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Dwell-time conditions for exponential stability and standard L_1-gain performance of discrete-time singular switched positive systems with time-varying delays

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ABSTRACT

This paper addresses the dwell-time issues on exponential stability and standard L_1-gain performance of discrete-time singular switched positive systems with time-varying delays. The dwell-time constraints refer to mode-dependent minimum dwell time(MDMDT), mode-dependent constant dwell time(MDCDT) and mode-dependent ranged dwell time(MDRDT). At first, a sufficient delay-dependent condition of exponential stability for delayed singular switched positive systems under three different kinds of dwell time constraints: MDMDT, MDCDT and MDRDT, is provided, respectively. Then, considering the disturbance attenuation performance, a sufficient condition of standard L_1-gain performance for delayed SSPSs with dwell-time constraints: MDMDT, MDCDT and MDRDT, is proposed, respectively. Meanwhile, the corresponding dwell-time results of exponential stability and standard L_1-gain performance for the system in delay-free case is also presented. Four examples are finally presented to show the feasibility and effectiveness of the obtained results.

1. Introduction

In the practical applications, many systems involve non-negative variables such as population level, material concentration, absolute temperature, and others. Such a system is expressed as a positive system [1], which often appear in actual fields [2,3]. It is known that many famous results of general systems are not suitable for positive systems, since positive system's states lie in not the whole linear space but the positive orthant. For positive systems, instead of the L_2-gain, the L_1-gain or L_∞-gain is more suitable to be used as the disturbance attenuation performance of the system, where L_1-gain represents the sum of quantities, while L_∞-gain represents the maximum of quantities [4]. Positive systems are mainly focused on stability and performance analysis, and many results can be found [5,6].

Switched system consists of a group of subsystems and a collection of logical rules, which coordinate the switches between the subsystems to determine which one to be valid [7,8]. As a special kind of constrained switching signals, dwell-time switching signal often appears in traffic system, telerobot system and other practical systems [9]. The dwell time is divided into three major categories: mode-dependent minimum dwell time(MDMDT), mode-dependent constant dwell time(MDCDT) and mode-dependent ranged dwell time(MDRDT), or to be more specifically, minimum dwell time(MDT),

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constant dwell time(CDT) and ranged dwell time(RDT). When a positive system's parameter undergoes the switching-type changes, this positive system can be modeled as a switched positive system(SPS), which has been widely utilized in congestion control of network communication systems [10], formation flying control of air traffic systems [11], isolation control of HIV virus in medical systems [12], and system theories [13–15]. Although many references on switched systems under dwell-time constraints exist [16,17], only a few results on SPSs under dwell-time constraints can be reached [18,19].

The above results are aimed at standard SPSs, that is, each subsystem is a standard system, however, in many actual engineering like power systems, biological systems, and economic systems, system models are necessary to be described as singular systems, which are generalizations of standard systems [20]. Compared with standard ones, singular systems are equipped with many advances, this is mainly due to their capacities to naturally model the real dynamical systems, to represent a wider range of systems than standard state-space systems, and to accurately describe the relationship between variables within the systems [21]. Especially in recent years, singular switched positive systems(SSPSs) have been utilized in numerous important domains like circuit network, computer controlling, communication, chemical reaction and so on. Due to that SSPS is coupling between the switching characteristic of subsystems, the non-negativity of valuables and the singularity of derivative matrix, the research of SPS in singular case is more complex than that in standard case. Although SSPSs have attracted wide attentions of scholars, only a few relevant results can be found [22–25], where [22] addressed the stability problem for SSPSs, [23] investigated the finite-time stability for discrete-time SSPSs, and [24,25] dealt with the stochastic stability issue for singular positive Markov jump systems.

It is widely recognized that the aforementioned literatures on SSPSs [22–25] are only based on arbitrary switching signals or average dwell time switching signals, and few are based on dwell-time switching signals. Different from the average dwell time switching signals, the dwell-time switching signals, as a special class of time-constrained switching signals, often appear in practical engineering, and meanwhile, the corresponding analyzing approaches with regard to these two kinds of switching signals are completely different, and obviously, the results resulting by these two kinds of switching signals cannot be extended to each other directly. Thus, it is of significance to undertake the research for SSPSs with dwell-time switching signals. When analyzing the stability of SSPSs with dwell-time switching signals, how to seek an effective approach to analyze the stability and how to deal with some mathematical problems in detail in the process of analysis and derivation become challenging, and the developed theory on dwell-time constraints is still not up to a quantitative level. Therefore, the first aim of this article is to investigate the stability of SSPSs under three kinds of dwell-time constraints: MDMDT, MDCDT and MDRDT, respectively.

Furthermore, the above mentioned achievements on SSPSs [22–25] are only concerned with the asymptotical stability problem. In practice, only demanding the system's asymptotical stability is often not enough, more hopefully, the system possesses a certain decay rate(DR) to converge quickly. Generally speaking, the asymptotical stability may converge slowly and be conservative to some extent, while the exponential stability can converge relatively fast with a convergence rate which can be described. When analyzing the exponential stability for the system with singularity constraints, determining the DR becomes the main difficulty. On the other hand, the results only focus on SSPSs without time-varying delays, which are commonly unavoidable in practice. Time-varying delays may result in instability and disruption of the performance, many related results can be found [26,27]. When considering this phenomenon, choosing an appropriate Lyapunov functional as well as analyzing the stability for SSPSs become the main difficulty, due to that delayed SSPSs are coupling between matrix difference equations and matrix delay differential equations, which makes it more complicated and challenging. Therefore, our second objective is to study the exponential stability of SSPSs with time-varying delays via determining the exponential DR with some effective approaches.

As mentioned earlier, the above results [22–25] only focus on SSPSs without exogenous disturbances, which are common in practical engineering and may inevitably affect the control outputs of the system [28,29]. Analyzing the disturbance attenuation performance for the system is an significant issue since one hope to judge whether the system can tolerate a certain degree of disturbances. Although weighted L_1-gain performance has been analyzed in some existing results on positive systems, the non-weighted L_1-gain performance or the standard L_1-gain performance remain to be studied. Compared with the standard L_1-gain performance, the weighted L_1-gain performance is not an anticipated performance in both mathematical analysis and practical application. Thus, the third aim of this article is to further investigate the disturbance attenuation performance i.e., the standard L_1-gain performance for SSPSs.

Thus, this paper addresses the dwell-time conditions for exponential stability and standard L_1-gain performance of discrete-time SSPSs with time-varying delays, and the dwell-time constraints refer to MDMDT, MDCDT and MDRDT. This is practically significant and meanwhile theoretically challenging. The main contributions are: (1) A sufficient delay-dependent condition of exponential stability for delayed SSPSs under three different kinds of dwell-time constraints: MDMDT, MDCDT and MDRDT, is provided, respectively; (2) Considering the disturbance attenuation performance, a sufficient condition of standard L_1-gain performance for delayed SSPSs with dwell-time constraints: MDMDT, MDCDT and MDRDT, is proposed, respectively; (3) The corresponding dwell-time results of exponential stability and standard L_1-gain performance for the system in delay-free case is presented.

In the rest of this article: Section 2 presents the preliminaries and problem formulation. Section 3 addresses the exponential stability for delayed SSPSs with dwell-time constraints: MDMDT, MDCDT and MDRDT, respectively. Considering the disturbance attenuation performance, the standard L_1-gain performance for delayed SSPSs with three different kinds of dwell-time constraints is analyzed in Section 4, respectively. Section 5 indicates the correctness and effectiveness of the obtained results through four examples. Section 6 finally gives the concluding remarks.

Notations For a matrix A, $A\preceq 0$ and $A\succeq 0$ indicate all elements of A to be non-positive and nonnegative, respectively. For a matrix A, its rank is represented as $rank\,(A)$, its determinant is denoted as $det\,(A)$, its degree of a polynomial is expressed as $deg\,(\cdot)$. The set of all real(positive real) numbers is denoted as $R(R_+)$. The set of all real(positive real) integers is represented as $(Z)Z_+$. The set of all n-dimensional real vectors and positive real vectors is defined as R^n and R^n_+, respectively. A collection of $m\times n$-dimensional real matrices is defined as $R^{m\times n}$. For a vector $x\in R^n$, x_i denotes its ith term, $i\in \underline{n}$. $\|x\|_1=\sum_{i=1}^n |x_i|$ is the 1-norm of a vector $x\in R^n$, $\|\nu\|_{L_1}=\sum_{k=k_0}^{\infty}\|\nu(k)\|_1$ is the L_1-norm of a vector valued function $\nu: R\to R^n$, we say $\nu(k)\in L_1[k_0,\infty)$ if $\|\nu\|_{L_1}<\infty$. For a vector $\nu\in R^n$, $\overline{\lambda}(\nu)$ denotes the largest entry of ν, $\underline{\lambda}(\nu)$ denotes the smallest entry of ν. $\mathbf{1}_n\in R^n$ means a n-dimensional column vector with all elements being 1.

2. Preliminaries and problem formulation

A discrete-time switched singular system with time-varying delays can be stated as:

$$\begin{cases} Ex(k+1) &= A_{\sigma(k)}x(k)+A_{d\sigma(k)}x(k-d(k))+F_{\sigma(k)}w(k),\\ z(k) &= C_{\sigma(k)}x(k),\\ x(k_0+\theta) &= \varphi(\theta),\quad \theta=-d_2,-d_2+1,\ldots,0, \end{cases} \tag{1}$$

where $x(k)\in R^n$ denotes the state, and x_0 is the initial state; $w(k)\in R^u$ is the exogenous disturbance signal; $z(k)\in R^m$ is the controlled output. $\sigma(k):[0,\infty)\to \underline{N}=\{1,2,\ldots,N\}$ is a switching signal with N being the total quantity of subsystems. The time-varying delay $d(k)$ satisfies $d(k)\in\{d_1,\ldots,d_2\}$ with d_1 and d_2 known, and $\varphi(\theta)$ is a given discrete vector-valued initial function. A_i, A_{di}, F_i, C_i, $i\in\underline{N}$ are known matrices, E is a singular matrix satisfying $rankE=r<n$. $k_0=0$ is the initial time and k_l is the lth switching instant in this paper.

Assumption 1.

1. The switching sequence of switching signal $\sigma(k)$ is $\{k_0,k_1,k_2,\ldots,k_l,k_{l+1},\ldots\}$, $l\in Z$;
2. The ith subsystem is activated during time interval $[k_l,k_{l+1})$, i.e., $\sigma(k)=i\in\underline{N}$, $k\in[k_l,k_{l+1})$;
3. The dwell time of ith subsystem is the holding time of time interval $[k_l,k_{l+1})$.

Definition 1 ([20]).

1. If each pair (E,A_i) is regular, i.e., $det(sE-A_i)\neq 0$, $\forall i\in\underline{N}$, then system (1) is regular;
2. If each pair (E,A_i) is causal, i.e., $deg(det(sE-A_i))=rank(E)$, $\forall i\in\underline{N}$, then system (1) is causal.

Definition 2 ([19]). For any switching rule $\sigma(k)$, if under any non-negative initial condition, the system state $x(k)\succeq 0$ and the controlled output $z(k)\succeq 0$ are always valid for all $k\geq 0$, then system (1) is called a positive system.

That system (1) is regular and causal implies the existence of $PEQ=\begin{bmatrix} I_r & 0\\ 0 & 0\end{bmatrix}$ with $P\in R^{n\times n}$ and $Q\in R^{n\times n}$ being non-singular matrices. For the sake of simplicity, in system (1), set $E=\begin{bmatrix} I_r & 0\\ 0 & 0\end{bmatrix}$ and

$$A_i=\begin{bmatrix} A_{i1} & A_{i2}\\ A_{i3} & A_{i4}\end{bmatrix},\quad A_{di}=\begin{bmatrix} A_{di1} & A_{di2}\\ A_{di3} & A_{di4}\end{bmatrix},\quad F_i=\begin{bmatrix} F_{i1}\\ F_{i2}\end{bmatrix},\quad C_i=\begin{bmatrix} C_{i1} & C_{i2}\end{bmatrix}, \tag{2}$$

where $\det(A_{i4})\neq 0$, $A_{i1}(A_{di1})\in R^{r\times r}$, $A_{i2}(A_{di2})\in R^{r\times(n-r)}$, $A_{i3}(A_{di3})\in R^{(n-r)\times r}$, $A_{i4}(A_{di4})\in R^{(n-r)\times(n-r)}$, $F_{i1}\in R^{r\times u}$, $F_{i2}\in R^{(n-r)\times u}$, $C_{i1}\in R^{m\times r}$, $C_{i2}\in R^{m\times(n-r)}$, $\forall i\in\underline{N}$. Assuming system (1) with (2) to be regular and causal, and letting $x(k)=\begin{bmatrix} x_1^T(k) & x_2^T(k)\end{bmatrix}^T$ with $x_1(k)\in R^r$ and $x_2(k)\in R^{n-r}$, it follows that, $\forall i\in\underline{N}$,

$$\begin{cases} x_1(k+1) &= \bar{A}_{i1}x_1(k)+\bar{A}_{di1}x_1(k-d(k))+\bar{A}_{di2}x_2(k-d(k))+\bar{F}_{i1}w(k),\\ x_2(k) &= \bar{A}_{i3}x_1(k)+\bar{A}_{di3}x_1(k-d(k))+\bar{A}_{di4}x_2(k-d(k))+\bar{F}_{i2}w(k),\\ z(k) &= C_{i1}x_1(k)+C_{i2}x_2(k),\\ x_1(k_0+\theta) &= \varphi_1(\theta),\quad \theta=-d_2,-d_2+1,\ldots,0,\\ x_2(k_0+\theta) &= \varphi_2(\theta),\quad \theta=-d_2,-d_2+1,\ldots,0, \end{cases} \tag{3}$$

where $\bar{A}_{i1}=A_{i1}-A_{i2}A_{i4}^{-1}A_{i3}$, $\bar{A}_{i3}=-A_{i4}^{-1}A_{i3}$, $\bar{A}_{di1}=A_{di1}-A_{i2}A_{i4}^{-1}A_{di3}$, $\bar{A}_{di2}=A_{di2}-A_{i2}A_{i4}^{-1}A_{di4}$, $\bar{A}_{di3}=-A_{i4}^{-1}A_{di3}$, $\bar{A}_{di4}=-A_{i4}^{-1}A_{di4}$, $\bar{F}_{i1}=F_{i1}-A_{i2}A_{i4}^{-1}F_{i2}$, $\bar{F}_{i2}=-A_{i4}^{-1}F_{i2}$.

Lemma 1 ([30]). *Assuming system (1) to be regular and causal, system (1) with (2) is positive if and only if* $\bar{A}_{i1}\succeq 0$, $\bar{A}_{i3}\succeq 0$, $\bar{A}_{di1}\succeq 0$, $\bar{A}_{di2}\succeq 0$, $\bar{A}_{di3}\succeq 0$, $\bar{A}_{di4}\succeq 0$, $\bar{F}_{i1}\succeq 0$, $\bar{F}_{i2}\succeq 0$, $C_{i1}\succeq 0$, $C_{i2}\succeq 0$, $\forall i\in\underline{N}$.

Definition 3 ([30]). If there exists a constant $0<\alpha<1$ so that, for any switching rule $\sigma(k)$ and any initial condition $x(k_0+\theta)=\varphi(\theta)$, $\theta=-d_2,-d_2+1,\ldots,0$, $x(k)$ meets

$$\|x(k)\|_1\leq c\alpha^{k-k_0}\|\varphi\|_{1c}=c\alpha^{k-k_0}\sup_{\theta=-d_2,-d_2+1,\ldots,0}\|\varphi(\theta)\|_1,\quad \forall k\geq 0, \tag{4}$$

then system (1) is exponentially stable(ES), where α and c are called the exponential DR and the decay coefficient, respectively.

Definition 4. Assuming system (1) to be regular, causal and ES(when $w(k) = 0$), for given scalars $0 < \alpha < 1$ and $\gamma > 0$, under zero initial conditions, system (1) possesses a standard L_1-gain performance level γ if

$$\sum_{k=k_0}^{\infty} \|z(k)\|_1 \le \gamma \sum_{k=k_0}^{\infty} \|w(k)\|_1, \quad \forall w(k) \in L_1.$$

Definition 5. For any switching signal $\sigma(k)$,

1. if $K_{il} \ge K_i^*$, $K_{il} = k_{l+1} - k_l$, $\sigma(k) = i \in \underline{N}$, $k \in [k_l, k_{l+1})$, $l \in Z$, $K_i^* \in R_+$, then it is called MDMDT switching signal;
2. if $K_{il} \equiv K_i^*$, $K_{il} = k_{l+1} - k_l$, $\sigma(k) = i \in \underline{N}$, $k \in [k_l, k_{l+1})$, $l \in Z$, $K_i^* \in R_+$, then it is called MDCDT switching signal;
3. if $\check{K}_i^* \le K_{il} \le \hat{K}_i^*$, $K_{il} = k_{l+1} - k_l$, $\sigma(k) = i \in \underline{N}$, $k \in [k_l, k_{l+1})$, $l \in Z$, $\check{K}_i^* \in R_+$, $\hat{K}_i^* \in R_+$, then it is called MDRDT switching signal.

Remark 1. In Definition 5, if we set $K_i^* = K_j^*$, $\forall i, j \in \underline{N}$, $i \ne j$, the MDMDT case will be equal to the MDT case, the MDCDT case will be degenerated into the CDT case, and if we set $\check{K}_i^* = \check{K}^*$, $\hat{K}_i^* = \hat{K}^*$, the MDRDT case will be transformed into the RDT case, which show that MDMDT(or MDCDT or MDRDT) case can be seen as a extension of MDT(or CDT or RDT) case, and it will be discussed in detail in the main results part.

The problem to be addressed in this paper can be formulated as follows:

1. Identify a class of switching signal $\sigma(k)$ and provide a sufficient Linear Programming(LP) condition of exponential stability for system (1) with dwell-time constraints: MDMDT, MDCDT and MDRDT, respectively. Moreover, give the corresponding dwell-time conditions of exponential stability for system (1) in delay-free case;
2. Considering the disturbance attenuation performance, provide a sufficient LP condition of standard L_1-gain performance for system (1) with dwell-time constraints: MDMDT, MDCDT and MDRDT, respectively. Moreover, give the corresponding dwell-time conditions of standard L_1-gain performance for system (1) in delay-free case;
3. Propose a convex optimization problem to determine the standard L_1-gain performance level optimum for system (1) with three different kinds of dwell-time constraints: MDMDT, MDCDT and MDRDT, respectively.

3. Exponential stability

In this section, the exponential stability problem of system (1) with three kinds of dwell-time constraints is considered. The dwell-time cases of MDMDT, MDCDT and MDRDT are considered in SubSection 3.1–3.3, respectively.

3.1. MDMDT case

In this subsection, by applying the discretized linear copositive Lyapunov-Krasovskii functional(DLCLKF) approach, the exponential stability issue for system (1) with MDMDT constraint is addressed.

Theorem 1. *Assume that system (1) satisfies the conditions in Lemma 1 and Assumption 1. Given constants* $0 < \alpha < 1$, $K_i^* \in Z_+$, $i \in \underline{N}$, *if there exist vectors* $\nu_{i,s} \in R_+^n$, $\vartheta \in R_+^n$, *such that,* $\forall (i,j) \in \underline{N} \times \underline{N}$, $i \ne j$, $s \in \{0, 1, \dots, K_i^*\}$,

$$A_i^T \nu_{i,s+1} - \alpha E^T \nu_{i,s} + (d_2 - d_1 + 1)\vartheta \preceq 0, \quad s \ne K_i^*, \tag{5}$$

$$A_{di}^T \nu_{i,s+1} - \alpha^{d_2} \vartheta \preceq 0, \quad s \ne K_i^*, \tag{6}$$

$$A_i^T \nu_{i,K_i^*} - \alpha E^T \nu_{i,K_i^*} + (d_2 - d_1 + 1)\vartheta \preceq 0, \tag{7}$$

$$\nu_{i,0} - \nu_{j,K_i^*} \preceq 0, \tag{8}$$

then system (1) is ES under any MDMDT switching signal $\sigma(k)$ *satisfying*

$$K_{il} \ge K_i^*, \quad K_{il} = k_{l+1} - k_l, \quad \sigma(k) = i, \quad k \in [k_l, k_{l+1}), \quad l \in Z. \tag{9}$$

Proof. Construct the following DLCLKF candidate for system (1),

$$V(x(k)) = x^T(k) E^T \nu_{\sigma(k)}(k) + \sum_{r=-d_2}^{-d_1} \sum_{m=k+r}^{k-1} \alpha^{(k-m-1)} x^T(m) \vartheta, \tag{10}$$

where the $\nu_{\sigma(k)}(k) \succ 0$ of DLCLKF is time-varying compared with the common $\nu_{\sigma(k)}$ of linear copositive Lyapunov-Krasovskii functional.

Since the switching signal satisfies the MDMDT condition in (9), the interval $[k_l, k_{l+1})$ can be divided into $[k_l, k_l + K_i^*)$ and $[k_l + K_i^*, k_{l+1})$.

Divide the interval $[k_l, k_l + K_i^*)$ into K_i^* segments described as $S_{is} = [k_l + s, k_l + (s + 1))$, $s = \{0, 1, \ldots, K_i^* - 1\}$. Set $\nu_{i,s} = \nu_i(k_l + s)$, then $\nu_i(k)$ can be described as follows:

$$\nu_i(k) = \begin{cases} \nu_{i,s}, & k \in S_{is}, \\ \nu_{i,K_i^*}, & k \in [k_l + K_i^*, k_{l+1}). \end{cases} \tag{11}$$

Then the differential of the vector function $\nu_i(k)$ can be obtained as follows:

$$\Delta\nu_i(k) = \begin{cases} \nu_{i,s+1} - \nu_{i,s}, & k \in S_{is}, \\ 0, & k \in [k_l + K_i^*, k_{l+1}). \end{cases} \tag{12}$$

Firstly, considering the case $k \in [k_l, k_l + K_i^*)$, one has from (1), (5) and (10)–(6) that

$$\begin{aligned} V_i(x(k+1)) - \alpha V_i(x(k)) &= x^T(k+1)E^T\nu_i(k+1) + \sum_{r=-d_2}^{-d_1}\sum_{m=k+1+r}^{k} \alpha^{(k-m)}x^T(m)\vartheta \\ &\quad - \alpha x^T(k)E^T\nu_i(k) - \alpha\sum_{r=-d_2}^{-d_1}\sum_{m=k+r}^{k-1} \alpha^{(k-m-1)}x^T(m)\vartheta \\ &= x^T(k+1)E^T\nu_i(k+1) - \alpha x^T(k)E^T\nu_i(k) \\ &\quad + \sum_{r=-d_2}^{-d_1}[x^T(k)\vartheta - \alpha^{-r}x^T(k+r)\vartheta] \\ &\le [x^T(k)A_i^T + x^T(k-d(k))A_{di}^T]\nu_{i,s+1} - \alpha x^T(k)E^T\nu_{i,s} \\ &\quad + x^T(k)(d_2 - d_1 + 1)\vartheta - \alpha^{d_2}x^T(k-d(k))\vartheta \\ &\le x^T(k)[A_i^T\nu_{i,s+1} - \alpha E^T\nu_{i,s} + (d_2 - d_1 + 1)\vartheta] \\ &\quad + x^T(k-d(k))[A_{di}^T\nu_{i,s+1} - \alpha^{d_2}\vartheta] \\ &\le 0. \end{aligned} \tag{13}$$

Next, the case $k \in [k_l + K_i^*, k_{l+1})$ is to be considered, one can get from (1), (7) and (10) that

$$\begin{aligned} V_i(x(k+1)) - \alpha V_i(x(k)) &= x^T(k+1)E^T\nu_i(k+1) + \sum_{r=-d_2}^{-d_1}\sum_{m=k+1+r}^{k} \alpha^{(k-m)}x^T(m)\vartheta \\ &\quad - \alpha x^T(k)E^T\nu_i(k) - \alpha\sum_{r=-d_2}^{-d_1}\sum_{m=k+r}^{k-1} \alpha^{(k-m-1)}x^T(m)\vartheta \\ &= x^T(k+1)E^T\nu_i(k+1) - \alpha x^T(k)E^T\nu_i(k) \\ &\quad + \sum_{r=-d_2}^{-d_1}[x^T(k)\vartheta - \alpha^{-r}x^T(k+r)\vartheta] \\ &\le [x^T(k)A_i^T + x^T(k-d(k))A_{di}^T]\nu_{i,K_i^*} - \alpha x^T(k)E^T\nu_{i,K_i^*} \\ &\quad + x^T(k)(d_2 - d_1 + 1)\vartheta - \alpha^{d_2}x^T(k-d(k))\vartheta \\ &\le x^T(k)[A_i^T\nu_{i,K_i^*} - \alpha E^T\nu_{i,K_i^*} + (d_2 - d_1 + 1)\vartheta] \\ &\quad + x^T(k-d(k))[A_{di}^T\nu_{i,K_i^*} - \alpha^{d_2}\vartheta] \\ &\le 0. \end{aligned} \tag{14}$$

Combining (13) and (14) leads to

$$V_i(x(k+1)) - \alpha V_i(x(k)) \le 0, \quad k \in [k_l, k_{l+1}), \tag{15}$$

which yields

$$V_{\sigma(k)}(x(k)) \le \alpha^{(k-k_l)}V_{\sigma(k_l)}(x(k_l)), \quad k \in [k_l, k_{l+1}). \tag{16}$$

Then, considering the switching instant k_l, and assuming that $\sigma(k)=j$ for $k\in[k_{l-1},k_l)$ and $\sigma(k)=i$ for $k\in[k_l,k_{l+1})$, $i,j\in\underline{N}$, we can get from (1) and (10) that

$$\begin{aligned} V_{\sigma(k_l^+)}(x(k_l^+))-V_{\sigma(k_l^-)}(x(k_l^-)) &= x^T(k_l^+)E^T\nu_{\sigma(k_l^+)}(k_l^+)+\sum_{r=-d_2}^{-d_1}\sum_{m=k_l^++r}^{k_l^+-1}\alpha^{(k_l^+-m-1)}x^T(m)\vartheta \\ &\quad -x^T(k_l^-)E^T\nu_{\sigma(k_l^-)}(k_l^-)-\sum_{r=-d_2}^{-d_1}\sum_{m=k_l^-+r}^{k_l^--1}\alpha^{(k_l^--m-1)}x^T(m)\vartheta \\ &= x^T(k_l)E^T(\nu_{i,0}-\nu_{j,K_i^*}) \\ &\quad +\sum_{r=-d_2}^{-d_1}\sum_{m=k_l+r}^{k_l-1}\alpha^{(k_l-m-1)}x^T(m)[\vartheta-\vartheta], \end{aligned} \tag{17}$$

and from (8), we have

$$V_{\sigma(k_l^+)}(x(k_l^+))-V_{\sigma(k_l^-)}(x(k_l^-))=V_{\sigma(k_l)}(x(k_l^+))-V_{\sigma(k_{l-1})}(x(k_l^-))\le 0,\quad \forall l\in Z_+. \tag{18}$$

It follows from (16) and (18) that

$$V_{\sigma(k)}(x(k))\le\alpha^{(k-k_0)}V_{\sigma(k_0)}(x(k_0)). \tag{19}$$

From the other side, denote $\nu_i(k)=\begin{bmatrix}\nu_i^{(1)}(k)\\ \nu_i^{(2)}(k)\end{bmatrix}$ with $\nu_i^{(1)}(k)\in R^r$ and $\nu_i^{(2)}(k)\in R^{n-r}$, then the DLCLKF (10) leads to

$$\begin{aligned} V_{\sigma(k)}(k) &= x^T(k)E^T\nu_i(k)+\sum_{r=-d_2}^{-d_1}\sum_{m=k+r}^{k-1}\alpha^{(k-m-1)}x^T(m)\vartheta \\ &= \begin{bmatrix} x_1^T(k) & x_2^T(k)\end{bmatrix}\begin{bmatrix} I_r & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\nu_i^{(1)}(k)\\ \nu_i^{(2)}(k)\end{bmatrix}+\sum_{r=-d_2}^{-d_1}\sum_{m=k+r}^{k-1}\alpha^{(k-m-1)}x^T(m)\vartheta \\ &= x_1^T(k)\nu_i^{(1)}(k)+\sum_{r=-d_2}^{-d_1}\sum_{m=k+r}^{k-1}\alpha^{(k-m-1)}x^T(m)\vartheta. \end{aligned} \tag{20}$$

According to (10) and (20), it is easy to get

$$V_{\sigma(k)}(x(k))\ge\beta_1\|x_1(k)\|_1, \tag{21}$$

$$V_{\sigma(k_0)}(x(k_0))\le\beta_2\|\varphi\|_{1c}, \tag{22}$$

where $\beta_1=\min_{\forall i\in\underline{N},s\in K_i^*}\underline{\lambda}(\nu_{i,s}^{(1)})$, $\beta_2=\max_{\forall i\in\underline{N}}\overline{\lambda}(\nu_{i,0}^{(1)})+(d_2-d_1+1)\frac{1-\alpha^{-d_2}}{1-\alpha}\overline{\lambda}(\vartheta)$, $\|\varphi\|_{1c}=\sup_{\theta=-d_2,-d_2+1,\ldots,0}\|x(k_0+\theta)\|_1$.

Combining (19) and (21)–(22) leads to, $\forall t\ge 0$,

$$\|x_1(k)\|_1\le\beta\alpha^{(k-k_0)}\|\varphi\|_{1c}, \tag{23}$$

where $\beta=\beta_2/\beta_1>1$, and it is obvious that $x_1(k)$ is ES with DR α. Also, (23) will lead to

$$\|x_1(k)\|_1\le\beta\alpha^{-d_2}\|\varphi\|_{1c}\alpha^{(k-k_0)},\quad\forall k\ge 0, \tag{24}$$

Next, $x_2(k)$ is proven to be ES with DR α. From (23), we can get

$$\|x_1(k-d(k))\|_1\le\beta\alpha^{(k-d(k)-k_0)}\|\varphi\|_{1c}\le\beta\alpha^{-d_2}\|\varphi\|_{1c}\alpha^{(k-k_0)},\quad\forall k>d(k), \tag{25}$$

$$\|x_1(k-d(k))\|_1\le\|\varphi\|_{1c}\le\|\varphi\|_{1c}\alpha^{(k-d(k))}\le\beta\alpha^{-d_2}\|\varphi\|_{1c}\alpha^{(k-k_0)},\quad\forall 0\le k\le d(k), \tag{26}$$

and it is obvious that

$$\|x_1(k-d(k))\|_1\le\beta\alpha^{-d_2}\|\varphi\|_{1c}\alpha^{(k-k_0)},\quad\forall k\ge 0. \tag{27}$$

The second equation of system (3) yields, $\forall k\ge 0$,

$$\begin{aligned} \|x_2(k)\|_1 &= \|\bar{A}_{i3}x_1(k)+\bar{A}_{di3}x_1(k-d(k))+\bar{A}_{di4}x_2(k-d(k))\|_1 \\ &\le \|\bar{A}_{i3}\|_1\|x_1(k)\|_1+\|\bar{A}_{di3}\|_1\|x_1(k-d(k))\|_1+\|\bar{A}_{di4}\|_1\|x_2(k-d(k))\|_1. \end{aligned} \tag{28}$$

For $k \in [0, d(k)]$, we have

$$\|x_2(k-d(k))\|_1 \le \|\varphi\|_{1c} \le \|\varphi\|_{1c}\alpha^{(k-d(k))} \le \alpha^{-d_2}\|\varphi\|_{1c}\alpha^{(k-k_0)}, \tag{29}$$

and from (24) and (27)–(29), we get

$$\begin{aligned}\|x_2(k)\|_1 &\le \|\bar{A}_{i3}\|_1\|x_1(k)\|_1 + \|\bar{A}_{di3}\|_1\|x_1(k-d(k))\|_1 + \|\bar{A}_{di4}\|_1\|x_2(k-d(k))\|_1\\ &\le \|\bar{A}_{i3}\|_1\beta\alpha^{-d_2}\|\varphi\|_{1c}\alpha^{(k-k_0)} + \|\bar{A}_{di3}\|_1\beta\alpha^{-d_2}\|\varphi\|_{1c}\alpha^{(k-k_0)} + \|\bar{A}_{di4}\|_1\alpha^{-d_2}\|\varphi\|_{1c}\alpha^{(k-k_0)}\\ &\le (\|\bar{A}_{i3}\|_1 + \|\bar{A}_{di3}\|_1)\beta\alpha^{-d_2}\|\varphi\|_{1c}\alpha^{(k-k_0)} + \|\bar{A}_{di4}\|_1\alpha^{-d_2}\|\varphi\|_{1c}\alpha^{(k-k_0)}\\ &\le (\|\bar{A}_{di4}\|_1\delta + \delta)\|\varphi\|_{1c}\alpha^{(k-k_0)},\end{aligned} \tag{30}$$

where $\delta = \max_{\forall i \in \underline{N}}\{(\|\bar{A}_{i3}\|_1 + \|\bar{A}_{di3}\|_1)\beta\alpha^{-d_2}, \alpha^{-d_2}\}$.

For $k \in [d(k), 2d(k)]$, we have from (30) that

$$\begin{aligned}\|x_2(k-d(k))\|_1 &\le (\|\bar{A}_{di4}\|_1\delta + \delta)\|\varphi\|_{1c}\alpha^{(k-d(k)-k_0)}\\ &\le (\|\bar{A}_{di4}\|_1\delta + \delta)\alpha^{-d_2}\|\varphi\|_{1c}\alpha^{(k-k_0)}\\ &\le (\|\bar{A}_{di4}\|_1\delta^2 + \delta^2)\|\varphi\|_{1c}\alpha^{(k-k_0)},\end{aligned} \tag{31}$$

and from (24) and (27)–(28) and (31), we get

$$\begin{aligned}\|x_2(k)\|_1 &\le \|\bar{A}_{i3}\|_1\|x_1(k)\|_1 + \|\bar{A}_{di3}\|_1\|x_1(k-d(k))\|_1 + \|\bar{A}_{di4}\|_1\|x_2(k-d(k))\|_1\\ &\le (\|\bar{A}_{i3}\|_1 + \|\bar{A}_{di3}\|_1)\beta\alpha^{-d_2}\|\varphi\|_{1c}\alpha^{(k-k_0)} + \|\bar{A}_{di4}\|_1(\|\bar{A}_{di4}\|_1\delta^2 + \delta^2)\|\varphi\|_{1c}\alpha^{(k-k_0)}\\ &\le (\|\bar{A}_{di4}\|_1^2\delta^2 + \|\bar{A}_{di4}\|_1\delta^2 + \delta)\|\varphi\|_{1c}\alpha^{(k-k_0)}.\end{aligned} \tag{32}$$

For any $k \in [(l-1)d(k), ld(k)]$, suppose that

$$\|x_2(k)\|_1 \le (\|\bar{A}_{di4}\|_1^l\delta^l + \|\bar{A}_{di4}\|_1^{l-1}\delta^l + \cdots + \|\bar{A}_{di4}\|_1\delta^2 + \delta)\|\varphi\|_{1c}\alpha^{(k-k_0)}, \tag{33}$$

then, through the inductive supposition method, when $k \in [ld(k), (l+1)d(k)]$, we have from (33) that

$$\begin{aligned}\|x_2(k-d(k))\|_1 &\le (\|\bar{A}_{di4}\|_1^l\delta^l + \|\bar{A}_{di4}\|_1^{l-1}\delta^l + \cdots + \|\bar{A}_{di4}\|_1\delta^2 + \delta)\|\varphi\|_{1c}\alpha^{(k-d(k)-k_0)}\\ &\le (\|\bar{A}_{di4}\|_1^l\delta^l + \|\bar{A}_{di4}\|_1^{l-1}\delta^l + \cdots + \|\bar{A}_{di4}\|_1\delta^2 + \delta)\alpha^{-d_2}\|\varphi\|_{1c}\alpha^{(k-k_0)},\end{aligned} \tag{34}$$

and from (24) and (27)–(28) and (34), we get

$$\begin{aligned}\|x_2(k)\|_1 &\le \|\bar{A}_{i3}\|_1\|x_1(k)\|_1 + \|\bar{A}_{di3}\|_1\|x_1(k-d(k))\|_1 + \|\bar{A}_{di4}\|_1\|x_2(k-d(k))\|_1\\ &\le (\|\bar{A}_{i3}\|_1 + \|\bar{A}_{di3}\|_1)\beta\alpha^{-d_2}\|\varphi\|_{1c}\alpha^{(k-k_0)}\\ &\quad + \|\bar{A}_{di4}\|_1(\|\bar{A}_{di4}\|_1^l\delta^l + \|\bar{A}_{di4}\|_1^{l-1}\delta^l + \cdots + \|\bar{A}_{di4}\|_1\delta^2 + \delta)\alpha^{-d_2}\|\varphi\|_{1c}\alpha^{(k-k_0)}\\ &\le (\|\bar{A}_{di4}\|_1^{(l+1)}\delta^{(l+1)} + \|\bar{A}_{di4}\|_1^l\delta^{(l+1)} + \cdots + \|\bar{A}_{di4}\|_1\delta^2 + \delta)\|\varphi\|_{1c}\alpha^{(k-k_0)}.\end{aligned} \tag{35}$$

If $0 < \|\bar{A}_{di4}\|_1\delta < 1$, through iterative method, we can get

$$\begin{aligned}\|x_2(t)\|_1 &\le [1 + \|\bar{A}_{di4}\|_1\delta + \|\bar{A}_{di4}\|_1^2\delta^2 + \cdots + \|\bar{A}_{di4}\|_1^l\delta^l + \cdots]\delta\|\varphi\|_{1c}\alpha^{(k-k_0)}\\ &\le \frac{\delta}{1 - \|\bar{A}_{di4}\|_1\delta}\|\varphi\|_{1c}\alpha^{(k-k_0)}.\end{aligned} \tag{36}$$

Finally, together (23) and (36) lead to $\|x(k)\|_1 \le \varpi\alpha^{(k-k_0)}\|\varphi\|_{1c}$, $\varpi = \max\{\beta, \frac{\delta}{1-\|\bar{A}_{di4}\|_1\delta}\}$, $\forall k \ge 0$. Thus, system (1) is ES with DR α based on Definition 3. $\square$

Remark 2. To prove the exponential stability, a novel Lyapunov-Krasovskii functional based approach, rather than the conventional Razumikhin method or the Halanay inequality method, is utilized here. In the literature [31], by solving a complex function, the exponential DR is calculated as a fixed value, which has a certain limitation in practical applications, to overcome this weakness, as shown in Theorem 1, by embedding a specific exponential term into the functional (10), the exponential DR is set as a free parameter, which could be selected based on diverse circumstances. All these features can truly introduce many flexibility in exponential stability analysis on SSPSs.

Remark 3. Given a MDMDT K_i^*, Theorem 1 has provide us a simple way to check the exponential stability of discrete-time SSPSs by solving a LP problem with constraints (5)–(9). Moreover, based on the LP problem, the minimal admissible MDMDT can be estimated with

$$K_{i\ \min}^* = \min_{K_i^*>0}\{K_i^* : (5)\text{–}(9) \quad hold\}.$$

Remark 4. In Theorem 1, as pointed out in the literature [16], the value of K_i^* is prescribed, and different K_i^* will cause different results. The conservativeness of the stability results is closely related to the value of K_i^*. Roughly speaking, the larger the value of K_i^* is, the less the conservativeness of the results will be. This has been discussed in detail in Example 4.

Remark 5. If $K_i^* = K_j^*$, $\forall i, j \in \underline{N}$, $i \neq j$, the MDMDT case will be equal to the MDT case, and the corresponding MDT switching signal $\sigma(k)$ satisfies

$$K_l \geq K^*, \quad K_l = k_{l+1} - k_l, \quad \sigma(k) = i, \quad k \in [k_l, k_{l+1}), \quad l \in Z.$$

In the MDT case, the exponential stability issue of discrete-time SSPSs can be solved by Theorem 1 via conditions (5)–(8) with $K_i^* = K^*$, $\forall i \in \underline{N}$. This indicates that the MDT case is only a special situation of the MDMDT case.

If $d(k) = 0$ in system (1), the time-varying delay case will be degenerated into the delay-free case, and the exponential stability issue of discrete-time SSPSs in MDMDT case is addressed.

Corollary 1. *Assume that system (1)($d(k) = 0$) satisfies the conditions in Lemma 1 and Assumption 1. Given constants $0 < \alpha < 1$, $K_i^* \in Z_+$, $i \in \underline{N}$, if there exist vectors $\nu_{i,s} \in R_+^n$, such that, $\forall (i, j) \in \underline{N} \times \underline{N}$, $i \neq j$, $s \in \{0, 1, \ldots, K_i^*\}$,*

$$\begin{aligned} A_i^T \nu_{i,s+1} - \alpha E^T \nu_{i,s} \preceq 0, \quad s \neq K_i^*, \\ A_i^T \nu_{i,K_i^*} - \alpha E^T \nu_{i,K_i^*} \preceq 0, \\ \nu_{i,0} - \nu_{j,K_i^*} \preceq 0, \end{aligned}$$

then system (1)($d(k) = 0$) is ES under any MDMDT switching signal $\sigma(k)$ satisfying (9).

3.2. MDCDT case

In this subsection, based on the DLCLKF approach, the exponential stability condition for system (1) with MDCDT constraint is provided.

Theorem 2. *Assume that system (1) satisfies the conditions in Lemma 1 and Assumption 1. Given constants $0 < \alpha < 1$, $K_i^* \in Z_+$, $i \in \underline{N}$, if there exist vectors $\nu_{i,s} \in R_+^n$, $\vartheta \in R_+^n$, such that, $\forall (i, j) \in \underline{N} \times \underline{N}$, $i \neq j$, $s \in \{0, 1, \ldots, K_i^*\}$,*

$$A_i^T \nu_{i,s+1} - \alpha E^T \nu_{i,s} + (d_2 - d_1 + 1)\vartheta \preceq 0, \quad s \neq K_i^*, \tag{37}$$

$$A_{di}^T \nu_{i,s+1} - \alpha^{d_2} \vartheta \preceq 0, \quad s \neq K_i^*, \tag{38}$$

$$\nu_{i,0} - \nu_{j,K_i^*} \preceq 0, \tag{39}$$

then system (1) is ES under any MDCDT switching signal $\sigma(k)$ satisfying

$$K_{il} \equiv K_i^*, \quad K_{il} = k_{l+1} - k_l, \quad \sigma(k) = i, \quad k \in [k_l, k_{l+1}), \quad l \in Z. \tag{40}$$

Proof. Construct the DLCLKF candidate (10) for system (1), and it is obvious that $\nu_{\sigma(t)}(t) \succ 0$ is time-varying.

Since the switching signal satisfies the MDCDT condition in (40), divide the interval $[k_l, k_{l+1})$ into K_i^* segments described as $S_{is} = [k_l + s, k_l + (s + 1))$, $s = \{0, 1, \ldots, K_i^* - 1\}$. Set $\nu_{i,s} = \nu_i(k_l + s)$, then $\nu_i(k)$ can be described as follows:

$$\nu_i(k) = \nu_{i,s}, \quad k \in S_{is}, \tag{41}$$

which leads to the differential of the vector function $\nu_i(k)$ as:

$$\Delta \nu_i(k) = \nu_{i,s+1} - \nu_{i,s}, \quad k \in S_{is}. \tag{42}$$

Consider $k \in [k_l, k_{l+1})$, and one get from (1), (10) and (37)–(38) that

$$\begin{aligned} V_i(x(k+1)) - \alpha V_i(x(k)) &= x^T(k+1)E^T \nu_i(k+1) + \sum_{r=-d_2}^{-d_1} \sum_{m=k+1+r}^{k} \alpha^{(k-m)} x^T(m)\vartheta \\ &\quad - \alpha x^T(k)E^T \nu_i(k) - \alpha \sum_{r=-d_2}^{-d_1} \sum_{m=k+r}^{k-1} \alpha^{(k-m-1)} x^T(m)\vartheta \\ &= x^T(k+1)E^T \nu_i(k+1) - \alpha x^T(k)E^T \nu_i(k) \\ &\quad + \sum_{r=-d_2}^{-d_1} [x^T(k)\vartheta - \alpha^{-r} x^T(k+r)\vartheta] \end{aligned}$$

$$\begin{aligned}
&\le [x^T(k)A_i^T + x^T(k-d(k))A_{di}^T]\nu_{i,s+1} - \alpha x^T(k)E^T\nu_{i,s} \\
&\quad + x^T(k)(d_2-d_1+1)\vartheta - \alpha^{d_2}x^T(k-d(k))\vartheta \\
&\le x^T(k)[A_i^T\nu_{i,s+1} - \alpha E^T\nu_{i,s} + (d_2-d_1+1)\vartheta] \\
&\quad + x^T(k-d(k))[A_{di}^T\nu_{i,s+1} - \alpha^{d_2}\vartheta] \\
&\le 0,
\end{aligned} \tag{43}$$

which yields

$$V_{\sigma(k)}(x(k)) \le \alpha^{(k-k_l)}V_{\sigma(k_l)}(x(k_l)), \quad k \in [k_l, k_{l+1}). \tag{44}$$

Then, considering the switching instant k_l, and assuming that $\sigma(k) = j$ for $k \in [k_{l-1}, k_l)$ and $\sigma(k) = i$ for $k \in [k_l, k_{l+1})$, $\forall i, j \in \underline{N}$, we can get from (1) and (10) that

$$\begin{aligned}
V_{\sigma(k_l^+)}(x(k_l^+)) - V_{\sigma(k_l^-)}(x(k_l^-)) &= x^T(k_l^+)E^T\nu_{\sigma(k_l^+)}(k_l^+) + \sum_{r=-d_2}^{-d_1}\sum_{m=k_l^++r}^{k_l^+-1}\alpha^{(k_l^+-m-1)}x^T(m)\vartheta \\
&\quad - x^T(k_l^-)E^T\nu_{\sigma(k_l^-)}(k_l^-) - \sum_{r=-d_2}^{-d_1}\sum_{m=k_l^-+r}^{k_l^--1}\alpha^{(k_l^--m-1)}x^T(m)\vartheta \\
&= x^T(k_l)E^T(\nu_{i,0} - \nu_{j,K_i^*}) \\
&\quad + \sum_{r=-d_2}^{-d_1}\sum_{m=k_l+r}^{k_l-1}\alpha^{(k_l-m-1)}x^T(m)[\vartheta - \vartheta],
\end{aligned} \tag{45}$$

and from (39), we have

$$V_{\sigma(k_l^+)}(x(k_l^+)) - V_{\sigma(k_l^-)}(x(k_l^-)) = V_{\sigma(k_l)}(x(k_l^+)) - V_{\sigma(k_{l-1})}(x(k_l^-)) \le 0, \quad \forall l \in Z_+. \tag{46}$$

Then it follows from (44) and (46) that

$$V_{\sigma(k)}(x(k)) \le \alpha^{(k-k_0)}V_{\sigma(k_0)}(x(k_0)). \tag{47}$$

The remaining is similar to the proof of Theorem 1.
This completes the proof. □

Remark 6. If $K_i^* = K_j^*$, $\forall i, j \in \underline{N}$, $i \ne j$, the MDCDT case will be degenerated into the CDT case, and the corresponding CDT switching signal $\sigma(k)$ satisfies

$$K_l \equiv K^*, \quad K_l = k_{l+1} - k_l, \quad \sigma(k) = i, \quad k \in [k_l, k_{l+1}), \quad l \in Z.$$

In the CDT case, the exponential stability issue of discrete-time SSPSs can be solved by Theorem 2 via conditions (37)–(39) with $K_i^* = K^*$, $\forall i \in \underline{N}$. This shows that the CDT case is a specific class of the MDCDT case.

Considering the delay-free case, i.e., $d(k) = 0$ in system (1), the exponential stability condition of discrete-time SSPSs in MDCDT case is provided.

Corollary 2. *Assume that system (1)($d(k) = 0$) satisfies the conditions in Lemma 1 and Assumption 1. Given constants $0 < \alpha < 1$, $K_i^* \in Z_+$, $i \in \underline{N}$, if there exist vectors $\nu_{i,s} \in R_+^n$, such that, $\forall (i, j) \in \underline{N} \times \underline{N}$, $i \ne j$, $s \in \{0, 1, \ldots, K_i^*\}$,*

$$\begin{aligned}
A_i^T\nu_{i,s+1} - \alpha E^T\nu_{i,s} \preceq 0, \quad s \ne K_i^*, \\
\nu_{i,0} - \nu_{j,K_i^*} \preceq 0,
\end{aligned}$$

then system (1)($d(k) = 0$) is ES under any MDCDT switching signal $\sigma(k)$ satisfying (40).

3.3. MDRDT case

In this subsection, on account of the DLCLKF approach, the exponential stability condition for system (1) with MDRDT constraint is proposed.

Theorem 3. *Assume that system (1) satisfies the conditions in Lemma 1 and Assumption 1. Given constants $0 < \alpha < 1$, $\hat{K}_i^* \in Z_+$, $\check{K}_i^* \in Z_+$, $i \in \underline{N}$, if there exist vectors $\nu_{i,s} \in R_+^n$, $\vartheta \in R_+^n$, such that, $\forall (i, j) \in \underline{N} \times \underline{N}$, $i \ne j$, $g \in \{\check{K}_i^*, \check{K}_i^* + 1, \ldots, \hat{K}_i^*\}$, $s \in \{0, 1, \ldots, \hat{K}_i^*\}$,*

$$A_i^T\nu_{i,s+1} - \alpha E_i^T\nu_{i,s} + (d_2 - d_1 + 1)\vartheta \preceq 0, \quad s \ne \hat{K}_i^*, \tag{48}$$

$$A_{di}^T\nu_{i,s+1} - \alpha^{d_2}\vartheta \preceq 0, \quad s \neq \hat{K}_i^*, \tag{49}$$

$$\nu_{i,0} - \nu_{j,g} \preceq 0, \tag{50}$$

then system (1) is ES under any MDRDT switching signal $\sigma(k)$ satisfying

$$\check{K}_i^* \le K_{il} \le \hat{K}_i^*, \quad K_{il} = k_{l+1} - k_l, \quad \sigma(k) = i, \quad k \in [k_l, k_{l+1}), \quad l \in Z. \tag{51}$$

Proof. Construct the DLCLKF candidate (10) for system (1), and it is obvious that $\nu_{\sigma(t)}(t) \succ 0$ is time-varying.

Since the switching signal satisfies the MDRDT condition in (51), divide the interval $[k_l, k_l + \hat{K}_i^*)$ into $\hat{K}_i^*$ segments described as $S_{is} = [k_l + s, k_l + (s+1))$, $s = \{0, 1, \ldots, \hat{K}_i^* - 1\}$. Set $\nu_{i,s} = \nu_i(k_l + s)$, then $\nu_i(k)$ can be described as follows:

$$\nu_i(k) = \nu_{i,s}, \quad k \in S_{is}, \tag{52}$$

which leads to the differential of the vector function $\nu_i(k)$ as:

$$\Delta\nu_i(k) = \nu_{i,s+1} - \nu_{i,s}, \quad k \in S_{is}. \tag{53}$$

Considering $k \in [k_l, k_{l+1})$, one get from (1), (10) and (48)–(49) that

$$\begin{aligned}
V_i(x(k+1)) - \alpha V_i(x(k)) &= x^T(k+1)E^T\nu_i(k+1) + \sum_{r=-d_2}^{-d_1}\sum_{m=k+1+r}^{k}\alpha^{(k-m)}x^T(m)\vartheta \\
&\quad - \alpha x^T(k)E^T\nu_i(k) - \alpha\sum_{r=-d_2}^{-d_1}\sum_{m=k+r}^{k-1}\alpha^{(k-m-1)}x^T(m)\vartheta \\
&= x^T(k+1)E^T\nu_i(k+1) - \alpha x^T(k)E^T\nu_i(k) \\
&\quad + \sum_{r=-d_2}^{-d_1}[x^T(k)\vartheta - \alpha^{-r}x^T(k+r)\vartheta] \\
&\le [x^T(k)A_i^T + x^T(k-d(k))A_{di}^T]\nu_{i,s+1} - \alpha x^T(k)E^T\nu_{i,s} \\
&\quad + x^T(k)(d_2 - d_1 + 1)\vartheta - \alpha^{d_2}x^T(k-d(k))\vartheta \\
&\le x^T(k)[A_i^T\nu_{i,s+1} - \alpha E^T\nu_{i,s} + (d_2 - d_1 + 1)\vartheta] \\
&\quad + x^T(k-d(k))[A_{di}^T\nu_{i,s+1} - \alpha^{d_2}\vartheta] \\
&\le 0,
\end{aligned} \tag{54}$$

which yields

$$V_{\sigma(k)}(x(k)) \le \alpha^{(k-k_l)}V_{\sigma(k_l)}(x(k_l)), \quad k \in [k_l, k_{l+1}). \tag{55}$$

Then, considering the switching instant k_l, and assuming that $\sigma(k) = j$ for $k \in [k_{l-1}, k_l)$ and $\sigma(k) = i$ for $k \in [k_l, k_{l+1})$, $\forall i, j \in \underline{N}$, we can get from (1) and (10) that

$$\begin{aligned}
V_{\sigma(k_l^+)}(x(k_l^+)) - V_{\sigma(k_l^-)}(x(k_l^-)) &= x^T(k_l^+)E^T\nu_{\sigma(k_l^+)}(k_l^+) + \sum_{r=-d_2}^{-d_1}\sum_{m=k_l^++r}^{k_l^+-1}\alpha^{(k_l^+-m-1)}x^T(m)\vartheta \\
&\quad - x^T(k_l^-)E^T\nu_{\sigma(k_l^-)}(k_l^-) - \sum_{r=-d_2}^{-d_1}\sum_{m=k_l^-+r}^{k_l^--1}\alpha^{(k_l^--m-1)}x^T(m)\vartheta \\
&= x^T(k_l)E^T(\nu_{i,0} - \nu_{j,g}) \\
&\quad + \sum_{r=-d_2}^{-d_1}\sum_{m=k_l+r}^{k_l-1}\alpha^{(k_l-m-1)}x^T(m)[\vartheta - \vartheta],
\end{aligned} \tag{56}$$

and from (50), we have

$$V_{\sigma(k_l^+)}(x(k_l^+)) - V_{\sigma(k_l^-)}(x(k_l^-)) = V_{\sigma(k_l)}(x(k_l^+)) - V_{\sigma(k_{l-1})}(x(k_l^-)) \le 0, \quad \forall l \in Z_+. \tag{57}$$

Then it follows from (55) and (57) that

$$V_{\sigma(k)}(x(k)) \le \alpha^{(k-k_0)}V_{\sigma(k_0)}(x(k_0)). \tag{58}$$

The remaining is similar to the proof of Theorem 1.

This completes the proof. □

Remark 7. If $\check{K}_i^* = \check{K}_j^*$, $\hat{K}_i^* = \hat{K}_j^*$, $\forall i, j \in \underline{N}$, $i \neq j$, the MDRDT case will be transformed into the RDT case, and the corresponding RDT switching signal $\sigma(k)$ satisfies

$$\check{K}^* \leq K_l \leq \hat{K}^*, \quad K_l = k_{l+1} - k_l, \quad \sigma(k) = i, \quad k \in [k_l, k_{l+1}), \quad l \in Z.$$

In the RDT case, the exponential stability issue of discrete-time SSPSs can be solved by Theorem 3 via conditions (48)–(50) with $\check{K}_i^* = \check{K}^*$, $\hat{K}_i^* = \hat{K}^*$, $\forall i \in \underline{N}$. This reveals that the RDT case is a special aspect of the MDRDT case.

When $d(k) = 0$ in system (1), the exponential stability condition of discrete-time SSPSs in MDRDT case is proposed.

Corollary 3. *Assume that system (1)($d(k) = 0$) satisfies the conditions in Lemma 1 and Assumption 1. Given constants $0 < \alpha < 1$, $\hat{K}_i^* \in Z_+$, $\check{K}_i^* \in Z_+$, $i \in \underline{N}$, if there exist vectors $\nu_{i,s} \in R_+^n$, such that, $\forall (i,j) \in \underline{N} \times \underline{N}$, $i \neq j$, $g \in \{\check{K}_i^*, \check{K}_i^* + 1, \ldots, \hat{K}_i^*\}$, $s \in \{0, 1, \ldots, \hat{K}_i^*\}$,*

$$\begin{aligned} A_i^T \nu_{i,s+1} - \alpha E_i^T \nu_{i,s} &\preceq 0, \quad s \neq \hat{K}_i^*, \\ \nu_{i,0} - \nu_{j,g} &\preceq 0, \end{aligned}$$

then system (1)($d(k) = 0$) is ES under any MDRDT switching signal $\sigma(k)$ satisfying (51).

4. Standard L_1-gain performance

In this section, the disturbance attenuation performance, i.e., the standard L_1-gain performance for system (1) with three kinds of dwell-time constraints: MDMDT, MDCDT and MDRDT is considered in SubSection 4.1–4.3, respectively.

4.1. MDMDT case

In this subsection, by applying the DLCLKF approach, the standard L_1-gain performance issue for system (1) with MDMDT constraint is addressed.

Theorem 4. *Assume that system (1) satisfies the conditions in Lemma 1 and Assumption 1. Given constants $0 < \alpha < 1$, $\gamma > 0$, $K_i^* \in Z_+$, $i \in \underline{N}$, if there exist vectors $\nu_{i,s} \in R_+^n$, $\vartheta \in R_+^n$, such that, $\forall (i,j) \in \underline{N} \times \underline{N}$, $i \neq j$, $s \in \{0, 1, \ldots, K_i^*\}$,*

$$A_i^T \nu_{i,s+1} - \alpha E^T \nu_{i,s} + (d_2 - d_1 + 1)\vartheta + C_i^T \mathbf{1}_m \preceq 0, \quad s \neq K_i^*, \tag{59}$$

$$A_{di}^T \nu_{i,s+1} - \alpha^{d_2} \vartheta \preceq 0, \quad s \neq K_i^*, \tag{60}$$

$$F_i^T \nu_{i,s+1} - \gamma \mathbf{1}_u \preceq 0, \quad s \neq K_i^*, \tag{61}$$

$$A_i^T \nu_{i,K_i^*} - \alpha E^T \nu_{i,K_i^*} + (d_2 - d_1 + 1)\vartheta + C_i^T \mathbf{1}_m \preceq 0, \tag{62}$$

$$\nu_{i,0} - \nu_{j,K_i^*} \preceq 0, \tag{63}$$

then system (1) is ES with a standard L_1-gain performance level γ under any MDMDT switching signal $\sigma(k)$ satisfying

$$K_{il} \geq K_i^*, \quad K_{il} = k_{l+1} - k_l, \quad \sigma(k) = i, \quad k \in [k_l, k_{l+1}), \quad l \in Z. \tag{64}$$

Proof. First, for $w(t) = 0$, system (1) can be directly proved to be ES based on Theorem 1, since (59)–(63) imply (5)–(8).

Next, for any $w(k) \in L_1$, system (1) satisfying an standard L_1-gain performance level is to be proved. Construct the DLCLKF candidate (10) for system (1), and meanwhile $\nu_{\sigma(t)}(t) \succ 0$ satisfies the conditions (11)–(12).

Since the switching signal satisfies the MDMDT condition in (64), the interval $[k_l, k_{l+1})$ can be divided into $[k_l, k_l + K_i^*)$ and $[k_l + K_i^*, k_{l+1})$.

Firstly, considering the case $k \in [k_l, k_l + K_i^*)$, one has from (1), (10) and (59)–(61) that

$$\begin{aligned} & V_i(x(k+1)) - \alpha V_i(x(k)) + \|z(k)\|_1 - \gamma \|w(k)\|_1 \\ &= x^T(k+1) E^T \nu_i(k+1) + \sum_{r=-d_2}^{-d_1} \sum_{m=k+1+r}^{k} \alpha^{(k-m)} x^T(m) \vartheta \\ &\quad - \alpha x^T(k) E^T \nu_i(k) - \alpha \sum_{r=-d_2}^{-d_1} \sum_{m=k+r}^{k-1} \alpha^{(k-m-1)} x^T(m) \vartheta + \|z(k)\|_1 - \gamma \|w(k)\|_1 \\ &= x^T(k+1) E^T \nu_i(k+1) - \alpha x^T(k) E^T \nu_i(k) \\ &\quad + \sum_{r=-d_2}^{-d_1} [x^T(k)\vartheta - \alpha^{-r} x^T(k+r)\vartheta] + x^T(k) C_i^T \mathbf{1}_m - \gamma w^T(k) \mathbf{1}_u \end{aligned}$$

$$\begin{aligned}
&\le [x^T(k)A_i^T + x^T(k-d(k))A_{di}^T + w^T(k)F_i^T]\nu_{i,s+1} - \alpha x^T(k)E^T\nu_{i,s} \\
&\quad + x^T(k)(d_2-d_1+1)\vartheta - \alpha^{d_2}x^T(k-d(k))\vartheta + x^T(k)C_i^T\mathbf{1}_m - \gamma w^T(k)\mathbf{1}_u \\
&\le x^T(k)[A_i^T\nu_{i,s+1} - \alpha E^T\nu_{i,s} + (d_2-d_1+1)\vartheta + C_i^T\mathbf{1}_m] \\
&\quad + x^T(k-d(k))[A_{di}^T\nu_{i,s+1} - \alpha^{d_2}\vartheta] \\
&\quad + w^T(k)[F_i^T\nu_{i,s+1} - \gamma\mathbf{1}_u] \\
&\le 0.
\end{aligned} \tag{65}$$

Next, the case $k \in [k_l + K_i^*, k_{l+1})$ is to be considered, one can get from (1), (10) and (60)–(62) that

$$\begin{aligned}
&V_i(x(k+1)) - \alpha V_i(x(k)) + \|z(k)\|_1 - \gamma\|w(k)\|_1 \\
&= x^T(k+1)E^T\nu_i(k+1) + \sum_{r=-d_2}^{-d_1}\sum_{m=k+1+r}^{k}\alpha^{(k-m)}x^T(m)\vartheta \\
&\quad - \alpha x^T(k)E^T\nu_i(k) - \alpha\sum_{r=-d_2}^{-d_1}\sum_{m=k+r}^{k-1}\alpha^{(k-m-1)}x^T(m)\vartheta + \|z(k)\|_1 - \gamma\|w(k)\|_1 \\
&= x^T(k+1)E^T\nu_i(k+1) - \alpha x^T(k)E^T\nu_i(k) \\
&\quad + \sum_{r=-d_2}^{-d_1}[x^T(k)\vartheta - \alpha^{-r}x^T(k+r)\vartheta] + x^T(k)C_i^T\mathbf{1}_m - \gamma w^T(k)\mathbf{1}_u \\
&\le [x^T(k)A_i^T + x^T(k-d(k))A_{di}^T + w^T(k)F_i^T]\nu_{i,K_i^*} - \alpha x^T(k)E^T\nu_{i,K_i^*} \\
&\quad + x^T(k)(d_2-d_1+1)\vartheta - \alpha^{d_2}x^T(k-d(k))\vartheta + x^T(k)C_i^T\mathbf{1}_m - \gamma w^T(k)\mathbf{1}_u \\
&\le x^T(k)[A_i^T\nu_{i,K_i^*} - \alpha E^T\nu_{i,K_i^*} + (d_2-d_1+1)\vartheta + C_i^T\mathbf{1}_m] \\
&\quad + x^T(k-d(k))[A_{di}^T\nu_{i,K_i^*} - \alpha^{d_2}\vartheta] \\
&\quad + w^T(k)[F_i^T\nu_{i,K_i^*} - \gamma\mathbf{1}_u] \\
&\le 0.
\end{aligned} \tag{66}$$

Combining (65) and (66) leads to

$$V_i(x(k+1)) - \alpha V_i(x(k)) + \|z(k)\|_1 - \gamma\|w(k)\|_1 \le 0, \quad k \in [k_l, k_{l+1}), \tag{67}$$

which yields

$$V_{\sigma(k)}(x(k)) \le \alpha^{(k-k_l)}V_{\sigma(k_l)}(x(k_l)) - \sum_{k_l}^{k-1}\alpha^{(k-s-1)}(\|z(s)\|_1 - \gamma\|w(s)\|_1), \quad k \in [k_l, k_{l+1}). \tag{68}$$

Then, considering the switching instant k_l, and assuming that $\sigma(k) = j$ for $k \in [k_{l-1}, k_l)$ and $\sigma(k) = i$ for $k \in [k_l, k_{l+1})$, $i, j \in \underline{N}$, we can get from (1) and (10) that

$$\begin{aligned}
V_{\sigma(k_l^+)}(x(k_l^+)) - V_{\sigma(k_l^-)}(x(k_l^-)) &= x^T(k_l^+)E^T\nu_{\sigma(k_l^+)}(k_l^+) + \sum_{r=-d_2}^{-d_1}\sum_{m=k_l^++r}^{k_l^+-1}\alpha^{(k_l^+-m-1)}x^T(m)\vartheta \\
&\quad - x^T(k_l^-)E^T\nu_{\sigma(k_l^-)}(k_l^-) - \sum_{r=-d_2}^{-d_1}\sum_{m=k_l^-+r}^{k_l^--1}\alpha^{(k_l^--m-1)}x^T(m)\vartheta \\
&= x^T(k_l)E^T(\nu_{i,0} - \nu_{j,K_i^*}) \\
&\quad + \sum_{r=-d_2}^{-d_1}\sum_{m=k_l+r}^{k_l-1}\alpha^{(k_l-m-1)}x^T(m)[\vartheta - \vartheta],
\end{aligned} \tag{69}$$

and from (63), we have

$$V_{\sigma(k_l^+)}(x(k_l^+)) - V_{\sigma(k_l^-)}(x(k_l^-)) = V_{\sigma(k_l)}(x(k_l^+)) - V_{\sigma(k_{l-1})}(x(k_l^-)) \le 0, \quad \forall l \in Z_+. \tag{70}$$

Then it follows from (68) and (70) that

$$
\begin{aligned}
V_{\sigma(k)}(x(k)) &\le \alpha^{(k-k_l)}V_{\sigma(k_{l-1})}(x(k_l^-)) - \sum_{k_l}^{k-1}\alpha^{(k-s-1)}(\|z(s)\|_1 - \gamma\|w(s)\|_1) \\
&\le \alpha^{(k-k_l)}[\alpha^{(k_l-k_{l-1})}V_{\sigma(k_{l-1})}(x(k_{l-1})) - \sum_{k_{l-1}}^{k_l}\alpha^{(k_l-s-1)}(\|z(s)\|_1 - \gamma\|w(s)\|_1)] \\
&\quad - \sum_{k_l}^{k-1}\alpha^{(k-s-1)}(\|z(s)\|_1 - \gamma\|w(s)\|_1) \\
&= \alpha^{(k-k_{l-1})}V_{\sigma(k_{l-1})}(x(k_{l-1})) - \sum_{k_{l-1}}^{k_l}\alpha^{(k-s-1)}(\|z(s)\|_1 - \gamma\|w(s)\|_1) \\
&\quad - \sum_{k_l}^{k-1}\alpha^{(k-s-1)}(\|z(s)\|_1 - \gamma\|w(s)\|_1) \\
&= \alpha^{(k-k_{l-1})}V_{\sigma(k_{l-1})}(x(k_{l-1})) - \sum_{k_{l-1}}^{k-1}\alpha^{(k-s-1)}(\|z(s)\|_1 - \gamma\|w(s)\|_1) \\
&\le \cdots \\
&\le \alpha^{(k-k_0)}V_{\sigma(k_0)}(x(k_0)) - \sum_{k_0}^{k-1}\alpha^{(k-s-1)}(\|z(s)\|_1 - \gamma\|w(s)\|_1).
\end{aligned} \tag{71}
$$

Under the zero initial condition, from (71), we have

$$
0 \le -\sum_{k_0}^{k-1}\alpha^{(k-s-1)}(\|z(s)\|_1 - \gamma\|w(s)\|_1), \tag{72}
$$

that is,

$$
\sum_{k_0}^{k-1}\alpha^{(k-s-1)}\|z(s)\|_1 \le \gamma\sum_{k_0}^{k-1}\alpha^{(k-s-1)}\|w(s)\|_1. \tag{73}
$$

Summing both sides of (73) from $k = k_0$ to ∞ leads to inequality:

$$
\sum_{k=k_0}^{\infty}\|z(k)\|_1 \le \gamma\sum_{k=k_0}^{\infty}\|w(k)\|_1, \tag{74}
$$

which implies that the effect of disturbance input $w(k)$ on the controlled output $z(k)$ is attenuated with a standard L_1-gain performance according to Definition 4.

This completes the proof. □

Remark 8. Compared with the weighted L_1-gain performance obtained in [4], the standard L_1-gain performance proposed in this paper can describe the system's ability of suppressing disturbance better. Specifically, according to the definitions of weighted L_1-gain performance and standard L_1-gain performance, it is easy to deduce that if the system is with a standard L_1 performance, then the system is with a weighted L_1 performance, but not vice versa.

Problem 1. By solving feasible conditions (59)–(63) in Theorem 4, the standard L_1-gain performance for system (1) under the MDMDT switching signal (64) can be analyzed, and it should be noted that these conditions are convex with regard to scalar γ, therefore, for given scalars α, d_1, d_2 and K_i^*, $i \in \underline{N}$, through the convex optimization problem below, the optimal standard L_1-gain performance level γ^* in MDMDT case is obtained:

$$
\min_{\nu_{i,s},\vartheta} \gamma
$$

$$
s.t. \quad (59)\text{–}(64), \quad \forall i \in \underline{N}.
$$

If $d(k) = 0$ in system (1), the time-varying delay case will be degenerated into the delay-free case, and the standard L_1-gain performance issue of discrete-time SSPSs in MDMDT case is addressed.

Corollary 4. *Assume that system (1)($d(k) = 0$) satisfies the conditions in Lemma 1 and Assumption 1. Given constants $0 < \alpha < 1$, $\gamma > 0$, $K_i^* \in Z_+$, $i \in \underline{N}$, if there exist vectors $\nu_{i,s} \in R_+^n$, such that, $\forall (i,j) \in \underline{N} \times \underline{N}$, $i \neq j$, $s \in \{0, 1, \dots, K_i^*\}$,*

$$\begin{aligned} A_i^T \nu_{i,s+1} - \alpha E^T \nu_{i,s} + C_i^T \mathbf{1}_m \preceq 0, \quad s \neq K_i^*, \\ F_i^T \nu_{i,s+1} - \gamma \mathbf{1}_u \preceq 0, \quad s \neq K_i^*, \\ A_i^T \nu_{i,K_i^*} - \alpha E^T \nu_{i,K_i^*} + C_i^T \mathbf{1}_m \preceq 0, \\ \nu_{i,0} - \nu_{j,K_i^*} \preceq 0, \end{aligned}$$

then system (1)($d(k) = 0$) is ES with a standard L_1-gain performance level γ under any MDMDT switching signal $\sigma(k)$ satisfying (64).

4.2. MDCDT case

In this subsection, based on the DLCLKF approach, the standard L_1-gain performance condition for system (1) with MDCDT constraint is provided.

Theorem 5. *Assume that system (1) satisfies the conditions in Lemma 1 and Assumption 1. Given constants $0 < \alpha < 1$, $\gamma > 0$, $K_i^* \in Z_+$, $i \in \underline{N}$, if there exist vectors $\nu_{i,s} \in R_+^n$, $\vartheta \in R_+^n$, such that, $\forall (i,j) \in \underline{N} \times \underline{N}$, $i \neq j$, $s \in \{0, 1, \dots, K_i^*\}$,*

$$A_i^T \nu_{i,s+1} - \alpha E^T \nu_{i,s} + (d_2 - d_1 + 1)\vartheta + C_i^T \mathbf{1}_m \preceq 0, \quad s \neq K_i^*, \tag{75}$$

$$A_{di}^T \nu_{i,s+1} - \alpha^{d_2} \vartheta \preceq 0, \quad s \neq K_i^*, \tag{76}$$

$$F_i^T \nu_{i,s+1} - \gamma \mathbf{1}_u \preceq 0, \quad s \neq K_i^*, \tag{77}$$

$$\nu_{i,0} - \nu_{j,K_i^*} \preceq 0, \tag{78}$$

then system (1) is ES with a standard L_1-gain performance level γ under any MDMDT switching signal $\sigma(k)$ satisfying

$$K_{il} \equiv K_i^*, \quad K_{il} = k_{l+1} - k_l, \quad \sigma(k) = i, \quad k \in [k_l, k_{l+1}), \quad l \in Z. \tag{79}$$

Proof. First, for $w(t) = 0$, system (1) can be directly proved to be ES based on Theorem 2, since (75)–(78) imply (37)–(39).

Next, for any $w(k) \in L_1$, system (1) satisfying a standard L_1-gain performance level is to be proved. Construct the DLCLKF candidate (10) for system (1), and meanwhile $\nu_{\sigma(t)}(t) \succ 0$ satisfies the conditions (41)–(42).

Consider $k \in [k_l, k_{l+1})$, and one get from (1), (10) and (75)–(77) that

$$\begin{aligned} & V_i(x(k+1)) - \alpha V_i(x(k)) + \|z(k)\|_1 - \gamma \|w(k)\|_1 \\ &= x^T(k+1) E^T \nu_i(k+1) + \sum_{r=-d_2}^{-d_1} \sum_{m=k+1+r}^{k} \alpha^{(k-m)} x^T(m) \vartheta \\ &\quad - \alpha x^T(k) E^T \nu_i(k) - \alpha \sum_{r=-d_2}^{-d_1} \sum_{m=k+r}^{k-1} \alpha^{(k-m-1)} x^T(m) \vartheta + \|z(k)\|_1 - \gamma \|w(k)\|_1 \\ &= x^T(k+1) E^T \nu_i(k+1) - \alpha x^T(k) E^T \nu_i(k) \\ &\quad + \sum_{r=-d_2}^{-d_1} [x^T(k)\vartheta - \alpha^{-r} x^T(k+r)\vartheta] + x^T(k) C_i^T \mathbf{1}_m - \gamma w^T(k) \mathbf{1}_u \\ &\le [x^T(k) A_i^T + x^T(k-d(k)) A_{di}^T + w^T(k) F_i^T] \nu_{i,s+1} - \alpha x^T(k) E^T \nu_{i,s} \\ &\quad + x^T(k)(d_2 - d_1 + 1)\vartheta - \alpha^{d_2} x^T(k-d(k))\vartheta + x^T(k) C_i^T \mathbf{1}_m - \gamma w^T(k) \mathbf{1}_u \\ &\le x^T(k)[A_i^T \nu_{i,s+1} - \alpha E^T \nu_{i,s} + (d_2 - d_1 + 1)\vartheta + C_i^T \mathbf{1}_m] \\ &\quad + x^T(k-d(k))[A_{di}^T \nu_{i,s+1} - \alpha^{d_2} \vartheta] \\ &\quad + w^T(k)[F_i^T \nu_{i,s+1} - \gamma \mathbf{1}_u] \\ &\le 0. \end{aligned} \tag{80}$$

which yields

$$V_{\sigma(k)}(x(k)) \le \alpha^{(k-k_l)} V_{\sigma(k_l)}(x(k_l)) - \sum_{k_l}^{k-1} \alpha^{(k-s-1)} (\|z(s)\|_1 - \gamma \|w(s)\|_1), \quad k \in [k_l, k_{l+1}). \tag{81}$$

Then, considering the switching instant k_l, and assuming that $\sigma(k) = j$ for $k \in [k_{l-1}, k_l)$ and $\sigma(k) = i$ for $k \in [k_l, k_{l+1})$, $i, j \in \underline{N}$, we can get from (1) and (10) that

$$\begin{aligned} V_{\sigma(k_l^+)}(x(k_l^+)) - V_{\sigma(k_l^-)}(x(k_l^-)) &= x^T(k_l^+)E^T\nu_{\sigma(k_l^+)}(k_l^+) + \sum_{r=-d_2}^{-d_1}\sum_{m=k_l^+ + r}^{k_l^+ - 1} \alpha^{(k_l^+ - m-1)}x^T(m)\vartheta \\ &\quad - x^T(k_l^-)E^T\nu_{\sigma(k_l^-)}(k_l^-) - \sum_{r=-d_2}^{-d_1}\sum_{m=k_l^- + r}^{k_l^- - 1} \alpha^{(k_l^- - m-1)}x^T(m)\vartheta \\ &= x^T(k_l)E^T(\nu_{i,0} - \nu_{j,K_i^*}) \\ &\quad + \sum_{r=-d_2}^{-d_1}\sum_{m=k_l+r}^{k_l-1} \alpha^{(k_l-m-1)}x^T(m)[\vartheta - \vartheta], \end{aligned} \tag{82}$$

and from (78), we have

$$V_{\sigma(k_l^+)}(x(k_l^+)) - V_{\sigma(k_l^-)}(x(k_l^-)) = V_{\sigma(k_l)}(x(k_l^+)) - V_{\sigma(k_{l-1})}(x(k_l^-)) \le 0, \quad \forall l \in Z_+. \tag{83}$$

Then it follows from (81) and (83) that

$$V_{\sigma(k)}(x(k)) \le \alpha^{(k-k_0)}V_{\sigma(k_0)}(x(k_0)) - \sum_{k_0}^{k-1}\alpha^{(k-s-1)}(\|z(s)\|_1 - \gamma\|w(s)\|_1). \tag{84}$$

The remaining is similar to the proof of Theorem 4.

This completes the proof. □

Problem 2. By solving feasible conditions (75)–(78) in Theorem 5, the standard L_1-gain performance for system (1) under the MDCDT switching signal (79) can be analyzed, and it should be noted that these conditions are convex with regard to scalar γ, therefore, for given scalars α, d_1, d_2 and K_i^*, $i \in \underline{N}$, through the convex optimization problem below, the optimal standard L_1-gain performance level γ^* in MDCDT case is obtained:

$$\min_{\nu_{i,s},\vartheta} \gamma$$

$$s.t. \quad (75)\text{–}(79), \quad \forall i \in \underline{N}.$$

Considering the delay-free case, i.e., $d(k) = 0$ in system (1), the standard L_1-gain performance condition of discrete-time SSPSs in MDCDT case is provided.

Corollary 5. *Assume that system (1)($d(k) = 0$) satisfies the conditions in Lemma 1 and Assumption 1. Given constants $0 < \alpha < 1$, $\gamma > 0$, $K_i^* \in Z_+$, $i \in \underline{N}$, if there exist vectors $\nu_{i,s} \in R_+^n$, such that, $\forall (i,j) \in \underline{N} \times \underline{N}$, $i \ne j$, $s \in \{0, 1, \ldots, K_i^*\}$,*

$$\begin{aligned} A_i^T\nu_{i,s+1} - \alpha E^T\nu_{i,s} + C_i^T\mathbf{1}_m \preceq 0, &\quad s \ne K_i^*, \\ F_i^T\nu_{i,s+1} - \gamma\mathbf{1}_u \preceq 0, &\quad s \ne K_i^*, \\ \nu_{i,0} - \nu_{j,K_i^*} \preceq 0, & \end{aligned}$$

then system (1)($d(k) = 0$) is ES with a standard L_1-gain performance level γ under any MDCDT switching signal $\sigma(k)$ satisfying (79).

4.3. MDRDT case

In this subsection, on account of the DLCLKF approach, the standard L_1-gain performance condition for system (1) with MDRDT constraint is proposed.

Theorem 6. *Assume that system (1) satisfies the conditions in Lemma 1 and Assumption 1. Given constants $0 < \alpha < 1$, $\gamma > 0$, $\hat{K}_i^* \in Z_+$, $\check{K}_i^* \in Z_+$, $i \in \underline{N}$, if there exist vectors $\nu_{i,s} \in R_+^n$, $\vartheta \in R_+^n$, such that, $\forall (i,j) \in \underline{N} \times \underline{N}$, $i \ne j$, $g \in \{\check{K}_i^*, \check{K}_i^* + 1, \ldots, \hat{K}_i^*\}$, $s \in \{0, 1, \ldots, \hat{K}_i^*\}$,*

$$A_i^T\nu_{i,s+1} - \alpha E^T\nu_{i,s} + (d_2 - d_1 + 1)\vartheta + C_i^T\mathbf{1}_m \preceq 0, \quad s \ne \hat{K}_i^*, \tag{85}$$

$$A_{di}^T\nu_{i,s+1} - \alpha^{d_2}\vartheta \preceq 0, \quad s \ne \hat{K}_i^*, \tag{86}$$

$$F_i^T\nu_{i,s+1} - \gamma\mathbf{1}_u \preceq 0, \quad s \ne \hat{K}_i^*, \tag{87}$$

$$\nu_{i,0} - \nu_{j,g} \preceq 0, \tag{88}$$

then system (1) is ES with a standard L_1-gain performance level γ under any MDRDT switching signal $\sigma(k)$ satisfying

$$\check{K}_i^* \le K_{il} \le \hat{K}_i^*, \quad K_{il} = k_{l+1} - k_l, \quad \sigma(k) = i, \quad k \in [k_l, k_{l+1}), \quad l \in Z. \tag{89}$$

Proof. First, for $w(t) = 0$, system (1) can be directly proved to be ES based on Theorem 3, since (85)–(88) imply (48)–(50).

Next, for any $w(k) \in L_1$, system (1) satisfying an standard L_1-gain performance level is to be proved. Construct the DLCLKF candidate (10) for system (1), and meanwhile $\nu_{\sigma(t)}(t) \succ 0$ satisfies the conditions (52)–(53).

Consider $k \in [k_l, k_{l+1})$, and one get from (1), (10) and (85)–(87) that

$$\begin{aligned}
&V_i(x(k+1)) - \alpha V_i(x(k)) + \|z(k)\|_1 - \gamma\|w(k)\|_1 \\
&= x^T(k+1)E^T\nu_i(k+1) + \sum_{r=-d_2}^{-d_1}\sum_{m=k+1+r}^{k} \alpha^{(k-m)}x^T(m)\vartheta \\
&\quad - \alpha x^T(k)E^T\nu_i(k) - \alpha\sum_{r=-d_2}^{-d_1}\sum_{m=k+r}^{k-1}\alpha^{(k-m-1)}x^T(m)\vartheta + \|z(k)\|_1 - \gamma\|w(k)\|_1 \\
&= x^T(k+1)E^T\nu_i(k+1) - \alpha x^T(k)E^T\nu_i(k) \\
&\quad + \sum_{r=-d_2}^{-d_1}[x^T(k)\vartheta - \alpha^{-r}x^T(k+r)\vartheta] + x^T(k)C_i^T\mathbf{1}_m - \gamma w^T(k)\mathbf{1}_u \\
&\le [x^T(k)A_i^T + x^T(k-d(k))A_{di}^T + w^T(k)F_i^T]\nu_{i,s+1} - \alpha x^T(k)E^T\nu_{i,s} \\
&\quad + x^T(k)(d_2-d_1+1)\vartheta - \alpha^{d_2}x^T(k-d(k))\vartheta + x^T(k)C_i^T\mathbf{1}_m - \gamma w^T(k)\mathbf{1}_u \\
&\le x^T(k)[A_i^T\nu_{i,s+1} - \alpha E^T\nu_{i,s} + (d_2-d_1+1)\vartheta + C_i^T\mathbf{1}_m] \\
&\quad + x^T(k-d(k))[A_{di}^T\nu_{i,s+1} - \alpha^{d_2}\vartheta] \\
&\quad + w^T(k)[F_i^T\nu_{i,s+1} - \gamma\mathbf{1}_u] \\
&\le 0.
\end{aligned} \tag{90}$$

which yields

$$V_{\sigma(k)}(x(k)) \le \alpha^{(k-k_l)}V_{\sigma(k_l)}(x(k_l)) - \sum_{k_l}^{k-1}\alpha^{(k-s-1)}(\|z(s)\|_1 - \gamma\|w(s)\|_1), \quad k \in [k_l, k_{l+1}). \tag{91}$$

Then, considering the switching instant k_l, and assuming that $\sigma(k) = j$ for $k \in [k_{l-1}, k_l)$ and $\sigma(k) = i$ for $k \in [k_l, k_{l+1})$, $i, j \in \underline{N}$, we can get from (1) and (10) that

$$\begin{aligned}
V_{\sigma(k_l^+)}(x(k_l^+)) - V_{\sigma(k_l^-)}(x(k_l^-)) &= x^T(k_l^+)E^T\nu_{\sigma(k_l^+)}(k_l^+) + \sum_{r=-d_2}^{-d_1}\sum_{m=k_l^++r}^{k_l^+-1}\alpha^{(k_l^+-m-1)}x^T(m)\vartheta \\
&\quad - x^T(k_l^-)E^T\nu_{\sigma(k_l^-)}(k_l^-) - \sum_{r=-d_2}^{-d_1}\sum_{m=k_l^-+r}^{k_l^--1}\alpha^{(k_l^--m-1)}x^T(m)\vartheta \\
&= x^T(k_l)E^T(\nu_{i,0} - \nu_{j,g}) \\
&\quad + \sum_{r=-d_2}^{-d_1}\sum_{m=k_l+r}^{k_l-1}\alpha^{(k_l-m-1)}x^T(m)[\vartheta - \vartheta],
\end{aligned} \tag{92}$$

and from (88), we have

$$V_{\sigma(k_l^+)}(x(k_l^+)) - V_{\sigma(k_l^-)}(x(k_l^-)) = V_{\sigma(k_l)}(x(k_l^+)) - V_{\sigma(k_{l-1})}(x(k_l^-)) \le 0, \quad \forall l \in Z_+. \tag{93}$$

Then it follows from (91) and (93) that

$$V_{\sigma(k)}(x(k)) \le \alpha^{(k-k_0)}V_{\sigma(k_0)}(x(k_0)) - \sum_{k_0}^{k-1}\alpha^{(k-s-1)}(\|z(s)\|_1 - \gamma\|w(s)\|_1). \tag{94}$$

The remaining is similar to the proof of Theorem 4.

This completes the proof. □

Problem 3. By solving feasible conditions (85)–(88) in Theorem 6, the standard L_1-gain performance for system (1) under the MDRDT switching signal (89) can be analyzed, and it should be noted that these conditions are convex with regard to scalar γ, therefore, for given scalars α, d_1, d_2 and K_i^*, $i \in \underline{N}$, through the convex optimization problem below, the optimal standard L_1-gain performance level γ^* in MDRDT case is obtained:

$$\min_{\nu_{i,s},\vartheta} \gamma$$

$$s.t. \quad (85)\text{–}(89), \quad \forall i \in \underline{N}.$$

When $d(k) = 0$ in system (1), the standard L_1-gain performance condition of discrete-time SSPSs in MDRDT case is proposed.

Corollary 6. *Assume that system (1)($d(k) = 0$) satisfies the conditions in Lemma 1 and Assumption 1. Given constants $0 < \alpha < 1$, $\gamma > 0$, $\hat{K}_i^* \in Z_+$, $\check{K}_i^* \in Z_+$, $i \in \underline{N}$, if there exist vectors $\nu_{i,s} \in R_+^n$, such that, $\forall (i,j) \in \underline{N} \times \underline{N}$, $i \neq j$, $g \in \{\check{K}_i^*, \check{K}_i^* + 1, \ldots, \hat{K}_i^*\}$, $s \in \{0, 1, \ldots, \hat{K}_i^*\}$,*

$$\begin{aligned} A_i^T \nu_{i,s+1} - \alpha E^T \nu_{i,s} + C_i^T \mathbf{1}_m \preceq 0, &\quad s \neq \hat{K}_i^*, \\ F_i^T \nu_{i,s+1} - \gamma\ \mathbf{1}_u \preceq 0, &\quad s \neq \hat{K}_i^*, \\ \nu_{i,0} - \nu_{j,g} \preceq 0, & \end{aligned}$$

then system (1)($d(k) = 0$) is ES with a standard L_1-gain performance level γ under any MDRDT switching signal $\sigma(k)$ satisfying (89).

Remark 9. The derivative matrix E is assumed to be switch-mode-independent in this paper. If E is switch-mode-dependent, then E is changed to E_i so that $E_i = \begin{bmatrix} I_{r_i} & 0 \\ 0 & 0 \end{bmatrix}$, $\forall i \in \underline{N}$. In this case, the state of the transformed system becomes $x(k) = [x_{i1}^T(k) \quad x_{i2}^T(k)]^T$ with $x_{i1}^T(k) \in R^{r_i}$ and $x_{i2}^T(k) \in R^{n-r_i}$, which means that there does not exist one common state space coordinate basis for different subsystems, thus it is rather complicated to discuss the transformed system. In this case, the method presented in this paper is also valid. Nonetheless, the general case with E being switch-mode-dependent is an interesting problem for future investigation via other methods.

5. Numerical examples

Four examples are given to exhibit the feasibility and effectiveness of developed results.

Example 1 (*MDMDT*)**.** Consider system (1) possessing two subsystems, i.e., $\sigma(t) \in \{1, 2\}$,

$$\begin{aligned} &A_1 = \begin{bmatrix} 0.7 & -1.9 \\ 0.4 & -1.8 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.02 & 0.2 \\ 0.01 & 0.1 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix}, \\ &A_2 = \begin{bmatrix} 0.6 & -1.9 \\ 0.3 & -1.8 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.02 & 0.3 \\ 0.01 & 0.1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, \end{aligned} \tag{95}$$

and $E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. By direct calculation, we have

$$\begin{aligned} &\det(sE - A_1) = 1.8s - 0.5 \neq 0, \quad \text{for} \quad s = 0, \\ &\det(sE - A_2) = 1.8s - 0.51 \neq 0, \quad \text{for} \quad s = 0, \\ &\deg(\det(sE - A_1)) = \deg(1.8s - 0.5) = \operatorname{rank} E = 1, \\ &\deg(\det(sE - A_2)) = \deg(1.8s - 0.51) = \operatorname{rank} E = 1. \end{aligned}$$

Hence, system (1) is regular and causal, and by the definitions in system (3), we can get

$$\begin{aligned} &\bar{A}_{11} = 0.2778, \quad \bar{A}_{d11} = 0.0094, \quad \bar{A}_{d12} = 0.0944, \quad \bar{F}_{11} = 0.1944, \\ &\bar{A}_{13} = 0.2222, \quad \bar{A}_{d13} = 0.0056, \quad \bar{A}_{d14} = 0.0556, \quad \bar{F}_{12} = 0.0556, \\ &\bar{A}_{21} = 0.2833, \quad \bar{A}_{d21} = 0.0094, \quad \bar{A}_{d22} = 0.1944, \quad \bar{F}_{21} = 0.0944, \\ &\bar{A}_{23} = 0.1667, \quad \bar{A}_{d23} = 0.0056, \quad \bar{A}_{d24} = 0.0556, \quad \bar{F}_{22} = 0.0556. \end{aligned}$$

Thus, system (1) with (95) is a positive system according to Lemma 1.

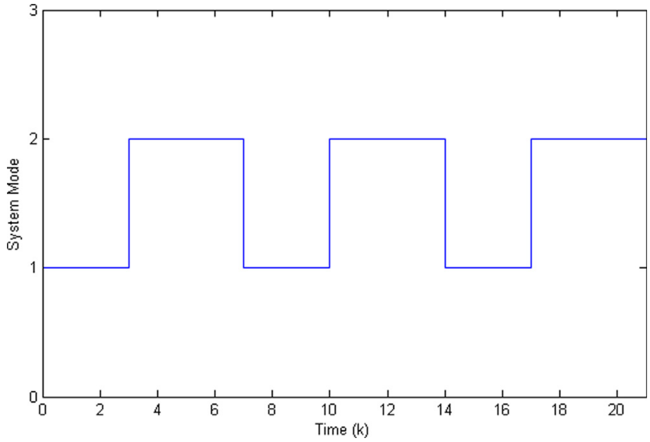

Fig. 1. Switching rule $\sigma(k)$ in MDMDT case.

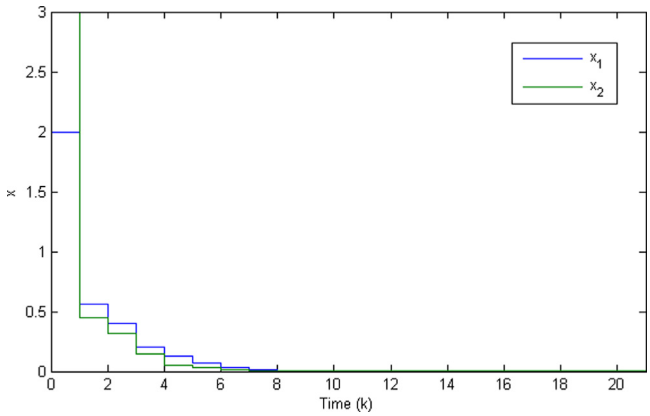

Fig. 2. The state $x(k)$ in MDMDT case.

Set $\alpha = 0.8$, $d(k) = 2 + \sin(k)$, $K_1^* = 2$ and $K_2^* = 3$. Solving the optimization Problem 1 leads to $\gamma^* = 1.1408$, and the corresponding solutions are

$$\nu_{10} = \begin{bmatrix} 0.0256 \\ 0.0357 \end{bmatrix}, \nu_{11} = \begin{bmatrix} 0.0604 \\ 0.0802 \end{bmatrix}, \nu_{12} = \begin{bmatrix} 0.0728 \\ 0.0909 \end{bmatrix}, \vartheta = \begin{bmatrix} 0.0909 \\ 0.0597 \end{bmatrix},$$
$$\nu_{20} = \begin{bmatrix} 0.0328 \\ 0.0454 \end{bmatrix}, \nu_{21} = \begin{bmatrix} 0.0368 \\ 0.0632 \end{bmatrix}, \nu_{22} = \begin{bmatrix} 0.0378 \\ 0.0630 \end{bmatrix}, \nu_{23} = \begin{bmatrix} 0.0520 \\ 0.0714 \end{bmatrix}.$$

Based on Theorem 4, system (1) with (95) is ES under the MDMDT switching signal satisfying:

$$\begin{cases} K_{1l} \geq K_1^* = 2, & l \in Z, \\ K_{2l} \geq K_2^* = 3, & l \in Z. \end{cases} \tag{96}$$

Figs. 1–3 show the simulation results, where $x(0) = [2 \quad 3]^T$, $x(k) = [0 \quad 0]^T$ with $k = -3, -2, -1, 0$, and the external disturbance is $w(k) = 0.05e^{-0.5k}$. Fig. 1 plots the MDMDT switching signal satisfying (96) with the sampling time $T = 1$. The system's state $x(k)$ and output $z(k)$ are shown in Figs. 2–3, from which one can see that system (1) with (95) is obviously positive and ES with switching signal plotted in Fig. 1. This illustrates the correctness and effectiveness of the obtained results.

Example 2 (*MDCDT*)**.** Consider system (1) with (95). By direct calculation, system (1) is regular and causal, and by the definitions in system (3), system (1) with (95) is a positive system according to Lemma 1.

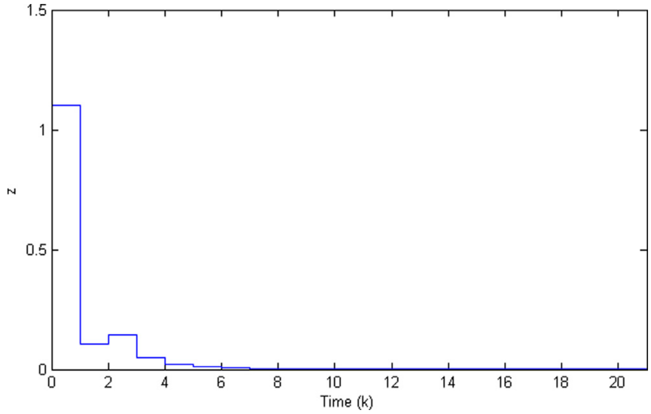

Fig. 3. The output $z(k)$ in MDMDT case.

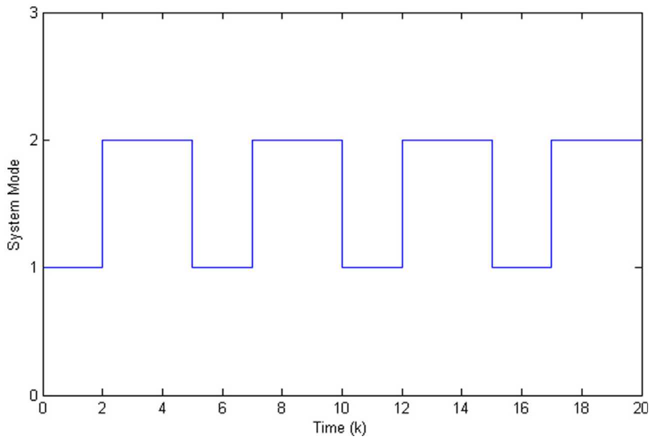

Fig. 4. Switching rule $\sigma(k)$ in MDCDT case.

Set $\alpha = 0.8$, $d(k) = 2 + \sin(k)$, $K_1^* = 2$ and $K_2^* = 3$. Solving the optimization Problem 2 leads to $\gamma^* = 1.1532$, and the corresponding solutions are

$$\nu_{10} = \begin{bmatrix} 0.0325 \\ 0.0468 \end{bmatrix}, \nu_{11} = \begin{bmatrix} 0.0625 \\ 0.0797 \end{bmatrix}, \nu_{12} = \begin{bmatrix} 0.0868 \\ 0.1063 \end{bmatrix}, \vartheta = \begin{bmatrix} 0.1001 \\ 0.0672 \end{bmatrix},$$
$$\nu_{20} = \begin{bmatrix} 0.0378 \\ 0.0532 \end{bmatrix}, \nu_{21} = \begin{bmatrix} 0.0361 \\ 0.0634 \end{bmatrix}, \nu_{22} = \begin{bmatrix} 0.0387 \\ 0.0622 \end{bmatrix}, \nu_{23} = \begin{bmatrix} 0.0702 \\ 0.0937 \end{bmatrix}.$$

Based on Theorem 5, system (1) with (95) is ES under the MDCDT switching signal satisfying:

$$\begin{cases} K_{1l} \equiv K_1^* = 1, & l \in Z, \\ K_{2l} \equiv K_2^* = 2, & l \in Z. \end{cases} \tag{97}$$

Figs. 4–6 show the simulation results, where $x(0) = [2 \quad 3]^T$, $x(k) = [0 \quad 0]^T$ with $k = -3, -2, -1, 0$, and the external disturbance is $w(k) = 0.05e^{-0.5k}$. Fig. 4 plots the MDCDT switching signal satisfying (97) with the sampling time $T = 1$. The system's state $x(k)$ and output $z(k)$ are shown in Figs. 5–6, from which one can see that system (1) with (95) is obviously positive and ES with switching signal plotted in Fig. 4. This illustrates the correctness and effectiveness of the obtained results.

Example 3 (*MDRDT*)**.** Consider system (1) with (95). By direct calculation, system (1) is regular and causal, and by the definitions in system (3), system (1) with (95) is a positive system according to Lemma 1.

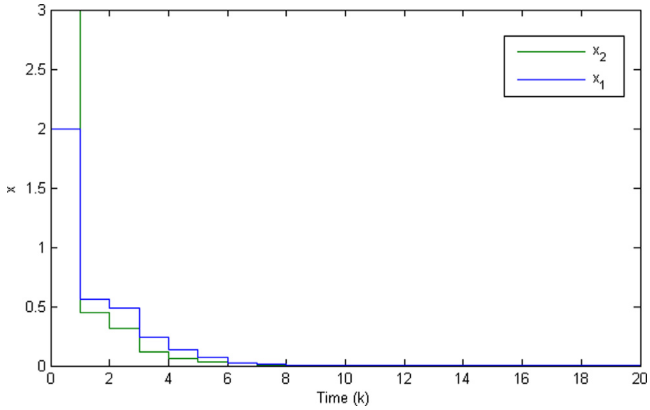

Fig. 5. The state $x(k)$ in MDCDT case.

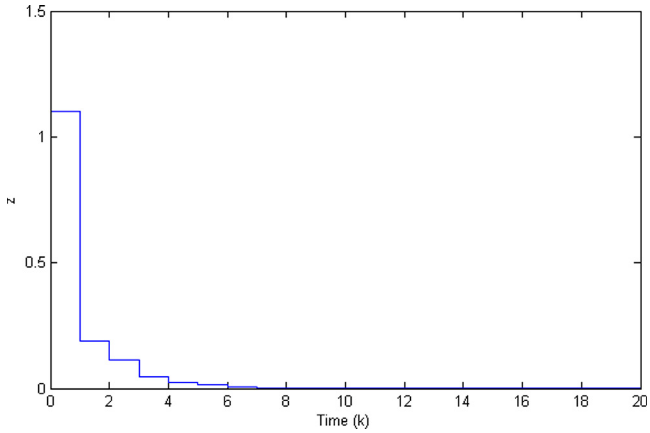

Fig. 6. The output $z(k)$ in MDCDT case.

Set $\alpha = 0.8$, $d(k) = 2 + \sin(k)$, $\check{K}_1^* = 1$, $\hat{K}_1^* = 3$, $\check{K}_2^* = 2$ and $\hat{K}_2^* = 4$. Solving the optimization Problem 3 leads to $\gamma^* = 22.9647$, and the corresponding solutions are

$$\nu_{10} = \begin{bmatrix} 4.2255 \\ 3.4544 \end{bmatrix}, \nu_{11} = \begin{bmatrix} 18.9859 \\ 7.8204 \end{bmatrix}, \nu_{12} = \begin{bmatrix} 38.1093 \\ 4.6189 \end{bmatrix}, \nu_{13} = \begin{bmatrix} 7.4887 \\ 20.9934 \end{bmatrix}, \vartheta = \begin{bmatrix} 5.3091 \\ 16.2107 \end{bmatrix},$$
$$\nu_{20} = \begin{bmatrix} 3.9691 \\ 2.0409 \end{bmatrix}, \nu_{21} = \begin{bmatrix} 6.0314 \\ 6.2806 \end{bmatrix}, \nu_{22} = \begin{bmatrix} 10.1086 \\ 10.8251 \end{bmatrix}, \nu_{23} = \begin{bmatrix} 10.3259 \\ 11.0062 \end{bmatrix}, \nu_{24} = \begin{bmatrix} 10.1071 \\ 11.0284 \end{bmatrix}.$$

Based on Theorem 6, system (1) with (95) is ES under the MDRDT switching signal satisfying:

$$\begin{cases} 1 = \check{K}_1^* \leq K_{1l} \leq \hat{K}_1^* = 3, & l \in Z, \\ 2 = \check{K}_2^* \leq K_{2l} \leq \hat{K}_1^* = 4, & l \in Z. \end{cases} \tag{98}$$

Figs. 7–9 show the simulation results, where $x(0) = [2 \quad 3]^T$, $x(k) = [0 \quad 0]^T$ with $k = -3, -2, -1, 0$, and the external disturbance is $w(k) = 0.05e^{-0.5k}$. Fig. 7 plots the MDRDT switching signal satisfying (98) with the sampling time $T = 1$. The system's state $x(k)$ and output $z(k)$ are shown in Figs. 8–9, from which one can see that system (1) with (95) is obviously positive and ES with switching signal plotted in Fig. 7. This illustrates the correctness and effectiveness of the obtained results.

Example 4. This example with comparison is given to show the influence of different value of parameters α, K_i^* and d_2 to the corresponding results of the system. Consider system (1) with (95), and for simplicity, the MDCDT case with $K_i^* = K^*$, $\forall i \in \{1, 2\}$, is taken into account here, then, the stability issue and the standard L_1-gain performance optimization problem can be solved via Theorem 5 in this paper.

Firstly, the stability issue is considered. Based on the conditions in Theorem 5, by choosing different value of parameters α, K_i^* and d_2, the results of the feasibility of corresponding solutions will be shown in the following Tables 1–2, in which

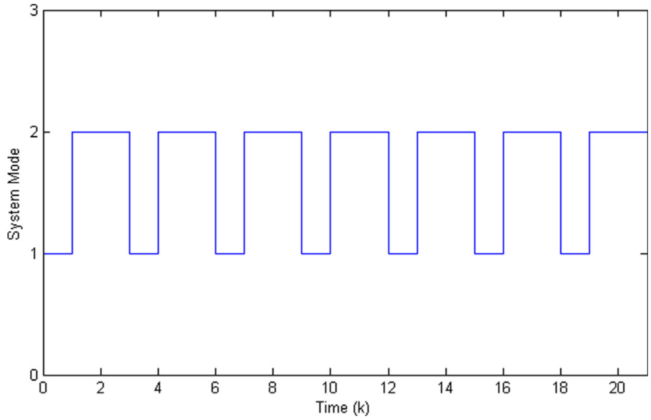

Fig. 7. Switching rule $\sigma(k)$ in MDRDT case.

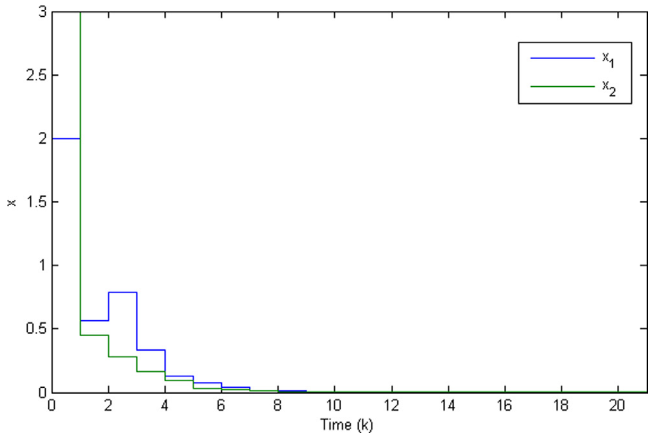

Fig. 8. The state $x(k)$ in MDRDT case.

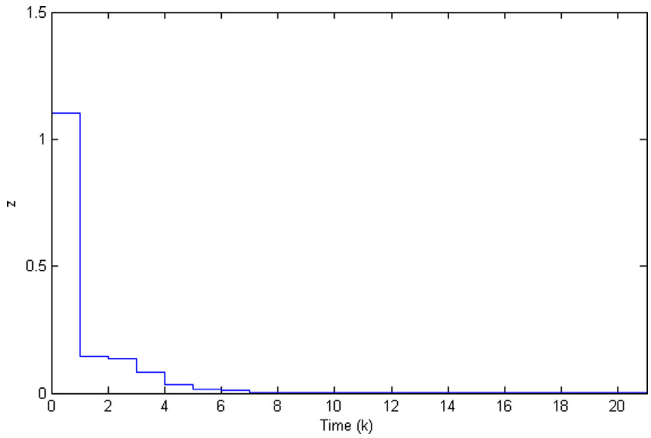

Fig. 9. The output $z(t)$ in MDRDT case.

'$*$' means that the corresponding feasible solutions can be found, while 'blank' means the reverse. That is, the influence of different parameters α, K_i^* and d_2 to the conservativeness of the results will be revealed in Tables 1–2.

Table 1 gives the relationship between α, K_i^* and the feasibility of corresponding solutions in the case that $d_1 = 1$, $d_2 = 3$, and Table 2 gives the relationship between α, d_2 and the feasibility of corresponding solutions in the case that $d_1 = 1$, $K_i^* = 3$. As indicated in Tables 1–2, the larger the value of α is, the easier the corresponding feasible solution

Table 1
The feasibility results for system (95) with $d_1 = 1$, $d_2 = 3$.

	$K_i^* = 2$	$K_i^* = 3$	$K_i^* = 4$	$K_i^* = 5$
$\alpha = 0.6$			*	*
$\alpha = 0.7$		*	*	*
$\alpha = 0.8$	*	*	*	*
$\alpha = 0.9$	*	*	*	*

Table 2
The feasibility results for system (95) with $d_1 = 1$, $K_i^* = 3$.

	$d_2 = 2$	$d_2 = 3$	$d_2 = 4$	$d_2 = 5$	$d_2 = 6$
$\alpha = 0.6$	*				
$\alpha = 0.7$	*	*			
$\alpha = 0.8$	*	*	*		
$\alpha = 0.9$	*	*	*	*	*

Table 3
The standard L_1-gain performance level optimum γ^* for system (95) with $d_1 = 1$, $d_2 = 3$.

	$K_i^* = 2$	$K_i^* = 3$	$K_i^* = 4$	$K_i^* = 5$
$\alpha = 0.6$			0.8317	0.4897
$\alpha = 0.7$		1.0760	0.5089	0.2387
$\alpha = 0.8$	1.3040	0.4259	0.2708	0.1981
$\alpha = 0.9$	0.6008	0.2674	0.1877	0.0945

Table 4
The standard L_1-gain performance level optimum γ^* for system (95) with $d_1 = 1$, $K_i^* = 3$.

	$d_2 = 2$	$d_2 = 3$	$d_2 = 4$	$d_2 = 5$	$d_2 = 6$
$\alpha = 0.6$	1.1107				
$\alpha = 0.7$	0.4976	1.0760			
$\alpha = 0.8$	0.2473	0.4259	0.7155		
$\alpha = 0.9$	0.1417	0.2674	0.3288	0.3873	0.4857

is obtained, i.e., the wider the range of feasible solution is, and the less the conservativeness of the result is. Moreover, Tables 1–2 show that if a larger dwell time K_i^* is chosen or a smaller delay bound d_2 is chosen, a corresponding feasible solution is easier to be obtained, that is, a wider range of feasible solution is found, and a less conservativeness of the result is achieved.

Furthermore, the standard L_1-gain performance optimization problem is considered. Based on the optimization Problem 2, by choosing different value of parameters α, K_i^* and d_2, the results of the standard L_1-gain performance level optimum γ^* will be shown in the following Tables 3–4, in which 'blank' means that the corresponding feasible solutions cannot be found. That is, the influence of different parameters α, K_i^* and d_2 to the disturbance attenuation performance of the system will be revealed in Tables 3–4.

Table 3 gives the relationship between α, K_i^* and the standard L_1-gain performance level optimum γ^* in the case that $d_1 = 1$, $d_2 = 3$, and Table 4 gives the relationship between α, d_2 and the standard L_1-gain performance level optimum γ^* in the case that $d_1 = 1$, $K_i^* = 3$. As indicated in Tables 3–4, the larger the value of α is, the less the value of γ^* is, i.e., the better the standard L_1-gain performance level is, and the better the disturbance attenuation performance of the system is. Moreover, Tables 3–4 show that if a larger dwell time K_i^* is chosen or a smaller delay bound d_2 is chosen, a smaller γ^* is obtained, that is, a better standard L_1-gain performance level is found and a better disturbance attenuation performance of the system is achieved.

6. Conclusions

This paper addresses the dwell-time issues of exponential stability and standard L_1-gain performance for discrete-time SSPSs with time-varying delays. At first, a sufficient delay-dependent condition of exponential stability for delayed SSPSs under dwell-time constraints: MDMDT, MDCDT and MDRDT, is provided, respectively. Then, the disturbance attenuation performance is further considered, and a sufficient condition of standard L_1-gain performance for delayed SSPSs under dwell-time constraints: MDMDT, MDCDT and MDRDT, is proposed, respectively. Meanwhile, the corresponding dwell-time results of exponential stability and standard L_1-gain performance for the system in delay-free case is presented. Four examples are finally given to illustrate the feasibility and effectiveness of the obtained results.

On the other hand, it is known that state estimation problem is frequently encountered in practice and has attracted much attentions in recent years [32–34]. To achieve more practical results, it would be interesting to generalize the obtained results to state estimation problem, like observation or filtering problem, for SSPSs in our future work. Moreover, future works will also be devoted to the extensions to finite-time control [35–37] or nonlinear systems [38,39], which is always a research hotspot for researchers due to its significance and challenge.

CRediT authorship contribution statement

Shuo Li: Conceptualization, Methodology, Writing - original draft. **Zhengrong Xiang:** Supervision, Writing - review & editing. **Junfeng Zhang:** Formal analysis, Writing - review & editing.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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