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A UNIFIED APPROACH TO STABILITY ANALYSIS OF SWITCHED LINEAR DESCRIPTOR SYSTEMS UNDER ARBITRARY SWITCHING

GUISHENG ZHAI *, XUPING XU **

* Department of Mathematical Sciences
Shibaura Institute of Technology, Saitama 337–8570, Japan
e-mail: zhai@shibaura-it.ac.jp

** Department of Electrical and Computer Engineering
California Baptist University, Riverside, CA 92504, USA
e-mail: xxu@calbaptist.edu

We establish a unified approach to stability analysis for switched linear descriptor systems under arbitrary switching in both continuous-time and discrete-time domains. The approach is based on common quadratic Lyapunov functions incorporated with linear matrix inequalities (LMIs). We show that if there is a common quadratic Lyapunov function for the stability of all subsystems, then the switched system is stable under arbitrary switching. The analysis results are natural extensions of the existing results for switched linear state space systems.

Keywords: switched linear descriptor systems, stability, arbitrary switching, linear matrix inequalities (LMIs), common quadratic Lyapunov functions.

1. Introduction

Bỏ qua

This paper is focused on analyzing stability properties for switched systems composed of a family of linear descriptor subsystems. As for descriptor systems, they are also known as singular systems or implicit systems and have good abilities concerning representing dynamical systems (Cobb, 1983; Lewis, 1986). Since they can preserve physical parameters in the coefficient matrices and describe the dynamic, static, and even improper part of the system in the same form, descriptor systems are much superior to those represented by state space models.

There have been many works on descriptor systems, which studied feedback stabilization (Cobb, 1983; Lewis, 1986), Lyapunov stability theory (Lewis, 1986; Takaba *et al.*, 1995; Ishida and Terra, 2001), the matrix inequality approach (Boyd *et al.*, 1994) for stabilization, $\mathcal{H}_2$ and/or $\mathcal{H}_\infty$ control (Masubuchi *et al.*, 1997; Uezato and Ikeda, 1999; Ikeda *et al.*, 2000), the infinite eigenvalue assignment by a feedback (Kaczorek, 2002),(2004).

On the other hand, there has been increasing interest recently in stability analysis and design for switched systems; see the survey papers (Liberzon and Morse, 1999; DeCarlo *et al.*, 2000; Sun and Ge, 2005a), the recent books (Liberzon, 2003; Sun and Ge, 2005b) and the references cited therein. One motivation for studying switched systems is that many practical systems are inherently multi-modal in the sense that several dynamical subsystems are required to describe their behavior which may depend on various environmental factors. Another important motivation is that switching among a set of controllers for a specified system can be regarded as a switched system, and that switching has been used in adaptive control to assure stability in situations where it cannot be proved otherwise, or to improve the transient response of adaptive control systems. Also, the methods of intelligent control design are based on the idea of switching among different controllers (Hespanha and Morse, 2002; Hu *et al.*, 2002).

We observe from the above that switched descriptor systems belong to an important class of systems that are interesting in both theoretic and practical sense. However, to the authors' best knowledge, there has not been many works dealing with such systems. The difficulty falls into two aspects. First, descriptor systems are not easy to tackle and there are not rich results available. Secondly, switching between several descriptor systems makes the problem more complicated and even not easy to make the motivation clear in some cases.

Next, let us review the classification of problems in switched systems. It is commonly recognized (Liberzon, 2003) that there are three basic problems in stability analysis and design of switched systems:

(i) find conditions for stability under arbitrary switching;

(ii) identify the limited but useful class of stabilizing switching laws;

(iii) construct a stabilizing switching law.

Specifically, Problem (i) deals with the case that all subsystems are stable. This problem seems trivial, but it is important since we can find many examples where all subsystems are stable but improper switchings can make the whole system unstable (Branicky, 1998). Furthermore, if we know that a switched system is stable under arbitrary switching, then we can consider higher control specifications for the system.

There have been several works for Problem (i) with state space systems. For example, Narendra and Balakrishnan (1994) showed that when all subsystems are stable and commutative pairwise, the switched linear system is stable under arbitrary switching. Liberzon *et al.* (1999) extended this result from the commutation condition to a Lie-algebraic condition. Zhai *et al.* (2001; 2002; 2006) extended the consideration to the case of $\mathcal{L}_2$ gain analysis and the case where both continuous-time and discrete-time subsystems exist, respectively. In our previous papers (Zhai *et al.*, 2009a; 2009b), we extended the existing result of Narendra and Balakrishnan (1994) to switched linear descriptor systems. In that context, we showed that in the case where all descriptor subsystems are stable, if the descriptor matrix and all subsystem matrices are commutative pairwise, then the switched system is stable under arbitrary switching. However, since the commutation condition is quite restrictive in real systems, alternative conditions are desired for the stability of switched descriptor systems under arbitrary switching.

In this paper, we propose a unified approach to the stability analysis of switched linear descriptor systems under arbitrary switching. The motivation is the same as in the case of switched state space systems. More precisely, even if all linear descriptor subsystems are stable, the switched system can be unstable when the switching is not done appropriately. A motivation example will be given later to illustrate this point. Since the existing results for the stability of switched state space systems suggest that the common Lyapunov functions condition should be less conservative than the commutation condition, we establish our approach based on common quadratic Lyapunov functions incorporated with linear matrix inequalities (LMIs). We show that if there is a common quadratic Lyapunov function for the stability of all descriptor subsystems, then the switched system is stable under arbitrary switching.

Since the results are consistent with those for switched state space systems when the descriptor matrix shrinks to an identity matrix, the results are natural but important extensions of the existing results. In addition, they establish reasonable extension of the results in (Zhai *et al.*, 2009a; 2009b), in the sense that if all descriptor subsystems are stable, and furthermore the descriptor matrix and all subsystem matrices are commutative pairwise, then there exists a common quadratic Lyapunov function for all subsystems, and thus the switched system is stable under arbitrary switching. We note that the approach is unified also in the sense that both continuous-time and discrete-time systems can be dealt with, except that the linear matrix inequalities are in different forms.

The rest of this paper is organized as follows. In Section 2, we formulate the problem under consideration and give some preliminaries. Section 3 states and proves the stability condition for switched linear continuous-time descriptor systems under arbitrary switching. The condition requires in fact a common quadratic Lyapunov function for the stability of all the subsystems, and includes the existing commutation conditions (Zhai *et al.*, 2009a; 2009b) as a special case. Section 4 establishes a parallel result in the discrete-time case. Finally, Section 5 concludes the paper.

Lần đầu phải đọc từ đây

2. Preliminaries and problem formulation

Let us first give some definitions on linear descriptor systems. Consider the linear continuous-time descriptor system

$$E\dot{x}(t) = Ax(t) \tag{1}$$

and the linear discrete-time descriptor system

$$Ex(k+1) = Ax(k)\,, \tag{2}$$

where $t \in \mathbb{R}$ denotes the continuous time, the nonnegative integer k denotes the discrete time, $x(t)(x(k)) \in \mathbb{R}^n$ is the descriptor variable, $E, A \in \mathbb{R}^{n\times n}$ are constant matrices. The matrix E may be singular and we denote its rank by $r = \operatorname{rank} E \le n$.

If $|sE - A| \not\equiv 0$ ($|zE - A| \not\equiv 0$), the linear descriptor system (1) ((2)) has a unique solution for any initial condition and is called *regular*. The finite eigenvalues of the matrix pair (E, A), that is, the solutions of $|sE - A| = 0$ ($|zE - A| = 0$), and the corresponding (generalized) eigenvectors define exponential modes of the system. If the finite eigenvalues lie in the open left half-plane of s (the open unit disc of z), the solution *decays exponentially*. The infinite eigenvalues of (E, A) with the eigenvectors satisfying the relations $Ex_1 = 0$ determines static modes. The infinite eigenvalues of (E, A) with generalized eigenvectors x_k satisfying the relations $Ex_1 = 0$ and $Ex_k = x_{k-1}$ ($k \ge 2$) create *impulsive modes*. The system has no impulsive mode if and only if rank $E = \deg|sE-A|$ ($\deg|zE-A|$). The system is said to be *stable* if it is regular and has only decaying exponential modes and static modes (without impulsive ones).

Lemma 1. (Weiertrass Form, (Cobb, 1983; Lewis, 1986)) *If the descriptor system (1) ((2)) is regular, then there exist two nonsingular matrices M and N such that*

$$MEN = \begin{bmatrix} I_d & 0 \\ 0 & J \end{bmatrix}, \quad MAN = \begin{bmatrix} \Lambda & 0 \\ 0 & I_{n-d} \end{bmatrix}, \tag{3}$$

where $d = \deg|sE-A|$ ($\deg|zE-A|$), J is composed of Jordan blocks for the finite eigenvalues. If the system (1) ((2)) is regular and there is no impulsive mode, then (3) holds with $d = r$ and $J = 0$. If the system (1) ((2)) is stable, then (3) holds with $d = r$, $J = 0$ and furthermore Λ is Hurwitz (Schur) stable.

Let the singular value decomposition (SVD) of E be

$$E = U \begin{bmatrix} E_{11} & 0 \\ 0 & 0 \end{bmatrix} V^T, \quad E_{11} = \mathrm{diag}\{\sigma_1, \ldots, \sigma_r\}, \tag{4}$$

where σ_i's are the singular values, U and V are orthonormal matrices ($U^TU = V^TV = I$). With the definitions

$$\bar{x} = V^T x \triangleq \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}, \; U^TAV = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \tag{5}$$

the differential (difference) equation in (1) ((2)) takes the form of

$$\begin{aligned} E_{11}\dot{\bar{x}}_1(t) &= A_{11}\bar{x}_1(t) + A_{12}\bar{x}_2(t), \\ 0 &= A_{21}\bar{x}_1(t) + A_{22}\bar{x}_2(t) \end{aligned} \tag{6}$$

or

$$\begin{aligned} E_{11}\bar{x}_1(k+1) &= A_{11}\bar{x}_1(k) + A_{12}\bar{x}_2(k), \\ 0 &= A_{21}\bar{x}_1(k) + A_{22}\bar{x}_2(k). \end{aligned} \tag{7}$$

From the above it is easy to obtain that the descriptor system is regular and has no impulsive modes if and only if A_{22} is nonsingular. Moreover, the system is stable if and only if A_{22} is nonsingular and furthermore $E_{11}^{-1}\left(A_{11} - A_{12}A_{22}^{-1}A_{21}\right)$ is Hurwitz (or Schur) stable. This discussion will be used again in the next section.

Next, we move to the problem formulation. In this paper, we consider a switched system composed of $\mathcal{N}$ linear continuous-time descriptor subsystems:

$$E\dot{x}(t) = A_i x(t) \tag{8}$$

or $\mathcal{N}$ linear discrete-time descriptor subsystems:

$$Ex(k+1) = A_i x(k), \tag{9}$$

where the vector $x \in \mathbb{R}^n$ and the descriptor matrix E are the same as in (1) and (2), the index i denotes the i-th subsystem and takes the value in the discrete set $\mathcal{I} = \{1, 2, \ldots, \mathcal{N}\}$, and thus the matrix A_i, together with E, represents the dynamics of the i-th subsystem.

Now we give the definition for the switched system. Given a switching sequence, the switched system (8) ((9)) is said to be *stable* if, starting from any initial value, the system's trajectories converge to the origin.

At the end of this section, we formulate the analysis problem, which will be dealt with in the next two sections.

Stability Analysis Problem: Assume that all the descriptor subsystems in (8) or (9) are stable. Establish the condition under which the switched system is stable under arbitrary switching.

Remark 1. There is a tacit assumption in the switched system described by (8) or (9) that the descriptor matrix E is the same in all the subsystems. Theoretically, this assumption is restrictive at present. However, as also discussed in (Zhai *et al.*, 2009a; 2009b)), the above problem settings and the results can later be applied to switching control problems for single linear descriptor systems. This is the main reason why we presently consider the same descriptor matrix E in the switched system. For example, if for a single descriptor system

$$E\dot{x}(t) = Ax(t) + Bu(t)$$

($Ex(k+1) = Ax(k) + Bu(k)$), where $u(t)$ ($u(k)$) is the control input, we have designed two stabilizing descriptor variable feedbacks $u = K_1x$, $u = K_2x$ and furthermore the switched system composed of the descriptor subsystems characterized by $(E, A+BK_1)$ and $(E, A+BK_2)$ are stable under arbitrary switching, then we can switch arbitrarily between the two controllers and thus consider higher control specifications. This kind of requirement is very important when we want more flexibility for multiple control specifications in real applications.

As mentioned in the introduction, the above-declared stability analysis problem is well posed (or practical) since a switched linear descriptor system can be unstable even if all the descriptor subsystems are stable. For better understanding, we give the following motivation example which is based on an example in (Branicky, 1998).

Example 1. Consider a switched system composed of two linear descriptor subsystems whose matrices are

$$\begin{aligned} E &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \\ A_1 &= \begin{bmatrix} -1 & 10 & 0 \\ -100 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -1 & 100 & 0 \\ -10 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \end{aligned} \tag{10}$$

Obviously, x_3 in both systems is always zero due to the algebraic equation constraint, and the pair (x_1, x_2) is actually dominated by switching among the differential equations

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} -1 & 10 \\ -100 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \\ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} -1 & 100 \\ -10 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \end{aligned} \tag{11}$$

As also pointed out in (Branicky, 1998), the elements x_1 and x_2 diverge very quickly when the descriptor subsystem (E, A_1) is activated in the second and fourth quadrants while the descriptor subsystem (E, A_2) is activated in the first and third quadrants. ♦

3. Stability analysis in the continuous-time domain

In this section, we first state and prove the stability condition, which is described by several LMIs. Then, we establish that the result is a nontrivial extension of the existing pairwise commutation stability condition.

3.1. LMI-based stability condition.

Theorem 1. *The switched system (8) is stable under arbitrary switching if there are matrices $P_i \in \mathbb{R}^{n\times n}$ satisfying*

$$E^T P_i = P_i^T E \geq 0, \tag{12}$$

$$A_i^T P_i + P_i^T A_i < 0, \tag{13}$$

$\forall i \in \mathcal{I}$, and furthermore

$$E^T P_i = E^T P_j\,, \quad \forall i, j \in \mathcal{I}\,, \quad i \neq j\,. \tag{14}$$

Proof. The conditions (12) and (13) guarantee that each descriptor system is stable (Masubuchi *et al.*, 1997). Thus, it is not difficult to see that the condition (14) deals with switching from the i-th subsystem to the j-th subsystem. This observation will be commented on more clearly later.

Since the rank of E is r, we can find nonsingular matrices M and N such that

$$MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}. \tag{15}$$

Then, we obtain from (12) that

$$N^T E^T M^T (M^{-1})^T P_i N = N^T P_i^T M^{-1} M E N \geq 0\,, \tag{16}$$

and define

$$(M^{-1})^T P_i N = \begin{bmatrix} P_{11}^i & P_{12}^i \\ P_{21}^i & P_{22}^i \end{bmatrix}$$

to reach

$$\begin{aligned} &\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_{11}^i & P_{12}^i \\ P_{21}^i & P_{22}^i \end{bmatrix} \\ &\quad = \begin{bmatrix} (P_{11}^i)^T & (P_{21}^i)^T \\ (P_{12}^i)^T & (P_{22}^i)^T \end{bmatrix} \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \geq 0\,. \end{aligned} \tag{17}$$

This results in $(P_{11}^i)^T = P_{11}^i \geq 0$, $P_{12}^i = 0$, and thus

$$(M^{-1})^T P_i N = \begin{bmatrix} P_{11}^i & 0 \\ P_{21}^i & P_{22}^i \end{bmatrix}. \tag{18}$$

Furthermore, from (13) we see that P_i must be nonsingular. This can be proved by contradiction: if P_i is singular, then there exists $x \neq 0$ such that $P_i x = 0$, which leads to $x^T(A_i^T P_i + P_i^T A_i)x = 0$. However, this is impossible due to (13). Since M and N are nonsingular, so is $(M^{-1})^T P_i N$, which implies that P_{11}^i is positive definite and P_{22}^i is nonsingular.

Similarly, from (14) we obtain that

$$N^T E^T M^T (M^{-1})^T P_i N = N^T E^T M^T (M^{-1})^T P_j N\,, \tag{19}$$

which is equivalent to

$$\begin{aligned} &\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_{11}^i & 0 \\ P_{21}^i & P_{22}^i \end{bmatrix} \\ &\quad = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_{11}^j & 0 \\ P_{21}^j & P_{22}^j \end{bmatrix}. \end{aligned} \tag{20}$$

Thus $P_{11}^i = P_{11}^j$, $\forall i \neq j \in \mathcal{I}$, and we modify (18) as

$$(M^{-1})^T P_i N = \begin{bmatrix} P_{11} & 0 \\ P_{21}^i & P_{22}^i \end{bmatrix}, \tag{21}$$

where P_{11} is positive definite and P_{22}^i is nonsingular.

Next, let

$$M A_i N = \begin{bmatrix} \bar{A}_{11}^i & \bar{A}_{12}^i \\ \bar{A}_{21}^i & \bar{A}_{22}^i \end{bmatrix} \tag{22}$$

and substitute it into the equivalent inequality of (13) as

$$N^T A_i^T M^T (M^{-1})^T P_i N + N^T P_i^T M^{-1} M A_i N < 0 \tag{23}$$

to reach

$$\begin{bmatrix} \Upsilon_{11} & \Upsilon_{12} \\ \Upsilon_{12}^T & \Upsilon_{22} \end{bmatrix} < 0\,, \tag{24}$$

where

$$\begin{aligned} \Upsilon_{11} &= (\bar{A}_{11}^i)^T P_{11} + P_{11} \bar{A}_{11}^i \\ &\quad + (\bar{A}_{21}^i)^T P_{21}^i + (P_{21}^i)^T \bar{A}_{21}^i, \\ \Upsilon_{12} &= (\bar{A}_{21}^i)^T P_{22}^i + P_{11} \bar{A}_{12}^i + (P_{21}^i)^T \bar{A}_{22}^i, \\ \Upsilon_{22} &= (\bar{A}_{22}^i)^T P_{22}^i + (P_{22}^i)^T \bar{A}_{22}^i\,. \end{aligned} \tag{25}$$

We declare that $\bar{A}_{22}^i$ is nonsingular from $\Upsilon_{22} < 0$. Otherwise, there is a nonzero vector v such that $\bar{A}_{22}^i v = 0$. Then, $v^T \Upsilon_{22} v < 0$ since $\Upsilon_{22} < 0$. However, by simple calculation,

$$v^T \Upsilon_{22} v = (\bar{A}_{22}^i v)^T P_{22}^i v + v^T (P_{22}^i)^T (\bar{A}_{22}^i v) = 0, \quad (26)$$

which results in a contradiction.

Premultiplying (24) by the nonsingular matrix

$$\begin{bmatrix} I_r & -(\bar{A}_{21}^i)^T \left((\bar{A}_{22}^i)^{-1}\right)^T \\ 0 & I_{n-r} \end{bmatrix}$$

and postmultiplying the result by its transpose, we obtain

$$\begin{bmatrix} (\tilde{A}_{11}^i)^T P_{11} + P_{11}\tilde{A}_{11}^i & * \\ \Upsilon_{12}^T - \Upsilon_{22}(\bar{A}_{22}^i)^{-1}\bar{A}_{21}^i & \Upsilon_{22} \end{bmatrix} < 0, \quad (27)$$

where $\tilde{A}_{11}^i = \bar{A}_{11}^i - \bar{A}_{12}^i(\bar{A}_{22}^i)^{-1}\bar{A}_{21}^i$.

Since the matrices E and A_i are transformed into (15) and (22), respectively, we use the well-known technique in stability analysis with the Weiertrass form (Lemma 1) to define the nonsingular transformation $\bar{x} = N^{-1}x = [\bar{x}_1^T \;\; \bar{x}_2^T]^T$, $\bar{x}_1 \in \mathbb{R}^r$. Then, all the descriptor subsystems in (8) take the form of

$$\begin{aligned} \dot{\bar{x}}_1 &= \bar{A}_{11}^i \bar{x}_1 + \bar{A}_{12}^i \bar{x}_2, \\ 0 &= \bar{A}_{21}^i \bar{x}_1 + \bar{A}_{22}^i \bar{x}_2\,, \end{aligned} \quad (28)$$

which is equivalent to

$$\dot{\bar{x}}_1 = \left[\bar{A}_{11}^i - \bar{A}_{12}^i(\bar{A}_{22}^i)^{-1}\bar{A}_{21}^i\right]\bar{x}_1 = \tilde{A}_{11}^i \bar{x}_1 \quad (29)$$

with

$$\bar{x}_2 = -(\bar{A}_{22}^i)^{-1}\bar{A}_{21}^i \bar{x}_1.$$

From (27) it is seen that

$$(\tilde{A}_{11}^i)^T P_{11} + P_{11}\tilde{A}_{11}^i < 0\,, \quad (30)$$

which means that all $\tilde{A}_{11}^i$'s are Hurwitz stable, and a common positive definite matrix P_{11} exists for the stability of all the subsystems in (29). Therefore, $\bar{x}_1$ converges to zero exponentially under arbitrary switching. The $\bar{x}_2$ part is dominated by $\bar{x}_1$ and thus also converges to zero exponentially. This completes the whole proof. ∎

Remark 2. When $E = I$ and all the subsystems are Hurwitz stable, the conditions (12)–(14) imply that there is a common positive definite matrix P satisfying $A_i^T P + PA_i < 0, \forall i \in \mathcal{I}$, which is exactly the existing stability condition for switched linear systems composed of $\dot{x}(t) = A_i x(t)$ under arbitrary switching (Narendra and Balakrishnan, 1994). Thus, Theorem 1 is an extension of the existing result for switched linear state space systems in the continuous-time domain.

Remark 3. From the proof of Theorem 1 it can be seen that $\bar{x}_1^T P_{11}\bar{x}_1$ is a common quadratic Lyapunov function for all the subsystems (29). Since the exponential convergence of $\bar{x}_1$ results in that of $\bar{x}_2$, we have enough reasons to regard $\bar{x}_1^T P_{11}\bar{x}_1$ as a common quadratic Lyapunov function for the whole switched system. In fact, this is rationalized by the following equation:

$$\begin{aligned} & x^T E^T P_i x \\ &= (N^{-1}x)^T (MEN)^T ((M^{-1})^T P_i N)(N^{-1}x) \\ &= \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}^T \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_{11} & 0 \\ P_{21}^i & P_{22}^i \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} \\ &= \bar{x}_1^T P_{11}\bar{x}_1. \end{aligned} \quad (31)$$

Therefore, although $E^T P_i$ is not positive definite and neither is $V_i(x) = x^T E^T P_i x$, we can regard this $V_i(x)$ as a common quadratic Lyapunov function for all the descriptor subsystems in the continuous-time domain. Moreover, if we consider

$$V = V_{\sigma(t)}(x) \triangleq x^T E^T P_{\sigma(t)} x, \quad (32)$$

where $\sigma(t)$ is the index of the activated subsystem at t, as a piecewise Lyapunov function candidate for the switched system, the condition (14) implies that there is no value jump when switchings occur. This is consistent with the existing results (Branicky, 1998) for general hybrid and switched systems.

Remark 4. The LMI conditions (12)–(14) include a non-strict matrix inequality, which may not be easy to solve using the existing LMI Control Toolbox in Matlab. As a matter of fact, the proof of Theorem 1 suggested an alternative method for solving it in the framework of strict LMIs:

(i) decompose E as in (15) using nonsingular matrices M and N;

(ii) compute MA_iN for each $i \in \mathcal{I}$ as in (22);

(iii) solve the strict LMIs (24) for all $i \in \mathcal{I}$ simultaneously with respect to $P_{11} > 0$, P_{21}^i and P_{22}^i;

(iv) compute the original P_i with

$$P_i = M^T \begin{bmatrix} P_{11} & 0 \\ P_{21}^i & P_{22}^i \end{bmatrix} N^{-1}$$

(motivated by (21)).

Remark 5. Note that the condition (14) should not be replaced with $P_i = P_j$, $\forall i \neq j$, as one might expect from the existing result for switched state space systems. The reason is that such setting leads to the obvious conservativeness of the result. For example, consider the switched system composed of two descriptor subsystems whose matrices are

$$E = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -I_r & 0 \\ 0 & I_{n-r} \end{bmatrix}, \; A_2 = \begin{bmatrix} -I_r & 0 \\ 0 & -I_{n-r} \end{bmatrix}. \tag{33}$$

It is easy to confirm that the switched system is stable under arbitrary switching, but we cannot find any common matrix P satisfying (13) for both A_1 and A_2. In fact, let

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}.$$

Then, the condition (13) requires that

$$\begin{bmatrix} -P_{11} - P_{11}^T & * \\ -P_{12}^T + P_{21} & P_{22} + P_{22}^T \end{bmatrix} < 0 \tag{34}$$

and

$$\begin{bmatrix} -P_{11} - P_{11}^T & * \\ -P_{12}^T - P_{21} & -P_{22} - P_{22}^T \end{bmatrix} < 0. \tag{35}$$

Focusing on the $(2,2)$-th block of the matrix on the left-hand side, one can easily see that the above two inequalities cannot hold simultaneously.

Although in the problem formulation we assumed that the descriptor matrix is the same for all the subsystems (as mentioned in Remark 1), from the proof of Theorem 1 it can be seen that what we really need is Eqn. (15). Therefore, Theorem 1 can be extended to the case where the subsystem descriptor matrices are different as in the following corollary.

Corollary 1. *Consider the switched system composed of $\mathcal{N}$ linear descriptor subsystems described by*

$$E_i \dot{x}(t) = A_i x(t), \tag{36}$$

where E_i is the descriptor matrix of the i-th subsystem and all the notation is the same as before. Assume that all the descriptor matrices have the same rank r and there are common nonsingular matrices M and N such that

$$ME_iN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad \forall i \in \mathcal{I}. \tag{37}$$

Then, the switched system (36) is stable under arbitrary switching if there are matrices $P_i \in \mathbb{R}^{n\times n}$ $(i = 1, \ldots, \mathcal{N})$ satisfying for every $i \in \mathcal{I}$

$$E_i^T P_i = P_i^T E_i \geq 0, \quad A_i^T P_i + P_i^T A_i < 0 \tag{38}$$

and, furthermore,

$$E_i^T P_i = E_j^T P_j, \quad \forall i, j \in \mathcal{I}, \; i \neq j. \tag{39}$$

3.2. Comparison with the pairwise commutation condition. In this subsection, we consider the relation of Theorem 1 with the result from (Zhai *et al.*, 2009a).

Lemma 2. (Zhai *et al.*, 2009a) *If all the descriptor subsystems are stable, and furthermore the matrices $E, A_1, \ldots, A_\mathcal{N}$ are commutative pairwise, i.e.,*

$$EA_i = A_iE, \quad A_iA_j = A_jA_i, \quad \forall i, j \in \mathcal{I}, \tag{40}$$

then the switched system is stable under arbitrary switching.

The above lemma establishes another sufficient condition for the stability of switched linear descriptor systems in the context of pairwise commutation. It is well known (Narendra and Balakrishnan, 1994) that in the case of switched linear systems composed of the state space subsystems

$$\dot{x}(t) = A_i x(t), \quad i \in \mathcal{I}, \tag{41}$$

where all subsystems are Hurwitz stable and the subsystem matrices commute pairwise ($A_iA_j = A_jA_i$, $\forall i, j \in \mathcal{I}$), there exists a common positive definite matrix P satisfying

$$A_i^T P + PA_i < 0. \tag{42}$$

One then tends to expect that if the commutation condition of Lemma 2 holds, then a common quadratic Lyapunov function $V(x) = x^T E^T P_i x$ should exist satisfying the condition of Theorem 1. This is exactly established in the following theorem.

Theorem 2. *If all the descriptor subsystems in (8) are stable, and furthermore the matrices $E, A_1, \ldots, A_\mathcal{N}$ are commutative pairwise, then there are matrices P_i ($i = 1, \ldots, \mathcal{N}$) satisfying (12)–(14), and thus the switched system is stable under arbitrary switching.*

Proof. For notational simplicity, we only prove the case of $\mathcal{N} = 2$. Since (E, A_1) is stable, according to Lemma 1, there exist two nonsingular matrices M, N such that

$$\begin{aligned} MEN &= \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \\ MA_1N &= \begin{bmatrix} \Lambda_1 & 0 \\ 0 & I_{n-r} \end{bmatrix}, \end{aligned} \tag{43}$$

where Λ_1 is a Hurwitz stable matrix. Here, without causing confusion, we use the same notations M, N as before.

Defining

$$N^{-1}M^{-1} = \begin{bmatrix} W_1 & W_2 \\ W_3 & W_4 \end{bmatrix} \tag{44}$$

and substituting it into the commutation condition $EA_1 = A_1E$ with

$$\begin{aligned} &(MEN)(N^{-1}M^{-1})(MA_1N) \\ &\quad = (MA_1N)(N^{-1}M^{-1})(MEN), \end{aligned} \tag{45}$$

we obtain

$$\begin{bmatrix} W_1\Lambda_1 & W_2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \Lambda_1 W_1 & 0 \\ W_3 & 0 \end{bmatrix}. \tag{46}$$

Thus, $W_1\Lambda_1 = \Lambda_1 W_1$, $W_2 = 0$, $W_3 = 0$.

Now, we use the same nonsingular matrices M, N for the transformation of A_2 and write

$$MA_2N = \begin{bmatrix} \Lambda_2 & X_1 \\ X_2 & X \end{bmatrix}. \tag{47}$$

According to another commutation condition $EA_2 = A_2E$,

$$\begin{bmatrix} W_1\Lambda_2 & W_1X_1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \Lambda_2 W_1 & 0 \\ X_2W_1 & 0 \end{bmatrix} \tag{48}$$

holds, and thus $W_1\Lambda_2 = \Lambda_2 W_1$, $W_1X_1 = 0$, $X_2W_1 = 0$. Since NM is nonsingular and $W_2 = 0, W_3 = 0$, W_1 has to be nonsingular. We then obtain $X_1 = 0, X_2 = 0$. Furthermore, since (E, A_2) is stable, Λ_2 is Hurwitz stable and X has to be nonsingular.

The third commutation condition $A_1A_2 = A_2A_1$ results in

$$\begin{bmatrix} \Lambda_1 W_1\Lambda_2 & 0 \\ 0 & W_4X \end{bmatrix} = \begin{bmatrix} \Lambda_2 W_1\Lambda_1 & 0 \\ 0 & XW_4 \end{bmatrix}. \tag{49}$$

We have $\Lambda_1W_1\Lambda_2 = \Lambda_2W_1\Lambda_1$. Combining this observation with $W_1\Lambda_1 = \Lambda_1W_1$, $W_1\Lambda_2 = \Lambda_2W_1$, we obtain

$$W_1\Lambda_1\Lambda_2 = \Lambda_1W_1\Lambda_2 = \Lambda_2W_1\Lambda_1 = W_1\Lambda_2\Lambda_1, \tag{50}$$

which implies that Λ_1 and Λ_2 are commutative ($\Lambda_1\Lambda_2 = \Lambda_2\Lambda_1$).

To summarize the above, we proceed to

$$MA_2N = \begin{bmatrix} \Lambda_2 & 0 \\ 0 & X \end{bmatrix}, \tag{51}$$

where Λ_2 is Hurwitz stable, X is nonsingular, and $\Lambda_1\Lambda_2 = \Lambda_2\Lambda_1$. According to the result from (Narendra and Balakrishnan, 1994), there is a common positive definite matrix P_{11} satisfying $\Lambda_i^T P_{11} + P_{11}\Lambda_i < 0$, $i = 1, 2$. Then, with the definition

$$\begin{aligned} P_1 &= M^T \begin{bmatrix} P_{11} & 0 \\ 0 & -I_{n-r} \end{bmatrix} N^{-1}, \\ P_2 &= M^T \begin{bmatrix} P_{11} & 0 \\ 0 & -X \end{bmatrix} N^{-1}, \end{aligned} \tag{52}$$

it is easy to confirm that

$$(MEN)^T((M^{-1})^T P_1 N) = \begin{bmatrix} P_{11} & 0 \\ 0 & 0 \end{bmatrix} \geq 0 \tag{53}$$

and

$$\begin{aligned} &(MA_1N)^T((M^{-1})^T P_1 N) \\ &\quad + ((M^{-1})^T P_1 N)^T (MA_1N) \\ &\quad = \begin{bmatrix} \Lambda_1^T P_{11} + P_{11}\Lambda_1 & 0 \\ 0 & -I_{n-r} \end{bmatrix} < 0, \\ &(MA_2N)^T((M^{-1})^T P_2 N) \\ &\quad + ((M^{-1})^T P_2 N)^T (MA_2N) \\ &\quad = \begin{bmatrix} \Lambda_2^T P_{11} + P_{11}\Lambda_2 & 0 \\ 0 & -X^T X \end{bmatrix} < 0. \end{aligned} \tag{54}$$

Since P_{11} is common for $i = 1, 2$ and N is nonsingular, (53) and (54) imply that the matrices in (52) satisfy the conditions (12)–(14). ■

3.3. Numerical example.

In this subsection, we provide a simple example illustrating the main result.

Example 2. Consider a switched system composed of two linear descriptor subsystems whose coefficient matrices are

$$\begin{aligned} E &= \begin{bmatrix} -2 & -5 & 3 \\ 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}, \\ A_1 &= \begin{bmatrix} -7 & 4 & -12 \\ 0 & -1 & 1 \\ 2 & -1 & 3 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 1 & 7 & -7 \\ -1 & -1 & 0 \\ 0 & -2 & 2 \end{bmatrix}. \end{aligned} \tag{55}$$

Note that these matrices do not satisfy the pairwise commutation condition required in Lemma 2, and thus the stability under arbitrary switching cannot be guaranteed by the result in (Zhai *et al.*, 2009a) or other references.

To solve the nonstrict LMIs (12) and (13), we use the procedure described in Remark 4. With the nonsingular matrices

$$\begin{aligned} M &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix}, \\ N &= \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix}, \end{aligned} \tag{56}$$

the descriptor matrix E is decomposed satisfying (15). Then, solving the strict LMI (24) for $i = 1, 2$ with respect to $P_{11} > 0$, P_{21}^i, P_{22}^i and computing the original P_i

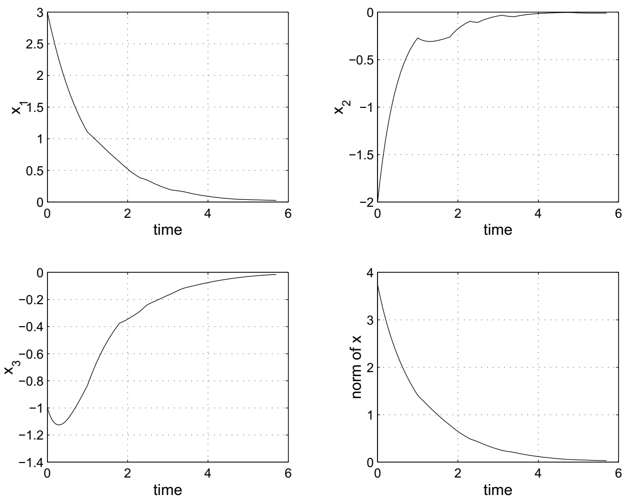

Fig. 1. Trajectories of x_1, x_2, x_3 and norm convergence.

with

$$P_i = M^T \begin{bmatrix} P_{11} & 0 \\ P_{21}^i & P_{22}^i \end{bmatrix} N^{-1},$$

we obtain

$$P_1 = \begin{bmatrix} 0.4344 & 0.1251 & 1.4708 \\ 1.9914 & 1.1964 & 3.1180 \\ 1.1268 & 0.5877 & 4.0235 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1.7292 & 1.3588 & 2.6677 \\ 4.5810 & 3.6638 & 5.5118 \\ 5.0111 & 4.2888 & 7.6142 \end{bmatrix}. \tag{57}$$

It is easy to confirm that these matrices satisfy the conditions (12) and (13). Therefore, according to Theorem 1, the switched system is stable under arbitrary switching.

In fact, activating the two systems alternately with a randomly generated time period series

$$1.0\,,\ 0.8\,,\ 0.5\,,\ 0.2\,,\ 0.6\,,\ 0.3\,,\ 1.4\,,\ 0.9\,,\ \dots \tag{58}$$

(activating (E, A_1) with the time period 1.0 and then (E, A_2) with the time period 0.8 and so on), the trajectories of the system states (with the initial state $x_0 = [3\ \ -2\ \ -1]^T$) and the norm convergence are shown in Fig. 1. Obviously, the switched system is stable. ♦

4. Stability analysis in the discrete-time domain

Theorem 3. *The switched system (9) is stable under arbitrary switching if there are nonsingular symmetric matrices $P_i \in \mathbb{R}^{n\times n}$ satisfying for every $i \in \mathcal{I}$*

$$E^T P_i E \geq 0, \tag{59}$$

$$A_i^T P_i A_i - E^T P_i E < 0, \tag{60}$$

and, furthermore,

$$E^T P_i E = E^T P_j E\,,\ \ \forall i, j \in \mathcal{I},\ i \neq j. \tag{61}$$

Proof. Similarly as in the proof of Theorem 1, the conditions (59) and (60) guarantee that each descriptor system is stable (Xu and Yang, 1999), while the condition (61) deals with switching from the i-th to the j-th subsystem.

Again, since the rank of E is r, we first find nonsingular matrices M and N such that (15) holds. Then, from (59) we obtain that

$$N^T E^T M^T (M^{-1})^T P_i M^{-1} M E N = \begin{bmatrix} P_{11}^i & 0 \\ 0 & 0 \end{bmatrix} \geq 0\,, \tag{62}$$

where

$$(M^{-1})^T P_i M^{-1} \triangleq \begin{bmatrix} P_{11}^i & P_{12}^i \\ (P_{12}^i)^T & P_{22}^i \end{bmatrix}. \tag{63}$$

Since P_i (and thus $(M^{-1})^T P_i M^{-1}$) is symmetric and nonsingular, we obtain $P_{11}^i > 0$.

Again, from (61) we get

$$N^T E^T M^T (M^{-1})^T P_i M^{-1} M E N = N^T E^T M^T (M^{-1})^T P_j M^{-1} M E N\,, \tag{64}$$

and thus $P_{11}^i = P_{11}^j$, $\forall i, j \in \mathcal{I}$. From now on, we let $P_{11}^i = P_{11}$ for notational simplicity.

Define $M A_i N$ as in (22) and substitute it into the equivalent inequality of (60) as

$$N^T A_i^T M^T (M^{-1})^T P_i M^{-1} M A_i N - N^T E^T M^T (M^{-1})^T P_i M^{-1} M E N < 0 \tag{65}$$

to reach

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ \Lambda_{12}^T & \Lambda_{22} \end{bmatrix} < 0\,, \tag{66}$$

where

$$\begin{aligned} \Lambda_{11} &= (\bar{A}_{11}^i)^T P_{11} \bar{A}_{11}^i - P_{11} + (\bar{A}_{21}^i)^T (P_{12}^i)^T \bar{A}_{11}^i \\ &\quad + (\bar{A}_{11}^i)^T P_{12}^i \bar{A}_{21}^i + (\bar{A}_{21}^i)^T P_{22}^i \bar{A}_{21}^i, \\ \Lambda_{12} &= (\bar{A}_{11}^i)^T P_{11} \bar{A}_{12}^i + (\bar{A}_{11}^i)^T P_{12}^i \bar{A}_{22}^i \\ &\quad + (\bar{A}_{21}^i)^T (P_{12}^i)^T \bar{A}_{12}^i + (\bar{A}_{21}^i)^T P_{22}^i \bar{A}_{22}^i, \\ \Lambda_{22} &= (\bar{A}_{12}^i)^T P_{11} \bar{A}_{12}^i + (\bar{A}_{22}^i)^T (P_{12}^i)^T \bar{A}_{12}^i \\ &\quad + (\bar{A}_{12}^i)^T P_{12}^i \bar{A}_{22}^i + (\bar{A}_{22}^i)^T P_{22}^i \bar{A}_{22}^i\,. \end{aligned} \tag{67}$$

At this point, we declare that $\bar{A}^i_{22}$ is nonsingular from $\Lambda_{22} < 0$. Otherwise, there is a nonzero vector v such that $\bar{A}^i_{22}v = 0$. Then, $v^T\Lambda_{22}v < 0$. However, by simple calculation,

$$v^T\Lambda_{22}v = v^T(\bar{A}^i_{12})^T P_{11}\bar{A}^i_{12}v \geq 0 \tag{68}$$

since P_{11} is positive definite. This results in a contradiction.

Similarly as in the proof of Theorem 1, premultiply (66) by

$$\begin{bmatrix} I_r & -(\bar{A}^i_{21})^T\left((\bar{A}^i_{22})^{-1}\right)^T \\ 0 & I_{n-r} \end{bmatrix}$$

and postmultiply the result by its transpose, and obtain

$$\begin{bmatrix} (\tilde{A}^i_{11})^T P_{11}\tilde{A}^i_{11} - P_{11} & * \\ \Lambda^T_{12} - \Lambda_{22}(\bar{A}^i_{22})^{-1}\bar{A}^i_{21} & \Lambda_{22} \end{bmatrix} < 0\,, \tag{69}$$

where $\tilde{A}^i_{11}$ is the same as in the proof of Theorem 1.

The remaining proof is almost the same as before. With the same nonsingular transformation $\bar{x}(k) = N^{-1}x(k) = [\bar{x}^T_1(k)\ \ \bar{x}^T_2(k)]^T$, $\bar{x}_1(k) \in \mathbb{R}^r$, all the descriptor subsystems in (9) take the form of

$$\begin{aligned} \bar{x}_1(k+1) &= \bar{A}^i_{11}\bar{x}_1(k) + \bar{A}^i_{12}\bar{x}_2(k), \\ 0 &= \bar{A}^i_{21}\bar{x}_1(k) + \bar{A}^i_{22}\bar{x}_2(k)\,, \end{aligned} \tag{70}$$

which is equivalent to

$$\bar{x}_1(k+1) = \tilde{A}^i_{11}\bar{x}_1(k) \tag{71}$$

with $\bar{x}_2(k) = -(\bar{A}^i_{22})^{-1}\bar{A}^i_{21}\bar{x}_1(k)$.

Also, from (69) it is seen that

$$(\tilde{A}^i_{11})^T P_{11}\tilde{A}^i_{11} - P_{11} < 0\,, \tag{72}$$

which means that all $\tilde{A}^i_{11}$'s are Schur stable, and a common positive definite matrix P_{11} exists for the stability of all the subsystems in (71). Therefore, $\bar{x}_1(k)$ converges to zero exponentially under arbitrary switching. The part $\bar{x}_2(k)$ is dominated by $\bar{x}_1(k)$ and thus also converges to zero exponentially. This completes the proof. ■

Remark 6. When $E = I$ and all the subsystems are Schur stable, the condition of Theorem 3 actually requires a common positive definite matrix P satisfying $A^T_i PA_i - P < 0, \forall i \in \mathcal{I}$, which is exactly the existing stability condition for switched linear systems composed of $x(k+1) = A_i x(k)$ under arbitrary switching (Narendra and Balakrishnan, 1994). Thus, Theorem 3 is an extension of the existing result for switched linear state space subsystems in the discrete-time domain.

Remark 7. From the proof of Theorem 2 it can be seen that $\bar{x}^T_1 P_{11}\bar{x}_1$ is a common quadratic Lyapunov function for all the subsystems (71). As in Remark 3, since the exponential convergence of $\bar{x}_1$ results in that of $\bar{x}_2$, we can regard $\bar{x}^T_1 P_{11}\bar{x}_1$ as a common quadratic Lyapunov function for the whole switched system. In fact, this is rationalized by the following equation:

$$\begin{aligned} & x^T E^T P_i E x \\ &= (N^{-1}x)^T(MEN)^T((M^{-1})^T P_i M^{-1}) \\ &\quad \times (MEN)(N^{-1}x) \\ &= \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}^T \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} P_{11} & P^i_{12} \\ (P^i_{12})^T & P^i_{22} \end{bmatrix} \\ &\quad \times \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix} \\ &= \bar{x}^T_1 P_{11}\bar{x}_1. \end{aligned} \tag{73}$$

Therefore, although $E^T P_i E$ is not positive definite and neither is $V_i(x) = x^T E^T P_i E x$, we can regard this $V_i(x)$ as a common quadratic Lyapunov function for all the descriptor subsystems in the discrete-time domain. Similarly as in Remark 3, if we consider (32) as a piecewise Lyapunov function candidate for the switched system, the condition (61) implies that there is no value jump when switchings occur.

We also state results parallel with Corollary 1 and Theorem 2, and omit the proofs since they can be proved similarly as in the continuous-time case.

Corollary 2. *Consider a switched system composed of $\mathcal{N}$ linear descriptor subsystems:*

$$E_i x(k+1) = A_i x(k)\,, \tag{74}$$

where E_i is the descriptor matrix of the i-th subsystem and the notation is the same as before. Assume that all the descriptor matrices have the same rank r and there are common nonsingular matrices M and N such that (37) holds. Then the switched system (74) is stable under arbitrary switching if there are symmetric nonsingular matrices $P_i \in \mathbb{R}^{n\times n}$ $(i = 1, \dots, \mathcal{N})$ satisfying for each $i \in \mathcal{I}$

$$E^T_i P_i E_i \geq 0\,,\ \ A^T_i P_i A_i - E^T_i P_i E_i < 0 \tag{75}$$

and furthermore

$$E^T_i P_i E_i = E^T_j P_j E_j\,,\ \ \forall i,j \in \mathcal{I}\,,\ \ i \neq j\,. \tag{76}$$

Theorem 4. *If all the descriptor subsystems in (9) are stable, and furthermore the matrices E, $A_1, \dots, A_{\mathcal{N}}$ are commutative pairwise, then there are nonsingular symmetric matrices P_i $(i = 1, \dots, \mathcal{N})$ satisfying (59)–(61), and thus the switched system is stable under arbitrary switching.*

5. Concluding remarks

We have established a unified approach to stability analysis for switched linear descriptor systems under arbitrary switching in both continuous-time and discrete-time domains. More precisely, we have shown that if there is a common quadratic Lyapunov function for the stability of all subsystems, then the switched system is stable under arbitrary switching. As has been mentioned in the remarks, the common quadratic Lyapunov functions proposed are not positive definite with respect to all states, but they actually play the role of Lyapunov functions as in the classical Lyapunov stability theory. The approach in this paper is unified in the sense that it is valid for both continuous-time and discrete-time systems, and it is a natural extension of the existing approach for switched linear state space systems. We also note that the approach is unified since it can be extended to $\mathcal{L}_2$ gain analysis of switched linear descriptor systems by modifying the matrix inequalities correspondingly (Zhai and Xu, 2009).

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Guisheng Zhai received the B.Sc. degree from Fudan University, China, in 1988, and the M.Eng. and Ph.D. degrees, both in systems science, from Kobe University, Japan, in 1993 and 1996, respectively. He was an assistant professor at Wakayama University, Japan, during the time period of 1998–2004, and an associate professor at Osaka Prefecture University, Japan, in the years 2004–2010. He has also held visiting professor positions at the University of Notre Dame, USA, and Central South University, China. Since April 2010, he has been an associate professor in the Department of Mathematical Sciences, Shibaura Institute of Technology, Japan. His research interests include large scale and decentralized control systems, robust control, switched systems and switching control, networked control systems, neural networks and signal processing, etc. He has published more than 180 international journal and conference papers and has been on editorial boards of several academic journals. He is a Senior Member of the IEEE, a member of the ISCIE, SICE, JSST and JSME.

Xuping Xu received B.Sc. degrees in electrical engineering and applied mathematics from Shanghai Jiao Tong University, Shanghai, China, in 1995. He received M.Sc. degrees in electrical engineering and applied mathematics, and the Ph.D. degree in electrical engineering from the University of Notre Dame, Notre Dame, IN, in 1998, 1999, and 2001, respectively. In 2008, Dr. Xu joined the School of Engineering at California Baptist University, where he is currently an associate professor. Between 2001 and 2008, he was an assistant professor and subsequently an associate professor in the Department of Electrical and Computer Engineering at Penn State Erie, Erie, PA. His current research interests include systems and control, hybrid systems, embedded systems, digital design, software/hardware enabled control applications, and optimization. Dr. Xu is a member of the IEEE and an associate editor of the Conference Editorial Board of the IEEE Control Systems Society. He also actively serves as a reviewer for a number of journals and conferences.

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