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Exponential stability of hybrid switched nonlinear singular systems with time-varying delay

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Abstract

We address exponential stability of switched nonlinear singular systems with time-delay in which delay is time varying and presents in the states. For switched nonlinear singular time-delay systems with average dwell-time switching signals, we provide sufficient conditions, in terms of linear matrix inequalities (LMIs) to guarantee the exponential stability of such systems. By using Lyapunov–like Krasovskii approach, the relationship between the average dwell-time of the switched nonlinear singular time-delay system and the exponential decay rate of differential and algebraic states is given. A numerical example is also included to illustrate the effectiveness of the results proposed in this paper.

1. Introduction

Switched system is an important class of hybrid dynamical systems, which is composed of a family of continuous-time or discrete-time subsystems and a rule orchestrating the switching among them. For switched systems, due to the complicated behavior caused by the interaction between the continuous dynamics and discrete switching, the problem of stability is more difficult to study and has a strong engineering background. In recent years, switched systems have been widely studied and many interesting results have been reported in

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the literature [1–6]. Since delays are often the main cause of instability and poor performance of dynamic systems, how to deal with time-delay has been also a hot topic in the stability analysis of switched time-delay systems. Several useful results have been reported in the literature such as the stability and stabilization analysis [7,8], observer-based tracking problem [9], output feedback control [10], H_∞ control design and filtering [11–14], exponential stability with stable and unstable subsystems [15], and exponential stability of delayed neural networks [16].

As a crucial factor, switching signals determine the dynamic behavior of a switched system in most cases. As a class of typical constrained switching signals, the average dwell-time switching means that the number of switches in a finite interval is bounded and the average time between consecutive switching is not less than a constant [17]. The average dwell-time switching can cover the dwell-time switching, and its extreme case is actually the arbitrary switching [17]. Therefore, it is of practical and theoretical significance to prove the stability of switched systems with average dwell-time, and the corresponding results have been available in [18,19] for discrete-time version, [20,21] for continuous-time version, and [22] for relevant applications. For more results of stability of switched systems, we refer the readers to [23,24] and the references cited therein.

Singular systems, such as robotics, power systems, networks, economical systems and highly interconnected large-scale systems, have been extensively studied in the past decades due to the fact that singular models describe physical systems more directly and generally than regular state-space ones [25]. Many fundamental system theories developed for regular state-space system have been successfully extended to their counterparts for singular system, for example, controllability and observability [26], the Lyapunov stability [27,28], robust stability and stabilization [29,30], singular time-delay systems [31–33], and optimal control [34].

When partial or total subsystems of a switched system are singular systems, the switched system becomes a class of switched singular systems. Owing to switches among multiple singular subsystems, it is inevitably difficult to analyze and synthesize such systems. There are some papers which have presented stability analysis of the switched singular system [35–40], but the simultaneous presence of switching signals with average dwell-time, time-delay, and nonlinearities have not been fully investigated. However, to the best of our knowledge, the delay range-dependent stability problem for switched nonlinear singular time-delay systems has not been fully investigated yet, which will be challenging due to the difficult extension of the existing stability results and has motivated us to carry out the present study.

In this paper, we are seeking for a switching signal based on the average dwell-time constraint able to exponentially stabilize the switched nonlinear singular time-delay system. The parameters of this signal are determined and the exponentially stability is proved. First, some properties of switched nonlinear singular time-delay systems are introduced and discussed. Then, the most important section is presented. In this section, with the help of the average dwell-time approach, a class of switching signals is found under which the switched nonlinear singular time-delay system is exponentially stable. The linear matrix inequality (LMI)-based existence conditions of such a stability analysis are derived by the introduction of free-weighting matrices. Some additional instrumental matrix variables are introduced which makes the stability analysis feasible.

The organization of the paper is as follows. The preliminaries are stated in Section 2, followed by the main result in Sections 3 and 4. A numerical example is presented in Section 5. Finally, we conclude the paper in Section 6 with a discussion of future work.

Notation: Throughout this paper, $\mathfrak{R}^n$ and $\mathfrak{R}^{n\times m}$ denote the n dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. The superscript "T" denotes the transpose matrix, and the notation $X \geq Y$, (respectively, $X > Y$), where X and Y are matrices, means that $X-Y$ is a positive semi-definite (positive definite) matrix. $\|A\|$ stands for *norm* of matrix A. Also "*min*" and "*max*" are abbreviations for minimum and maximum operators, ($\lambda_{min}(A)$,$\lambda_{max}(A)$) denotes the minimum and maximum eigenvalues of A, respectively, and $diag(M_1,\ldots,M_n)$ is a diagonal matrix such that its (i,i)th entry is $M_i (i=1,2,\ldots,n)$. Also $|a|$ represents the absolute value of scalar a.

2. Preliminaries

Let the dynamics of a class of switched nonlinear singular systems with time-delay be described by the following equations:

$$\begin{cases} E\dot{x}(t) = A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t-d(t)) + f_{\sigma(t)}(t,x(t),x(t-d(t))) \\ x(\theta) = \phi(\theta) \quad \forall \theta \in [-h_2,\ 0] \end{cases} \tag{1}$$

where $x(t) \in \Re^n$ denotes the state, $rank(E)=r\leq n$ (i.e. E may be singular), ($A_{\sigma(t)}$,$A_{d\sigma(t)}$) are known constant matrices with appropriate dimensions, and $f_{\sigma(t)}(t,x(t),x(t-d(t)))$ is a nonlinear function satisfying certain conditions stated below. $\phi(\theta)$ is an initial vector-valued continuous function and $\|\phi(\theta)\|_c$ is defined as $\|\phi(\theta)\|_c = \sup_{-h_2 \leq s \leq 0}\|\phi(s)\|$ which stands for the norm of initial condition $\phi(\theta)$, and $d(t)$ is a time-varying continuous function that satisfies $0<h_1 \leq d(t) \leq h_2$, $\dot{d}(t) \leq \overline{d} < 1$ in which h_2 and h_1 are scalars representing the upper and lower bound of delay, respectively. The index set $\mathcal{P}$ is finite: $\mathcal{P}=\{1,2,\ldots,m\}$. The piecewise right continuous (and constant) function $\sigma(t): [0,\infty) \to \mathcal{P}$ is a switching signal to specify, at each time instant t, the index $\sigma(t) \in \mathcal{P}$ of the active subsystem, i.e. $\sigma(t)=i$ ($i \in \mathcal{P}$) denotes that the ith subsystem. The following assumption is made through the paper:

Assumption 1. For any symmetric positive-definite matrix G_i, the nonlinear function $f_i(t,x(t),x(t-d(t)))$ satisfies for all i ($i \in \mathcal{P}$)

$$\kappa_i f_i(t,x(t),x(t-d(t)))^T f_i(t,x(t),x(t-d(t))) \leq x(t)^T U_{i_1}^T U_{i_1} x(t) + x(t-d(t))^T U_{i_2}^T U_{i_2} x(t-d(t))$$

in which (U_{i_1}, U_{i_2}) are symmetric positive definite matrices and $\kappa_i=(\lambda_{max}(G_i))^2$. □

Remark 1. Assumption 1 is referred to as Lipschitz condition. It is easy to see that Assumption 1 implies that the nonlinear function $f_i(t,x(t),x(t-d(t)))$ satisfies a Lipschitz condition which is usually assumed in literature; see, for instance [41–43]. Also, Assumption 1 guarantees that $f_i(t,\mathbf{0},\mathbf{0})=\mathbf{0}$ which implies the origin ($x(t)\equiv\mathbf{0}$) is the trivial solution of (1).

Now, for all i ($i \in \mathcal{P}$) choose two non-singular matrices M_i and N such that $\overline{E} = M_iEN = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}$ and define

$$\overline{A}_i = M_iA_iN = \begin{pmatrix} A_{i_{11}} & A_{i_{12}} \\ A_{i_{21}} & A_{i_{22}} \end{pmatrix}, \quad \overline{A}_{di} = M_iA_{di}N = \begin{pmatrix} A_{di_{11}} & A_{di_{12}} \\ A_{di_{21}} & A_{di_{22}} \end{pmatrix}$$

$$\overline{f}_i\left(t,N\xi\left(t\right),N\xi\left(t-d\left(t\right)\right)\right) = M_if_i\left(t,x\left(t\right),x\left(t-d\left(t\right)\right)\right) = \begin{pmatrix} f_{i_1}(t,N\xi(t),N\xi(t-d(t))) \\ f_{i_2}(t,N\xi(t),N\xi(t-d(t))) \end{pmatrix} \tag{2}$$

where $\xi(t) = N^{-1}x(t) = \begin{pmatrix} \xi_1(t) \\ \xi_2(t) \end{pmatrix}$. By considering (2), the switched nonlinear singular time-delay system (1) at time t can be rewritten as follows:

$$\dot{\xi}_1(t) = \mathcal{D}_{\sigma(t)}(t) \tag{3a}$$

$$0 = \mathcal{S}_{\sigma(t)}(t) \tag{3b}$$

where

$$\mathcal{D}_{\sigma(t)}(t) = A_{\sigma(t)_{11}}\xi_1(t) + A_{\sigma(t)_{12}}\xi_2(t) + A_{d\sigma(t)_{11}}\xi_1(t-d(t)) + A_{d\sigma(t)_{12}}\xi_2(t-d(t)) + f_{\sigma(t)_1}(t, N\xi(t), N\xi(t-d(t)))$$
$$\mathcal{S}_{\sigma(t)}(t) = A_{\sigma(t)_{21}}\xi_1(t) + A_{\sigma(t)_{22}}\xi_2(t) + A_{d\sigma(t)_{21}}\xi_1(t-d(t)) + A_{d\sigma(t)_{22}}\xi_2(t-d(t)) + f_{\sigma(t)_2}(t, N\xi(t), N\xi(t-d(t)))$$

Consider the initial condition $\begin{cases} \xi_1(t) = \psi_1(t) \\ \xi_2(t) = \psi_2(t) \end{cases}$, $t \in [-h_2, 0]$. By substituting initial condition $\psi(t) = N^{-1}\phi(t) = col\begin{pmatrix} \psi_1(t) & \psi_2(t) \end{pmatrix}$ into (3b), we have $\mathcal{S}_{\sigma(0)}(0) = 0$ as follows:

$$0 = A_{\sigma(0)_{21}}\psi_1(0) + A_{\sigma(0)_{22}}\psi_2(0) + A_{d\sigma(0)_{21}}\psi_1(-d(0)) + A_{d\sigma(0)_{22}}\psi_2(-d(0)) + f_{\sigma(0)_2}(0, N\psi(0), N\psi(-d(0))) \tag{4}$$

Eq. (4) is a compatible initial condition of the switched nonlinear singular time-delay system (1). Furthermore, compatibility conditions at switching points (τ_k) can be written as $\mathcal{S}_{\sigma(\tau_k^-)}(\tau_k^-) = \mathcal{S}_{\sigma(\tau_k)}(\tau_k^+)$ which guarantee the continuity of states at switching points. Here, $t^- = \lim_{\varepsilon\downarrow 0}(t-\varepsilon)$ and $t^+ = \lim_{\varepsilon\downarrow 0}(t+\varepsilon)$. In singular time-delay systems, if the pair matrices (E, A_i) are regular and impulse-free, the system can still have finite discontinuities due to incompatible initial conditions [28,44]. Thus, it will be assumed in this paper the following:

Assumption 2. Each pair matrix (E, A_i) is regular and impulse-free.

In continuation, we can see that regularity and impulse-free condition of each subsystem, which have been stated by some researches, [45], can be also checked by the conditions of Lemma 2 stated in the next section.

Remark 2. In Eq. (2) we have considered two matrices M_i and N. It means that there is a family of matrices M_i such that $M_iEN = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}$ and we can use different M_i for each subsystem. This decomposition is independent on the form of E and it is always possible to choose two non-singular matrices M_i and N such that $M_iEN = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}$ [25]. The motivation to consider different M_i for each subsystem is mainly twofold. First, to show that M_i is not unique. So we can maintain the general set-up. Secondly, to give more flexibility to satisfy Assumption 3 stated in the next section. However, it is not a restrictive constraint and can be considered as a common M_i for all subsystems. However, matrix N is a common matrix for all subsystems to preserve the concept of states in the switched case.

Now, the following definitions and lemma are given to prove the main results. Since we need the exponential stability of switched nonlinear singular time-delay systems, motivated from [44], the following definition is given.

Definition 1. Switched nonlinear singular time-delay system (1) is said to be exponentially stable if there exist $\sigma > 0$ and $\gamma > 0$ such that, for any compatible initial condition $\phi(t)$, the solution $x(t)$ *t*o the switched nonlinear singular time-delay system satisfies $\|x(t)\| \leq \gamma e^{-\sigma(t-t_0)}\|\phi(t)\|_c$.

Definition 2. System (1) is impulse-free and regular if (1) has a unique real-valued smooth solution for each initial condition.

Lemma 1. *Suppose that a positive continuous function $f(t)$ satisfies*:

$$f(t) \leq \xi_1 \sup_{t-\tau \leq s \leq t} f(s) + \xi_2 e^{-\epsilon(t-t_i)} \quad \forall t \geq t_i \tag{a}$$

where $\epsilon > 0$, $0 < \xi_1 < 1, \xi_2 > 0$, *and* $\tau > 0$. *Then*

$$f(t) \leq \sup_{t_i-\tau \leq s \leq t_i} f(s) e^{-\xi_0(t-t_i)} + \frac{\xi_2}{1-\xi_1 e^{\xi_0 \tau}} e^{-\xi_0(t-t_i)} \quad \forall t \geq t_i \tag{b}$$

where $\xi_0 = \min\{\epsilon, \zeta\}$, $0 \leq \zeta \leq -(1/\tau)\ln\xi_1$.

Proof. The proof is given in Appendix A. □

It should be noted that Lemma 1 is an extension of Lemma 2 in [30] and the proof is similar to the proof of the same lemma in [30].

3. Stability analysis of subsystems

In this section, the stability analysis of each subsystem is given. Here, $\mathcal{P}$ represents the stable subsystems with exponential stable differential variables and satisfying certain conditions stated below as Lemma 2. Lemma 2 is used to establish which subsystems belong to $\mathcal{P}$. The characterization of the each subsystem is carried out by using the following Lyapunov–Krasovskii type functional.

$$V_i(t,x(t)) = \sum_{l=1}^{3} V_{i_l}(t,x(t)) \tag{5}$$

where

$$V_{i_1}\Big(t,x\Big(t\Big)\Big) = x(t)^T E^T P_i x(t), \quad V_{i_2}\Big(t,x(t)\Big) = \sum_{l=1}^{2}\left\{\int_{t-h_l}^{t} x(s)^T\ e^{2\alpha(s-t)} Q_{i_l} x(s) ds\right\}$$
$$+ \int_{t-d(t)}^{t} x(s)^T\ e^{2\alpha(s-t)} Q_{i_3} x(s) ds$$
$$V_{i_3}(t,x(t)) = \int_{-h_2}^{0} \int_{t+\theta}^{t} \dot{x}(s)^T\ e^{2\alpha(s-t)} E^T Z_{i_1} E \dot{x}(s) ds\, d\theta$$
$$+ \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} \dot{x}(s)^T\ e^{2\alpha(s-t)} E^T Z_{i_2} E \dot{x}(s) ds\, d\theta$$

P_i is a non-singular matrix in which $E^T P_i = P_i^T E \geq 0$, Q_{i_l} $(l = 1,2,3)$ and $Z_{i_l} (l = 1,2)$ are symmetric positive-definite matrices, and $\alpha > 0$ denotes the decay convergence rate of the differential states.

Remark 3. Here, we want to characterize each subsystem without considering switching sequence. So index "i" is used instead of $\sigma(t)$ for some parameters.

Lemma 2. *For the ith subsystem which satisfies Assumption 1, given* $0<h_1\leq d(t)\leq h_2$, $\dot{d}(t)\leq \overline{d}<1$, *and scalar* $\alpha>0$, *if there exist non-singular matrices* P_i, *symmetric and positive-definite matrices* $Q_{i_l}(l=1,2,3)$, $Z_{i_l}(l=1,2)$, *and* R_{i_l} $(l=1,2)$, *matrices* T_i, L_{i_l}, Y_{i_l} $(l=1,2)$, *and positive scalars* ϵ_i *such that the following matrix inequalities hold*:

$$\mathcal{G}_i<0 \tag{6a}$$

with the following constraint

$$E^T P_i = P_i^T E \geq 0 \tag{6b}$$

then

$$V_i(t,x(t))\leq e^{-2\alpha(t-t_0)}V_i(t_0,x(t_0)) \tag{6c}$$

which yields $\|\xi_1(t)\|\leq\sqrt{\left(\lambda_{i_2}/\lambda_{i_1}\right)}e^{-\alpha(t-t_0)}\|\phi(t)\|_c$ *in which* $\begin{pmatrix}\xi_1(t)\\ \xi_2(t)\end{pmatrix}=N^{-1}x(t)$, $\lambda_{i_1}=\lambda_{min}\left(P_{i_{11}}\right)$, λ_{i_2} *is a sufficiently large scalar, and*

$$\mathcal{G}_i=\begin{pmatrix} \Upsilon_{i_{11}} & P_i^T A_{di} & A_i^T T_i & A_i^T\mathcal{L}_{i_1} & P_i^T \\ * & \Upsilon_{i_{22}} & A_{di}^T T_i & A_{di}^T\mathcal{L}_{i_1} & 0 \\ * & * & \Upsilon_{i_{33}} & -\mathcal{L}_{i_1} & T_i^T \\ * & * & * & \Upsilon_{i_{44}} & \mathcal{L}_{i_1}^T \\ * & * & * & * & -\epsilon_i I \end{pmatrix},$$

$$\Upsilon_{i_{11}}=A_i^T P_i+P_i^T A_i+\sum_{l=1}^{3}Q_{i_l}+2\alpha E^T P_i+\epsilon_i U_{i_1}^T U_{i_1}+R_{i_1}$$

$$\Upsilon_{i_{22}}=-\left(1-\overline{d}\right)e^{-2\alpha h_2}Q_{i_3}+\epsilon_i U_{i_2}^T U_{i_2}+R_{i_2},\quad \Upsilon_{i_{33}}=h_2 Z_{i_1}+(h_2-h_1)Z_{i_2}-T_i^T-T_i,$$

$$\Upsilon_{i_{44}}=-diag\left(e^{-2\alpha h_1}Q_{i_1},e^{-2\alpha h_2}Q_{i_2},\frac{e^{-2\alpha h_2}}{h_2}Z_{i_1},\frac{e^{-2\alpha h_2}}{(h_2-h_1)}Z_{i_2}\right),$$

$$\mathcal{L}_{i_1}=\begin{pmatrix} L_{i_1} & L_{i_2} & Y_{i_1} & Y_{i_2}\end{pmatrix},\ \begin{pmatrix} P_{i_{11}} & P_{i_{12}} \\ P_{i_{21}} & P_{i_{22}}\end{pmatrix}=M_i^{-T}P_i N$$

where "" denotes the matrix entries implied by the symmetry of a matrix. Hence, the ith subsystem belongs to* $\mathcal{P}$.

Proof. Taking the time derivative of $V_i(t,x(t))$ for the ith subsystem and using (6b) yields

$$\begin{aligned}\dot{V}_{i_1}(t,x(t))&=2x(t)^T E^T P_i\dot{x}(t)=2x(t)^T P_i^T(A_i x(t)+A_{di}x(t-d(t))\\ &\quad+f_i(t,x(t),x(t-d(t))))\end{aligned} \tag{7a}$$

$$\begin{aligned}\dot{V}_{i_2}(t,x(t))&=-2\alpha V_{i_2}(t,x(t))+\sum_{l=1}^{3}(x(t)^T Q_{i_l}x(t))-x(t-h_1)^T e^{-2\alpha h_1}Q_{i_1}x(t-h_1)\\ &\quad-x(t-h_2)^T e^{-2\alpha h_2}Q_{i_2}x(t-h_2)-\left(1-\dot{d}(t)\right)x(t-d(t))^T e^{-2\alpha d(t)}Q_{i_3}x(t-d(t))\end{aligned} \tag{7b}$$

$$\dot{V}_{i_3}(t,x(t)) = -2\alpha V_{i_3}(t,x(t)) + h_2\dot{x}(t)^T E^T Z_{i_1} E\dot{x}(t) - \int_{-h_2}^{0} \dot{x}(t+\theta)^T e^{2\alpha\theta} E^T Z_{i_1} E\dot{x}(t+\theta)d\theta$$
$$+(h_2-h_1)\dot{x}(t)^T E^T Z_{i_2} E\dot{x}(t) - \int_{-h_2}^{-h_1} \dot{x}(t+\theta)^T e^{2\alpha\theta} E^T Z_{i_2} E\dot{x}(t+\theta)d\theta \tag{7c}$$

by noting that

$$-\left(1-\dot{d}(t)\right)x(t-d(t))^T e^{-2\alpha d(t)} Q_{i_3} x(t-d(t)) \le -\left(1-\overline{d}\right)x(t-d(t))^T e^{-2\alpha h_2} Q_{i_3} x(t-d(t)) \tag{8}$$

and by using Jensen's inequality (Appendix B, Lemma A.1) as follows:

$$-\int_{-h_2}^{0} \dot{x}(t+\theta)^T e^{2\alpha\theta} E^T Z_{i_1} E\dot{x}(t+\theta)d\theta \le -e^{-2\alpha h_2}\left(\int_{-h_2}^{0} E\dot{x}(t+\theta)d\theta\right)^T \frac{Z_{i_1}}{h_2}\left(\int_{-h_2}^{0} E\dot{x}(t+\theta)d\theta\right) \tag{9a}$$

$$-\int_{-h_2}^{-h_1} \dot{x}(t+\theta)^T e^{2\alpha\theta} E^T Z_{i_2} E\dot{x}(t+\theta)d\theta \le -e^{-2\alpha h_2}\left(\int_{-h_2}^{-h_1} E\dot{x}(t+\theta)d\theta\right)^T$$
$$\frac{Z_{i_2}}{(h_2-h_1)}\left(\int_{-h_2}^{-h_1} E\dot{x}(t+\theta)d\theta\right) \tag{9b}$$

by considering Assumption 1 and adding the following free matrices

$$0 = 2\left(x(t)^T,\ (E\dot{x}(t))^T,\ x(t-h_1)^T,\ x(t-h_2)^T,\ \left(\int_{-h_2}^{0} E\dot{x}(t+\theta)d\theta\right)^T,\ \left(\int_{-h_2}^{-h_1} E\dot{x}(t+\theta)d\theta\right)^T\right)\mathcal{L}_i^T\{-E\dot{x}(t)$$
$$+A_i x(t) + A_{di}x(t-d(t)) + f_i(t,x(t),\ x(t-d(t)))\} \tag{10}$$

in which $\mathcal{L}_i = \begin{pmatrix} 0 & T_i & L_{i_1} & L_{i_2} & Y_{i_1} & Y_{i_2} \end{pmatrix}$, we can get $\dot{V}_i(t,x(t)) + 2\alpha V_i(t,x(t)) \le \mathcal{X}_i^T \mathcal{G}_i \mathcal{X}_i$ where

$$\mathcal{X}_i^T(t) = \left(x(t)^T,\ x(t-d(t))^T,\ (E\dot{x}(t))^T,\ x(t-h_1)^T,\ x(t-h_2)^T,\right.$$
$$\left.\left(\int_{-h_2}^{0} E\dot{x}(t+\theta)d\theta\right)^T,\ \left(\int_{-h_2}^{-h_1} E\dot{x}(t+\theta)d\theta\right)^T,\ f_i(t,x(t),x(t-d(t)))^T\right)$$

and $\mathcal{G}_i$ was defined in (6a). It should be noted that, $\mathcal{L}_i = \begin{pmatrix} 0 & T_i & L_{i_1} & L_{i_2} & Y_{i_1} & Y_{i_2} \end{pmatrix}$ is considered to check some regularity and impulse properties of the system which have been introduced by [45] and stated below as Remark 10. Inequality (6a) implies that $\dot{V}_i(t,x(t)) \le -2\alpha V_i(t,x(t))$. Now, by integrating this equation it can be concluded that

$$V_i(t,x(t)) \le e^{-2\alpha(t-t_0)} V_i(t_0,x(t_0)) \tag{11}$$

From (6b), we conclude that $P_{i_{12}} = 0$ and $P_{i_{11}} = P_{i_{11}}^T > 0$ and since $V_i(t,x(t))$ is a bounded quadratic function, we can find scalar λ_{i_1} and a sufficiently large scalar λ_{i_2} [21,44] such that

$$\lambda_{i_1}\|\xi_1(t)\|^2 \le V_i(t,x(t)), \quad V_i\left(t_0,x(t_0)\right) \le \lambda_{i_2}\|\phi(t)\|_c^2 \tag{12}$$

which leads to $\|\xi_1(t)\| \le \sqrt{\left(\lambda_{i_2}/\lambda_{i_1}\right)} e^{-\alpha(t-t_0)}\|\phi(t)\|_c$ in which $\lambda_{i_1} = \lambda_{min}\left(P_{i_{11}}\right)$. Then the differential variables are upper-bound by a decreasing exponential function and the subsystem belongs to $\mathcal{P}$. Thus, the proof is completed. □

Remark 4. Notice that Lemma 2 is used to establish which subsystem belongs to $\mathcal{P}$. This characterization of each subsystem is considered in order to prove the stability of the complete switched system.

4. Stability analysis of the switched case

In this section, based on the Lemmas 1 and 2 and the following definitions, the exponential stability of the switched case is presented.

Let $t_0=0<\tau_1<\tau_2<\cdots<\tau_k<t, \quad k=1,2,\ldots$ denote the switching points within $[0,t)$. Constructing switching sequence $\alpha=\{(\sigma(t_0),t_0),(\sigma(\tau_1),\tau_1),\ldots,(\sigma(\tau_k),\tau_k),\ldots|k=0,1,2,\ldots, \tau_0=t_0\}$ means that the $\sigma(\tau_k)$th subsystem is activated during $[\tau_k,\tau_{k+1})$.

Definition 3. For any $T_2>T_1\geq 0$, let $N_\sigma(T_1,T_2)$ denote the number of switching of $\sigma(t)$ over (T_1,T_2). If $N_\sigma(T_1,T_2)\leq N_0+(T_2-T_1)/T_a$ holds for $T_a>0$, $N_0\geq 0$. Then T_a is called the average dwell-time [46].

Remark 5. By the average dwell-time switching, we mean a class of switching signals such that the average time interval between consecutive switching is at least T_a. Then, a basic problem for such systems is how to specify the minimal T_a and, thereby, get the admissible switching signals such that the underlying system is stable and satisfies a prescribed performance if the system dynamics meet some conditions. The average dwell-time switching strategy may contain signals that occasionally have consecutive discontinuities separated by less than a constant T_a (it should be compensated by switching sufficiently slowly later). As commonly used in the literature, we choose $N_0=0$ in Definition 3. Therefore, the average dwell-time switching signal has been recognized to be a flexible and efficient in system stability analysis.

Assumption 3. In this paper, it is assumed that $\mathcal{S}_{\sigma(0)}(0)=0$ and $\mathcal{S}_{\sigma\left(\tau_k^-\right)}(t)=\mathcal{S}_{\sigma(\tau_k)}(t)$ which guarantee the continuity of states at switching points. $\mathcal{S}_{\sigma(t)}(t)$ was defined in (3b).

As an example, we can consider nonlinear singular subsystems with a common algebraic equation or consider a switched nonlinear singular time-delay system in which its switching points occur when the trajectories intersect the compatibility space of the new subsystems.

Now, we want to define an average dwell-time switching signal such that the exponential stability of the switched nonlinear singular system under Assumptions 1–3 is guaranteed. The characterization of the switching signal and the exponential stability of the overall system are carried out simultaneously by using a piecewise Lyapunov functional candidate and linear matrix inequality approach. The main result is given in the following Theorem 1.

Theorem 1. *For given* $0<h_1\leq d(t)\leq h_2$, $\dot{d}(t)\leq\overline{d}<1$, *and scalar* $\alpha>0$, *if there exist non-singular matrices* P_i, *symmetric and positive-definite matrices* $Q_{i_l}(l=1,2,3)$, $Z_{i_l}(l=1,2)$, *and* $R_{i_l}(l=1,2)$, *matrices* T_i, $L_{i_l}, Y_{i_l}(l=1,2)$, *and positive scalars* ϵ_i *and* $\mu\geq 1$ *satisfying*

$$P_{s_{11}}\leq\mu^2P_{l_{11}}, \quad Q_{s_k}\leq\mu^2Q_{l_k}, \quad Z_{s_n}\leq\mu^2Z_{l_n}, \quad k=1,2,3,\ n=1,2\ \forall s,\ l\in\mathcal{P} \tag{13a}$$

such that (6a) and (6b) hold and the switching signal satisfying the average dwell-time as

$$T_a\geq(\ln\sigma/\alpha) \tag{13b}$$

then, the switched nonlinear singular time-delay system (1) which satisfies Assumptions 1, 2 and 3 is impulse-free, regular, and exponentially stable, and the state convergence can be estimated as $\|x(t)\| \le \mathcal{D}e^{-\lambda(t-t_0)}\|\phi(t)\|_c$ *in which* $\mathcal{D}$ *is a sufficiently large scalar,* $0<\lambda\le\alpha-(\ln\sigma/T_a)$, $\upsilon=\max_i\{|\sqrt{\kappa_i}|\}$, $\kappa_i=\lambda_{max}\left(Q_{i_{3_{22}}}\right)/\lambda_{min}\left(Q_{i_{3_{22}}}\right)$ *is condition number of* $Q_{i_{3_{22}}}$, $\begin{pmatrix} Q_{i_{3_{11}}} & Q_{i_{3_{12}}} \\ * & Q_{i_{3_{22}}} \end{pmatrix}=N^TQ_{i_3}N$, *and* σ *is a scalar in which* $\sigma\ge\max\{\upsilon,\mu\}$.

Proof. The proof is divided into three parts: (1) to show the exponential stability of differential variables of the switched nonlinear singular time-delay system: (2) to show the exponential stability of algebraic variables of the switched nonlinear singular time-delay system: (3) to show the exponential behavior of the switched system (1).

Parts: (1) Exponential stability of differential variables of the switched nonlinear singular time-delay system: From (5), (6c) and (12), it is obtained on the switching points τ_k that $V_{\sigma(\tau_k)}\left(\tau_k^-\right)\le\mu^2V_{\sigma(\tau_{k-1})}\left(\tau_k^-\right)$. Based on (6c) and (12), we obtain by induction that

$$\begin{aligned} V_{\sigma(\tau_k)}(t) &\le e^{-2\alpha(t-\tau_k)}V_{\sigma(\tau_k)}(\tau_k)\le\left(\mu^2\right)e^{-2\alpha(t-\tau_k)}V_{\sigma(\tau_{k-1})}\left(\tau_k^-\right) \\ &\le\left(\mu^2\right)e^{-2\alpha(t-\tau_{k-1})}V_{\sigma(\tau_{k-1})}(\tau_{k-1})\ldots\le\left(\mu^2\right)^{N_\sigma(t_0,t)}e^{-2\alpha(t-t_0)}V_{\sigma(t_0)}(t_0) \end{aligned} \tag{14}$$

By considering (6c), (12) and (14), we get $\|\xi_1(t)\|\le\mathcal{M}\mu^{N_\sigma(t_0,t)}e^{-\alpha(t-t_0)}\|\phi(t)\|_c$ in which $\mathcal{M}=\sqrt{\max_i\{\lambda_{i_2}\}/\max_i\{\lambda_{i_1}\}}$. Using average dwell-time (13b) yields $\|\xi_1(t)\|\le\mathcal{D}e^{-\lambda(t-t_0)}\|\phi(t)\|_c$. Until now, we have proved the exponential stability of differential variables. Our next step is to prove the exponential stability of algebraic variables.

Parts: (2) Exponential stability of algebraic variables of the switched nonlinear singular time-delay system: By using the state transformation $\xi(t)=N^{-1}x(t)$, the switched nonlinear singular time-delay system (1) at time t can be rewritten as follows:

$$\dot{\xi}_1(t)=\mathcal{D}_{\sigma(t)}(t) \tag{15a}$$

$$0=\mathcal{S}_{\sigma(t)}(t) \tag{15b}$$

in which $\mathcal{D}_{\sigma(t)}$ and $\mathcal{S}_{\sigma(t)}(t)$ were defined in (3a) and (3b), respectively. To study the exponential stability of $\xi_2(t)$, we define

$$\overline{Q}_{i_s}=N^TQ_{i_s}N=\begin{pmatrix} Q_{i_{s_{11}}} & Q_{i_{s_{12}}} \\ * & Q_{i_{s_{22}}} \end{pmatrix},\quad \overline{R}_{i_l}=N^TR_{i_l}N=\begin{pmatrix} R_{i_{l_1}} & R_{i_{l_0}} \\ * & R_{i_{l_2}} \end{pmatrix},$$
$$N^TU_{i_l}^TU_{i_l}N=\begin{pmatrix} U_{i_{l_{11}}} & U_{i_{l_{12}}} \\ * & U_{i_{l_{22}}} \end{pmatrix},\quad s=1,2,3,\ l=1,2$$

and a function as

$$J_{\sigma(t)}(t)=\xi_2(t)^TQ_{\sigma(t)_{3_{22}}}\xi_2(t)-\left(1-\overline{d}\right)e^{-2\alpha h_2}\xi_2(t-d(t))^TQ_{\sigma(t)_{3_{22}}}\xi_2(t-d(t)) \tag{16}$$

Here, we are interested in the exponential stability of algebraic variables by using Lemma 1. We defined $J_{\sigma(t)}(t)$ to construct $f_{\sigma(t)}(t)$ in (32) as stated below to use Lemma 1. From (15b), by pre-multiplying $2\xi_2(t)^TP^T_{\sigma(t)_{22}}$, we obtain that

$$0=2\xi_2(t)^T P^T_{\sigma(t)_{22}} A_{\sigma(t)_{22}}\xi_2(t)+2\xi_2(t)^T P^T_{\sigma(t)_{22}} A_{d\sigma(t)_{22}}\xi_2(t-d(t))$$
$$+2\xi_2(t)^T P^T_{\sigma(t)_{22}} f_{\sigma(t)_2}(t,N\xi(t),N\xi(t-d(t)))+2\xi_2(t)^T P^T_{\sigma(t)_{22}}\left(A_{\sigma(t)_{21}}\xi_1(t)\right.$$
$$\left.+A_{d\sigma(t)_{21}}\xi_1(t-d(t))\right) \tag{17}$$

From Assumption 1, we know that

$$0\le -\epsilon_{\sigma(t)}\lambda_{max}\left(M^{-T}_{\sigma(t)}M^{-1}_{\sigma(t)}\right) f_{\sigma(t)_2}(t,N\xi(t),N\xi(t-d(t)))^T f_{\sigma(t)_2}(t,N\xi(t),N\xi(t-d(t)))$$
$$+\epsilon_{\sigma(t)}\left(\xi_1(t)^T U_{\sigma(t)_{1_{11}}}\xi_1(t)+2\xi_1(t)^T U_{\sigma(t)_{1_{12}}}\xi_2(t)+\xi_2(t)^T U_{\sigma(t)_{1_{22}}}\xi_2(t)+\xi_1(t-d(t))^T\right.$$
$$\left.U_{\sigma(t)_{2_{11}}}\xi_1(t-d(t))+2\xi_1(t-d(t))^T U_{\sigma(t)_{2_{12}}}\xi_2(t-d(t))+\xi_2(t-d(t))^T U_{\sigma(t)_{2_{22}}}\xi_2(t-d(t))\right) \tag{18}$$

where $\epsilon_{\sigma(t)}$ is a positive scalar. Adding (17) and (18) to (16) yields

$$J_{\sigma(t)}\le \xi_2(t)^T\left(P^T_{\sigma(t)_{22}}A_{\sigma(t)_{22}}+A^T_{\sigma(t)_{22}}P_{\sigma(t)_{22}}+Q_{\sigma(t)_{3_{22}}}+\epsilon_{\sigma(t)}U_{\sigma(t)_{1_{22}}}\right)\xi_2(t)$$
$$+2\xi_2(t)^T P^T_{\sigma(t)_{22}}A_{d\sigma(t)_{22}}\xi_2(t-d(t))$$
$$+\xi_2(t-d(t))^T\left(-\left(1-\overline{d}\right)e^{-2\alpha h_2}Q_{\sigma(t)_{3_{22}}}+\epsilon_{\sigma(t)}U_{\sigma(t)_{2_{22}}}\right)\xi_2(t-d(t))$$
$$+2\xi_2(t)^T P^T_{\sigma(t)_{22}} f_{\sigma(t)_2}(t,N\xi(t),N\xi(t-d(t)))$$
$$-\epsilon_{\sigma(t)}\lambda_{max}\left(M^{-T}_{\sigma(t)}M^{-1}_{\sigma(t)}\right) f_{\sigma(t)_2}(t,N\xi(t),N\xi(t-d(t)))^T f_{\sigma(t)_2}(t,N\xi(t),N\xi(t-d(t)))$$
$$+2\xi_2(t)^T g_{\sigma(t)}(t)+2\epsilon_{\sigma(t)}\xi_2(t-d(t))^T U_{\sigma(t)_{2_{12}}}\xi_1(t-d(t))$$
$$+\epsilon_{\sigma(t)}\xi_1(t)^T U_{\sigma(t)_{1_{11}}}\xi_1(t)+\epsilon_{\sigma(t)}\xi_1(t-d(t))^T U_{\sigma(t)_{2_{11}}}\xi_1(t-d(t)) \tag{19}$$

where $g_{\sigma(t)}(t)=2\epsilon_{\sigma(t)}U_{\sigma(t)_{1_{12}}}\xi_1(t)+P^T_{\sigma(t)_{22}}\left(A_{\sigma(t)_{21}}\xi_1(t)+A_{d\sigma(t)_{21}}\xi_1(t-d(t))\right)$. By using the following known inequality for two vectors x and y [47]

$$2x^T y\le \epsilon x^T P^{-1}x+\epsilon^{-1}y^T Py \tag{20}$$

in which P is a real non-singular matrix, we have

$$2\xi_2(t)^T g_{\sigma(t)}(t)\le \eta_{\sigma(t)_1}\xi_2(t)^T\xi_2(t)+\frac{1}{\eta_{\sigma(t)_1}}g_{\sigma(t)}(t)^T g_{\sigma(t)}(t) \tag{21a}$$

$$2\epsilon_{\sigma(t)}\xi_2(t-d(t))^T U_{\sigma(t)_{2_{12}}}\xi_1(t-d(t))\le \eta_{\sigma(t)_2}\epsilon_{\sigma(t)}\xi_2(t-d(t))^T\xi_2(t-d(t))$$
$$+\frac{\epsilon_{\sigma(t)}}{\eta_{\sigma(t)_2}}\xi_1(t-d(t))^T U^T_{\sigma(t)_{2_{12}}}U_{\sigma(t)_{2_{12}}}\xi_1(t-d(t)) \tag{21b}$$

From (6a), we can get $\begin{pmatrix} \Upsilon_{\sigma(t)_{11}} & P^T_{\sigma(t)}A_{d\sigma(t)} & P^T_{\sigma(t)} \\ * & \Upsilon_{\sigma(t)_{22}} & 0 \\ * & * & -\epsilon_{\sigma(t)}I \end{pmatrix}<0$. Pre- and post-multiplying this inequality by $diag\{N^T, N^T, M_{\sigma(t)}^{-T}\}$ and $diag\{N, N, M_{\sigma(t)}^{-1}\}$, respectively, we can conclude that

$$J_{\sigma(t)}(t) \le \mathcal{X}_{\sigma(t)_2}^T(t)\mathcal{G}_{\sigma(t)_2}\mathcal{X}_{\sigma(t)_2}(t) + \eta_{\sigma(t)_1}\xi_2(t)^T\xi_2(t) + \frac{1}{\eta_{\sigma(t)_1}}g_{\sigma(t)}(t)^T g_{\sigma(t)}(t) + \frac{\epsilon_{\sigma(t)}}{\eta_{\sigma(t)_2}}\xi_1(t-d(t))^T U_{\sigma(t)_{2_{12}}}^T U_{\sigma(t)_{2_{12}}}\xi_1(t-d(t)) \tag{22}$$

where

$$\mathcal{G}_{\sigma(t)_2} = \begin{pmatrix} \Upsilon_{\sigma(t)_{2_{11}}} & P_{\sigma(t)_{22}}^T A_{d_{\sigma(t)_{22}}} & P_{\sigma(t)_{22}}^T \\ * & \Upsilon_{\sigma(t)_{2_{22}}} & 0 \\ * & * & -\epsilon_{\sigma(t)}\lambda_{max}\left(M_{\sigma(t)}^{-T}M_{\sigma(t)}^{-1}\right)I \end{pmatrix}$$

and $\Upsilon_{\sigma(t)_{2_{11}}} = P_{\sigma(t)_{22}}^T A_{\sigma(t)_{22}} + A_{\sigma(t)_{22}}^T P_{\sigma(t)_{22}} + Q_{\sigma(t)_{3_{22}}} + \epsilon_{\sigma(t)}U_{\sigma(t)_{1_{22}}}$, $\Upsilon_{\sigma(t)_{2_{22}}} = -(1-\overline{d})e^{-2\alpha h_2} Q_{\sigma(t)_{3_{22}}} + \epsilon_{\sigma(t)}U_{\sigma(t)_{2_{22}}} + \epsilon_{\sigma(t)}\eta_{\sigma(t)_2}I$,
$\mathcal{X}_{\sigma(t)_2}^T(t) = \left(\xi_2(t)^T,\ \xi_2(t-d(t))^T,\ f_{\sigma(t)_2}(t, N\xi(t), N\xi(t-d(t)))^T\right)$. Since $\epsilon_{\sigma(t)}$ and $\eta_{\sigma(t)_2}$ can be chosen arbitrarily, they can be thus chosen small enough such that $R_{\sigma(t)_{2_2}} \ge \epsilon_{\sigma(t)}\eta_{\sigma(t)_2}I$. By considering (21a), (21b) and (22), we get

$$J_{\sigma(t)}(t) \le -\xi_2(t)^T\left(R_{\sigma(t)_{1_2}} - \eta_{\sigma(t)_1}I\right)\xi_2(t) + \frac{1}{\eta_{\sigma(t)_1}}g_{\sigma(t)}(t)^T g_{\sigma(t)}(t) + \frac{\epsilon_{\sigma(t)}}{\eta_{\sigma(t)_2}}\xi_1(t-d(t))^T U_{\sigma(t)_{2_{12}}}^T U_{\sigma(t)_{2_{12}}}\xi_1(t-d(t)) \tag{23}$$

where $\eta_{\sigma(t)_1}$ is a positive scalar. Since $\eta_{\sigma(t)_1}$ can be also chosen arbitrarily, $\eta_{\sigma(t)_1}$ can be thus chosen small enough such that $R_{\sigma(t)_{1_2}} - \eta_{\sigma(t)_1}I \ge 0$. If $\eta_{\sigma(t)_1}$ is fixed such that $R_{\sigma(t)_{1_2}} - \eta_{\sigma(t)_1}I \ge 0$, then another constant $\eta_{\sigma(t)_3}$ can be found such that $Q_{\sigma(t)_{3_{22}}} + R_{\sigma(t)_{1_2}} - \eta_{\sigma(t)_1}I \ge \left(1 + \eta_{\sigma(t)_3}\right)Q_{\sigma(t)_{3_{22}}}$. Therefore, combining (23) and (16), we obtain

$$\begin{aligned} &\xi_2(t)^T Q_{\sigma(t)_{3_{22}}}\xi_2(t) - (1-\overline{d})e^{-2\alpha h_2}\xi_2(t-d(t))^T Q_{\sigma(t)_{3_{22}}}\xi_2(t-d(t)) \\ &\quad \le -\xi_2(t)^T\left(R_{\sigma(t)_{1_2}} - \eta_{\sigma(t)_1}I\right)\xi_2(t) + \frac{1}{\eta_{\sigma(t)_1}}g_{\sigma(t)}(t)^T g_{\sigma(t)}(t) \\ &\quad + \frac{\epsilon_{\sigma(t)}}{\eta_{\sigma(t)_2}}\xi_1(t-d(t))^T U_{\sigma(t)_{2_{12}}}^T U_{\sigma(t)_{2_{12}}}\xi_1(t-d(t)) \end{aligned} \tag{24}$$

Eq. (24) is rewritten as

$$\begin{aligned} \xi_2(t)^T Q_{\sigma(t)_{3_{22}}}\xi_2(t) \le{}& \frac{(1-\overline{d})e^{-2\alpha h_2}}{1+\eta_{\sigma(t)_3}}\xi_2(t-d(t))^T Q_{\sigma(t)_{3_{22}}}\xi_2(t-d(t)) \\ &+ \frac{1}{\eta_{\sigma(t)_1}\left(1+\eta_{\sigma(t)_3}\right)}g_{\sigma(t)}(t)^T g_{\sigma(t)}(t) \\ &+ \frac{\epsilon_{\sigma(t)}}{\eta_{\sigma(t)_2}\left(1+\eta_{\sigma(t)_3}\right)}\xi_1(t-d(t))^T U_{\sigma(t)_{2_{12}}}^T U_{\sigma(t)_{2_{12}}}\xi_1(t-d(t)) \end{aligned} \tag{25}$$

See (32) which is stated below. To construct (32), $g_{\sigma(t)}(t)^T g_{\sigma(t)}(t)$ and $\xi_1(t-d(t))^T U^T_{\sigma(t)_{2_{12}}} U_{\sigma(t)_{2_{12}}} \xi_1(t-d(t))$ in (25) should be rewritten in exponential form. So, one can obtain

$$\begin{aligned} g_{\sigma(t)}(t)^T g_{\sigma(t)}(t) = &\Big(\Big(2\epsilon_{\sigma(t)} U_{\sigma(t)_{1_{12}}} + P^T_{\sigma(t)_{22}} A_{\sigma(t)_{21}}\Big) \\ &\xi_1(t) + P^T_{\sigma(t)_{22}} A_{d\sigma(t)_{21}} \xi_1(t-d(t))\Big)^T \Big(\Big(2\epsilon_{\sigma(t)} U_{\sigma(t)_{1_{12}}} + P^T_{\sigma(t)_{22}} A_{\sigma(t)_{21}}\Big) \\ &\xi_1(t) + P^T_{\sigma(t)_{22}} A_{d\sigma(t)_{21}} \xi_1(t-d(t))\Big) = \xi_1(t)^T \Big(2\epsilon_{\sigma(t)} U_{\sigma(t)_{1_{12}}} + P^T_{\sigma(t)_{22}} A_{\sigma(t)_{21}}\Big)^T \\ &\qquad\qquad \times \Big(2\epsilon_{\sigma(t)} U_{\sigma(t)_{1_{12}}} + P^T_{\sigma(t)_{22}} A_{\sigma(t)_{21}}\Big) \\ &\xi_1(t) + 2\xi_1(t)^T \Big(2\epsilon_{\sigma(t)} U_{\sigma(t)_{1_{12}}} + P^T_{\sigma(t)_{22}} A_{\sigma(t)_{21}}\Big)^T P^T_{\sigma(t)_{22}} A_{d\sigma(t)_{21}} \xi_1(t-d(t)) \\ &+\xi_1(t-d(t))^T \Big(P^T_{\sigma(t)_{22}} A_{d\sigma(t)_{21}}\Big)^T \Big(P^T_{\sigma(t)_{22}} A_{d\sigma(t)_{21}}\Big) \xi_1(t-d(t)) \end{aligned} \tag{26}$$

Now, define

$$\begin{aligned} &\|\Big(2\epsilon_{\sigma(t)} U_{\sigma(t)_{1_{12}}} + P^T_{\sigma(t)_{22}} A_{\sigma(t)_{21}}\Big)^T \Big(2\epsilon_{\sigma(t)} U_{\sigma(t)_{1_{12}}} + P^T_{\sigma(t)_{22}} A_{\sigma(t)_{21}}\Big)\| = \rho_{\sigma(t)_1} \\ &\|\Big(2\epsilon_{\sigma(t)} U_{\sigma(t)_{1_{12}}} + P^T_{\sigma(t)_{22}} A_{\sigma(t)_{21}}\Big)^T P^T_{\sigma(t)_{22}} A_{d\sigma(t)_{21}}\| = \rho_{\sigma(t)_2} \\ &\|\Big(P^T_{\sigma(t)_{22}} A_{d\sigma(t)_{21}}\Big)^T \Big(P^T_{\sigma(t)_{22}} A_{d\sigma(t)_{21}}\Big)\| = \rho_{\sigma(t)_3} \end{aligned} \tag{27}$$

Then, from (26) and (27), we have

$$\|g_{\sigma(t)}(t)\|^2 \le \rho_{\sigma(t)_1} \|\xi_1(t)\|^2 + 2\rho_{\sigma(t)_2} \|\xi_1(t)\| \|\xi_1(t-d(t))\| + \rho_{\sigma(t)_3} \|\xi_1(t-d(t))\|^2 \tag{28}$$

In Part 1, we showed the exponential stability of $\xi_1(t)$. So, from (14), we can get

$$\begin{aligned} \|g_{\sigma(t)}(t)\|^2 \le &\Big(\rho^{max}_{\sigma(t)} \mathscr{M}^2 \|\phi(t)\|_c^2\Big) \Big(\big(\mu^{N_\sigma(t,t_0)} e^{-\alpha(t,t_0)}\big)^2 + 2\big(\mu^{N_\sigma(t,t_0)} e^{-\alpha(t,t_0)}\big)\big(\mu^{N_\sigma(t,t_0)} e^{-\alpha(t-d(t)-t_0)}\big) \\ &+\big(\mu^{N_\sigma(t,t_0)} e^{-\alpha(t-d(t)-t_0)}\big)^2\Big) \end{aligned} \tag{29}$$

where $\rho^{max}_{\sigma(t)} = max\{\rho_{\sigma(t)_1}, \rho_{\sigma(t)_2}, \rho_{\sigma(t)_3}\}$. Therefore, it can be concluded

$$\|g_{\sigma(t)}(t)\|^2 \le \rho^{max}_{\sigma(t)} \beta \mathscr{M}^2 \big(\mu^{N_\sigma(t,t_0)} e^{-\alpha(t-t_0)} \|\phi(t)\|_c\big)^2 \tag{30a}$$

in which $\beta = \big(1 + e^{2\alpha h_2}\big)^2$. Also from (14), we have

$$\begin{aligned} \frac{\epsilon_{\sigma(t)}}{\eta_{\sigma(t)_2}} \xi_1(t-d(t))^T U^T_{\sigma(t)_{2_{12}}} U_{\sigma(t)_{2_{12}}} \xi_1(t-d(t)) \le &\frac{\epsilon_{\sigma(t)} \lambda_{max}\Big(U^T_{\sigma(t)_{2_{12}}} U_{\sigma(t)_{2_{12}}}\Big)}{\eta_{\sigma(t)_2}} \\ \times \mathscr{M}^2 e^{2\alpha h_2} e^{-2\alpha(t-t_0)} \|\phi(t)\|_c^2 & \end{aligned} \tag{30b}$$

By substituting (30a) and (30b) into (25), (25) can be rewritten as

$$\xi_2(t)^T Q_{\sigma(t)_{3_{22}}} \xi_2(t) \le \frac{(1-\overline{d}) e^{-2\alpha h_2}}{1 + \eta_{\sigma(t)_3}} \xi_2(t-d(t))^T Q_{\sigma(t)_{3_{22}}} \xi_2(t-d(t))$$

$$+\mathscr{M}^2\left(\frac{\beta\rho^{max}_{\sigma(t)}\eta_{\sigma(t)_2}+\epsilon_{\sigma(t)}\eta_{\sigma(t)_1}e^{2\alpha h_2}\lambda_{max}\left(U^T_{\sigma(t)_{2_{12}}}U_{\sigma(t)_{2_{12}}}\right)}{\eta_{\sigma(t)_1}\eta_{\sigma(t)_2}\left(1+\eta_{\sigma(t)_3}\right)}\right)\left(\mu^{N_\sigma(t,t_0)}e^{-\alpha(t-t_0)}\|\phi(t)\|_c\right)^2 \tag{31}$$

Now, define

$$\vartheta_{\sigma(t)_1}=\frac{(1-\overline{d})e^{-2\alpha h_2}}{1+\eta_{\sigma(t)_3}},\quad \vartheta_{\sigma(t)_2}=\mathscr{M}^2\left(\frac{\beta\rho^{max}_{\sigma(t)}\eta_{\sigma(t)_2}+\epsilon_{\sigma(t)}\eta_{\sigma(t)_1}e^{2\alpha h_2}\lambda_{max}\left(U^T_{\sigma(t)_{2_{12}}}U_{\sigma(t)_{2_{12}}}\right)}{\eta_{\sigma(t)_1}\eta_{\sigma(t)_2}\left(1+\eta_{\sigma(t)_3}\right)}\right)$$

and let $f_{\sigma(t)}(t)=\xi_2(t)^TQ_{\sigma(t)_{3_{22}}}e^{2\alpha\tau_k}\xi_2(t)$. From (31), we have

$$f_{\sigma(t)}(t)\le\vartheta_{\sigma(t)_1}\sup_{t-h_2\le s\le t}f_{\sigma(t)}(s)+\vartheta_{\sigma(t)_2}e^{2\alpha\tau_k}\left(\mu^{N_\sigma(t,t_0)}e^{-\alpha(t-t_0)}\|\phi(t)\|_c\right)^2 \tag{32}$$

Now, we have obtained an inequality in respect of $\xi_2(t)$ and Lemma 1 can be used to show the exponential stability of $\xi_2(t)$. Using Lemma 1, one can obtain

$$f_{\sigma(t)}(t)\le\sup_{\tau_k-h_2\le s\le\tau_k}f_{\sigma(t)}(s)e^{-\boldsymbol{\varepsilon}_{\sigma(t)_0}t}+\frac{e^{2\alpha\tau_k}\vartheta_{\sigma(t)_2}\left(\mu^{N_\sigma(t,t_0)}\|\phi(t)\|_c\right)^2}{\left(1-\vartheta_{\sigma(t)_1}e^{-\boldsymbol{\varepsilon}_{\sigma(t)_0}h_2}\right)}e^{-\boldsymbol{\varepsilon}_{\sigma(t)_0}(t-t_0)} \tag{33}$$

where $\boldsymbol{\varepsilon}_{\sigma(t)_0}=min\{2\alpha,\epsilon_{\sigma(t)}\},\quad 0\le\epsilon_{\sigma(t)}\le-1/(h_2)\ln\vartheta_{\sigma(t)_1}$. Therefore,

$$\|\xi_2(t)\|^2\le\left(\kappa_{\sigma(t)}\sup_{\tau_k-h_2\le s\le\tau_k}\|\xi_2(s)\|^2+\mu^{2N_\sigma(t,t_0)}m_{\sigma(t)}\right)e^{\psi(t_0,t)} \tag{34}$$

in which $\kappa_{\sigma(t)}=\lambda_{max}\left(Q_{\sigma(t)_{3_{22}}}\right)/\lambda_{min}\left(Q_{\sigma(t)_{3_{22}}}\right)$ is condition number of $Q_{\sigma(t)_{3_{22}}}$, $m_{\sigma(t)}=\vartheta_{\sigma(t)_2}\|\phi(t)\|_c^2/\left(\lambda_{min}\left(Q_{\sigma(t)_{3_{22}}}\right)\left(1-\vartheta_{\sigma(t)_1}e^{-\boldsymbol{\varepsilon}_{\sigma(t)_0}h_2}\right)\right)$, $\psi(t_0,t)=-\boldsymbol{\varepsilon}_{\sigma(t)_0}(t-t_0)$.

Now, assume $t\in[\tau_k,\tau_{k+1})$, by an iterative method we show the exponential stability of the algebraic variables. From (34), we have

$$\sup_{\tau_k-h_2\le s\le\tau_k}\|\xi_2(s)\|^2\le\left(\kappa_{k-1}\sup_{\tau_{k-1}-h_2\le s\le\tau_{k-1}}\|\xi_2(s)\|^2+\mu^{2N_\sigma(t,t_0)}m_k\right)e^{\psi\left(t_0,t^*_{k-1}\right)} \tag{35}$$

where $\psi\left(t_0,\ t^*_{k-1}\right)$ is a the maximum amount of $\psi(t_0,t)$ in $[\tau_{k-1},\tau_k)$. By substituting (35) in (34), we have

$$\|\xi_2(t)\|^2\le\left(\left(\kappa_k\kappa_{k-1}\sup_{\tau_{k-1}-h_2\le s\le\tau_{k-1}}\|\xi_2(s)\|^2+\kappa_km_{k-1}\right)e^{\psi\left(t_0,t^*_{k-1}\right)}+m_k\right)e^{\psi(t_0,t)} \tag{36}$$

by induction and repeating (35) for the previous subsystems, we can conclude that

$$\begin{aligned}\|\xi_2(t)\|^2\le\Bigg(&\kappa_1\kappa_2\cdots\kappa_k\|\phi(t)\|_c^2e^{\overbrace{\psi\left(t_0,t^*_1\right)+\psi\left(t_0,t^*_1\right)+\cdots+\psi\left(t_0,t^*_k\right)}^{N_\sigma(t_0,t)}}\\&+\kappa_2\cdots\kappa_km_1e^{\overbrace{\psi\left(t_0,t^*_1\right)+\psi\left(t_0,t^*_1\right)+\cdots+\psi\left(t_0,t^*_{k-1}\right)}^{N_\sigma(t_0,t)-1}}\\&+\cdots+\kappa_km_{k-1}e^{\psi\left(t_0,t^*_{k-1}\right)}+\mu^{2N_\sigma(t,t_0)}m_k\Bigg)e^{\psi(t_0,t)}\end{aligned} \tag{37}$$

Since, $\psi(t_0,t)$ is a continuous and $\psi(t_0,\infty)=-\infty$, so $e^{\psi(t_0,t)}$ is a positive and continuous with $e^{\psi(t_0,\infty)}=0$. Certainly, after $\overline{n}$ switching, amount of $e^{\psi(t_0,t)}$ becomes less that $q<1$. In other words, there exist $\overline{t}$ such that for $t\geq\overline{t}$, $e^{\psi(t_0,\overline{t})}\leq q$. So,

$$\begin{aligned}\|\xi_2(t)\|^2 &\leq (\|\phi(t)\|_c^2 e^{\sum_{j=1}^{\overline{n}}\psi\left(t_0,t_j^*\right)} q^{N_\sigma(t_0,t)-\overline{n}} + m_1 e^{\sum_{j=1}^{\overline{n}-1}\psi\left(t_0,t_j^*\right)} q^{N_\sigma(t_0,t)-\overline{n}} \\ &+\cdots+q^{N_\sigma(t_0,t)-\overline{n}}e^{\psi\left(t_0,\overline{t}\right)}+q^{N_\sigma(t_0,t)-\overline{n}-1}+q^{N_\sigma(t_0,t)-\overline{n}-2}+\cdots+1)\sigma^{2N_\sigma(t,t_0)}e^{\psi(t_0,t)} \end{aligned} \tag{38}$$

where $\upsilon=\max_i\{|\sqrt{\kappa_i}|\}$ and $\sigma\geq\max\{\upsilon,\mu\}$. Therefore, $\|\xi_2(t)\|^2\leq\mathcal{N}\left(\overline{n}e^{\sum_{j=1}^{\overline{n}}}\psi(t_0,t_j^*)q^{N_\sigma(t_0,t)-\overline{n}}+1/(1-q)\right)\sigma^{2N_\sigma(t,t_0)}e^{\psi(t_0,t)}$ where $\mathcal{N}=\max_i\{\|\phi(t)\|_c^2,m_i\}$. It should be noted that $1/(1-q)=1+q+q^2+\cdots\geq q^{N_\sigma(t_0,t)-\overline{n}-2}+\cdots+1$ where $\sum_{j=1}^{\infty}q^j$ is a geometric series. So we can find a positive scalar $\overline{\mathscr{M}}$ such that $\|\xi_2(t)\|^2\leq\overline{\mathscr{M}}^2e^{-2\lambda(t-t_0)}\|\phi(t)\|_c^2$.
Part 3: to show the exponential behavior of the switched system (1): From Parts 1 and 2, it can be concluded that

$$\begin{aligned}\|\xi(t)\| &\leq \|\xi_1(t)\|+\|\xi_2(t)\|\leq\mathscr{M}e^{-\lambda(t-t_0)}\|\phi(t)\|_c+\overline{\mathscr{M}}e^{-\lambda(t-t_0)}\|\phi(t)\|_c \\ &\leq\{\mathscr{M}+\overline{\mathscr{M}}\}e^{-\lambda(t-t_0)}\|\phi(t)\|_c\end{aligned} \tag{39}$$

Noting that $\xi(t)=N^{-1}x(t)$ yields $\|x(t)\|\leq\mathcal{D}e^{-\lambda(t-t_0)}\|\phi(t)\|_c$ in which $\mathcal{D}=\{\|N\|\left(\mathscr{M}+\overline{\mathscr{M}}\right)\}$. This completes the proof. □

Remark 6. From the proof of Theorem 1 stated above, it is easy to conclude that

(i) By letting $\varepsilon_{i_0}=2\alpha$ in (33) and (34) and noting that $\ln\vartheta_{i_1}<0$, an estimation of the maximum amount of α is obtained as $2\alpha\leq-1/(h_2)\ln\vartheta_{i_1}$, $(i\in\mathcal{P})$. So $\alpha\leq-1/(2h_2)\ln\left(\left(1-\overline{d}\right)e^{-2\alpha h_2}/1+\max_i\eta_{i_3}\right)$ and an estimation for upper bound for the maximum decay rate λ is $\lambda_{max}=-1/(2h_2)\ln\left(\left(1-\overline{d}\right)e^{-2\alpha h_2}/\left(1+\max_i\eta_{i_3}\right)\right)-\left(\ln\sigma/T_a\right)$.

(ii) In Theorem 1 "sufficiently large" means that $\mathcal{D}\geq\|N\|\{\left(\mathscr{M}+\overline{\mathscr{M}}\right)\}$ in which $\mathscr{M}$ and $\overline{\mathscr{M}}$ were defined in (37) and (38).

(iii) By letting $Q_{\sigma(t)_{3_{22}}}=I$, if (6a) and (6b) hold, then we can conclude that $T_a\geq\left(\ln\mu/\alpha\right)$.

(iv) It is noted that conditions (6a) and (6b) are non-strict LMIs, which contain equality constraints. Considering that (6a) and (6b) can be combined into a single strict LMI. Let $P_i>0$ and $S_i\in R^{n\times(n-r)}$ be any matrix with a full column rank and satisfied $E^TS_i=0$ [25]. Changing P_i to $(P_iE+S_iQ_i)$ in (6a) yields the strict LMI.

Remark 7. As mentioned before, if the pair matrices (E,A_i) are regular and impulse-free, it can still have finite discontinuities due to incompatible initial conditions (see [28] and [44] for more details). Therefore, unlike standard switched time-delay systems, discontinuities in switched (nonlinear) singular time-delay systems can propagate between different times due to the existence of delayed solution terms and discontinuities of the previous switching points. For having continuous trajectories, switching must occur when the states of the activated subsystem satisfy the compatibility condition of the new one. So, regular and impulsive free properties of each subsystem and Assumption 3 simultaneously guarantee the continuity of states. Similar to [28] and [44], and by considering (3a) and (3b), we can conclude that each subsystem of (1) has a unique continuous solution on $[\tau_k,\tau_{k+1})$. Since each subsystem has unique continuous solution and switching points satisfy Assumption 3, it can be concluded that the switched system (1) has unique continuous solution with respect to compatible initial condition. Thus, switched system (1) has unique real-valued smooth solution which implies that switched system (1) is regular and impulse-free.

Remark 8. Similar to the discussion in [28] for ordinary singular time-delay systems with incompatible initial conditions, the jump can occur only in the algebraic variables ($\xi_2(t)$) or in the derivatives of the differential variables $\left(\dot{\xi}_1(t)\right)$, i.e., the differential variables are always continuous. If Assumption 3 is not satisfied, then the switched singular time-delay system (1) has discontinuities at switching points. Since $V_{i_1}(t,x(t)) = \xi_1(t)^T P_{i_{11}} \xi_1(t)$ and because of integral structure of $V_{i_2}(t,x(t))$ and $V_{i_3}(t,x(t))$, we can conclude that $V_i(t,x(t))$ is continuous everywhere. Also, without loss of generality we can assume that $\xi_2(t)$ is left-continues, then (33) can be easily obtained. Thus, conditions of Theorem 1 guarantee the exponential stability of switched nonlinear singular system (1) with regularity. In this case, the switched nonlinear singular may not have impulse-free solution.

Remark 9. Note that the above conditions can only be met when all the subsystems belong to $\mathcal{P}$. It means that all subsystems satisfy conditions of Lemma 2. In continuation, we can see that, regularity and impulse-free properties of each subsystem can be checked by the conditions of Theorem 1.

Remark 10. From (6b), it was mentioned that we can conclude that $P_{i_{12}} = 0$ and $P_{i_{11}} = P_{i_{11}}^T > 0$. Also, from (6b), $\Upsilon_{i_{11}} < 0$ is obtained. By pre- and post-multiplying this inequality by N^T and N, respectively, we have $N^T \Upsilon_{i_{11}} N \leq 0$, which implies $A_{i_{22}}^T P_{i_{22}} + P_{i_{22}}^T A_{i_{22}} + \sum_{l=1}^{3} Q_{i_{l_{22}}} + \epsilon_i U_{i_{1_{22}}}^T U_{i_{1_{22}}} + R_{i_{1_2}} < 0$ and leads to $A_{i_{22}}^T P_{i_{22}} + P_{i_{22}}^T A_{i_{22}} < 0$. Thus, $A_{i_{22}}$ is non-singular which implies that $det(sE-A_i)$ is not identically zero and that $deg(det(sE-A_i)) = r = rank(E)$. Then the pair of (E,A_i) is regular and impulse-free [25]. By considering the conditions of regularity and impulse-free of nonlinear singular systems introduced by [45], it can be concluded that (6a) satisfies these conditions.

In the case of $E=I$, the following corollary is obtained.

Corollary 1. *Consider the switched nonlinear time-delay system* (1), *for given* $0 < h_1 \leq d(t) \leq h_2$, $\dot{d}(t) \leq \overline{d} < 1$, *scalar* $\alpha > 0$, *and* $E=I$, *if there exist symmetric and positive definite matrices* P_i, $Q_{i_l}(l=1,2,3)$, $Z_{i_l}(l=1,2)$, *and* R_{i_l} $(l=1,2)$, *matrices* T_{i_l}, L_{i_l}, Y_{i_l} $(l=1,2)$, *positive scalars* ϵ_i *and* $\mu \geq 1$ *satisfying*

$$P_s \leq \mu^2 P_l, \quad Q_{s_k} \leq \mu^2 Q_{l_k}, \quad Z_{s_n} \leq \mu^2 Z_{l_n},\ k=1,2,3,\ n=1,2\ \forall s,l \in \mathcal{P} \tag{40a}$$

such that the following matrix inequalities hold:

$$\mathcal{G}_i < 0 \tag{40b}$$

and the switching signal satisfying the average dwell-time as $T_a \geq \left(\ln\mu/\alpha\right)$. *Then, the switched nonlinear time-delay system* (1) *which satisfy* Assumption 1 *is exponentially stable, and the state convergence can be estimated as* $x(t) \leq \mathscr{M} e^{-\lambda(t-t_0)} \|\phi(t)\|_c$ *in which* $0 < \lambda \leq \alpha - \left(\ln \mu/\alpha\right)$,

$\mathscr{M}$ *was defined in* (14), *and*

$$\mathcal{G}_i = \begin{pmatrix} \Upsilon_{i_{11}} & P_i^T A_{di} + T_{i_1}^T A_{di} & -T_{i_1}^T + A_i^T T_{i_2} & A_i^T \mathscr{L}_{i_1} & P_i^T + T_{i_1}^T \\ * & \Upsilon_{i_{22}} & A_{di}^T T_{i_2} & A_{di}^T \mathscr{L}_{i_1} & 0 \\ * & * & \Upsilon_{i_{33}} & -\mathscr{L}_{i_1} & T_{i_2}^T \\ * & * & * & \Upsilon_{i_{44}} & \mathscr{L}_{i_1}^T \\ * & * & * & * & -\epsilon_i I \end{pmatrix}$$

$$\Upsilon_{i_{11}} = A_i^T P_i + P_i^T A_i + \sum_{k=1}^{3} Q_{i_k} + 2\alpha P_i + \epsilon_i U_{i_1}^T U_{i_1} + T_{i_1}^T A_i + A_i^T T_{i_1} + R_{i_1}$$

Note that the remaining notations used here are similar to Theorem 1.

Proof. The proof is similar to the proof of Theorem 1. By letting $\mathscr{L}_i = \begin{pmatrix} T_{i_1} & T_{i_2} & L_{i_1} & L_{i_2} & Y_{i_1} & Y_{i_2} \end{pmatrix}$ in (10), we can get

$$\dot{V}_i(t,x(t)) + 2\alpha V_i(t,x(t)) \le \mathcal{X}_i^T \mathcal{G}_i \mathcal{X}_i \tag{41}$$

In this case $\lambda_{i_1}\|x(t)\|^2 \le V_i(t,x(t))$, $V_i(t_0,x(t_0)) \le \lambda_{i_2}\|\phi(t)\|_c^2$. Now, set $E=I$, then from (5) and (40a), it is obtained on the switching points τ_k that $V_{\sigma(\tau_k)}(\tau_k^-) \le \mu^2 V_{\sigma(\tau_{k-1})}(\tau_k^-)$. Thus, we obtain by induction that

$$\begin{aligned} V_{\sigma(\tau_k)}(t) &\le e^{-2\alpha(t-\tau_k)} V_{\sigma(\tau_k)}(\tau_k) \\ &\le (\mu^2) e^{-2\alpha(t-\tau_k)} V_{\sigma(\tau_{k-1})}(\tau_k^-) \\ &\le (\mu^2) e^{-2\alpha(t-\tau_{k-1})} V_{\sigma(\tau_{k-1})}(\tau_{k-1}) \\ &\vdots \\ &\le (\mu^2)^{N_\sigma(t_0,t)} e^{-2\alpha(t-t_0)} V_{\sigma(t_0)}(t_0) \end{aligned} \tag{42}$$

which leads $\|x(t)\| \le \mathscr{M}\mu^{N_\sigma(t_0,t)} e^{-\alpha(t-t_0)}\|\phi(t)\|_c$ in which $\mathscr{M} = (\max_i\{\lambda_{i_2}\})^{0.5}(\max_i\{\lambda_{i_1}\})^{-0.5}$ and λ_{i_2}, λ_{i_1} were defined in (12). So, no need to check regularity, impulse property, and (15)–(39). Therefore, by considering $T_a \ge (\ln\mu/\alpha)$, we can conclude that $\|x(t)\| \le \mathscr{M}e^{-\lambda(t-t_0)}\|\phi(t)\|_c$. This completes the proof. □

It is worth pointing out that for the case $f_i(t,x(t),x(t-d(t)))=0$, the following corollary is obtained.

Corollary 2. *Consider the switched singular time-delay system* (1) *with* $f_i(t,x(t), x(t-d(t)))=0$, *for given* $0<h_1\le d(t)\le h_2$, $\dot{d}(t)\le \overline{d}<1$, *and scalar* $\alpha>0$, *if there exist matrices* $\mathcal{Q}_i$, *full column rank matrices* S_i, *symmetric and positive-definite matrices* P_i, $Q_{i_l}(l=1,2,3)$, $Z_{i_l}(l=1,2)$, *and* R_{i_l} $(l=1,2)$, *matrices* T_i, L_{i_l}, Y_{i_l} $(l=1,2)$, *and scalars* $\mu\ge 1$ *satisfying* (13a) *such that the following matrix inequalities hold*:

$$\mathcal{G}_i < 0 \tag{43}$$

with the constraint $E^T S_i=0$ *and the switching signal satisfying the average dwell-time* $T_a \ge (\ln\sigma/\alpha)$, *then the switched singular time-delay system* (1) *which satisfies Assumption 3 is impulse-free, regular, and exponentially stable, and the state convergence can be estimated as*

$\|x(t)\| \le \mathcal{D}e^{-\lambda(t-t_0)}\|\phi(t)\|_c$, *and*

$$\mathcal{G}_i = \begin{pmatrix} \Upsilon_{i_{11}} & (P_iE+S_i\mathcal{Q}_i)^TA_{di} & A_i^TT_i & A_i^T\mathscr{L}_{i_1} \\ * & \Upsilon_{i_{22}} & A_{di}^TT_i & A_{di}^T\mathscr{L}_{i_1} \\ * & * & \Upsilon_{i_{33}} & -\mathscr{L}_{i_1} \\ * & * & * & \Upsilon_{i_{44}} \end{pmatrix},$$

$$\Upsilon_{i_{11}} = A_i^T(P_iE+S_i\mathcal{Q}_i)+(P_iE+S_i\mathcal{Q}_i)^TA_i+\sum_{k=1}^{3}Q_{i_k}+2\alpha E^T(P_iE+S_i\mathcal{Q}_i)+R_{i_1},$$

$$\Upsilon_{i_{22}} = -\left(1-\overline{d}\right)e^{-2\alpha h_2}Q_{i_3}+R_{i_2}$$

Note that the notations used in here are similar to Theorem 1.

Proof. The proof is similar to the proof of Theorem 1, the only thing is that, in here we have used (43) and $E^TS_i=0$ instead of (6a) and (6b) by considering Remark 6. So, it is omitted here. □

Remark 11. It should be noted that the conditions required Lemma 2 imply that all the subsystems must be globally stable. Since singular systems have solution only for compatible initial conditions, "globally stable" means that "system is stable for all compatible initial conditions". Also, a major trend in the development of stability theory of nonlinear switched systems is the study of stability within a finite region. In the case where global stability does not hold, we need to restrict our attention to a finite region in the state space, where a sector that is narrower than the global sector can be used to bound the nonlinear function. In the finite region, a guaranteed domain of attraction can be then obtained by using some invariant level set of a quadratic type Lyapunov function. In singular system, since "domain of attraction" have not been fully definite and investigated, definition and estimation of "domain of attraction" can be considered as a new open problem for both singular system and switched nonlinear singular systems. However, to the best of our knowledge, some definitions for "domain of attraction" have been given [48], but till now, a unique definition has not been given. So it seems considering "domain of attraction" in stability analysis of singular systems will be challenging due to the difficult extension of the existing stability results. Also, using some less conservative conditions for nonlinear terms can be challenging problem in stability analysis. One of these conditions can be used instead of Lipschitz condition has been introduced by [49] as $f_i(t,x(t))^TP_ix(t) \le \varphi_i(t)x(t)^TP_ix(t)$ in which $\varphi_i(t)>0$ is continuous function and P_i is a positive-definite matrix. In this issue, it was shown that this condition is less conservative than the Lipschitz condition which is usually assumed in literature.

Remark 12. As mentioned before, the stability analyses based on the average dwell-time of switched nonlinear singular time-delay systems are closely related to the exponential stability of algebraic equations and there are some features in switched nonlinear singular systems with time-delay which are neither found in singular nor in switched time-delay systems. Concerning the comparison with other well-known results in the literature [35–40], all these results have been presented for singular systems without considering simultaneously average dwell-time, time-delay, and nonlinear terms. Therefore, it is interesting and challenging to investigate the stability problem of switched nonlinear singular time-delay systems.

5. Numerical example

In this section, a numerical example is presented to verify the results of the proposed procedure in switched nonlinear singular time-delay systems. Consider the following switched nonlinear singular time-delay system, composed of two subsystems described by

- Subsystem $i=1$:

$$E_1=\begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix},\quad A_1=\begin{pmatrix}-1.5&0.1&0\\-0.2&-2&0\\-0.5&1&1\end{pmatrix},\quad A_{d1}=\begin{pmatrix}-0.8&-0.4&0.6\\-0.5&0.2&0.3\\-0.1&0.3&0.5\end{pmatrix}$$

$$f_1(t,x(t),x(t-d(t)))^T = [x_1(t-d(t))\sin\left(x_1(t-d(t))\right) \quad x_2(t-d(t))\cos\left(x_3(t-d(t))\right)^2 \quad x_1(t-d(t))e^{-\left|x_2(t-d(t))\right|}]$$

- Subsystem $i=2$:

$$E_2=\begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix},\quad A_2=\begin{pmatrix}-1.5&-0.1&0\\0.2&-1.6&0\\-0.5&1&1\end{pmatrix},\quad A_{d2}=\begin{pmatrix}-0.6&0.3&0.9\\0.5&0.2&0.1\\-0.1&0.3&0.5\end{pmatrix},$$

$$f_2(t,x(t),x(t-d(t)))^T=\left[x_1(t-d(t))\sin\left(x_3(t-d(t))\right)\ 0\ x_1(t-d(t))e^{-\left|x_2(t-d(t))\right|}\right].$$

where $d(t)=0.1+0.01\sin(t)$ and

$$U_{1_1}=\begin{pmatrix}0.01&0&0\\0&0.01&0\\0&0&0.01\end{pmatrix},\quad U_{1_2}=\begin{pmatrix}2&0&0\\0&1&0\\0&0&0.01\end{pmatrix},$$

$$U_{2_1}=\begin{pmatrix}0.01&0&0\\0&0.01&0\\0&0&0.01\end{pmatrix},\quad U_{2_2}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

It is found that both subsystems are stable for $\alpha=1.25$. Hence, given $\mu=2.2395$, it follows from Theorem 1 and Remark 6 that the proposed system is exponentially stable with average dwell-time $T_a\geq 0.6450$ s. For $T_a=1.6450$ s, the state decay of the switched nonlinear singular time-delay system can be estimated as $\|x(t)\|\leq \mathcal{D}e^{-0.76(t-t_0)}\|\phi(t)\|_c$. Furthermore, the corresponding matrices can be obtained by using Matlab software. For the first subsystem, we have

$$P_1=\begin{pmatrix}0.0016&-0.0020&-0.0021\\-0.0020&0.0106&0.0040\\-0.0021&0.0040&0.0039\end{pmatrix}\qquad Q_{1_1}=\begin{pmatrix}0.0077&-0.0109&-0.0113\\-0.0109&0.0231&0.0206\\-0.0113&0.0206&0.0235\end{pmatrix}$$

$$Q_{1_2}=\begin{pmatrix}0.0064&-0.0085&-0.0088\\-0.0085&0.0186&0.0161\\-0.0088&0.0161&0.0189\end{pmatrix}\qquad Q_{1_3}=\begin{pmatrix}0.0142&-0.0240&-0.0263\\-0.0240&0.0511&0.0522\\-0.0263&0.0522&0.0589\end{pmatrix}$$

$$Z_{1_1} = \begin{pmatrix} 0.0259 & -0.0010 & -0.0001 \\ -0.0010 & 0.0314 & -0.0002 \\ -0.0001 & -0.0002 & 0.0409 \end{pmatrix} \quad Z_{1_2} = \begin{pmatrix} 0.0257 & -0.0009 & -0.0001 \\ -0.0009 & 0.0308 & -0.0002 \\ -0.0001 & -0.0002 & 0.0389 \end{pmatrix}$$

$$T_1 = \begin{pmatrix} 0.0005 & -0.0004 & -0.0000 \\ -0.0004 & 0.0026 & -0.0002 \\ -0.0000 & -0.0002 & 0.0084 \end{pmatrix} \quad R_{1_1} = \begin{pmatrix} 0.0044 & -0.0047 & -0.0050 \\ -0.0047 & 0.0114 & 0.0089 \\ -0.0050 & 0.0089 & 0.0115 \end{pmatrix}$$

$$R_{1_2} = \begin{pmatrix} 0.0049 & -0.0058 & -0.0053 \\ -0.0058 & 0.0156 & 0.0092 \\ -0.0053 & 0.0092 & 0.0112 \end{pmatrix} \quad L_{1_1} = \begin{pmatrix} 0.0022 & -0.0009 & -0.0008 \\ -0.0009 & 0.0049 & 0.0018 \\ -0.0008 & 0.0018 & 0.0074 \end{pmatrix}$$

$$L_{1_2} = \begin{pmatrix} 0.0003 & -0.0005 & -0.0004 \\ -0.0005 & 0.0018 & 0.0011 \\ -0.0004 & 0.0011 & 0.0034 \end{pmatrix} \quad Y_{1_2} = \begin{pmatrix} -0.0023 & 0.0016 & 0.0004 \\ 0.0016 & -0.0103 & 0.0002 \\ 0.0004 & 0.0002 & -0.0238 \end{pmatrix}$$

$$Y_{1_2} = \begin{pmatrix} 0.0025 & -0.0016 & -0.0004 \\ -0.0016 & 0.0108 & -0.0002 \\ -0.0004 & -0.0002 & 0.0243 \end{pmatrix} \quad \mathcal{Q}_1 = [0.0633 \ \ -0.1217 \ \ -0.1227]$$

$$S_1 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \quad \epsilon_1 = 0.0017$$

For the second subsystem, we have

$$P_2 = \begin{pmatrix} 0.0030 & -0.0008 & -0.0014 \\ -0.0008 & 0.0046 & 0.0028 \\ -0.0014 & 0.0028 & 0.0026 \end{pmatrix} \quad Q_{2_1} = \begin{pmatrix} 0.0094 & -0.0153 & -0.0143 \\ -0.0153 & 0.0336 & 0.0294 \\ -0.0143 & 0.0294 & 0.0293 \end{pmatrix}$$

$$Q_{2_2} = \begin{pmatrix} 0.0079 & -0.0123 & -0.0115 \\ -0.0123 & 0.0274 & 0.0238 \\ -0.0115 & 0.0238 & 0.0242 \end{pmatrix} \quad Q_{2_3} = \begin{pmatrix} 0.0177 & -0.0344 & -0.0358 \\ -0.0344 & 0.0769 & 0.0780 \\ -0.0358 & 0.0780 & 0.0833 \end{pmatrix}$$

$$Z_{2_1} = \begin{pmatrix} 22.2534 & -0.0010 & -0.0003 \\ -0.0010 & 0.0013 & -0.0001 \\ -0.0003 & -0.0001 & 0.0652 \end{pmatrix} \quad Z_{2_2} = \begin{pmatrix} 0.0344 & -0.0011 & -0.0003 \\ -0.0011 & 0.0369 & -0.0006 \\ -0.0003 & -0.0006 & 0.0674 \end{pmatrix}$$

$$T_2 = \begin{pmatrix} 0.0007 & -0.0002 & -0.0001 \\ -0.0002 & 0.0013 & -0.0001 \\ -0.0001 & -0.0001 & 0.0102 \end{pmatrix} \quad R_{2_1} = \begin{pmatrix} 0.0051 & -0.0064 & -0.0060 \\ -0.0064 & 0.0152 & 0.0123 \\ -0.0060 & 0.0123 & 0.0134 \end{pmatrix}$$

$$R_{2_2} = \begin{pmatrix} 0.0082 & -0.0107 & -0.0090 \\ -0.0107 & 0.0259 & 0.0181 \\ -0.0090 & 0.0181 & 0.0165 \end{pmatrix} \quad L_{2_1} = \begin{pmatrix} 4.8349 & -0.0011 & -0.0011 \\ -0.0011 & 0.0044 & 0.0020 \\ -0.0011 & 0.0020 & 0.0081 \end{pmatrix}$$

$$L_{2_2} = \begin{pmatrix} 0.0004 & -0.0006 & -0.0006 \\ -0.0006 & 0.0015 & 0.0012 \\ -0.0006 & 0.0012 & 0.0038 \end{pmatrix} \quad Y_{2_1} = \begin{pmatrix} -0.0038 & 0.0015 & 0.0005 \\ 0.0015 & -0.0074 & 0.0002 \\ 0.0005 & 0.0002 & -0.0348 \end{pmatrix}$$

$$Y_{2_2} = \begin{pmatrix} 0.0043 & -0.0016 & -0.0005 \\ -0.0016 & 0.0083 & -0.0002 \\ -0.0005 & -0.0002 & 0.0372 \end{pmatrix} \quad \begin{array}{l} Q_2 = \begin{bmatrix} 0.0827 & -0.1761 & -0.1687 \end{bmatrix} \\ S_2 = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}, \ \epsilon_2 = 0.0022. \end{array}$$

The complete simulation results with compatible initial conditions are shown in Fig. 1. In addition, the switching signal with average dwell-time property is shown in the same figure.

6. Conclusion

In this paper, stability analysis of a class of continuous-time switched nonlinear singular time-delay systems consisting of a family of stable subsystems with time-varying delay was investigated. With the help of the average dwell-time approach, a class of switching signals is found under which the switched nonlinear singular time-delay system was exponentially stable. An example was put worth to demonstrate the applicability and effectiveness of the proposed approach. It was mentioned that using some less conservative conditions for nonlinear term can be challenging problem in stability analysis and can be considered as future works.

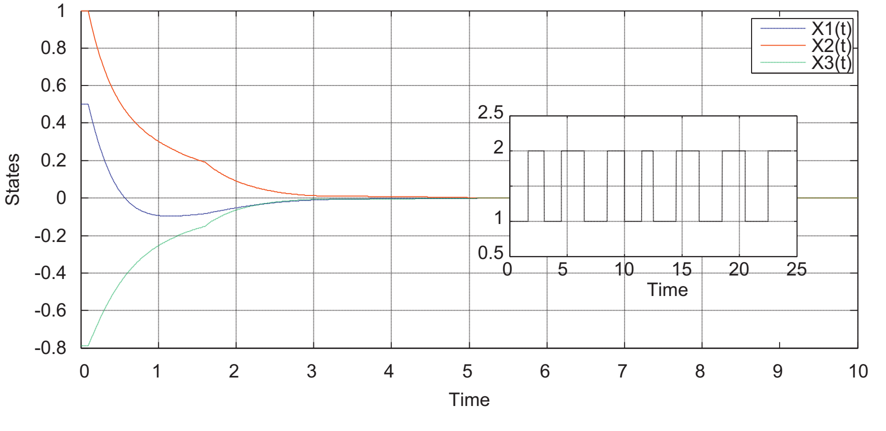

Fig. 1. Responses of the switched nonlinear singular time-delay system and Switching signal with average dwell-time property.

Appendix A. Proof of Lemma 1

From Lemma 1 (part (a)), we know that

$$f(t) \leq \xi_1 \sup_{t-\tau \leq s \leq t} f(s) + \xi_2 e^{-\xi_0(t-t_i)} \quad \forall t \geq t_i \tag{A.1}$$

Now, we want to prove part (b) of Lemma 1. On one hand, we prove that for any $\epsilon_0 > 0$,

$$f(t) < \sup_{t_i-\tau \leq s \leq t_i} f(s) e^{-\xi_0(t-t_i)} + \frac{\xi_2}{1-\xi_1 e^{\xi_0 \tau}} e^{-\xi_0(t-t_i)} + \epsilon_0 \quad \forall t \geq t_i \tag{A.2}$$

From (A.1), it can be concluded that

$$f(t_i) \leq \xi_1 \sup_{t_i-\tau \leq s \leq t_i} f(s) + \frac{\xi_2}{1-\xi_1 e^{\xi_0 \tau}} + \epsilon_0 \tag{A.3}$$

If part (b) in Lemma 1 is not true, then $\bar{t}$ exists such that

$$f(\bar{t}) = \sup_{t_i-\tau \leq s \leq t_i} f(s) e^{-\xi_0(\bar{t}-t_i)} + \frac{\xi_2}{1-\xi_1 e^{\xi_0 \tau}} e^{-\xi_0(\bar{t}-t_i)} + \epsilon_0 \tag{A.4}$$

and

$$f(t) < \sup_{t_i-\tau \leq s \leq t_i} f(s) e^{-\xi_0(t-t_i)} + \frac{\xi_2}{1-\xi_1 e^{\xi_0 \tau}} e^{-\xi_0(t-t_i)} + \epsilon_0 \quad \forall t < \bar{t} \tag{A.5}$$

In fact, for $t \in [t_i - \tau, t_i]$, we have

$$f(t) \leq \sup_{t_i-\tau \leq s \leq t_i} f(s) < \sup_{t_i-\tau \leq s \leq t_i} f(s) e^{-\xi_0(t-t_i)} + \frac{\xi_2}{1-\xi_1 e^{\xi_0 \tau}} e^{-\xi_0(t-t_i)} + \epsilon_0 \tag{A.6}$$

Therefore, (A.5) holds for any $t \in [t_i - \tau, \bar{t}]$. However, from parts (a) in Lemmas 1, (A.4), and (A.5), we can see that

$$\begin{aligned} f(\bar{t}) &\leq \xi_1 \sup_{\bar{t}-\tau \leq s \leq \bar{t}} f(s) + \xi_2 e^{-\xi_0(\bar{t}-t_i)} \\ &\leq \xi_1 e^{\xi_0 \tau} \sup_{t_i-\tau \leq s \leq t_i} f(s) e^{-\xi_0(\bar{t}-t_i)} + \frac{\xi_1 e^{\xi_0 \tau} \xi_2}{1-\xi_1 e^{\xi_0 \tau}} e^{-\xi_0(\bar{t}-t_i)} + \xi_1 \epsilon_0 + \xi_2 e^{-\xi_0(\bar{t}-t_i)} \\ &< \sup_{t_i-\tau \leq s \leq t_i} f(s) e^{-\xi_0(\bar{t}-t_i)} + \frac{\xi_2}{1-\xi_1 e^{\xi_0 \tau}} e^{-\xi_0(\bar{t}-t_i)} + \epsilon_0 \end{aligned} \tag{A.7}$$

which is contradicts to (A.4). By letting $\epsilon_0 \to 0$ in (A.2), we obtain part (b) in Lemma 1. □

Appendix B. Jensen inequality

Lemma A.1. (Jensen Inequality) [50]: *Let $x(t)$ be a vector-valued function with first-order continuous-derivative entries. Then the following descriptor integral-inequality holds for any matrices E and $R = R^T > 0$, and a scalar $\tau > 0$*

$$-\int_{t-\tau}^{t} \dot{x}(s)^T \left(E^T R E\right) \dot{x}(s) ds \leq -\frac{1}{\tau} \int_{t-\tau}^{t} \dot{x}(s)^T \, ds \, E^T R E \int_{t-\tau}^{t} \dot{x}(s) ds$$

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