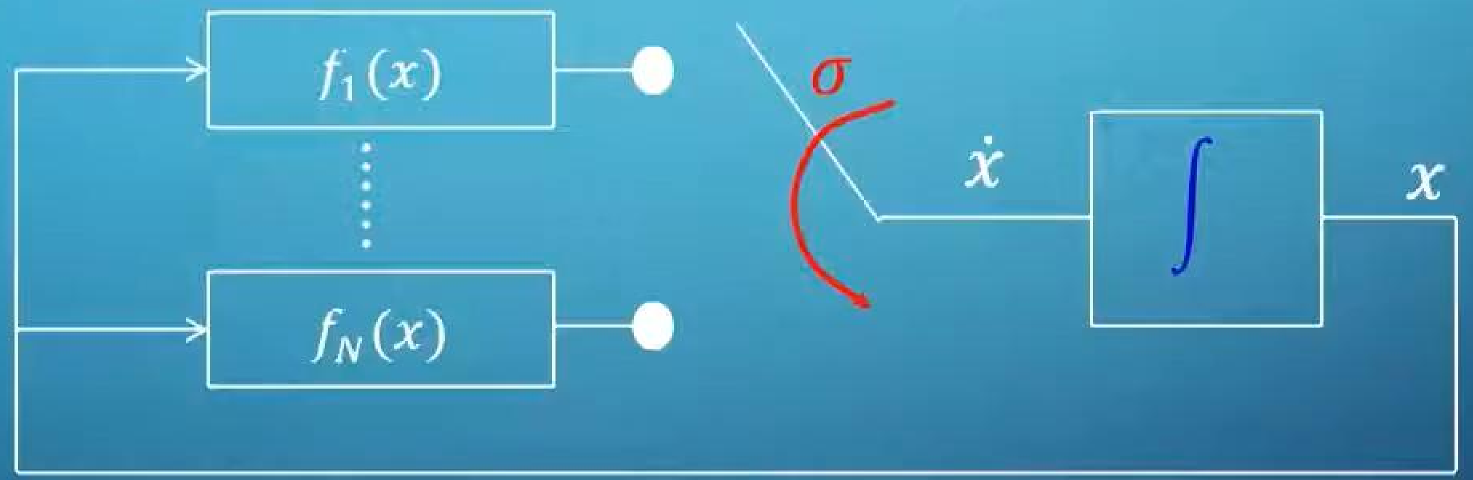


EXAMPLE#2- BOUNCING BALL

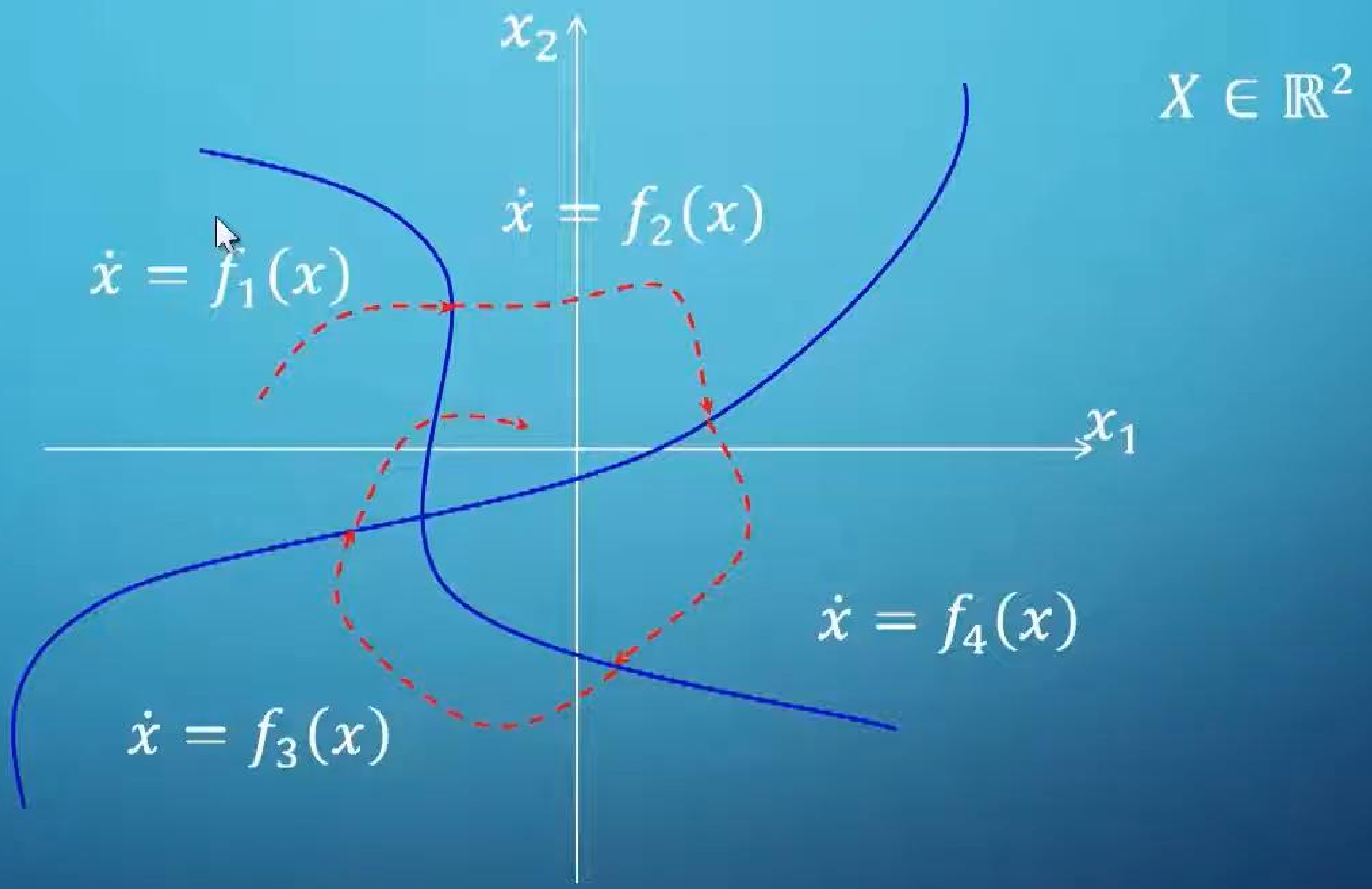


SWITCHED SYSTEMS

- a type a model of hybrid systems
- $\dot{x}(t) = f_{q(t)}\big(x(t)\big),\ q(t) \in \mathbb{Q} = \{1, 2, \dots, N\},$



STATE-DEPENDENT SWITCHING



COMMON LYAPUNOV FUNCTION

- Consider the system

$$\dot{x}(t) = A_{q(t)}x(t),\ \ q(t) \in \mathbb{Q} = \{1, 2, \ldots, N\},$$

- eq. pt. $x = 0$, for all operation modes q.
- Asymptotic stability of this eq. pt. can be proved, by constructing a Lyapunov function $V(x)$ that satisfies

$$\frac{dV(x)}{dt} < 0, as\ long\ as\ x \neq 0.$$

COMMON LYAPUNOV FUNCTION

- Condition for existence of common Lyapunov function?

- *If there exists a matrix P, $P = P^T > 0$ as a solution of the LMIs*

$$A_q^{\,T} P + PA_q < 0, \ \forall q = 1, \dots, N,$$

the quadratic function $V(x) = x^T Px$ is a Lyaponov function for the system, and the origin $x = 0$ is stable.

HOWEVER…

- The existence of a common quadratic Lyapunov function is only a sufficient condition, not a necessary one.

- It is sometimes too conservative and even infeasible to find a common Lyapunov function.

SWITCHING BETWEEN TWO UNSTABLE SYSTEMS

- There are two unstable subsystems

$$\dot{x}(t) = A_1 x(t),\ \dot{x}(t) = A_2 x(t),$$

and $\lambda(A_1),\ \lambda(A_2) > 0.$

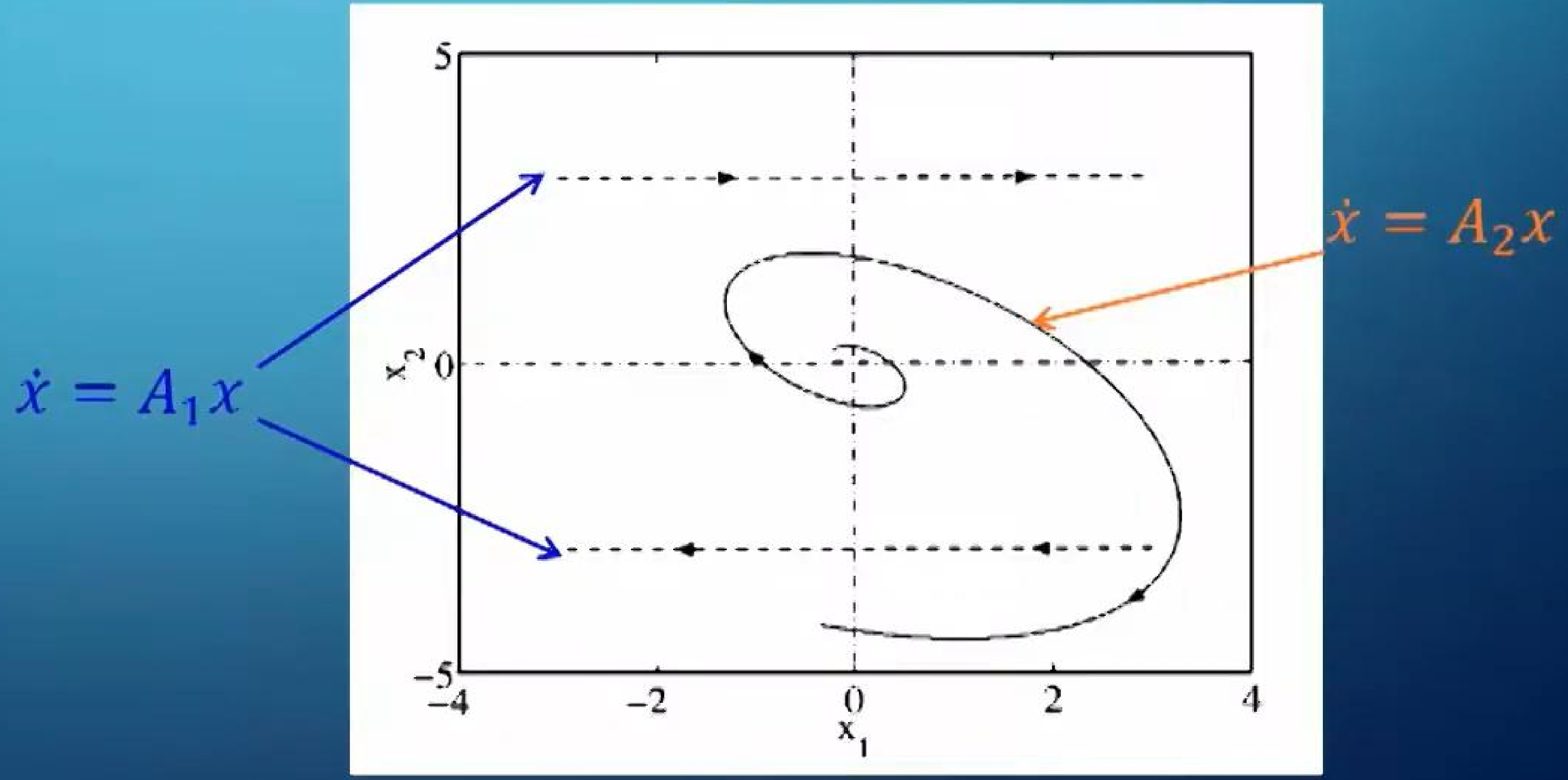
- Could the overall system still be stable by switching ?

It could be!!

SWITCHING BETWEEN TWO UNSTABLE SYSTEMS

$$\dot{x}(t) = A_{q(t)}x(t), q \in \{1, 2\}$$

$$A_1 = \begin{bmatrix} 0 & 10 \\ 0 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} 1.5 & 2 \\ -2 & -0.5 \end{bmatrix},$$



Phase portrait

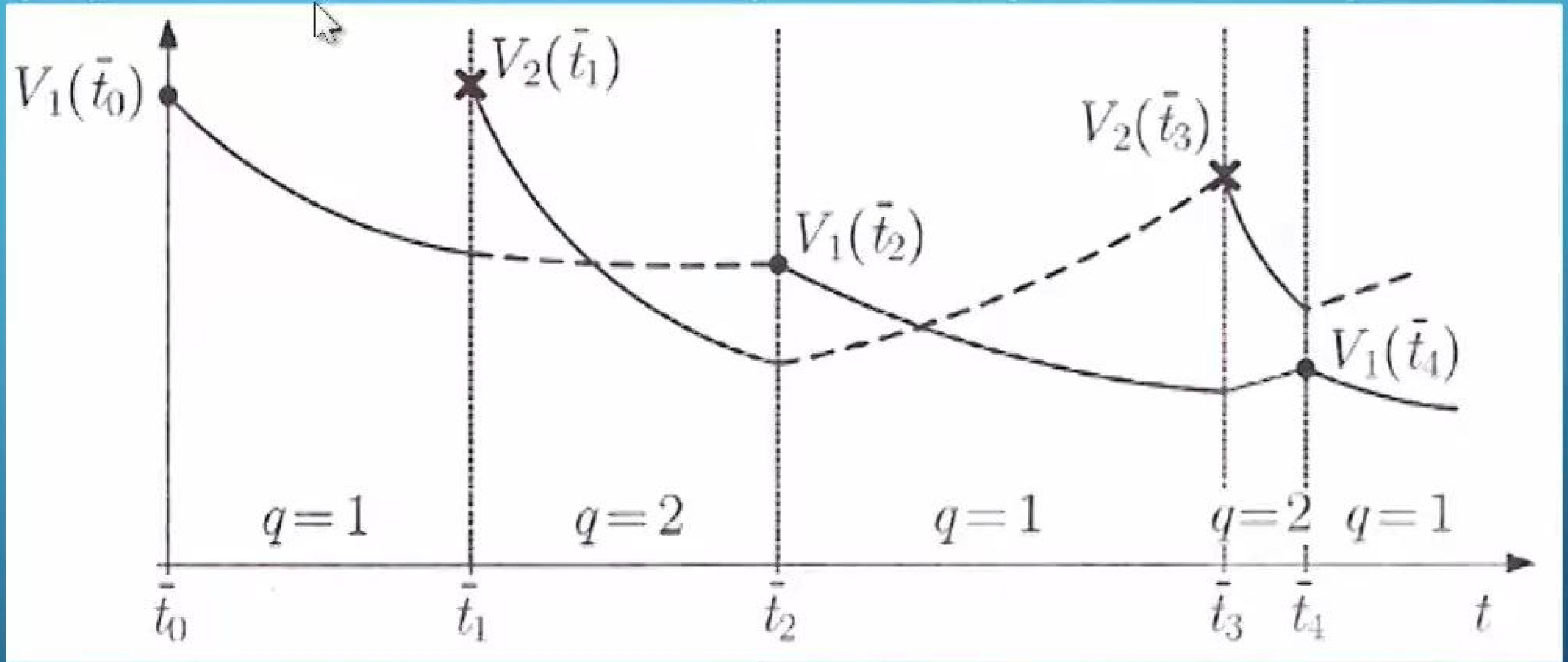
MULTIPLE LYAPUNOV-LIKE FUNCTIONS

- There are families of piecewise continuous and piecewise differentiable functions that concatenated together to produce a single non-traditional Lyapunov function.

- A family of Lyapunov-like functions $\{V_i(x) = x^T P_i x, \mathrm{i} \subset \mathbb{Q}\}$, $\mathbb{Q}=\{1, \ldots, Q\}$, such that each vector field $A_i x, i \in \mathbb{Q}$ has its own Lyapunov function been used.

MULTIPLE LYAPUNOV-LIKE FUNCTIONS

- Decay of the Lyapunov-like function is required only when the system is activated.



MULTIPLE LYAPUNOV-LIKE FUNCTIONS

Consider a family of Lyapunov functions V_q*, each associated with a vector field* $A_q x$*. For* $i < j$*, let* $t_i < t_j$ *be the switching times for which* $q(t_i) = q(t_j)$*.*
If there exists a $\gamma > 0$ *such that*

$$V_{q(t_j)}\big(x(t_{j+1})\big) - V_{q(t_i)}\big(x(t_{i+1})\big) \leq -\gamma \|x(t_{i+1})\|^2$$

then the switched system is stable.

MULTIPLE LYAPUNOV-LIKE FUNCTIONS

$$\dot{x}(t) = A_{q(t)}x(t), q \in \{1, 2\}$$

$$A_1 = \begin{bmatrix} 0 & 10 \\ 0 & 0 \end{bmatrix}, P_1 = \begin{bmatrix} 0.468 & -1.875 \\ -1.875 & 15 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1.5 & 2 \\ -2 & -0.5 \end{bmatrix}, P_2 = \begin{bmatrix} 1 & 1.2 \\ 1.2 & 1.6 \end{bmatrix}$$