Các bài 2.40 đến 2.42 làm như Ánh là được

Bài tập:

$f(x) = \dfrac{1}{x^2 - 3x + 2} = \dfrac{1}{(x-1)(x-2)}$ tính $f^{(2008)}(0)$

2.40. Ta có $f(x) = \dfrac{-1}{x-1} + \dfrac{1}{x-2} = \dfrac{1}{x-2} - \dfrac{1}{x-1}$

Ta có $f^{(n)}(x) = \left(\dfrac{1}{x-2}\right)^n - \left(\dfrac{1}{x-1}\right)^n$

$\Rightarrow f^{(2008)}(x) = (-1)^{2008} \cdot 2008! \dfrac{1}{(x-2)^{2009}} - (-1)^{2008} \cdot 2008! \dfrac{1}{(x-1)^{2009}}$

$\Rightarrow f^{(2008)}(x) = \dfrac{2008!}{(-2)^{2009}} + 2008!$

2.41. Cho hàm số $f(x) = \dfrac{1}{x^2}$ tính $f^{(2008)}(1)$

$\cancel{f(x) = \dfrac{1}{x^2}} = \left(\dfrac{1}{x}\right)^2$ mà $\cancel{\left(\dfrac{1}{x}\right)^{(n)}} = \cancel{(-1)^n} . n! \cancel{\dfrac{1}{x^{n+1}}}$

Ta có $f(x) = \dfrac{1}{x^2} = x^{-2}$

$f'(x) = -2x^{-3} = -2! . x^{-3}$

$f''(x) = -2.(-3)x^{-4} = 3! \ x^{-4}$

$f'''(x) = -2(-3).(-4)x^{-5} = -4! \ x^{-5}$

.....

$f^{n}(x) = (-1)^n (n+1)! \ x^{-(n+2)}$

$\Rightarrow f^{(2008)}(1) = (-1)^{2008} . 2009! \ 1^{2010}$

$= 2009!$

2.42. $f(x) = \ln \frac{x+1}{x-1}$ tính $f^{(2008)}(2)$

Có: $f(x) = \ln(x+1) - \ln(x-1)$

$\Rightarrow f^{(n)}(x) = [\ln(x+1)]^{(n)} - [\ln(x-1)]^{(n)}$

$= \frac{(-1)^{n-1}.(n-1)!\,1^n}{(x+1)^n} - \frac{(-1)^{n-1}.(n-1)!\,1^n}{(x-1)^n}$

$\Rightarrow f^{(2008)}(2) = \frac{(-1)^{2008-1}.(2008-1)!.}{3^{2008}} - \frac{(-1)^{2007}.\,2007!}{1^{2008}}$

$= \frac{-2007!}{3^{2008}} + 2007!$

2.43 $f(x) = \frac{1+x}{1-x}$ tính $f^{(n)}(0)$

$f(x) = \frac{1}{1-x} + \frac{x}{1-x} \Rightarrow f^{(n)}(x) = \left(\frac{1}{1-x}\right)^n + \left(\frac{x}{1-x}\right)^n$

Ta có: $\left(\frac{1}{1-x}\right)^n = \frac{(-1)^n.\,n!\,(-1)^n}{(1-x)^n} \Rightarrow x=0 \Rightarrow \left(\frac{1}{1-x}\right)^n = \frac{(-1)^n.\,n!\,(-1)^n}{1^n}$

+) $\left(\frac{x}{(1-x)}\right)^n = y$

$y' = \frac{1}{(1-x)^2} = (1-x)^{-2}$

$y'' = (-2).(-1)(1-x)^{-3} = 2!(1-x)^{-2-1}$

$y''' = (-1).(-2).(-3).(-1)(1-x)^{-4} = 3!(1-x)^{-3-1}$

$y^{(4)} = (-1)(-2)(-3)(-4).(-1).(-1)(1-x)^{-5} = 4!(1-x)^{-4-1}$

....

$y^{(n)} = n!(1-x)^{-n-1}$

$\Rightarrow \left(\frac{x}{(1-x)}\right)^n = n!(1-x)^{-n-1} \Rightarrow$ vs $x = 0$

có $n!\,1^{-n-1} = n!$

$\Rightarrow f^{n}(0) = (-1)^n.\,n!\,(-1)^n + n! = 2.\,n!$

2.44: $f(x) = \ln \frac{1}{1-x}$ tính $f^{(n)}(0)$

Ta có $f(x) = \ln \frac{1}{1-x} = \ln(1) - \ln(1-x) = -\ln(1-x)$

$$f^{(n)}(x) = \left[-\ln(1-x)\right]^{(n)} = -\frac{(-1)^{n-1}(n-1)!\,(-1)^{n}}{(1-x)^{n}}$$

$$\Rightarrow f^{n}(0) = (-1)^{2n-1}(n-1)!$$

2.45 Tính $y''(x)$ với $y(x) = (1+x)^{\frac{1}{x}}$

Ta có $\ln y(x) = \ln(1+x)^{1/x} = \frac{1}{x}\ln(1+x)$

$$\Rightarrow \frac{y'(x)}{y(x)} = \frac{-1}{x^2}\cdot\ln(1+x) + \frac{1}{x}\frac{1}{1+x}$$

$$\Rightarrow y'(x) = y(x)\cdot\frac{-1}{x^2}\ln(1+x) + y(x)\cdot\frac{1}{x(x+1)}$$

$$y'' = y'(x)\cdot\frac{-1}{x^2}\cdot\ln(1+x) + y(x)\cdot\frac{2}{x^3}\ln(1+x) - y(x)\cdot\frac{1}{x^2}\cdot\frac{1}{1+x} + y'(x)\cdot\frac{1}{x(x+1)}$$

$$+ y(x)\cdot\frac{-2x-1}{(x^2+x)^2}$$

$$= -(1+x)^{1/x}\cdot\frac{1}{x^4}\left[\ln(1+x)\right]^2 + (1+x)^{1/x}\frac{2}{x^3}\ln(1+x) - (1+x)^{1/x}\cdot\frac{1}{x^2}\cdot\frac{1}{x+1}$$

$$- (1+x)^{1/x}\cdot\frac{1}{x^2}\cdot\ln(1+x)\frac{1}{x(x+1)} + (1+x)^{1/x}\cdot\frac{2x-1}{(x^2+x)^2}$$

2.89 $f(x) = (x^2+1)\sin x$. Tính $f^{(20)}(x)$

Đặt $g(x) = \sin x$ $h(x) = x^2+1$

Có $g^{(n)}(x) = \sin\left(x + \frac{n\pi}{2}\right)$

$h'(x) = 2x$

$h''(x) = 2$

... $h^{(k)}(x) = 0$ với $k \geq 3$

ADCT Newton - Leibnitz

$$f^{(n)}(x) = \sum_{k=0}^{n} C_n^k \, g^{(n-k)}(x) \, . \, h^{(k)}(x)$$

$$= C_n^0 \sin\left(x + \frac{n\pi}{2}\right) . (x^2+1) + C_n^1 . \sin\left(x + \frac{(n-1)\pi}{2}\right) . 2x$$

$$+ C_n^2 \sin\left(x + \frac{(n-2)\pi}{2}\right) . 2 + 0 \ldots + 0$$

$$\Rightarrow f^{(20)}(x) = \sin\left(x + \frac{20\pi}{2}\right)(x^2+1) + 20 . \sin\left(x + \frac{19\pi}{2}\right) . 2x$$

$$+ 190 . \sin(x + 9\pi) . 2 .$$

Bài tập trong đề thi

Câu 2: $y = (x^3 - 2x + 5) \cos 3x$ tìm đạo hàm cấp cao $y^{(2016)}(\pi)$

Đặt $g(x) = \cos 3x$, $h(x) = x^3 - 2x + 5$

$$g^{(n)}(x) = 3^n \cos\left(3x + \frac{n\pi}{2}\right)$$

$h'(x) = 3x^2 - 2$ $\quad$ $h''(x) = 6x$ $\quad$ $h'''(x) = 6$.

AD CT Newton - Leibnitz.

$$y^{(n)} = \sum_{k=0}^{n} C_n^k \, g^{(n-k)}(x) . \, h^{(k)}(x)$$

$$= C_n^0 \, 3^n \cos\left(3x + \frac{n\pi}{2}\right)(x^3 - 2x + 5) + C_n^1 \, 3^{n-1} . \cos\left(3x + \frac{(n-1)\pi}{2}\right) . (3x^2 - 2)$$

$$+ C_n^2 \, 3^{n-2} . \cos\left(3x + \frac{(n-2)\pi}{2}\right) . 6x + C_n^3 . 3^{n-3} . \cos\left(3x + \frac{(n-3)\pi}{2}\right) . 6 + 0 \ldots + 0$$

$$\Rightarrow y^{(2016)}(\pi) = -3^{2016}(\pi^3 - 2\pi + 5) + 6\pi . 3^{2014} . C_{2016}^2$$

Câu 3 Cho hàm số $f(x) = (3x^2 + 1) \sin^2 x$. Tìm đạo hàm cấp cao $f^{(2017)}(\pi)$

<u>Giải</u>

ta có $f(x) = (3x^2 + 1) \sin^2 x = \frac{1}{2}(3x^2 + 1)(1 - \cos 2x)$

$(3x^2+1)' = 6x$; $(3x^2+1)'' = 6$ $\quad (3x^2+1)^{(n)} = 0$

$$(1-\cos 2x)^{(n)} = -2^n \cdot \cos\left(2x + \frac{n\pi}{2}\right)$$

2. $\cancel{2 f^{(2017)}(x) = C^0_{2017}(3x+1)(1-\cos 2x}$

$$f^{(2017)}(x) = \frac{1}{2}\sum_{k=0}^{2017}\left[C^k_{2017}(3x^2+1)^{(k)}\cdot(1-\cos 2x)^{(2017-k)}\right]$$

$$\Rightarrow 2f^{(2017)}(x) = C^0_{2017}(3x^2+1)\cdot -2^{2017}\cdot\cos\left(2x+2017\frac{\pi}{2}\right)$$

$$+ C^1_{2017}\cdot 6x\cdot -2^{2016}\cdot\cos\left(2x+\frac{2016\pi}{2}\right) + C^2_{2017}\cdot 6\cdot -2^{2015}\cdot\cos\left(2x+2015\cdot\frac{\pi}{2}\right)$$

$$= 2^{2017}\cdot(3x^2+1)\sin(2x) - 2^{2016}\cdot C^1_{2017}\, 6x\cdot\cos(2x) - 3\cdot 2^{2016}\cdot C^2_{2017}\cdot\sin 2x$$

$$= 2^{2016}\cdot\left[2(3x^2+1)\sin 2x - 6C^1_{2017}\cdot x\cos 2x - 3C^2_{2017}\cdot\sin 2x\right]$$

$$\Rightarrow 2f^{(2017)}\pi = 2^{2016}\cdot\left[2(3\pi^2+1)\cdot\sin 2\pi - 6\cdot C^1_{2017}\cdot\pi\cdot\cos 2\pi - 3\cdot C^2_{2017}\sin 2\pi\right]$$

$$\Rightarrow f^{(2017)}(\pi) = 6051\cdot 2^{2016}\cdot\pi.$$

Câu 3. Tính đạo hàm $y''(x)$ của hls

$$\begin{cases} x = t - \sin t. \\ y = 1 - \cos t. \end{cases}$$

Ta có $\begin{cases} x'(t) = 1 - \cos t. \\ y'(t) = 1 - \sin t. \end{cases} \Rightarrow y'(x) = \dfrac{y'(t)}{x'(t)} = \dfrac{1-\sin t}{1-\cos t}.$

sai rồi

$\cancel{\Rightarrow y''(x) = \left(\dfrac{1-\sin t}{1-\cos t}\right)'}$

y''_xx = (y'_x)'_t / x'_t

Ánh quên chưa chia cho x'_t

$$\Rightarrow y'' = \frac{-\cos t(1-\cos t) - (1-\sin t)\cdot\sin t}{(1-\cos t)^2} = \frac{1-\sin t-\cos t}{(1-\cos t)^2}.$$