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Stabilization of discrete-time switched singular time-delay systems under asynchronous switching ☆

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Abstract

This paper is concerned with the problem of state feedback stabilization of a class of discrete-time switched singular systems with time-varying state delay under asynchronous switching. The asynchronous switching considered here means that the switching instants of the candidate controllers lag behind those of the subsystems. The concept of mismatched control rate is introduced. By using the multiple Lyapunov function approach and the average dwell time technique, a sufficient condition for the existence of a class of stabilizing switching laws is first derived to guarantee the closed-loop system to be regular, causal and exponentially stable in the presence of asynchronous switching. The stabilizing switching laws are characterized by a upper bound on the mismatched control rate and a lower bound on the average dwell time. Then, the corresponding solvability condition for a set of mode-dependent state feedback controllers is established by using the linear matrix inequality (LMI) technique. Finally, two numerical examples are provided to illustrate the effectiveness of the proposed method.

1. Introduction

Switched systems have drawn considerable attention since the 1990s, due to their great flexibility in modeling and control of practical systems, for example, event-driven systems,

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logic-based systems, parameter- or structure-varying systems, and highly complex nonlinear systems, etc.; for details, see [1–3] and the references therein. A switched system consists of a collection of continuous- or discrete-time subsystems and a switching rule specifying the switching among them. According to the features of the switching rule, switched systems can be classified into two classes: systems under uncontrolled switching and systems under controlled switching. In the first class the attention is focused on stability analysis and synthesis of stabilizing controllers with given switching signals, including arbitrary switching and stochastic switching governed by Markov chains. The other, which is of interest in this paper, is on synthesizing a stabilizing switching signal or even corresponding controllers for a given collection of subsystems. It has been proved that the multiple Lyapunov function approach [4] and average dwell time technique [5] are two powerful and effective tools to deal with switched systems under controlled switching. Besides switching properties, many engineering systems always involve time-delay phenomenon due to various reasons such as inherent phenomena like mass transport flow and recycling and/or by-products of computational delays. Therefore, the study of switched systems with time delays has become a hot topic in control community during the last decade, see, e.g. [6–12] and the references therein.

As an important class of switched systems, switched singular (SS) systems have found many practical application in industry, for example, electrical networks [13], DC motor [13], networked control systems [14], etc., and even in economic systems [15]. Singular systems, also known as descriptor, implicit or differential-algebraic systems, are much superior to systems represented by state-space models due to their capacity to describe the algebraic constraints between physical variables [16]. Compared with switched state-space models, the study of SS systems is more arduous, since not only stability, but also regularity and impulse elimination (for continuous-time SS systems) and causality (for discrete-time SS systems) should be considered simultaneously. The last decade has witnessed a rapidly growing interest in SS systems, and many important results have been reported in [17–23], and references therein. Specifically, the control problems for discrete-time SS systems with or without time delays under arbitrary switching is investigated in [18–20], and for discrete-time SS time-delay systems under stochastic switching is addressed in [22].

It should be pointed out that, in all the afore-mentioned results on control of SS systems, it is implicitly assumed that the controllers are switched synchronously with the switching of the subsystems. In actual operation of switched systems, however, this assumption may be unfeasible. The reason is mainly twofold. Firstly, the temporary failure of component or the transmission delay will inevitably impede detecting the change of the subsystem's switching signal instantly, but after a time period, which results in the switching signals available to the controller be a delayed version of the subsystem's switching signals [24,25]. A typical example can be found in networked switched control systems, where the switched plant and the switched controllers are separated by a communication channel [26]. Due to transmission delay, there inevitably exists asynchronous switching phenomena in the closed-loop system. Secondly, in some situation, it may be necessary to design a robust switching signal that can adapt the uncertain environment. One good example is the switched control of nonlinear chemical systems whose mode of operation changes according to a given time-depending switching rule [27]. So, from reliability as well as performance point of view, it is quite necessary to design a switched control system that could tolerate asynchronous switching between the controllers and subsystems while still retaining certain properties. In the past few years, some interesting results on stabilization of switched state-space systems under asynchronous switching have been presented in the literature; see [24,25,28–33] for non-delay cases and [34,35] for time-delay cases. However, to the authors' knowledge, the problem of stabilization of discrete-time SS time-delay systems under asynchronous switching has not yet been investigated. Moreover, the procedures given in [34,35] cannot be applied to the discrete-time case. This motivates the present study.

In this paper, we investigate the state feedback stabilization problem for a class of discrete-time SS time-delay systems under asynchronous switching. The concept of mismatched control rate is introduced. By using the multiple Lyapunov function approach and the average dwell time technique, a sufficient condition for the existence of a stabilizing switching law is derived at a first attempt to guarantee the closed-loop system to be regular, causal and exponentially stabile in the presence of asynchronous switching. The stabilizing switching law is characterized by a upper bound on the mismatched control rate and a lower bound on the average dwell time. Then, the corresponding solvability condition for a set of mode-dependent state feedback controllers is established by using the linear matrix inequality (LMI) technique. Finally, two numerical examples are provided to illustrate the effectiveness of the proposed method.

Notation: For real symmetric matrices P, $P>0$ ($P\geq 0$) means that matrix P is positive definite (semi-positive definite). $\lambda_{\max}(P)$ ($\lambda_{\min}(P)$) denotes the largest (smallest) eigenvalue of the positive definite matrix P. $\mathbf{R}^n$ is the n-dimensional real Euclidean space. $\mathbf{R}^{m\times n}$ is the set of all real $m\times n$ matrices. $\mathbf{Z}^+$ represents the sets of all non-negative integers. The superscript 'T' represents matrix transposition, and '$*$' in a matrix is used to represent the term which is induced by symmetry. $\|\cdot\|$ refers to the Euclidean vector norm. diag$\{\cdots\}$ stands for a block-diagonal matrix. Sym$\{A\}$ is the shorthand notation for $A+A^T$.

2. Description of problem and preliminaries

2.1. Problem formulation

Consider a class of discrete-time SS time-delay systems of the form

$$\begin{cases} Ex_{k+1} = A_{\sigma(k)}x_k + A_{d\sigma(k)}x_{k-d(k)} + B_{\sigma(k)}u_k \\ x_k = \phi_k, \quad k = -\overline{d}, \ldots, -1, 0 \end{cases} \tag{1}$$

where $x_k \in \mathbf{R}^n$ is the system state, $u_k \in \mathbf{R}^p$ is the control input, and ϕ_k, $k=-\overline{d},\ldots,-1,0$ is the initial condition sequence. $d(k)$ is a time-varying delay and satisfies $\underline{d}\leq d(k)\leq \overline{d}$, where $\underline{d}$ and $\overline{d}$ are constant positive scalars representing the minimum and maximum delays, respectively. $\sigma(k): \mathbf{Z}^+ \to \mathcal{I} = \{1,2,\ldots,N\}$ (N is the number of subsystems or modes) is the switching signal, which is a piecewise continuous (from the right) function of time. The matrix E is singular and rank $E=r<n$. A_i, A_{di} and B_i, $\forall i \in \mathcal{I}$, are constant matrices. As often assumed in the switched system literature, we exclude Zeno behavior for the switching signal here. Corresponding to the switching signal $\sigma(k)$, we denote the switching sequence by $\mathcal{S}=\{(i_0,k_0),(i_1,k_1),\ldots,(i_p,k_p),\ldots | i_p \in \mathcal{I}, p \in \mathbf{Z}^+\}$ with $k_0=0$, which means that the i_jth subsystem is activated when $k_j \leq k < k_{j+1}$, or equivalently, $\sigma(k)=i_j$ when $k_j\leq k<k_{j+1}$. The quadruple-matrix $(E, A_{i_j}, A_{di_j}, B_{i_j})$, $i_j \in \mathcal{I}$, represents the i_jth subsystem or i_jth mode of system (1).

In this paper, the mode-dependent state-feedback control is considered and formed as $u_k = K_{\sigma(k)}x_k$, where K_i, $\forall\sigma(k)=i\in\mathcal{I}$, is the controller gain to be determined. However, due to the asynchronous switching, the switches of $K_{\sigma(k)}$ do not coincide with those of system modes. Without loss of generality, the asynchronous switching considered here means that the switches of $K_{\sigma(k)}$ lag behind the switches of system modes. Then, the real control input will become

$$u_k = K_{\sigma(k-\Delta_{k_l})}x_k, \quad \forall k\in[k_l,k_{l+1}) \tag{2}$$

where $\Delta_{k_l} < k_{l+1}-k_l$ represents the time lag from the switch of k_{i_l} to that of i_lth system mode, and is said to be mismatched control time during the interval $[k_l,k_{l+1})$.

Remark 1. The mismatched control time $\Delta_{k_l} < k_{l+1}-k_l$, $l\in\mathbf{Z}^+$, guarantees that there exists a period during the interval $[k_l,k_{l+1})$ such that the system mode and the desirably mode-dependent controller operate synchronously.

Definition 1. Given switching sequence $\mathcal{S}$ and any $k_l+\Delta_{k_l} < k < k_{l+1}$, let $\Delta_{[0,k)} = \Delta_{k_0} + \cdots + \Delta_{k_l}$ denote the total mismatched control time during the interval $[0,k)$, and call the ratio $\Delta_{[0,k)}/k$ the mismatched control rate of the switched controllers in the system.

Applying the control (2) to system (1), the resulting closed-loop system is given by $\forall\sigma(k-\Delta_{k_l}) = i_{l-1}\in\mathcal{I}$, $\sigma(k)=i_l\in\mathcal{I}$, $i_{l-1}\neq i_l$

$$\begin{cases} Ex_{k+1} = \overline{A}_{i_l i_{l-1}}x_k + A_{di_l}x_{k-d(k)}, & \forall k\in[k_l,k_l+\Delta_{k_l}) \\ Ex_{k+1} = \overline{A}_{i_l i_l}x_k + A_{di_l}x_{k-d(k)}, & \forall k\in[k_l+\Delta_{k_l},k_{l+1}) \end{cases} \tag{3}$$

where $\overline{A}_{i_l i_{l-1}} = A_{i_l} + B_{i_l}K_{i_{l-1}}$ and $\overline{A}_{i_l i_l} = A_{i_l} + B_{i_l}K_{i_l}$. To describe the main objective of this paper more precisely, we also introduce the following definitions.

Definition 2 (*Zhang and Shi [31]*). For a switching signal $\sigma(k)$ and any $k_v > k_s > k_0$, let $N_{\sigma(k)}(k_s,k_v)$ be the switching numbers of $\sigma(k)$ over the interval $[k_s,k_v)$. If for any given $N_0\geq 1$ and $\tau_a > 0$, we have $N_{\sigma(k)}(k_s,k_v)\leq N_0 + (k_v-k_s)/\tau_a$, then τ_a and N_0 are called the average dwell time and the chatter bound, respectively.

Definition 3 (*Dai [16] and Ma et al. [18]*). System (1) with $u_k=0$ is said to be:

(i) regular if $\det(zE-A_i)$ is not identically zero, $\forall\sigma(k)=i\in\mathcal{L}$,
(ii) causal if $\deg(\det(zE-A_i)) = \operatorname{rank} E$, $\forall\sigma(k)=i\in\mathcal{L}$.

Remark 2. The regularity and causality of system (1) with $u_k=0$ ensure that the solution to this system exists and is unique and causal, $\forall\sigma(k)=i\in\mathcal{L}$ [18].

Definition 4 (*Zhang and Yu [8]*). System (1) with $u_k=0$ is said to be exponentially stable with decay rate λ $(0<\lambda<1)$ under switching signal $\sigma(k)$ if, for any initial conditions ϕ_k, $k=k_0-\overline{d},\ldots,k_0$, its solution x_k satisfies $\|x_k\|\leq c\lambda^{(k-k_0)}\|\phi\|_{\overline{d}}$ for all $k\geq k_0$, where $\|\phi\|_{\overline{d}} = \sup_{k_0-\overline{d}\leq l\leq k_0}\|\phi_l\|$, k_0 is the initial time step, and $c>0$ is the decay coefficient.

Therefore, the purpose of this paper is to identify a class of stabilizing switching laws and design a set of mode-dependent state-feedback controllers such that the closed-loop system (3) is regular, causal and exponentially stable under asynchronous switching.

2.2. Preliminary results

In this subsection, several lemmas are offered, which will be useful to present our main results.

Define

$$\xi_k = [x_k^T \;\; x_{k-d(k)}^T \;\; x_{k-\underline{d}}{}^T \;\; x_{k-\overline{d}}^T]^T \tag{4}$$

$$y_k = x_{k+1} - x_k \tag{5}$$

Lemma 1. *For any appropriately dimensioned matrices $R>0$ and N, two positive time-varying integer $d(k_1)$ and $d(k_2)$ satisfying $d(k_1)+1 \le d(k_2) \le \overline{d}$, and a scalar $\lambda>0$, the following equality holds*:

$$-\sum_{l=k-d(k_2)}^{k-d(k_1)-1} y_l^T E^T \lambda^{k-l} R E y_l = c\xi_k^T N R^{-1} N^T \xi_k + 2\xi_k^T N \sum_{l=k-d(k_2)}^{k-d(k_1)-1} E y_l$$
$$-\sum_{l=k-d(k_2)}^{k-d(k_1)-1} (\xi_k^T N + \lambda^{k-l} y_l^T E^T R)(\lambda^{k-l} R)^{-1} (N^T \xi_k + \lambda^{k-l} R E y_l) \tag{6}$$

where $c = (\lambda^{-d(k_2)} - \lambda^{-d(k_1)})/(1-\lambda)$.

Proof. See the Appendix.

Consider the closed-loop system (3) running in a mismatched control period $[k_i, k_i + \Delta_{k_i})$, i.e.

$$\begin{cases} E x_{k+1} = \overline{A}_{ij} x_k + A_{di} x_{k-d(k)} \\ x_k = \varphi_k, \quad k = k_i - \overline{d}, \ldots, k_i - 1, k_i \end{cases} \tag{7}$$

where $\overline{A}_{ij} = A_i + B_i K_j$ and φ_k, $k = k_i - \overline{d}, \ldots, k_i - 1, k_i$ is the initial condition sequence. For the sake of simplicity, in Eq. (7), we use the subscripts i and j to substitute for i_i and i_{i-1}, respectively. Choose the following Lyapunov-like function for system (7):

$$V_{i1}(x_k) = \sum_{s=1}^{4} V_{i1s}(x_k) \tag{8}$$

where

$$V_{i11}(x_k) = x_k^T E^T P_i E x_k$$

$$V_{i12}(x_k) = \sum_{l=k-d(k)}^{k-1} x_l^T (1+\beta)^{k-1-l} Q_{i1} x_l + \sum_{\theta=-\overline{d}+1}^{-\underline{d}} \sum_{l=k+\theta}^{k-1} \tilde{x}_l^T (1+\beta)^{k-1-l} Q_{i1} x_l$$

$$V_{i13}(x_k) = \sum_{l=k-\underline{d}^{k-1}} x_l^T (1+\beta)^{k-1-l} Q_{i2} x_l + \sum_{l=k-\overline{d}}^{k-1} x_l^T (1+\beta)^{k-1-l} Q_{i3} x_l$$

$$V_{i14}(x_k) = \sum_{\theta=-\overline{d}}^{-1}\sum_{l=k+\theta}^{k-1} y_l^T E^T (1+\beta)^{k-1-l} Z_{i1} E y_l + \sum_{\theta=-\overline{d}}^{-\underline{d}-1}\sum_{l=k+\theta}^{k-1} y_l^T E^T (1+\beta)^{k-1-l} Z_{i2} E y_l$$

with $P_i > 0$, $Q_{il} > 0$, $l = 1,2,3$, and $Z_{iv} > 0$, $v = 1,2$, are matrices to be determined, and $\beta > 0$ is a given constant. The following lemma provides a sufficient condition on the regularity, causality for system (7) and an increase estimation of $V_{i1}(x_k)$ in Eq. (8) along the state trajectory of system (7).

Lemma 2. *Given constants $\beta > 0$ and $0 < \underline{d} < \overline{d}$. If there exist matrices $P_i > 0$, $Q_{il} > 0$, $l = 1,2,3$, $Z_{iv} > 0$, $v = 1,2$, $S_i = S_i^T$, $M_{i1} = [M_{i11}^T\ M_{i12}^T\ M_{i13}^T\ M_{i14}^T]^T$, $N_{i1} = [N_{i11}^T\ N_{i12}^T\ N_{i13}^T\ N_{i14}^T]^T$ and $T_{i1} = [T_{i11}^T\ T_{i12}^T\ T_{i13}^T\ T_{i14}^T]^T$ such that the following inequality holds:*

$$\begin{bmatrix} \Phi_{ij} & \Psi_{i1} \\ * & \Gamma_{i1} \end{bmatrix} < 0 \tag{9}$$

where

$$\begin{aligned}\Phi_{ij} = {} & \mathrm{diag}\{-(1+\beta)E^T P_i E + (1+\tilde{d})Q_{i1} + Q_{i2} + Q_{i3}, -(1+\beta)^{\underline{d}} Q_{i1}, -(1+\beta)^{\underline{d}} Q_{i2}, \\ & -(1+\beta)^{\overline{d}} Q_{i3}\} + \mathrm{Sym}\{M_{i1}\Pi_1 + N_{i1}\Pi_2 + T_{i1}\Pi_3\} \\ & + \mathcal{A}_{ij}^T (P_i - R^T S_i R)\mathcal{A}_{ij} + (\mathcal{A}_{ij} - \Pi_4)^T U_i (\mathcal{A}_{ij} - \Pi_4)\end{aligned}$$

$$\Psi_{i1} = [\rho_1 M_{i1}\ \ \rho_2 N_{i1}\ \ \rho_2 T_{i1}], \quad \Gamma_{i1} = \mathrm{diag}\{-\rho_1 Z_{i1}, -\rho_2(Z_{i1} + Z_{i2}), -\rho_2 Z_{i2}\}$$

$$\mathcal{A}_{ij} = [\overline{A}_{ij}\ \ A_{di}\ \ 0\ \ 0], \quad \Pi_1 = [E\ \ -E\ \ 0\ \ 0]$$

$$\Pi_2 = [0\ \ E\ \ 0\ \ -E], \quad \Pi_3 = [0\ \ -E\ \ E\ \ 0]$$

$$\Pi_4 = [E\ \ 0\ \ 0\ \ 0], \quad \tilde{d} = \overline{d} - \underline{d}, \quad U_i = \overline{d} Z_{i1} + \tilde{d} Z_{i2}$$

$$\rho_1 = (1 - (1+\beta)^{-\overline{d}})/\beta, \quad \rho_2 = ((1+\beta)^{-\underline{d}} - (1+\beta)^{-\overline{d}})/\beta$$

and $R \in \mathbf{R}^{n\times n}$ is any constant matrix satisfying $RE = 0$ with $\mathrm{rank}(R) = n - r$, *then system* (7) *is regular and causal*, *and along any state trajectory of system* (7), *the function $V_{i1}(x_k)$ in Eq.* (8) *ensures the following increase estimation*:

$$V_{i1}(x_{k+1}) < (1+\beta)V_{i1}(x_k) \tag{10}$$

Moreover, *if Eq.* (9) *holds*, *there exists a constant $\alpha_{i1} > 0$ such that*

$$\alpha_{i1}\|x_k\|^2 \le V_{i1}(x_k) \tag{11}$$

Proof. See the Appendix.

Consider the closed-loop system (3) running in a matched control period $[k_i + \Delta_{k_i}, k_{i+1})$, i.e.

$$\begin{cases} Ex_{k+1} = \overline{A}_{ii} x_k + A_{di} x_{k-d(k)} \\ x_k = \psi_k, \quad k = k_i' - \overline{d}, \ldots, k_i' - 1, k_i' \end{cases} \tag{12}$$

where $\overline{A}_{ii} = A_i + B_i K_i$, $k_i' = k_i + \Delta_{k_i}$, and ψ_k, $k = k_i' - \overline{d}, \ldots, k_i'-1, k_i'$, is the initial condition sequence. For conciseness, in Eq. (12), we also use the subscript i instead of i_i. Choose the following Lyapunov function for system (12):

$$V_{i2}(x_k) = \sum_{s=1}^{4} V_{i2s}(x_k) \tag{13}$$

where

$$V_{i21}(x_k) = x_k^T E^T P_i E x_k$$

$$V_{i22}(x_k) = \sum_{l=k-d(k)}^{k-1} x_l^T (1-\alpha)^{k-1-l} Q_{i1} x_l + \sum_{\theta=-\overline{d}+1}^{-\underline{d}} \sum_{l=k+\theta}^{k-1} x_l^T (1-\alpha)^{k-1-l} Q_{i1} x_l$$

$$V_{i23}(x_k) = \sum_{l=k-\underline{d}^{k-1}} x_l^T (1-\alpha)^{k-1-l} Q_{i2} x_l + \sum_{l=k-\overline{d}}^{k-1} x_l^T (1-\alpha)^{k-1-l} Q_{i3} x_l$$

$$V_{i24}(x_k) = \sum_{\theta=-\overline{d}}^{-1} \sum_{l=k+\theta}^{k-1} y_l^T E^T (1-\alpha)^{k-1-l} Z_{i1} E y_l + \sum_{\theta=-\overline{d}}^{-\underline{d}-1} \sum_{l=k+\theta}^{k-1} y_l^T E^T (1-\alpha)^{k-1-l} Z_{i2} E y_l$$

with $P_i > 0$, $Q_{il} > 0$, $l=1,2,3$ and $Z_{iv} > 0$, $v=1,2$ are matrices to be determined, and $0 < \alpha < 1$ is a given constant. The following lemma provides a sufficient condition on the regularity, causality for system (12) and a decay estimation of $V_{i2}(x_k)$ in Eq. (13) along the state trajectory of system (12).

Lemma 3. *Given constants $0 < \alpha < 1$ and $0 < \underline{d} < \overline{d}$, if there exist matrices $P_i > 0$, $Q_{il} > 0$, $l=1,2,3$, $Z_{iv} > 0$, $v=1,2$, $S_i = S_i^T$, $M_{i2} = [M_{i21}^T \ M_{i22}^T \ M_{i23}^T \ M_{i24}^T]^T$, $N_{i2} = [N_{i21}^T \ N_{i22}^T \ N_{i23}^T \ N_{i24}^T]^T$ and $T_{i2} = [T_{i21}^T \ T_{i22}^T \ T_{i23}^T \ T_{i24}^T]^T$ such that the following inequality holds*:

$$\begin{bmatrix} \Phi_{ii} & \Psi_{i2} \\ * & \Gamma_{i2} \end{bmatrix} < 0 \tag{14}$$

where

$$\begin{aligned} \Phi_{ii} = {} & \mathrm{diag}\{-(1-\alpha)E^T P_i E + (1+\tilde{d})Q_{i1} + Q_{i2} + Q_{i3}, -(1-\alpha)^{\overline{d}} Q_{i1}, -(1-\alpha)^{\underline{d}} Q_{i2}, \\ & -(1-\alpha)^{\overline{d}} Q_{i3}\} + \mathrm{Sym}\{M_{i2}\Pi_1 + N_{i2}\Pi_2 + T_{i2}\Pi_3\} + \mathcal{A}_{ii}^T (P_i - R^T S_i R)\mathcal{A}_{ii} \\ & + (\mathcal{A}_{ii} - \Pi_4)^T U_i (\mathcal{A}_{ii} - \Pi_4) \end{aligned}$$

$$\Psi_{i2} = [\varrho_1 M_{i2} \ \ \varrho_2 N_{i2} \ \ \varrho_2 T_{i2}], \quad \Gamma_{i2} = \mathrm{diag}\{-\varrho_1 Z_{i1}, -\varrho_2 (Z_{i1} + Z_{i2}), -\varrho_2 Z_{i2}\}$$

$$\mathcal{A}_{ii} = [\overline{A}_{ii} \ \ A_{di} \ \ 0 \ \ 0], \quad \varrho_1 = ((1-\alpha)^{-\overline{d}} - 1)/\alpha, \quad \varrho_2 = ((1-\alpha)^{-\overline{d}} - (1-\alpha)^{-\underline{d}})/\alpha$$

Π_1, Π_2, Π_3, Π_4, $\tilde{d}$ and U_i are defined in Eq. (9), *and $R \in \mathbf{R}^{n \times n}$ is any constant matrix satisfying $RE = 0$ with $\mathrm{rank}(R) = n - r$, then system* (12) *is regular and causal, and along any state trajectory of system* (12), *the function $V_{i2}(x_k)$ in Eq.* (13) *ensures the following decay estimation*:

$$V_{i2}(x_{k+1}) < (1-\alpha) V_{i2}(x_k) \tag{15}$$

Moreover, if Eq. (14) *holds, there exists a constant* $\alpha_{i2}>0$ *such that*

$$\alpha_{i2}\|x_k\|^2 \le V_{i2}(x_k) \tag{16}$$

Proof. Similar to the proof of Lemma 2, Lemma 3 can be easily obtained. So the proof is omitted. □

Remark 3. Lemmas 2 and 3 provide sufficient conditions of the growth estimation and decay estimation of Lyapunov functions (8) and (13), respectively. It is worth pointing out that the LMIs in Eqs. (9) and (14) are obtained without decomposition of the original systems, which is in contrast with the results in [9,23], where the conditions are formulated in terms of the coefficient matrices of the state-augmented systems or state-transformed systems; thus possibly computational problem can be avoided.

Lemma 4 (*Ma et al. [22]*). *Given matrices* X, Y *and* Z *with appropriate dimensions, and* Y *is symmetric. Then there exists a scalar* $\rho>0$, *such that* $\rho I+Y>0$ *and* $-\mathrm{Sym}\{X^TZ\}-Z^TYZ\le X^T(\rho I+Y)^{-1}X+\rho Z^TZ$.

3. Main results

In this section, we will first identify and design the stabilizing switching law for the closed-loop system (3), and further give the corresponding controllers design.

Based on Lemmas 2 and 3, the following theorem presents a sufficient condition for the existence of a stabilizing switching law for the closed-loop system (3) under asynchronous switching.

Theorem 1. *Let* $0<\alpha<1$, $\beta>0$, $0<\underline{d}<\overline{d}$, *and* $\mu\ge 1$ *be given constants. If there exist matrices* $P_i>0$, $Q_{il}>0$, $l=1,2,3$, $Z_{iv}>0$, $v=1,2$, $S_i=S_i^T$, $M_{vi}=[M_{vi1}^T\ M_{vi2}^T\ M_{vi3}^T\ M_{vi4}^T]^T$, $N_{vi}=[N_{vi1}^T\ N_{vi2}^T\ N_{vi3}^T\ N_{vi4}^T]^T$ *and* $T_{vi}=[T_{vi1}^T\ T_{vi2}^T\ T_{vi3}^T\ T_{vi4}^T]^T$, $\forall i\in\mathcal{I}$, *such that* $\forall(i,j)\in\mathcal{I}\times\mathcal{I}$, $i\neq j$, *inequalities* (9) *and* (14) *hold, then the closed-loop system* (3) *is regular, causal and exponentially stable with decay rate* λ ($\sqrt{1-\alpha}<\lambda<1$) *under the switching signal* $\sigma(k)$ *with the following conditions*:

$$Cond_1:\quad \frac{\Delta_{[0,k)}}{k}\le\frac{\ln\lambda^*-\ln(1-\alpha)}{\ln(1+\beta)-\ln(1-\alpha)} \tag{17}$$

$$Cond_2:\quad N_0=\frac{\ln\upsilon}{2\ln\mu+\ln\mu_1},\quad \tau_a\ge\tau_a^*=\frac{2\ln\mu+\ln\mu_1}{2\ln\lambda-\ln\lambda^*} \tag{18}$$

where $0<1-\alpha<\lambda^*<\lambda^2<1$, $\upsilon>0$, $\mu_1=((1+\beta)/(1-\alpha))^{\overline{d}-1}$, *and* μ *satisfies* $\forall(i,j)\in\mathcal{I}\times\mathcal{I}$, $i\neq j$

$$P_i\le\mu P_j,\quad Q_{il}\le\mu Q_{jl},\quad Z_{iv}\le\mu Z_{jv},\ l=1,2,3,\ v=1,2 \tag{19}$$

Proof. For any given $k\ge 1$, we let $k_1<\cdots<k_t$, $t\ge 1$, denote the switching instants of the switching signal $\sigma(k)$ over the interval $[0,k)$. As mentioned earlier, $\sigma(k)=i_i$ when $k\in[k_i,k_{i+1})$. Choose the following piecewise Lyapunov-like function for system (3)

$$V(x_k)=V_{\sigma(k)}(x_k)=\begin{cases}V_{i_i1}(x_k), & \forall k\in[k_i,k_i+\Delta_{k_i}),\ i=0,1,\ldots,t\\ V_{i_i2}(x_k), & \forall k\in[k_i+\Delta_{k_i},k_{i+1}),\ i=0,1,\ldots,t\end{cases} \tag{20}$$

where $V_{i_i1}(x_k)$ and $V_{i_i2}(x_k)$ follow the same definitions as those in Eqs. (8) and (13), respectively, except the subscript i is instead of i_i. Then, it holds from Eqs. (20), (10) and

(15) that

$$V_{\sigma(k)}(x_k) \leq \begin{cases} (1+\beta)^{k-k_i} V_{i_i 1}(x_{k_i}), & \forall k \in [k_i, k_i+\Delta_{k_i}),\ i=0,1,\ldots,t \\ (1-\alpha)^{k-k_i-\Delta_{k_i}} V_{i_i 2}(x_{k_i+\Delta_{k_i}}), & \forall k \in [k_i+\Delta_{k_i}, k_{i+1}),\ i=0,1,\ldots,t \end{cases} \tag{21}$$

Moreover, from Eqs. (19), (8) and (13), it can be deduced that $\forall (i_{i-1}, i_i) \in \mathcal{I} \times \mathcal{I},\ i_{i-1} \neq i_i$

$$V_{i_i 1}(x_{k_i}) \leq \mu_1 \mu V_{i_{i-1} 2}(x_{k_i}), \quad V_{i_i 2}(x_{k_i+\Delta_{k_i}}) \leq \mu V_{i_i 1}(x_{k_i+\Delta_{k_i}}) \tag{22}$$

where $\mu_1 = ((1+\beta)/(1-\alpha))^{\overline{d}-1}$.

When $k \in [k_l+\Delta_{k_l}, k_{l+1})$, it follows from Eqs. (21) and (22) and Definition 1 that

$$\begin{aligned} V_{\sigma(k)}(x_k) &\leq (1-\alpha)^{k-(k_l+\Delta_{k_l})} V_{\sigma(k_l+\Delta_{k_l})}(x_{k_l+\Delta_{k_l}}) \leq (1-\alpha)^{k-(k_l+\Delta_{k_l})} \mu V_{\sigma(k_l+\Delta_{k_l}-1)}(x_{k_l+\Delta_{k_l}}) \\ &\leq \mu(1-\alpha)^{k-(k_l+\Delta_{k_l})}(1+\beta)^{\Delta_{k_l}} V_{\sigma(k_l)}(x_{k_l}) \\ &\leq \mu(1-\alpha)^{k-(k_l+\Delta_{k_l})}(1+\beta)^{\Delta_{k_l}} \mu\mu_1 V_{\sigma(k_l-1)}(x_{k_l}) \\ &\leq \mu^2\mu_1(1-\alpha)^{k-(k_l+\Delta_{k_l})}(1+\beta)^{\Delta_{k_l}}(1-\alpha)^{k_l-(k_{l-1}+\Delta_{k_{l-1}})} V_{\sigma(k_{l-1}+\Delta_{k_{l-1}})}(x_{k_{l-1}+\Delta_{k_{l-1}}}) \\ &\leq \mu^2\mu_1(1-\alpha)^{k-(k_{l-1}+\Delta_{k_l}+\Delta_{k_{l-1}})}(1+\beta)^{\Delta_{k_l}} \mu V_{\sigma(k_{l-1}+\Delta_{k_{l-1}}-1)}(x_{k_{l-1}+\Delta_{k_{l-1}}}) \\ &\leq \mu^3\mu_1(1-\alpha)^{k-(k_{l-1}+\Delta_{k_l}+\Delta_{k_{l-1}})}(1+\beta)^{\Delta_{k_l}}(1+\beta)^{\Delta_{k_{l-1}}} V_{\sigma(k_{l-1})}(x_{k_{l-1}}) \\ &\leq \ldots \\ &\leq \mu^{2N_{\sigma(k)}(0,k)+1} \mu_1^{N_{\sigma(k)}(0,k)} (1-\alpha)^{k-\Delta_{[0,k)}} (1+\beta)^{\Delta_{[0,k)}} V_{\sigma(0)}(x_0) \end{aligned} \tag{23}$$

From Eq. (17), it holds that

$$\Delta_{[0,k)}(\ln(1+\beta)-\ln(1-\alpha)) < k(\ln \lambda^* - \ln(1-\alpha))$$

that is

$$(1-\alpha)^{k-\Delta_{[0,k)}}(1+\beta)^{\Delta_{[0,k)}} < (\lambda^*)^k \tag{24}$$

In view of Eq. (18), it is obtained that

$$\begin{aligned} \mu^{2N_{\sigma(k)}(0,k)+1} \mu_1^{N_{\sigma(k)}(0,k)} &= e^{(2N_{\sigma(k)}(0,k)+1)\ln \mu + N_{\sigma(k)}(0,k)\ln \mu_1} \\ &= e^{(2N_0+1)\ln\mu + N_0 \ln\mu_1} e^{k/\tau_a(2\ln\mu+\ln\mu_1)} \\ &\leq \mu\upsilon e^{k(2\ln\lambda - \ln\lambda^*)} = \mu\upsilon\left(\frac{\lambda^2}{\lambda^*}\right)^k \end{aligned} \tag{25}$$

Substituting Eqs. (24) and (25) into Eq. (23) yields

$$V_{\sigma(k)}(x_k) \leq \mu\upsilon\lambda^{2k} V_{\sigma(0)}(x_0) \tag{26}$$

Moreover, according to Eqs. (16) and (20), we have

$$a\|x_k\|^2 \leq V_{\sigma(k)}(x_k), \quad V_{\sigma(0)}(x_0) \leq b\|\phi\|_{\overline{d}}^2 \tag{27}$$

where $a = \min_{\forall i \in \mathcal{I}}\{\alpha_{i2}\}$ and $b = \max_{\forall i \in \mathcal{I}}\{\lambda_{\max}(E^T P_i E) + ((\overline{d}-1)(1+\beta)^{\overline{d}-1} + \tilde{d}(\overline{d}-1)(1+\beta)^{\overline{d}-2})\lambda_{\max}(Q_{i1}) + (\underline{d}-1)(1+\beta)^{\underline{d}-1}\lambda_{\max}(Q_{i2}) + (\overline{d}-1)(1+\beta)^{\overline{d}-1}\lambda_{\max}(Q_{i3}) + \overline{d}^2(1+\beta)^{\overline{d}-1}\lambda_{\max}(Z_{i1}) + \tilde{d}^2(1+\beta)^{\overline{d}-1}\lambda_{\max}(Z_{i2})\}$. Combining Eqs. (26) and (27) gives rise to

$$\|x_k\| \leq \sqrt{\frac{b}{a}\mu\upsilon}\lambda^k \|\phi\|_{\overline{d}} \tag{28}$$

which leads to $\|x_k\| \le c\lambda^k \|\phi\|_{\overline{d}}$, where $c=\sqrt{(b/a)\mu\upsilon}$. Then, by Definition 4 with $k_0=0$, we can conclude that system (3) is regular, causal and exponentially stable with decay rate $0<\lambda<1$. This completes the proof. $\square$

Remark 4. $Cond_1$ is used to restrict the upper bound of the ratio between the total mismatched control period and the overall time; $Cond_2$ is used to restrict the lower bound of average dwell time of the subsystems. Theorem 1 shows that within a relatively small mismatched control rate and a relatively large average dwell time, the closed-loop system (3) can be exponentially stable despite the fact that it may be unstable during some certain period.

Remark 5. From Theorem 1, it is easy to see that a larger β and a smaller α will be favorable to the solvability of inequalities (17) and (18). On the contrary, a smaller β and a larger α are more desirable to relax $Cond_1$. Considering these, we can first select a larger β and a smaller α to guarantee the feasible solution of inequalities (17) and (18), and then decrease β and increase α to obtain the suitable β and α.

Now, we will present the sufficient condition on the existence of a set of mode-dependent state-feedback controllers for the switched singular time-delay system (1) under asynchronous switching via Theorem 1.

Theorem 2. *Let $\alpha>0$, $\beta>0$, $0<\underline{d}<\overline{d}$, $\mu\ge 1$, $\rho_i>0$, ε_{if}, $f=1,2,\ldots,5$, $\forall i\in\mathcal{I}$, be given constants. If there exist matrices $X_i>0$, $Q_{il}>0$, $l=1,2,3$, $Z_{iv}>0$, $v=1,2$, $S_i=S_i^T$, $M_{iv}=[M_{iv1}^T\ M_{iv2}^T\ M_{iv3}^T\ M_{iv4}^T]^T$, $N_{iv}=[N_{iv1}^T\ N_{iv2}^T\ N_{iv3}^T\ N_{iv4}^T]^T$ and $T_{iv}=[T_{iv1}^T\ T_{iv2}^T\ T_{iv3}^T\ T_{iv4}^T]^T$, $\forall i\in\mathcal{I}$, such that $\forall (i,j)\in\mathcal{I}\times\mathcal{I}$, $i\ne j$*

$$\begin{bmatrix} \overline{\Phi}_{ij} & \Psi_{i1} & \overline{d}\Pi_{ij} & \tilde{d}\Pi_{ij} & \Xi_i & \rho_i\Upsilon_{ij}R^T & \Upsilon_{ij} \\ * & \Gamma_{i1} & 0 & 0 & 0 & 0 & 0 \\ * & * & \overline{d}(\varepsilon_{i4}^2 Z_{i1}-2\varepsilon_{i4}I) & 0 & 0 & 0 & 0 \\ * & * & * & \tilde{d}(\varepsilon_{i5}^2 Z_{i2}-2\varepsilon_{i5}I) & 0 & 0 & 0 \\ * & * & * & * & -\rho_i I - S_i & 0 & 0 \\ * & * & * & * & * & -\rho_i I & 0 \\ * & * & * & * & * & * & -X_i \end{bmatrix} < 0 \tag{29}$$

$$\begin{bmatrix} \overline{\Phi}_{ii} & \Psi_{i2} & \overline{d}\Pi_{ii} & \tilde{d}\Pi_{ii} & \Xi_i & \rho_i\Upsilon_{ii}R^T & \Upsilon_{ii} \\ * & \Gamma_{i2} & 0 & 0 & 0 & 0 & 0 \\ * & * & \overline{d}(\varepsilon_{i4}^2 Z_{i1}-2\varepsilon_{i4}I) & 0 & 0 & 0 & 0 \\ * & * & * & \tilde{d}(\varepsilon_{i5}^2 Z_{i2}-2\varepsilon_{i5}I) & 0 & 0 & 0 \\ * & * & * & * & -\rho_i I - S_i & 0 & 0 \\ * & * & * & * & * & -\rho_i I & 0 \\ * & * & * & * & * & * & -X_i \end{bmatrix} < 0 \tag{30}$$

$$X_j \le \mu X_i, \quad Q_{il} \le \mu Q_{jl}, \quad Z_{iv} \le \mu Z_{jv}, \ l=1,2,3, \ v=1,2 \tag{31}$$

where

$$\overline{\Phi}_{ij} = \text{diag}\{(1+\beta)(\varepsilon_{i3}\,\text{Sym}\{E\}+\varepsilon_{i3}^2 X_i)+(1+\tilde{d})Q_{i1}+Q_{i2}+Q_{i3}, -(1+\beta)^{\underline{d}}Q_{i1},$$

$$-(1+\beta)^{\underline{d}}Q_{i2},-(1+\beta)^{\overline{d}}Q_{i3}\}+\mathrm{Sym}\{M_{i1}\Pi_1+N_{i1}\Pi_2+T_{i1}\Pi_3+\Xi_i R\mathcal{A}_{ij}\}$$

$$\Pi_{ij}=[\overline{A}_{ij}-E\ \ A_{di}\ \ 0\ \ 0]^T,\quad \Upsilon_{ij}=[\overline{A}_{ij}\ \ A_{di}\ \ 0\ \ 0]^T$$

$$\overline{\Phi}_{ii}=\mathrm{diag}\{(1-\alpha)(\varepsilon_{i3}\mathrm{Sym}\{E\}+\varepsilon_{i3}^2X_i)+(1+\tilde{d})Q_{i1}+Q_{i2}+Q_{i3},-(1-\alpha)^{\overline{d}}Q_{i1},$$
$$-(1-\alpha)^{\underline{d}}Q_{i2},-(1-\alpha)^{\overline{d}}Q_{i3}\}+\mathrm{Sym}\{M_{i2}\Pi_1+N_{i2}\Pi_2+T_{i2}\Pi_3+\Xi_i R\mathcal{A}_{ii}\}$$

$$\Pi_{ii}=[\overline{A}_{ii}-E\ \ A_{di}\ \ 0\ \ 0]^T,\quad \Upsilon_{ii}=[\overline{A}_{ii}\ \ A_{di}\ \ 0\ \ 0]^T$$

$$\Xi_i=[\varepsilon_{i1}I\ \ \varepsilon_{i2}I\ \ 0\ \ 0]^T$$

Ψ_{i1}, Γ_{i1}, Π_1, Π_2, Π_3, $\mathcal{A}_{ij}$, $\mathcal{A}_{ii}$, Ψ_{i2} and Γ_{i2} are defined in Eqs. (9) *and* (14), *respectively*, *and $R\in\mathbf{R}^{n\times n}$ is any constant matrix satisfying $RE=0$ with* $\mathrm{rank}(R)=n-r$, *then*, *under the state-feedback controllers $u_k=K_{\sigma(k)}x_k$, the closed-loop system* (3) *is regular*, *causal and exponentially stable with decay rate λ ($\sqrt{1-\alpha}<\lambda<1$) for any switching signal $\sigma(k)$ satisfying Eqs.* (17) *and* (18). *Moreover*, *if Eqs.* (29)–(31) *have feasible solutions*, *the admissible controllers gains are given by K_i, $\forall i\in\mathcal{I}$.*

Proof. By Schur complement, Eq. (29) is equivalent to

$$\begin{bmatrix}\overline{\Phi}_{ij}+\Upsilon_{ij}X_i^{-1}\Upsilon_{ij}^T+\rho_i\Upsilon_{ij}R^TR\Upsilon_{ij}^T+\Xi_i(\rho_iI+S_i)^{-1}\Xi_i^T & \Psi_{i1} & \overline{d}\Pi_{ij} & \tilde{d}\Pi_{ij}\\ * & \Gamma_{i1} & 0 & 0\\ * & * & \overline{d}(\varepsilon_{i4}^2Z_{i1}-2\varepsilon_{i4}I) & 0\\ * & * & * & \tilde{d}(\varepsilon_{i5}^2Z_{i2}-2\varepsilon_{i5}I)\end{bmatrix}<0 \tag{32}$$

According to Lemma 4, for any scalars ε_{i1} and ε_{i2}, there exists a scalar $\rho_i>0$ such that

$$\rho_iI+S_i>0 \tag{33}$$

$$-\begin{bmatrix}\overline{A}_{ij}^T\\ A_{di}^T\end{bmatrix}R^TS_iR[\overline{A}_{ij}\ \ A_{di}]\le\mathrm{Sym}\left\{\begin{bmatrix}\overline{A}_{ij}^T\\ A_{di}^T\end{bmatrix}R^T[\varepsilon_{i1}I\ \ \varepsilon_{i2}I]\right\}$$
$$+\begin{bmatrix}\varepsilon_{i1}I\\ \varepsilon_{i2}I\end{bmatrix}(\rho_iI+S_i)^{-1}[\varepsilon_{i1}I\ \ \varepsilon_{i2}I]+\rho_i\begin{bmatrix}\overline{A}_{ij}^T\\ A_{di}^T\end{bmatrix}R^TR[\overline{A}_{ij}\ \ A_{di}] \tag{34}$$

On the other hand, since $P_i>0$ and $Z_{iv}>0$, $v=1,2$, the following inequalities hold:

$$-E^TP_iE\le\mathrm{Sym}\{\varepsilon_{i3}E\}+\varepsilon_{i3}^2P_i^{-1} \tag{35}$$

$$-Z_{i1}^{-1}\le-2\varepsilon_{i4}I+\varepsilon_{i4}^2Z_{i1} \tag{36}$$

$$-Z_{i2}^{-1}\le-2\varepsilon_{i5}I+\varepsilon_{i5}^2Z_{i2} \tag{37}$$

for arbitrary scalars ε_{i3}, ε_{i4} and ε_{i5}. Setting $X_i = P_i^{-1}$, and using Eqs. (33)–(37), it follows from Eq. (32) that

$$\begin{bmatrix} \hat{\Phi}_{ij} & \Psi_{i1} & \overline{d}\Pi_{ij} & \tilde{d}\Pi_{ij} \\ * & \Gamma_{i1} & 0 & 0 \\ * & * & -\overline{d}Z_{i1}^{-1} & 0 \\ * & * & * & -\tilde{d}Z_{i2}^{-1} \end{bmatrix} < 0 \tag{38}$$

where

$$\hat{\Phi}_{ij} = \mathrm{diag}\{-(1+\beta)E^T P_i E + (1+\tilde{d})Q_{i1} + Q_{i2} + Q_{i3}, -(1+\beta)^{\underline{d}} Q_{i1}, -(1+\beta)^{\underline{d}} Q_{i2},$$
$$-(1+\beta)^{\overline{d}} Q_{i3}\} + \mathrm{Sym}\{M_{i1}\Pi_1 + N_{i1}\Pi_2 + T_{i1}\Pi_3\} + \mathcal{A}_{ij}^T (P_i - R^T S_i R)\mathcal{A}_{ij}$$

By Schur complement, if Eq. (9) holds, then Eq. (38) holds. Similarly, we can also obtain that, if Eq. (14) holds, then Eq. (30) is satisfied. In addition, $P_i \le \mu P_j$ ensures $X_j \le \mu X_i$, which means that if Eq. (19) holds, then Eq. (31) holds. Therefore, by Theorem 1, the desired result follows immediately. The proof is thus over. □

If there is no asynchronous switching in system (3), i.e. $\Delta_{k_i} = 0$, $\forall i \in \mathcal{I}$, then Theorem 2 reduces to the following corollary.

Corollary 1. *Let* $\alpha > 0$, $0 < \underline{d} < \overline{d}$, $\mu \ge 1$, $\rho_i > 0$, $\varepsilon_{if}, f = 1, 2, \ldots, 5$, $\forall i \in \mathcal{I}$, *be given constants. If there exist matrices* $X_i > 0$, $Q_{il} > 0$, $l = 1,2,3$, $Z_{iv} > 0$, $v = 1,2$, $S_i = S_i^T$, $M_{i2} = [M_{i21}^T \; M_{i22}^T \; M_{i23}^T \; M_{i24}^T]^T$, $N_{i2} = [N_{i21}^T \; N_{i22}^T \; N_{i23}^T \; N_{i24}^T]^T$ *and* $T_{i2} = [T_{i21}^T \; T_{i22}^T \; T_{i23}^T \; T_{i24}^T]^T$, $\forall i \in \mathcal{I}$, *such that* $\forall (i,j) \in \mathcal{I} \times \mathcal{I}$, $i \ne j$ *inequalities* (30) *and* (31), *and*

$$\tau_a \ge \tau_a^* = -\frac{\ln \mu}{\ln(1-\alpha)} \tag{39}$$

hold, then, under the state-feedback controllers $u_k = K_{\sigma(k)} x_k$, *the closed-loop system* (3) *is regular, causal and exponentially stable with decay rate* λ ($\sqrt{1-\alpha} < \lambda < 1$) *for any switching signal* $\sigma(k)$ *with the average dwell time* τ_a *satisfying Eq.* (39). *Moreover, if Eqs.* (30) *and* (31) *have feasible solutions, the admissible controller gains are given by* K_i, $\forall i \in \mathcal{I}$.

Remark 6. Scalars $\varepsilon_{if}, f = 1,2,\ldots,5$, $\forall i \in \mathcal{I}$, in Theorem 2 are tuning parameters which need to be given first. In fact, Eqs. (29) and (30), for fixed $\alpha > 0$, $\beta > 0$, $0 < \underline{d} < \overline{d}$ and $\mu \ge 1$, are bilinear matrix inequalities (BMIs) regarding to these tuning parameters. If one can accept more computational burden, the optimal values of these parameters can be obtained by applying some global optimization algorithms [36] to solve the BMIs.

4. Numerical examples

In this section, two examples are given to show the effectiveness of the proposed method.

Example 1 (*Stability*). Consider the switched non-singular time-delay system (1) with $E = I$, $N = 2$ (i.e., two subsystems) and the following parameters [8]:

$$A_1 = \begin{bmatrix} 0 & 0.3 \\ -0.2 & 0.1 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0 & 0.1 \\ 0 & 0.2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0.3 \\ -0.2 & -0.1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0 & 0.1 \\ 0 & 0 \end{bmatrix}$$

The time delay is $0 \le d(k) \le \overline{d}$. Suppose the average dwell time $\tau_a = 2$ and $\mu = 1.1$. For different decay rate λ, the calculated values of the delay upper bound $\overline{d}$ by the methods of Theorem 1 in [8] and Corollary 1 ($Q_{i2} = Q_{i3} = Z_{i2} = 0$, $S_i = 0$, $R=0$, $M_{i2} = [M_{i21}^T \ M_{i22}^T]^T$, $N_{i2} = T_{i2} = 0, \forall i \in \mathcal{I}$) in this paper are listed in Table 1. It can be seen from Table 1 that the values of $\overline{d}$ decreases as λ decreases. On the other hand, it is clearly shown that the condition in Corollary 1 provides better results than that in [8], since on state transformation is involved in Corollary 1.

Example 2 (*Stabilisability*). Consider the switched singular time-delay system (1) with $N=2$ and the following parameters:

$$\text{Subsystem 1}: E = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 3 & -3 \\ 2 & 1 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.1 & -0.1 \\ 0 & -0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -2 \\ -3 \end{bmatrix}$$

$$\text{Subsystem 2}: E = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0 & -0.2 \\ 0.1 & -0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.2 \\ 1.5 \end{bmatrix}$$

The time-varying delay is $d(k) = 2 + \sin(2.5\pi k)$. By simulation, it can be checked that both the above subsystems with $u_k = 0$ are unstable, and the state responses of the two subsystems are shown in Fig. 1(a) and (b) with given initial condition $\phi_k = [-1, 1.3]^T$, $k = -3, -2, \ldots, 0$. In view of this, our aim is to design a set of mode-dependent state feedback controllers and find the admissible switching law such that the resulting closed-loop system is regular, causal and exponentially stable.

Table 1
Allowable delay upper bound $\overline{d}$ for different decay rate λ.

λ	0.9310	0.8535	0.7878	0.7315	0.5690
Theorem 1 in [8]	14	8	4	3	1
Corollary 1	16	8	5	4	2

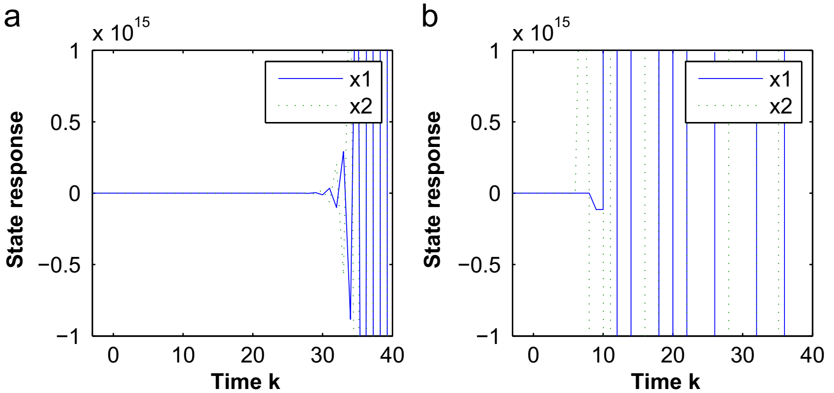

Fig. 1. State responses of the subsystems with $u_k = 0$. (a) Subsystem 1. (b) Subsystem 2.

First, based on Corollary 1, we shall study the synchronous control problem of the above system. Setting $\mu=1.2$ and $\alpha=0.2$ (thus $\tau_a^*=-\ln\mu/(\ln(1-\alpha))=0.8171$), letting $\kappa_{211}=-26$, $\kappa_{212}=-0.02$, $\kappa_{213}=-15$, $\kappa_{214}=\kappa_{215}=2$, $\kappa_{221}=-39$, $\kappa_{222}=0.08$, $\kappa_{223}=-12$, $\kappa_{224}=\kappa_{225}=2$ and $R=\mathrm{diag}\{0,1\}$, and solving the LMIs in Corollary 1, one obtain the controllers gains $K_1=[1.4109\ -1.4247]$ and $K_2=[0.8955\ 8.7859]$. By simulation, it can be checked that closed-loop subsystem $(E,A_1+B_1K_2,A_{d1})$ is unstable, which shows that if there exists asynchronous switching between the above subsystems, we cannot determine whether the corresponding closed-loop system with $\tau_a \ge \tau_a^*$ is stable by Corollary 1.

Now, we consider the asynchronous switching control problem based on Theorem 2. Setting $\mu=1.2$, $\alpha=0.2$ and $\beta=0.1$, letting $\varepsilon_{11}=-277$, $\varepsilon_{12}=-1.02$, $\varepsilon_{13}=-209$, $\varepsilon_{14}=\varepsilon_{15}=2$, $\varepsilon_{21}=-156$, $\varepsilon_{22}=-1.18$, $\varepsilon_{23}=-501$, $\varepsilon_{24}=\varepsilon_{25}=2$ and $R=\mathrm{diag}\{0,1\}$, and solving the LMIs in Theorem 2, one obtain the controllers gains as follows:

$$K_1=[1.1079\ \ -0.6623], \quad K_2=[1.2900\ \ -0.4538]$$

Let $\lambda^*=0.82$, $\lambda=0.955$ and $v=0$. Then, according to Eqs. (35) and (36), the switching law require

$$\tau_a^* \ge 12.4103, \quad \frac{\Delta_{[0,k)}}{k} \le 0.0775$$

Denote by $(E,A_1+B_1K_1,A_{d1})$, $(E,A_2+B_2K_2,A_{d2})$, $(E,A_1+B_1K_2,A_{d1})$ and $(E,A_2+B_2K_1,A_{d2})$ the closed-loop subsystems S_1, S_2, S_1' and S_2', respectively. Supposing that the four subsystems are activated in the following sequence:

$$S_1'\underbrace{S_1\cdots S_1}_{12}S_2'\underbrace{S_2\cdots S_2}_{12}S_1'\underbrace{S_1\cdots S_1}_{12}S_2'\underbrace{S_2\cdots S_2}_{12}\cdots$$

that is, the average dwell time $\tau_a=13$ and $\Delta_{k_l}=1$, $l=0,1,\ldots$, the state response of the resulting closed-loop system is shown in Fig. 2 under the given initial condition $\phi_k=[-1,\ 1.3]^T$, $k=-3,-2,\ldots,0$. It can be seen from Fig. 2 that the designed controllers is effective despite asynchronous switching.

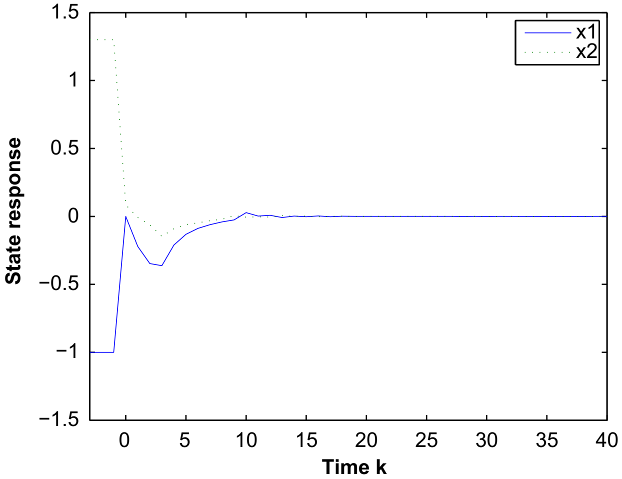

Fig. 2. State response of the closed-loop system under the switching sequence $S_1S_2S_1S_2\cdots$ ($\tau_a=1$).

5. Conclusions

In this paper, the problem of state feedback stabilization of a class of discrete-time switched singular systems with time-varying state delay under asynchronous switching has been studied. A class of stabilizing switching laws has been derived for the closed-loop system to be regular, causal and exponentially stable in the presence of asynchronous switching. The corresponding solvability condition for a set of mode-dependent state feedback controllers has also been presented by using the LMI technique. The effectiveness of main results has been illustrated by numerical examples. As a future research, the following work can be considered:

(1) In the present work, it is assumed that the switchings of system modes are exact, but the switchings of candidate controllers are not timely due to the delay in the detection of the switching signals of the system modes. However, for the stochastic switched systems such as Markov jump systems, some recent research in [37] has shown that the likelihood to obtain the complete knowledge on the change of system modes is quite unpractical. So, generalizing the obtained results on deterministic switched systems to stochastic switched systems deserves further research via other methods.
(2) Considering the transfer delays of sensor to controller and controller to actuator that arise in many practical control systems [25], more attention should be paid to the study of SS systems where delays are present in both the feedback state and the switching signals of the switched controllers.
(3) In a recent work [38], a delay partitioning technique has been introduced to derive less conservative results for stability and stabilization problems of discrete-time singular time-delay systems. Generalizing this technique to here may be addressed.

Appendix

Proof of Lemma 1. For given ξ_k and y_k defined in Eqs. (4) and (5), the following equation holds:

$$-\sum_{l=k-d(k_2)}^{k-d(k_1)-1}(\xi_k^T N+\lambda^{k-l}y_l^T E^T R)(\lambda^{k-l}R)^{-1}(N^T\xi_k+\lambda^{k-l}REy_l)$$
$$=-\xi_k^T NR^{-1}N^T\xi_k\sum_{l=k-d(k_2)}^{k-d(k_1)-1}\lambda^{l-k}-2\xi_k^T N\sum_{l=k-d(k_2)}^{k-d(k_1)-1}Ey_l-\sum_{l=k-d(k_2)}^{k-d(k_1)-1}y_l^T E^T\lambda^{k-l}REy_l$$

Rearranging the above equation yields Eq. (6). This completes the proof. □

Proof of Lemma 2. First, let us show that system (7) is regular and causal. Since E is singular and $\mathrm{rank}(E)=r$, there exist two non-singular matrices $G,H\in\mathbf{R}^{n\times n}$ such that

$$GEH=\begin{bmatrix} I_r & 0\\ 0 & 0\end{bmatrix} \tag{40}$$

According to Eq. (40), let

$$G\overline{A}_{ij}H = \begin{bmatrix} \overline{A}_{ij1} & \overline{A}_{ij2} \\ \overline{A}_{ij3} & \overline{A}_{ij4} \end{bmatrix}, \quad G^{-T}P_iG^{-1} = \begin{bmatrix} P_{i1} & P_{i2} \\ P_{i2}^T & P_{i3} \end{bmatrix}$$

$$H^{-T}M_{i11}G^{-1} = \begin{bmatrix} M_{1i11} & M_{1i12} \\ M_{i13} & M_{i14} \end{bmatrix}, \quad RG^{-1} = [R_1 \ \ R_2] \tag{41}$$

From $RE=0$ and $\text{rank}(R)=n-r$, it can be deduced that $R_1=0$ and $\text{rank}(R_2)=n-r$, $R_2 \in \mathbf{R}^{n\times(n-r)}$, that is

$$RG^{-1} = [0 \ \ R_2] \tag{42}$$

Noting $Q_{il}>0$, $l=1,2,3$, $Z_{il}>0$, $Z_{i2}>0$, $\underline{d}>0$ and $\tilde{d}>0$, it is easily obtained from Eq. (9) that

$$-\overline{A}_{ij}^TR^TS_iR\overline{A}_{ij}-(1+\beta)E^TP_iE+\text{Sym}\{M_{i11}E\}<0 \tag{43}$$

Substituting Eqs. (40)–(42) into Eq. (43) yields

$$\begin{bmatrix} \star & \star \\ \star & -\overline{A}_{ij4}^TR_2^TS_iR_2\overline{A}_{ij4} \end{bmatrix}<0$$

where $\star$ represents matrices that are not relevant in the following discussion, which implies

$$-\overline{A}_{ij4}^TR_2^TS_iR_2\overline{A}_{ij4}<0 \tag{44}$$

Now, we assume that the matrix $\overline{A}_{ij4}$ is singular, then, there exists a vector $\eta_i \in \mathbf{R}^{n-r}$ and $\eta_i \neq 0$ such that $\overline{A}_{ij4}\eta_i=0$. Pre- and post-multiplying Eq. (44) by η_i^T and η_i result in $\eta_i^T\overline{A}_{ij4}^TR_2^TS_iR_2\overline{A}_{ij4}\eta_i=0$ which contradicts $-\overline{A}_{ij4}^TR_2^TS_iR_2\overline{A}_{ij4}<0$. Thus, $\overline{A}_{ij4}$ is non-singular, which implies that system (7) is regular and causal by Definition 3.

Next, we will show the increase estimation of $V_{i1}(x_k)$ in Eq. (8) along the state trajectory of system (7). To this end, define

$$\Delta V_{i1}(x_k) = \sum_{s=1}^{4}[V_{i1s}(x_{k+1})-(1+\beta)V_{i1s}(x_k)] \tag{45}$$

Then, it follows from Eq. (8) that

$$\begin{aligned} \Delta V_{i11}(x_k) &= x_{k+1}^TE^TP_iEx_{k+1}-(1+\beta)x_k^TE^TP_iEx_k \\ &= (\overline{A}_{ij}x_k+A_{di}x_{k-d(k)})^TP_i(\overline{A}_{ij}x_k+A_{di}x_{k-d(k)})-(1+\beta)x_k^TE^TP_iEx_k \end{aligned} \tag{46}$$

$$\Delta V_{i12}(x_k) \le (1+\tilde{d})x_k^TQ_{i1}x_k-x_{k-d(k)}^T(1+\beta)^{d(k)}Q_{i1}x_{k-d(k)} \tag{47}$$

$$\Delta V_{i13}(x_k) = x_k^T(Q_{i2}+Q_{i3})x_k-x_{k-\underline{d}^T(1+\beta)^{\underline{d}}Q_{i2}x_{k-\underline{d}}-x^T_{k-\overline{d}}(1+\beta)^{\overline{d}}Q_{i3}x_{k-\overline{d}}} \tag{48}$$

$$\Delta V_{i14}(x_k) = y_k^TE^T(\overline{d}Z_{i1}+\tilde{d}Z_{i2})Ey_k-\sum_{l=k-\overline{d}}^{k-1}y_l^TE^T(1+\beta)^{k-l}Z_{i1}Ey_l-\sum_{l=k-\overline{d}}^{k-\underline{d}-1}y_l^TE^T(1+\beta)^{k-l}Z_{i2}Ey_l$$

$$=y_k^T E^T(\overline{d}Z_{i1}+\tilde{d}Z_{i2})Ey_k-\sum_{l=k-d(k)}^{k-1} y_l^T E^T(1+\beta)^{k-l}Z_{i1}Ey_l$$
$$-\sum_{l=k-\overline{d}}^{k-d(k)-1} y_l^T E^T(1+\beta)^{k-l}(Z_{i1}+Z_{i2})Ey_l-\sum_{l=k-d(k)}^{k-\underline{d}-1} y_l^T E^T(1+\beta)^{k-l}Z_{i2}Ey_l \tag{49}$$

Using Lemma 1 for the last three terms of $\Delta V_{i14}(x_k)$, respectively, and noting $\underline{d}\le d(k)\le\overline{d}$, we have

$$-\sum_{l=k-d(k)}^{k-1} y_l^T E^T(1+\beta)^{k-l}Z_{i1}Ey_l\le\rho_1\xi_k^T M_{i1}Z_{i1}^{-1}M_{i1}^T\xi_k+2\xi_k^T M_{i1}\sum_{l=k-d(k)}^{k-1}Ey_l$$
$$-\sum_{l=k-d(k)}^{k-1}[\xi_k^T M_{i1}+(1+\beta)^{k-l}y_l^T E^T Z_{i1}]((1+\beta)^{k-l}Z_{i1})^{-1}[M_{i1}^T\xi_k+(1+\beta)^{k-l}Z_{i1}Ey_l] \tag{50}$$

$$-\sum_{l=k-\overline{d}}^{k-d(k)-1} y_l^T E^T(1+\beta)^{k-l}(Z_{i1}+Z_{i2})Ey_l\le\rho_2\xi_k^T N_{i1}(Z_{i1}+Z_{i2})^{-1}N_{i1}^T\xi_k+2\xi_k^T N_{i1}\sum_{l=k-\overline{d}}^{k-d(k)-1}Ey_l$$
$$-\sum_{l=k-\overline{d}}^{k-d(k)-1}[\xi_k^T N_{i1}+(1+\beta)^{k-l}y_l^T E^T(Z_{i1}+Z_{i2})]((1+\beta)^{k-l}(Z_{i1}+Z_{i2}))^{-1}$$
$$\cdot[N_{i1}^T\xi_k+(1+\beta)^{k-l}(Z_{i1}+Z_{i2})Ey_l] \tag{51}$$

$$-\sum_{l=k-d(k)}^{k-\underline{d}-1} y_l^T E^T(1+\beta)^{k-l}Z_{i2}Ey_l\le\rho_2\xi_k^T T_{i1}Z_{i2}^{-1}T_{i1}^T\xi_k+2\xi_k^T T_{i1}\sum_{l=k-d(k)}^{k-\underline{d}-1Ey_l}$$
$$-\sum_{l=k-d(k)}^{k-\underline{d}-1}[\xi_k^T T_{i1}+(1+\beta)^{k-l}y_l^T E^T Z_{i2}]((1+\beta)^{k-l}Z_{i2})^{-1}[T_{i1}^T\xi_k+(1+\beta)^{k-l}Z_{i2}Ey_l] \tag{52}$$

Noting Eq. (5), one can obtain

$$\sum_{l=k-d(k)}^{k-1}Ey_l=[E\ \ -E\ \ 0\ \ 0]\xi_k \tag{53}$$

$$\sum_{l=k-\overline{d}}^{k-d(k)-1}Ey_l=[0\ \ E\ \ 0\ \ -E]\xi_k \tag{54}$$

$$\sum_{l=k-d(k)}^{k-\underline{d}-1}Ey_l=[0\ \ -E\ \ E\ \ 0]\xi_k \tag{55}$$

On the other hand, from $RE=0$, the following equation holds for any symmetric matrix S_i with appropriate dimension:

$$0=-x_{k+1}^T E^T R^T S_i R E x_{k+1}$$

$$= -(\overline{A}_{ij}x_k + A_{di}x_{k-d(k)})^T R^T S_i R(\overline{A}_{ij}x_k + A_{di}x_{k-d(k)}) \tag{56}$$

Then, substituting Eqs. (46)–(54) into Eq. (55) and using Eq. (56) yield that

$$\begin{aligned}
\Delta V_{i1}(x_k) \le\ & \xi_k^T \Lambda_{ij}\xi_k - \sum_{l=k-d(k)}^{k-1} [\xi_k^T M_{i1} \\
& + (1+\beta)^{k-l} y_l^T E^T Z_{i1}]((1+\beta)^{k-l}Z_{i1})^{-1}[M_{i1}^T\xi_k + (1+\beta)^{k-l}Z_{i1}Ey_l] \\
& - \sum_{l=k-\overline{d}}^{k-d(k)-1} [\xi_k^T N_{i1} + (1+\beta)^{k-l} y_l^T E^T (Z_{i1}+Z_{i2})][(1+\beta)^{k-l}(Z_{i1}+Z_{i2})]^{-1} \\
& \cdot [N_{i1}^T\xi_k + (1+\beta)^{k-l}(Z_{i1}+Z_{i2})Ey_l] \\
& - \sum_{l=k-d(k)}^{k-\underline{d}-1} [\xi_k^T T_{i1} + (1+\beta)^{k-l} y_l^T E^T Z_{i2}]((1+\beta)^{k-l}Z_{i2})^{-1}[T_{i1}^T\xi_k + (1+\beta)^{k-l}Z_{i2}Ey_l] \\
& \le \xi_k^T \Lambda_{ij}\xi_k
\end{aligned} \tag{57}$$

where $\Lambda_{ij} = \Phi_{ij} + \rho_1 M_{i1}Z_{i1}^{-1}M_{i1}^T + \rho_2 N_{i1}(Z_{i1}+Z_{i2})^{-1}N_{i1}^T + \rho_2 T_{i1}Z_{i2}^{-1}T_{i1}^T$, and the second inequality holds from the fact $1+\beta > 0$, $Z_{i1} > 0$ and $Z_{i2} > 0$. By Schur complement, Eq. (9) is equivalent to $\Lambda_{ij} < 0$. Thus, we can obtain $\Delta V_{i1}(x_k) < 0$, which means that Eq. (10) holds. Furthermore, let $\lambda_{\min}(-\Lambda_{ij}) = \lambda_{i1}$; then $\lambda_{i1} > 0$, and for any $x_k \neq 0$, it follows from Eqs. (45), (57) and (4) that

$$\begin{aligned}
V_{i1}(x_{k+1}) - (1+\beta)V_{i1}(x_k) &\le -\lambda_{i1}(\|x_k\|^2 + \|x_{k-d(k)}\|^2 + \|x_{k-\underline{d}}\|^2 + \|x_{k-\overline{d}}\|^2) \\
&\le -\lambda_{i1}\|x_k\|^2
\end{aligned}$$

which implies that

$$\lambda_{i1}\|x_k\|^2 \le -V_{i1}(x_{k+1}) + (1+\beta)V_{i1}(x_k) \le (1+\beta)V_{i1}(x_k)$$

Then, Eq. (11) holds for $\alpha_{i1} = \lambda_{i1}(1+\beta)^{-1}$. The proof is completed. □

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