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Asynchronous Output Feedback Control Design for Nonlinear Switched Singular Systems with Time Varying Delay

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Abstract— **This work discusses the problem of static output feedback controller design for a class of switched singular systems under asynchronous switching and subject of time-varying delay and nonlinearity. Based on mode-dependent average dwell time (MDADT) and an appropriate Lyapunov-Krasovskii function with triple sum, the existence of stabilizing switching signals and the static output feedback controllers are derived in terms of linear matrix inequalities to ensure the exponential admissibility of the closed loop system under not only the matching but also the mismatching between the system and the controller modes. Moreover, a numerical example is simulated to verify the merits of the proposed method.**

I. Introduction

Recently, the study of delayed switched singular systems has become a topic of great interest on both application and theoretical level. The main motivation behind this fact comes from the following conditions. First, many real systems are affected by sudden variation of their parameters and structures, such as abrupt environment changes, components repairs and interconnection changes of subsystems. Second, singular systems bring more suitable representation than standard state space ones. Generally speaking, this kind of systems is composed of a set of subsystems described by continuous or discrete time dynamics and logic rules that manage the switching among them. For most cases, switching rules play a crucial role in characterization of the system dynamic behavior. A great number of interesting results related to delayed switched singular systems have been reported in the literatures. Thus, in [2], the problem of l_1 and l_∞ stability analysis for positive switched linear singular systems with constant time delay has been addressed. Using the average dwell time (ADT) switching technique, the finite-time H_∞ control problem for a class of discrete-time switched singular systems with constant time-delay and actuator saturation has been developed in [7] and [6]. The designed strategy in [6] is developed using an iterative algorithm. The issue of robust H_∞ guaranteed cost control for discrete-time switched singular systems with time-varying delay under MDADT switching has been studied in [8]. Using the SVD decomposition of the output matrix, both state feedback and static output feedback controller have been designed to cope with linear switched singular systems in [1].

However, all the previous cited works consider only linear switched singular systems with synchronous switching mode between controller and the system. Although often, in real process, there exists a lag time between the switching instant of the controller and the system. Some appreciable works have been investigated in the literature to deal with the problem of asynchronous switching control. For example the problem of asynchronous state feedback control design for discrete time varying switched linear singular systems using ADT approach has been developed in [4] and [5].
To the best of our Knowledge, the problem of asynchronous static output feedback control design for switched singular systems with time varying delay has not been fully investigated, especially when the system is subject to nonlinearities.

In extension of these efforts, this paper investigates the problem of static output feedback control design of discrete-time nonlinear switched singular systems with time-varying delay. The main contributions can be summarized as follows:

(i) Based on the MDADT approach, with uncommon parameters for all subsystems, and an appropriate Lypunov function with triple sum, the issue of asynchronous control for the delayed switched singular systems with nonlinearity terms is studied. (ii) The proposed strategy is developed in terms of linear matrix inequalities, such that the resulting closed-loop switched nonlinear system working on asynchronous switching mode is exponentially admissible.

II. System description and problem formulation

Consider a class of discrete switched singular systems with time-varying delay described by:

$$\begin{cases} E_{\sigma(k)}x(k+1) = A_{\sigma(k)}x(k) + A_{d\sigma(k)}x(k-d(k)) \\ \qquad\qquad\qquad + B_{\sigma(k)}u(k) + f(x(k)) \\ y(k) = C_{\sigma(k)}x(k) \\ x(k) = \phi(k),\ k \in [-d_M, 0] \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $u(k) \in \mathbb{R}^{m_u}$ is the control input vector, $y(k) \in \mathbb{R}^p$ is the measured output and $\phi(k)$ is a given initial condition sequence. Delay $d(k)$ is time-varying and satisfies

$$0 < d_m \le d(k) \le d_M \tag{2}$$

where d_m and d_M are positive integers representing the bounds of the delay. $\sigma(k) : \{1, 2, \} \to \mathbb{I} = \{1, 2 \cdots, N\}$ is a piecewise constant switching signal with N being the

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number of subsystems. Moreover, $\sigma(k) = i \in \mathbb{I}$ denotes the ith subsystem is activated. $f(x(k))$ is an unknown but bounded nonlinear real-valued function which represents any model uncertainty in the system including external disturbances and satisfy the following sector condition:

$$(f(\omega) - S_1\omega)^T (f(\omega) - S_2\omega) \leq 0 \tag{3}$$

where $S_1 \geq 0$, $S_2 \geq 0$ are diagonal matrices with $S_2 > S_1$. For technical convenience and inspired by [11], nonlinear function $f(x(k))$ can be decomposed into linear and nonlinear parts as

$$f(x(k)) = f_n(x(k)) + S_1 x(k) \tag{4}$$

where nonlinearity $f_n(x(k))$ satisfies

$$f_n^T(x(k)) \Big[f_n(x(k)) - Sx(k) \Big] \leq 0 \tag{5}$$

with $S = S_2 - S_1 > 0$.

Consider the following unforced switched singular systems with delay:

$$E_i x(k+1) = A_i x(k) + A_{di} x(k - d(k)) \tag{6}$$

Definition *1:* [9]

1) For a given $i \in \mathbb{I}$, pair (E_i, A_i) is said to be regular if $\det(zE_i - A_i) \neq 0$.
2) For a given $i \in \mathbb{I}$, pair (E_i, A_i) is said to be causal, if it is regular and $deg\Big(det(zE_i - A_i)\Big) = rank(E_i)$.
3) For given positive scalars d_m and d_M, system (6) is said to be regular and causal for any time delay $d(k)$ satisfying (2), if pair (E_i, A_i) is regular and causal.
4) System (6) is said to be admissible if it is regular, causal and stable.

Definition *2:* [16] For switching signal $\sigma(k)$ and any $k_s > k_a > k_0$, let $N_{\sigma p}(k_a, k_s)$ be the switching numbers that the pth subsystem is activated over interval $[k_a, k_s]$ and $T_p(k_a, k_s)$ denotes the total running time of the pth subsystem over interval $[k_a, k_s]$ with $p \in \mathbb{I}$. We say that $\sigma(k)$ has a mode dependent average dwell time τ_{ap} if the exist mode dependent chatter bounds N_{0p} such that

$$N_{\sigma p}(k_a, k_s) \leq N_{0p} + \frac{T_p(k_a, k_s)}{\tau_{ap}} \tag{7}$$

Definition *3:* [14] Assume that a switching signal $\sigma(k)$ is given. Equilibrium $x^* = 0$ of system (6) is exponentially stable with marginal δ under switching signal $\sigma(k)$ if for any initial conditions $x(k_0)$, there exist constants $\mathcal{L} > 0$, $\chi > 0$, and there is a solution of system $x(k)$ such that

$$\| x(k) \| \leq \mathcal{L} e^{-\chi(k-k_0)} \| x(k_0) \| \tag{8}$$

The following lemmas are provided to illustrate our main results.

Lemma *2.1:* For any matrices $V > 0$, R_1, R_2 and a scalar $d > 0$, the following inequality holds

$$\begin{aligned}
&-\sum_{n=-d}^{1} \sum_{s=k+n}^{k-1} \eta(s)^T E_i^T V E_i \eta(s) \\
&\leq \zeta_1^T(k) \begin{bmatrix} dR_1^T E_i + dE_i^T R_1 & -R_1^T + dE_i^T R_2 \\ * & -R_2^T - R_2 \end{bmatrix} \zeta_1(k) \\
&+ \frac{d(d+1)}{2} \zeta_1^T(k) \begin{bmatrix} R_1^T \\ R_2^T \end{bmatrix} V^{-1} \begin{bmatrix} R_1 & R_2 \end{bmatrix} \zeta_1(k)
\end{aligned}$$

where $\eta(k) = x(k+1) - x(k)$ and $\zeta_1 = \begin{bmatrix} x^T(k) & (\sum_{s=k-d}^{k-1} E_i x(s))^T \end{bmatrix}^T$.

Proof: The proof of this Lemma can be justified through $\sum_{n=-d}^{1} \sum_{s=k+n}^{k-1} \begin{bmatrix} E_i \eta(k) \\ \zeta_1 \end{bmatrix}^T \mathbb{V}^T \mathbb{V} \begin{bmatrix} E_i \eta(k) \\ \zeta_1 \end{bmatrix} \geq 0$, with $\mathbb{R} = \begin{bmatrix} R_1 & R_2 \end{bmatrix}$ and $\mathbb{V} = \begin{bmatrix} V^{\frac{1}{2}} & V^{-\frac{1}{2}} \mathbb{R} \\ 0 & 0 \end{bmatrix}$. ■

Lemma *2.2:* [10] For given real matrices X, N and M with appropriate dimensions, the following statements are equivalent

1)
$$\begin{bmatrix} X & N \\ N^T & 0 \end{bmatrix} + \mathrm{sym}\Big\{ \begin{bmatrix} H \\ F \end{bmatrix} \begin{bmatrix} M^T & -I \end{bmatrix} \Big\} < 0 \tag{9}$$
is feasible in variable F and H
2) X, N and M satisfy
$$X + \mathrm{sym}(NM^T) < 0 \tag{10}$$

III. ASYNCHRONOUS STATIC OUTPUT FEEDBACK DESIGN

In many engineering processes, it takes time to distinguish the subsystems, which can bring some delay, called lag time, between the switching of the controller and the system modes. Consequently, the closed loop system may have an asynchronous switching caused through the mismatch among the switching instances of the controller and those of the subsystems. Hence, to deal with such switching, the controller is considered to have the following form:

$$u(k) = K_{\bar{\sigma}(k)} y(k), \forall k \in [k_m, k_{m+1}) \tag{11}$$

where $\bar{\sigma}(k) = \sigma(k - \Delta_m)$ is the switching signal of the controller with $\Delta_0 = 0$ and $\Delta_m < k_{m+1} - k_m$ represents the delayed period.

Remark *3.1:* The delayed period $\Delta_m > 0$ ensures that switching instants of the controllers lag behind the switches of system modes and also, there exists a period called synchronous period during which the system mode and the controller operate synchronously. Let the ith subsystem be activated at the switching instant k_m, and the jth subsystem be activated at the switching instant k_{m+1}. Then, the corresponding controllers are activated at the switching instants $k_m + \Delta_m$ and $k_{m+1} + \Delta_{m+1}$, respectively.

Applying controller (11) to system (1) and respecting the nonlinearity decomposition in (4), the resulting closed-loop system is given by

$$\begin{cases} E_ix(k+1)=\tilde{A}_ix(k)+A_{di}x(k-d(k))+f_n(x(k)), \\ \qquad\qquad k\in[k_m+\Delta_m,k_{m+1}) \\ E_ix(k+1)=\tilde{A}_{ij}x(k)+A_{di}x(k-d(k))+f_n(x(k)), \\ \qquad\qquad k\in[k_m,k_m+\Delta_m) \end{cases} \tag{12}$$

where $\tilde{A}_i=A_i+B_iK_iC_i+S_1I_n$, $\tilde{A}_{ij}=A_i+B_iK_jC_i+S_1I_n$

Theorem *3.1:* Given tunable scalars $0<\alpha_i<1$, $\beta_i\geq 1$, $\mu_{1i}>1$, $\mu_{2i}>1$ and positive integers d_m and d_M. Switched singular system (12) is exponentially admissible, if there exist matrices $P_i>0$, $P_{ij}>0$, $Q_{1i}>0$, $Q_{1ij}>0$, $Q_{2i}>0$, $Q_{2ij}>0$, $Q_{3i}>0$, $Q_{3ij}>0$, $Z_{1i}>0$, $Z_{1ij}>0$, $Z_{2i}>0$, $Z_{2ij}>0$, $Z_{3i}>0$, $Z_{3ij}>0$, T_1, T_2, R_1, R_2, X_i, X_{ij}, Y_i, Y_{ij}, $\mathcal{S}_i$, F_s, $s=1,2,3$ and positive scalars ε_{1i} such that the following inequalities hold for all $(i,j)\in\mathbb{I}\times\mathbb{I}$, $i\neq j$,

$$\begin{cases}\Sigma_{Xi}(\tilde{A}_i)<0,\ \Sigma_{Yi}(\tilde{A}_i)<0 \\ \Sigma_{Xij}(\tilde{A}_{ij})<0,\ \Sigma_{Yij}(\tilde{A}_{ij})<0\end{cases} \tag{13}$$

and for any switching rule with the following MDADT condition

$$\tau_{ai}>\tau_{ai}^*=-\frac{\ln(\mu_{1i}\mu_{2i})+\Delta_{ni}ln(\frac{\beta_i}{\alpha_i})}{\ln\alpha_i} \tag{14}$$

where Δ_{ni} denotes the maximum lag periods, $\mu_{0i}=(\frac{\alpha_i}{\beta_i})^{d_M-2}$ and $\mu_{1i}\mu_{2i}\geq 1$ satisfying

$$\begin{aligned}&P_i-\mu_{1i}P_{ij}<0,\ Q_{1i}-\mu_{1i}Q_{1ij}<0,\ Q_{2i}-\mu_{1i}Q_{2ij}<0,\\&Q_{3i}-\mu_{1i}Q_{3ij}<0,\ Z_{1i}-\mu_{1i}Z_{1ij}<0,\ Z_{2i}-\mu_{1i}Z_{2ij}<0,\\&Z_{3i}-\mu_{1i}Z_{3ij}<0,\ \beta_iQ_{1ij}-\mu_{2i}\mu_{0i}Q_{1j}<0,\\&\beta_iQ_{2ij}-\mu_{2i}\mu_{0i}Q_{2j}<0,\ \beta_iQ_{3ij}-\mu_{2i}\mu_{0i}Q_{3j}<0,\\&\beta_iZ_{1ij}-\mu_{2i}\mu_{0i}Z_{1j}<0,\ \beta_iZ_{2ij}-\mu_{2i}\mu_{0i}Z_{2j}<0,\\&\beta_iZ_{3ij}-\mu_{2i}\mu_{0i}Z_{3j}<0\end{aligned} \tag{15}$$

and

$$\Sigma_{Xij}(\tilde{A}_{ij})=\begin{bmatrix}\bar{\Upsilon}_{ij} & * & * & * \\ \sqrt{d_M}\mathbb{T}\bar{H}_T & -\beta_i^{d_M}Z_{1ij} & * & * \\ \sqrt{d_r}X_{ij}^T & 0 & -\beta_i^{d_M}Z_{2ij} & * \\ \sqrt{\tilde{d}_M}\mathbb{R}\bar{H}_R & 0 & 0 & -\beta_i^{d_M}Z_{3ij}\end{bmatrix}$$

$$\Sigma_{Yij}(\tilde{A}_{ij})=\begin{bmatrix}\bar{\Upsilon}_{ij} & * & * & * \\ \sqrt{d_M}\mathbb{T}\bar{H}_T & -\beta_i^{d_M}Z_{1ij} & * & * \\ \sqrt{d_r}Y_{ij}^T & 0 & -\beta_i^{d_M}Z_{2ij} & * \\ \sqrt{\tilde{d}_M}\mathbb{R}\bar{H}_R & 0 & 0 & -\beta_i^{d_M}Z_{3ij}\end{bmatrix} \tag{16}$$

$$\begin{aligned}\bar{\Upsilon}_{ij}=&\bar{\Gamma}_{ij}+sym(\bar{\Gamma}_{1ij})+\bar{H}_{1i}^TP_{ij}\bar{H}_{1i}-\beta_i\bar{H}_{2i}^TP_{ij}\bar{H}_{2i}+\bar{H}_3^T(d_mZ_{1ij}\\&+d_rZ_{2ij}+\tilde{d}_MZ_{3ij})\bar{H}_3+sym(\bar{H}_4\mathcal{S}_i\mathcal{R}_i^T\bar{H}_3)+sym(\bar{\mathbb{F}}\bar{\mathbb{A}}_{ij})\\&+\bar{H}_T^T\Pi_T\bar{H}_T+\bar{H}_R^T\Pi_R\bar{H}_R-\varepsilon_{1i}sym(H_f\bar{H}_f)\\ \bar{\Gamma}_{ij}=&diag\big(Q_{1ij}+(d_r+1)Q_{3ij};\ \beta_i^{d_m}(-Q_{1ij}+Q_{2ij});\\&-\beta_i^{d_M}Q_{3ij};\ -\beta_i^{d_M}Q_{2ij}\ ;0_n;\ 0_n;0_n\big)\\ \bar{\Gamma}_{1ij}=&\begin{bmatrix}0 & Y_{ij}E_i & X_{ij}E_i-Y_{ij}E_i & -X_{ij}E_i & 0 & 0 & 0\end{bmatrix},\\ \bar{H}_{1i}=&\begin{bmatrix}E_i & 0_{n\times 3n} & I_n & 0_n & 0_n\end{bmatrix},\ \bar{H}_{2i}=\begin{bmatrix}E_i & 0_{n\times 6n}\end{bmatrix},\\ \bar{H}_3=&\begin{bmatrix}0_{n\times 4n} & I_n & 0_{n\times 2n}\end{bmatrix},\ \bar{H}_4^T=\begin{bmatrix}I_n & 0_{n\times 6n}\end{bmatrix},\\ \bar{H}_T=&\begin{bmatrix}I_n & 0_{n\times 6n} & \\ 0_{n\times 3n} & I_n & 0_{n\times 3n}\end{bmatrix},\ H_f=\begin{bmatrix}0_{n\times 6n} & I_n\end{bmatrix},\\ \bar{H}_R=&\begin{bmatrix}I_n & 0_{n\times 6n} & \\ 0_{n\times 5n} & I_n & 0_n\end{bmatrix},\ \bar{H}_f=\begin{bmatrix}-SI_n & 0_{n\times 5n} & I_n\end{bmatrix},\\ \Pi_T=&\begin{bmatrix}T_1^TE_i+E_i^TT_1 & -T_1^TE_i+E_i^TT_2 \\ * & -T_2^TE_i-E_i^TT_2\end{bmatrix},\ \mathbb{T}=\begin{bmatrix}T_1 & T_2\end{bmatrix},\\ \Pi_R=&\begin{bmatrix}d_MR_1^TE_i+d_ME_i^TR_1 & -R_1^T+d_ME_i^TR_2 \\ * & -R_2^T-R_2\end{bmatrix},\\ \bar{\mathbb{F}}^T=&\begin{bmatrix}F_1 & 0_n & F_2 & 0_n & F_3 & 0_n & 0_n\end{bmatrix},\\ \bar{\mathbb{A}}_{ij}=&\begin{bmatrix}\tilde{A}_{ij}-E_i & 0_n & A_{di} & 0_n & -I_n & 0_n & I_n\end{bmatrix},\\ \mathbb{R}=&\begin{bmatrix}R_1 & R_2\end{bmatrix},\ d_r=d_M-d_m,\ \tilde{d}_M=\frac{d_M(d_M+1)}{2}\end{aligned}$$

$\mathcal{R}_i$ are any matrices with full column rank satisfying $\mathcal{R}_i^TE_i=0$.

Noting that matrices Σ_{Xi} and Σ_{Yi} have the same form as Σ_{Xij} and Σ_{Yij}, respectively, by replacing β_i with α_i and $j=i$.

Proof: This proof can be organized into two parts. The first one treats the regularity and the causality, while the second one deals with the stability of system (12).

Since $rank(E_i)=r\leq n$, there exist two nonsingular matrices $\mathbb{N}_i$ and $\mathbb{L}_i\in\mathbb{R}^{n\times n}$ such that

$$\begin{aligned}&\bar{E}_i=\mathbb{N}_iE_i\mathbb{L}_i=\begin{bmatrix}I_r & 0\\ 0 & 0\end{bmatrix},\bar{A}_{di}=\mathbb{N}_iA_{di}\mathbb{L}_i=\begin{bmatrix}\bar{A}_{d11}^i & \bar{A}_{d12}^i\\ \bar{A}_{d21}^i & \bar{A}_{d22}^i\end{bmatrix},\\&\bar{A}_i=\mathbb{N}_i\tilde{A}_{ij}\mathbb{L}_i=\begin{bmatrix}\bar{A}_{11}^{ij} & \bar{A}_{12}^{ij}\\ \bar{A}_{21}^{ij} & \bar{A}_{22}^{ij}\end{bmatrix},\bar{S}_i=\mathbb{L}_i^T\mathcal{S}_i=\begin{bmatrix}\bar{S}_{11}^i\\ \bar{S}_{21}^i\end{bmatrix},\end{aligned} \tag{17}$$

and, $\mathcal{R}_i$ can be described as $\mathcal{R}_i=\mathbb{N}_i^T\begin{bmatrix}0\\ \Theta_i\end{bmatrix}$, where $\Theta_i\in\mathbb{R}^{(n-r)\times(n-r)}$ is any nonsingular matrix.

From (13) and (16), we can easy verify that

$$\begin{aligned}&\begin{bmatrix}\Lambda_{11} & \Lambda_{12}\\ * & \Lambda_{22}\end{bmatrix}<0,\\&\Lambda_{11}=(1-\beta_i)E_i^TP_{ij}E_i+sym(F_1^T(\tilde{A}_{ij}-E_i)\\&\quad+E_i^TT_1+d_ME_i^TR_1)\\&\Lambda_{12}=E_i^TP_{ij}+\mathcal{S}_i\mathcal{R}_i^T+(\tilde{A}_{ij}-E_i)^TF_3-F_1^T\\&\Lambda_{22}=P_{ij}-sym(F_3)\end{aligned} \tag{18}$$

Let $\mathcal{A}=\begin{bmatrix}I & \tilde{A}_{ij}^T\end{bmatrix}^T$. Pre- and post-multiplying inequality (18) by $\mathcal{A}^T$ and $\mathcal{A}$, respectively, yields

$$\begin{aligned}&(1-\beta_i)E_i^TP_{ij}E_i+\tilde{A}_{ij}^TP_{ij}\tilde{A}_{ij}+sym(E_i^T(P_{ij}-F_3)\tilde{A}_{ij}\\&+E_i^T(-F_1+T_1+d_MR_1)+\mathcal{S}_i\mathcal{R}_i^T\tilde{A}_{ij})<0\end{aligned} \tag{19}$$

By checking a congruence transformation to (19) by $\mathbb{L}_i$ and using (17), we get

$$sym(\bar{S}_{21}^i\Theta_i^T\bar{A}_{22}^{ij})<0 \tag{20}$$

with $\bar{A}_{22}^{ij}$ is nonsingular. Otherwise, we suppose that the matrix $\bar{A}_{22}^{ij}$ is singular. Then, there exists a non-zero vector ϑ_i ensuring $\bar{A}_{22}^{ij}\vartheta_i=0$. Consequently, we can deduce that $\vartheta_i^Tsym(\bar{S}_{21}^i\Theta_i^T\bar{A}_{22}^{ij})\vartheta_i=0$, which contradict (20). Then, $\bar{A}_{22}^{ij}$

is nonsingular. We conclude that pair $(E_i, \tilde{A}_{ij})$ is regular and causal. The same conclusion can be deduced for the matched period.

To develop our results, we choose the following switched Lyapunov-Krasovskii functional candidate:

$$V_{ij}(x(k)) = \sum_{s=1}^{5} V_{ijs}(k),\ V_{ij1}(k) = x^T(k)E_i^T P_{ij} E_i x(k)$$
$$V_{ij2}(k) = \sum_{s=k-d_m}^{k-1} x^T(s)\beta_i^{k-1-s} Q_{1ij} x(s) + \sum_{s=k-d_M}^{k-1-d_m} x^T(s)\beta_i^{k-1-s} Q_{2ij} x(s)$$
$$V_{ij3}(k) = \sum_{s=k-d(k)}^{k-1} x^T(s)\beta_i^{k-1-s} Q_{3ij} x(s) + \sum_{n=-d_M+1}^{-d_m} \sum_{s=k+n}^{k-1} x^T(s)\beta_i^{k-1-s} Q_{3ij} x(s)$$
$$V_{ij4}(k) = \sum_{n=-d_M}^{-1} \sum_{s=k+n}^{k-1} \eta^T(s) E_i^T \beta_i^{k-1-s} Z_{1ij} E_i \eta(s) + \sum_{n=-d_M}^{-d_m-1} \sum_{s=k+n}^{k-1} \eta^T(s) E_i^T \beta_i^{k-1-s} Z_{2ij} E_i \eta(s)$$
$$V_{ij5}(k) = \sum_{s=-d_M}^{-1} \sum_{n=s}^{-1} \sum_{m=k+n}^{k-1} \eta^T(m) E_i^T \beta_i^{k-1-m} Z_{3ij} E_i \eta(m)$$

Define $\eta(k) = x(k+1) - x(k)$, and

$$\zeta(k) = \begin{bmatrix} x^T(k) & x^T(k-d_m) & x^T(k-d(k)) & x^T(k-d_M) \\ & \eta^T(k)E_i^T & (\sum_{s=k-d_M}^{k-1} E_i x(s)))^T & f_n^T(x(k)) \end{bmatrix}^T$$

Along the solution of system (12) and taking the forward difference of $V_{ij}(k)$, we have

$$\Delta_\beta V(k) = V_{ij}(x(k+1)) - \beta_i V_{ij}(x(k)),\ k \in [k_m, k_m + \Delta_m) \tag{21}$$

$$\begin{aligned}
&\Delta_\beta V_{ij1}(k) = \zeta^T(k)(\bar{H}_{1i}^T P_{ij} \bar{H}_{1i} - \beta_i \bar{H}_{2i}^T P_{ij} \bar{H}_{2i})\zeta(k) \\
&\Delta_\beta V_{ij2}(k) = x^T(k) Q_{1ij} x(k) - x^T(k-d_m)\beta_i^{d_m} Q_{1ij} x(k-d_m) \\
&+ x^T(k-d_m)\beta_i^{d_m} Q_{2ij} x(k-d_m) \\
&- x^T(k-d_M)\beta_i^{d_M} Q_{2ij} x(k-d_M) \\
&\Delta_\beta V_{ij3}(k) \le x^T(k) Q_{3ij} x(k) \\
&- x^T(k-d(k))\beta_i^{d_M} Q_{3ij} x(k-d(k)) + x^T(k) d_r Q_{3ij} x(k)
\end{aligned} \tag{22}$$

Using Lemma 2.1 in [15] and defining $\zeta_0 = \begin{bmatrix} x^T(k) & x^T(k-d_M) \end{bmatrix}^T$, we get

$$\begin{aligned}
&\Delta_\beta V_{ij4}(k) = \eta^T(k) E_i^T (d_M Z_{1ij} + d_r Z_{2ij}) E_i \eta(k) \\
&+ \zeta_0^T(k) \Pi_T \zeta_0(k) + d_M \zeta_0^T(k) \mathbb{T}^T (\beta_i^{d_M} Z_{1ij})^{-1} \mathbb{T} \zeta_0(k) \\
&- \sum_{s=k-d(k)}^{k-d_m-1} \eta^T(s) E_i^T \beta_i^{d_M} Z_{2ij} E_i \eta(s) \\
&- \sum_{s=k-d_M}^{k-d(k)-1} \eta^T(s) E_i^T \beta_i^{d_M} Z_{2ij} E_i \eta(s)
\end{aligned}$$

Adopting any appropriately dimensioned matrices X_{ij}, we introduce

$$\sum_{s=k-d_M}^{k-d(k)-1} \begin{bmatrix} \zeta(k) \\ E_i \eta(s) \end{bmatrix}^T \begin{bmatrix} X_{ij} \beta_i^{-d_M} Z_{2ij}{}^{-1} X_{ij}^T & X_{ij} \\ X_{ij}^T & \beta_i^{d_M} Z_{2ij} \end{bmatrix} \begin{bmatrix} \zeta(k) \\ E_i \eta(s) \end{bmatrix} \ge 0$$

Then, we can write

$$\begin{aligned}
&- \sum_{s=k-d_M}^{k-d(k)-1} \eta^T(s) E_i^T \beta_i^{d_M} Z_{2ij} E_i \eta(s) \\
&\quad \le (d_M - d(k)) \zeta^T(k) X_{ij} \beta_i^{-d_M} Z_{2ij}{}^{-1} X_{ij}^T \zeta(k) \\
&\quad + 2\zeta^T(k) X_{ij} E_i [x(k-d(k)) - x(k-d_M)]
\end{aligned} \tag{23}$$

The same for any Y_{ij}, we get

$$\begin{aligned}
&- \sum_{s=k-d(k)}^{k-\tau m-1} \eta^T(s) E_i^T \beta_i^{d_M} Z_{2ij} E_i \eta(s) \\
&\quad \le (d(k) - \tau m) \zeta^T(k) Y_{ij} \beta_i^{-d_M} Z_{2ij}{}^{-1} Y_{ij}^T \zeta(k) \\
&\quad + 2\zeta^T(k) Y_{ij} E_i [x(k-\tau m) - x(k-d(k))]
\end{aligned} \tag{24}$$

According to (23) and (24) and letting $\overline{d}(k) = \frac{d_M - d(k)}{d_r}$, we can identify that

$$\begin{aligned}
&- \sum_{s=k-d_M}^{k-\tau m-1} \eta^T(s) E_i^T \beta_i^{d_M} Z_{2ij} E_i \eta(s) \\
&\le \zeta^T(k) \Big(d_r \overline{d}(k) X_{ij} \beta_i^{-d_M} Z_{2ij}{}^{-1} X_{ij}^T \\
&+ d_r (1 - \overline{d}(k)) Y_{ij} \beta_i^{-d_M} Z_{2ij}{}^{-1} Y_{ij}^T + 2\bar{\Gamma}_{1ij} \Big) \zeta(k)
\end{aligned} \tag{25}$$

Next, based on lemma 2.1, the following inequality holds

$$\begin{aligned}
&\Delta_\beta V_{ij5}(k) \le \frac{d_M(d_M+1)}{2} \eta^T(k) E_i^T Z_{3ij} E_i \eta(k) \\
&+ \zeta_1^T(k) \Pi_R \zeta_1(k) + \frac{d_M(d_M+1)}{2} \zeta_1^T(k) \mathbb{R}^T (\beta_i^{d_M} Z_{3ij})^{-1} \mathbb{R} \zeta_1(k)
\end{aligned} \tag{26}$$

In addition, for any free-weighting matrices F_s, $s = 1,2,3$ with appropriate dimensions, we have

$$\begin{aligned}
&2\zeta^T(k) \bar{\mathbb{F}} \times \big[(\bar{A}_{ij} - E_i) x(k) \\
&\quad + A_{di} x(k-d(k)) + f_n(x(k)) - E_i \eta(k) \big] = 0
\end{aligned} \tag{27}$$

On the other hand, it is clear that

$$2\zeta^T(k) \bar{H}_4 \mathcal{S}_i \mathcal{R}_i^T \bar{H}_3 \zeta(k) = 0 \tag{28}$$

Furthermore, we can get from (5) that for any scalars $\varepsilon_{1i} > 0$

$$\tilde{f}(k) = 2\varepsilon_{1i} f_n^T(x(k)) \Big[f_n(x(k)) - Sx(k) \Big] \le 0$$

From (22) to (28) and since $-\tilde{f}(k) > 0$, we can see that

$$\Delta_\beta V(k) \le \zeta^T(k) \Big(\overline{d}(k) \Xi_{1ij} + (1 - \overline{d}(k)) \Xi_{2ij} \Big) \zeta(k) \tag{29}$$

where

$$\begin{aligned}
\Xi_{1ij} &= \bar{\Upsilon}_{ij} + d_M (\mathbb{T} \bar{H}_T)^T (\beta_i^{d_M} Z_{1ij})^{-1} (\mathbb{T} \bar{H}_T) \\
&+ \tilde{d}_M (\mathbb{R} \bar{H}_R)^T (\beta_i^{d_M} Z_{3ij})^{-1} (\mathbb{R} \bar{H}_R) + d_r X_{ij}^T (\beta_i^{d_M} Z_{2ij})^{-1} X_{ij} \\
\Xi_{2ij} &= \bar{\Upsilon}_{ij} + d_M (\mathbb{T} \bar{H}_T)^T (\beta_i^{d_M} Z_{1ij})^{-1} (\mathbb{T} \bar{H}_T) \\
&+ \tilde{d}_M (\mathbb{R} \bar{H}_R)^T (\beta_i^{d_M} Z_{3ij})^{-1} (\mathbb{R} \bar{H}_R) + d_r Y_{ij}^T (\beta_i^{d_M} Z_{2ij})^{-1} Y_{ij}
\end{aligned}$$

We have $0 \leq \bar{d}(k) \leq 1$, which main that $(\bar{d}(k)\Xi_{1ij} + (1-\bar{d}(k))\{\Xi_{2ij})$ is a convex combination of Ξ_{1ij} and Ξ_{2ij}. If inequalities in (13) are verified, by applying the Schur complement, it is possible to prove that $(\bar{d}(k)\Xi_{1ij} + (1-\bar{d}(k))\Xi_{2ij}) < 0$ and $\Delta_\beta V(k) < 0$. On the other hand, when $k \in [k_m + \Delta_m, k_{m+1})$, the system and the controller are activated synchronously. Then, following the same previous proof for the matched intervals, with $\Delta_\alpha V(k) = V_i(x(k+1)) - \alpha_i V_i(x(k))$, we can easily get $\Delta_\alpha V(k) < 0$.

From (14), (15) and the expression of $\Delta_\alpha V(k)$ and $\Delta_\beta V(k)$, for $k \geq k_m + \Delta_m$, we have

$$\begin{aligned}
V_{\sigma(k)}(x(k)) \leq & \alpha_{\sigma(k_m)}^{k-k_m-\Delta_m} V_{\sigma(k_m)}(x(k_m+\Delta_m)) \\
\leq & \alpha_{\sigma(k_m)}^{k-k_m-\Delta_m} \mu_{1\sigma(k_m)} V_{\sigma(k_m)\bar{\sigma}(k_m)}(x(k_m+\Delta_m)) \\
\leq & \alpha_{\sigma(k_m)}^{k-k_m-\Delta_m} \mu_{1\sigma(k_m)} \mu_{2\sigma(k_m)} \beta_{\sigma(k_m)}^{\Delta_m} V_{\sigma(k_m-1)}(x(k_m-1)) \\
\leq & \alpha_{\sigma(k_m)}^{k-k_m-\Delta_m} \alpha_{\sigma(k_{m-1})}^{k_m-k_{m-1}-1-\Delta_{m-1}} \mu_{1\sigma(k_m)} \mu_{1\sigma(k_{m-1})} \\
& \mu_{2\sigma(k_m)} \mu_{2\sigma(k_{m-1})} \beta_{\sigma(k_m)}^{\Delta_m} \beta_{\sigma(k_{m-1})}^{\Delta_{m-1}} V_{\sigma(k_{m-1}-1)}(x(k_{m-1}-1)) \\
\leq \ldots \leq & exp\Big\{ \sum_{i=1}^{N} (ln(\alpha_i) \sum_{s\in\theta(i)} (k_{s+1}-k_s-\Delta_{ni}) + ln(\beta_i) \sum_{s\in\theta(i)} \Delta_{ni}) \Big\} \\
& \prod_{i=1}^{N} (\mu_{1i}\mu_{2i})^{N_{\sigma i}(0,k)} V_{\sigma(0)}(x(0)) \\
\leq & exp\Big\{ \sum_{i=1}^{N} N_{0i} ln(\mu_{1i}\mu_{2i}) \Big\} exp\Big\{ \sum_{i=1}^{N} ln(\mu_{1i}\mu_{2i}) T_i(0,k)/\tau_{ai} \\
& \sum_{i=1}^{N} (ln(\alpha_i)(-T_i(0,k) - \Delta_{ni} N_{\sigma i}(0,k)) \\
& + ln(\beta_i)\Delta_{ni} N_{\sigma i}(0,k)) \Big\} V_{\sigma(0)}(x(0)) \\
\leq & exp\Big\{ \sum_{i=1}^{N} N_{0i}(ln(\mu_{1i}\mu_{2i}) + ln(\frac{\beta_i}{\alpha_i})\Delta_{ni}) \Big\} \\
& exp\Big\{ \sum_{i=1}^{N} \Big(ln(\mu_{1i}\mu_{2i})/\tau_{ai} - ln(\alpha_i) + ln(\frac{\beta_i}{\alpha_i})\Delta_{ni}/\tau_{ai} \Big) \\
& \times T_i(0,K) \Big\} V_{\sigma(0)}(x(0))
\end{aligned}$$

where $\theta(i) = \{s : \sigma(k_s) = i, k_s \in \{k_0, k_1, \ldots, k_{N_\sigma(0,k)}\}\}$. Setting $\mathcal{L} = exp\Big\{ \sum_{i=1}^{N} N_{0i}(ln(\mu_{1i}\mu_{2i}) + ln(\frac{\beta_i}{\alpha_i})\Delta_{ni}) \Big\}$ and $-\chi = \max_{i\in\mathbb{I}} \Big(ln(\mu_{1i}\mu_{2i})/\tau_{ai} - ln(\alpha_i) + ln(\frac{\beta_i}{\alpha_i})\Delta_{ni}/\tau_{ai} \Big)$, we obtain

$$V_{\sigma(k)}(x(k)) \leq \mathcal{L} e^{-\chi k} V_{\sigma(0)}(x(0)) \tag{30}$$

Then, from Definition 3, we can conclude that system (12) is exponentially admissible. ∎

By applying Lemma 2.2, we obtain the following result.

Theorem *3.2:* Given tunable scalars $0 < \alpha_i < 1$, $\beta_i \geq 1$, $\mu_{1i} > 1$, $\mu_{2i} > 1$, positive integers d_m and d_M and K_{ci} given matrices with appropriate dimensions. Switched singular system (12) is exponentially admissible, if there exist matrices $P_i > 0$, $P_{ij} > 0$, $Q_{1i} > 0$, $Q_{1ij} > 0$, $Q_{2i} > 0$, $Q_{2ij} > 0$, $Q_{3i} > 0$, $Q_{3ij} > 0$, $Z_{1i} > 0$, $Z_{1ij} > 0$, $Z_{2i} > 0$, $Z_{2ij} > 0$, $Z_{3i} > 0$, $Z_{3ij} > 0$, T_1, T_2, R_1, R_2, X_i, X_{ij}, Y_i, Y_{ij}, S_i, F_s, $s = 1,2,3$, X_{ck}, W_{ki} and positive scalars ε_{1i} such that the following inequalities hold for all $(i,j) \in \mathbb{I}\times\mathbb{I}$, $i \neq j$,

$$\begin{aligned}
&\begin{bmatrix} \Sigma_{Xi}(A_{ci}) & \mathbb{F}_{Bi} \\ * & 0 \end{bmatrix} + sym\{\mathcal{I}\ \mathbb{W}_{ki}\} < 0 \\
&\begin{bmatrix} \Sigma_{Yi}(A_{ci}) & \mathbb{F}_{Bi} \\ * & 0 \end{bmatrix} + sym\{\mathcal{I}\ \mathbb{W}_{ki}\} < 0 \\
&\begin{bmatrix} \Sigma_{Xij}(A_{cij}) & \mathbb{F}_{Bi} \\ * & 0 \end{bmatrix} + sym\{\mathcal{I}\ \mathbb{W}_{kj}\} < 0 \\
&\begin{bmatrix} \Sigma_{Yij}(A_{cij}) & \mathbb{F}_{Bi} \\ * & 0 \end{bmatrix} + sym\{\mathcal{I}\ \mathbb{W}_{kj}\} < 0
\end{aligned} \tag{31}$$

and any switching rule satisfying (14) and (15).

$$\begin{aligned}
&\mathcal{I} = \begin{bmatrix} 0 \\ \mathbb{O}^T \\ I \end{bmatrix}, \ \mathbb{W}_{ki} = \begin{bmatrix} (W_{ki} - X_{ck}K_{ci})C_i & \mathbb{O} & -X_{ck} \end{bmatrix}, \\
&\mathbb{F}_{Bi}^T = \begin{bmatrix} B_i^T F_1 & 0 & B_i^T F_2 & 0 & B_i^T F_3 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
&\mathbb{O} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\
&A_{ci} = A_i + B_i K_{ci} C_i + S_1 I_n, \ A_{cij} = A_i + B_i K_{cj} C_i + S_1 I_n.
\end{aligned} \tag{32}$$

The controller gains K_i are given by :

$$K_i = X_{ck}^{-1} W_{ki} \tag{33}$$

Proof: To design static output feedback controller, we introduce some auxiliary variables K_{ci} and we get

$$\begin{aligned}
\bar{A}_{ki} &= A_i + B_i(K_i - K_{ci})C_i + B_i K_{ci} C_i + S_1 I_n, \\
\bar{A}_{kij} &= A_i + B_i(K_j - K_{cj})C_i + B_i K_{cj} C_i + S_1 I_n
\end{aligned} \tag{34}$$

Performing Theorem 3.1 to (34), the following inequalities hold

$$\begin{aligned}
&\Sigma_{Xi}(A_{ci}) + sym(\mathbb{F}_{Bi}\mathbb{K}_i) < 0, \ \Sigma_{Yi}(A_{ci}) + sym(\mathbb{F}_{Bi}\mathbb{K}_i) < 0 \\
&\Sigma_{Xij}(A_{cij}) + sym(\mathbb{F}_{Bi}\mathbb{K}_j) < 0, \ \Sigma_{Yij}(A_{cij}) + sym(\mathbb{F}_{Bi}\mathbb{K}_j) < 0
\end{aligned} \tag{35}$$

where $\mathbb{K}_i = \begin{bmatrix} (K_i - K_{ci})C_i & \mathbb{O} \end{bmatrix}$. By applying Lemma 2.2 to (35), conditions in (31) hold for $W_{ki} = X_{ck}K_i$. ∎

Remark *3.2:* Differently from [12], [3], [13], the presented LMIs in Theorem 3.2 are independent. Therefore, it can be solved in one step and the controller gains can be calculated without using an iterative algorithm.

IV. Numerical Example

To evaluate the effectiveness of the proposed controller design approach, we consider the following example:

$$E_1 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, B_{11} = \begin{bmatrix} 0.16 \\ 0.16 \\ 0.21 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} 0.3 & 0.2 & 0.4 \\ -0.3 & -0.2 & 0.15 \\ -0.3 & 0.1 & 0.4 \end{bmatrix}, A_2 = \begin{bmatrix} 0.4 & 0.2 & 0.1 \\ 0 & -0.2 & 0.15 \\ -0.3 & 0.1 & 0.9 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} 0.1 & 0.01 & 0 \\ 0.1 & 0.03 & -0.1 \\ 0.1 & 0.02 & 0.01 \end{bmatrix}, B_{12} = \begin{bmatrix} 0.35 \\ 0.35 \\ 0.37 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -0.01 & 0.01 & 0 \\ 0.01 & 0.03 & -0.03 \\ 0.1 & 0.02 & 0.05 \end{bmatrix}, C_1 = C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

and the nonlinearity is chosen by $f(x(k)) = \begin{bmatrix} x_1 - 0.04x_2 + 0.2(S_2 - S_1)tanh(x_1) \\ -0.6x_3 + (S_2 - S_1)tanh(x_2) \\ 0.2x_1 + (S_2 - S_1)tanh(x_3) \end{bmatrix}$, with $S_2 = 0.4$ and $S_1 = 0.1$. Set $d_m = 2$, $d_M = 3$, $K_{c1} = [-1.4\ -1]$, $K_{c2} = [-1\ 0]$ and taking parameters $\mu_{11} = 1.15$, $\mu_{12} = 1.0203$, $\mu_{21} = 1.5$, $\mu_{22} = 3$, $\alpha_1 = 0.1$, $\alpha_2 = 0.5$, $\beta_1 = 1.03$, $\beta_2 = 1.01$ which implies respectively $\tau^*_{a1} = 1.2496$ and $\tau^*_{a2} = 2.1211$, a feasible solution is given by Theorem 3.2 with the following associated controller gains

$$K_1 = \begin{bmatrix} -1.8621 & -1.3301 \end{bmatrix}, \ K_2 = \begin{bmatrix} -1.0001 & -0.0001 \end{bmatrix} \tag{36}$$

The simulation results are shown in Figs.1-2 with initial states $\phi_0(k) = [-1\ -4\ 2]^T$, $k = -3, \cdots, 0$. Fig.1 depicts the state trajectories and the switching signal of the system with $\tau_{a1} = 1.3$ and $\tau_{a2} = 2.3$ and the controller with lag periods $\Delta_{n1} = 0.8$, $\Delta_{n2} = 0.4$. The control input is shown in Fig.2 and the time varying delay is considered as a repeating sequence in Fig.3.

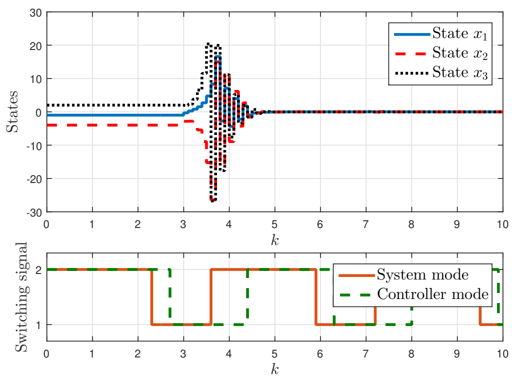

Fig. 1. Trajectories of state $x(k)$ and switching signals

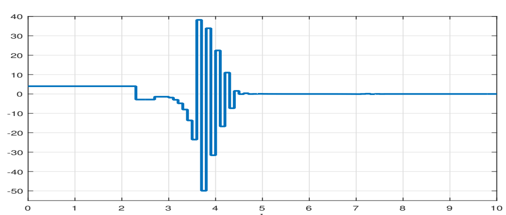

Fig. 2. Control signal $u(k)$

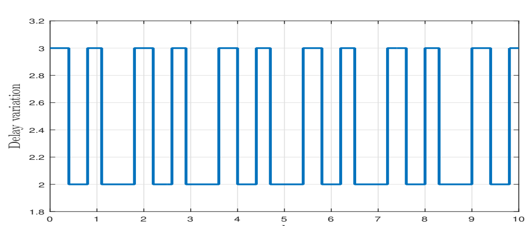

Fig. 3. Time varying delay

It is observed from these Figures that the proposed asynchronous control guarantees the convergence of the system state to zero and scheme effectively eliminates the effect of both nonlinearity and time delay. So, simulation results have confirmed the effectiveness of the proposed approach.

V. CONCLUSION

In this paper, the problem of admissibilization for a class of discrete-time switched singular systems subject to time-varying delay and nonlinearity has been investigated. Under asynchronous switching and by solving a set of LMIs, sufficient conditions have been derived to synthesis a static output feedback controller based on mode-dependent average dwell time approach and using an appropriate Lyapunov-Krasovskii functional. The merit and the effectiveness of the proposed controller have been verified through a numerical example. How to extend the proposed method to deal with fault-tolerant control of nonlinear singular systems is a work worthy of future study.

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