Delay-Dependent Exponential Stability Analysis of Non-linear Switched Singular Systems with Average Dwell Time Approach

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This manuscript analyzed the delay-dependent stability problem for non-linear switched singular time-delay systems based on average dwell time and delay decomposition approach. Both cases of time-delays namely constant and time-varying delays are treated in the switched singular systems. Based on piecewise Lyapunov–Krasovskii functional, linear matrix inequality technique and an average dwell time approach, sufficient conditions which ensures that the delay-dependent exponential stability conditions for uncertain switched singular systems are discussed. Finally, numerical examples and simulations are provided to demonstrate the effectiveness of the proposed techniques.

Key Words: switched singular systems; average dwell time; delay decomposition; linear matrix inequality; time-varying delay

1. INTRODUCTION

Singular systems such as descriptor systems, implicit systems, differential-algebraic systems are found in engineering systems (electrical circuit network, power systems, aerospace engineering and chemical processing), social systems, economic systems, biological systems, network analysis, time-series analysis, and singularly perturbed systems. Their form also makes them useful in system modeling [1–3]. It is worth mentioning that a singular system usually has three kinds of modes that is, finite dynamic modes, infinite dynamic modes and infinite nondynamic modes. Infinite dynamic modes can generate undesired impulsive behavior.

In the state-space case, neither infinite dynamic modes nor infinite non-dynamic modes can arise. In addition, regularity and non-impulsiveness are satisfied automatically in the state-space systems. It should be pointed out that the stability problem of singular systems is much more complicated than that for state-space systems, because it requires not only stability but also regularity and impulse-free conditions for continuous systems [4].

A switched system is an important class of hybrid dynamical system consisting of a family of continuous-

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time subsystems or discrete-time subsystems and a rule that manages the switching between them. Many dynamical systems are modeled as switched systems by means of special class of hybrid systems [5]. Switched systems have a strong background in engineering part and studying the problem of stability is difficult to study when the complicated behavior caused by the interaction between the continuous dynamics and discrete switching. Recently, many researchers have given importance to study about the controllability, observability, stability & stabilization and filter designs of the switched systems. Among these research topics, stability & stabilization analysis for switched systems have attracted more researchers [6–13].

Time-delays are often occurring in many dynamical systems, and they are the main cause of instability and poor performance of dynamical systems. Hence, it is crucial to analyze the stability of switched dynamical systems with time-delays. Recently, more importance have been paid to analyze these types of problems in different areas such as stability and stabilization, H_∞ control design & filtering, output feedback control problem, and neural networks [14–25].

The average dwell time (ADT) switching is attractive and more flexible in system stability analysis. ADT switching means that the number of switches in a finite interval is bounded and the average time between consecutive switching is not less than a constant. Many advanced results for the switched systems with ADT switching have been reported within the continuous-time context including both linear and non-linear cases [9,26–33]. In continuous-time-domain approach, Xing et al. [24], analyzed the stability analysis of continuous-time switched singular time-delay systems with stable subsystems and Zamani et al. [34,35], discussed the switched singular system with ADT as well as time delay. Recently, researchers have paid attention to investigate the delay-dependent stability analysis using the delay-decomposition approach [36,37]. At the same time, a number of interesting new ideas have been proposed recently to improve the delay-dependent stability criteria for time-delay systems with stochastic, feedback times, switched system, synchronization, and sampled data control [38–43].

Motivated by the above discussions, the problem of non-linear switched singular systems with ADT is discussed based on delay decomposition approach. By constructing suitable piecewise Lyapunov–Krasovskii functionals (LKFs), sufficient delay-dependent exponential stability conditions are derived in terms of linear matrix inequality (LMI) technique. Furthermore, these results are extended to uncertain systems based on norm bounded uncertainty conditions. Finally, two numerical examples are given to demonstrate the effectiveness and the benefits of the proposed method.

In Section 2, the problem is stated and the basic definitions, preliminaries, lemmas are provided. Section 3 deals with the main results on exponential stability analysis of switched singular systems. Section 4 investigates the robustness problem of the main results in Section 3. In Section 5, numerical examples are dealt to demonstrate the effectiveness and less conservativeness of the derived theoretical results and Section 6 concludes the article.

Notation: Throughout this article, $X \in \mathbb{R}^n$ and $X \in \mathbb{R}^{n\times p}$ denotes the n-dimensional Euclidean space and the set of all $n \times p$ real matrices, respectively. $\mathbb{C}([a,b],\mathbb{R}^n)$ denotes the space of all continuous functions from $\phi : [a,b] \to \mathbb{R}^n$. The superscript T denotes the transposition and the notation $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). I_n is the $n \times n$ identity matrix. $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. The notation * always denotes the symmetric block in one symmetric matrix.

2. PROBLEM DESCRIPTION AND PRELIMINARIES

Consider the following switched non-linear singular systems with time-varying delay

$$\begin{aligned} E_{\sigma(t)}\dot{x}(t) &= A_{\sigma(t)}x(t)+B_{\sigma(t)}x(t-\tau(t))+F_{1\sigma(t)}f_{1\sigma(t)}(t,x(t)) \\ &\quad +F_{2\sigma(t)}f_{2\sigma(t)}(t,x(t-\tau(t))), \\ x(t) &= \phi(t),\ t \in [-\tau_2, 0], \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $\tau(t)$ is the time-varying delay in the state, τ_2 is an upper bound of the delay $\tau(t)$ and it is assumed to satisfy the condition $0 \leq \tau(t) \leq \tau_2$ and $\dot{\tau}(t) \leq \mu$, where τ_2 and μ are constants. $\phi(t)$ is a continuous initial vector function defined on the delay interval $[-\tau_2, 0]$, with the norm $\|\phi(t)\|_{\mathbb{C}} = \sup_{-\tau_2 \leq s \leq 0} \|\phi(s)\|$ which stands for the norm of initial condition $\phi(t)$. The switching law $\sigma(t) : \mathbb{R}^+ \to U = \{1,2,\ldots,u\}$ is a piecewise constant and right continuous function, where u is the number of modes of the overall switched system. The sequence of ordered pairs (t_0, i_0), (t_1, i_1), ... is said to be the switching sequence of $\sigma(t)$ over $[t_0, t_k)$. It is assumed that the value of $\sigma(t)$ is unknown but its instantaneous value is available in real time. The model (1) can be written in the following form. For the sake of convenience, $\sigma(t)$ is referred as σ

$$\begin{aligned} &E_\sigma\dot{x}(t)=A_\sigma x(t)+B_\sigma x(t-\tau(t))+F_{1\sigma}f_{1\sigma}(t,x(t))+F_{2\sigma}f_{2\sigma}(t,x(t-\tau(t))), \\ &x(t)=\phi(t),\ t \in [-\tau_2, 0]. \end{aligned} \tag{2}$$

For each possible value $\sigma(t) = i$, $i \in U$, denote the system matrices associated with mode i by $E_\sigma = E_i$, $A_\sigma = A_i$, $B_\sigma = B_i$, $F_{1\sigma} = F_{1i}$, $F_{2\sigma} = F_{2i}$ where $E_\sigma \in \mathbb{R}^{n\times n}$ is the coefficient matrix of the system and $\text{rank}(E_i) = r < n$. $A_i, B_i \in \mathbb{R}^{n\times n}$ are known real constant matrices. F_{1i} and $F_{2i} \in \mathbb{R}^{n\times n}$ are the coefficients of the non-linear functions f_{1i}

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and f_{2i} respectively. The following assumption, definitions, and lemmas which will be used in the sequel.

Assumption 2.1

The non-linear functions $f_{li}: \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^{n_l} (l=1,2)$ are continuous, satisfy $f_{li}(t,0) \equiv 0$ and the Lipschitz condition:

$$||f_{li}(t,x_0)-f_{li}(t,y_0)|| \le ||u_{li}(x_0-y_0)||, \forall t, \forall x_0, y_0 \in \mathbb{R}^n \text{ and } i \in U,$$

such that u_{li} are some known matrices of appropriate dimensions.

Definition 1 [4]

i. *The switched system (2) is said to be regular if $det(sE_i - A_i)$ is not identically zero.*
ii. *The switched system (2) is said to be impulse-free if $deg(det(sE_i - A_i)) = rank(E_i)$.*

Definition 2 [26]

The switched system (2) is said to be exponentially stable for every initial state $\phi(\cdot)$ under switching signal $\sigma(t)$ with convergence rate $\lambda > 0$ if there exist some positive scalars k, λ such that $||x(t,\phi)|| \le ke^{-\lambda(t-t_0)}||\phi||_{\mathbb{C}}, \forall t \ge t_0$ where $||\phi||_{\mathbb{C}} = \max_{-\tau_2 \le \theta \le 0} ||\phi(\theta)||$.

The continuous switched singular system in (2) is said to be exponentially admissible if it is regular, impulse-free and exponentially stable. Here, we consider that the each subsystem has a common equilibrium point, such as origin, then the equilibrium point of the switched singular system (2) is the origin.

Definition 3 [44]

For any $T_2 > T_1 \ge 0$, let $N_\sigma(T_1, T_2)$ denote the number of switchings of $\sigma(t)$ over (T_1, T_2). If $N_\sigma(T_1, T_2) \le N_0 + (T_2 - T_1)/T_a$ holds for $T_a > 0$, $N_0 \ge 0$ then T_a is called the ADT where N_0 is the chatter bound.

As commonly used in the literature, let $N_0=0$ in Definition 3.

Lemma 1 [45] (Schur Complement)

Given constant matrices Θ_1, Θ_2 and Θ_3 with appropriate dimensions, where $\Theta_1^T=\Theta_1$ and $\Theta_2^T=\Theta_2>0$, the inequality

$$\Theta_1+\Theta_3^T\Theta_2^{-1}\Theta_3 < 0,$$

holds, if and only if

$$\begin{bmatrix} \Theta_1 & \Theta_3^T \\ * & -\Theta_2 \end{bmatrix} < 0, \quad or \quad \begin{bmatrix} -\Theta_2 & \Theta_3 \\ * & \Theta_1 \end{bmatrix} < 0.$$

This lemma is used to work with the time-varying structured uncertainties in the system.

Lemma 2 [46]

Given matrices $Q=Q^T$, M, N, with appropriate dimensions, then $Q+MF_i(t)N+N^TF_i^T(t)M^T<0$ for all $F_i(t)$ satisfying $F_i^T(t)F_i(t) \le I$, if and only if there exists an $\epsilon_i > 0$ such that $Q+\epsilon_i^{-1}MM^T+\epsilon_iN^TN<0$.

Lemma 3 [46]

For any constant matrix $M>0$, any scalars a and b with $a<b$ and a vector function $x(t):[a,b] \to \mathbb{R}^n$ such that the integrals concerned as well defined, the following condition holds:

$$\left(\int_a^b x^T(s)ds\right)M\left(\int_a^b x(s)ds\right) \le (b-a)\left(\int_a^b x^T(s)Mx(s)ds\right).$$

3. MAIN RESULTS

In this section, the main results are given. Under Assumption 2.1, Definition 1, 2, 3, Lemma 1, 2, and 3 sufficient conditions for the exponential stability of the switched non-linear singular time-delay system are given using a piecewise LKF. The following theorem mainly consider the piecewise LKF, delay decomposition approach and ADT approach.

Theorem 3.1

For given scalars $\tau_2>0$, tuning parameter η $(0<\eta<1)$, μ and $\alpha>0$, the equilibrium point of system (2) is exponentially admissible and for any switching signal σ with ADT satisfying

$$T_a > T_a^* = \frac{\ln \mu_1}{\alpha}, \quad \mu_1 \ge 1, \tag{3}$$

if there exist symmetric positive-definite matrices P_i, Q_{1i}, Q_{2i}, Q_{3i}, R_{1i}, R_{2i}, R_{3i}, real matrices K_1, K_2, for any matrix S with appropriate dimension, some known matrices u_{1i}, u_{2i}, positive scalars $\epsilon_{1i}, \epsilon_{2i}$ and the constant matrix $R \in \mathbb{R}^{n\times(n-r)}$ satisfying $E_i^TR=0$ with $rank(R)=n-r$ such that the following symmetric LMIs hold:

$$\Theta_i^1 = \begin{bmatrix} \Theta_{1,1,i}^1 & \Theta_{1,2,i}^1 & 0 & 0 & -K_1^T+A_i^TK_2 & \Theta_{1,6,i}^1 & \Theta_{1,7,i}^1 \\ * & \Theta_{2,2,i}^1 & 0 & \frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^TR_{1i}E_i & B_i^TK_2 & 0 & 0 \\ * & * & \Theta_{3,3,i}^1 & \frac{e^{-\alpha\eta\tau_2}}{\tau_2-\eta\tau_2}E_i^TR_{2i}E_i & 0 & 0 & 0 \\ * & * & * & \Theta_{4,4,i}^1 & 0 & 0 & 0 \\ * & * & * & * & \Theta_{5,5,i}^1 & K_2^TF_{1i} & K_2^TF_{2i} \\ * & * & * & * & * & -\epsilon_{1i}I & 0 \\ * & * & * & * & * & * & -\epsilon_{2i}I \end{bmatrix} < 0 \tag{4}$$

$$
\Theta_i^2=\begin{bmatrix}
\Theta_{1,1,i}^2 & \Theta_{1,2,i}^2 & 0 & \frac{e^{-\alpha\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}R_{1i}E_i & -K_1^{\mathrm{T}}+A_i^{\mathrm{T}}K_2 & \Theta_{1,6,i}^2 & \Theta_{1,7,i}^2\\
* & \Theta_{2,2,i}^2 & \frac{e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i & \frac{e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i & B_i^{\mathrm{T}}K_2 & 0 & 0\\
* & * & \Theta_{3,3,i}^2 & 0 & 0 & 0 & 0\\
* & * & * & \Theta_{4,4,i}^2 & 0 & 0 & 0\\
* & * & * & * & \Theta_{5,5,i}^2 & K_2^{\mathrm{T}}F_{1i} & K_2^{\mathrm{T}}F_{2i}\\
* & * & * & * & * & -\epsilon_{1i}I & 0\\
* & * & * & * & * & * & -\epsilon_{2i}I
\end{bmatrix}<0 \tag{5}
$$

where

$$
\begin{gathered}
\Theta_{1,1,i}^1=(E_i^{\mathrm{T}}P_i+SR^{\mathrm{T}}+K_1^{\mathrm{T}})A_i+A_i^{\mathrm{T}}(P_iE_i+RS^{\mathrm{T}}+K_1)+Q_{1i}+Q_{3i}+\alpha E_i^{\mathrm{T}}P_iE_i+\epsilon_{1i}u_{1i}^{\mathrm{T}}u_{1i}\\
-\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}(R_{1i}+R_{3i})E_i,\quad \Theta_{1,2,i}^1=(E_i^{\mathrm{T}}P_i+SR^{\mathrm{T}}+K_1^{\mathrm{T}})B_i+\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}(R_{1i}+R_{3i})E_i,\\
\Theta_{1,6,i}^1=(E_i^{\mathrm{T}}P_i+SR^{\mathrm{T}}+K_1^{\mathrm{T}})F_{1i},\Theta_{1,7,i}^1=(E_i^{\mathrm{T}}P_i+SR^{\mathrm{T}}+K_1^{\mathrm{T}})F_{2i},\\
\Theta_{2,2,i}^1=-(1-\mu)e^{-\alpha\eta\tau_2}Q_{3i}-\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}(R_{1i}+R_{3i})E_i-\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}R_{1i}E_i+\epsilon_{2i}u_{2i}^{\mathrm{T}}u_{2i},\\
\Theta_{3,3,i}^1=-e^{-\alpha\tau_2}Q_{2i}-\frac{e^{-\alpha\eta\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i,\quad \Theta_{4,4,i}^1=-e^{-\alpha\eta\tau_2}(Q_{1i}-Q_{2i})-\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}R_{1i}E_i\\
-\frac{e^{-\alpha\eta\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i,\quad \Theta_{5,5,i}^1=\eta\tau_2R_{1i}+(\tau_2-\eta\tau_2)R_{2i}+\eta\tau_2R_{3i}-K_2-K_2^{\mathrm{T}},\\
\Theta_{1,1,i}^2=(E_i^{\mathrm{T}}P_i+SR^{\mathrm{T}}+K_1^{\mathrm{T}})A_i+A_i^{\mathrm{T}}(P_iE_i+RS^{\mathrm{T}}+K_1)+Q_{1i}+Q_{3i}+\alpha E_i^{\mathrm{T}}P_iE_i+\epsilon_{1i}u_{1i}^{\mathrm{T}}u_{1i}\\
-\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}R_{1i}E_i-\frac{e^{-\alpha\tau_2}}{\tau_2}E_i^{\mathrm{T}}R_{3i}E_i,\quad \Theta_{1,2,i}^2=(E_i^{\mathrm{T}}P_i+SR^{\mathrm{T}}+K_1^{\mathrm{T}})B_i+\frac{e^{-\alpha\tau_2}}{\tau_2}E_i^{\mathrm{T}}R_{3i}E_i,\\
\Theta_{1,6,i}^2=(E_i^{\mathrm{T}}P_i+SR^{\mathrm{T}}+K_1^{\mathrm{T}})F_{1i},\quad \Theta_{1,7,i}^2=(E_i^{\mathrm{T}}P_i+SR^{\mathrm{T}}+K_1^{\mathrm{T}})F_{2i},\\
\Theta_{2,2,i}^2=-(1-\mu)e^{-\alpha\tau_2}Q_{3i}-\frac{2e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i-\frac{e^{-\alpha\tau_2}}{\tau_2}E_i^{\mathrm{T}}R_{3i}E+\epsilon_{2i}u_{2i}^{\mathrm{T}}u_{2i},\\
\Theta_{3,3,i}^2=-e^{-\alpha\tau_2}Q_{2i}-\frac{e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i,\quad \Theta_{4,4,i}^2=-e^{-\alpha\eta\tau_2}(Q_{1i}-Q_{2i})-\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}R_{1i}E_i\\
-\frac{e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i,\quad \Theta_{5,5,i}^2=\eta\tau_2R_{1i}+(\tau_2-\alpha\tau_2)R_{2i}+\tau_2R_{3i}-K_2-K_2^{\mathrm{T}},\\
\Theta_{5,6,i}^2=K_2^{\mathrm{T}}F_{1i},\quad \Theta_{5,7,i}^2=K_2^{\mathrm{T}}F_{2i},\quad \Theta_{6,6,i}^2=-\epsilon_{1i}I,\quad \Theta_{7,7,i}^2=-\epsilon_{2i}I.
\end{gathered}
$$

and the remaining terms are zero. Moreover, the estimate of state decay is given by

$$
||x(t)||\le\sqrt{\frac{b}{a}}e^{-\lambda(t-t_0)}||\phi||_{\mathbb{C}} \tag{6}
$$

where μ_1 satisfies

$$
P_i\le\mu_1P_j,\quad Q_{ri}\le\mu_1Q_{rj},\quad R_{ri}\le\mu_1R_{rj},\quad \forall i,j\in U,r=1,2,3. \tag{7}
$$

$$
\begin{gathered}
\lambda=\frac{1}{2}(\alpha-\frac{\ln\mu_1}{T_a}),a=\min_{\forall i\in U}(\lambda_{\min}P_i),\\
b=\max_{\forall i\in U}(\lambda_{\max}P_i)+\eta\tau_2\max_{\forall i\in U}(\lambda_{\max}Q_{1i})+\tau_2\max_{\forall i\in U}(\lambda_{\max}Q_{2i})+\tau_2\max_{\forall i\in U}(\lambda_{\max}Q_{3i})\\
+\frac{(\eta\tau_2)^2}{2}\max_{\forall i\in U}(\lambda_{\max}R_{1i})+\frac{(\tau_2-\eta\tau_2)^2}{2}\max_{\forall i\in U}(\lambda_{\max}R_{2i})+\frac{\tau_2^2}{2}\max_{\forall i\in U}(\lambda_{\max}R_{3i}).
\end{gathered}
$$

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Proof

The proof of this theorem is organized into two parts. In the first part, the regular and impulse-free conditions are considered, second part is concerned with the exponential stability conditions under delay decomposition approach and ADT approach. In the first part, consider the inequality (4), it follows that

$$\begin{bmatrix} \Theta_{1,i} & \Theta_{2,i} & \Theta_{3,i} \\ * & \Theta_{4,i} & \Theta_{5,i} \\ * & * & \Theta_{6,i} \end{bmatrix} < 0, \tag{8}$$

where

$$\Theta_{1,i} = (E_i^{\mathrm{T}} P_i + SR^{\mathrm{T}} + K_1^{\mathrm{T}})A_i + A_i^{\mathrm{T}}(P_i E_i + RS^{\mathrm{T}} + K_1) - \frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^{\mathrm{T}} R_{1i} E_i - \frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^{\mathrm{T}} R_{3i} E_i,$$

$$\Theta_{2,i} = (E_i^{\mathrm{T}} P_i + SR^{\mathrm{T}} + K_1^{\mathrm{T}})B_i, \quad \Theta_{3,i} = -K_1^{\mathrm{T}} + A_i^{\mathrm{T}} K_2,$$

$$\Theta_{4,i} = -(1-\mu)e^{-\alpha\eta\tau_2} Q_{3i} - 2\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^{\mathrm{T}} R_{1i} E_i - \frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^{\mathrm{T}} R_{3i} E_i, \quad \Theta_{5,i} = B_i^{\mathrm{T}} K_2,$$

$$\Theta_{6,i} = -K_2 - K_2^{\mathrm{T}}.$$

Let $V = \begin{bmatrix} I & 0 & A_i^{\mathrm{T}} \\ 0 & I & B_i^{\mathrm{T}} \end{bmatrix}$. Pre and post multiplying the inequality (8) by V and V^T respectively, the output matrix Υ_i is $\begin{bmatrix} \Upsilon_{1i} & \Upsilon_{2i} \\ * & \Upsilon_{3i} \end{bmatrix} < 0$, where

$$\Upsilon_{1i} = (E_i^{\mathrm{T}} P_i + SR^{\mathrm{T}})A_i + A_i^{\mathrm{T}}(P_i E_i + RS^{\mathrm{T}}) - \frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^{\mathrm{T}} R_{1i} E_i - \frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^{\mathrm{T}} R_{3i} E_i,$$

$$\Upsilon_{2i} = (E_i^{\mathrm{T}} P_i + SR^{\mathrm{T}})B_i, \quad \Upsilon_{3i} = -(1-\mu)e^{-\alpha\eta\tau_2} Q_{3i} - \frac{2e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^{\mathrm{T}} R_{1i} E_i - \frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^{\mathrm{T}} R_{3i} E_i.$$

Since $\mathrm{rank}(E_i) < n$, there must exist two invertible matrices $\hat{G}$ and $\hat{H} \in \mathbb{R}^{n\times n}$ such that $\hat{E}_i = \hat{G} E_i \hat{H} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$. Then R, can be parameterized as $R = \hat{G}^{\mathrm{T}} \begin{bmatrix} 0 \\ \Phi \end{bmatrix}$ where $\Phi \in \mathbb{R}^{(n-r)\times(n-r)}$ is any non-singular matrix. Similarly, one can define

$$\hat{A}_i = \hat{G} A_i \hat{H} = \begin{bmatrix} \hat{A}_{11i} & \hat{A}_{12i} \\ \hat{A}_{21i} & \hat{A}_{22i} \end{bmatrix}, \quad \hat{B}_i = \hat{G} B_i \hat{H} = \begin{bmatrix} \hat{B}_{11i} & \hat{B}_{12i} \\ \hat{B}_{21i} & \hat{B}_{22i} \end{bmatrix},$$

$$\hat{P}_i = \hat{G}^{-\mathrm{T}} P_i \hat{G}^{-1} = \begin{bmatrix} \hat{P}_{11i} & \hat{P}_{12i} \\ \hat{P}_{21i} & \hat{P}_{22i} \end{bmatrix}, \quad \hat{Q}_{3i} = \hat{G}^{-\mathrm{T}} Q_{3i} \hat{G}^{-1} = \begin{bmatrix} \hat{Q}_{311i} & \hat{Q}_{312i} \\ \hat{Q}_{321i} & \hat{Q}_{322i} \end{bmatrix},$$

$$\hat{R}_{1i} = \hat{G}^{-\mathrm{T}} R_{1i} \hat{G}^{-1} = \begin{bmatrix} \hat{R}_{111i} & \hat{R}_{112i} \\ \hat{R}_{121i} & \hat{R}_{122i} \end{bmatrix},$$

$$\hat{R}_{3i} = \hat{G}^{-\mathrm{T}} R_{3i} \hat{G}^{-1} = \begin{bmatrix} \hat{R}_{311i} & \hat{R}_{312i} \\ \hat{R}_{321i} & \hat{R}_{322i} \end{bmatrix}, \quad \hat{S} = \hat{H}^{\mathrm{T}} S = \begin{bmatrix} \hat{S}_{11} \\ \hat{S}_{21} \end{bmatrix}.$$

Pre and post multiplying Υ_{1i} by $\hat{H}^{\mathrm{T}}$ and $\hat{H}$, respectively, which yields

$$\hat{\Upsilon}_{1i} = \hat{H}^{\mathrm{T}} \Upsilon_{1i} \hat{H} = \begin{bmatrix} \hat{\Upsilon}_{11i} & \hat{\Upsilon}_{12i} \\ * & \hat{A}_{22i}^{\mathrm{T}} \Phi \hat{S}_{21}^{\mathrm{T}} + \hat{S}_{21} \Phi^{\mathrm{T}} \hat{A}_{22i} \end{bmatrix} < 0. \tag{9}$$

From inequality (9), it is easy to see the condition that

$$\hat{A}_{22i}^{\mathrm{T}} \Phi \hat{S}_{21}^{\mathrm{T}} + \hat{S}_{21} \Phi^{\mathrm{T}} \hat{A}_{22i} < 0, \tag{10}$$

and $\hat{A}_{22i}$ is non-singular. Suppose that $\hat{A}_{22i}$ is singular, there must exist a non-zero vector $\rho \in \mathbb{R}^{(n-r)}$, which ensures that $\hat{A}_{22i}\rho = 0$. This implies that $\rho^T(\hat{A}_{22i}^{\mathrm{T}} \Phi \hat{S}_{21}^{\mathrm{T}} + \hat{S}_{21} \Phi^{\mathrm{T}} \hat{A}_{22i})\rho = 0$, this contradicts to the inequality (10). Therefore, $\hat{A}_{22i}$ is non-singular. Then, it can be shown that

$$\begin{aligned} \det(sE_i - A_i) &= \det(\hat{G}^{-1}) \det(s\hat{E}_i - \hat{A}_i) \det(\hat{H}^{-1}) \\ &= \det(\hat{G}^{-1}) \det((sI_r - \hat{A}_{11i})(-\hat{A}_{22i}) - \hat{A}_{12i}\hat{A}_{21i}) \det(\hat{H}^{-1}) \\ &= \det(\hat{G}^{-1}) \det(-\hat{A}_{22i}) \det(sI_r - \hat{A}_{11i} + \hat{A}_{12i}\hat{A}_{21i}\hat{A}_{22i}^{-1}) \det(\hat{H}^{-1}), \end{aligned}$$

which implies that $\det(sE_i - A_i)$ is not identically zero and $\deg(\det(sE_i - A_i)) = r = \mathrm{rank}(E_i)$. Thus, the pair (E_i, A_i) is regular and impulse-free.

Pre and post multiplying Υ_i by $[I \ I]$ and $[I \ I]^{\mathrm{T}}$ respectively yields,

$$\bar{\Upsilon}_{1i} = (E_i^{\mathrm{T}} P_i + SR^{\mathrm{T}})(A_i + B_i) + (A_i^{\mathrm{T}} + B_i^{\mathrm{T}})(P_i E_i + RS^{\mathrm{T}}) - (1-\mu)e^{-\alpha\eta\tau_2} Q_{3i} - \frac{3e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^{\mathrm{T}} R_{1i} E_i - \frac{2e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^{\mathrm{T}} R_{3i} E_i < 0. \tag{11}$$

Pre-and post-multiplying the inequality (11) by $\hat{H}^{\mathrm{T}}$ and $\hat{H}$, clearly, the pairs $(E_i, (A_i + B_i))$ are regular and impulse-free. Hence, from Definition 1, the system (2) is regular and impulse-free for $0 \le \tau(t) \le \eta\tau_2$. Similarly, one can prove that the regular and impulse-free conditions for the rest case $\eta\tau_2 \le \tau(t) \le \tau_2$. Hence, system (2) is regular and impulse free.

Next, the characterization of the system is carried out using the following piecewise LKF candidate as follows:

$$\begin{aligned} V(t, x(t), \sigma) &= x^{\mathrm{T}}(t) E_\sigma^{\mathrm{T}} P_\sigma E_\sigma x(t) + \int_{t-\eta\tau_2}^{t} e^{\alpha(s-t)} x^{\mathrm{T}}(s) Q_{1\sigma} x(s) ds \\ &+ \int_{t-\tau_2}^{t-\eta\tau_2} e^{\alpha(s-t)} x^{\mathrm{T}}(s) Q_{2\sigma} x(s) ds + \int_{t-\tau(t)}^{t} e^{\alpha(s-t)} x^{\mathrm{T}}(s) Q_{3\sigma} x(s) ds \\ &+ \int_{-\eta\tau_2}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} \dot{x}^{\mathrm{T}}(s) E_i^{\mathrm{T}} R_{1\sigma} E_\sigma \dot{x}(s) ds d\theta \\ &+ \int_{-\tau_2}^{-\eta\tau_2} \int_{t+\theta}^{t} e^{\alpha(s-t)} \dot{x}^{\mathrm{T}}(s) E_\sigma^{\mathrm{T}} R_{2\sigma} E_\sigma \dot{x}(s) ds d\theta \\ &+ \int_{-\tau(t)}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} \dot{x}^{\mathrm{T}}(s) E_\sigma^{\mathrm{T}} R_{3\sigma} E_\sigma \dot{x}(s) ds d\theta. \end{aligned} \tag{12}$$

Assume that $P_\sigma = P_i$, $Q_{1\sigma} = Q_{1i}$, $Q_{2\sigma} = Q_{2i}$, $Q_{3\sigma} = Q_{3i}$, $R_{1\sigma} = R_{1i}$, $R_{2\sigma} = R_{2i}$, $R_{3\sigma} = R_{3i}$, for some $\sigma(=i) \in U$, are symmetric positive-definite matrices of appropriate dimensions to be determined. Taking the time derivative of V $(t, x(t), i)$ along the trajectory of system (2) yields

$$\begin{aligned}\dot{V}(t,x(t),i) \le& -\alpha V(t,x(t),i)+\alpha x^{\mathrm{T}}(t)E_i^{\mathrm{T}}P_iE_ix(t)+2x^{\mathrm{T}}(t)E_i^{\mathrm{T}}P_iE_i\dot{x}(t)\\ &+x^{\mathrm{T}}(t)(Q_{1i}+Q_{3i})x(t)-e^{-\alpha\eta\tau_2}x^{\mathrm{T}}(t-\eta\tau_2)Q_{1i}x(t-\eta\tau_2)\\ &+e^{-\alpha\eta\tau_2}x^{\mathrm{T}}(t-\eta\tau_2)Q_{2i}x(t-\eta\tau_2)-e^{-\alpha\tau_2}x^{\mathrm{T}}(t-\tau_2)Q_{2i}x(t-\tau_2)\\ &-(1-\mu)e^{-\alpha\eta\tau_2}x^{\mathrm{T}}(t-\tau(t))Q_{3i}x(t-\tau(t))+\tau(t)\dot{x}^{\mathrm{T}}(t)E_i^{\mathrm{T}}R_{3i}E_i\dot{x}(t)\\ &+\dot{x}^{\mathrm{T}}(t)E_i^{\mathrm{T}}(\eta\tau_2R_{1i}+(\tau_2-\eta\tau_2)R_{2i})E_i\dot{x}(t)-\int_{t-\eta\tau_2}^{t}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{1i}E_i\dot{x}(s)ds\\ &-\int_{t-\tau_2}^{t-\eta\tau_2}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{2i}E_i\dot{x}(s)ds-\int_{t-\tau(t)}^{t}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{3i}E_i\dot{x}(s)ds,\end{aligned} \tag{13}$$

For non-linear functions $f_{li}(\cdot)$ $(l=1,\ 2)$, given $\epsilon_{1i}>0$, $\epsilon_{2i}>0$, the inequalities are hold.

$$0 \le -\epsilon_{1i}f_{1i}^{\mathrm{T}}(t,x(t))f_{1i}(t,x(t))+\epsilon_{1i}x^{\mathrm{T}}(t)u_{1i}^{\mathrm{T}}u_{1i}x(t), \tag{14}$$

$$0 \le -\epsilon_{2i}f_{2i}^{\mathrm{T}}(t,x(t-\tau(t)))f_{2i}(t,x(t-\tau(t)))+\epsilon_{2i}x^{\mathrm{T}}(t-\tau(t))u_{2i}^{\mathrm{T}}u_{2i}x(t-\tau(t)). \tag{15}$$

Furthermore, noting that $E_i^{\mathrm{T}}R=0$, which yields,

$$2x^{\mathrm{T}}(t)SR^{\mathrm{T}}[A_ix(t)+B_ix(t-\tau(t))+F_{1i}f_{1i}(t,x(t))+F_{2i}f_{2i}(t,x(t-\tau(t)))]=0. \tag{16}$$

Using the free-weighting matrix technique, the following zero equation is valid one.

$$\begin{aligned}0=&2[x^{\mathrm{T}}(t)K_1^{\mathrm{T}}+(E_i\dot{x}(t))^{\mathrm{T}}K_2^{\mathrm{T}}]\\ &\times[-E_i\dot{x}(t)+A_ix(t)+B_ix(t-\tau(t))+F_{1i}f_{1i}(t,x(t))+F_{2i}f_{2i}(t,x(t-\tau(t)))].\end{aligned} \tag{17}$$

Now, estimating the upper bound of the last three terms in inequality (13) under two cases as follows:

Case I

If $0 \le \tau(t) \le \eta\tau_2$ and using Lemma 3, then

$$\begin{aligned}&-\int_{t-\eta\tau_2}^{t}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{1i}E_i\dot{x}(s)ds-\int_{t-\tau_2}^{t-\eta\tau_2}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{2i}E_i\dot{x}(s)ds\\ &\quad-\int_{t-\tau(t)}^{t}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{3i}E_i\dot{x}(s)ds\\ &=-\int_{t-\eta\tau_2}^{t-\tau(t)}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{1i}E_i\dot{x}(s)ds\\ &\quad-\int_{t-\tau(t)}^{t}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}(R_{1i}+R_{3i})E_i\dot{x}(s)ds\\ &\quad-\int_{t-\tau_2}^{t-\eta\tau_2}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{2i}E_i\dot{x}(s)ds\\ &\le-\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}[x^{\mathrm{T}}(t-\tau(t))-x^{\mathrm{T}}(t-\eta\tau_2)]E_i^{\mathrm{T}}R_{1i}E_i[x(t-\tau(t))-x(t-\eta\tau_2)]\\ &\quad-\frac{e^{-\alpha\eta\tau_2}}{\tau_2-\eta\tau_2}[x^{\mathrm{T}}(t-\eta\tau_2)-x^{\mathrm{T}}(t-\tau_2)]E_i^{\mathrm{T}}R_{2i}E_i[x(t-\eta\tau_2)-x(t-\tau_2)]\\ &\quad-\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}[x^{\mathrm{T}}(t)-x^{\mathrm{T}}(t-\tau(t))]E_i^{\mathrm{T}}(R_{1i}+R_{3i})E_i[x(t)-x(t-\tau(t))].\end{aligned} \tag{18}$$

Using the Lemma 1 and combining the Eqs. (13)–(18), which yields

$$\dot{V}(t,x(t),i)+\alpha V(t,x(t),i) \le \xi^{\mathrm{T}}(t)\Theta_i^1\xi(t)<0, \tag{19}$$

where

$$\xi^{\mathrm{T}}(t)=[x^{\mathrm{T}}(t)\,x^{\mathrm{T}}(t-\tau(t))\quad x^{\mathrm{T}}(t-\tau_2)\quad x^{\mathrm{T}}(t-\eta\tau_2)\quad (E_i\dot{x}(t))^{\mathrm{T}}\quad f_{1i}^{\mathrm{T}}(t,x(t))\quad f_{2i}^{\mathrm{T}}(t,x(t-\tau(t)))].$$

From (19), it is clear that the inequality $\Theta_i^1<0$ is satisfied.

Case II

If $\eta\tau_2 \le \tau(t) \le \tau_2$ and using Lemma 3, then

$$\begin{aligned}&-\int_{t-\eta\tau_2}^{t}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{1i}E_i\dot{x}(s)ds-\int_{t-\tau_2}^{t-\eta\tau_2}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{2i}E_i\dot{x}(s)ds\\ &\quad-\int_{t-\tau(t)}^{t}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{3i}E_i\dot{x}(s)ds\\ &=-\int_{t-\eta\tau_2}^{t}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{1i}E_i\dot{x}(s)ds-\int_{t-\tau_2}^{t-\tau(t)}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{2i}E_i\dot{x}(s)ds\\ &\quad-\int_{t-\tau(t)}^{t-\eta\tau_2}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{2i}E_i\dot{x}(s)ds-\int_{t-\tau(t)}^{t}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{3i}E_i\dot{x}(s)ds\\ &\le-\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}[x^{\mathrm{T}}(t)-x^{\mathrm{T}}(t-\eta\tau_2)]E_i^{\mathrm{T}}R_{1i}E_i[x(t)-x(t-\eta\tau_2)]\\ &\quad-\frac{e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2}[x^{\mathrm{T}}(t-\tau(t))-x^{\mathrm{T}}(t-\tau_2)]E_i^{\mathrm{T}}R_{2i}E_i[x(t-\tau(t))-x(t-\tau_2)]\\ &\quad-\frac{e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2}[x^{\mathrm{T}}(t-\eta\tau_2)-x^{\mathrm{T}}(t-\tau(t))]E_i^{\mathrm{T}}R_{2i}E_i[x(t-\eta\tau_2)-x(t-\tau(t))]\\ &\quad-\frac{e^{-\alpha\tau_2}}{\tau_2}[x^{\mathrm{T}}(t)-x^{\mathrm{T}}(t-\tau(t))]E_i^{\mathrm{T}}R_{3i}E_i[x(t)-x(t-\tau(t))]\end{aligned} \tag{20}$$

Using the Lemma 1, combining the Eqs. (13)–(17) and (20), which yields

$$\dot{V}(t,x(t),i)+\alpha V(t,x(t),i) \le \xi^{\mathrm{T}}(t)\Theta_i^2\xi(t)<0, \tag{21}$$

From (21), it is clear that the inequality $\Theta_i^2<0$ is satisfied.

For an arbitrary piecewise constant switching signal $\sigma(t)$ and for $t>0$, let $0=t_0<t_1<\ldots<t_k<\ldots$, $k=1, 2, 3,\ldots$, denote the switching points of α (t) over the interval $(0,\ t)$, and suppose the i_kth subsystem is activated when $t\in[t_k,t_{k+1})$. Integrating the inequalities (19) and (21) from t_k to t yields,

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$$V(t,x(t),\sigma) \leq e^{-\alpha(t-t_k)} V(t_k,x(t_k),\sigma(t_k)). \tag{22}$$

From the inequality (7) and (22), at any switching instant t_k, we get

$$V(t,x(t),\sigma_{t_k}) \leq \mu_1 e^{-\alpha(t-t_k^-)} V(t_k^-,x(t_k^-),\sigma(t_k^-)). \tag{23}$$

Hence, from (22), (23) and using the relation $\delta = N_\sigma(t_0,\ t) \leq (t-t_0)/T_a$, we have

$$\begin{aligned} V(t,x(t),\sigma) &\leq \mu_1 e^{-\alpha(t-t_k^-)} V(t_k^-,x(t_k^-),\sigma(t_k^-)) \\ &\leq \cdots \leq \mu_1^{\delta} e^{-\alpha(t-t_0)} V(t_0,x(t_0),\sigma(t_0)) \\ &\leq e^{-(t-t_0)(\sigma-\ln\mu_1/T_a)} V(t_0,x(t_0),\sigma(t_0)). \end{aligned} \tag{24}$$

Further from (12), we get

$$a||x(t)||^2 \leq V(t,x(t),\sigma), \quad V(t_0,x(t_0),\sigma(t_0)) \leq b||\phi||_{\mathbb{C}}^2 \tag{25}$$

Thus Eqs. (24) and (25) yield

$$||x(t)|| \leq \sqrt{\frac{b}{a}} e^{-\frac{1}{2}(t-t_0)(\alpha-\ln\mu_1/T_a)} ||\phi||_{\mathbb{C}} \tag{26}$$

which implies that system (2) is exponentially stable by the Eqs. (19), (21), Definition 2 and Definition 3. According to Definition 1 and Definition 2, we conclude that the system (2) is exponentially admissible. This completes the proof.

Remark 1

It should be noted that the proposed stability condition is based on the ADT of switched non-linear time-delay system via delay decomposition method. Here, the time-delay $[t-\tau_2,\ t]$ is divided into two unequal subintervals such as $[t-\tau_2,\ t-\eta\tau_2]$ and $[t-\eta\tau_2,\ t]$, where η is a tuning parameter. Also, the information of delayed state $x(t-\eta\tau_2)$ is also taken into account. Concerning the comparison with some well-known results in [9,30,32], all these results have been proposed for singular system without considering the tuning parameter η. To the best of the authors' knowledge, there are no results available in the existing literature for the admissible condition of switched non-linear singular systems based on delay decomposition approach. This article aims to fulfill the gap in existing literature.

Assume that the non-linear functions $f_{1i}=0=f_{2i}$ in the non-linear switched singular system (2). The switched linear singular time-delay system can be described as follows:

$$\begin{aligned} E_i\dot{x}(t) &= A_i x(t) + B_i x(t-\tau(t)), \\ x(t) &= \phi(t), t \in [-\tau_2, 0]. \end{aligned} \tag{27}$$

The exponential admissible conditions for system (27) are stated in the following Corollary.

Corollary 1

For given scalars $\tau_2 > 0$, tuning parameter η $(0<\eta<1)$, μ, and $\alpha>0$, the equilibrium point of system (27) is exponentially admissible and for any switching signal σ with ADT satisfying

$$T_a > T_a^* = \frac{\ln\mu_1}{\alpha}, \quad \mu_1 \geq 1 \tag{28}$$

if there exist symmetric positive-definite matrices P_i, Q_{1i}, Q_{2i}, Q_{3i}, R_{1i}, R_{2i}, R_{3i}, real matrices K_1, K_2, for any matrix S with appropriate dimension, and the constant matrix $R \in \mathbb{R}^{n\times(n-r)}$ satisfying $E_i^T R=0$ with rank(R) $= n-r$ such that the following symmetric LMIs hold:

$$\bar{\Theta}_i^1 = \begin{bmatrix} \bar{\Theta}_{1,1,i}^1 & \bar{\Theta}_{1,2,i}^1 & 0 & 0 & -K_1^T + A_i^T K_2 \\ * & \bar{\Theta}_{2,2,i}^1 & 0 & \frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^T R_{1i} E_i & B_i^T K_2 \\ * & * & \bar{\Theta}_{3,3,i}^1 & \frac{e^{-\alpha\eta\tau_2}}{\tau_2-\eta\tau_2} E_i^T R_{2i} E_i & 0 \\ * & * & * & \bar{\Theta}_{4,4,i}^1 & 0 \\ * & * & * & * & \bar{\Theta}_{5,5,i}^1 \end{bmatrix} < 0, \tag{29}$$

$$\bar{\Theta}_i^2 = \begin{bmatrix} \bar{\Theta}_{1,1,i}^2 & \bar{\Theta}_{1,2,i}^2 & 0 & \frac{e^{-\alpha\tau_2}}{\eta\tau_2} E_i^T R_{1i} E_i & -K_1^T + A_i^T K_2 \\ * & \bar{\Theta}_{2,2,i}^2 & \frac{e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2} E_i^T R_{2i} E_i & \frac{e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2} E_i^T R_{2i} E_i & B_i^T K_2 \\ * & * & \bar{\Theta}_{3,3,i}^2 & 0 & 0 \\ * & * & * & \bar{\Theta}_{4,4,i}^2 & 0 \\ * & * & * & * & \bar{\Theta}_{5,5,i}^2 \end{bmatrix} < 0, \tag{30}$$

where

$$\begin{aligned} \bar{\Theta}_{1,1,i}^1 &= (E_i^T P_i + SR^T + K_1^T)A_i + A_i^T(P_i E_i + RS^T + K_1) + Q_{1i} + Q_{3i} \\ &\quad + \alpha E_i^T P_i E_i - \frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^T (R_{1i}+R_{3i}) E_i, \\ \bar{\Theta}_{1,2,i}^1 &= (E_i^T P_i + SR^T + K_1^T)B_i + \frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^T (R_{1i}+R_{3i}) E_i, \\ \bar{\Theta}_{2,2,i}^1 &= -(1-\mu)e^{-\alpha\eta\tau_2} Q_{3i} - \frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^T (R_{1i}+R_{3i}) E_i - \frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^T R_{1i} E_i, \\ \bar{\Theta}_{3,3,i}^1 &= -e^{-\alpha\tau_2} Q_{2i} - \frac{e^{-\alpha\eta\tau_2}}{\tau_2-\eta\tau_2} E_i^T R_{2i} E_i, \\ \bar{\Theta}_{4,4,i}^1 &= -e^{-\alpha\eta\tau_2}(Q_{1i}-Q_{2i}) - \frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^T R_{1i} E_i \\ &\quad - \frac{e^{-\alpha\eta\tau_2}}{\tau_2-\eta\tau_2} E_i^T R_{2i} E_i, \quad \bar{\Theta}_{5,5,i}^1 = \eta\tau_2 R_{1i} + (\tau_2-\eta\tau_2)R_{2i} + \eta\tau_2 R_{3i} - K_2 - K_2^T, \\ \bar{\Theta}_{1,1,i}^2 &= (E_i^T P_i + SR^T + K_1^T)A_i + A_i^T(P_i E_i + RS^T + K_1) + Q_{1i} + Q_{3i} + \alpha E_i^T P_i E_i \\ &\quad - \frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2} E_i^T R_{1i} E_i - \frac{e^{-\alpha\tau_2}}{\tau_2} E_i^T R_{3i} E_i, \\ \bar{\Theta}_{1,2,i}^2 &= (E_i^T P_i + SR^T + K_1^T)B_i + \frac{e^{-\alpha\tau_2}}{\tau_2} E_i^T R_{3i} E_i, \end{aligned}$$

$$\bar{\Theta}^2_{2,2,i}=-(1-\mu)e^{-\alpha\tau_2}Q_{3i}-\frac{2e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i-\frac{e^{-\alpha\tau_2}}{\tau_2}E_i^{\mathrm{T}}R_{3i}E,$$

$$\bar{\Theta}^2_{3,3,i}=-e^{-\alpha\tau_2}Q_{2i}-\frac{e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i,$$

$$\bar{\Theta}^2_{4,4,i}=-e^{-\alpha\eta\tau_2}(Q_{1i}-Q_{2i})-\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}R_{1i}E_i$$

$$-\frac{e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i,\quad \bar{\Theta}^2_{5,5,i}=\eta\tau_2R_{1i}+(\tau_2-\alpha\tau_2)R_{2i}+\tau_2R_{3i}-K_2-K_2^{\mathrm{T}}.$$

and the remaining terms are zero. Moreover, the estimate of state decay is given by

$$||x(t)||\le\sqrt{\frac{b}{a}}e^{-\lambda(t-t_0)}||\phi||_{\mathbb{C}} \tag{31}$$

where μ_1 *satisfies*

$$P_i\le\mu_1P_j,Q_{ri}\le\mu_1Q_{rj},R_{ri}\le\mu_1R_{rj},\forall i,j\in U,r=1,2,3. \tag{32}$$

$$\lambda=\frac{1}{2}\left(\alpha-\frac{\ln\mu_1}{T_a}\right),a=\min_{\forall i\in U}(\lambda_{\min}P_i),$$

$$b=\max_{\forall i\in U}(\lambda_{\max}P_i)+\eta\tau_2\max_{\forall i\in U}(\lambda_{\max}Q_{1i})+\tau_2\max_{\forall i\in U}(\lambda_{\max}Q_{2i})$$
$$+\tau_2\max_{\forall i\in U}(\lambda_{\max}Q_{3i})+\frac{(\eta\tau_2)^2}{2}\max_{\forall i\in U}(\lambda_{\max}R_{1i})$$
$$+\frac{(\tau_2-\eta\tau_2)^2}{2}\max_{\forall i\in U}(\lambda_{\max}R_{2i})+\frac{\tau_2^2}{2}\max_{\forall i\in U}(\lambda_{\max}R_{3i}).$$

Proof

The proof of this Corollary is similar to Theorem 3.1.

Remark 2

In system (2), consider the switched linear singular constant time-delay system as follows

$$E_i\dot{x}(t)=A_ix(t)+B_ix(t-\tau),$$
$$x(t)=\phi(t),t\in[-\tau,0]. \tag{33}$$

The exponential admissible conditions for switched linear system (33) are derived in the following Corollary.

Corollary 2

For given scalars $\tau>0$*, tuning parameter* η $(0<\eta<1)$*, and* $\alpha>0$*, the equilibrium point of system (33) is exponentially admissible and for any switching signal* σ *with ADT satisfying*

$$T_{\mathrm{a}}>T_{\mathrm{a}}^*=\frac{\ln\mu_1}{\alpha},\quad \mu_1\ge1 \tag{34}$$

if there exist symmetric positive-definite matrices P_i, Q_{1i}, Q_{2i}, Q_{3i}, R_{1i}, R_{2i}, R_{3i}, *real matrices* K_1, K_2, *for any matrix* S *with appropriate dimension, and the constant matrix* $R\in\mathbb{R}^{n\times(n-r)}$ *satisfying* $E_i^{\mathrm{T}}R=0$ *with rank(R)* $=n-r$ *such that the following symmetric LMIs hold:*

$$\hat{\Theta}=\begin{bmatrix}\hat{\Theta}_{1,1} & (E_i^{\mathrm{T}}P_i+K_1^{\mathrm{T}})B_i+\frac{e^{-\alpha\tau}}{\tau}E_i^{\mathrm{T}}R_{3i}E_i & \frac{e^{-\alpha\eta\tau}}{\eta\tau}E_i^{\mathrm{T}}R_{1i}E_i & -K_1^{\mathrm{T}}+A_i^{\mathrm{T}}K_2\\ * & \hat{\Theta}_{2,2} & \frac{e^{-\alpha\tau}}{\tau-\eta\tau}E_i^{\mathrm{T}}R_{2i}E_i & B_i^{\mathrm{T}}K_2\\ * & * & \hat{\Theta}_{3,3} & 0\\ * & * & * & \hat{\Theta}_{4,4}\end{bmatrix}<0 \tag{35}$$

where

$$\hat{\Theta}_{1,1}=(E_i^{\mathrm{T}}P_i+K_1^{\mathrm{T}})A_i+A_i^{\mathrm{T}}(P_iE_i+K_1)+Q_{1i}+Q_{3i}+\alpha E_i^{\mathrm{T}}P_iE_i$$
$$-\frac{e^{-\alpha\eta\tau}}{\eta\tau}E_i^{\mathrm{T}}R_{1i}E_i-\frac{e^{-\alpha\tau}}{\eta\tau}E_i^{\mathrm{T}}R_{3i}E_i,$$

$$\hat{\Theta}_{2,2}=-e^{-\alpha\tau}(Q_{2i}+Q_{3i})-\frac{e^{-\alpha\tau}}{\tau-\eta\tau}E_i^{\mathrm{T}}R_{2i}E_i-\frac{e^{-\alpha\tau}}{\tau}E_i^{\mathrm{T}}R_{3i}E_i,$$

$$\hat{\Theta}_{3,3}=-e^{-\alpha\tau}Q_{1i}-\frac{e^{-\alpha\eta\tau}}{\eta\tau}E_i^{\mathrm{T}}R_{1i}E_i$$

$$-\frac{e^{-\alpha\tau}}{\tau-\eta\tau}E_i^{\mathrm{T}}R_{2i}E_i,\quad \hat{\Theta}_{4,4}=\eta\tau R_{1i}+(\tau-\alpha\tau)R_{2i}+\tau R_{3i}-K_2-K_2^{\mathrm{T}}.$$

and the remaining terms are zero. Moreover, the estimate of state decay is given by

$$||x(t)||\le\sqrt{\frac{b}{a}}e^{-\lambda(t-t_0)}||\phi||_{\mathbb{C}} \tag{36}$$

where μ_1 *satisfies*

$$P_i\le\mu_1P_j,Q_{ri}\le\mu_1Q_{rj},R_{ri}\le\mu_1R_{rj},\forall i,j\in U,r=1,2,3. \tag{37}$$

$$\lambda=\frac{1}{2}\left(\alpha-\frac{\ln\mu_1}{T_a}\right),a=\min_{\forall i\in U}(\lambda_{\min}P_i),$$

$$b=\max_{\forall i\in U}(\lambda_{\max}P_i)+\eta\tau\max_{\forall i\in U}(\lambda_{\max}Q_{1i})+\tau\max_{\forall i\in U}(\lambda_{\max}Q_{2i})$$
$$+\tau\max_{\forall i\in U}(\lambda_{\max}Q_{3i})+\frac{(\eta\tau)^2}{2}\max_{\forall i\in U}(\lambda_{\max}R_{1i})$$
$$+\frac{(\tau-\eta\tau)^2}{2}\max_{\forall i\in U}(\lambda_{\max}R_{2i})+\frac{\tau^2}{2}\max_{\forall i\in U}(\lambda_{\max}R_{3i}).$$

Proof

To derive the exponential admissible conditons, we considered the following LKF candidate:

$$V(t,x(t),i)=x^{\mathrm{T}}(t)E_i^{\mathrm{T}}P_iE_ix(t)+\int_{t-\eta\tau}^{t}e^{\alpha(s-t)}x^{\mathrm{T}}(s)Q_{1i}x(s)ds$$
$$+\int_{t-\tau}^{t-\eta\tau}e^{\alpha(s-t)}x^{\mathrm{T}}(s)Q_{2i}x(s)ds+\int_{t-\tau}^{t}e^{\alpha(s-t)}x^{\mathrm{T}}(s)Q_{3i}x(s)ds$$
$$+\int_{-\eta\tau}^{0}\int_{t+\theta}^{t}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{1i}E_i\dot{x}(s)dsd\theta$$
$$+\int_{-\tau}^{-\eta\tau}\int_{t+\theta}^{t}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{2i}E_i\dot{x}(s)dsd\theta$$
$$+\int_{-\tau}^{0}\int_{t+\theta}^{t}e^{\alpha(s-t)}\dot{x}^{\mathrm{T}}(s)E_i^{\mathrm{T}}R_{3i}E_i\dot{x}(s)dsd\theta.$$

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The remaining part of the proof is similar to that of Theorem 3.1.

Remark 3

In system (1), the pair matrices (E_i, A_i), $(E_i, (A_i+B_i))$ are regular and impulse-free then it can still have finite discontinuities due to incompatible initial conditions (see [34,47,48] for more details). For having continuous trajectories, switching must occur when the states of the activated subsystem satisfy the compatibility condition of the new one. So, regular and impulse-free properties of each subsystem with Assumption 2.1 simultaneously guarantee has a unique continuous solution on the interval $[\tau_k, \tau_{k+1})$ in (1).

The switched singular system (1) is normal one while system considered as $E_i = \mathrm{I}$. For this case, the condition $E_i^{\mathrm{T}}R=0$ is relaxed and the corresponding stability conditions can be obtained. Also, it is free from the regular and impulse-free conditions.

4. ROBUST STABILITY CRITERIA FOR SWITHCED NON-LINEAR SINGULAR SYSTEMS

In this section, robust exponential admissible conditions for the uncertain swithced non-linear singular systems with time-varying delays are developed.

The time-varying uncertainties are of the form assumed to be $A_i(t) = A_i + \Delta A_i(t)$, $B_i(t) = B_i + \Delta B_i(t)$. The non-linear functions $f_{1i}(t, x(t))$ and $f_{2i}(t, x(t–\tau(t)))$ are assumed to be norm bounded. The time-varying uncertainties $\Delta A_i(t)$, $\Delta B_i(t)$ are assumed to be norm bounded, and can be described as $[\Delta A_i(t)\ \ \Delta B_i(t)] = D_iF_i(t)[H_{1i}\ \ H_{2i}]$, where D_i, H_{1i} and H_{2i} are constant matrices of appropriate dimensions. $F_i(t)$ is an unknown and possibly time-varying real matrix with Lebesgue measurable elements, which is assumed to satisfy $F_i^{\mathrm{T}}(t)F_i(t) \le I, \forall t>0$. When $F_{1i} = F_{2i} = I$, the switched singular time-delay system (2) can be written in the following form

$$E_i\dot{x}(t)=(A_ix(t)+B_ix(t-\tau(t)))+(\Delta A_i(t)x(t)+\Delta B_i(t)x(t-\tau(t))) \\ x(t)=\phi(t), t\in[-\tau_2, 0]. \tag{38}$$

Then, the system (38) becomes

$$E_i\dot{x}(t)=(A_i+\Delta A_i(t))x(t)+(B_i+\Delta B_i(t))(x(t-\tau(t)), \\ x(t)=\phi(t), t\in[-\tau_2, 0]. \tag{39}$$

Delay-dependent robust exponential admissible criterion for system (39) is derived in the following Theorem.

Theorem 4.1

For given scalars $\tau_2>0$, tuning parameter η $(0<\eta<1)$, μ and $\alpha>0$, the equilibrium point of system (39) is robustly exponentially admissible and for any switching signal σ with ADT satisfying

$$T_{\mathrm{a}} > T_{\mathrm{a}}^{*}=\frac{\ln\mu_1}{\alpha}, \quad \mu_1\ge 1 \tag{40}$$

if there exist symmetric positive-definite matrices P_i, Q_{1i}, Q_{2i}, Q_{3i}, R_{1i}, R_{2i}, R_{3i}, real matrices K_1, K_2, for any matrix S with appropriate dimension, some known matrices u_{1i}, u_{2i}, unknown scalars δ_i, and the constant matrix $R \in \mathbb{R}^{n\times(n-r)}$ satisfying $E_i^{\mathrm{T}}R=0$ with rank$(R) = n - r$ such that the following symmetric LMIs hold:

$$\begin{bmatrix} \tilde{\Theta}^1 & \Lambda_1D_i & \delta_i\Lambda_2 \\ * & -\delta_iI & 0 \\ * & * & -\delta_iI \end{bmatrix}<0, \quad \begin{bmatrix} \tilde{\Theta}^2 & \Lambda_1D_i & \delta_i\Lambda_2 \\ * & -\delta_iI & 0 \\ * & * & -\delta_iI \end{bmatrix}<0, \tag{41}$$

where $\tilde{\Theta}^1=\tilde{\Theta}^1_{5\times5\times i}<0,\ \tilde{\Theta}^2=\tilde{\Theta}^2_{5\times5\times i}<0,$

$$\tilde{\Theta}^1_{1,1,i}=(E_i^{\mathrm{T}}P_i+SR^{\mathrm{T}}+K_1^{\mathrm{T}})A_i+A_i^{\mathrm{T}}(P_iE_i+RS^{\mathrm{T}}+K_1)+Q_{1i}+Q_{3i} \\ +\alpha E_i^{\mathrm{T}}P_iE_i-\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}(R_{1i}+R_{3i})E_i,$$

$$\tilde{\Theta}^1_{1,2,i}=(E_i^{\mathrm{T}}P_i+SR^{\mathrm{T}}+K_1^{\mathrm{T}})B_i+\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}(R_{1i}+R_{3i})E_i,$$

$$\tilde{\Theta}^1_{1,5,i}=-K_1^{\mathrm{T}}+A_i^{\mathrm{T}}K_2, \quad \tilde{\Theta}^1_{2,2,i}=-(1-\mu)e^{-\alpha\eta\tau_2}Q_{3i} \\ -\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}(R_{1i}+R_{3i})E_i-\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}R_{1i}E_i,$$

$$\tilde{\Theta}^1_{2,4,i}=\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}R_{1i}E_i, \quad \tilde{\Theta}^1_{2,5,i}=B_i^{\mathrm{T}}K_2,$$

$$\tilde{\Theta}^1_{3,3,i}=-e^{-\alpha\tau_2}Q_{2i}-\frac{e^{-\alpha\eta\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i,$$

$$\tilde{\Theta}^1_{3,4,i}=\frac{e^{-\alpha\eta\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i, \quad \tilde{\Theta}^1_{4,4,i}=-e^{-\alpha\eta\tau_2}(Q_{1i}-Q_{2i}) \\ -\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}R_{1i}E_i-\frac{e^{-\alpha\eta\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i,$$

$$\tilde{\Theta}^1_{5,5,i}=\eta\tau_2R_{1i}+(\tau_2-\eta\tau_2)R_{2i}+\eta\tau_2R_{3i}-K_2-K_2^{\mathrm{T}},$$

$$\tilde{\Theta}^2_{1,1,i}=(E_i^{\mathrm{T}}P_i+SR^{\mathrm{T}}+K_1^{\mathrm{T}})A_i+A_i^{\mathrm{T}}(P_iE_i+RS^{\mathrm{T}}+K_1)+Q_{1i}+Q_{3i} \\ +\alpha E_i^{\mathrm{T}}P_iE_i-\frac{e^{-\alpha\eta\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}R_{1i}E_i-\frac{e^{-\alpha\tau_2}}{\tau_2}E_i^{\mathrm{T}}R_{3i}E_i,$$

$$\tilde{\Theta}^2_{1,2,i}=(E_i^{\mathrm{T}}P_i+SR^{\mathrm{T}}+K_1^{\mathrm{T}})B_i+\frac{e^{-\alpha\tau_2}}{\tau_2}E_i^{\mathrm{T}}R_{3i}E_i, \quad \tilde{\Theta}^2_{1,4,i}=\frac{e^{-\alpha\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}R_{1i}E_i,$$

$$\tilde{\Theta}^2_{1,5,i}=-K_1^{\mathrm{T}}+A_i^{\mathrm{T}}K_2, \quad \tilde{\Theta}^2_{2,2,i}=-(1-\mu)e^{-\alpha\tau_2}Q_{3i} \\ -\frac{2e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i-\frac{e^{-\alpha\tau_2}}{\tau_2}E_i^{\mathrm{T}}R_{3i}E_i,$$

$$\tilde{\Theta}^2_{2,3,i}=\frac{e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i, \quad \tilde{\Theta}^2_{2,4,i}=\frac{e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i, \quad \tilde{\Theta}^2_{2,5,i}=B_i^{\mathrm{T}}K_2,$$

$$\tilde{\Theta}^2_{3,3,i}=-e^{-\alpha\tau_2}Q_{2i}-\frac{e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i,$$

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$$\tilde{\Theta}^2_{4,4,i} = -e^{-\alpha\tau_2}(Q_{1i}-Q_{2i}) - \frac{e^{-\alpha\tau_2}}{\eta\tau_2}E_i^{\mathrm{T}}R_{1i}E_i - \frac{e^{-\alpha\tau_2}}{\tau_2-\eta\tau_2}E_i^{\mathrm{T}}R_{2i}E_i,$$

$$\tilde{\Theta}^2_{5,5,i} = \eta\tau_2 R_{1i} + (\tau_2-\alpha\tau_2)R_{2i} + \tau_2 R_{3i} - K_2 - K_2^{\mathrm{T}},$$
$$\Lambda_1 = [(E_i^{\mathrm{T}}P_i + SR^{\mathrm{T}}) \quad K_1^{\mathrm{T}} \quad 0 \quad 0 \quad K_2^{\mathrm{T}}]^T, \Lambda_2 = [H_{1i}^{\mathrm{T}} \quad H_{2i}^{\mathrm{T}} \quad 0 \quad 0 \quad 0]^{\mathrm{T}}.$$

and the remaining terms are zero. Moreover, the estimate of state decay is given by

$$||x(t)|| \le \sqrt{\frac{b}{a}}e^{-\lambda(t-t_0)}||\phi||_{\mathbb{C}} \tag{42}$$

where μ_1 satisfies

$$P_i \le \mu_1 P_j, Q_{ri} \le \mu_1 Q_{rj}, R_{ri} \le \mu_1 R_{rj}, \forall i,j \in U, r=1,2,3. \tag{43}$$
$$\lambda = \frac{1}{2}\left(\alpha - \frac{\ln\mu_1}{T_a}\right), a = \min_{\forall i \in U}(\lambda_{\min}P_i),$$
$$b = \max_{\forall i\in U}(\lambda_{\max}P_i) + \eta\tau_2 \max_{\forall i\in U}(\lambda_{\max}Q_{1i}) + \tau_2 \max_{\forall i\in U}(\lambda_{\max}Q_{2i})$$
$$+\tau_2 \max_{\forall i\in U}(\lambda_{\max}Q_{3i}) + \frac{(\eta\tau_2)^2}{2}\max_{\forall i\in U}(\lambda_{\max}R_{1i})$$
$$+\frac{(\tau_2-\eta\tau_2)^2}{2}\max_{\forall i\in U}(\lambda_{\max}R_{2i}) + \frac{\tau_2^2}{2}\max_{\forall i\in U}(\lambda_{\max}R_{3i}).$$

Proof

The proof of this theorem is similar to Theorem 3.1.

Remark 4

In system (39), consider the time-varying delay is constant, now we can discuss the robust stability criterion for the following system

$$E_i\dot{x}(t) = A_i x(t) + B_i x(t-\tau),$$
$$x(t) = \phi(t), t \in [-\tau, 0]. \tag{44}$$

Now, we shall analyze the system (44) in the following Corollary.

Corollary 3

For given scalars $\tau > 0$, tuning parameter η ($0 < \eta < 1$), and $\alpha > 0$, the equilibrium point of system (44) is robustly exponentially admissible and for any switching signal σ with ADT satisfying

$$T_a > T_a^* = \frac{\ln\mu_1}{\alpha}, \quad \mu_1 \ge 1 \tag{45}$$

if there exist symmetric positive-definite matrices P_i, Q_{1i}, Q_{2i}, Q_{3i}, R_{1i}, R_{2i}, R_{3i}, real matrices K_1, K_2, for any matrix S with appropriate dimension, and the constant matrix $R \in \mathbb{R}^{n\times(n-r)}$ satisfying $E_i^T R = 0$ with rank(R) = $n - r$ such that the following symmetric LMIs hold:

$$\bar{\Theta} = \begin{bmatrix} \bar{\Theta}_{1,1} & \bar{\Theta}_{1,2} & \frac{e^{-\alpha\eta\tau}}{\eta\tau}E_i^{\mathrm{T}}R_{1i}E_i & -K_1^{\mathrm{T}} + A_i^{\mathrm{T}}K_2 & (E_i^{\mathrm{T}}P_i + SR^{\mathrm{T}})D_i & \epsilon_i H_{1i} \\ * & \bar{\Theta}_{2,2} & \frac{e^{-\alpha\tau}}{\tau-\eta\tau}E_i^{\mathrm{T}}R_{2i}E_i & B_i^{\mathrm{T}}K_2 & K_1 & \epsilon_i H_{2i} \\ * & * & \bar{\Theta}_{3,3} & 0 & 0 & 0 \\ * & * & * & \bar{\Theta}_{4,4} & K_2 & 0 \\ * & * & * & * & -\epsilon_i I & 0 \\ * & * & * & * & 0 & -\epsilon_i I \end{bmatrix} < 0 \tag{46}$$

where

$$\bar{\Theta}_{1,1} = (E_i^{\mathrm{T}}P_i + K_1^{\mathrm{T}})A_i + A_i^{\mathrm{T}}(P_iE_i + K_1) + Q_{1i} + Q_{3i} + \alpha E_i^{\mathrm{T}}P_iE_i$$
$$-\frac{e^{-\alpha\eta\tau}}{\eta\tau}E_i^{\mathrm{T}}R_{1i}E_i - \frac{e^{-\alpha\tau}}{\eta\tau}E_i^{\mathrm{T}}R_{3i}E_i, \quad \bar{\Theta}_{1,2} = (E_i^{\mathrm{T}}P_i + K_1^{\mathrm{T}})B_i + \frac{e^{-\alpha\tau}}{\tau}E_i^{\mathrm{T}}R_{3i}E_i,$$
$$\bar{\Theta}_{2,2} = -e^{-\alpha\tau}(Q_{2i}+Q_{3i}) - \frac{e^{-\alpha\tau}}{\tau-\eta\tau}E_i^{\mathrm{T}}R_{2i}E_i - \frac{e^{-\alpha\tau}}{\tau}E_i^{\mathrm{T}}R_{3i}E_i,$$
$$\bar{\Theta}_{3,3} = -e^{-\alpha\tau}Q_{1i} - \frac{e^{-\alpha\eta\tau}}{\eta\tau}E_i^{\mathrm{T}}R_{1i}E_i - \frac{e^{-\alpha\tau}}{\tau-\eta\tau}E_i^{\mathrm{T}}R_{2i}E_i,$$
$$\bar{\Theta}_{4,4} = \eta\tau R_{1i} + (\tau-\alpha\tau)R_{2i} + \tau R_{3i} - K_2 - K_2^{\mathrm{T}}.$$

and the remaining terms are zero. Moreover, the estimate of state decay is given by

$$||x(t)|| \le \sqrt{\frac{b}{a}}e^{-\lambda(t-t_0)}||\phi||_{\mathbb{C}} \tag{47}$$

where μ_1 satisfies

$$P_i \le \mu_1 P_j, Q_{ri} \le \mu_1 Q_{rj}, R_{ri} \le \mu_1 R_{rj}, \forall i,j \in U, r=1,2,3. \tag{48}$$
$$\lambda = \frac{1}{2}\left(\alpha - \frac{\ln\mu_1}{T_a}\right), a = \min_{\forall i \in U}(\lambda_{\min}P_i),$$
$$b = \max_{\forall i\in U}(\lambda_{\max}P_i) + \eta\tau \max_{\forall i\in U}(\lambda_{\max}Q_{1i}) + \tau \max_{\forall i\in U}(\lambda_{\max}Q_{2i})$$
$$+\tau \max_{\forall i\in U}(\lambda_{\max}Q_{3i}) + \frac{(\eta\tau)^2}{2}\max_{\forall i\in U}(\lambda_{\max}R_{1i})$$
$$+\frac{(\tau-\eta\tau)^2}{2}\max_{\forall i\in U}(\lambda_{\max}R_{2i}) + \frac{\tau^2}{2}\max_{\forall i\in U}(\lambda_{\max}R_{3i}).$$

5. NUMERICAL EXAMPLES

In this section, numerical examples are provided to demonstrate the effectiveness and applicability of the proposed method.

Example 1

Consider the switched non-linear singular time-delay system

$$E_i\dot{x}(t) = A_i x(t) + B_i x(t-\tau(t)) + F_{1i}f_{1i}(t, x(t)) + F_{2i}f_{2i}(t, x(t-\tau(t))), \tag{49}$$

where

Subsystem 1

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$$E_1=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\quad A_1=\begin{bmatrix}-5.5&1&0\\-5.2&0.9&0\\1&0&-4.2\end{bmatrix},\quad B_1=\begin{bmatrix}-1.3&0&-1.1\\0&-1.7&0\\0&0.2&-1\end{bmatrix},\quad R=\begin{bmatrix}0\\0\\1\end{bmatrix},$$

$$F_{11}=\begin{bmatrix}1&1&0\\0&1&0\\0&0&1\end{bmatrix},\quad F_{21}=\begin{bmatrix}1&0&0\\1&1&0\\0&0&1\end{bmatrix},\quad u_{11}=\begin{bmatrix}0.04&0&0\\0&0.04&0\\0&0&0.04\end{bmatrix},\quad u_{21}=\begin{bmatrix}2&0&0\\0&1&0\\0&0&0.01\end{bmatrix},$$

Subsystem 2

$$E_2=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\quad A_2=\begin{bmatrix}-4.5&0.1&0\\0.2&-2.6&0\\-0.5&1&-2\end{bmatrix},\quad B_2=\begin{bmatrix}-0.8&0.3&0.9\\-0.7&0.2&0\\-0.1&0&0.5\end{bmatrix},\quad F_{12}=\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix},$$

$$F_{22}=\begin{bmatrix}1&0&1\\0&1&0\\0&0&1\end{bmatrix},\quad u_{12}=\begin{bmatrix}0.03&0&0\\0&0.03&0\\0&0&0.03\end{bmatrix},\quad u_{22}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix},\quad R=\begin{bmatrix}0\\0\\1\end{bmatrix}.$$

Considering the values $\alpha=0.15$, $\mu=0.1$, $\tau_2=0.2$, $\eta=0.1$ and $\mu_1=1.5$. The smallest ADT T_a^* can be calculated from (3) as 2.7031 from which the ADT can be taken as $T_a=3$. Solving the inequalities from (4) to (7) obtained in Theorem 3.1 using MATLAB LMI Control Toolbox, we can get the following feasible solutions. Due to the page consideration, we listed few matrices here,

$$P_1=\begin{bmatrix}1.7454&-0.4230&0.2718\\-0.4230&0.6414&-0.0748\\0.2718&-0.0748&3.1649\end{bmatrix},\quad P_2=\begin{bmatrix}1.6422&-0.3406&-0.1256\\-0.3406&0.8371&0.0559\\-0.1256&0.0559&3.1707\end{bmatrix},\quad Q_{11}=\begin{bmatrix}2.3556&-0.0485&-0.8416\\-0.0485&0.2751&-0.0006\\-0.8416&-0.0006&1.0295\end{bmatrix},$$

$$Q_{21}=\begin{bmatrix}2.0746&-0.2237&-0.4614\\-0.2237&0.2530&-0.0165\\-0.4614&-0.0165&0.6266\end{bmatrix},\quad Q_{31}=\begin{bmatrix}3.9152&0.0750&-0.5054\\0.0750&0.1687&0.1967\\-0.5054&0.1967&2.5143\end{bmatrix},\quad Q_{32}=\begin{bmatrix}3.3399&0.0209&-0.9019\\0.0209&0.2009&0.1392\\-0.9019&0.1392&2.2696\end{bmatrix},$$

$$R_{11}=\begin{bmatrix}0.0427&-0.0149&-0.0058\\-0.0149&0.0725&0.0007\\-0.0058&0.0007&2.2062\end{bmatrix},\quad R_{21}=\begin{bmatrix}0.4207&-0.1715&-0.1718\\-0.1715&0.5007&0.0461\\-0.1718&0.0461&0.8209\end{bmatrix},$$

$$R_{31}=\begin{bmatrix}0.1027&-0.0587&-0.0386\\-0.0587&0.1694&0.0144\\-0.0386&0.0144&0.8326\end{bmatrix},\quad R_{32}=\begin{bmatrix}0.0947&-0.0423&-0.0387\\-0.0423&0.1395&0.0140\\-0.0387&0.0140&0.8785\end{bmatrix}$$

and $\epsilon_{11}=3.7978$, $\epsilon_{12}=6.1053$, $\epsilon_{21}=0.8761$, $\epsilon_{22}=1.1191$. Solving the inequality (6), we get $a=0.4979$, $b=4.5922$ and $||x(t)||\leq 3.0371e^{-0.0074(t-t_0)}||\phi||_{\mathbb{C}}$ then the system (2) is exponentially stable with ADT $T_a=3$.

If $E_i=I$ then the system (49) becomes a normal one. For this case, the condition $E_i^T R=0$ can be relaxed and the corresponding stability conditions can be obtained. Also, regular and impulse-free conditions can be dropped. In the

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above example, consider the matrix E_i to be identity and all other parameters are as defined in Example 5.1. Solving the inequalities (4)–(7) which obtained from Theorem 3.1 using MATLAB LMI Control Toolbox, we can get the following feasible solutions. Due to the page consideration, few matrices are listed here

$$P_1=\begin{bmatrix} 4.8852 & -1.0969 & -1.0482 \\ -1.0969 & 1.3060 & 0.6190 \\ -1.0482 & 0.6190 & 2.8623 \end{bmatrix},\quad P_2=\begin{bmatrix} 5.1477 & -0.9399 & -1.4865 \\ -0.9399 & 1.6158 & 0.8816 \\ -1.4865 & 0.8816 & 3.2644 \end{bmatrix},\quad Q_{11}=\begin{bmatrix} 4.8300 & -0.1837 & -1.3437 \\ -0.1837 & 0.5585 & 0.1045 \\ -1.3437 & 0.1045 & 2.4455 \end{bmatrix},$$

$$Q_{21}=\begin{bmatrix} 4.4681 & -0.4998 & -1.0725 \\ -0.4998 & 0.5465 & 0.0081 \\ -1.0725 & 0.0081 & 2.1092 \end{bmatrix},\quad Q_{31}=\begin{bmatrix} 8.4871 & 0.0881 & -1.4455 \\ 0.0881 & 0.3186 & 0.1700 \\ -1.4455 & 0.1700 & 3.6986 \end{bmatrix},\quad Q_{32}=\begin{bmatrix} 6.9750 & -0.0190 & -1.9550 \\ -0.0190 & 0.3960 & 0.1984 \\ -1.9550 & 0.1984 & 3.6738 \end{bmatrix},$$

$$R_{11}=\begin{bmatrix} 0.0783 & -0.0177 & 0.0068 \\ -0.0177 & 0.1384 & -0.0109 \\ 0.0068 & -0.0109 & 0.0772 \end{bmatrix},\quad R_{21}=\begin{bmatrix} 0.7898 & -0.2794 & 0.0956 \\ -0.2794 & 1.0672 & -0.0696 \\ 0.0956 & -0.0696 & 0.7131 \end{bmatrix},$$

$$R_{31}=\begin{bmatrix} 0.1845 & -0.0747 & 0.0154 \\ -0.0747 & 0.3065 & -0.0358 \\ 0.0154 & -0.0358 & 0.1501 \end{bmatrix},$$

$$R_{32}=\begin{bmatrix} 0.1779 & -0.0562 & 0.0104 \\ -0.0562 & 0.2622 & -0.0252 \\ 0.0104 & -0.0252 & 0.1486 \end{bmatrix}$$

and $\epsilon_{11}=7.3652$, $\epsilon_{12}=10.7971$, $\epsilon_{21}=1.6914$, $\epsilon_{22}=2.2906$. Solving the inequality (6), we get $a=0.9361$, $b=9.2082$ and $||x(t)||\leq 3.1364e^{-0.0074(t-t_0)}||\phi||_{\mathbb{C}}$ then the system (2) is exponentially stable with ADT $T_a=3$. The simulation results for the proposed system with compatible initial conditions and different modes are shown in Figures 1 and 2 while $E_i=I$. In addition, the response of switching signal is shown in Figure 3.

Example 2

Consider the switched singular system (39) with uncertainties in the following known matrices of

Subsystem 1

$$E_1=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\quad A_1=\begin{bmatrix} -5.5 & 1 & 0 \\ -5.2 & 0.9 & 0 \\ 1 & 0 & -4.2 \end{bmatrix},\quad B_1=\begin{bmatrix} -1.3 & 0 & -1.1 \\ 0 & -1.7 & 0 \\ 0 & 0.2 & -1 \end{bmatrix},\quad R=\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$D_1=\begin{bmatrix} 0.5 & 2 & 0.1 \\ -1.8 & 0.2 & 0.5 \\ -2 & -3.5 & 3.2 \end{bmatrix},\quad H_{11}=\begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix},\quad H_{21}=\begin{bmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{bmatrix},$$

Subsystem 2

$$E_2=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix},\quad A_2=\begin{bmatrix} -4.5 & 0.1 & 0 \\ 0.2 & -2.6 & 0 \\ -0.5 & 1 & -2 \end{bmatrix},\quad B_2=\begin{bmatrix} -0.8 & 0.3 & 0.9 \\ -0.7 & 0.2 & 0 \\ -0.1 & 0 & 0.5 \end{bmatrix},\quad R=\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$D_2=\begin{bmatrix} 1.1 & 2.8 & -2.5 \\ 2.4 & -1.4 & 3.6 \\ -3.2 & 2.1 & 3.8 \end{bmatrix},\quad H_{12}=\begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix},\quad H_{22}=\begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}.$$

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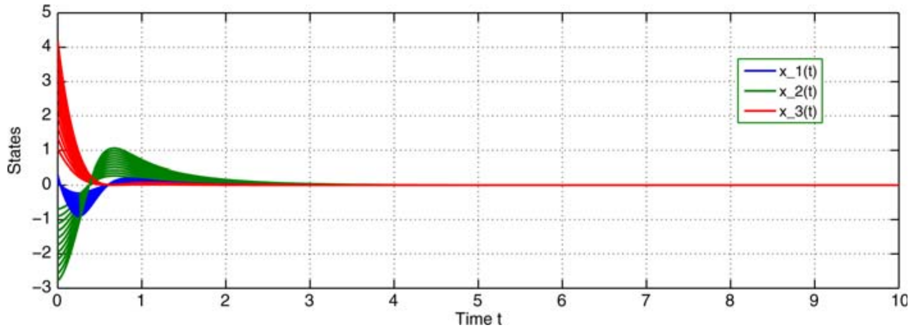
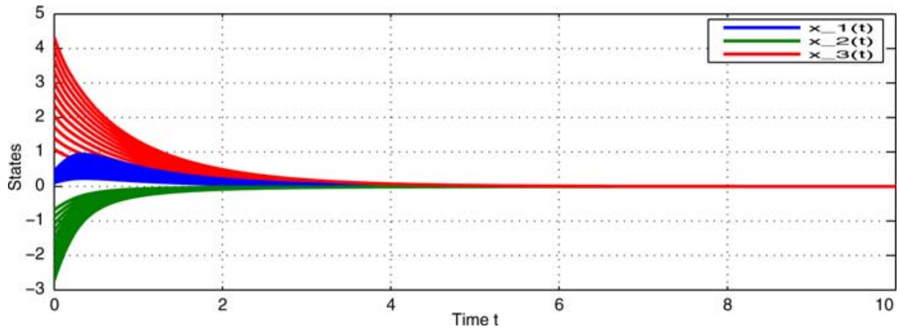
FIGURE 1



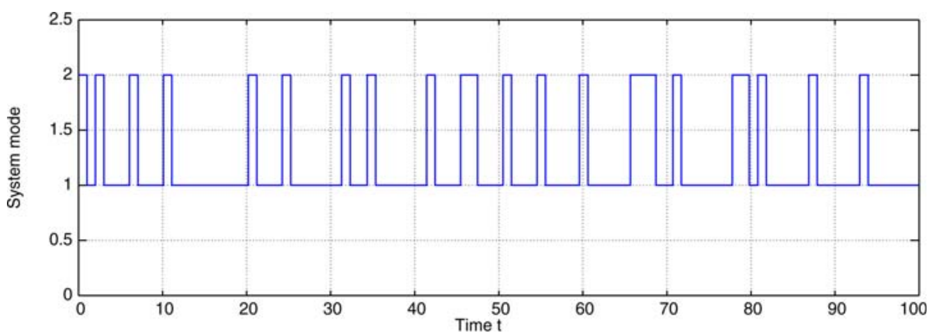
The state trajectories of the proposed system with different modes and $E_1 = I$.

FIGURE 2



The state trajectories of the proposed system with different modes and $E_2 = I$.

FIGURE 3



Switching signal with average dwell time when $E_i = I$.

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Considering the values $\alpha = 0.14$, $\mu = 0.1$, $\tau_2 = 0.2$, $\eta = 0.1$ and $\mu_1 = 1.9$. The smallest ADT T_a^* can be calculated from (3) as 4.5847 from which the ADT can be taken as $T_a = 5$. Solving the inequalities (41) to (43) which obtained Theorem 4.1 using MATLAB LMI Control Toolbox, we can get the following feasible solutions. Due to the page consideration, few matrices are listed here

$$P_1 = \begin{bmatrix} 1.2250 & -0.2051 & -0.0597 \\ -0.2051 & 0.3893 & 0.2434 \\ -0.0597 & 0.2434 & 3.6703 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 0.9296 & -0.1159 & 0.0515 \\ -0.1159 & 0.5576 & 0.4334 \\ 0.0515 & 0.4334 & 3.7510 \end{bmatrix}, \quad Q_{11} = \begin{bmatrix} 1.6581 & 0.0566 & -0.0555 \\ 0.0566 & 0.0531 & -0.0404 \\ -0.0555 & -0.0404 & 0.1046 \end{bmatrix},$$

$$Q_{21} = \begin{bmatrix} 1.6445 & 0.0423 & -0.0481 \\ 0.0423 & 0.0271 & -0.0273 \\ -0.0481 & -0.0273 & 0.0701 \end{bmatrix}, \quad Q_{31} = \begin{bmatrix} 1.4607 & 0.0314 & 0.0821 \\ 0.0314 & 0.0195 & -0.0918 \\ 0.0821 & -0.0918 & 0.8235 \end{bmatrix}, \quad Q_{32} = \begin{bmatrix} 1.7858 & 0.0574 & -0.1017 \\ 0.0574 & 0.0194 & -0.0661 \\ -0.1017 & -0.0661 & 0.5520 \end{bmatrix},$$

$$R_{11} = \begin{bmatrix} 0.0480 & -0.0107 & 0.0082 \\ -0.0107 & 0.1055 & -0.0118 \\ 0.0082 & -0.0118 & 0.8780 \end{bmatrix}, \quad R_{21} = \begin{bmatrix} 0.3307 & -0.0610 & 0.0710 \\ -0.0610 & 0.4328 & -0.0610 \\ 0.0710 & -0.0610 & 0.1516 \end{bmatrix},$$

$$R_{31} = \begin{bmatrix} 0.1134 & -0.0710 & 0.0331 \\ -0.0710 & 0.3476 & -0.0596 \\ 0.0331 & -0.0596 & 0.1494 \end{bmatrix}, \quad R_{32} = \begin{bmatrix} 0.1115 & -0.0499 & 0.0273 \\ -0.0499 & 0.2464 & -0.0366 \\ 0.0273 & -0.0366 & 0.1742 \end{bmatrix}$$

and $\delta_1 = 3.6651$, $\delta_2 = 10.9315$. Solving the inequality (42), we get $a = 0.3267$, $b = 4.5742$ and $||x(t)|| \leq 3.7420\, e^{-0.0102(t-t_0)}||\phi||_{\mathbb{C}}$ then the system (2) is exponentially stable with ADT $T_a = 5$.

6. CONCLUSION

The problem of delay-dependent exponential stability analysis of non-linear switched singular systems has been studied. The exponential stability criterion have been obtained, which can guaranteed that the non-linear switched singular time-delay system to be regular, impulse-free, and exponentially stable. Sufficient delay-dependent exponential stability conditions have been derived based on ADT approach and delay decomposition approach which yields the less conservative results. Finally, numerical examples have been illustrated which shows that the effectiveness of the proposed results. Furthermore, the delay decomposition approach and ADT approach can be used to study the delay-dependent robust stability, robust control problem for singular time-delay systems such as H_∞ control, feed forward control, feed backward control, guaranteed cost control variable structure control and Takagi–Sugeno fuzzy model, Markov jump systems on neural networks.

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