1. Tìm khai triển Taylor tại $x_0 = 2$ đến cấp 3 của hàm

$$f(x) = \frac{2x-1}{x-1} \Rightarrow f(2) = 3$$

$$f'(x) = \frac{-1}{(x-1)^2} \Rightarrow f'(2) = -1$$

$$f''(x) = \frac{2x-2}{(x^2-2x+1)^2} \Rightarrow f''(2) = 2$$

$$f'''(x) = \frac{2(x-1)^4 - (2x-2)(4x^3+12x-5x^2-2)}{(x-1)^8}$$

$$\Rightarrow f'''(2) = 30$$

AD khai triển Taylor :

$$f(x) = \frac{2x-1}{x-1} = f(2) + \frac{f'(2)}{1!}(x-2) + \frac{f''(2)}{2!}(x-2)^2 + \frac{f'''(2)}{3!}(x-2)^3$$

$$= 3 + \frac{-1}{1!}(x-2) + \frac{2}{2!}(x-2)^2 + \frac{30}{3!}(x-2)^3 + o(x-2)$$

$$= 3 - (x-2) - (x-2)^2 + 5(x-2)^3 + o(x-2)^3$$

2. Tìm khai triển Taylor tại $x_0 = 2$ đến cấp 3 của hàm.

$$f(x) = \frac{x-1}{x^2-5x+6} = \frac{x-1}{(x-3)(x-2)}$$

$$= \frac{A}{(x-3)} + \frac{B}{(x-2)} = \frac{2}{x-2} + \frac{-1}{x-3}$$

$$f^{(n)}(x) = \left(\frac{2}{x-2}\right)^{(n)} + \left(\frac{-1}{x-3}\right)^{n}$$

$$= 2 \cdot \frac{(-1)^n n!}{(x-2)^{n+1}} - \frac{(-1)^n n!}{(x-3)^{n+1}}$$

$$\Rightarrow f'(x) = \frac{-2}{(x-2)^2} + \frac{1}{(x-3)^2} \Rightarrow f'(2) =$$

Có ĐK $f(x)$: $\begin{cases} x \neq 3 \\ x \neq 2 \end{cases}$ mà $x_0 = 2$

$\Rightarrow$ ĐK để bài ? thoả mãn

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3. Tìm khai triển Taylor tại $x_0 = 1$ đến cấp 3 của hàm.

$$f(x) = \ln(2+3x)$$

$$f^{(n)}(x) = (-1)^{n-1}(n-1)! \cdot \frac{3^n}{(3x+2)^n}$$

$$f'(x) = \frac{3}{3x+2} \Rightarrow f'(1) = \frac{3}{5}$$

$$f''(x) = \frac{-9}{(3x+2)^2} \Rightarrow f''(1) = \frac{-9}{25}$$

$$f'''(x) = \frac{54}{(3x+2)^3} \Rightarrow f'''(1) = \frac{54}{125}$$

AD Khai triển Taylor

$$f(x) = \ln(2+3x) = f(1) + \frac{f'(1)}{1!}(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + O(x-1)^3$$

$$= \ln 5 + \frac{3}{5}(x-1) + \frac{-9}{50}(x-1)^2 + \frac{9}{125}(x-1)^3 + O(x-1)^3$$

Bài 4 Tìm đạo hàm cấp n

1, $(x-1).2^{x-1} = y$

Đặt $u = x-1$, $v = 2^{x-1}$

Có $u' = 1$, $u'' = 0$

$$v^{(n)} = 2^{x-1}.\ln^n(2)$$

ADCT Newton - Leibniz:

$$y^{(n)} = (u.v)^n = \sum_{k=0}^{n} C_n^k . u^k . v^{(n-k)}$$

$$= C_n^0 (x-1).2^{x-1}\ln^n 2 + C_n^1 2^{x-1}(\ln 2)^{n-1} + 0 + 0 \dots$$

$$= (x-1)2^{x-1}(\ln 2)^n + n.2^{x-1}(\ln 2)^{n-1} + 0 \dots + 0$$

2. $y = x \ln \frac{3+x}{3-x}$.

Đặt $u = x \Rightarrow u' = 1$

$v = \ln \frac{3+x}{3-x} = \ln(3+x) - \ln(3-x)$

$$\Rightarrow v^{(n)} = (-1)^{n-1} \cdot \frac{(n-1)!}{(3+x)^n} - \frac{(-1)^{n-1}(n-1)!(-1)^n}{(3-x)^n}$$

$$= (-1)^{n-1} \frac{(n-1)!}{(3+x)^n} - \cancel{(-1)} \frac{(n-1)!}{(3-x)^n}$$

ADCT Newton - Leibniz:

$$y^{(n)} = C_n^0 \cdot x \left[\frac{(-1)^{n-1} \cdot (n-1)!}{(3+x)^n} - \frac{(n-1)!}{(3-x)^n} \right]$$

$$+ C_n^1 \cdot \left[\frac{(-1)^{n-1}(n-1)!}{(3+x)^n} - \frac{(n-1)!}{(3-x)^n} \right]$$

3. $y = x \ln(x^2 - 3x + 2) = x . \ln[(x-2)(x-1)]$

$= x [\ln(x-2) + \ln(x-1)]$

Đặt $u = x \Rightarrow u' = 1$

$v = \ln(x^2 - 3x + 2) = \ln(x-2) + \ln(x-1)$

$$\Rightarrow v^{(n)} = \frac{(-1)^{n-1}(n-1)!}{(x-2)^n} + \frac{(-1)^{n-1} \cdot (n-1)!}{(x-1)^n}$$

ADCT Newton - leibniz:

$$y^{(n)} = C_n^0 . x . (-1)^{n-1}(n-1)! . \left(\frac{1}{(x-2)^n} + \frac{1}{(x-1)^n} \right)$$

$$+ C_n^1 . (-1)^{n-1} . (n-1)! . \left(\frac{1}{(x-2)^n} + \frac{1}{(x-1)^n} \right)$$

4, $(x^2 + x) . \cos^2 x$

Đặt $u = x^2 + x \Rightarrow u' = 2x + 1$, $u'' = 2$.

$v = \cos^2 x = \frac{\cos 2x + 1}{2}$

$$\Rightarrow v^{(n)} = \frac{1}{2} . 2^n \cos\left(2x + \frac{n\pi}{2}\right) + \left(\frac{1}{2}\right)^n$$

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ADCT Newton leibniz

$$y^{(n)} = C_n^0 (x^2+x)\left[\frac{1}{2}\, 2^n \cos\left(2x+\frac{n\pi}{2}\right) - \left(\frac{1}{2}\right)^n\right]$$

$$+ C_n^1 (2x+1)\left[\frac{1}{2}\, 2^{n-1} \cos\left(2x+\frac{(n-1)\pi}{2}\right) - \left(\frac{1}{2}\right)^{n-1}\right] + C_n^2 \left[2^{n-1} \cos\left(2x+\frac{(n-2)\pi}{2}\right) - \left(\frac{1}{2}\right)^{n-2}\right]$$

5. $y = (3-2x)^2 \cdot e^{2-3x}$

Đặt $u = (3-2x)^2 = 4x^2 - \cancel{8x}\ 12x + 9$

$\Rightarrow u' = 8x - 12$, $u'' = 8$

$v = e^{2-3x}$

$v^{(n)} = (-3)^n \cdot e^{2-3x}$

ADCT Newton leibniz:

$$y^{(n)} = C_n^0 (4x^2-12x+9)\cdot(-3)^n e^{2-3x} + C_n^1 (8x-12)\cdot(-3)^{n-1} \cdot e^{2-3x}$$

$$+ C_n^2 \cdot 8 \cdot (-3)^{n-2} \cdot e^{2-3x}$$

$$= (4x^2-12x+9)(-3)^n \cdot e^{2-3x} + n(8x-12)\cdot(-3)^{n-1} \cdot e^{2-3x}$$

$$+ n(n-1) \cdot 4 \cdot (-3)^{n-2}\, e^{2-3x}$$

Bài 5. Tìm khai triển Maclaurin đến cấp n

1. $y = \frac{x^2 + 3e^x}{e^{2x}}$, $n = 3$

$y(0) = 3$

$y' = \frac{2x + x^2}{e^{2x}} - \frac{3}{e^x} \Rightarrow y'(0) = -3$

$y'' = \frac{-2x^2 - 2x + 2}{e^{2x}} + \frac{3}{e^x} \Rightarrow y''(0) = 5$

$y''' = \frac{4x^2 - 2}{e^{2x}} - \frac{3}{e^x} \Rightarrow y'''(0) = -5$

KT Maclaurin $\Rightarrow y = y(0) + \frac{y'(0)}{1!}x + \frac{y''(0)}{2!}x^2 + \frac{y'''(0)}{3!}x^3 + o($

$= 3 - 3x + \frac{5}{2!}x^2 - \frac{5}{3!}x^3 + o(x^3) \approx O(x^3)$

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2. $\ln \dfrac{2-3x}{3+2x}$, $n = 3$

$y(0) = \dfrac{2}{3}$

$y = \ln \dfrac{2-3x}{3+2x} = \ln(2-3x) - \ln(3+2x)$

$\Rightarrow y^{(n)} = \dfrac{(-1)^{n-1} \cdot (n-1)! \, (-3)^n}{(2-3x)^n} - \dfrac{(-1)^{n-1} \cdot (n-1)! \cdot (2)^n}{(3+2x)^n}$

$y^{(1)}(0) = -\dfrac{13}{6}$, $y^{(2)}(0) = \dfrac{-65}{36}$, $y^{(3)}(0) = -\dfrac{793}{103}$

$\Rightarrow y = \dfrac{2}{3} + \dfrac{-13/6}{1!} x - \dfrac{65/36}{2!} \cdot x^2 - \dfrac{793/103}{3!} x^3 + o(x^3)$

$= \cancel{-3,68} + o(\cancel{x^3})$

3. $\ln(x^2 + 3x + 2)$, $n = 4$.

$y(0) = \ln 2$

$y = \ln(x^2 + 3x + 2) = \ln(x+2) + \ln(x+1)$

$y^{(n)} = \dfrac{(-1)^{n-1} (n-1)!}{(x+2)^n} + \dfrac{(-1)^{n-1} (n-1)!}{(x+1)^n}$

$\Rightarrow y^{(1)}(0) = \dfrac{3}{2}$, $y^{(2)}(0) = \dfrac{-5}{4}$, $y^{(3)}(0) = \dfrac{9}{4}$, $y^{(4)}(0) = \dfrac{-51}{8}$

Ma laurin

$\Rightarrow y = \ln 2 + \dfrac{3/2}{1!} x - \dfrac{5/4}{2!} x^2 + \dfrac{9/4}{3!} x^3 - \dfrac{51/8}{4!} x^4 + o(x^4)$

$= \cancel{\ln 2} + \cancel{\dfrac{63}{64}} + \cancel{o(x^4)}$

Bài 6 Tìm khai triển Taylor.

1. $(x^2 - 1) e^{2x} = y$

Đặt $u = x^2 - 1$, $u' = 2x$, $u'' = 2$

$v = e^{2x}$, $v^{(n)} = 2^n \cdot e^{2x}$

ADCT Newton Leibniz:

$y^{(n)} = C_n^0 (x^2 - 1) 2^n \cdot e^{2x} + C_n^1 \cdot 2x \cdot 2^{n-1} \cdot e^{2x} + C_n^2 \cdot 2 \cdot 2^{n-2} \cdot e^{2x}$

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$\Rightarrow y^{(n)} = (x^2-1)\,2^n e^{2x} + n.2x.2^{n-1}.e^{2x} + n(n-1).2^{n-2}e^{2x}$

$\Rightarrow y'(-1) = -2e^{-2}$, $y'''(-1) = -12e^{-2}.$

$y''(-1) = -4e^{-2}$

Khai triển Taylor:

$\cancel{y = y(-1) + \frac{y'(-1)}{1!}(x+1) + \frac{y''(-1)}{2!}x+1} + \frac{y'''(-1)}{3!} + o(x^3)$

$$y = y(-1) + \frac{y'(-1)}{1!}(x+1) + \frac{y''(-1)}{2!}(x+1)^2 + \frac{y'''(-1)}{3!}(x+1)^3 + o(x+1)^3$$

$$= 0 + \frac{-2e^{-2}}{1!}(x+1) + \frac{-4.e^{-2}}{2!}(x+1)^2 + \frac{-12.e^{-2}}{3!}(x+1)^3 + o(x+1)^3.$$

2. $y = \ln(2x+1)$, $x = 1/2$, $n = 3$.

$$y^{(n)} = \frac{(-1)^{n-1}.(n-1)!\,2^n}{(2x+1)^n}$$

$y^{(1)}(1/2) = 1$ $y^{(2)}(1/2) = -1$, $y^{(3)}(1/2) = 2$

KT Maclaurin $y = y(1/2) + \frac{y^{(1)}(1/2)}{1!}\left(x-\frac{1}{2}\right) + \frac{y^{(2)}(1/2)}{2!}\left(x-\frac{1}{2}\right)^2 + \frac{y^{(3)}(1/2)}{3!}\left(x-\frac{1}{2}\right)^3 + o\left(x-\frac{1}{2}\right)^3$

$$= \ln 2 + x - \frac{1}{2} - 2\left(x-\frac{1}{2}\right)^2 + \frac{1}{3}\left(x-\frac{1}{2}\right)^3 + o\left(x-\frac{1}{2}\right)^3$$

Bài 7. Tính các gh sau

$$\lim_{x\to 0} \frac{\cos x - 1 + \frac{x^2}{2}}{x^4}$$

Khai triển Maclaurin với $\cos x$: $\cos x = 1 - \frac{x^2}{2} + o(x^2)$

$$\Rightarrow L = \lim_{x\to 0} \frac{1 - \frac{x^2}{2} - 1 + \frac{x^2}{2} + o(x^2)}{x^4}$$

$$= 0$$

2. $\lim_{x\to 0} \dfrac{\arctan x - \arcsin x}{\tan x - \sin x}$

ADCT Maclaurin của $\arctan x$, $\tan x$, $\sin x$ và $\arcsin x$,

có $L = \lim_{x\to 0} \dfrac{x - \frac{x^3}{3} - \left(x + \frac{x^3}{6}\right)}{x + \frac{x^3}{3} - \left(x - \frac{x^3}{6}\right)} = \lim_{x\to 0} \dfrac{-1/2\, x^3}{1/2\, x^3}$

$= -1.$

3. $\lim_{x\to 0} \dfrac{1 + x\cos x - \sqrt{1+2x}}{\ln(1+x) - x}.$

AD Maclaurin của $\cos x$ và $\ln(1+x)$

$\lim_{x\to 0} \dfrac{1 + x\left(1 - \frac{x^2}{2}\right) - \sqrt{1+2x}}{x - \frac{x^2}{2} - x}$

$= \lim_{x\to 0} \dfrac{2 + 2x - \frac{x^3}{2} - 2\sqrt{1+2x}}{x^2}.$

Đề thi.

Câu 3: Khai triển theo Maclaurin tới số hạng chứa x^2

$f(x) = \ln(1 + 2x + 3x^2)$

ADCT Khai triển Maclaurin hàm cơ bản

$\ln(1+u)$ với $u = 2x + 3x^2$

$f(x) = \ln(1 + 2x + 3x^2) = (2x + 3x^2) - \dfrac{(2x + 3x^2)^2}{2} + o[(2x+3x^2)^2]$

$= 2x + 3x^2 - 2x^2 + o(x^2)$

$= 2x + x^2 + o(x^2)$

Câu 3 Hãy sd khai triển Maclaurin để tìm $g^{(2016)}(0)$ của hàm số.

$g(x) = (1 + x^3)\, e^{x^3}$

ADCT Newton Leibniz:

$g^{(2016)}(x) = \sum_{k=0}^{2016} \left[C_{2016}^{k} (1+x^3)^{(k)} \left(e^{x^3}\right)^{(2016-k)} \right]$

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$$= C_{2016}^{0}(1+x^3)(e^{x^3})^{(2016)} + C_{2016}^{1}(1+x^3)^{(1)}(e^{x^3})^{(2015)}$$

$$+ C_{2016}^{2}(1+x^3)^{(2)}(e^{x^3})^{2014} + C_{2016}^{3}(1+x^3)^{(3)}(e^{x^3})^{(2013)}$$

$$+ C_{2016}^{2016}(1+x^3)^{2016} e^{x^3}$$

mà $y' = (1+x^3)^{(1)} = 3x^2$, $y'' = 6x$, $y''' = 6$.

$$\Rightarrow g^{(2016)}(x) = C_{2016}^{0}(1+x^3)(e^{x^3})^{(2016)} + 3C_{2016}^{1} \cdot x^2 \cdot (e^{x^3})^{(2015)}$$

$$+ 6C_{2016}^{2} \cdot x(e^{x^3})^{(2014)} + 6C_{2016}^{3} \cdot (e^{x^3})^{(2013)}$$

mà $e^{x^3} = 1 + x^3 + \frac{x^6}{2!} + \frac{x^9}{3!} + \frac{x^{12}}{4!} + \dots + \frac{x^{3n}}{n!} + o(x^{3n})$

$$\Rightarrow g^{(2016)}(0) = C_{2016}^{0}(1+0^3)\frac{2016!}{672!} + 3C_{2016}^{1} \cdot 0^2 \cdot 0 + 6C_{2016}^{2} \cdot 0 \cdot 0$$

$$+ 6C_{2016}^{3}\frac{2013!}{671!}$$

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