Automatica 44 (2008) 2635–2641

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Control for discrete singular hybrid systems☆

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ARTICLE INFO

Article history:
Received 26 July 2007
Received in revised form
6 January 2008
Accepted 28 February 2008
Available online 19 September 2008

Keywords:
Discrete-time systems
Linear matrix inequality (LMI)
Hybrid systems
State feedback control
Static output feedback control
Singular systems

ABSTRACT

The problems of stability, state feedback control and static output feedback control for a class of discrete-time singular hybrid systems are investigated in this paper. A new sufficient and necessary condition for a class of discrete-time singular hybrid systems to be regular, causal and stochastically stable is proposed in terms of a set of coupled strict linear matrix inequalities (LMIs). Sufficient conditions are proposed for the existence of state feedback controller and static output feedback controller in terms of a set of coupled strict LMIs, respectively. Finally, two illustrative examples are provided to demonstrate the applicability of the proposed approach.

1. Introduction

Many physical systems can have different structures due to random abrupt changes, which may be caused by random failures and repairs of the components, changes in the interconnections of subsystems, sudden environment changes, modification of the operating point of a linearized model of a nonlinear system, etc. Such systems can be modeled as hybrid ones with two components in the state vector. One is system state, the other is a discrete variable called system mode. A special class of hybrid systems is the so-called Jump Linear System (JLS). JLS is a hybrid system with many operation modes. In the JLS, each mode corresponds to a deterministic dynamical behavior, and the random jumps in system parameters are governed by a Markov process which takes values in a finite set. A number of control problems related to continuous- or discrete-time JLS has been analyzed by several authors since the mid 1960s, see, e.g. Aberkane, Ponsart, and Sauter (2006), Blair and Sworder (1975), Boukas, Liu, and Liu (2001), Boukas and Shi (1998), de Souza (2006), Shi and Boukas (1997), Shi, Boukas, and Agarwal (1999), Xie, Ogai, Inoe, and Ohata (2006) and Zhang, Basin, and Skliar (2006) and the references therein.

As far as we know, a singular system is also a natural representation of dynamic systems and describes a larger class of systems than the normal linear system model. The singular form is useful to represent and handle systems such as mechanical systems, electric circuits, interconnected systems, and so on. In the past decades, control for singular systems has been extensively studied and many notions and results in state-space systems have been extended to singular systems, such as stability and stabilization (see, e.g. Boukas, Xu, and Lam (2005), Dai (1989), Xu and Lam (2004) and Xu and Yang (1999)), H_∞ control problem (see, e.g. Masubuchi, Kamime, Ohara, and Suda (1997) and Xia and Jia (2003)), mixed H_2/H_∞ control (see, e.g. Xia, Shi, Liu, and Rees (2005) and Zhang, Huang, and Lam (2003)), filtering problem (see, e.g. Xu, Lam, and Zou (2003) and Yue and Han (2004)), and so on. In recent years, more and more attention has been paid to deriving strict LMI condition for stability analysis and controller design, see, e.g. Uezato and Ikeda (1999), Xu, Van Dooren, Stefan, and Lam (2002) and Zhang et al. (2003) for continuous singular system, and Xu and Lam (2004) and Zhang, Xia, and Shi (2008) for discrete singular system. The strict LMI conditions, that is, definite LMIs without equality constraints, are highly tractable and reliable when we use recent popular softwares for solving LMIs. More recently, continuous singular systems with Markovian switching have been extensively studied, see for example Boukas (2005, 2006a,b, 2007) and references therein, and the LMI conditions are not in the strict LMI settings. Moreover, to date and to the best of our knowledge, for a discrete singular system with Markovian jump parameters, the problem of stability, stabilization

☆ This paper was not presented at any IFAC meeting. This paper was recommended for publication in revised form by Associate Editor George Yin under the direction of Editor Ian R. Petersen.

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doi:10.1016/j.automatica.2008.02.027

and feedback control has not been fully investigated yet Lam, Shu, Xu, and Boukas (2007). This problem is important and challenging in both theory and practice, which motivated us for this study.

In this paper, firstly, we consider the problems of stability for discrete-time singular hybrid systems. A sufficient and necessary condition for a discrete-time singular hybrid system to be regular, causal and stochastically stable is proposed in terms of strict linear matrix inequalities (LMIs). Next, the state feedback controller and static output feedback controller for discrete-time singular hybrid systems are proposed. Finally, two illustrative examples are given to show the effectiveness of the proposed approach.

2. Problem formulation

Let the dynamics of the class of discrete-time singular systems with Markovian jumps be described by the following:

$$Ex_{k+1} = A(r_k)x_k + B(r_k)u_k \tag{1}$$

$$y_k = C(r_k)x_k \tag{2}$$

where, for $k \in \mathcal{Z}$, $x_k \in \mathbb{R}^n$ is the descriptor variable, $u_k \in \mathbb{R}^m$ is the control input and $y_k \in \mathbb{R}^q$ is the controlled output. $\{r_k, k \in \mathcal{Z}\}$ is a time homogeneous Markov chain taking values in a finite set $\mathcal{S} = \{1, 2, \dots, N\}$, with stationary transition probability matrix $\Pi = [\pi_{ij}]_{N\times N}$, where $\pi_{ij} = \Pr\{r_{k+1} = j | r_k = i\}$ with $\pi_{ij} \geq 0$, for $i, j \in \mathcal{S}$, and $\sum_{j=1}^{N} \pi_{ij} = 1$. The matrix $E \in \mathbb{R}^{n\times n}$ is supposed to be singular with $\operatorname{rank}(E) = r < n$. $A(r_k) \in \mathbb{R}^{n\times n}$, $B(r_k) \in \mathbb{R}^{n\times m}$ and $C(r_k) \in \mathbb{R}^{q\times n}$, for $r_k = i$, $i \in \mathcal{S}$ are known real-valued constant matrices of appropriate dimensions that describe the nominal system and $C(r_k)$ for $r_k = i$, $i \in \mathcal{S}$ are assumed to be of full row rank for simplicity.

Definition 1 (*Xu & Lam, 2006*).

I. The discrete singular hybrid system in (1) is said to be regular if, for each $i \in \mathcal{S}$, $\det(sE - A(i))$ is not identically zero.
II. The discrete singular hybrid system in (1) is said to be causal if, for each $i \in \mathcal{S}$, $\deg(\det(sE - A(i))) = \operatorname{rank}(E)$.
III. The discrete singular hybrid system in (1) is said to be stochastically stable if for any $x_0 \in \mathbb{R}^n$ and $r_0 \in \mathcal{S}$, there exists a scalar $M(x_0, r_0) > 0$ such that
$$\lim_{N\to\infty} \mathbb{E}\left\{\sum_{k=0}^{N} \|x(k, x_0, r_0)\|^2 | x_0, r_0\right\} \leq M(x_0, r_0),$$
where $x(k, x_0, r_0)$ denote the solution to system (1) at time k under the initial conditions x_0 and r_0.
IV. The discrete singular hybrid system in (1) is said to be stochastically admissible if it is regular, causal and stochastically stable.

Definition 2. System (1) is said to be regular, causal and stochastically stabilizable via state feedback (static output feedback) if there exists a control

$$u_k = K(r_k)x_k \tag{3}$$

or

$$u_k = K(r_k)y_k \tag{4}$$

with $K(i)$, when $r_k = i$, is a constant matrix, such that the closed-loop system is stochastically admissible.

The objective of this paper is to:

I. develop LMI-based conditions for system (1) with $u(t) \equiv 0$ to check if system (1) is stochastically admissible;
II. design a state feedback controller of the form (3) that renders the closed-loop system to be stochastically admissible; and
III. design a static output feedback controller of the form (4) that makes the closed-loop system stochastically admissible.

3. Stability analysis

In this section, we analyze the stochastic stability of system (1). Our attention will be paid to establishing strict LMI conditions to check the regularity, causality and stochastic stability of system (1). Firstly, we recall the stability results based on nonstrict conditions, then change them into strict ones.

Lemma 3 (*Xu & Lam, 2006*). *System (1) is stochastically admissible if and only if there exist symmetric matrices $P(i)$, $i \in \mathcal{S}$, such that the following coupled LMIs hold for each $i \in \mathcal{S}$:*

$$E^{\mathrm{T}}P(i)E \geq 0, \tag{5}$$

$$A^{\mathrm{T}}(i)\bar{P}(i)A(i) - E^{\mathrm{T}}P(i)E < 0, \tag{6}$$

where $\bar{P}(i) = \sum_{j=1}^{N} \pi_{ij}P(j)$.

Define $R \in \mathbb{R}^{n\times n}$ as the matrix with the properties of $E^{\mathrm{T}}R^{\mathrm{T}} = 0$ and $\operatorname{rank} R = n - r$, which is used in all the subsequent results.

Theorem 4. *System (1) is stochastically admissible if and only if there exist a set of symmetric-positive-definite matrices $P(i)$, $i \in \mathcal{S}$ and a symmetric and nonsingular matrix Φ, such that the following coupled LMIs hold for each $i \in \mathcal{S}$:*

$$A^{\mathrm{T}}(i)\left(\bar{P}(i) - R^{\mathrm{T}}\Phi R\right)A(i) - E^{\mathrm{T}}P(i)E < 0, \tag{7}$$

where $\bar{P}(i) = \sum_{j=1}^{N} \pi_{ij}P(j)$.

Proof. *Sufficiency*. Let $Y(i) = P(i) - R^{\mathrm{T}}\Phi R$ in (7), we can get

$$E^{\mathrm{T}}Y(i)E = E^{\mathrm{T}}(P(i) - R^{\mathrm{T}}\Phi R)E = E^{\mathrm{T}}P(i)E \geq 0, \tag{8}$$

$$A^{\mathrm{T}}(i)\bar{Y}(i)A(i) - E^{\mathrm{T}}Y(i)E < 0, \tag{9}$$

where $\bar{Y}(i) = \sum_{j=1}^{N} \pi_{ij}Y(j)$.

Necessity. Suppose that system (1) is stochastically admissible. Now, we choose two nonsingular matrices M and N such that

$$E = M\begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}N, \quad A(i) = M\begin{bmatrix} A_1(i) & A_2(i) \\ A_3(i) & A_4(i) \end{bmatrix}N. \tag{10}$$

From Xu and Lam (2006), we know that the regularity and causality of (1) imply that $A_4(i)$ is nonsingular for any $i \in \mathcal{S}$. Then, select a nonsingular matrix as

$$\mathcal{L}(i) = \begin{bmatrix} I & -A_3^{\mathrm{T}}(i)A_4^{-\mathrm{T}}(i) \\ 0 & I \end{bmatrix},$$

and let $\tilde{N}(i) = N^{-1}\mathcal{L}^{\mathrm{T}}(i)$. Then, it can be verified that

$$\tilde{E} = M^{-1}E\tilde{N}(i) = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \tag{11}$$

$$\tilde{A}(i) = M^{-1}A(i)\tilde{N}(i) = \begin{bmatrix} \tilde{A}_1(i) & A_2(i) \\ 0 & A_4(i) \end{bmatrix}, \tag{12}$$

where $\tilde{A}_1(i) = A_1(i) - A_2(i)A_4^{-1}(i)A_3(i)$.

Therefore, the stochastic stability of system (1) implies that the discrete Markovian jump system

$$\xi(k+1) = \tilde{A}_1(r_k)\xi(k),$$

is stochastically stable. It follows that there exist matrices $\tilde{P}(i) > 0$, $i \in \mathcal{S}$, such that

$$\tilde{A}_1^{\mathrm{T}}(i)\bar{\tilde{P}}(i)\tilde{A}_1(i) - \tilde{P}(i) < 0$$

where $\bar{\tilde{P}}(i) = \sum_{j=1}^{N} \pi_{ij}\tilde{P}(i)$.

Therefore, we can always find a sufficiently large scalar $\kappa > 0$ such that, for $i \in \mathcal{S}$,

$$\tilde{A}^{\mathrm{T}}(i)\begin{bmatrix}\bar{\tilde{P}}(i) & 0\\ 0 & \kappa I\end{bmatrix}\tilde{A}(i) - \tilde{E}^{\mathrm{T}}\begin{bmatrix}\tilde{P}(i) & 0\\ 0 & \kappa I\end{bmatrix}\tilde{E}$$
$$-\begin{bmatrix}0\\ A_4^{\mathrm{T}}(i)\end{bmatrix}2\kappa I\begin{bmatrix}0 & A_4(i)\end{bmatrix} = \begin{bmatrix}\Sigma_{(1,1)} & \Sigma_{(1,2)}\\ \Sigma_{(1,2)}^{\mathrm{T}} & \Sigma_{(2,2)}\end{bmatrix} < 0,$$

where $\Sigma_{(1,1)} = \tilde{A}_1^{\mathrm{T}}(i)\bar{\tilde{P}}(i)\tilde{A}_1(i) - \tilde{P}(i)$, $\Sigma_{(1,2)} = \tilde{A}_1^{\mathrm{T}}(i)\bar{\tilde{P}}(i)A_2(i)$, $\Sigma_{(2,2)} = A_2^{\mathrm{T}}(i)\bar{\tilde{P}}(i)A_2(i) - \kappa A_4^{\mathrm{T}}(i)A_4(i)$.

Let $M^{-1} = G = \begin{bmatrix}G_1^{\mathrm{T}} & G_2^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}}$, we obtain

$$G_2E = \begin{bmatrix}0 & 0\end{bmatrix}, G_2A(i)\tilde{N}(i) = \begin{bmatrix}0 & A_4(i)\end{bmatrix}.$$

Now, define $P(i) = G^{\mathrm{T}}\begin{bmatrix}\tilde{P}(i) & 0\\ 0 & \kappa I\end{bmatrix}G$, $R = WG_2$, $\Phi = 2\kappa W^{-\mathrm{T}}W^{-1}$, then we can get

$$\tilde{N}^{\mathrm{T}}(i)\left(A^{\mathrm{T}}(i)\bar{P}(i)A(i) - E^{\mathrm{T}}P(i)E - A^{\mathrm{T}}(i)R^{\mathrm{T}}\Phi RA(i)\right)\tilde{N}(i) < 0,$$

which is equivalent to (7). This completes the proof.

Remark 5. Theorem 4 provides a sufficient and necessary condition for the discrete-time singular hybrid system (1) to be stochastically admissible. It is noted that the condition in (7) is strict LMI, which is in contrast to that in Boukas (2006a) where a nonstrict LMI was reported.

Remark 6. Theorem 4 in this paper not only extends previous results in singular systems, but also provides a useful way to design the controller of state feedback and static feedback. The approach applied in this paper can be further extended to investigate the H_∞ control or filtering problem for singular hybrid system, which has not been fully investigated yet.

4. Controller design

In this section, we are in a position to present a solution to the problem of controller design for discrete-time singular hybrid systems (1). The following lemma is needed to derive the main results of this section.

Lemma 7. *For each $i \in \mathcal{S}$, $A^{\mathrm{T}}(i)(\bar{P}(i) - R^{\mathrm{T}}\Phi R)A(i) - E^{\mathrm{T}}P(i)E < 0$ if*

$$\begin{bmatrix}\Upsilon(i) & A^{\mathrm{T}}(i)L(i) - L^{\mathrm{T}}(i)\\ L^{\mathrm{T}}(i)A(i) - L(i) & \bar{P}(i) - L(i) - L^{\mathrm{T}}(i)\end{bmatrix} < 0, \tag{13}$$

where $\Upsilon(i) = A^{\mathrm{T}}(i)L(i) + L^{\mathrm{T}}(i)A(i) - A^{\mathrm{T}}(i)R^{\mathrm{T}}\Phi RA(i) - E^{\mathrm{T}}P(i)E$.

Proof. Since $\begin{bmatrix}I & A^{\mathrm{T}}(i)\end{bmatrix}$ is of full row rank, the desired result can be seen from the fact that

$$A^{\mathrm{T}}(i)(\bar{P}(i) - R^{\mathrm{T}}\Phi R)A(i) - E^{\mathrm{T}}P(i)E$$
$$= \begin{bmatrix}I & A^{\mathrm{T}}(i)\end{bmatrix}\begin{bmatrix}\Upsilon(i) & A^{\mathrm{T}}(i)L(i) - L^{\mathrm{T}}(i)\\ L^{\mathrm{T}}(i)A(i) - L(i) & \bar{P}(i) - L(i) - L^{\mathrm{T}}(i)\end{bmatrix}\begin{bmatrix}I\\ A(i)\end{bmatrix},$$

where $\Upsilon(i) = A^{\mathrm{T}}(i)L(i) + L^{\mathrm{T}}(i)A(i) - A^{\mathrm{T}}(i)R^{\mathrm{T}}\Phi RA(i) - E^{\mathrm{T}}P(i)E$. This completes the proof.

Defining $G(i) = L^{-1}(i)$ and $\Psi = \Phi^{-1}$, then pre- and post-multiplying (13) by $\begin{bmatrix}G^{\mathrm{T}}(i) & 0\\ 0 & G^{\mathrm{T}}(i)\end{bmatrix}$ and its transpose, respectively, we can get

$$\begin{bmatrix}\Gamma(i) & G^{\mathrm{T}}(i)A^{\mathrm{T}}(i) - G(i)\\ A(i)G(i) - G^{\mathrm{T}}(i) & G^{\mathrm{T}}(i)\bar{P}(i)G(i) - G^{\mathrm{T}}(i) - G(i)\end{bmatrix} < 0,$$

where

$$\Gamma(i) = G^{\mathrm{T}}(i)A^{\mathrm{T}}(i) + A(i)G(i) - G^{\mathrm{T}}(i)E^{\mathrm{T}}P(i)EG(i) - G^{\mathrm{T}}(i)A^{\mathrm{T}}(i)R^{\mathrm{T}}\Psi^{-1}RA(i)G(i).$$

Defining $\mathcal{X}_i(Y) = \mathrm{diag}\{Y(1), Y(2), \ldots, Y(N)\}$, $\mathcal{W}_i(G) = \begin{bmatrix}\sqrt{\pi_{i1}}G^{\mathrm{T}}(i) & \sqrt{\pi_{i2}}G^{\mathrm{T}}(i) & \cdots & \sqrt{\pi_{iN}}G^{\mathrm{T}}(i)\end{bmatrix}^{\mathrm{T}}$ and $Y(i) = P^{-1}(i)$, by using the Schur complement lemma, it follows that

$$\begin{bmatrix}\Sigma(i) & G^{\mathrm{T}}(i)A^{\mathrm{T}}(i) - G(i) & 0\\ A(i)G(i) - G^{\mathrm{T}}(i) & -G^{\mathrm{T}}(i) - G(i) & \mathcal{W}_i^{\mathrm{T}}(G)\\ 0 & \mathcal{W}_i(G) & -\mathcal{X}_i(Y)\end{bmatrix} < 0, \tag{14}$$

where

$$\Sigma(i) = G^{\mathrm{T}}(i)A^{\mathrm{T}}(i) + A(i)G(i) - G^{\mathrm{T}}(i)E^{\mathrm{T}}Y^{-1}(i)EG(i) - G^{\mathrm{T}}(i)A^{\mathrm{T}}(i)R^{\mathrm{T}}\Psi^{-1}RA(i)G(i).$$

Note that, for any scalars ε and δ, the following inequalities hold

$$\begin{aligned}0 &\le (G^{\mathrm{T}}(i)E^{\mathrm{T}} - \varepsilon Y(i))Y^{-1}(i)(EG(i) - \varepsilon Y(i))\\ &= G^{\mathrm{T}}(i)E^{\mathrm{T}}Y^{-1}(i)EG(i) - \varepsilon EG(i) - \varepsilon G^{\mathrm{T}}(i)E^{\mathrm{T}} + \varepsilon^2 Y(i),\\ 0 &\le (G^{\mathrm{T}}(i)A^{\mathrm{T}}(i)R^{\mathrm{T}} - \delta\Psi)\Psi^{-1}(RA(i)G(i) - \delta\Psi)\\ &= G^{\mathrm{T}}(i)A^{\mathrm{T}}(i)R^{\mathrm{T}}\Psi^{-1}RA(i)G(i) - \delta RA(i)G(i)\\ &\quad - \delta G^{\mathrm{T}}(i)A^{\mathrm{T}}(i)R^{\mathrm{T}} + \delta^2\Psi.\end{aligned}$$

Therefore, we have

$$\begin{aligned}&-G^{\mathrm{T}}(i)E^{\mathrm{T}}Y^{-1}(i)EG(i)\\ &\quad\le -\varepsilon EG(i) - \varepsilon G^{\mathrm{T}}(i)E^{\mathrm{T}} + \varepsilon^2 Y(i),\\ &\quad -G^{\mathrm{T}}(i)A^{\mathrm{T}}(i)R^{\mathrm{T}}\Psi^{-1}RA(i)G(i)\\ &\quad\le -\delta RA(i)G(i) - \delta G^{\mathrm{T}}(i)A^{\mathrm{T}}(i)R^{\mathrm{T}} + \delta^2\Psi.\end{aligned}$$

Then, (14) together with the above inequalities results in

$$\begin{bmatrix}\Xi(i) & G^{\mathrm{T}}(i)A^{\mathrm{T}}(i) - G(i) & 0\\ A(i)G(i) - G^{\mathrm{T}}(i) & -G^{\mathrm{T}}(i) - G(i) & \mathcal{W}_i^{\mathrm{T}}(G)\\ 0 & \mathcal{W}_i(G) & -\mathcal{X}_i(Y)\end{bmatrix}$$
$$\le \begin{bmatrix}\Omega(i) & G^{\mathrm{T}}(i)A^{\mathrm{T}}(i) - G(i) & 0\\ A(i)G(i) - G^{\mathrm{T}}(i) & -G^{\mathrm{T}}(i) - G(i) & \mathcal{W}_i^{\mathrm{T}}(G)\\ 0 & \mathcal{W}_i(G) & -\mathcal{X}_i(Y)\end{bmatrix},$$

where

$$\begin{aligned}\Xi(i) &= G^{\mathrm{T}}(i)A^{\mathrm{T}}(i) + A(i)G(i) - G^{\mathrm{T}}(i)A^{\mathrm{T}}(i)R^{\mathrm{T}}\Psi^{-1}RA(i)G(i)\\ &\quad - G^{\mathrm{T}}(i)E^{\mathrm{T}}Y^{-1}(i)EG(i),\\ \Omega(i) &= G^{\mathrm{T}}(i)A^{\mathrm{T}}(i) + A(i)G(i) - \delta RA(i)G(i)\\ &\quad - \delta G^{\mathrm{T}}(i)A^{\mathrm{T}}(i)R^{\mathrm{T}} + \delta^2\Psi\\ &\quad - \varepsilon G^{\mathrm{T}}(i)E^{\mathrm{T}} - \varepsilon EG(i) + \varepsilon^2 Y(i).\end{aligned}$$

Therefore, the following theorem summarizes the results of the above development.

Theorem 8. *Let ε and δ be given scalars. System (1) is regular, causal and stochastically stable if there exist symmetric and positive-definite matrices $Y(i)$ and nonsingular matrices $G(i)$ such that the following coupled of set of LMIs holds for each $i \in \mathcal{S}$*

$$\begin{bmatrix}\Omega(i) & G^{\mathrm{T}}(i)A^{\mathrm{T}}(i) - G(i) & 0\\ A(i)G(i) - G^{\mathrm{T}}(i) & -G^{\mathrm{T}}(i) - G(i) & \mathcal{W}_i^{\mathrm{T}}(G)\\ 0 & \mathcal{W}_i(G) & -\mathcal{X}_i(Y)\end{bmatrix} < 0, \tag{15}$$

where $\Omega(i) = G^{\mathrm{T}}(i)A^{\mathrm{T}}(i) + A(i)G(i) - \delta RA(i)G(i) - \delta G^{\mathrm{T}}(i)A^{\mathrm{T}}(i)R^{\mathrm{T}} - \varepsilon G^{\mathrm{T}}(i)E^{\mathrm{T}} - \varepsilon EG(i) + \delta^2\Psi + \varepsilon^2 Y(i)$.

Remark 9. In Theorem 8, scalars ε and δ are introduced to reduce the conservativeness, which can be seen from Example 17. It is easy to see that scalars ε and δ may be positive or negative, even zero. Obviously, when $\varepsilon = 1$ and $\delta = 1$, we have the Lemma 2.3 in Boukas (2006a).

4.1. State feedback control

Now, we consider the following form for the state feedback control

$$u_k = K(r_k)x_k, \tag{16}$$

where $K(i)$ is the feedback gain to be determined. Then, the closed-loop system obtained by applying controller (16) to system (1) is

$$\begin{aligned} Ex_{k+1} &= (A(r_k) + B(r_k)K(r_k))\,x_k, \\ y_k &= C(r_k)x_k. \end{aligned} \tag{17}$$

According to Theorems 4 and 8, we immediately have the following corollary.

Corollary 10. *Consider system (1), for each $i \in \mathcal{S}$ and given scalars ε and δ, if there exist symmetric and positive-definite matrices $Y(i)$, a symmetric and nonsingular matrix Ψ and matrices $G(i)$ and $K(i)$ satisfying the following matrix inequalities:*

$$\begin{bmatrix} \Omega_K(i) & G^{\mathrm{T}}(i)A_K^{\mathrm{T}}(i) - G(i) & 0 \\ A_K(i)G(i) - G^{\mathrm{T}}(i) & -G^{\mathrm{T}}(i) - G(i) & \mathcal{W}_i^{\mathrm{T}}(G) \\ 0 & \mathcal{W}_i(G) & -\mathcal{X}_i(Y) \end{bmatrix} < 0, \tag{18}$$

where

$$A_K(i) = A(i) + B(i)K(i),$$

$$\begin{aligned} \Omega_K(i) = {} & G^{\mathrm{T}}(i)A_K^{\mathrm{T}}(i) + A_K(i)G(i) - \delta R A_K(i)G(i) \\ & - \delta G^{\mathrm{T}}(i)A_K^{\mathrm{T}}(i)R^{\mathrm{T}} - \varepsilon G^{\mathrm{T}}(i)E^{\mathrm{T}} - \varepsilon EG(i) \\ & + \delta^2\Psi + \varepsilon^2 Y(i). \end{aligned}$$

Then system (17) is regular, causal and stochastically stabilizable via the state feedback controller $u_k = K(r_k)x_k$.

Now, we are in a position to give the solution to the stochastic stabilization problem based on Corollary 10.

Theorem 11. *Let ε and δ be given scalars. There exists a state feedback controller of the form (16) such that the closed-loop system is stochastically admissible if there exist symmetric and positive-definite matrices $Y(i)$, a symmetric and nonsingular matrix Ψ, nonsingular matrices $G(i)$, and matrices $H(i)$ such that the following coupled LMIs hold for each $i \in \mathcal{S}$*

$$\begin{bmatrix} \Sigma_K^{(1,1)}(i) & \Sigma_K^{(1,2)}(i) & 0 \\ \Sigma_K^{(1,2)}(i)^{\mathrm{T}} & -G^{\mathrm{T}}(i) - G(i) & \mathcal{W}_i^{\mathrm{T}}(G) \\ 0 & \mathcal{W}_i(G) & -\mathcal{X}_i(Y) \end{bmatrix} < 0, \tag{19}$$

where $\Sigma_K^{(1,1)}(i) = G^{\mathrm{T}}(i)A^{\mathrm{T}}(i) + H^{\mathrm{T}}(i)B^{\mathrm{T}}(i) + A(i)G(i) + B(i)H(i) - \delta RA(i)G(i) - \delta RB(i)H(i) - \delta G^{\mathrm{T}}(i)A^{\mathrm{T}}(i)R^{\mathrm{T}} - \delta H^{\mathrm{T}}(i)B^{\mathrm{T}}(i)R^{\mathrm{T}} - \varepsilon G^{\mathrm{T}}(i)E^{\mathrm{T}} - \varepsilon EG(i) + \delta^2\Psi + \varepsilon^2 Y(i)$, $\Sigma_K^{(1,2)}(i) = G^{\mathrm{T}}(i)A^{\mathrm{T}}(i) + H^{\mathrm{T}}(i)B^{\mathrm{T}}(i) - G(i)$. The gain of the stabilizing state feedback controller is given by: $K(i) = H(i)G^{-1}(i)$.

Proof. The desired result follows immediately by applying Corollary 10 to the closed-loop system (17) and setting $H(i) = K(i)G(i)$.

4.2. Static output feedback control

In this section, we consider the following static output feedback

$$u_k = F(r_k)y_k, \tag{20}$$

where $F(i) \in R^{m\times q}$ is the feedback gain to be determined. Then the closed-loop system is described as follows:

$$Ex_{k+1} = (A(r_k) + B(r_k)F(r_k)C(r_k))x_k. \tag{21}$$

It should be noted that the conditions developed in Boukas (2006a) and Gao, Lam, Wang, and Wang (2004) may not be easy to solve due to the presence of the equality constraints. However, the following method will make it much easier for controller design based on LMIs.

Since it is assumed that $C(i)$ is of full row rank, obviously, we can select a nonsingular $M(i)$ such that $C(i)M(i) = \begin{bmatrix} I_q & 0 \end{bmatrix}$ hold for $i \in \mathcal{S}$. Then pre- and post-multiplying the corresponding inequality to (7) for the closed-loop dynamics by $M^{\mathrm{T}}(i)$ and $M(i)$ respectively, yield

$$\begin{aligned} & \left(A(i)M(i) + B(i)\begin{bmatrix} F(i) & 0 \end{bmatrix}\right)^{\mathrm{T}} \left(\bar{P}(i) - R^{\mathrm{T}}\Phi R\right) \\ & \times \left(A(i)M(i) + B(i)\begin{bmatrix} F(i) & 0 \end{bmatrix}\right) - M^{\mathrm{T}}(i)E^{\mathrm{T}}P(i)EM(i) < 0. \end{aligned} \tag{22}$$

Using Lemma 8, inequality (22) can be guaranteed by

$$\begin{bmatrix} \Pi(i) & G^{\mathrm{T}}(i)\Theta(i)^{\mathrm{T}} - G(i) & 0 \\ \Theta(i)G(i) - G^{\mathrm{T}}(i) & -G^{\mathrm{T}}(i) - G(i) & \mathcal{W}_i^{\mathrm{T}}(G) \\ 0 & \mathcal{W}_i(G) & -\mathcal{X}_i(Y) \end{bmatrix} < 0,$$

where

$$\begin{aligned} \Pi(i) = {} & G^{\mathrm{T}}(i)\left(A(i)M(i) + B(i)\begin{bmatrix} F(i) & 0 \end{bmatrix}\right)^{\mathrm{T}} \\ & + \left(A(i)M(i) + B(i)\begin{bmatrix} F(i) & 0 \end{bmatrix}\right)G(i) \\ & - \delta R\left(A(i)M(i) + B(i)\begin{bmatrix} F(i) & 0 \end{bmatrix}\right)G(i) \\ & - \delta G^{\mathrm{T}}(i)\left(A(i)M(i) + B(i)\begin{bmatrix} F(i) & 0 \end{bmatrix}\right)^{\mathrm{T}} R^{\mathrm{T}} \\ & - \varepsilon G^{\mathrm{T}}(i)M^{\mathrm{T}}(i)E^{\mathrm{T}} - \varepsilon EM(i)G(i) + \delta^2\Psi + \varepsilon^2 Y(i) \end{aligned}$$

$$\Theta(i) = \left(A(i)M(i) + B(i)\begin{bmatrix} F(i) & 0 \end{bmatrix}\right)G(i).$$

Therefore, we have the following corollary.

Corollary 12. *Consider system (1) and (2), for each $i \in \mathcal{S}$ and given matrices $M(i)$ and scalars ε and δ, if there exist symmetric and positive-definite matrices $Y(i)$, a symmetric and nonsingular matrix Ψ and matrices $G(i)$ and $F(i)$ satisfying the following matrix inequalities:*

$$\begin{bmatrix} \Pi(i) & G^{\mathrm{T}}(i)\Theta(i)^{\mathrm{T}} - G(i) & 0 \\ \Theta(i)G(i) - G^{\mathrm{T}}(i) & -G^{\mathrm{T}}(i) - G(i) & \mathcal{W}_i^{\mathrm{T}}(G) \\ 0 & \mathcal{W}_i(G) & -\mathcal{X}_i(Y) \end{bmatrix} < 0.$$

Then system (1) is regular, causal and stochastically stabilizable via the static output feedback controller $u_k = F(r_k)y_k$.

The following theorem summarizes the results of the above development.

Theorem 13. *Let $M(i)$, ε and δ be given matrices and scalars, respectively. There exists a static output feedback controller of the form (20) such that the closed-loop system is stochastically admissible if there exist symmetric and positive-definite matrices $Y(i)$, a symmetric and nonsingular matrix Ψ, matrices $J(i)$, and nonsingular matrices $G(i)$ such that the following coupled set of LMIs holds for each $i \in \mathcal{S}$:*

$$\begin{bmatrix} \Pi_F(i) & \Lambda_F^{\mathrm{T}} & 0 \\ \Lambda_F & -G^{\mathrm{T}}(i) - G(i) & \mathcal{W}_i^{\mathrm{T}}(G) \\ 0 & \mathcal{W}_i(G) & -\mathcal{X}_i(Y) \end{bmatrix} < 0, \tag{23}$$

where $G(i)$ is in the form of $G(i) = \begin{bmatrix} G_{11}(i) & 0 \\ G_{21}(i) & G_{22}(i) \end{bmatrix}$, and $G_{11}(i) \in R^{q\times q}$ is nonsingular, $G_{21}(i) \in R^{(n-q)\times q}$ and $G_{22}(i) \in R^{(n-q)\times(n-q)}$ are arbitrary matrices,

$$\begin{aligned} \Pi_F(i) = {} & \left(A(i)M(i)G(i) + B(i)\begin{bmatrix} J(i) & 0 \end{bmatrix}\right)^{\mathrm{T}} \\ & + \left(A(i)M(i)G(i) + B(i)\begin{bmatrix} J(i) & 0 \end{bmatrix}\right) \\ & - \delta R\left(A(i)M(i)G(i) + B(i)\begin{bmatrix} J(i) & 0 \end{bmatrix}\right) \\ & - \delta\left(A(i)M(i)G(i) + B(i)\begin{bmatrix} J(i) & 0 \end{bmatrix}\right)^{\mathrm{T}} R^{\mathrm{T}} \\ & - \varepsilon G^{\mathrm{T}}(i)M^{\mathrm{T}}(i)E^{\mathrm{T}} - \varepsilon EM(i)G(i) \\ & + \delta^2\Psi + \varepsilon^2 Y(i), \end{aligned}$$

$$\Lambda_F = \left(A(i)M(i)G(i) + B(i)\begin{bmatrix} J(i) & 0 \end{bmatrix}\right) - G^{\mathrm{T}}(i).$$

The gain of the stabilizing static output feedback controller is given by: $F(i) = J(i)G_{11}^{-1}(i)$.

Proof. Similarly to the proof of Theorem 11, and defining $J(i) = F(i)G_{11}(i)$, we can get the desired result based on the above analysis immediately.

Remark 14. Theorem 13 presents a new method to design a static output feedback controller in the LMI settings. It should be noticed that the use of $G(i)$ with the following structure:

$$G(i) = \begin{bmatrix} G_{11}(i) & 0 \\ G_{21}(i) & G_{22}(i) \end{bmatrix}$$

will provide less conservatism compared to the structure with the following form:

$$G(i) = \begin{bmatrix} G_{11}(i) & 0 \\ 0 & G_{22}(i) \end{bmatrix}$$

since extra matrix variable $G_{21}(i)$ is introduced, which has been discussed in Xia, Liu, Shi, Rees, and Thomas (2007).

Remark 15. Theorem 13 presents a sufficient condition for static output feedback stabilization of discrete-time singular hybrid systems (1). It should be pointed that the conditions of Theorem 13 are linear in the variables $Y(i)$, $G(i)$ and $J(i)$, for $i \in \mathcal{S}$ and hence they are easily tractable by convex optimization techniques, such as the recently developed interior-point methods available in Matlab LMI toolbox.

5. Numerical example

In this section, we present two illustrative examples to demonstrate the applicability and effectiveness of the proposed approach.

First, we give an example to show the effectiveness of the state feedback control problem.

Example 16. Consider a discrete-time singular hybrid system (1) with parameters as

$$E = \begin{bmatrix} 1.4 & 0.5 & 0 \\ 0 & 1.2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A(1) = \begin{bmatrix} 1.5 & 0.4 & 0.6 \\ 0.5 & 0.6 & 0 \\ 0.4 & 0.5 & 0.7 \end{bmatrix},$$

$$A(2) = \begin{bmatrix} 0.9 & 0.5 & 0.7 \\ -0.9 & 0.5 & -0.7 \\ 0.4 & 0.5 & -0.2 \end{bmatrix}, \quad B(1) = \begin{bmatrix} 0.4 & 0 \\ 0.3 & 0.6 \\ 0.3 & 0.8 \end{bmatrix},$$

$$B(2) = \begin{bmatrix} 0.5 & 0 \\ 0.3 & 0.6 \\ 0.7 & 0.1 \end{bmatrix}, \quad R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1.5 \end{bmatrix}, \quad \delta = 2, \varepsilon = 1.$$

The transition probability matrix that relates the two operation modes is given as $\Pi = \begin{bmatrix} 0.5 & 0.5 \\ 0.4 & 0.6 \end{bmatrix}$.

Using Theorem 11 and LMI control toolbox in Matlab, the solutions of LMI (19) are given as follows (for simplicity, here, we only present some items needed for the calculation of controller matrices):

$$G(1) = \begin{bmatrix} 9.3022 & -0.8059 & -1.3804 \\ 0.9460 & 7.9089 & 1.0583 \\ 0.8709 & -1.4456 & 8.4564 \end{bmatrix},$$

$$G(2) = \begin{bmatrix} 8.9603 & -0.5555 & -1.6586 \\ 0.6026 & 9.3002 & 0.9357 \\ 3.8562 & -0.6974 & 7.1362 \end{bmatrix},$$

$$H(1) = \begin{bmatrix} -14.5631 & 1.4491 & -4.9284 \\ -3.5413 & -0.1108 & 1.8549 \end{bmatrix},$$

$$H(2) = \begin{bmatrix} -6.2756 & -4.6859 & 9.7824 \\ 19.5553 & 6.4333 & -1.0010 \end{bmatrix}.$$

Then, we can obtain the following state feedback controller matrices:

$$K(1) = H(1)G^{-1}(1) = \begin{bmatrix} -1.4780 & -0.1154 & -0.8096 \\ -0.3930 & -0.0251 & 0.1583 \end{bmatrix},$$

$$K(2) = H(2)G^{-1}(2) = \begin{bmatrix} -1.1682 & -0.4864 & 1.1631 \\ 2.0307 & 0.8297 & 0.2229 \end{bmatrix}.$$

From Theorem 8, we know that the given values of parameters δ and ε have an important effect on the feasible solution of the coupled LMIs (15). The coupled LMIs (15) may have no feasible solutions if the values of parameters δ and ε are not selected properly.

Next, we are in a position to give another example to show the effectiveness of the static output feedback control problem.

Example 17. Consider a discrete-time singular hybrid system (1) and (2) with parameters as

$$E = \begin{bmatrix} 4.1 & 1.5 & 1.2 & 0 \\ 0 & 0 & 0 & 0 \\ 3.5 & 7.2 & 5.2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A(1) = \begin{bmatrix} 5.5 & 1.2 & 2.7 & 2.3 \\ 3.7 & 5.1 & -3.9 & 3.6 \\ -1.4 & -2.5 & 6.5 & 4.7 \\ -5.3 & -1.5 & 2.2 & 4.6 \end{bmatrix},$$

$$A(2) = \begin{bmatrix} 6.2 & -2.9 & -3.1 & 2.8 \\ 2.4 & 5.9 & -2.3 & 1.9 \\ 1.9 & -3.7 & 4.3 & 1.5 \\ 1.6 & 3.1 & 2.2 & 5.5 \end{bmatrix},$$

$$A(3) = \begin{bmatrix} 3.5 & -2.8 & -1.7 & 0.9 \\ 1.7 & 6.5 & 3.2 & 2.4 \\ 2.3 & 1.6 & 4.5 & 4.7 \\ 1.1 & 2.4 & 3.7 & 3.9 \end{bmatrix},$$

$$B(1) = \begin{bmatrix} 0.8 & 2.1 & 2.4 \\ -0.6 & 2.5 & 2.8 \\ 1.9 & -1.5 & 1.4 \\ 1.6 & 3.8 & 1.2 \end{bmatrix}, \quad B(2) = \begin{bmatrix} 2.5 & 3.7 & 2.9 \\ 3.6 & 2.1 & 1.8 \\ -3.4 & 4.2 & -2.7 \\ 2.7 & 2.5 & -2.4 \end{bmatrix},$$

$$B(3) = \begin{bmatrix} 3.4 & 1.0 & 1.6 \\ 2.7 & 1.9 & -3.2 \\ 4.3 & 1.1 & 2.7 \\ 1.9 & 3.4 & 3.6 \end{bmatrix}, \quad R = \begin{bmatrix} 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 8 \\ 0 & 0 & 0 & 2 \\ 0 & 5.5 & 0 & 6.4 \end{bmatrix},$$

$$C(1) = \begin{bmatrix} 0.2 & -0.5 & 0.6 & 0.9 \\ -7.0 & -2.5 & 9.0 & 8.5 \end{bmatrix},$$

$$C(2) = \begin{bmatrix} 1.25 & 7.75 & -1.75 & -7.5 \\ 0.625 & -2.125 & 1.125 & 1.25 \end{bmatrix},$$

$$C(3) = \begin{bmatrix} 0.41 & -0.075 & -0.26 & -0.025 \\ 0.05 & -0.375 & 0.7 & -0.125 \end{bmatrix},$$

$\delta = 1.2, \varepsilon = 2.$

The transition probability matrix that relates the three operation modes is given as $\Pi = \begin{bmatrix} 0.3 & 0.5 & 0.2 \\ 0.4 & 0.3 & 0.3 \\ 0.2 & 0.4 & 0.4 \end{bmatrix}$.

Then, we can select $M(1)$ and $M(2)$ such that $C(1)M(1) = \begin{bmatrix} I_2 & 0 \end{bmatrix}$, $C(2)M(2) = \begin{bmatrix} I_2 & 0 \end{bmatrix}$ and $C(3)M(3) = \begin{bmatrix} I_2 & 0 \end{bmatrix}$ hold.

$$M(1) = \begin{bmatrix} 1.5 & 0.2 & 1 & 0.5 \\ -0.5 & 0.5 & 2.5 & 2.0 \\ 1.5 & 0.5 & 1.0 & 0 \\ -0.5 & -0.1 & 0.5 & 1 \end{bmatrix},$$

$$M(2) = \begin{bmatrix} 1.2 & 0.2 & 0 & 0.5 \\ 1.5 & 0.5 & 0.5 & 1.0 \\ 0.5 & 1.5 & 0.5 & 0.5 \\ 1.5 & 0.2 & 0.4 & 1.0 \end{bmatrix}, \quad M(3) = \begin{bmatrix} 2.5 & 1.0 & 0.5 & 1.0 \\ 0 & 0.1 & 1.0 & 1.5 \\ 0 & 1.5 & 0.5 & 1.0 \\ 1.0 & 0.5 & 0 & 1.5 \end{bmatrix}.$$

The solutions of LMI (23) are given as follows (for simplicity, here, we only present some items needed for the calculation of controller matrices):

$$G(1) = \begin{bmatrix} 0.2472 & -0.3658 & 0 & 0 \\ -0.1299 & 1.8729 & 0 & 0 \\ -0.0930 & 0.0092 & 0.1099 & -0.1246 \\ 0.1645 & -0.1055 & -0.0810 & 0.2007 \end{bmatrix},$$

$$G(2) = \begin{bmatrix} 0.6449 & -0.2414 & 0 & 0 \\ -0.6257 & 0.9673 & 0 & 0 \\ -0.2112 & -0.0614 & 0.4149 & -0.1511 \\ 0.0405 & 0.0427 & -0.1715 & 0.1229 \end{bmatrix},$$

$$G(3) = \begin{bmatrix} 0.6491 & 0.2004 & 0 & 0 \\ -0.1580 & 0.3436 & 0 & 0 \\ -0.0930 & -0.0404 & 0.2617 & -0.0594 \\ 0.0510 & -0.0146 & -0.0828 & 0.0417 \end{bmatrix},$$

$$J(1) = \begin{bmatrix} 0.4656 & 0.1706 \\ 0.6665 & -0.0628 \\ -1.1147 & -0.2876 \end{bmatrix},$$

$$J(2) = \begin{bmatrix} -1.6299 & 0.2746 \\ -0.7014 & -0.0035 \\ -0.1804 & 1.3254 \end{bmatrix},$$

$$J(3) = \begin{bmatrix} -1.1093 & -0.9770 \\ -0.1792 & -0.4813 \\ -0.1385 & -0.1847 \end{bmatrix}.$$

The static output feedback controller gains are

$$F(1) = J(1)G_{11}^{-1}(1) = \begin{bmatrix} 2.1523 & 0.5115 \\ 2.9849 & 0.5495 \\ -5.1150 & -1.1526 \end{bmatrix},$$

$$F(2) = J(2)G_{11}^{-1}(2) = \begin{bmatrix} -2.9714 & -0.4577 \\ -1.4397 & -0.3629 \\ 1.3850 & 1.7159 \end{bmatrix}$$

$$F(3) = J(3)G_{11}^{-1}(3) = \begin{bmatrix} -2.1026 & -1.6171 \\ -0.5403 & -1.0856 \\ -0.3014 & -0.3617 \end{bmatrix}.$$

6. Conclusions

The problems of stability, state feedback and static output feedback control for discrete-time singular hybrid systems have been studied. Some new sufficient conditions for a discrete-time singular hybrid system to be stochastically admissible have been proposed in terms of strict LMIs. An explicit construction of a desired state feedback control and static output feedback control law has also been given. Numerical examples have been used to illustrate the main results.

Acknowledgements

The authors would like to thank the reviewers for their very helpful comments and suggestions which have improved the presentation of the paper. The work of Yuanqing Xia was supported by the National Natural Science Foundation of China under Grant 60504020 and Excellent young scholars Research Fund of Beijing Institute of Technology 2006y0103, respectively.

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