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Stability Analysis and Design for Switched Descriptor Systems

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Abstract: In this paper, we consider stability analysis and design for switched systems consisting of linear descriptor systems that have the same descriptor matrix. When all descriptor systems are stable, we show that if the descriptor matrix and all the system matrices are commutative pairwise, then the switched system is stable under arbitrary switching. This is an extension of the existing well known result in [1] for switched linear systems with state space models to switched descriptor systems. Under the same commutation condition, we also show that in the case where all descriptor systems are not stable, if there is a stable convex combination of the unstable descriptor systems, then we can establish a class of switching laws which stabilize the switched system. We finally make some discussion about obtaining the stable convex combination by solving a matrix inequality efficiently, and about relaxing the commutation condition for stabilizability of the switched system.

Keywords: Arbitrary switching, commutation, convex combination, descriptor systems, stability analysis, stabilization, switched systems.

1. INTRODUCTION

It is well known that descriptor systems (also known as singular systems or implicit systems) have high abilities in representing dynamical systems [2]. They can preserve physical parameters in the coefficient matrices, and describe the dynamic part, static part, and even improper part of the system in the same form. In this sense, descriptor systems are much superior to systems represented by state space models. There have been reported many works on descriptor systems, which studied feedback stabilization [2], Lyapunov stability theory [2,3], the matrix inequality approach [4] for stabilization, H_2 and/or H_∞ control [5,6].

Recently, there has been increasing interest in stability analysis and controller design for switched systems; see the survey papers [7-9] and the references cited therein. The motivation for studying switched systems is from many aspects. It is known that many practical systems are inherently multimodal in the sense that several dynamical subsystems are required to describe their behavior which may depend on various environmental factors. Since these systems are essentially switched systems, powerful analysis or design results of switched systems are helpful for dealing with real systems. Another important observation is that switching among a set of controllers for a specified system can be regarded as a switched system, and that switching has been used in adaptive control to assure stability in situations where stability can not be proved otherwise, or to improve transient response of adaptive control systems. Also, the methods of intelligent control design are based on the idea of switching among different controllers. Therefore, study on switched systems contributes greatly in switching controller and intelligent controller design.

When focusing on stability analysis of switched systems, there are three basic problems in stability and design of switched systems: (i) find conditions for stability under arbitrary switching; (ii) identify the limited but useful class of stabilizing switching laws; and (iii) construct a stabilizing switching law. There are many existing works studying these problems for switched linear time-invariant systems. For Problem (i), [1] showed that when all subsystems are stable and commutative pairwise, the switched linear system is stable under arbitrary switching. [10] extended this result from the commutation condition to a Lie-algebraic condition. [11] considered Problem (ii) using piecewise Lyapunov functions. [12] considered Problem (iii) by dividing the state space associated with appropriate switching depending on state, and [13] considered quadratic stabilization, which belongs to Problem (iii), for switched systems composed of a pair of unstable linear subsystems by using a linear stable combination of unstable subsystems.

However, to the authors' best knowledge, there is very few reference dealing with switched descriptor systems. There are many practical examples where we need to switch among several descriptor systems. Consider an electrical circuit composed of a voltagesource $u(t)$, a resistance $R(t)$ and a condenser with capacity 1, where all the elements are cascaded. The resistance $R(t)$ can take several different values depending on the temperature,

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i.e., $R(t) \in \{R_1, R_2, \cdots, R_m\}$. Let the circuit current be $x_1(t)$, the voltage of the condenser be $x_2(t)$. Then, the equation $\dot{x}_2(t) = x_1(t) = \frac{u(t) - x_2(t)}{R(t)}$ leads to

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ R(t) & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} u(t), \tag{1}$$

which is a typical descriptor system. Since the resistance $R(t)$ takes one value from $\{R_1, R_2, \cdots, R_m\}$, the system (1) is a switched descriptor system. The difficulty of dealing with such kind of switched descriptor systems falls into two aspects. First, descriptor systems are not easy to tackle and there are not rich results available up to know. Secondly, switching among several descriptor systems makes the problem much more complicated and in some cases it is even difficult to discuss the well-posedness of the system's solutions.

Motivated by the above observations, we study in this paper the stability analysis and design problem for switched descriptor systems that have the same descriptor matrix. We first focus our attention on Problem (i). More precisely, when all descriptor systems are stable, we show that if the descriptor matrix and all system matrices are commutative pairwise, then the switched system is stable under arbitrary switching. This result includes the existing well known result [1] for switched LTI systems as a special case. We also give some discussion on the application to switching control problem for a single descriptor system with control input (which is the main reason for the assumption of the same descriptor matrix). Then, we consider Problem (ii) for switched descriptor systems, under the assumption that none of the descriptor systems is stable and the descriptor matrix and all system matrices are commutative pairwise. We show that if there is a stable convex combination of the descriptor systems, then we can propose a class of switching laws which stabilize the switched system. Finally, we give some remarks to discuss how to obtain the stable convex combination by solving a matrix inequality efficiently, and how to relax the commutation condition for stabilizability of the switched system.

2. PROBLEM FORMULATION & PRELIMINARIES

Without losing generality, we consider the switched system composed of two descriptor systems described by

$$\begin{aligned} E\dot{x} &= A_1 x, \\ E\dot{x} &= A_2 x, \end{aligned} \tag{2}$$

where $x \in R^n$ is the descriptor variable, and E, A_1, A_2 are constant square matrices of appropriate size. The matrix E may be singular and we denote its rank by $r = \operatorname{rank} E \leq n.$

To define stability/stabilizability of the switched system (2), we need to introduce some definitions [6] and a preliminary result for linear descriptor systems.

Definition 1: Consider the descriptor system

$$E\dot{x} = Ax, \tag{3}$$

where all the notations are the same as in the system (2) except that A_1 (or A_2) in (2) is replaced with A here. The system (3) has a unique solution for any initial condition and is called *regular*, if $|sE - A| \not\equiv 0$. The finite eigenvalues of the matrix pair (E, A), that is, the solutions of $|sE - A| = 0$, and the corresponding (generalized) eigenvectors define exponential modes of (3). If the finite eigenvalues lie in the open left half-plane of s, the solution *decays exponentially*. The infinite eigenvalues of (E, A) with the eigenvectors satisfying the relations $Ex_1 = 0$ determines static modes. The infinite eigenvalues of (E, A) with generalized eigenvectors x_k satisfying the relations $Ex_1 = 0$ and $Ex_k = x_{k-1}$ $(k \geq 2)$ create *impulsive modes*. The system (3) has no impulsive mode if and only if $\operatorname{rank} E = \deg |sE - A|$. The system (3) is said to be *stable* if it is regular and has only decaying exponential modes and static modes (without impulsive modes).

Lemma 1 (Weiertrass Form) [2,14]**:** If the descriptor system (3) is regular, then there exist two nonsingular matrices M and N such that

$$\begin{aligned} M^{-1} E N^{-1} &= \begin{bmatrix} I_d & 0 \\ 0 & J \end{bmatrix}, \\ M^{-1} A N^{-1} &= \begin{bmatrix} \Lambda & 0 \\ 0 & I_{n-d} \end{bmatrix}, \end{aligned} \tag{4}$$

where $d = \deg |sE - A|$, J is composed of Jordan blocks for the finite eigenvalues. If the system (3) is regular and there is no impulsive mode, then (4) holds with $d = r$ and $J = 0$. If the system (3) is stable, then (4) holds with $d = r$, $J = 0$ and furthermore Λ being (Hurwitz) stable.

Now we give the definition for the switched system (2).

Definition 2: Given a switching law, the switched system (2) is said to be *stable* if starting from any initial value the system's trajectories converge to the origin exponentially. If there exists a switching law under which the switched system is stable, the switched system (2) is said to be *stabilizable* (under appropriate switching).

In this paper, we consider Problem (i) and (ii) for the switched system (2). More precisely, we study the following problems:

Problem A (Stability under arbitrary switching): "Assume the two descriptor systems in (2) are stable. Establish the condition under which the switched system is stable under arbitrary switching."

Problem B (Stabilization under appropriate switch-

ing): “Assume that neither of the two descriptor systems in (2) is stable but both of them are regular and have no impulsive mode. Establish the condition and the switching law for stability of the switched system.”

We will consider these problems respectively in Sections 3 and 4.

3. STABILITY UNDER ARBITRARY SWITCHING

We study Problem A in this section. Since arbitrary switching includes the case that one of the two descriptor systems is activated for all time, it is necessary to assume that both the two descriptor systems are stable (i.e., regular, no impulsive mode, decaying exponential modes). Note that in this case, for any initial value and any fixed switching signal, the trajectory of the switched system is determined uniquely.

For the system (2) with $E = I_n$ (state space model), [1] showed that if A_1 and A_2 are (Hurwitz) stable and commutative ($A_1A_2 = A_2A_1$), then the switched system is stable under arbitrary switching. We extend this result to the switched descriptor system (2) in almost the same form.

Theorem 1: If the two descriptor systems in (2) are stable, and furthermore the descriptor matrix E and the two system matrices A_1, A_2 are commutative pairwise, i.e.,

$$EA_1 = A_1E,\ EA_2 = A_2E,\ A_1A_2 = A_2A_1, \tag{5}$$

then the switched system (2) is stable under arbitrary switching.

Proof: Since (E, A_1) is stable, according to Lemma 1, there exist two nonsingular matrices M, N such that

$$M^{-1}EN^{-1} = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix},\quad M^{-1}A_1N^{-1} = \begin{bmatrix} \Lambda_1 & 0 \\ 0 & I_{n-r} \end{bmatrix}, \tag{6}$$

where Λ_1 is a (Hurwitz) stable matrix. Here, without causing confusion, we use the same notations M, N as in Lemma 1. Defining

$$NM = \begin{bmatrix} W_1 & W_2 \\ W_3 & W_4 \end{bmatrix} \tag{7}$$

and substituting it into the commutation condition $EA_1 = A_1E$, we obtain easily

$$\begin{bmatrix} W_1\Lambda_1 & W_2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \Lambda_1W_1 & 0 \\ W_3 & 0 \end{bmatrix}. \tag{8}$$

Thus, $W_1\Lambda_1 = \Lambda_1W_1$, $W_2 = 0$, $W_3 = 0$.

Now, we use the same nonsingular matrices M, N for the transformation of A_2 and write

$$M^{-1}A_2N^{-1} = \begin{bmatrix} \Lambda_2 & X_1 \\ X_2 & X \end{bmatrix}. \tag{9}$$

According to another commutation condition $EA_2 = A_2E$,

$$\begin{bmatrix} W_1\Lambda_2 & W_1X_1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \Lambda_2W_1 & 0 \\ X_2W_1 & 0 \end{bmatrix} \tag{10}$$

holds, and thus $W_1\Lambda_2 = \Lambda_2W_1$, $W_1X_1 = 0$, $X_2W_1 = 0$. Since NM is nonsingular and $W_2 = 0$, $W_3 = 0$, W_1 has to be nonsingular. We obtain then $X_1 = 0, X_2 = 0$. Furthermore, since (E, A_2) is stable, Λ_2 is a (Hurwitz) stable matrix and X has to be nonsingular.

The third commutation condition $A_1A_2 = A_2A_1$ results in

$$\begin{bmatrix} \Lambda_1W_1\Lambda_2 & 0 \\ 0 & W_4X \end{bmatrix} = \begin{bmatrix} \Lambda_2W_1\Lambda_1 & 0 \\ 0 & XW_4 \end{bmatrix}. \tag{11}$$

We have $\Lambda_1W_1\Lambda_2 = \Lambda_2W_1\Lambda_1$. Combining with $W_1\Lambda_1 = \Lambda_1W_1$, $W_1\Lambda_2 = \Lambda_2W_1$, we obtain that

$$W_1\Lambda_1\Lambda_2 = \Lambda_1W_1\Lambda_2 = \Lambda_2W_1\Lambda_1 = W_1\Lambda_2\Lambda_1, \tag{12}$$

which implies Λ_1 and Λ_2 are commutative ($\Lambda_1\Lambda_2 = \Lambda_2\Lambda_1$).

To summarize the above discussion, we obtain that under the same nonsingular transformation $\bar{x} = Nx$, the two descriptor systems in (2) take the form of

$$\dot{\bar{x}}_1 = \Lambda_1\bar{x}_1,\quad \bar{x}_2 = 0 \tag{13}$$

and

$$\dot{\bar{x}}_1 = \Lambda_2\bar{x}_1,\quad X\bar{x}_2 = 0 \tag{14}$$

respectively, where $\bar{x} = Nx = [\bar{x}_1^T\ \bar{x}_2^T]^T$, $\bar{x}_1 \in R^r$, Λ_1 and Λ_2 are (Hurwitz) stable and commutative, and X is nonsingular. Then, under arbitrary switching, $\bar{x}_2 = 0$, and $\bar{x}_1$ converges to the origin exponentially according to the existing result [1]. Therefore, the switched system composed of (13)-(14) is stable under arbitrary switching, and so is the original switched system (2). This completes the proof.

Remark 1: Using the same technique as in the proof of Theorem 1, we can easily establish and prove the result for the case where there are more than three descriptor systems involved. More precisely, for the switched system composed of L descriptor systems described by

$$E\dot{x} = A_ix,\quad i = 1, \cdots, L, \tag{15}$$

if all (E, A_i), $i = 1, \cdots, L$, are stable, and furthermore E and A_i’s are commutative pairwise, then the switched system is stable under arbitrary switching.

Remark 2: In the special case of $E = I_n$, the commutation condition (5) shrinks to $A_1A_2 = A_2A_2$, and thus Theorem 1 (or Remark 1) becomes the existing result in [1]. This implies that the result in this section is an extension of [1] to switched descriptor systems.

Remark 3: In the proof of the theorem, we obtained that the original switched system is equivalent to the switched system composed of (13) and (14). Since Λ_1 and Λ_2 are commutative, we obtain a common positive definite matrix P_Λ satisfying

$$\Lambda_1^T P_\Lambda + P_\Lambda \Lambda_1 < 0, \quad \Lambda_2^T P_\Lambda + P_\Lambda \Lambda_2 < 0, \tag{16}$$

which implies that $V(\bar{x}) = \bar{x}^T \mathrm{diag}\{P_\Lambda, I\}\bar{x}$ is a common Lyapunov function for the systems (13) and (14). Since $\bar{x} = Nx$, $V(x) = x^T N^T \mathrm{diag}\{P_\Lambda, I\} Nx$ is a common Lyapunov function for the original systems in (2). Therefore, under the condition in Theorem 1, there is a common Lyapunov function for the two descriptor systems.

In the end of this section, we point out that the result can be applied for switching control problem for linear descriptor systems. This is the main reason and motivation that we assume the same descriptor matrix E in the switched system (2). For example, if for a single descriptor system

$$E\dot{x} = Ax + Bu, \tag{17}$$

where u is the control input, we have designed two stabilizing state feedbacks $u = K_1x, u = K_2x$, and furthermore E, $A + BK_1$, $A + BK_2$ are commutative pairwise, then the system is stable no matter how we switch between the two controllers. This kind of requirement is very important when we want more flexibility for multiple control specifications in real applications.

4. STABILIZATION UNDER APPROPRIATE SWITCHING

We study Problem B in this section. That is, under the assumption that the two descriptor systems in (2) are not stable, we consider the condition for stabilizability under appropriate switching and establish the switching law.

For the system (2) with $E = I_n$ (state space model), [13] showed that if there is a convex combination of A_1 and A_2, i.e., $\lambda A_1 + (1-\lambda)A_2$ $(\lambda \in (0,1))$, which is (Hurwitz) stable, then there is a state dependent switching law achieving quadratic stability for the switched system. Later, [15] proved that if the number of systems is two, then the existence of such stable convex combination is both necessary and sufficient for quadratic stabilizability of the switched system.

Here, we extend the idea of stable convex combination of systems to the switched descriptor system (2), although we are not considering quadratic stabilizability presently.

As described in Problem B in Section 2, we assume that both descriptor systems are regular and do not have impulsive mode. According to Lemma 1, there exist two nonsingular matrices M, N satisfying (6), where now Λ_1 is not (Hurwitz) stable. If we assume that E, A_1, A_2 are commutative pairwise (satisfying (5)), as done in the previous section, then we also obtain

$$M^{-1}A_2N^{-1} = \begin{bmatrix} \Lambda_2 & 0 \\ 0 & X \end{bmatrix}. \tag{18}$$

Since (E, A_2) is not stable, Λ_2 is not (Hurwitz) stable now. X is nonsingular since (E, A_2) does not have impulsive mode.

Next, we define a convex combination of the two descriptor systems as

$$(E, A_\lambda) \triangleq \left(E, \lambda A_1 + (1-\lambda)A_2\right), \tag{19}$$

where $\lambda \in (0,1)$. Then,

$$\begin{aligned} M^{-1}EN^{-1} &= \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \\ M^{-1}A_\lambda N^{-1} &= \begin{bmatrix} \lambda\Lambda_1 + (1-\lambda)\Lambda_2 & 0 \\ 0 & \lambda I + (1-\lambda)X \end{bmatrix}. \end{aligned} \tag{20}$$

If we can find a scalar $\lambda \in (0,1)$ such that (E, A_λ) is stable, then we obtain from (20) that $\lambda\Lambda_1 + (1-\lambda)\Lambda_2$ is (Hurwitz) stable and $\lambda I + (1-\lambda)X$ is nonsingular.

Therefore, as discussed in the previous section, the switched system's stability can be studied for the switched system (13)-(14) instead, with the nonsingular transformation $\bar{x} = Nx$. The $\bar{x}_2$ part remains zero no matter how we switch. Concerning the $\bar{x}_1$ part, we know that $\lambda\Lambda_1 + (1-\lambda)\Lambda_2$ is (Hurwitz) stable and Λ_1 and Λ_2 are commutative. Then, we propose the following class of switching laws [16]:

Switching Law: *Activate the two descriptor systems in (7) in sequence with the duration time periods λT and $(1-\lambda)T$, respectively, where T is an arbitrarily specified positive scalar.*

Since $\Lambda_\lambda \triangleq \lambda\Lambda_1 + (1-\lambda)\Lambda_2$ is (Hurwitz) stable, and Λ_1 and Λ_2 are (Hurwitz) unstable, there always exist positive scalars α_s, α_u and k_s, k_u such that

$$\| e^{\Lambda_\lambda t} \| \le k_s e^{-\alpha_s t}, \quad \| e^{\Lambda_i t} \| \le k_u e^{\alpha_u t} (i = 1, 2) \tag{21}$$

hold for $\forall\ t \ge 0$. For any $t > 0$, we divide the interval $[0, t)$ according to the switching law as $t = lT + \hat{t}$, where l is a nonnegative integer, $0 \le \hat{t} < T$. Since Λ_1 and Λ_2 are commutative, for any initial condition $\bar{x}_1(0)$, we have

$$
\begin{aligned}
|\bar{x}_1(t)| &\le k_s e^{-\alpha_s(lT)} \times k_u e^{\alpha_u \hat{t}} |\bar{x}_1(0)| \\
&= (k_s k_u) e^{-\alpha_s(lT+\hat{t})} \times e^{(\alpha_s+\alpha_u)\hat{t}} |\bar{x}_1(0)| \\
&\le \left(k_s k_u e^{(\alpha_s+\alpha_u)T} \right) e^{-\alpha_s t} |\bar{x}_1(0)|,
\end{aligned} \tag{22}
$$

which implies that the $\bar{x}_1$ part converges exponentially. For more detailed discussion and other switching strategy, refer to [16].

We summarize the above discussion in the following theorem.

Theorem 2: Assume that the two descriptor systems in (2) are not stable, and the descriptor matrix E and the two system matrices A_1, A_2 are commutative pairwise. If there is a stable convex combination (E, A_λ) of the descriptor systems, then the switched system (2) is stable under the above switching law.

Remark 4: Theorem 1 is also valid for switched systems composed of more than three descriptor systems. For example, if the descriptor systems are described by $E\dot{x} = A_i x, i = 1, \cdots, L,$ then the convex combination of the descriptor systems under consideration is $(E, \sum_{i=1}^{L} \lambda_i A_i),$ where $\lambda_i \ge 0,$ $\sum_{i=1}^{L} \lambda_i = 1.$

We observe that the stabilization problem is now reduced to finding a stable convex combination (E, A_λ). As pointed out in the literature, this is generally a very hard task. A practical method of finding such a combination is to use the matrix inequality based condition proposed in [6]. More precisely, we introduce matrices $V, U \in R^{n\times(n-r)}$ which are of full column rank and composed of bases of Null E, Null E^T respectively. Then, according to [6], (E, A_λ) is stable if and only if there exist a positive definite matrix P and a matrix $S \in R^{(n-r)\times(n-r)}$ satisfying the matrix inequality

$$
\begin{aligned}
&\left[\lambda A_1 + (1-\lambda) A_2\right](PE^T + VSU^T) \\
&+ (PE^T + VSU^T)\left[\lambda A_1 + (1-\lambda) A_2\right] < 0.
\end{aligned} \tag{23}
$$

Thus, the control problem is reduced to solving (23) with respect to $P > 0,$ S and $0 < \lambda < 1.$

The condition (23) is a bilinear matrix inequality (BMI) in λ and $(P, S).$ As is well known, it is not an easy task to solve BMI. Although the branch and bound method [17] and the homotopy-based algorithm [18] have been proposed for solving BMIs, they either are sufficient (not necessary) or need very large efforts dealing with general BMIs. Before trying these methods, we suggest a discretization method for λ. More precisely, since the matrix inequality (23) is "continuous"[1] with respect to $\lambda,$ we discretize the interval $(0,1)$ into a sequence of values $d_1, d_2, \cdots, d_{K-1}$ with $d_i = \frac{i}{K}$ and K large enough, and solve the linear matrix inequality (23) with λ fixed as $d_i,$ using the existing LMI software. If the original matrix inequality is feasible, we can expect that this method converges with some d_i when the discretization is fine enough (or K is large enough). We suggest that this is a more efficient and practical method in most cases.

Remark 5: It can be obtained from the matrix inequality (23) that (E, A_λ) is stable if and only if there are two scalars $\lambda_1 \ge 0$ and $\lambda_2 \ge 0$ such that $(E, \lambda_1 A_1 + \lambda_2 A_2)$ is stable. Therefore, the proposed convex combination $(E, A_\lambda) \triangleq (E, \lambda A_1 + (1-\lambda) A_2),$ where $\lambda \in (0,1)$, can be replaced with the positive combination $(E, A_\lambda) \triangleq (E, \lambda_1 A_1 + \lambda_2 A_2),$ where $\lambda_1 \ge 0,$ $\lambda_2 \ge 0.$ This is also true for the case involving more than three descriptor systems, as described in Remark 4.

In the end of this section, we make some discussion about the commutation condition. It is observed that the commutation condition among the descriptor matrix E and the system matrices A_i's leads to the Weiertrass forms (6) and (18) and the commutation between Λ_1 and $\Lambda_2,$ with the same nonsingular matrices M and N. In real applications, the commutation condition may be quite restrictive, and thus it is desirable to consider the possibility of relaxing this condition.

We note that even if the commutation condition is not satisfied, (6) is true since (E, A_1) is regular. Thus, the commutation condition can be relaxed as: "with the same nonsingular matrices M and N, the matrix A_2 takes the form of (18)." Furthermore, by observing the transformation of A_λ in (20), we are enlightened that the right side of (18) does not have to be block diagonal. More precisely, if

$$
M^{-1} A_2 N^{-1} = \begin{bmatrix} \Lambda_2 & X_1 \\ 0 & X \end{bmatrix}, \tag{24}
$$

where X_1 is not necessarily zero, then (20) becomes

$$
\begin{aligned}
M^{-1} E N^{-1} &= \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \\
M^{-1} A_\lambda N^{-1} &= \begin{bmatrix} \lambda \Lambda_1 + (1-\lambda)\Lambda_2 & X_1 \\ 0 & \lambda I + (1-\lambda) X \end{bmatrix}.
\end{aligned} \tag{25}
$$

In this case, the condition that (E, A_λ) is stable also implies that $\lambda \Lambda_1 + (1-\lambda)\Lambda_2$ is (Hurwitz) stable and $\lambda I + (1-\lambda) X$ is nonsingular. Although Λ_1 and Λ_2 are not commutative, according to the existing literature [13], there exist a state dependent switching law stabilizing the system.

We summarize the above in the following corollary.

Corollary: Assume that the two descriptor systems in (2) are regular and have no impulsive mode, and A_2 is upper-triangularized (as (24)) by the nonsingular

[1] Here, we use the word "continuous" to mean that if there is a scalar λ_0 satisfying (23) with some (P,S), then any λ in a small neighborhood of λ_0 also satisfies (23) with the same (P,S).

matrices M and N in the Weiertrass form (6) of (E, A_1). If there is a stable convex combination (E, A_λ) of the descriptor systems, then there is a switching law asymptotically stabilizing the switched system.

Remark 6: Although we have focused our attention on continuous-time systems, all the results in this paper can be applied to switched discrete-time descriptor systems with some notation change.

5. CONCLUSIONS

In this paper, we have considered stability analysis and design for switched descriptor systems, focusing on the case where the descriptor matrix and the system matrices are commutative pairwise. We have shown that if all descriptor systems are stable, then the switched system is stable under arbitrary switching, which is a natural extension of the existing result [1] to switched descriptor systems. We have also shown that in the case where none of the descriptor systems is stable, if there is a stable convex combination of the descriptor systems, then we can construct a class of switching laws stabilizing the switched system. This is also a natural and important extension of existing results to switched descriptor systems.

We note finally that there is much space for future research on this line. For stability under arbitrary switching, we suggest that less conservative condition is possible by taking advantage of the descriptor matrix's structure. For stabilization under appropriate switching, we certainly have to consider how to relax the commutation condition in a more practical form in real applications.

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