ĐẠI HỌC QUỐC GIA HÀ NỘI
TRƯỜNG ĐẠI HỌC KHOA HỌC TỰ NHIÊN

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Additional excersises

Bài 1. Consider the equation

$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = Bu(t) \qquad \forall t \geq 0 \tag{1}$$

$$y = Cx \tag{2}$$

$$x(0) = x_0 \tag{3}$$

$$\dot{x}(0) = \dot{x}_0 \tag{4}$$

where $M \in \mathbb{R}^{n,n}$ is an invertible matrix.

a, *Necessary and sufficient condition for C-controllable*

We introduce here 3 definitions:

Definition 1. Two second order systems of the form (1) with system matrices (M, D, K, B) and $(\hat{M}, \hat{D}, \hat{K}, \hat{B})$ are called strongly equivalent if there exist nonsingular matrices $P \in \mathbb{R}^{n,n}$, $Q \in \mathbb{R}^{n,n}$, and $V \in \mathbb{R}^{m,m}$, such that

$$\hat{M} = PMQ, \hat{D} = PDQ, \hat{K} = PKQ, \hat{B} = PBV$$

We write $(M, D, K, B)\tilde{(}\hat{M}, \hat{D}, \hat{K}, \hat{B})$.

Definition 2. Systems $M\ddot{x} + D\dot{x} + Kx = Bu(t)$ and $\hat{M}\ddot{x} + \hat{D}\dot{x} + \hat{K}x = \hat{B}u(t)$ with $M, G, K, \hat{M}, \hat{G}, \hat{K} \in \mathbb{R}n, n, B, \hat{B} \in \mathbb{R}n, m$ are called opu-equivalent if there exists $P \in \mathbb{R}[D_2]^{n,n}$ with constant nonzero determinant such that

hóa ra là dùng bài của Loose/Mehrmann
như Bảo hỏi

$$P(M\ddot{x} + D\dot{x} + Kx - Bu) = \hat{M}\ddot{x} + \hat{D}\dot{x} + \hat{K}x - \hat{B}u$$

Here, $\mathbb{R}[D_2]^{n,n} = \{a_0 + a_1\frac{d}{dt} + a_2\frac{d^2}{dt^2} | a_i \in \mathbb{R}, i = 0, 1, 2\}$
We have the results :

1. Consider system (1)(2)(3)(4). Then there exists a sequence of strong and opu-equivalence transformations such that the transformed system
$$\hat{M}\ddot{\hat{x}} + \hat{D}\dot{\hat{x}} + \hat{K}\hat{x} = \hat{B}\hat{u}$$
has the form
$$\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ * & * & * & * \end{bmatrix} \ddot{\hat{x}} + \begin{bmatrix} \hat{G_{11}} & 0 & \hat{G_{13}} & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \dot{\hat{x}} + \begin{bmatrix} \hat{K_{11}} & \hat{K_{12}} & \hat{K_{13}} & 0 \\ \hat{K_{21}} & \hat{K_{22}} & \hat{K_{23}} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix} = \begin{bmatrix} \hat{B_1} \\ \hat{B_2} \\ \hat{B_3} \\ 0 \end{bmatrix} \hat{u}.$$
$$y = \begin{bmatrix} \hat{C_1} & \hat{C_2} & \hat{C_3} & \hat{C_4} \end{bmatrix} \hat{x}$$
where
2.

We will prove that the systems $(1) - (4)$ is C controllable if and only if it is controllable in the reachable set and $rank[M \;\; D \;\; B] = 2n$. Consider a strong equivalence transformation with
$$P = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -I \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \end{bmatrix}, Q = \begin{bmatrix} I & -\hat{G_{11}} & 0 & -\hat{G_{13}} & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \end{bmatrix}$$
We obtain the system :
$$\begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \dot{\hat{\xi}} + \begin{bmatrix} 0 & \hat{K_{11}} & \hat{K_{12}} & \hat{K_{13}} & 0 \\ -I & \hat{G_{11}} & 0 & \hat{G_{13}} & 0 \\ 0 & \hat{K_{21}} & \hat{K_{22}} & \hat{K_{23}} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \xi = \begin{bmatrix} \hat{B_1} \\ 0 \\ \hat{B_2} \\ \hat{B_3} \\ 0 \end{bmatrix} \hat{u}.$$
where $\xi = Q\hat{\xi}$.

b, *Necessary and sufficient condition for C2-controllable*

Set $\xi = \begin{bmatrix} \dot{x} \\ x \end{bmatrix}$ then $\dot{\xi} = \begin{bmatrix} \ddot{x} \\ \dot{x} \end{bmatrix}$ and $\xi(0) = \xi_0 = \begin{bmatrix} \dot{x_0} \\ x_0 \end{bmatrix}$

Rewrite the equation under the form :

$$\begin{bmatrix} M \\ 0 \end{bmatrix} \ddot{x} + \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} \dot{x} + \begin{bmatrix} K & 0 \\ 0 & 0 \end{bmatrix} x = \begin{bmatrix} B \\ 0 \end{bmatrix} u$$

which is equivalent to : $\begin{bmatrix} M & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} -D & -K \\ I & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u$

and $y = \begin{bmatrix} C & 0 \end{bmatrix} \begin{bmatrix} \dot{x} \\ x \end{bmatrix}$ where I is the elementary matrix.

From here we have the system :

$$E\dot{\xi} = A\xi + B_0 u \tag{5}$$

$$y = \hat{C}\xi$$

which is a general state space system. From L.Dai's book, we already have the following result : The system (5) can be rewrite under the 2 systems

$$\dot{\xi}_1 = A_1\xi_1 + B_1 u \tag{6}$$

$$y_1 = C_1\xi_1 \tag{7}$$

$$N\dot{\xi}_2 = xi_2 + B_2 u \tag{8}$$

$$y_2 = C_2\xi_2 \tag{9}$$

$$y = C_1\xi_1 + C_2\xi_2 = y_1 + y_2 \tag{10}$$

The first one called slow system and the later called fast system. In here, we have use some notations :

$$\begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix} = P^{-1}\xi, \quad \xi_1 \in \mathbb{R}^{n_1,n_1}, \quad \xi_2 \in \mathbb{R}^{n_2,n_2}, \quad N \in \mathbb{R}^{n_2,n_2} \text{ is nilpotent,}$$
$$n_1 + n_2 = n$$

Then the original second order systems is C2-controllable

$\Longleftrightarrow$ the system (5) is controllable
$\Longleftrightarrow$ the slow system and the fast system are controllable
$\Longleftrightarrow$ $rank[\lambda E - A \;\; B_0] = 2n \quad \forall \lambda \in \mathbb{C}$ and $rank[E \;\; B_0] = 2n$.
Following from theorem 2-2.1 from L.Dai's book.

c, *Necessary and sufficient condition for C-observable*

Again, we use the system in the form (5). As a result, the C-observability of the second order system is equivalent to the observability of (5), i.e, both its the slow and fast system are observable. From theorem 2-3.1 from L.Dai's book, we arrive at the condition of obserbility of (5) :

$$rank[\lambda E - A, \hat{C}] = 2n \quad \forall \lambda \in \mathbb{C} \tag{11}$$

$$rank[E, \hat{C}] = 2n. \tag{12}$$

d, *The dual condition*

The system in (1) - (4) is C-observable
$\Longleftrightarrow$ the corresponding first order system (5) is observable
$\Longleftrightarrow$ its dual dual first order system is controllable
$\Longleftrightarrow$ the dual second order system is C2-controllable.

Bài 2. Using Octave. With n = 3, p = 1, nn here G stands for $\hat{C}$, H stands for B_0 .

```
> n = 3  > M = rand(n,n)
> D = rand(n,n)
> K = rand(n,n)
> B = rand(n,1)
> C = rand(1,n)
> O = zeros(n,n)
> hcat1 = [M,O]
> hcat2 = [O, eye(3)]
> E = [hcat1; hcat2]
> hcat3 = [-D, -K]
> hcat3 = [eye(3), O]
> A = [hcat3; hcat4]
```

```
> H = [B; zeros(3,1)]
> G =  [C, O]
```

Check for C-observability `> rank(rand(1)*E - A, G)`

```
> rank(E, G)
```

Check for C2-controbability with n = 3.

```
> rank(rand(1)*E - A, H)
> rank(E, H)
```

Duality check :

```
> n = 3  > M = rand(n,n)
> D = rand(n,n)
> K = rand(n,n)
> B = rand(n,1)
> C = rand(1,n)
> O = zeros(n,n)
> hcat1 = [M',O]
> hcat2 = [O, eye(3)]
> E = [hcat1; hcat2]
> hcat3 = [-D', -K']
> hcat3 = [eye(3), O]
> A = [hcat3; hcat4]
> H = [B'; zeros(3,1)]
> G =  [C', O]
> rank(rand(1)*E - A, H)
> rank(E, H)
```