

Buổi 3/04

Tiết 1: Chia BT giới hạn

Tiết 2: Hồi về haminh lực, trái/phải

Thực hành

Tiết 3: Day về quy tắc L'Hospital

Chia BT: 3 → 8 (NĐT)

$$3) \textcircled{1} I = \lim_{x \rightarrow 2} \frac{(x^2 - x - 2)^{20}}{(x^3 - 12x + 16)^{10}} \quad \left( \frac{0}{0} \right) \quad \begin{aligned} & \stackrel{x^2 - 2^2 - 2 - 2 = 0}{\text{màu}} \\ & \stackrel{2^3 - 12 \cdot 2 + 16 = 0}{\text{màu}} \end{aligned}$$

$$\begin{aligned} x^2 - x - 2 &= (x-2)(x+1) \\ (x^3 - 12x + 16) &= (x-2)(x^2 + 2x + 8) \end{aligned}$$

$$\begin{aligned} &= (x-2)(x-2)(x+4) \\ &= (x-2)^2(x+4). \end{aligned}$$

Do đó

$$\begin{aligned} I &= \lim_{x \rightarrow 2} \frac{(x-2)^{20}}{(x+4)^{10}} \\ &= \lim_{x \rightarrow 2} \frac{(x+1)^{20}}{(x+4)^{10}} = \frac{3^{20}}{6^{10}} = \frac{3^{10}}{2^{10}} \end{aligned}$$

$$\textcircled{2} I = \lim_{x \rightarrow 1} \frac{x + x^2 + \dots + x^n - n}{x-1}$$

$$I = \lim_{x \rightarrow 1} \frac{x-1 + (x^2-1) + (x^3-1) + \dots + (x^n-1)}{x-1}$$

Chú ý:  $x^k - 1 = (x-1)(x^{k-1} + x^{k-2} + \dots + 1)$

Dò dò

$$I = \lim_{\substack{x \rightarrow 1 \\ }} 1 + (x+1) + (x^2+x+1) + \dots + (x^{n-1}+x^{n-2}+\dots+1)$$

$$= 1 + 2 + 3 + \dots + n$$

$$= \frac{n(n+1)}{2}$$

③  $\lim_{x \rightarrow 1} \frac{x^{100} - 2x + 1}{x^{60} - 2x + 1} \left( \text{deg } \frac{0}{0} \right)$

$$\begin{aligned} TS &= x^{100} - 2x + 1 = x^{100} - 1 - (2x - 2) \\ &= (x-1) [x^{99} + x^{98} + \dots + 1] - 2(x-1), \end{aligned}$$

$$= (x-1) [x^{99} + x^{98} + \dots + 1 - 2]$$

$$\begin{aligned} MS &= x^{60} - 2x + 1 \\ &\equiv (x-1) [x^{59} + x^{58} + \dots + 1 - 2] \end{aligned}$$

I =  $\lim_{x \rightarrow 1} \frac{TS}{MS} = \lim_{x \rightarrow 1} \frac{x^{99} + x^{98} + \dots + 1 - 2}{x^{59} + x^{58} + \dots + 1 - 2}$

$$= \frac{100-2}{60-2} = \frac{98}{58}$$

④  $I = \lim_{x \rightarrow a} \frac{x^n - a^n - (x-a)n a^{n-1}}{(x-a)^2} \left( \frac{0}{0} \right)$

$$TS = x^n - a^n - (x-a) n a^{n-1}$$

~~Lý thuyết xác suất và thống kê~~

[c1] Tìm cách phân tích tesser thành

$$(x-a)^2 \left( \dots \right) \Rightarrow \text{lỗi dài.}$$

[c2] Dùng quy tắc L'Hospital  $\left( \frac{0}{0}, \frac{\infty}{\infty} \right)$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} \quad \left( \text{vì } \deg \frac{0}{0}, \frac{\infty}{\infty} \right)$$

$$= \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} \quad \left( \because \text{đạo hàm theo } x \right)$$

Áp dụng:  $I = \lim_{x \rightarrow a} \frac{x^n - a^n - xna^{n-1} + na^n}{(x-a)^2}$

$$L'H = \lim_{x \rightarrow a} \frac{nx^{n-1} - na^{n-1}}{2(x-a)} \quad \left( \frac{0}{0} \right)$$

$$L''H = \lim_{x \rightarrow a} \frac{n(n-1)x^{n-2}}{2} = \frac{n(n-1)a^{n-2}}{2}$$

4) ①  $I = \lim_{x \rightarrow \infty} \frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}} \quad (\text{dạng } \frac{\infty}{\infty})$

$$= \lim_{x \rightarrow \infty} \frac{\sqrt{1 + \sqrt{\frac{x+1}{x^2}}}}{\sqrt{1 + \frac{1}{x}}} = 1$$

$$\textcircled{2} \quad I = \lim_{x \rightarrow +\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{2x+1}} = \lim_{x \rightarrow +\infty} \frac{x^{1/2} + x^{1/3} + x^{1/4}}{\sqrt{2x+1}}$$

$$= \lim_{x \rightarrow +\infty} \frac{1 + x^{-1/6} + x^{-1/4}}{\sqrt{2 + \frac{1}{x}}} \quad (\text{chia chia & nhân}\ \text{do } x)$$

$$= \frac{1}{2}$$

6)  $\textcircled{1} \quad I = \lim_{x \rightarrow a} \frac{\sin x - \sin a}{x - a} = \lim_{x \rightarrow a} \frac{2 \cos \frac{x+a}{2} \sin \frac{x-a}{2}}{2 \cdot \frac{x-a}{2}}$

$$= \lim_{x \rightarrow a} \cos \frac{x+a}{2} \cdot \frac{\sin \frac{x-a}{2}}{\left(\frac{x-a}{2}\right)} = \cos a \cdot 1 = \cos a$$

$$\textcircled{2} \quad \lim_{x \rightarrow 0} \frac{\sqrt{1+\tan x} - \sqrt{1+\sin x}}{x^3} = \lim_{x \rightarrow 0} \frac{1 + \tan x - (1 + \sin x)}{x^3 (\sqrt{...} + \sqrt{...})}$$

$$= \frac{\sin x - \sin x}{\cos x} = \lim_{x \rightarrow 0} \frac{\sin x}{x} \cdot \frac{1 - \cos x}{x^2 (\sqrt{...} + \sqrt{...})}$$

$$= 1 \cdot \lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} \cdot \frac{1}{(\sqrt{1+\tan x} + \sqrt{1+\sin x})}$$

$$= 1 \cdot \frac{1}{2}$$

$$\textcircled{3} \quad \lim_{x \rightarrow 0} \frac{1 - \cos x + \cos x - \cos x \cos 2x + \cos x \cos 2x (1 - \cos 3x)}{1 - \cos x}$$

$$\lim_{x \rightarrow 0} \frac{1 + \cos x \cdot \frac{1 - \cos 2x}{1 - \cos x} + \cos x \cos 2x \cdot \frac{1 - \cos 3x}{1 - \cos x}}{1 - \cos x}$$

$$\text{Chứng: } \frac{1-\cos(kx)}{1-\cos x} = \frac{2 \sin^2 \frac{kx}{2}}{2 \sin^2 \frac{x}{2}}$$

$$\Rightarrow \lim_{x \rightarrow 0} \frac{1-\cos(kx)}{1-\cos x} = \lim_{x \rightarrow 0} \dots = k^2$$

$$\text{Do đó } I = 1 + 1 \cdot 2^2 + 1 \cdot 1 \cdot 3^2 = 14.$$

$$④ I = \lim_{x \rightarrow 0} \frac{\sqrt[3]{\cos x} - 1}{\sin^2 x} = \lim_{x \rightarrow 0} \frac{\sqrt[3]{\cos x} - 1}{\sin^2 x} + \frac{1 - \sqrt[3]{\cos x}}{\sin^2 x}$$

$$\times \text{ lhop} \lim_{x \rightarrow 0} \frac{\cos x - 1}{\sin^2 x \cdot (\sqrt[3]{\cos x} + 1)}$$

$$+ \frac{1 - \cos x}{\sin^2 x \left( (\sqrt[3]{\cos x})^2 + \sqrt[3]{\cos x} + 1 \right)}$$

$$= \frac{1}{(1 + \sqrt[3]{1})} \cdot \frac{-1}{2} + \frac{+1}{2} \cdot \frac{1}{3} = \frac{-1}{12}$$

F) Cả 2 căn đều x liệm hợp, để quá nên các em thử  
xem giải.

$$① I = \lim_{x \rightarrow +\infty} \left( \frac{3x^2 - x + 1}{2x^2 + x + 1} \right)^{\frac{x^3}{1-x}}$$

$$\text{Vì } \lim_{x \rightarrow +\infty} \frac{3x^2 - x + 1}{2x^2 + x + 1} = \frac{3}{2}$$

$\Rightarrow I$  li<sup>o</sup>c<sup>o</sup> dạng vô định

$$\text{Vì } \lim_{x \rightarrow +\infty} \frac{x^3}{1-x} = -\infty.$$

$$\Rightarrow I = \left(\frac{3}{2}\right)^{-\infty} = 0.$$

(2) Cung cấp phái dạng vô định.

$$I = 1^1 = 1.$$

(3)  $I = \lim_{x \rightarrow 0} \sqrt[1]{1-2x}$  (Dạng  $1^\infty$ )

$$= \lim_{x \rightarrow 0} (1-2x)^{1/x}$$

Tac'  $I = \lim_{x \rightarrow 0} (1 + (-2x))^{1/-2x} = e^{-2x/x}$

$$= \lim_{x \rightarrow 0} (1+u)^{\frac{1}{u}} \cdot 2 = e^2$$

(4)  $I = \lim_{x \rightarrow 0} \sqrt[1]{\cos \sqrt{x}}$  ( $u = -2x$ )  $= \lim_{x \rightarrow 0} (\cos \sqrt{x})^{1/\sqrt{x}}$  (Dạng  $1^\infty$ )

Đặt  $\sqrt{x} = y \Rightarrow x = y^2 \Rightarrow \frac{1}{x} = \frac{1}{y^2}$

$$I = \lim_{y \rightarrow 0} (\cos y)^{1/y^2} = \lim_{y \rightarrow 0} (1 - (1 - \cos y))^{\frac{1}{y^2}}$$

$$= \lim_{y \rightarrow 0} \left(1 - \frac{2\sin^2 \frac{y}{2}}{2}\right)^{1/y^2} - \lim_{y \rightarrow 0} e^{-\frac{2\sin^2 \frac{y}{2}}{y^2}}$$

$$= e^{-\frac{1}{2}}$$

(5)  $I = \lim_{x \rightarrow \frac{\pi}{2}} (\sin x)^{\tan x}$  (Dạng  $1^\infty$ )

$$\text{Đổi biến } y = \frac{\pi}{2} - x \Rightarrow x \rightarrow \frac{\pi}{2} \text{ khi } y \rightarrow 0$$

$$\sin x = \sin\left(\frac{\pi}{2} - y\right) = \cos y = 1 - 2\left(\sin \frac{y}{2}\right)^2$$

$$\tan x = \tan\left(\frac{\pi}{2} - y\right) = \cot y = \frac{\cos y}{\sin y}$$

$$\begin{aligned} \text{Do đó } I &= \lim_{y \rightarrow 0} \left[ 1 - 2\left(\sin \frac{y}{2}\right)^2 \right] \frac{\cos y}{\sin y} \\ &= \lim_{y \rightarrow 0} \frac{-2\left(\sin \frac{y}{2}\right)^2 \cdot \cos y}{\sin y} \\ &= e \end{aligned}$$

T<sub>2</sub> C Thử:  $\sin y = 2 \sin \frac{y}{2} \cos \frac{y}{2}$

~~$$I = \lim_{y \rightarrow 0} \frac{\sin y}{e^y} = \lim_{y \rightarrow 0} \frac{2 \sin \frac{y}{2} \cos \frac{y}{2}}{e^y}$$~~

$$I = \lim_{y \rightarrow 0} \frac{\sin \frac{y}{2} \cos \frac{y}{2}}{e^{\frac{y}{2}}} = \frac{0 \cdot 1}{1} = 1$$

⑥  $I = \lim_{x \rightarrow +\infty} \dots \Rightarrow$  T<sub>2</sub> cũng làm như sách giải của thầy Trí thi

⑦  $I = \lim_{x \rightarrow 0} \frac{e^{\alpha x} - e^{\beta x}}{\sin(\alpha x) - \sin(\beta x)}$  Dùng quy tắc L'Hospital  
tាមnhất

L'H  $\lim_{x \rightarrow 0} \frac{\alpha e^{\alpha x} - \beta e^{\beta x}}{\alpha \cos(\alpha x) - \beta \cos(\beta x)}$

Thay  $x=0$   $\frac{\alpha - \beta}{\alpha - \beta} = 1.$

⑧ Thầy cũng làm như sách giải thầy Trí! HONG HA

$$\textcircled{8} \quad I = \lim_{n \rightarrow \infty} n^2 \cdot x^{\frac{1}{n+1}} \left( x^{\frac{1}{n(n+1)}} - 1 \right)$$

$$= \lim_{n \rightarrow \infty} x^{\frac{1}{n+1}} \cdot \frac{n^2}{n(n+1)} \cdot \lim_{n \rightarrow \infty} \frac{x^{\frac{1}{n(n+1)}} - 1}{\left( \frac{1}{n(n+1)} \right)}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$x^0 = 1 \qquad 1 \qquad \text{dạng } \lim_{y \rightarrow 0} \frac{x^y - 1}{y}$$

Chú ý: các đẳng thức giới hạn dùng để thay cho

$$\boxed{\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1}, \quad \boxed{\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a}$$

Do đó ta có  $I = 1 \cdot 1 \cdot \ln(x) = \ln(x)$ .

Bài tip lô sung:

5)  $I = \lim_{x \rightarrow 0} \frac{1}{(1 - \tan^2 x)^{\frac{1}{\sin^2(2x)}}}$

$= e^{\lim_{x \rightarrow 0} \frac{-\tan^2 x}{\sin^2(2x)}}$

( $\sqrt{\lim_{x \rightarrow 0} \tan x} = 0$ )

vì ta s.đy  $\lim_{u \rightarrow 0} (1+u)^{\frac{1}{u}} = e$ )

Đo đó ta cđtln  $I_2 = \lim_{x \rightarrow 0} \frac{-\tan^2 x}{\sin^2(2x)}$

$= \lim_{x \rightarrow 0} \frac{\sin^2(x)}{2 \sin^2(x) \cos^2(x)} = \lim_{x \rightarrow 0} \frac{1}{2 \cos^4(x)} = \frac{1}{2}$

Vậy  $I_2 = e^{I_2} = e^{-\frac{1}{2}}$

6)  $I = \lim_{x \rightarrow 0} \left(1 - 2 \sin \frac{x}{2}\right)^{\frac{1}{x^2}}$

$= e^{\lim_{x \rightarrow 0} \frac{-2 \sin^2 \frac{x}{2}}{x^2}} = e^{-\frac{1}{2}}$

7) ~~cách 8, 10 lô~~ Câu 7 lô vi chia cosh.

$I = \lim_{x \rightarrow \infty} \left(\frac{2x^2 + 3}{2x^2 - 1}\right)^x$

$= \lim_{x \rightarrow \infty} \left(1 + \frac{4}{2x^2 - 1}\right)^x$

Ghi chú:

$$\begin{aligned} &= \lim_{x \rightarrow \infty} \left( 1 + \frac{1}{\frac{(2x^2-1)}{4}} \right)^{\frac{1}{x^2}} \\ &= e^{\lim_{x \rightarrow \infty} \frac{x^2}{(2x^2-1)/4}} \\ &= e^{\lim_{x \rightarrow \infty} \frac{4x^2}{2x^2-1}} = e^2. \end{aligned}$$

9)  $\lim_{x \rightarrow 2} \frac{2^x - x^2}{x-2} \stackrel{\text{L'Hospital}}{=} \lim_{x \rightarrow 2} \frac{2^x \ln 2 - 2x}{1}$   
 $= 4 \ln 2 - 4.$

10)  $I = \lim_{x \rightarrow \infty} \left( e^{\frac{1}{x}} + \frac{1}{x} \right)^x$   
 $\Rightarrow \ln I = \lim_{x \rightarrow \infty} x \ln \left( e^{\frac{1}{x}} + \frac{1}{x} \right)$

Đặt biến  $y = \frac{1}{x} \rightarrow \ln I = \lim_{u \rightarrow 0} \frac{\ln(e^u + u)}{u}$   
 $\stackrel{\text{L'H}}{=} \lim_{u \rightarrow 0} \frac{e^u + 1}{e^u + u} = \frac{e^0 + 1}{e^0 + 0} = 2$

$$\Rightarrow I = e^2.$$