Câu 3a:

$$\begin{cases} 2x_1 + 4x_2 + 3x_3 = 3 \\ 3x_1 + x_2 - 2x_3 = 3 \\ 4x_1 + 11x_2 + 7x_3 = 4 \end{cases}$$

$$[A|b] = \left[\begin{array}{ccc|c} 2 & 4 & 3 & 3 \\ 3 & 1 & -2 & 3 \\ 4 & 11 & 7 & 4 \end{array}\right] \xrightarrow[H_3 \to H_3 - 2H_1]{H_2 \to H_2 - \frac{3}{2}H_1} \left[\begin{array}{ccc|c} 2 & 4 & 3 & 3 \\ 0 & -5 & -\frac{13}{2} & -\frac{3}{2} \\ 0 & 3 & 1 & -2 \end{array}\right] \xrightarrow{H_3 + \frac{3}{5}H_2} \left[\begin{array}{ccc|c} 2 & 4 & 3 & 3 \\ 0 & -5 & -\frac{13}{2} & -\frac{3}{2} \\ 0 & 0 & -\frac{29}{10} & -\frac{29}{10} \end{array}\right]$$

U | $\tilde{b}$

Giải hệ từ dưới lên:

$$\begin{cases} x_3 = 2 \\ x_2 = -1 \\ x_1 = 1 \end{cases}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ 2 & -\frac{3}{5} & 1 \end{bmatrix}$$

$Ux = \tilde{b}$

$Ax = b \xrightarrow{①} L_1 Ax = L_1 b$ với $L_1 = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{2} & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$\xrightarrow{②} L_2 L_1 Ax = L_2 L_1 b$ với $L_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{3}{5} & 1 \end{bmatrix}$

Thấy dạng như thế nào

$$L_2 L_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{3}{5} & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{2} & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{2} & 1 & 0 \\ -\frac{29}{10} & \frac{3}{5} & 1 \end{bmatrix}$$

$$L = (L_2 L_1)^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ 2 & -\frac{3}{5} & 1 \end{bmatrix}$$

$$\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21} & u_{22} & u_{23} \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$$

$$U = L_2 L_1 A = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{3}{2} & 1 & 0 \\ -\frac{29}{10} & \frac{3}{5} & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 3 \\ 3 & -1 & -2 \\ 4 & 11 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 4 & 3 \\ 0 & -5 & -\frac{13}{2} \\ 0 & 0 & -\frac{29}{10} \end{bmatrix}$$