Applied Mathematics and Computation 372 (2020) 124981

Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Positivity, exponential stability and disturbance attenuation performance for singular switched positive systems with time-varying distributed delays

Shuo Li[a], Zhengrong Xiang[b,*]

[a] School of Automation, Hangzhou Dianzi University, Hangzhou 310018, People's Republic of China
[b] School of Automation, Nanjing University of Science and Technology, Nanjing 210094, People's Republic of China

ARTICLE INFO

Article history:
Received 18 May 2019
Revised 25 October 2019
Accepted 15 December 2019

Keywords:
Singular switched positive systems
Time-varying distributed delays
Positivity
Exponential stability
Disturbance attenuation performance

ABSTRACT

This article analyzes the singular switched positive systems with time-varying distributed delays from the perspective of positivity, exponential stability and disturbance attenuation performance referring to both L_1-gain and L_∞-gain. For the system, a sufficient and necessary positivity condition is firstly developed by using the singular value decomposition technique. Then, a sufficient condition of exponential stability, which makes the considered system exponentially stable, is proposed on the basis of co-positive Lyapunov–Krasovskii functional and average dwell time techniques, and the obtained exponential decay rate can be adjusted in the light of various actual situations. Furthermore, the article analyzes the disturbance attenuation performance referring to both L_1-gain and L_∞-gain, and through the convex optimization approach, the optimal L_1-gain and L_∞-gain performance level could be established, respectively. Three examples are finally presented to show the feasibility and effectiveness of the obtained results.

1. Introduction

In the practical applications, many systems involve non-negative variables such as population level, material concentration, absolute temperature, and others. Such a system is expressed as a positive system [1,2], which often appear in actual fields [3–5]. It is known that many famous results of general systems are not suitable for positive systems, since positive system's states lie in not the whole linear space but the positive orthant. Because the co-positive Lyapunov function(CLF) produces less conservativeness than the quadratic Lyapunov function, the CLF is usually applied in stability analysis of positive systems [6]. Besides, for positive systems, instead of the L_2-gain, the L_1-gain or L_∞-gain is more suitable to be used as the system performance, where L_1-gain represents the sum of quantities, while L_∞-gain represents the maximum of quantities [7]. Positive systems are mainly focused on stability and performance analysis, and many results can be found [8–11].

Switched system consisits of a group of subsystems and a collection of logical rules, which can coordinate the switches between the subsystems to determine which subsystem to be valid [12–19]. When a positive system's parameter undergoes

* Corresponding author.
E-mail addresses: lishuo@hdu.edu.cn (S. Li), xiangzr@mail.njust.edu.cn (Z. Xiang).

https://doi.org/10.1016/j.amc.2019.124981
0096-3003/

the switching-type changes, this positive system can be modelled as a switched positive system, which has been widely utilized in congestion control of network communication systems [20], formation flying control of air traffic systems [21], isolation control of HIV virus in medical systems [22]. The related problems of switched positive systems, especially on stability and stabilization, have attracted the attention of many researchers [23–28].

The above results are aimed at standard switched positive systems, that is, each subsystem is a standard system, however, in many actual engineering like power systems, biological systems, and economic systems, system models are necessary to be described by singular systems, which are generalizations of standard systems [29,30]. Compared with standard ones, singular systems are equipped with many advances, this is mainly due to their capacities to naturally model the real dynamical systems, to represent a wider range of systems than standard state-space systems, and to accurately describe the relationship between variables within the systems [31]. Especially in recent years, singular switched positive systems(SSPSs) have been utilized in numerous important domains like circuit network, computer controlling, communication, chemical reaction and so on. Due to that SSPS is coupling between the switching characteristic of subsystems, the non-negativity of valuables and the singularity of derivative matrix, the research of switched positive system in singular case is more complex than that in standard case. Thus, although SSPSs have attracted wide attentions of scholars, only a few relevant results can be found [32–36], including some results on singular positive systems, where Xia et al. [32] addressed the stability problem for SSPSs, Liu et al. [33] investigated the finite-time stability for discrete-time SSPSs, and [34,35] dealt with the stochastic stability issue for singular positive Markov jump systems.

It is widely recognized that the above mentioned achievements on SSPSs [32–35] are only concerned with the asymptotical stability problem. In practice, only demanding the system's asymptotical stability is often not enough, more hopefully, the system possesses a certain decay rate(DR) to converge quickly. For the purpose of discussing how stability changes with emphasis on the DR, investigating the exponential stability of SSPSs becomes interesting. Generally speaking, the asymptotical stability may converge slowly and be conservative to some extent, while the exponential stability can converge relatively fast with a convergence rate which can be described. When analyzing the exponential stability for the system with singularity constraints, determining the DR becomes the main difficulty. Therefore, our first objective is to study the exponential stability for SSPSs via determining the exponential DR with some effective approaches.

Furthermore, the above results [32–35] only focus on SSPSs without time-varying delays as well as exogenous disturbances, which are commonly unavoidable in practice. Time-varying delays may result in instability and disruption of the performance, in addition, exogenous disturbances may inevitably affect the system's outputs. When considering these two phenomena, choosing an appropriate Lyapunov functional as well as analyzing the positivity, the stability and the disturbance attenuation performance(DAP) for SSPSs become the main difficulties, owing to that delayed SSPSs are coupling between matrix difference equations and matrix delay differential equations, which makes it more complicated and challenging. Besides, time-varying distributed delays(TDDs), as a classical type of time-varying delays, exist in many fields and disciplines like engineering, chemistry and mechanics, meanwhile because of the complex form of distributed delay, how to deal with some mathematical problems in detail in the process of analysis and derivation becomes challenging, and the developed theory on such kind of delays is still not up to a quantitative level. Therefore, the second aim of this article is to investigate SSPSs with TDDs and external disturbances.

Thus, for SSPSs with TDDs, this article considers the issue of positivity, exponential stability as well as DAP referring to both L_1-gain and L_∞-gain, which is practically significant and meanwhile possesses theoretical challenges. The main contributions are: (1) a sufficient and necessary positivity condition is firstly provided for the SSPSs with TDDs via singular value decomposition(SVD) approach; (2) on the basis of co-positive Lyapunov–Krasovskii functional(CLKF) and average dwell time(ADT) techniques, a sufficient condition of exponential stability is derived for the considered SSPSs with TDDs for the first time. Moreover, the obtained exponential DR could be adjusted in the light of various actual situations; (3) the DAP for the SSPSs is analyzed in the sense of both L_1-gain and L_∞-gain, and through convex optimization approach, the optimal L_1-gain and L_∞-gain performance level could be established, respectively.

In the rest of this article: Section 2 presents the preliminaries and problem formulation. For the considered SSPSs, Section 3 addresses the issue of positivity, exponential stability and DAP referring to both L_1-gain and L_∞-gain. Section 4 indicates the correctness of obtained results through three examples. Section 5 finally gives the concluding remarks.

Notations For a matrix A, $A\preceq 0$ and $A\succeq 0$ indicate all elements of A to be non-positive and nonnegative, respectively. For a matrix A, its rank is represented as $rank(A)$, its determinant is denoted as $det(A)$, its degree of a polynomial is expressed as $deg(\cdot)$. The set of all n-dimensional real vectors and positive real vectors is defined as R^n and R^n_+, respectively. A collection of $m \times n$-dimensional real matrices is defined as $R^{m\times n}$. $1_n \in R^n$ means a n-dimensional column vector with all elements being 1. For a vector $x \in R^n$, x_i denotes its ith term, $i \in \underline{n}$; and for a matrix $A \in R^{m\times n}$, $[A]_{ij}$ denotes its (i, j)th term, $i \in \underline{m}$, $j \in \underline{n}$. $\|x\|_1 = \sum_{i=1}^n |x_i|$ is the 1-norm of a vector $x \in R^n$, $\|A\|_1 = max_{1\le j\le n} \sum_{i=1}^m \left|A_{ij}\right|$ is the 1-norm of a matrix $A \in R^{m\times n}$, $\|\nu\|_{L_1} = \int_{t=t_0}^{\infty} \|\nu(t)\|_1 dt$ is the L_1-norm of a vector valued function ν: $R \to R^n$, we say $\nu(t) \in L_1[t_0, \infty)$ if $\|\nu\|_{L_1} < \infty$. $\|x\|_\infty = \max_{1\le i\le n} |x_i|$ is the ∞-norm of a vector $x \in R^n$, $\|A\|_\infty = max_{1\le i\le m} \sum_{j=1}^n \left|A_{ij}\right|$ is the ∞-norm of a matrix $A \in R^{m\times n}$, $\|\nu\|_{L_\infty} = \sup_{t\in[t_0,\infty)} \|\nu(t)\|_\infty$ is the L_∞-norm of a vector valued function ν: $R \to R^n$, we say $\nu(t) \in L_\infty[t_0, \infty)$ if $\|\nu\|_{L_\infty} < \infty$. For a vector $\nu \in R^n$, $\overline{\rho}(\nu)$ denotes the largest entry of ν, $\underline{\rho}(\nu)$ denotes the smallest entry of ν.

2. Preliminaries and problem formulation

The singular switched systems with TDDs is considered as follows:

$$\begin{cases} E\dot{x}(t) = A_{\sigma(t)}x(t) + G_{\sigma(t)}\int_{t-h(t)}^{t} x(s)ds + F_{\sigma(t)}w(t), \\ z(t) = C_{\sigma(t)}x(t) + D_{\sigma(t)}w(t), \\ x(t_0+\theta) = \varphi(\theta), \quad \theta = [-h_u, 0], \end{cases} \tag{1}$$

where $x(t) \in R^n$ denotes the state, $w(t) \in R^u$ denotes the disturbance input, $z(t) \in R^m$ denotes the controlled output. The TDD $h(t)$ satisfying $0 < h(t) \le h_u$ and $\dot{h}(t) \le h_d < 1$ is differentiable everywhere with h_u and h_d known, and $\varphi(\theta)$ is a vector-valued initial function. $\sigma(t) : [0, \infty) \to \underline{N} = \{1, 2, \ldots, N\}$ is the switching rule with N being the total quantity of subsystems, and $\sigma(t) = i \in \underline{N}$ indicates the ith subsystem to be valid. A_i, G_i, F_i, C_i and D_i, $i \in \underline{N}$ are known matrices, E is a singular matrix satisfying $rankE = r < n$. $t_0 = 0$ is the initial time, and t_k is the kth switching instant in this paper.

Definition 1 [36].

1. If each pair (E, A_i) is regular, i.e., $det(sE - A_i) \neq 0$, $\forall i \in \underline{N}$, then system (1) is regular;
2. If each pair (E, A_i) is impulse-free, i.e., $deg(det(sE - A_i)) = rank(E)$, $\forall i \in \underline{N}$, then system (1) is impulse-free.

Definition 2 [11]. For any switching rule $\sigma(t)$, if under any initial condition $\varphi(\theta) \succeq 0$ and disturbance input $w(t) \succeq 0$, the system state $x(t) \succeq 0$ and the system output $z(t) \succeq 0$ are always valid for all $t \ge 0$, then system (1) is called a positive system.

The regularity and impulse-free of system (1) imply the existence of $PEQ = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ with $P \in R^{n \times n}$ and $Q \in R^{n \times n}$ to be non-singular matrices. For the sake of simplicity, in system (1), set $E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ and

$$A_i = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix}, G_i = \begin{bmatrix} G_{i1} & G_{i2} \\ G_{i3} & G_{i4} \end{bmatrix}, F_i = \begin{bmatrix} F_{i1} \\ F_{i2} \end{bmatrix}, C_i = \begin{bmatrix} C_{i1} & C_{i2} \end{bmatrix}, \tag{2}$$

where $\det(A_{i4}) \neq 0$, A_{i1}, $G_{i1} \in R^{r \times r}$, $A_{i2}, G_{i2} \in R^{r\times(n-r)}$, $A_{i3}, G_{i3} \in R^{(n-r)\times r}$, $A_{i4}, G_{i4} \in R^{(n-r)\times(n-r)}$, $F_{i1} \in R^{r \times u}$, $F_{i2} \in R^{(n-r)\times u}$, $C_{i1} \in R^{m \times r}$, $C_{i2} \in R^{m\times(n-r)}$, $\forall i \in \underline{N}$.

Lemma 1 [30]. *System (1) is impulse-free if and only if A_{i4} is a non-singular matrix, i.e., $rank(A_{i4}) = n - r$ or $\det(A_{i4}) \neq 0$, $\forall i \in \underline{N}$.*

Assuming system (1) with (2) to be regular and impulse-free, letting $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ with $x_1(t) \in R^r$ and $x_2(t) \in R^{n-r}$, it follows that, $\forall i \in \underline{N}$,

$$\begin{cases} \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}\begin{pmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{pmatrix} = \begin{bmatrix} A_{i1} & A_{i2} \\ A_{i3} & A_{i4} \end{bmatrix}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + \begin{bmatrix} G_{i1} & G_{i2} \\ G_{i3} & G_{i4} \end{bmatrix}\int_{t-h(t)}^{t}\begin{pmatrix} x_1(s) \\ x_2(s) \end{pmatrix}ds + \begin{bmatrix} F_{i1} \\ F_{i2} \end{bmatrix}w(t), \\ z(t) = \begin{bmatrix} C_{i1} & C_{i2} \end{bmatrix}\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} + D_i w(t), \\ \begin{pmatrix} x_1(t_0+\theta) \\ x_2(t_0+\theta) \end{pmatrix} = \begin{pmatrix} \varphi_1(\theta) \\ \varphi_2(\theta) \end{pmatrix}, \quad \theta = [-h_u, 0], \end{cases} \tag{3}$$

which leads to

$$\begin{cases} \dot{x}_1(t) = A_{i1}x_1(t) + A_{i2}x_2(t) + G_{i1}\int_{t-h(t)}^{t} x_1(s)ds + G_{i2}\int_{t-h(t)}^{t} x_2(s)ds + F_{i1}w(t), \\ 0 = A_{i3}x_1(t) + A_{i4}x_2(t) + G_{i3}\int_{t-h(t)}^{t} x_1(s)ds + G_{i4}\int_{t-h(t)}^{t} x_2(s)ds + F_{i2}w(t), \\ z(t) = C_{i1}x_1(t) + C_{i2}x_2(t) + D_i w(t), \\ x_1(t_0+\theta) = \varphi_1(\theta), \quad \theta = [-h_u, 0], \\ x_2(t_0+\theta) = \varphi_2(\theta), \quad \theta = [-h_u, 0]. \end{cases} \tag{4}$$

According to Lemma 1, since $\det(A_{i4}) \neq 0$, $\forall i \in \underline{N}$, system (4) can be transformed into:

$$\begin{cases} \dot{x}_1(t) = \overline{A}_{i1}x_1(t) + \overline{G}_{i1}\int_{t-h(t)}^{t} x_1(s)ds + \overline{G}_{i2}\int_{t-h(t)}^{t} x_2(s)ds + \overline{F}_{i1}w(t), \\ x_2(t) = \overline{A}_{i3}x_1(t) + \overline{G}_{i3}\int_{t-h(t)}^{t} x_1(s)ds + \overline{G}_{i4}\int_{t-h(t)}^{t} x_2(s)ds + \overline{F}_{i2}w(t), \\ z(t) = C_{i1}x_1(t) + C_{i2}x_2(t) + D_i w(t), \\ x_1(t_0+\theta) = \varphi_1(\theta), \quad \theta = [-h_u, 0], \\ x_2(t_0+\theta) = \varphi_2(\theta), \quad \theta = [-h_u, 0], \end{cases} \tag{5}$$

where

$$\overline{A}_{i1} = A_{i1} - A_{i2}A_{i4}^{-1}A_{i3}, \quad \overline{G}_{i1} = G_{i1} - A_{i2}A_{i4}^{-1}G_{i3},$$

$$\overline{G}_{i2} = G_{i2} - A_{i2}A_{i4}^{-1}G_{i4}, \quad \overline{F}_{i1} = F_{i1} - A_{i2}A_{i4}^{-1}F_{i2},$$

$$\overline{A}_{i3} = -A_{i4}^{-1}A_{i3}, \quad \overline{G}_{i3} = -A_{i4}^{-1}G_{i3}, \quad \overline{G}_{i4} = -A_{i4}^{-1}G_{i4}, \quad \overline{F}_{i2} = -A_{i4}^{-1}F_{i2}.$$

Lemma 2 [36]. *Assuming system (1) to be regular and impulse-free, the positivity of system (1) is equivalent to that of system (5).*

Remark 1. It should be noted that the SVD technique applied in the above process is one of the most useful and effective tools in linear algebra. Compared with other decomposition methods, the SVD approach can be applied to arbitrary matrix without limitations on dimension or rank. By applying the SVD technique, a complex matrix can be decomposed into products of other small matrices with simple structures or familiar properties, and meanwhile these small matrices can still describe the important characteristics of the original matrix. In this paper, based on the SVD method, system (1) with (2) is equivalently transformed into system (5) with simple form and distinct features, which is conducive to the subsequent positivity analysis. Thus, the positivity condition of system (1) can be obtained via the equivalently transformed system (5).

Definition 3 [11]. If there exists a constant $\alpha > 0$ so that, for any switching rule $\sigma(t)$ and any initial condition $x(t_0+\theta) = \varphi(\theta)$, $\theta \in [-h_u, 0]$, $x(t)$ meets

$$\|x(t)\|_1 \le ce^{-\alpha(t-t_0)}\|\varphi\|_{1c} = ce^{-\alpha(t-t_0)} \sup_{-h_u \le s \le 0} \|\varphi(s)\|_1, \quad \forall t \ge 0, \tag{6}$$

then system (1)($w(t) = 0$) is exponentially stable(ES), α and c are called the exponential DR and decay coefficient, respectively.

Definition 4 [12]. For any $T_2 > T_1 \ge 0$, if the following inequality is satisfied,

$$N_\sigma(T_1, T_2) \le N_0 + \frac{(T_2 - T_1)}{T_a}, \tag{7}$$

where $N_\sigma(T_1, T_2)$ means the total switching number of the switched rule $\sigma(t)$ on the interval $[T_1, T_2)$, then $T_a > 0$ is known as ADT, and $N_0 \ge 0$ is known as chattering bound.

This paper chooses $N_0 = 0$ as commonly used in other literatures.

Definition 5. Assuming system (1) to be regular, impulse-free and ES(when $w(t) = 0$), for given scalars $\alpha > 0$ and $\gamma > 0$, under zero initial conditions,

- (i) system (1) possesses an specified L_1-gain performance γ if, $\forall w(t) \in L_1$,

$$e^{-\alpha(t-t_0)}\|z(t)\|_{L_1} \le \gamma\|w(t)\|_{L_1},$$

- (ii) system (1) possesses an specified L_∞-gain performance γ if, $\forall w(t) \in L_\infty$,

$$e^{-\alpha(t-t_0)}\|z(t)\|_{L_\infty} \le \gamma\|w(t)\|_{L_\infty}.$$

The goals of this paper are, for system (1),

1. Derive a sufficient and necessary positivity condition;
2. Develop a set of switching rules $\sigma(t)$ and a sufficient condition of exponential stability;
3. Analyze the DAP referring to both L_1-gain and L_∞-gain;
4. Determine the optimal L_1-gain and L_∞-gain performance level, respectively.

3. Main results

This section is mainly focused on analyzing the positivity, the exponential stability and the DAP for SSPSs.

3.1. Positivity

In this subsection, a necessary and sufficient condition of positivity is presented for system (1).

Theorem 1. *Assuming system (1) to be regular and impulse-free, system (1) with (2) is positive if and only if* $\overline{A}_{i3}\succeq 0$, $\overline{G}_{i1}\succeq 0$, $\overline{G}_{i2}\succeq 0$, $\overline{G}_{i3}\succeq 0$, $\overline{G}_{i4}\succeq 0$, $\overline{F}_{i1}\succeq 0$, $\overline{F}_{i2}\succeq 0$, $C_{i1}\succeq 0$, $C_{i2}\succeq 0$, $D_i\succeq 0$, $\forall i\in \underline{N}$.

Proof. *First, sufficiency is to be proved.*

When $t_0=0$ and $T=0$, $x(T)=x(0)\succeq 0$ can be obviously obtained due to the non-negative initial conditions.

When $t_0=0$ and $T>0$, denote the switching instances as $t_1,\dots t_k, t_{k+1}\dots t_{N_\sigma(t_0,T)}$ on the interval $[t_0, T)$, $k=1,2,3,\dots$. For t belongs to the interval $[t_k,t_{k+1})$, the positivity of $x(t)=[x_1^T(t)\quad x_2^T(t)]^T$ is proven. In light of the Lagrange's formula, from the first equation of system (5), we get

$$exp\left\{-\overline{A}_{\sigma(t_k)1}t\right\}(\dot{x}_1(t)-\overline{A}_{\sigma(t_k)1}x_1(t))=exp\left\{-\overline{A}_{\sigma(t_k)1}t\right\}[\overline{G}_{\sigma(t_k)1}\int_{t-h(t)}^{t}x_1(\tau)d\tau+\overline{G}_{\sigma(t_k)2}\int_{t-h(t)}^{t}x_2(\tau)d\tau+\overline{F}_{\sigma(t_k)1}w(t)], \tag{8}$$

and it leads to

$$\begin{aligned}x_1(t)=&\int_{t_k}^{t}\exp\left\{\overline{A}_{\sigma(t_k)1}(t-s)\right\}\overline{G}_{\sigma(t_k)1}\int_{s-h(s)}^{s}x_1(\tau)d\tau ds\\&+\int_{t_k}^{t}\exp\left\{\overline{A}_{\sigma(t_k)1}(t-s)\right\}\overline{G}_{\sigma(t_k)2}\int_{s-h(s)}^{s}x_2(\tau)d\tau ds\\&+\int_{t_k}^{t}\exp\left\{\overline{A}_{\sigma(t_k)1}(t-s)\right\}\overline{F}_{\sigma(t_k)1}w(s)ds+\exp\left\{\overline{A}_{\sigma(t_k)1}t\right\}x_1(t_0),\end{aligned} \tag{9}$$

if $\overline{G}_{\sigma(t_k)1}(\overline{G}_{\sigma(t_k)2},\overline{F}_{\sigma(t_k)1})\succeq 0$, $\overline{A}_{\sigma(t_k)1}$ is a Metzler matrix, then we have $x_1(t)\succeq 0$, $t\in[t_k,t_{k+1})$, and $x_1(t_k)=x_1(t_k^-)\succeq 0$.

Also, from the second equation of system (5), if $\overline{A}_{\sigma(t_k)3}(\overline{G}_{\sigma(t_k)3},\overline{G}_{\sigma(t_k)4},\overline{F}_{\sigma(t_k)2})\succeq 0$, then $x_2(t)\succeq 0$, $t\in[t_k,t_{k+1})$.

Furthermore, from the third equation of system (5), if $C_{\sigma(t_k)1}(C_{\sigma(t_k)2},D_{\sigma(t_k)})\succeq 0$, then $z(t)\succeq 0$, $t\in[t_k,t_{k+1})$.

Recursively, for any $i\in\underline{N}$, if $\overline{G}_{i1}(\overline{G}_{i2},\overline{F}_{i1},\overline{A}_{i3},\overline{G}_{i3},\overline{G}_{i4},\overline{F}_{i2},C_{i1},C_{i2},D_i)\succeq 0$, $\overline{A}_{i1}$ is a Metzler matrix, we have $x_1(t)\succeq 0$, $x_2(t)\succeq 0$, $z(t)\succeq 0$, $t\in[t_0,T)$.

Then, necessity is to be proved.

Prove by contradiction, along the proof line of Lemma 2 in literature [11], it is easy to draw the conclusion, the details of the intermediate process is omitted here.

Therefore, from these two aspects of sufficiency and necessity, system (5) is positive when and only when $\overline{G}_{i1}(\overline{G}_{i2},\overline{F}_{i1},\overline{A}_{i3},\overline{G}_{i3},\overline{G}_{i4},\overline{F}_{i2},C_{i1},C_{i2},D_i)\succeq 0$, $\overline{A}_{i1}$ is a Metzler matrix, $\forall i\in\underline{N}$. Furthermore, according to Lemma 2, Theorem 1 can be obtained directly.

The proof is completed. □

3.2. Exponential stability

This subsection presents a sufficient delay-dependent condition of exponential stability for system (1)($w(t)=0$).

Theorem 2. *Assume that system (2) is regular and impulse-free, and satisfies conditions in Theorem 1, for a given scalar* $\alpha>0$, *if there exist vectors* ν_i, $\eta_i\in R_+^n$, *so that,* $\forall i\in\underline{N}$,

$$A_i^T\nu_i+\alpha E^T\nu_i+h_u\eta_i\preceq 0, \tag{10}$$

$$G_i^T\nu_i-(1-h_d)e^{-\alpha h_u}\eta_i\preceq 0, \tag{11}$$

$$\|\overline{G}_{i4}\|_1\delta<1, \tag{12}$$

where $\delta=\max\{\tau_1, He^{\beta h_u}\}$, $\tau_1=\tau He^{\beta h_u}(\|\overline{A}_{i3}\|_1+\|\overline{G}_{i3}\|_1)$, $H=\max\{h_u,1\}$, $\tau=\beta_2/\beta_1=(\max_{\forall i\in\underline{N}}\overline{\rho}(\nu_{i1})+h_u^2\max_{\forall i\in\underline{N}}\overline{\rho}(\eta_i))/\min_{\forall i\in\underline{N}}\underline{\rho}(\nu_{i1})$ *with* $\nu_{i1}\in R^r$, *then system (2) is positive and ES, meanwhile the exponential DR is estimated as* $\beta=\alpha-(\ln\mu)/T_a$ *for any switching rule* $\sigma(t)$ *satisfying the ADT condition*

$$T_a>T_a^*=\frac{\ln\mu}{\alpha}, \tag{13}$$

where $\mu \geq 1$ and satisfies

$$\nu_i \preceq \mu\nu_j, \quad \eta_i \preceq \mu\eta_j, \quad \forall i, j \in \underline{N}. \tag{14}$$

Proof. Here, we will prove that system (1) with $w(t) = 0$ is exponentially stable. To prove the exponential stability, an appropriate CLKF candidate is constructed as follows,

$$V_i(t) = V_{1i}(t) + V_{2i}(t), \quad \sigma(t) = i \in \underline{N}, \quad t \in [t_k, t_{k+1}), \tag{15}$$

where

$$\begin{aligned} V_{1i}(t) &= x^T(t)E^T\nu_i, \\ V_{2i}(t) &= \int_{t-h(t)}^{t} (s - (t - h(t)))e^{\alpha(-t+s)}x^T(s)\eta_i ds. \end{aligned}$$

The CLKF candidate (15) is constructed based on the copositive Lyapunov–Krasovskii functional approach, which is commonly used for analyzing the positive delayed systems.

The derivation of $V_{1i}(t)$ and $V_{2i}(t)$ can be obtained as

$$\dot{V}_{1i}(t) = \dot{x}^T(t)E^T\nu_i = [E\dot{x}(t)]^T\nu_i = x^T(t)A_i^T\nu_i + \int_{t-h(t)}^{t} x^T(s)G_i^T\nu_i ds, \tag{16}$$

$$\begin{aligned} \dot{V}_{2i}(t) &= \left[\int_{t-h(t)}^{t} se^{\alpha(-t+s)}x^T(s)\eta_i ds - \int_{t-h(t)}^{t} (t - h(t))e^{\alpha(-t+s)}x^T(s)\eta_i ds\right]' \\ &= \left[e^{-\alpha t}\int_{t-h(t)}^{t} se^{\alpha s}x^T(s)\eta_i ds\right]' - \left[(t - h(t))e^{-\alpha t}\int_{t-h(t)}^{t} e^{\alpha s}x^T(s)\eta_i ds\right]' \\ &= \left[e^{-\alpha t}\right]'\int_{t-h(t)}^{t} se^{\alpha s}x^T(s)\eta_i ds + e^{-\alpha t}\left[\int_{t-h(t)}^{t} se^{\alpha s}x^T(s)\eta_i ds\right]' \\ &\quad -\left[(t - h(t))e^{-\alpha t}\right]'\int_{t-h(t)}^{t} e^{\alpha s}x^T(s)\eta_i ds - (t - h(t))e^{-\alpha t}\left[\int_{t-h(t)}^{t} e^{\alpha s}x^T(s)\eta_i ds\right]' \\ &= -\alpha\int_{t-h(t)}^{t} (s - (t - h(t)))e^{\alpha(-t+s)}x^T(s)\eta_i ds + h(t)x^T(t)\eta_i - \left(1 - \dot{h}(t)\right)\int_{t-h(t)}^{t} x^T(s)e^{\alpha(-t+s)}\eta_i ds, \end{aligned} \tag{17}$$

which lead to

$$\dot{V}_{1i}(t) + \alpha V_{1i}(t) = x^T(t)A_i^T\nu_i + \int_{t-h(t)}^{t} x^T(s)G_i^T\nu_i ds + \alpha x^T(t)E^T\nu_i, \tag{18}$$

$$\begin{aligned} \dot{V}_{2i}(t) + \alpha V_{2i}(t) &= -\alpha\int_{t-h(t)}^{t} (s - (t - h(t)))e^{\alpha(-t+s)}x^T(s)\eta_i ds \\ &\quad + h(t)x^T(t)\eta_i - (1 - \dot{h}(t))\int_{t-h(t)}^{t} x^T(s)e^{\alpha(-t+s)}\eta_i ds \\ &\quad + \alpha\int_{t-h(t)}^{t} (s - (t - h(t)))e^{\alpha(-t+s)}x^T(s)\eta_i ds \\ &= h(t)x^T(t)\eta_i - (1 - \dot{h}(t))\int_{t-h(t)}^{t} x^T(s)e^{\alpha(-t+s)}\eta_i ds \\ &\leq h_u x^T(t)\eta_i - (1 - h_d)e^{-\alpha h_u}\int_{t-h(t)}^{t} x^T(s)\eta_i ds. \end{aligned} \tag{19}$$

Add (18) and (19) into the term $\dot{V}_i(t) + \alpha V_i(t)$, and from (10)–(11), we have

$$\begin{aligned} \dot{V}_i(t) + \alpha V_i(t) &= \dot{V}_{1i}(t) + \alpha V_{1i}(t) + \dot{V}_{2i}(t) + \alpha V_{2i}(t) \\ &\leq x^T(t)\left[A_i^T\nu_i + \alpha E^T\nu_i + h_u\eta_i\right] + \int_{t-h(t)}^{t} x^T(s)\left[G_i^T\nu_i - (1 - h_d)e^{-\alpha h_u}\eta_i\right]ds \\ &\leq 0. \end{aligned} \tag{20}$$

We have from (20) that, $\forall t \in [t_k, t_{k+1})$,

$$V_{\sigma(t)}(t) \le e^{-\alpha(t-t_k)} V_{\sigma(t_k)}(t_k), \tag{21}$$

meanwhile, it is obvious from (14)–(15) that, $\forall t = t_k,\ k = 1, 2, \ldots,$

$$V_{\sigma(t_k)}(t_k) \le \mu V_{\sigma(t_k^-)}(t_k^-), \tag{22}$$

where t_k^- is the left limitation of t_k.

Then, based on ADT relation $N_\sigma(t_0, t) \le \frac{t-t_0}{T_a}$, (13) and (21)–(22) result in

$$V_{\sigma(t)}(t) \le e^{-\alpha(t-t_{k-1})} \mu V_{\sigma(t_{k-1})}(t_{k-1}) \le \cdots \le e^{-\alpha(t-t_0)} \mu^{N_\sigma(t_0,t)} V_{\sigma(t_0)}(t_0) \le e^{-(\alpha - \frac{\ln\mu}{T_a})(t-t_0)} V_{\sigma(t_0)}(t_0). \tag{23}$$

From the other side, denoting $\nu_i = [\nu_{i1}^T \quad \nu_{i2}^T]^T$, $\nu_{i1} \in R^r$, functional (15) will lead to

$$V_{1i}(t) = x^T(t) E^T \nu_i = \begin{bmatrix} x_1^T(t) & x_2^T(t) \end{bmatrix} \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \nu_{i1} \\ \nu_{i2} \end{bmatrix} = x_1^T(t) \nu_{i1}. \tag{24}$$

According to (15) and (24), it is easy to get

$$V_{\sigma(t)}(t) \ge \beta_1 \|x_1(t)\|_1, \tag{25}$$

$$\begin{aligned} V_{\sigma(t_0)}(t_0) &\le \max_{\forall i \in \underline{N}} \overline{\rho}(\nu_{i1}) \|x_1(t_0)\|_1 + \max_{\forall i \in \underline{N}} \overline{\rho}(\eta_i) h_u \int_{t_0-h_u}^{t_0} \|x(s)\|_1 ds \\ &= \varepsilon_2 \|x_1(t_0)\|_1 + \varepsilon_5 h_u \int_{t_0-h_u}^{t_0} \|x(s)\|_1 ds \\ &\le (\varepsilon_2 + \varepsilon_5 h_u^2) \sup_{-h_u \le \theta \le 0} \|x(t_0+\theta)\|_1 \\ &= \beta_2 \|\varphi\|_{1c}, \end{aligned} \tag{26}$$

where $\|\varphi\|_{1c} = \sup_{-h_u \le \theta \le 0} \|x(t_0+\theta)\|_1$, β_1 and β_2 have been defined in Theorem 2.

Then, combing (23) and (25)–(26) leads to, $\forall t \ge 0$,

$$\|x_1(t)\|_1 \le \tau e^{-\beta(t-t_0)} \|\varphi\|_{1c}, \tag{27}$$

where τ and β have been defined in Theorem 2. Moreover, condition (13) yields $\beta > 0$, which indicates the first component solution $x_1(t)$ is ES, and the exponential DR is estimated as β. Also, (27) yields, $\forall t \ge 0$,

$$\|x_1(t)\|_1 \le \tau e^{-\beta t} \|\varphi\|_{1c} \le \tau e^{\beta h_u} \|\varphi\|_{1c} e^{-\beta t}. \tag{28}$$

Furthermore, $x_2(t)$ is proven to be ES, and its exponential DR is the same as that of $x_1(t)$. From (28), we can get

$$\int_{t-h(t)}^{t} \|x_1(s)\|_1 ds \le \int_{t-h(t)}^{t} \tau e^{-\beta s} \|\varphi\|_{1c} ds \le \tau h_u e^{\beta h_u} \|\varphi\|_{1c} e^{-\beta t}, \quad \forall t > h(t), \tag{29}$$

$$\int_{t-h(t)}^{t} \|x_1(s)\|_1 ds = \|\varphi_1\|_{1c} \le \tau e^{\beta h_u} \|\varphi\|_{1c} e^{-\beta t}, \quad \forall t \in [0, h(t)]. \tag{30}$$

Together (29)–(30) yield

$$\int_{t-h(t)}^{t} \|x_1(s)\|_1 ds \le \tau H e^{\beta h_u} \|\varphi\|_{1c} e^{-\beta t}, \quad \forall t \ge 0, \tag{31}$$

where $H = \max\{h_u, 1\}$.

Let $p(t) = \overline{A}_{i3} x_1(t) + \overline{G}_{i3} \int_{t-h(t)}^{t} x_1(s) ds$. Substituting (28) and (31) into the term $p(t)$, we get, $\forall t \ge 0$,

$$\| p(t)\|_1 = \| \overline{A}_{i3} x_1(t) + \overline{G}_{i3} \int_{t-h(t)}^{t} x_1(s) ds \|_1 \le \| \overline{A}_{i3} \|_1 \| x_1(t) \|_1 + \| \overline{G}_{i3} \|_1 \int_{t-h(t)}^{t} \| x_1(s) \|_1 ds \le \tau_1 \| \varphi \|_{1c} e^{-\beta t}, \tag{32}$$

where τ_1 has been defined in Theorem 2.

On the other hand, plugging the term $p(t)$ into the second equation of system (5) yields

$$x_2(t) = \overline{G}_{i4} \int_{t-h(t)}^{t} x_2(s) ds + p(t), \tag{33}$$

which indicates, $\forall t \geq 0$,

$$\| x_2(t)\|_1 =\| \overline{G}_{i4} \int_{t-h(t)}^{t} x_2(s)ds + p(t)\|_1 \leq \| \overline{G}_{i4}\|_1 \int_{t-h(t)}^{t} \| x_2(s)\|_1 ds + \| p(t)\|_1. \tag{34}$$

For $t \in [0, h(t)]$, we have from (32) and (34) that

$$\begin{aligned}\|x_2(t)\|_1 &\leq \|\overline{G}_{i4}\|_1 \|\varphi\|_{1c} + \|p(t)\|_1 \\ &\leq \|\overline{G}_{i4}\|_1 \|\varphi\|_{1c} e^{-\beta(t-h(t))} + \|p(t)\|_1 \\ &\leq \|\overline{G}_{i4}\|_1 e^{\beta h_u} \|\varphi\|_{1c} e^{-\beta t} + \tau_1 \|\varphi\|_{1c} e^{-\beta t} \\ &\leq (\|\overline{G}_{i4}\|_1 \delta + \delta) \|\varphi\|_{1c} e^{-\beta t},\end{aligned} \tag{35}$$

where δ has been defined in Theorem 2.

For $t \in [h(t), 2h(t)]$, from (32) and (34)–(35), we get

$$\begin{aligned}\|x_2(t)\|_1 &\leq \|\overline{G}_{i4}\|_1 \int_{t-h(t)}^{t} (\|\overline{G}_{i4}\|_1 \delta + \delta) \|\varphi\|_{1c} e^{-\beta s} ds + \|p(t)\|_1 \\ &\leq \|\overline{G}_{i4}\|_1 (\|\overline{G}_{i4}\|_1 \delta + \delta) h_u e^{\beta h_u} \|\varphi\|_{1c} e^{-\beta t} + \tau_1 \|\varphi\|_{1c} e^{-\beta t} \\ &\leq (\|\overline{G}_{i4}\|_1^2 \delta^2 + \|\overline{G}_{i4}\|_1 \delta^2 + \delta) \|\varphi\|_{1c} e^{-\beta t}.\end{aligned} \tag{36}$$

For any $t \in [(l-1)h(t), lh(t)]$, suppose that

$$\|x_2(t)\|_1 \leq (\|\overline{G}_{i4}\|_1^l \delta^l + \|\overline{G}_{i4}\|_1^{l-1} \delta^l + \|\overline{G}_{i4}\|_1^{l-2} \delta^{l-1} + \cdots + \|\overline{G}_{i4}\|_1 \delta^2 + \delta) \|\varphi\|_{1c} e^{-\beta t}, \tag{37}$$

then, through the inductive supposition method, when $t \in [lh(t), (l+1)h(t)]$, we have from (32), (34) and (37) that

$$\begin{aligned}\|x_2(t)\|_1 &\leq \|\overline{G}_{i4}\|_1 \int_{t-h(t)}^{t} \|x_2(s)\|_1 ds + \|p(t)\|_1 \\ &\leq \|\overline{G}_{i4}\|_1 \int_{t-h(t)}^{t} (\|\overline{G}_{i4}\|_1^l \delta^l + \|\overline{G}_{i4}\|_1^{l-1} \delta^l + \cdots + \|\overline{G}_{i4}\|_1 \delta^2 + \delta) \|\varphi\|_{1c} e^{-\beta s} ds + \|p(t)\|_1 \\ &\leq \|\overline{G}_{i4}\|_1 (\|\overline{G}_{i4}\|_1^l \delta^l + \|\overline{G}_{i4}\|_1^{l-1} \delta^l + \cdots + \|\overline{G}_{i4}\|_1 \delta^2 + \delta) h_u e^{\beta h_u} \|\varphi\|_{1c} e^{-\beta t} + \tau_1 \|\varphi\|_{1c} e^{-\beta t} \\ &\leq (\|\overline{G}_{i4}\|_1^{l+1} \delta^{l+1} + \|\overline{G}_{i4}\|_1^{l} \delta^{l+1} + \|\overline{G}_{i4}\|_1^{l-1} \delta^{l} + \cdots + \|\overline{G}_{i4}\|_1 \delta^2 + \delta) \|\varphi\|_{1c} e^{-\beta t}.\end{aligned} \tag{38}$$

If $\|\overline{G}_{i4}\|_1 \delta < 1$, through iterative method, we can get

$$\begin{aligned}\|x_2(t)\|_1 &\leq [1 + \|\overline{G}_{i4}\|_1 \delta + \cdots + \|\overline{G}_{i4}\|_1^l \delta^l + \cdots] \delta \|\varphi\|_{1c} e^{-\beta t} \\ &\leq \frac{\delta}{1 - \|\overline{G}_{i4}\|_1 \delta} \|\varphi\|_{1c} e^{-\beta t}.\end{aligned} \tag{39}$$

Finally, together (27) and (39) lead to the conclusion that system (1)($w(t) = 0$) is ES with DR β on the basis of Definition 3.

The proof is completed. □

Remark 2. To prove the exponential stability, a Lyapunov–Krasovskii functional based approach, rather than the conventional Razumikhin method and the Halanay inequality method, is utilized here. In the literature [37], by solving a complex function, the exponential DR is calculated as a fixed value, which is with a certain limitation in practical applications, to overcome this weakness, as shown in Theorem 2, by embedding a specific exponential term into the functional (13), the exponential DR is obtained as a free parameter, which could be selected based on diverse circumstances. In addition, once the ADT T_a is selected, different DR β can be obtained by tuning α and μ, and a larger β can be achieved if a larger value of α or a smaller value of μ is chosen. All these features can truly introduce many flexibility on exponential stability analysis of SSPSs.

Remark 3. If there exists no parameter switching in system (1), i.e., $N = 1$, the corresponding exponential stability condition can be obtained by removing (13)–(14). In such a circumstance, the exponential DR β is merely concerned with α, which can also be adjusted based on various situations.

The exponential stability condition in the non-switching case is provided in the following corollary.

Corollary 1. *Assume that system (1)($N = 1$) is regular and impulse-free, and satisfies conditions in Theorem 1, for a given scalar $\alpha > 0$, if there exist vectors ν, $\eta \in R_+^n$, so that,*

$$\begin{aligned}A^T \nu + \alpha E^T \nu + h_u \eta &\preceq 0, \\ G^T \nu - (1 - h_d) e^{-\alpha h_u} \eta &\preceq 0, \\ \|\overline{G}_4\|_1 \delta &< 1,\end{aligned}$$

where $\delta = \max\{\tau_1, He^{\beta h_u}\}$, $\tau_1 = \tau He^{\beta h_u}(\|\overline{A}_3\|_1 + \|\overline{G}_3\|_1)$, $H = \max\{h_u, 1\}$, $\tau = \beta_2/\beta_1 = (\overline{\rho}(\nu_1) + h_u^2\overline{\rho}(\eta))/\underline{\rho}(\nu_1)$ with $\nu_1 \in R^r$, then system (1)($w(t) = 0$, $N = 1$) is positive and ES, meanwhile the exponential DR is estimated as $\beta = \alpha$.

When $h(t) = 0$ in system (1), that is, system (1) becomes a delay-free case, a corresponding exponential stability condition is proposed.

Corollary 2. *Assume that system (1) is regular and impulse-free, and satisfies conditions in Theorem 1, for a given scalar $\alpha > 0$, if there exist vectors $\nu_i \in R^n_+$, so that, $\forall i \in \underline{N}$,*

$$A_i^T\nu_i + \alpha E^T\nu_i \preceq 0, \tag{40}$$

then system (1)($h(t) = 0$, $w(t) = 0$) is positive and ES, meanwhile the exponential DR is estimated as $\beta = \alpha - (\ln\mu)/T_a$ for any switching rule $\sigma(t)$ satisfying the ADT condition

$$T_a > T_a^* = \frac{\ln\mu}{\alpha}, \tag{41}$$

where $\mu \geq 1$ and satisfies

$$\nu_i \preceq \mu\nu_j, \quad \forall i, j \in \underline{N}. \tag{42}$$

3.3. Disturbance attenuation performance

In this subsection, we mainly focus on analyzing the DAP referring to both L_1-gain and L_∞-gain for system (1).

Theorem 3. *Assume that system (1) is regular and impulse-free, and satisfies conditions in Theorem 1, for given scalars $\alpha > 0$ and $\gamma > 0$, if there exist vectors ν_i, $\eta_i \in R^n_+$, so that, $\forall i \in \underline{N}$,*

$$A_i^T\nu_i + \alpha E^T\nu_i + h_u\eta_i + C_i^T 1_m \preceq 0, \tag{43}$$

$$G_i^T\nu_i - (1 - h_d)e^{-\alpha h_u}\eta_i \preceq 0, \tag{44}$$

$$F_i^T\nu_i + D_i^T 1_m - \gamma 1_u \preceq 0, \tag{45}$$

$$\|\overline{G}_{i4}\|_1\delta < 1, \tag{46}$$

where δ is defined the same as in Theorem 2, then, for any switching rule $\sigma(t)$ satisfying (13), system (1) is positive and ES, meanwhile possesses a specified L_1-gain performance level γ.

Proof. First, for $w(t) = 0$, system (1) can be directly proved to be positive and ES based on Theorem 2, since (43)–(44) imply (10)–(12).

Next, for any $w(t) \in L_1$, system (1) satisfying an specified L_1-gain performance level is the only thing to be proved. Constructing functional (15), and along the proof line of Theorem 2, we get

$$\dot{V}_{1i}(t) + \alpha V_{1i}(t) = x^T(t)A_i^T\nu_i + \alpha x^T(t)E^T\nu_i + \int_{t-h(t)}^t x^T(s)G_i^T\nu_i ds + w^T(t)F_i^T\nu_i, \tag{47}$$

$$\begin{aligned}\dot{V}_{2i}(t) + \alpha V_{2i}(t) &= -\alpha\int_{t-h(t)}^t (s - (t - h(t)))e^{\alpha(-t+s)}x^T(s)\eta_i ds \\ &\quad + h(t)x^T(t)\eta_i - (1 - \dot{h}(t))\int_{t-h(t)}^t x^T(s)e^{\alpha(-t+s)}\eta_i ds \\ &\quad + \alpha\int_{t-h(t)}^t (s - (t - h(t)))e^{\alpha(-t+s)}x^T(s)\eta_i ds \\ &\leq h_u x^T(t)\eta_i - (1 - h_d)e^{-\alpha h_u}\int_{t-h(t)}^t x^T(s)\eta_i ds.\end{aligned} \tag{48}$$

Letting $\Theta(t) = \|z(t)\|_1 - \gamma\|w(t)\|_1$, and adding (47)–(48) into the following term, (43)–(45) leads to

$$\begin{aligned}&\dot{V}_i(t) + \alpha V_i(t) + \Theta(t) \\ &\leq x^T(t)\left[A_i^T\nu_i + \alpha E^T\nu_i + h_u\eta_i\right] + w^T(t)F_i^T\nu_i + \int_{t-h(t)}^t x^T(s)\left[G_i^T\nu_i - (1 - h_d)e^{-\alpha h_u}\eta_i\right]ds + z^T(t)1_m - \gamma w^T(t)1_u \\ &\leq x^T(t)\left[A_i^T\nu_i + \alpha E^T\nu_i + h_u\eta_i + C_i^T 1_m\right] + \int_{t-h(t)}^t x^T(s)\left[G_i^T\nu_i - (1 - h_d)e^{-\alpha h_u}\eta_i\right]ds + w^T(t)\left[F_i^T\nu_i + D_i^T 1_m - \gamma 1_u\right] \\ &\leq 0.\end{aligned} \tag{49}$$

Then, for any $t \in [t_k, t_{k+1})$, (49) yields

$$V_{\sigma(t)}(t) \le e^{-\alpha(t-t_k)} V_{\sigma(t_k)}(t_k) - \int_{t_k}^{t} e^{-\alpha(t-s)} \Theta(s) ds. \tag{50}$$

And at the switching time t_k, from (13)–(14), we get

$$V_{\sigma(t_k)}(t_k) \le \mu V_{\sigma t_k^-}(t_k^-), \quad \forall k = 1, 2, \ldots \tag{51}$$

Under the zero initial conditions, combing (50)–(51) yileds

$$0 \le -\int_{t_0}^{t} e^{-\alpha(t-s)+N_\sigma(s,t)\ln\mu} \Theta(s) ds. \tag{52}$$

Multiplying two sides of (52) by $e^{-N_\sigma(t_0,t)\ln\mu}$ leads to

$$\int_{t_0}^{t} e^{-\alpha(t-t_0)} \|z(s)\|_1 ds \le \gamma \int_{t_0}^{t} e^{-\alpha(t-s)} \|w(s)\|_1 ds. \tag{53}$$

Integrate two sides of (53) from $t = t_0$ to ∞, then we get

$$\int_{t_0}^{\infty} e^{-\alpha(t-t_0)} \|z(t)\|_1 dt \le \gamma \int_{t_0}^{\infty} \|w(t)\|_1 dt, \tag{54}$$

on the basis of the definition of L_1-norm, it leads to

$$e^{-\alpha(t-t_0)} \|z(t)\|_{L_1} \le \gamma \|w(t)\|_{L_1}. \tag{55}$$

The proof is completed. □

Remark 4. When $\mu = 1$ in Theorem 3, the L_1-gain performance (55) will reduce into a standard L_1-gain performance as

$$\|z(t)\|_{L_1} \le \gamma \|w(t)\|_{L_1}.$$

Problem 1. By solving feasible conditions (43)–(45) and (13)–(14) in Theorem 3, the DAP referring to L_1-gain can be analyzed, and it should be noted that these conditions are convex in regard to scalar γ, therefore, for given scalars α, h_u and h_d, though the convex optimization problem below, the optimal L_1-gain performance level γ is obtained:

$$\min_{\nu_i, \eta_i} \gamma$$

$$s.t. \quad (43) - (46), (13) - (14), \quad \forall i \in \underline{N}.$$

Furthermore, a sufficient L_∞-gain performance condition is proposed for system (1).

Theorem 4. *Assume that system (1) is regular and impulse-free, and satisfies the conditions in Theorem 1, for given scalars $\alpha > 0$ and $\gamma > 0$, if there exist vectors ν_i, $\eta_i \in R_+^n$, so that, $\forall i \in \underline{N}$,*

$$A_i^T \nu_i + \alpha E^T \nu_i + h_u \eta_i + C_i^T 1_m \preceq 0, \tag{56}$$

$$G_i^T \nu_i - (1 - h_d) e^{-\alpha h_u} \eta_i \preceq 0, \tag{57}$$

$$1_u^T F_i^T \nu_i + 1_u^T D_i^T 1_m - \gamma 1_u^T \preceq 0, \tag{58}$$

$$\|\overline{G}_{i4}\|_1 \delta < 1, \tag{59}$$

where δ is defined the same as in Theorem 2, then, for any switching rule $\sigma(t)$ satisfying (13), system (1) is positive and ES, meanwhile posseses a specified L_∞-gain performance level $\overline{\gamma} = \gamma/\alpha$.

Proof. First, for $w(t) = 0$, system (1) can be directly proved to be positive and ES based on Theorem 2, since (56)–(57) imply (10)–(12).

Next, for any $w(t) \in L_\infty$, system (1) satisfying a specified L_∞-gain performance level is the only thing to be proved. System (1) yileds

$$\begin{aligned} \|z(t)\|_\infty &\le \|C_i x(t)\|_\infty + \|D_i w(t)\|_\infty \\ &\le \|C_i x(t)\|_1 + \|D_i w(t)\|_1 \\ &\le x^T(t) C_i^T 1_m + \|w(t)\|_\infty 1_u^T D_i^T 1_m. \end{aligned} \tag{60}$$

Constructing functional (15) and letting $\Lambda(t) = \|z(t)\|_\infty - \gamma\|w(t)\|_\infty$, (56)–(58) leads to

$$\begin{aligned}\dot{V}_i(t) + \alpha V_i(t) + \Lambda(t) &\le x^T(t)[A_i^T\nu_i + \alpha E^T\nu_i + h_u\eta_i] + w^T(t)F_i^T\nu_i \\ &\quad + \int_{t-h(t)}^{t} x^T(s)[G_i^T\nu_i - (1-h_d)e^{-\alpha h_u}\eta_i]ds \\ &\quad + x^T(t)C_i^T 1_m + \|w(t)\|_\infty 1_u^T D_i^T 1_m - \gamma\|w(t)\|_\infty 1_u^T \\ &\le x^T(t)[A_i^T\nu_i + \alpha E^T\nu_i + h_u\eta_i + C_i^T 1_m] \\ &\quad + \int_{t-h(t)}^{t} x^T(s)[G_i^T\nu_i - (1-h_d)e^{-\alpha h_u}\eta_i]ds \\ &\quad + \|w(t)\|_\infty[1_u^T F_i^T\nu_i + 1_u^T D_i^T 1_m - \gamma 1_u^T] \\ &\le 0. \end{aligned} \tag{61}$$

Along the proof process of (50)–(53), we can get

$$\int_{t_0}^{t} e^{-\alpha(t-t_0)}\|z(s)\|_\infty ds \le \gamma\int_{t_0}^{t} e^{-\alpha(t-s)}\|w(s)\|_\infty ds, \tag{62}$$

which implies

$$\|z(t)\|_\infty \le \gamma\int_{t_0}^{t} e^{\alpha(s-t_0)}\|w(s)\|_\infty ds. \tag{63}$$

Besides,

$$\|w(t)\|_\infty \le \sup_{t\in[t_0,\infty)}\|w(t)\|_\infty, \tag{64}$$

then, (63)–(64) lead to

$$\| z(t)\|_\infty \le \gamma \sup_{t\in[t_0,\infty)} \| w(t)\|_\infty \int_{t_0}^{t} e^{\alpha(s-t_0)}ds \le \frac{\gamma}{\alpha}\left(e^{\alpha(t-t_0)} - 1\right)\sup_{t\in[t_0,\infty)} \| w(t)\|_\infty \le \frac{\gamma}{\alpha}e^{\alpha(t-t_0)}\sup_{t\in[t_0,\infty)} \| w(t)\|_\infty, \tag{65}$$

which means that

$$\sup_{t\in[t_0,\infty)}\|z(t)\|_\infty \le \frac{\gamma}{\alpha}e^{\alpha(t-t_0)}\sup_{t\in[t_0,\infty)}\|w(t)\|_\infty. \tag{66}$$

Then, it leads to

$$e^{-\alpha(t-t_0)}\sup_{t\in[t_0,\infty)}\|z(t)\|_\infty \le \frac{\gamma}{\alpha}\sup_{t\in[t_0,\infty)}\|w(t)\|_\infty,$$

according to the definition of L_∞-norm, it follows

$$e^{-\alpha(t-t_0)}\|z(t)\|_{L_\infty} \le \overline{\gamma}\|w(t)\|_{L_\infty},$$

where $\overline{\gamma} = \gamma/\alpha$.

The proof is completed. □

Problem 2. By solving feasible conditions (56)–(58) and (13)–(14) in Theorem 4, the DAP referring to L_∞-gain can be analyzed, and it should be noted that these conditions are convex in regard to scalar γ, therefore, for given scalars α, h_u and h_d, through the convex optimization problem below, the optimal L_∞-gain performance level $\overline{\gamma}$ is obtained:

$$\min_{\nu_i,\eta_i} \overline{\gamma}$$

$$s.t. \quad (56)-(59), (13)-(14), \quad \forall i \in \underline{N}.$$

If no TDDs exist in system (1), i.e., $h(t)=0$, the L_1-gain and L_∞-gain performance are analyzed, respectively.

Corollary 3. *Assume that system (1) is regular and impulse-free, and satisfies the conditions in Theorem 1, for given scalars $\alpha > 0$ and $\gamma > 0$, if there exist vectors $\nu_i \in R^n_+$, so that, $\forall i \in \underline{N}$,*

$$A_i^T \nu_i + \alpha E^T \nu_i + C_i^T 1_m \preceq 0, \tag{67}$$

$$F_i^T \nu_i + D_i^T 1_m - \gamma 1_u \preceq 0, \tag{68}$$

then, for any switching rule $\sigma(t)$ satisfying (41), system (1)($h(t)=0$) is positive and ES, meanwhile possesses a specified L_1-gain performance level γ.

Corollary 4. *Assume that system (1) is regular and impulse-free, and satisfies the conditions in Theorem 1, for given scalars $\alpha > 0$ and $\gamma > 0$, if there exist vectors $\nu_i \in R^n_+$, so that, $\forall i \in \underline{N}$,*

$$A_i^T \nu_i + \alpha E^T \nu_i + C_i^T 1_m \preceq 0, \tag{69}$$

$$1_u^T F_i^T \nu_i + 1_u^T D_i^T 1_m - \gamma 1_u^T \preceq 0, \tag{70}$$

then, for any switching rule $\sigma(t)$ satisfying (41), system (1)($h(t)=0$) is positive and ES, meanwhile possesses a specified L_∞-gain performance level $\overline{\gamma} = \gamma/\alpha$.

4. Numerical examples

Three examples are presented to exhibit the feasibility of developed results.

Example 1. Consider system (1) possessing two subsystems,

$$A_1 = \begin{bmatrix} 0.2 & -0.4 \\ 0.1 & -0.6 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, \quad D_1 = 0.1,$$
$$A_2 = \begin{bmatrix} 0.1 & -0.3 \\ 0.2 & -1.2 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.2 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, \quad D_2 = 0.2,$$

and $E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$. By direct calculation, we have

$$\begin{aligned} &\det(sE - A_1) = 0.6s - 0.08 \neq 0, \quad \text{for} \quad s = 0, \\ &\det(sE - A_2) = 1.2s - 0.06 \neq 0, \quad \text{for} \quad s = 0, \\ &\deg(\det(sE - A_1)) = \deg(0.6s - 0.08) = \text{rank}E = 1, \\ &\deg(\det(sE - A_2)) = \deg(1.2s - 0.06) = \text{rank}E = 1. \end{aligned}$$

Hence, system (1) is regular and impulse-free, and by the definitions in system (5), we can get

$$\begin{aligned} &\overline{A}_{11} = 0.1333, \quad \overline{G}_{11} = 0.1, \quad \overline{G}_{12} = 0.0333, \quad \overline{F}_{11} = 0.0333, \\ &\overline{A}_{13} = 0.1667, \quad \overline{G}_{13} = 0, \quad \overline{G}_{14} = 0.1667, \quad \overline{F}_{12} = 0.1667, \\ &\overline{A}_{21} = 0.0500, \quad \overline{G}_{21} = 0.1, \quad \overline{G}_{22} = 0.0500, \quad \overline{F}_{21} = 0.1750, \\ &\overline{A}_{23} = 0.1667, \quad \overline{G}_{23} = 0, \quad \overline{G}_{24} = 0.1667, \quad \overline{F}_{22} = 0.0833. \end{aligned}$$

Thus, system (1) with (2) is a positive system according to Theorem 1.

Setting $\alpha = 0.5$ and $h(t) = 0.2 + 0.1\sin t$, solving Problem 1 leads to

$$\nu_1 = \begin{bmatrix} 1.2912 \\ 0.3484 \end{bmatrix}, \nu_2 = \begin{bmatrix} 1.0790 \\ 0.2231 \end{bmatrix}, \eta_1 = \begin{bmatrix} 0.4469 \\ 0.6110 \end{bmatrix}, \eta_2 = \begin{bmatrix} 0.3765 \\ 0.5947 \end{bmatrix},$$

and $\gamma^* = 0.4403$. By analysis, we can verify that $\|\overline{G}_{i4}\|_1 \delta = 0.1682 < 1$, and in this case, $\mu = 1.5983$, $T_a^* = 0.6765$. When we choose $\mu = 1.6$ and $T_a = 1.0$, the corresponding exponential DR can be determined as $\beta = 0.03$. Therefore, in light of the results in Theorem 3, system (1) is ES and meanwhile possesses an optimum $\gamma^* = 0.4403$ under switching rules fulfilling $T_a > 0.6765$.

Figs. 1–3 show the simulation results, where $x(0) = [1 \quad 2]^T$, $x(t) = [0 \quad 0]^T$, $t = [-0.3, 0)$, denotes the initial condition, and $w(t) = 0.5e^{-0.5t}$ is the external disturbance. Fig. 1 plots the switching rule $\sigma(t)$; The system's state x and output z

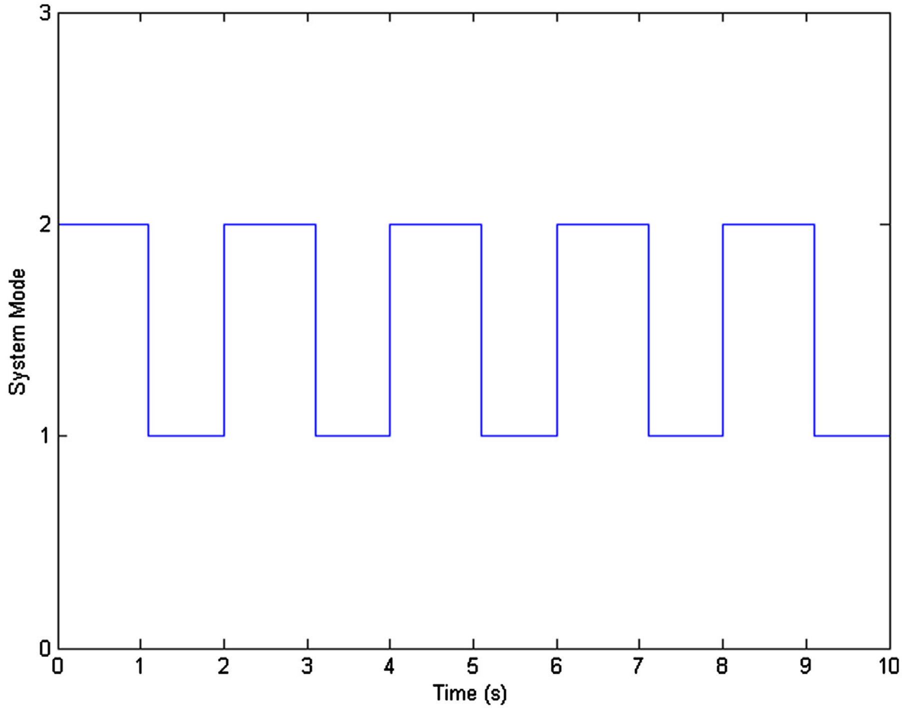

Fig. 1. Switching rule $\sigma(t)$.

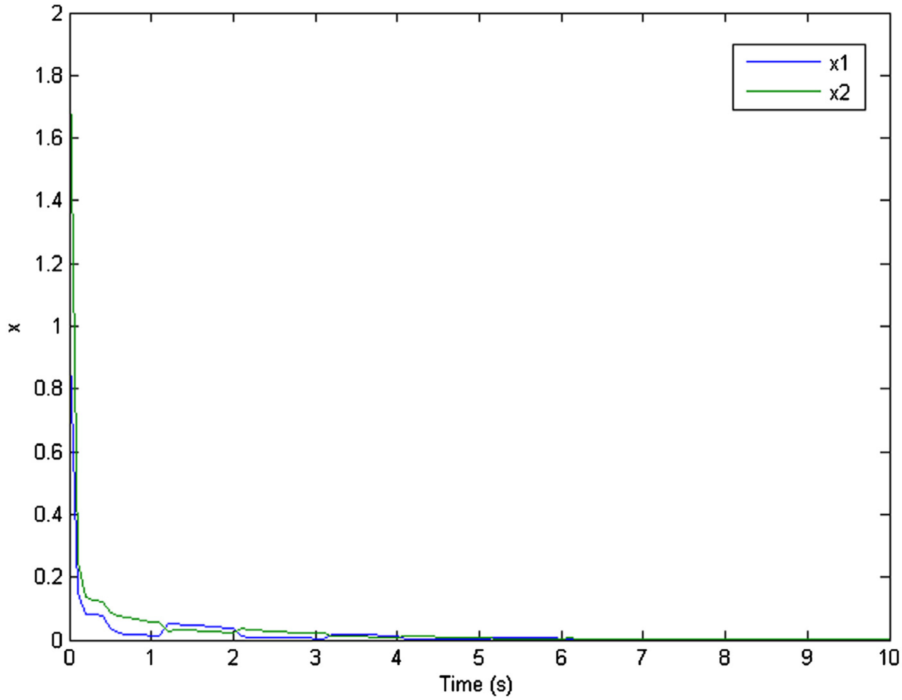

Fig. 2. The state $x(t)$.

Table 1
Relationship of α and γ^* with $\mu = 1.6$.

α	0.4	0.45	0.5	0.55	0.6	0.65
T_a^*	0.5129	0.5886	0.6781	0.7862	0.9201	1.0910
γ^*	0.5055	0.4582	0.4403	0.3834	0.3350	0.3068

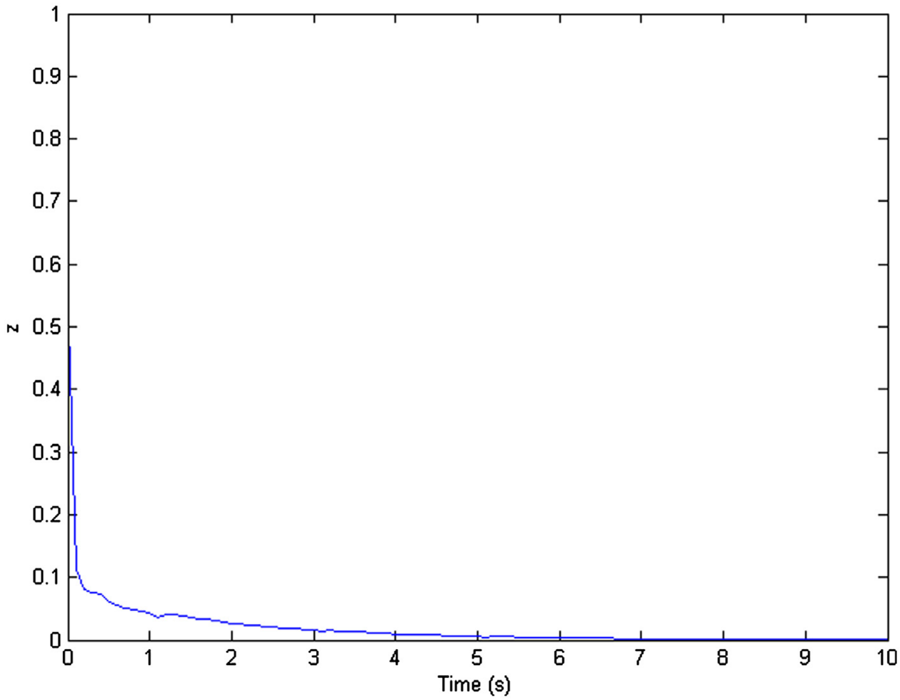

Fig. 3. The output $z(t)$.

Table 2
Relationship of h_u and γ^* with $\alpha = 0.5$, $\mu = 1.6$, and $h_d = 0.1$.

h_u	0.1	0.15	0.2	0.25	0.3	0.35
γ^*	0.3246	0.3664	0.3800	0.4295	0.4403	0.4514

are exhibited in Figs. 2 and 3, respectively. From Figs. 1–3, system (1) is obviously positive and ES, which illustrates the correctness of the obtained results.

By choosing various α and μ, the relationship between T_a^* and γ^* will be shown in the following Tables 1 and 2. Firstly, Table 1 gives the relationship between α and γ^* in the case that $\mu = 1.6$, $h_u = 0.3$, $h_d = 0.1$. As indicated in Table 1, the faster and more frequent the system's switching is, the worse the L_1-gain performance level is. Furthermore, Table 2 gives the relationship between h_u and γ^* in the case that $\alpha = 0.5$, $\mu = 1.6$, and $h_d = 0.1$. It is shown from Table 2 that if a smaller delay bound h_u is chosen, a better L_1-gain performance level is obtained.

Example 2. Consider system (1) possessing two subsystems,

$$A_1 = \begin{bmatrix} -0.4 & 0.16 & 0.7 \\ 0.16 & -0.5 & 0.8 \\ 0.1 & 0.1 & -1.0 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 0.1 & 0 & 0.2 \\ 0 & 0.2 & 0.3 \\ 0 & 1.0 & 0.1 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.1 & 0.1 & 0.2 \end{bmatrix}, \quad D_1 = 0.1,$$

$$A_2 = \begin{bmatrix} -0.6 & 0.15 & 0.8 \\ 0.15 & -0.6 & 0.9 \\ 0.1 & 0.1 & -1.0 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.2 & 0 & 0.3 \\ 0 & 0.1 & 0.2 \\ 0 & 1.0 & 0.2 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.2 & 0.2 & 0.1 \end{bmatrix}, \quad D_2 = 0.1,$$

and $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. By direct calculation, we have

$$\begin{aligned}
&\det(sE - A_1) = s^2 + 0.75s + 0.0834 \neq 0, \quad \text{for} \quad s = 0, \\
&\det(sE - A_2) = s^2 + 1.03s + 0.306 \neq 0, \quad \text{for} \quad s = 0, \\
&\deg(\det(sE - A_1)) = \deg(s^2 + 0.75s + 0.0834) = \operatorname{rank} E = 2, \\
&\deg(\det(sE - A_2)) = \deg(s^2 + 1.03s + 0.306) = \operatorname{rank} E = 2.
\end{aligned}$$

Hence, system (1) is regular and impulse-free, and by the definitions in system (5), we can get

$$\bar{A}_{11} = \begin{bmatrix} -0.33 & 0.23 \\ 0.24 & -0.42 \end{bmatrix}, \quad \bar{G}_{11} = \begin{bmatrix} 0.1 & 0.7 \\ 0 & 1.0 \end{bmatrix}, \quad \bar{G}_{12} = \begin{bmatrix} 0.27 \\ 0.38 \end{bmatrix}, \quad \bar{F}_{11} = \begin{bmatrix} 0.17 \\ 0.28 \end{bmatrix},$$

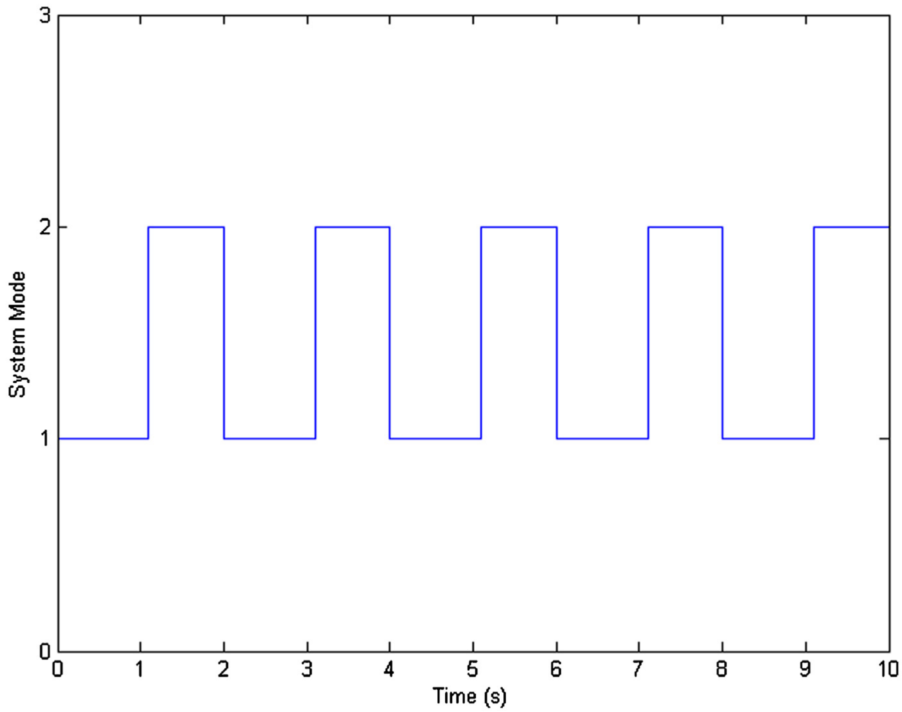

Fig. 4. Switching rule $\sigma(t)$.

$$\bar{A}_{13} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \quad \bar{G}_{13} = \begin{bmatrix} 0 & 1.0 \end{bmatrix}, \quad \bar{G}_{14} = 0.1, \quad \bar{F}_{12} = 0.1,$$
$$\bar{A}_{21} = \begin{bmatrix} -0.52 & 0.23 \\ 0.24 & -0.51 \end{bmatrix}, \quad \bar{G}_{21} = \begin{bmatrix} 0.2 & 0.8 \\ 0 & 1.0 \end{bmatrix}, \quad \bar{G}_{22} = \begin{bmatrix} 0.46 \\ 0.38 \end{bmatrix}, \quad \bar{F}_{21} = \begin{bmatrix} 0.28 \\ 0.19 \end{bmatrix},$$
$$\bar{A}_{23} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, \quad \bar{G}_{23} = \begin{bmatrix} 0 & 1.0 \end{bmatrix}, \quad \bar{G}_{24} = 0.2, \quad \bar{F}_{22} = 0.1.$$

Thus, system (1) with (2) is a positive system according to Theorem 1.

Setting $\alpha = 0.5$ and $h(t) = 0.2 + 0.1\sin t$, solving Problem 2 leads to

$$\nu_1 = \begin{bmatrix} 1.0154 \\ 2.0421 \\ 3.1163 \end{bmatrix}, \nu_2 = \begin{bmatrix} 1.0607 \\ 2.1935 \\ 3.7001 \end{bmatrix}, \eta_1 = \begin{bmatrix} 0.3850 \\ 4.7031 \\ 1.6625 \end{bmatrix}, \eta_2 = \begin{bmatrix} 0.6007 \\ 5.3049 \\ 2.2294 \end{bmatrix},$$

and $\gamma^* = 0.9516$. By analysis, we can verify that $\|\bar{G}_{i4}\|_1 \delta = 0.5839 < 1$, and in this case, $\mu = 1.5665$, $T_a^* = 0.8977$. When we choose $\mu = 1.6$ and $T_a = 1.0$, the corresponding exponential DR can be determined as $\beta = 0.03$. Therefore, in light of the results in Theorem 4, system (1) is ES and meanwhile possesses an optimum $\bar{\gamma} = 1.9024$ under switching rules fulfilling $T_a > 0.8977$.

Figs. 4–6 show the simulation results, where $x(0) = [0.2 \quad 0.3 \quad 0.8]^T$, $x(t) = [0 \quad 0 \quad 0]^T$, $t = [-0.3, 0)$, denotes the initial condition, and $w(t) = 0.5e^{-0.5t}$ is the external disturbance. Fig. 4 depicts the switching rule $\sigma(t)$; The system's state x and output z are exhibited in Figs. 5 and 6, respectively. From Figs. 4–6, system (1) is obviously positive and ES, which illustrates the correctness of the obtained results.

Example 3. To illustrate better the correctness and effectiveness of the obtained results, we consider a practical example. Fig. 7 introduces an four-mesh circuit system which is a practical electrical circuit consisting of resistances, inductances and voltages. Since the actual system may be affected by factors such as external environment or random faults, their parameters or structures changed abruptly rather than time-invariant. To facilitate analysis, we consider a four-mesh circuit system as shown in Fig. 8, in which the switch occupies two positions and the switching between inductance L_1 and inductance L_3 follows a certain switching rule.

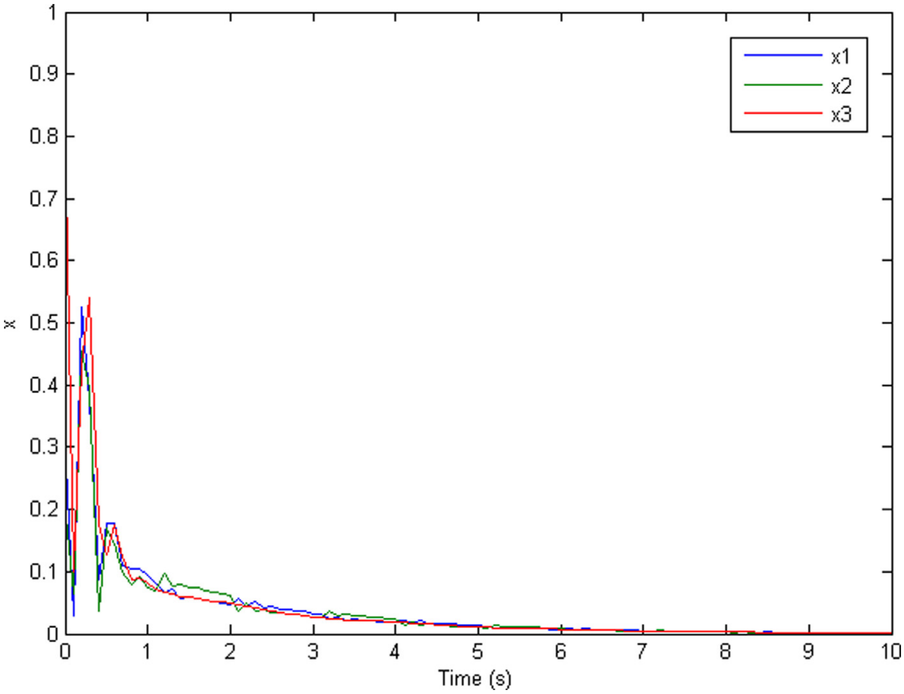

Fig. 5. The state $x(t)$.

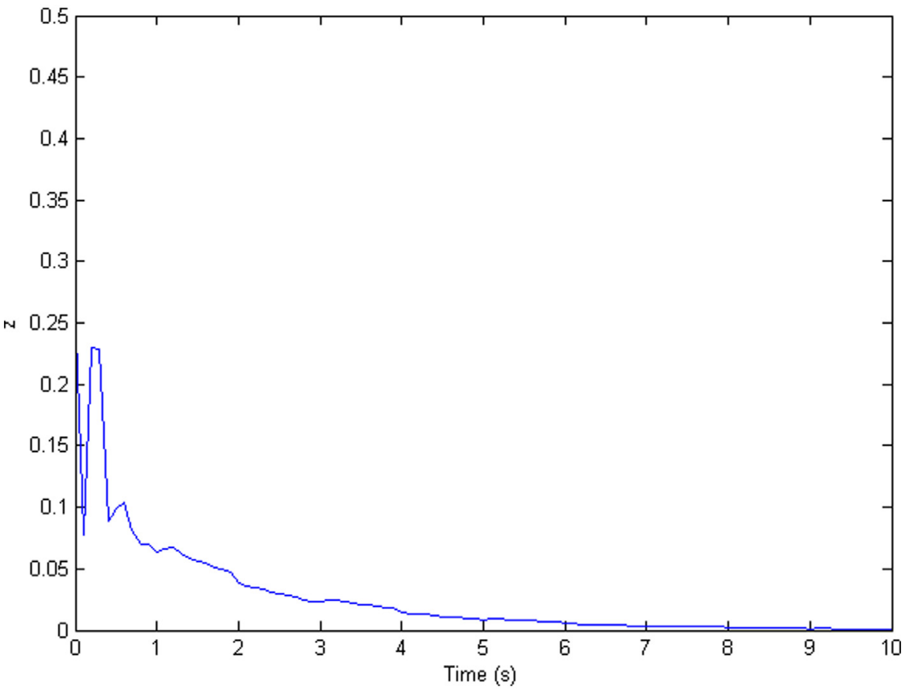

Fig. 6. The output $z(t)$.

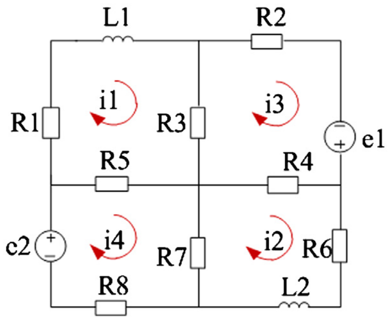

Fig. 7. An four-mesh circuit.

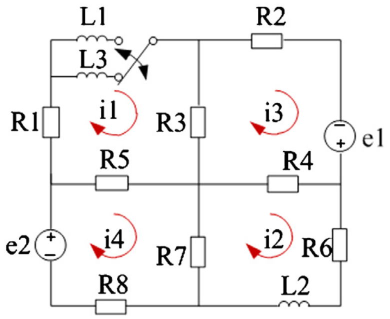

Fig. 8. An four-mesh circuit.

If the inductance L_1 is closed, subsystem 1 operates, i.e. $\sigma(t)=1$; If the inductance L_3 is closed, subsystem 2 operates, i.e. $\sigma(t)=2$. By using the mesh analysis method and Kirchhoff's law, the following equation can be obtained:

$$\begin{cases} a(\sigma(t))\frac{di_1}{dt} = -(R_1+R_3+R_5)i_1 + R_3 i_3 + R_5 i_4, \\ L_2\frac{di_2}{dt} = -(R_4+R_6+R_7)i_2 + R_4 i_3 + R_7 i_4, \\ 0 = R_3 i_1 + R_4 i_2 - (R_2+R_3+R_4)i_3 + e_1, \\ 0 = R_5 i_1 + R_7 i_2 - (R_5+R_7+R_8)i_4 + e_2, \end{cases}$$

where $a(\sigma(t)) = \begin{cases} L_1, & \sigma(t)=1 \\ L_3, & \sigma(t)=2 \end{cases}$.

Choose the state variables $x_1=i_1$, $x_2=i_2$, $x_3=i_3$, $x_4=i_4$, then the system can be written as follows:

$$E\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t),$$

where

$$A_{\sigma(t)} = \begin{bmatrix} -\frac{(R_1+R_3+R_5)}{a(\sigma(t))} & 0 & \frac{R_3}{a(\sigma(t))} & \frac{R_5}{a(\sigma(t))} \\ 0 & -\frac{(R_4+R_6+R_7)}{L_2} & \frac{R_4}{L_2} & \frac{R_7}{L_2} \\ R_3 & R_4 & -(R_2+R_3+R_4) & 0 \\ R_5 & R_7 & 0 & -(R_5+R_7+R_8) \end{bmatrix},$$

$$E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B_{\sigma(t)} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad x(t) = \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix}, \quad u(t) = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}.$$

Let $R_1=R_2=R_3=R_4=R_5=R_6=R_7=R_8=1$, $L_1=1$, $L_2=L_3=2$. Suppose we give a distributed delay $h(t)=0.2+0.1\sin t$, then the above system becomes

$$E\dot{x}(t) = A_{\sigma(t)}x(t) + G_{\sigma(t)}\int_{t-h(t)}^{t} x(s)ds + B_{\sigma(t)}u(t), \tag{71}$$

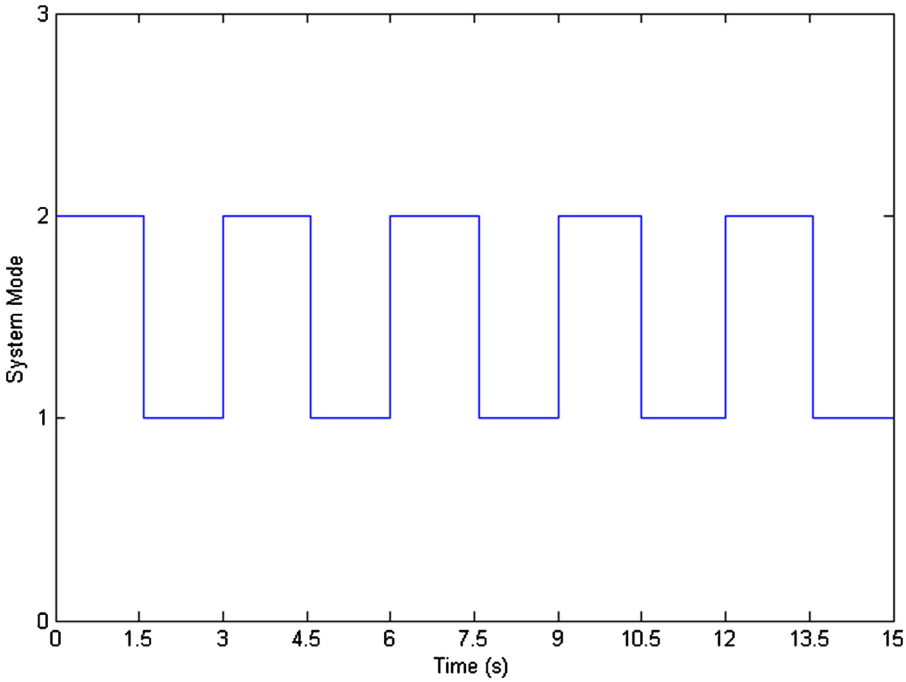

Fig. 9. Switching rule $\sigma(t)$.

where

$$A_1 = \begin{bmatrix} -3 & 0 & 1 & 1 \\ 0 & -\frac{3}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 1 & -3 & 0 \\ 1 & 1 & 0 & -3 \end{bmatrix}, \quad G_1 = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -\frac{3}{2} & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & -\frac{3}{2} & \frac{1}{2} & \frac{1}{2} \\ 1 & 1 & -3 & 0 \\ 1 & 1 & 0 & -3 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.2 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Since the objective is to judge the stability for this system, we let $u(t) = 0$ here. By direct calculation, we have

$$\begin{aligned} &\det(sE - A_1) = 9s^2 + 45s + 63 \neq 0, \quad \text{for} \quad s = 0, \\ &\det(sE - A_2) = 9s^2 + 48s + 67.5 \neq 0, \quad \text{for} \quad s = 0, \\ &\deg(\det(sE - A_1)) = \deg(9s^2 + 45s + 63) = \mathrm{rank}E = 2, \\ &\deg(\det(sE - A_2)) = \deg(9s^2 + 48s + 67.5) = \mathrm{rank}E = 2. \end{aligned}$$

Hence, this system is regular and impulse-free, and by the definitions in system (5), we can get

$$\bar{A}_{11} = \begin{bmatrix} -2.3333 & 0.6667 \\ 0.3333 & -1.1667 \end{bmatrix}, \quad \bar{G}_{11} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad \bar{G}_{12} = \begin{bmatrix} 0.0333 & 0.0667 \\ 0.0167 & 0.0333 \end{bmatrix},$$

$$\bar{A}_{13} = \begin{bmatrix} 0.3333 & 0.3333 \\ 0.3333 & 0.3333 \end{bmatrix}, \quad \bar{G}_{13} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{G}_{14} = \begin{bmatrix} 0.0333 & 0 \\ 0 & 0.0667 \end{bmatrix},$$

$$\bar{A}_{21} = \begin{bmatrix} -1.1667 & 0.3333 \\ 0.3333 & -1.1667 \end{bmatrix}, \quad \bar{G}_{21} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad \bar{G}_{22} = \begin{bmatrix} 0.0333 & 0.0167 \\ 0.0333 & 0.0167 \end{bmatrix},$$

$$\bar{A}_{23} = \begin{bmatrix} 0.3333 & 0.3333 \\ 0.3333 & 0.3333 \end{bmatrix}, \quad \bar{G}_{23} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{G}_{24} = \begin{bmatrix} 0.0667 & 0 \\ 0 & 0.0333 \end{bmatrix}.$$

Thus, system (71) is a positive system according to Theorem 1.

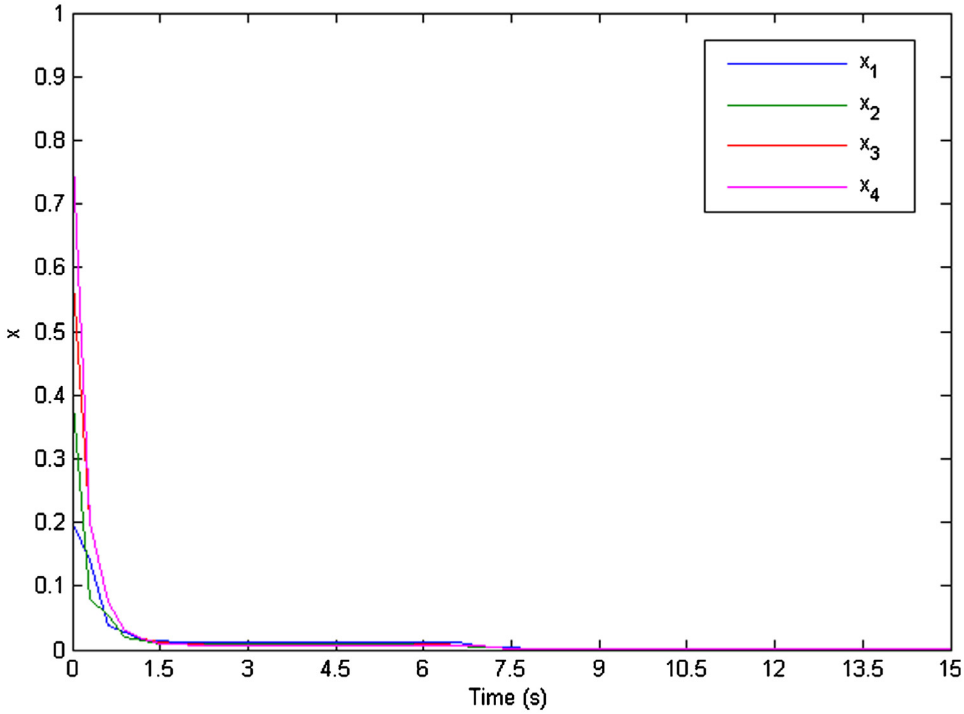

Fig. 10. The state $x(t)$.

Setting $\alpha = 0.5$ and computing by Theorem 2, we get

$$\nu_1 = \begin{bmatrix} 8.4336 \\ 20.4061 \\ 7.8963 \\ 7.9346 \end{bmatrix}, \nu_2 = \begin{bmatrix} 15.8456 \\ 15.7231 \\ 6.4187 \\ 6.3887 \end{bmatrix}, \eta_1 = \begin{bmatrix} 7.3088 \\ 8.7524 \\ 7.1595 \\ 7.7061 \end{bmatrix}, \eta_2 = \begin{bmatrix} 7.4786 \\ 6.4040 \\ 6.8431 \\ 6.4237 \end{bmatrix},$$

and in this case, $\mu = 1.9296$, $T_a^* = 1.3146$. When we choose $\mu = 2.0$ and $T_a = 1.5$, the corresponding exponential DR can be determined as $\beta = 0.46$. Therefore, in light of the results in Theorem 2, system (71) is ES under the switching rule satisfying $T_a > 1.3146$.

Figs. 9 and 10 show the simulation results, where $x(0) = [0.2 \quad 0.4 \quad 0.6 \quad 0.8]^T$, $x(t) = [0 \quad 0 \quad 0 \quad 0]^T$, $t = [-0.3, 0)$, denote the initial condition. Fig. 9 depicts the switching rule $\sigma(t)$, and the system's state x is exhibited in Fig. 10. From Figs. 9 and 10, system (71) is obviously positive and ES, which illustrates the effectiveness of the obtained results.

5. Conclusions

This paper solves the issue of positivity, exponential stability and DAP analysis for SSPSs with TDDs. Firstly, a sufficient and necessary positivity condition of SSPSs has been given through the SVD method. Then, a set of switching rules is identified and a sufficient exponential stability condition is developed making the SSPSs to be ES. Considering the external disturbances, the DAP referring to both L_1-gain and L_∞-gain is analyzed for the SSPSs, and furthermore, the optimal L_1-gain and L_∞-gain performance level is obtained, respectively. Three examples are finally given to illustrate the feasibility and effectiveness of the obtained results.

Acknowledgements

This work was partially supported by the National Natural Science Foundation of China under Grant (61803134, 61873128).

References

[1] L. Farina, S. Rinaldi, Positive Linear Systems, Wiley, New York, 2000.
[2] T. Kaczorek, Positive 1D and 2D Systems, Springer-Verlag, Berlin, 2002.
[3] I. Sandberg, On the mathematical foundations of compartmental analysis in biology and medicine, and ecology, IEEE Trans. Circuits Syst. 25 (5) (1978) 273–279.

[4] G. Silva-Navarro, J. Alvarez-Gallegos, Sign and stability of equilibria in quasi-monotone positive nonlinear systems, IEEE Trans. Autom. Control 42 (3) (1997) 403–407.
[5] R. Shorten, F. Wirth, D. Leith, A positive systems model of TCP-like congestion control: asymptotic results, IEEE/ACM Trans. Netw. 14 (3) (2006) 616–629.
[6] F. Knorn, O. Mason, R. Shorten, On linear co-positive Lyapunov functions for sets of linear positive systems, Automatica 45 (2009) 1943–1947.
[7] C. Briat, Robust stability and stabilization of uncertain linear positive systems via integral linear constraints: l_1-gain and l_∞-gain characterization, Int. J. Robust Nonlinear Control 23 (2013) 1932–1954.
[8] J. Shen, J. Lam, On l_∞ and l_∞ gains for positive systems with bounded time-varying delays, Int. J. Syst. Sci. 46 (11) (2015) 1953–1960.
[9] X. Chen, M. Chen, J. Shen, A novel approach to l_1-induced controller synthesis for positive systems with interval uncertainties, J. Franklin Inst. 354 (8) (2017) 3364–3377.
[10] X. Chen, M. Chen, W. Qi, J. Shen, Dynamic output-feedback control for continuous-time interval positive systems under l_1 perforamance, Appl. Math. Comput. 289 (2016) 48–59.
[11] S. Li, Z. Xiang, H. Karimi, Stability and l_1-gain controller design for positive switched systems with mixed time-varying delays, Appl. Math. Comput. 222 (2013) 507–518.
[12] W. Xiang, On equivalence of two stability criteria for continuous-time switched systems with dwell time constraint, Automatica 54 (2015) 36–40.
[13] W. Xiang, Necessary and sufficient condition for stability of switched uncertain linear systems under dwell-time constraint, IEEE Trans. Autom. Control 61 (11) (2016) 3619–3624.
[14] Y. Li, H. Zhang, L. Zhang, Equivalence of several stability conditions for switched linear systems with dwell time, Int. J. Robust Nonlinear Control 29 (2019) 306–331.
[15] J. Cheng, J.H. Park, J. Cao, W. Qi, Hidden Markov model-based nonfragile state estimation of switched neural network with probabilistic quantized outputs, IEEE Trans. Cybern. (2019), doi:10.1109/TCYB.2019.2909748.
[16] J. Cheng, C.K. Ahn, H.R. Karimi, J. Cao, W. Qi, An event-based asynchronous approach to Markov jump systems with hidden mode detections and missing measurements, IEEE Trans. Syst. Man Cybern.: Syst. 49 (9) (2019) 1749–1758.
[17] B. Wang, D. Zhang, J. Cheng, J.H. Park, Fuzzy model-based nonfragile control of switched discrete-time systems, Nonlinear Dyn. 93 (4) (2018) 2461–2471.
[18] Z. Wang, J. Sun, G. Chen, Stability analysis of nonlinear switched systems with sampled-data controllers, Appl. Math. Comput. (2019), doi:10.1016/j.amc.2019.04.006.
[19] Z. Wang, J. Sun, G. Chen, J. Chen, Stability analysis of switched nonlinear delay systems with sampled-data inputs, Int. J. Robust Nonlinear Control (2019), doi:10.1002/rnc.4653.
[20] R.N. Shorten, D.J. Leith, J. Foy, R. Kilduff, Analysis and design of AIMD congestion control algorithms in communication networks, Automatica 41 (4) (2005) 725–730.
[21] A. Jadbabaie, J. Lin, A. Morse, Coordination of groups of mobile autonomous agents using nearest neighbor rules, IEEE Trans. Autom. Control 48 (6) (2003) 988–1001.
[22] E. Hernandez-Vargas, P. Colaneri, R. Middleton, F. Blanchini, Discrete-time control for switched positive systems with application to mitigating viral escape, Int. J. Robust Nonlinear Control 21 (10) (2011) 1093–1111.
[23] W. Xiang, J. Lam, J. Shen, Stability analysis and l_1-gain characterization for switched positive systems under dwell-time constraint, Automatica 85 (2017) 1–8.
[24] J. Zhang, M. Li, R. Zhang, New computation method for average dwell time of general switched systems and positive switched systems, IET Control Theory Appl. 12 (16) (2018) 2263–2268.
[25] J. Zhang, X. Zhao, X. Cai, Absolute exponential l_1-gain analysis and synthesis of switched nonlinear positive systems with time-varying delay, Appl. Math. Comput. 284 (2016) 24–36.
[26] W. Qi, G. Zong, J. Cheng, T. Jiao, Robust finite-time stabilization for positive delayed semi-Markovian switching systems, Appl. Math. Comput. 351 (2019) 139–152.
[27] X. Zhao, Y. Yin, L. Liu, X. Sun, Stability analysis and delay control for switched positive linear systems, IEEE Trans. Autom. Control 63 (7) (2018) 2184–2190.
[28] X. Zhao, Y. Yin, X. Zheng, State-dependent switching control of switched positive fractional-order systems, ISA Trans. 62 (2016) 103–108.
[29] L. Dai, Singular Control Systems, Springer, Berlin, 1989.
[30] S. Xu, J. Lam, Robust Control and Filtering of Singular Systems, Springer, Berlin, 2006.
[31] S. Xu, J. Lam, Y. Zou, An improved characterization of bounded realness for singular delay systems and its applications, Int. J. Robust Nonlinear Control 18 (3) (2008) 263–277.
[32] B. Xia, J. Lian, X. Yuan, Stability of switched positive descriptor systems with average dwell time switching, J. Shanghai Jiaotong Univ. 20 (2) (2015) 177–184.
[33] T. Liu, B. Wu, L. Liu, Finite-time stability of discrete switched singular positive systems, Circuits Syst. Signal Process. 36 (6) (2017) 2243–2255.
[34] W. Qi, X. Gao, State feedback controller design for singular positive Markovian jump systems with partly known transition rates, Appl. Math. Lett. 46 (2015) 111–116.
[35] D. Zhang, Q. Zhang, B. Du, Positivity and stability of positive singular Markovian jump time-delay systems with partially unknown transition rates, J. Franklin Inst. 354 (2) (2017) 627–649.
[36] V. Phat, N. Sau, On exponential stability of linear singular positive delayed systems, Appl. Math. Lett. 38 (2014) 67–72.
[37] D. Yue, Q. Han, Robust h_∞ filter design of uncertain descriptor systems with discrete and distributed delays, IEEE Trans. Signal Process. 52 (2004) 3200–3212.