

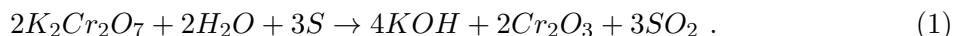
**Câu 1** Water flows from an inverted conical tank with circular orifice at the rate

$$x'(t) = -0.6\pi r^2 \sqrt{2g} \frac{\sqrt{x}}{Ax},$$

where  $r$  is the radius of the orifice,  $x$  is the height of the liquid level from the vertex of the cone, and  $A(x)$  is the area of the cross section of the tank  $x$  units above the orifice. Suppose  $r = 0.1$  ft,  $g = 32.1$  ft/s<sup>2</sup>, and the tank has an initial water level of 8 ft and initial volume of  $512(\pi/3)$  ft<sup>3</sup>.

- Compute the water level after 10 min with  $h = 20$  s.
- Determine, to within 1 min, when the tank will be empty.

**Câu 2** The irreversible chemical reaction in which two molecules of solid potassium dichromate ( $K_2Cr_2O_7$ ), two molecules of water ( $H_2O$ ), and three atoms of solid sulfur ( $S$ ) combine to yield three molecules of the gas sulfur dioxide ( $SO_2$ ), four molecules of solid potassium hydroxide ( $KOH$ ), and two molecules of solid chromic oxide ( $Cr_2O_3$ ) can be represented symbolically by the stoichiometric equation:



If  $n_1$  molecules of  $K_2Cr_2O_7$ ,  $n_2$  molecules of  $H_2O$ , and  $n_3$  molecules of  $S$  are originally available, the following differential equation describes the amount  $x(t)$  of  $KOH$  after time  $t$ :

$$x'(t) = k \left(n_1 - \frac{x}{2}\right)^2 \left(n_2 - \frac{x}{2}\right)^2 \left(n_3 - \frac{3x}{4}\right)^3$$

where  $k$  is the velocity constant of the reaction. If  $k = 6.22 \cdot 10^{-19}$ ,  $n_1 = n_2 = 2 \cdot 10^3$ , and  $n_3 = 3 \cdot 10^3$ , how many units of potassium hydroxide will have been formed after 0.2 s?

**Câu 3** Viết các hàm trong Python để tính tích phân dựa trên 5 phương pháp ẩn và hiện cơ sở, ví dụ

```
def Euler_exp(f, t0, tf, y_0, h):  
    return y, t
```

trong đó  $f = f(t, y)$  là hàm ;  $t \in [t_0, t_f]$  là khoảng thời gian,  $h$  là độ rộng mỗi bước và được cho trước. Chú ý các nút được sử dụng là cách đều, tức là  $t_0 < t_1 < \dots < t_f$ ,  $t_i = a + i * h$ ,

$$h = \frac{tf - t0}{n}.$$

Hết