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CHƯƠNG 4: ĐẠO HÀM VÀ VI PHÂN CỦA HÀM SỐ MỘT BIẾN SỐ

Bài 20 (139-NĐT)

1, y = $x\sqrt{1+x^2}$, $y' = \sqrt{1+x^2} + \frac{x^2}{\sqrt{1+x^2}} = \frac{2+2x^2}{\sqrt{1+x^2}}$

$y'' = \frac{(1+4x)\sqrt{1+x^2} - x(1+x^2)^{-1/2}}{1+x^2} = \frac{x(3+2x^2\ )}{(1+x^2)^{3/2}}$

2, y = $\frac{x}{\sqrt{x-x^2}}$, $y' = \frac{\sqrt{x-x^2} - \frac{x}{2}(1+2x)(x-x^2)^{-1/2}}{x-x^2} = \frac{1}{(1-x^2)^{3/2}}$

$y'' = \frac{3x}{(1-x^2)^{5/2}}$, /x/<1.

3, y= $2e^{-x^2}$,$y' = -2xe^{-x^2}$, $y'' = 2e^{-x^2}(2x^2-1)$,

4, y= lnf(x) , $y' = \frac{f'(x)}{f(x)}$,$y'' = \frac{f''(x)f(x) - (f'(x))^2}{(f(x))^2}$

Bài 21(139-NĐT)

1, dx = (2-2t)dt = 2(1-t)dt

dy = $(3-3t^2)$dt=$3(1-t^2)$dt

$y'_x = \frac{dy}{dx} = \frac{3(1-t^2)dt}{2(1-t)dt} = \frac{3}{2}(1+t)$

$y''_{xx} = \frac{d^2y}{dx^2} = \frac{3}{2}(1+t)'_t t'_x = \frac{3}{2}t'_x$

vì dx = 2(1-t)dt => t'_x=dt/dx = 1/2(1-t)

$y''_{xx} = \frac{3}{2} \cdot \frac{1}{2(1-t)} = \frac{3}{4(1-t)}$, t$\neq$ 1.

2, dx = -asintdt, $t'_x = -\frac{1}{asint}$

Dy = acostdt,

$Y'_x = -\text{cotgt}$

$Y''_{xx} = -(\text{cotgt})'_t\ t'_x = \frac{1}{sin^2t} \cdot \left(-\frac{1}{asint}\right) = -\frac{1}{asin^3t}$; $t \neq k\pi, k \in Z$

3, $dx = a(1-\cos t)dt$, $t'_x = \frac{1}{a(1-cost)} = \frac{1}{2asin^2\frac{t}{2}}$

$dy = \text{asintdt} = 2\text{asin}\frac{t}{2} cos\frac{t}{2}\text{dt}$

$y'_x = \frac{dy}{dx} = \frac{2asin\frac{t}{2}cos\frac{t}{2}dt}{2asin^2\frac{t}{2}} = \text{cotgt}/2$

$y''_{xx} = \frac{d^2y}{dx^2} = (\text{cotgt}/2)'_t . t'_x = -\frac{2}{2sin^2\frac{t}{2}} . \frac{1}{2asin^2\frac{t}{2}} = \frac{1}{4asin^4\frac{t}{2}}$,$t \neq 2k\pi, k \in$

Bài 22(139-NĐT)

1, $y = \frac{x^2}{1-x}$, *tính* $y^{(8)}$

$y = \frac{x^2}{1-x} = \frac{1-(1-x^2)}{1-x} = \frac{1}{1-x} - (1+x^2)$

$y^{(8)} = \left(\frac{1}{1-x}\right)^{(8)} - (1+x^2)^{(8)} = ((1-x)^{-1})^{(8)} - 0$

$= \frac{8!}{(1-X)^9}$, $x \neq 1$

2,$y = \frac{1+x}{\sqrt{1-x}}$, tính $y^{(100)}$

$y = \frac{1+x}{\sqrt{1-x}} = (1+x)(1-x)^{-\frac{1}{2}}$

Theo quy tắc Leibnitz:

$y^{(100)} = ((1-x)^{-\frac{1}{2}})^{(100)}(1+x) + 100((1-x)^{-\frac{1}{2}})^{(99)}(1+x)'$

Ta có: $((1-x)^{-\frac{1}{2}})^{(n)} = \frac{(2n-1)!!}{2^n} . \frac{1}{(1-x)^n\sqrt{1-x}}$

Thay lần lượt n=99 và n=100 vào biểu thức ta được:

$Y^{(100)} = \frac{199!!}{2^{100}} \cdot \frac{1+x}{(1-x)^{100}\sqrt{1-x}} + \frac{197!!}{2^{99}} \cdot \frac{100}{(1-x)^{99}\sqrt{1-x}}$

3, $y = x^2e^{2x}$, tính $y^{(20)}$

$Y^{(20)} = (e^{2x})^{(20)}x^2 + 20(e^{2x})^{(19)}(x^2)' + 190(e^{2x})^{(18)}(x^2)''$

Ta có : $(e^{2x})^{(n)} = 2^n e^{2x}$

Thay lần lượt n=18,n=19 và n=20 vào biểu thức ta được:

$y^{(20)} = 2^{20}e^{2x}x^2 + 20.2^{19}e^{2x}2x + 190.2^{18}e^{2x}.2$

$= 2^{20}e^{2x}(x^2 + 20x + 95)$

4, $y = x^2\sin 2x$, tính $y^{(50)}$

$y^{(50)} = (\sin 2x)^{(50)}\, x^2 + 50.(sin2x)^{49}(x^2)' + \frac{50.49}{2!}(sin2x)^{48}(x^2)''$ (1)

Ta có: $(sin2x)^{50} = 2^{50}(-1)^{25}sin2x = -2^{50}sin2x$ (2)

$(sin2x)^{49} = 2^{49}(-1)^{24}cos2x = 2^{49}cos2x$ (3)

$(sin2x)^{48} = 2^{48}(-1)^{24}sin2x = 2^{48}sin2x$ (4)

Thế 2,3,4 vào 1 ta được:

$y^{(50)} = -2^{50}sin2x.x^2 + 50.2^{49}cos2x.2x + \frac{50.49}{2!}2^{48}sin2x.2$

$=2^{50}(-sin2x.x^2 + 50\, cos2x.x + \frac{1225}{2}\, sin2x)$

Bài 23(139-NĐT): tính $y^{(n)}$

1, $y = \frac{1}{x(1-x)} = \frac{x+(1-x)}{x(1-x)} = \frac{1}{1-x} + \frac{1}{x}$

$y^{(n)} = (\frac{1}{1-x})^{(n)} + \left(\frac{1}{x}\right)^{(n)} = \frac{n!}{(1-x)^{n+1}} + \frac{(-1)^n}{x^{n+1}}$

2, $y = \frac{1}{x^2-3x+2} = \frac{1}{(1-x)(2-x)} = \frac{(2-x)-(1-x)}{(1-x)(2-x)} = \frac{1}{1-x} - \frac{1}{2-x}$

$y^{(n)} = (\frac{1}{1-x})^{(n)} + \left(\frac{1}{2-x}\right)^{(n)} = \frac{n!}{(1-x)^{n+1}} + \frac{n!}{(2-x)^{n+1}} = n!(\frac{1}{(1-x)^{n+1}} + \frac{1}{(2-x)^{n+1}})$; $x \neq 1; 2$

3, y= $\frac{x}{\sqrt[3]{1+x}} = (1+x)^{-\frac{1}{3}}.x$

$Y^{(n)} = ((1+x)^{-\frac{1}{3}}.x)^{(n)} = ((1+x)^{-\frac{1}{3}})^{(n)}.x + n.((1+x)^{-\frac{1}{3}})^{(n-1)}$

$= (-1)^n \frac{1}{3^n}(1.4 \ldots (3n-2)).\frac{1}{(1+x)^{n+\frac{1}{3}}}$

Do đó: $y^{(n)} = \frac{(-1)^{n-1}}{3^n}(1.4 \ldots (3n-2)).\frac{(3n+2x)}{(1+x)^{n+\frac{1}{3}}}$; $n \geq 2, x \neq -1$

Bài tập bổ sung

2.40 f(x) = $\frac{1}{x^2-3x+2}$,tính $f^{2008}(0)$

$f(x) = \frac{1}{x^2-3x+2} = \frac{1}{(1-x)(2-x)} = \frac{(2-x)-(1-x)}{(1-x)(2-x)} = \frac{1}{1-x} - \frac{1}{2-x}$

$f^{(2008)}(x) = (\frac{1}{1-x})^{(2008)} + \left(\frac{1}{2-x}\right)^{(2008)} = \frac{2008!}{(1-x)^{2008+1}} + \frac{2008!}{(2-x)^{2008+1}} =$

$=2008!(\frac{1}{(1-x)^{2009}} + \frac{1}{(2-x)^{2009}})$; $x \neq 1; 2$

$\Rightarrow f^{2008}(0) = 2008!(1 + \frac{1}{2^{2009}})$

2.41 f(x)= $\frac{1}{x^2}$, tính $f^{(2008)}(1)$

$f(x) = \frac{1}{x^2} = x^{-2}$

$f^{2008}(x) = (x^{-2})^{2008} = -2(-2-1)\ldots(-2-2008+1)x^{-2+2008}$

$= -2.(-3)\ldots.(-2009).x^{2006} = 2009!.x^{2006}$

$\Rightarrow f^{2008}(1) = 2009!$

2.42 f(x)=ln $\frac{x+1}{x-1}$, tính $f^{(2008)}(2)$

F(x) = ln(x+1)-ln(x-1)

$F^{2008}(x)=(\ln(x+1))^{(2008)} - (\ln(x-1))^{(2008)}$

$= (-1)^{2008-1}\frac{(2008-1)!}{(x+1)^{2008}} - (-1)^{2008-1}\frac{(2008-1)!}{(x-1)^{2008}}$

$= -\frac{2007!}{(x+1)^{2008}} + \frac{2007!}{(x-1)^{2008}}$

$\Rightarrow F^{2008}(2) = -\frac{2007!}{3^{2008}} + 2007!$

2.43 $f(x)=\frac{1+x}{1-x}$,tính $f^{(n)}(0)$

$F(x)=\frac{1+x}{1-x} = (1+x).\frac{1}{1-x}$

$f^{(n)}(x)=(1+x).\left(\frac{1}{1-x}\right)^{2008} + 2008(1+x)'\left(\frac{1}{1-x}\right)^{2008-1}$

$=(1+x).(\frac{1}{1-x})^{2008} + 2008.(\frac{1}{1-x})^{2007}$ (1)

Ta có: $(\frac{1}{1-x})^{2008} = \frac{2008!}{(1-x)^{2009}}$ (2)

$(\frac{1}{1-x})^{2007} = \frac{2007!}{(1-x)^{2008}}$ (3)

Thay 2,3 vào 1 ta được:

$f^{(n)}(x)=(1+x).\frac{2008!}{(1-x)^{2009}} + 2008.\frac{2007!}{(1-x)^{2008}}$

⇨ $f^{(n)}(0)=2.2008!$

2.44 $f(x)=\ln\frac{1}{x-1}$, tính $f^{(n)}(0)$

$f(x)=\ln\frac{1}{x-1} = ln1 - ln(1-x) = -ln(1-x)$

$f^{(n)}(x) = -(-1)^{n-1}\frac{(n-1)!}{(1-x)^{n}}(-1)^{n}$

⇨ $f^{(n)}(0) = -\frac{(n-1)!}{1^{n}}$