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Global stabilizing controller design for linear time-varying systems and its application on BTT missiles*

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Abstract: A parametric method for the gain-scheduled controller design of a linear time-varying system is given. According to the proposed scheduling method, the performance between adjacent characteristic points is preserved by the invariant eigenvalues and the gradually varying eigenvectors. A sufficient stability criterion is given by constructing a series of Lyapunov functions based on the selected discrete characteristic points. An important contribution is that it provides a simple and feasible approach for the design of gain-scheduled controllers for linear time-varying systems, which can guarantee both the global stability and the desired closed-loop performance of the resulted system. The method is applied to the design of a BTT missile autopilot and the simulation results show that the method is superior to the traditional one in sense of either global stability or system performance.

Keywords: linear time-varying systems, global stability, gain-scheduled control , eigenstructure assignment, BTT.

1. Introduction

Gain-scheduled control is a widely used engineering design approach for linear time-varying systems. It has been proved effective in several practical applications[1−3]. According to the method, a global time-varying controller is obtained by the scheduling among some local controllers. Despite the past success of gain-scheduled control in practice, little has been known about it theoretically as a time-varying technique. Also, determining the actual scheduling routine is more of an art than a science.

During the recent years, the gain-scheduled control via LMIs (linear matrix inequalities), especially that for LPV (linear parameter varying) systems, has been under deep research. The basic idea is to construct a convex hull that covers the whole operating range, and then the controller can be obtained by solving a convex optimization problem, which consists of LMI constraints corresponding only to a finite number of vertices. To allow arbitrary large changing rates of gain-scheduled parameters and to keep simple synthesis procedure, a common Lyapunov function has been used for the whole operating range[4−5], which leads to conservatism of design. Robust stability is overemphasized at the expense of performance. A new technique that allows parameter-dependent Lyapunov functions for gain-scheduled controllers has also been reported[6]. However, this technique restricts the changing rates of gain-scheduled parameters and requires additional LMI constraints that may introduce additional conservatism. Furthermore, its design procedure may be very complicated for practice. The design of a gain-scheduled controller to guarantee stability remains an open problem.

In this article, we propose a new scheduling method for linear time-varying systems based on the eigenstructure assignment theory of linear invariant systems. A time-varying controller is constructed artfully in a way that is proved to guarantee the global stability of the closed-loop system. A sufficient stability criterion is given by constructing a series of Lyapunov functions based on some selected characteristic time points. The stability criterion proposed in this article is less conservative and is easy to be verified.

Consequently, the global stabilizing controller can

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be derived by simply introducing an additional constraint to the design procedure of the gain-scheduled controller. The approach is applied to the design of a BTT (bank to turn) missile autopilot. The simulation results are given to show its effect.

2. Problem statement

Consider an MIMO (multi-input multi-output) linear time-varying system represented by

$$\dot{x} = A(t)x + B(t)u \ , \ t \in [t_0, \, t_e] \tag{1}$$

where $x \in R^n$, $u \in R^r$, $y \in R^m$ are respectively the state vector, the input vector and the output vector. $A(t)$, $B(t)$ are system coefficient matrices of appropriate dimensions which are continuously time-varying. Throughout this paper, the following assumption is made:

Assumption A1 $(A(t), \, B(t))$ is controllable for all t.

The problem is to find a controller for system (1), which guarantees both global stability and desired performance of the resulted closed-loop system. In the following, state feedback controllers are designed for the local subsystems, and a new justified technique for scheduling is presented to construct the global controller.

Suppose t_i, $i = 1, 2, \ldots, r_0$ are the characteristic time points selected in the operating range; then, the local subsystems are

$$\Sigma_i \ : \ \dot{x} = A_i x + B_i u \ , \ \ i = 1, \, 2, \ldots, r_0 \tag{2}$$

where

$$A_i = A(t_i), \, B_i = B(t_i)$$

The linear system theory can be used to derive local controllers for each subsystem Σ_i. State feedback controllers in the following form are employed to stabilize the subsystems

$$u = K_i x \ , \ \ i = 1, 2, \ldots, r_0 \tag{3}$$

In order to construct the global controller, certain interpolation method should be adopted to link K_i, $i = 1, \, 2, \ldots, r_0$ together. The problem here is how to construct the global state feedback controller $K(t)$ such that the resulted closed-loop system

$$\dot{x} = A_c(t)x \tag{4}$$

where

$$A_c(t) = A(t) + B(t)K(t), \ t \geqslant t_0$$

can be stabilized and at the same time the desired performance can be guaranteed. $K(t)$ is a time-varying controller, which can be obtained by combining the local controllers using certain gain-scheduled method.

3. Controller design

3.1 Gain-scheduled controller design

We first design the local controllers $K_i, i = 1, 2, \ldots,$ r_0 according to the eigenstructure assignment theory. The following lemma is introduced.

Lemma 1[7] Suppose $A \in R^{n\times n}$, $B \in R^{n\times r}$, $(A, \, B)$ is controllable. s_i, $i = 1, \, 2, \ldots, n$ are a set of complex numbers, which are symmetric about the real axis. Then the matrices $K \in R^{r\times n}$ and $W \in C^{n\times n}$ satisfying

$$A_c = A + BK = W\mathrm{diag}(s_1, \, s_2, \ldots, \, s_n)W^{-1} \tag{5}$$

are given by

$$K = [D(s_1)f_1 \ D(s_2)f_2 \ \ldots \ D(s_n)f_n] \, W^{-1} \tag{6}$$

$$W = [w_1 \ w_2 \ \ldots \ w_n] \tag{7}$$

$$w_j = N(s_j)f_j, \, j = 1, \, 2, \ldots, \, n \tag{8}$$

where $f_j \in C^r$, $j = 1, \, 2, \ldots, \, n$ are arbitrary vectors which satisfy

Constraint 1

$$f_i = \bar{f}_j \text{ if } s_i = \bar{s}_j$$

Constraint 2

$$\det(W) \neq 0$$

$N(s)$, $D(s)$ are right comprime polynomial matrices satisfying

$$(sI - A)^{-1}B = N(s)D^{-1}(s) \tag{9}$$

Lemma 1 gives a complete parametric approach for the eigenstructure assignment of controllable linear systems via static state feedback. Accordingly, we can derive the local controller gain matrices K_i, $i = 1, \, 2, \ldots, r_0$ by properly assigning the closed-loop eigenvalues s_j^i and the free vectors f_j^i, $j = 1, 2, \ldots, \, n$. The eigenvalues can be determined by desired system performance such as overshoot and so on, and additional requirements can also be fulfilled by extra freedom proposed by f_j^i.

The next procedure is linking the local controllers together to construct a global controller. In order to maintain the desired performance of the closed-loop system between adjacent characteristic points, the eigenvalues of the closed-loop system for all fixed time points should stay in a small neighborhood of the desired area. Some algorithms have been put forward to achieve this goal[1,8] and in this article, it can be fulfilled by eigenstructure assignment.

The local controllers of two adjacent characteristic points t_i and t_{i+1} are noted respectively by $K(s_j^i, f_j^i, j = 1, 2, \ldots, n)$ and $K(s_j^{i+1}, f_j^{i+1}, j = 1, 2, \ldots, n)$. Denote $s_j(t), j = 1, 2, \ldots, n$ as the pointwise eigenvalues of the closed-loop system. Since the performance requirements on the whole operating range are usually similar, we can simply choose the same set of desired eigenvalues between characteristic points, that is

$$s_j(t) = s_j^i = s_j^{i+1} = s_j, j = 1, 2, \ldots, n, \; t \in [t_i, t_{i+1}) \quad (10)$$

where $\mathrm{Re}(s_j) < 0$. Then, the controller between characteristic points can be obtained by the following scheduling procedure

$$\begin{aligned} K(t) = K(s_j(t),\, f_j(t),\, j = 1, 2, \ldots, n) = \\ [D(s_1)f_1 \; D(s_2)f_2 \; \ldots \; D(s_n)f_n] \cdot \\ [N(s_1)f_1 \; N(s_2)f_2 \; \ldots \; N(s_n)f_n]^{-1} \end{aligned} \quad (11)$$

where

$$f_j(t) = \frac{t_{i+1} - t}{t_{i+1} - t_i} f_j^i + \frac{t - t_i}{t_{i+1} - t_i} f_j^{i+1} \quad (12)$$

Remark 1 According to the proposed scheduling method, the pointwise closed-loop eigenvalues of the system are time invariant for the whole operating range, as shown in (10). The eigenvalues can also be set time-varying as long as they meet the performance requirements.

Remark 2 In (12), linear interpolation, for simplicity, is used. Other interpolation methods are as well applicable.

It is known that the response characteristics of a linear system depend on the eigenvalues s_j and the corresponding eigenvectors w_j which respectively determine the damping factor and the shape of the state response. Following from the scheduling procedure proposed in this section, the eigenvalues of the closed-loop system remain invariant and the eigenvectors chang gradually, which implies that the performance of the system between adjacent characteristic points can be preserved to a great extent. Moreover, since our method takes account of the internal characteristics of the system, it is considerably more reasonable than the traditional scheduling method, which is simply implemented by the interpolation of the feedback gain matrices. In this sense, the scheduling method proposed in this article may help to obtain global stability of the resulted system for most cases, which will be verified by the application example given in Section 4.

According to the given approach, since the closed-loop eigenvalues of the resulted system are assigned to the left half complex plane, the resulted system is pointwise stable for any fixed $t \in [t_i, t_{i+1}]$. However, unfortunately, being pointwise stable only guarantees the local stability or the pointwise stability of the resulted time-varying system. Deriving global stability based on this property is known to be hazardous. Therefore, it is necessary to verify the global stability of the closed-loop time-varying system (4). To solve this problem, a sufficient stability criterion for linear time-varying systems is proposed in the following.

3.2 Stability criterion

In this section, we give a sufficient condition for the global stability of the gain-scheduled system designed above.

According to the design procedure introduced in Subsection 3.1, the system matrix $A_c(t)$ of the resulted closed-loop system is pointwise Hurwitz for all t; thus, the matrix equation

$$A_c^{\mathrm{T}}(t_i)P_i + P_i A_c(t_i) = -I \quad (13)$$

has a unique positive definite solution P_i for $i \in \{0, 1, \ldots, r_0\}$. The following lemma provides the parametric expression for P_i based on the eigenstucture of the system.

Lemma 2[8] The solution to Eq. (13) has the following parametric representation

$$P_i = W_i^{-\mathrm{T}} Q_i W_i^{-1} \quad (14)$$

where

$$W_i = \begin{matrix} w_1^i & w_2^i & \dots & w_n^i \end{matrix}$$

$$Q_i = \left[q_{kj}^i\right]_{n\times n}$$

$$q_{kj}^i = -\frac{(w_k^i)^{\mathrm{T}} w_j^i}{s_k^i + s_j^i}$$

s_j^i, w_j^i, $j = 1, \dots, n$ are respectively the eigenvalues and corresponding eigenvectors of $A_c(t_i)$.

Without loss of generality, we give the following assumption.

Assumption A2 $A_c(t)$ is bounded from above, i.e. $\exists \zeta > 0$ such that $\|A_c(t)\| < \zeta, \forall t \geqslant t_0$.

The Lyapunov function valid on the time interval $[t_i,\, t_{i+1})$ is defined as

$$V(t) = x^{\mathrm{T}}(t) P_i x(t),\ t \in [t_i,\, t_{i+1}) \tag{15}$$

Since P_i is positive definite, there exists a invertible matrix D_i such that $D_i^{\mathrm{T}} D_i = P_i$. Denote

$$\alpha_i(t) = \max\left\{eig(D_i^{-\mathrm{T}} A^{\mathrm{T}}(t) D_i^{\mathrm{T}} + D_i A(t) D_i^{-1})\right\} \tag{16}$$

Then we obtain the following result.

Theorem 1 The linear time-varying system (4) is uniformly aymptotically stable in the sense of Lyapunov if the following conditions hold

$$\int_{t_i}^{t} \alpha_i(\tau)\mathrm{d}\tau < 0, t \in [t_i + t_{i+1}), i = 0, 1, \dots, \infty \tag{17}$$

$$R_i = \frac{\exp\left\{\displaystyle\int_{t_i}^{t_{i+1}} \alpha_i(\tau)\mathrm{d}\tau\right\}}{\min\{eig(D_{i+1}^{-\mathrm{T}} P_i D_{i+1}^{-1})\}} < 1, i = 0, 1, \dots, \infty \tag{18}$$

Proof Over the time interval $[t_i,\, t_{i+1})$, we have

$$\dot{V}(t) = x^{\mathrm{T}}(t)(A^{\mathrm{T}}(t) P_i + P_i A(t)) x(t) \tag{19}$$

We introduce the change of coordinates $z(t) = D_i x(t)$; then, it holds that

$$\frac{\dot{V}(t)}{V(t)} = \frac{z^{\mathrm{T}}(t)(D_i^{-T} A^{\mathrm{T}}(t) D_i^{\mathrm{T}} + D_i A(t) D_i^{-1}) z(t)}{z^{\mathrm{T}}(t) z(t)} \leqslant \alpha_i(t)$$

Thus, for $t \in [t_i,\, t_{i+1})$

$$V(t) \leqslant \exp\left\{\int_{t_i}^{t} \alpha_i(\tau)\mathrm{d}\tau\right\} V(t_i) \leqslant V(t_i)$$

Accordingly, $V(t)$ has an upper bound $V(t_i)$ over the time interval $[t_i,\, t_{i+1})$. Further, let $P_{i+1} = D_{i+1}^{\mathrm{T}} D_{i+1}$, $z(t_{i+1}) = D_{i+1} x(t_{i+1})$, we have

$$V(t_{i+1}^-) = x^{\mathrm{T}}(t_{i+1}) P_i x(t_{i+1}) =$$

$$z^{\mathrm{T}}(t_{i+1}) D_{i+1}^{-\mathrm{T}} P_i D_{i+1}^{-1} z(t_{i+1}) \leqslant$$

$$\exp\left\{\int_{t_i}^{t_{i+1}} \alpha_i(\tau)\mathrm{d}\tau\right\} V(t_i)$$

Thus,

$$V(t_{i+1}) = z^{\mathrm{T}}(t_{i+1}) z(t_{i+1}) \leqslant$$

$$\frac{\exp\left\{\displaystyle\int_{t_i}^{t_{i+1}} \alpha_i(\tau)\mathrm{d}\tau\right\}}{\min\{eig(D_{i+1}^{-\mathrm{T}} P_i D_{i+1}^{-1})\}} V(t_i) =$$

$$R_i V(t_i) < V(t_i)$$

Accordingly, we have a decreasing sequence $\{V(t_i),\ i = 0, 1, \dots\}$. Following from Assumption A2, there exists a $c > 0$ such that

$$\min\{eig(P_i)\} > \frac{c}{\zeta},\ i = 0, 1, \dots, \infty$$

Then, as long as $x(t_0)$ is chosen such that $x^{\mathrm{T}}(t_0) P_0 x(t_0) < V_0$, for any $t > t_0$

$$\|x(t)\|^2 \leqslant \frac{V(t)\zeta}{c} \leqslant \frac{V(t_0)\zeta}{c} < \frac{V_0\zeta}{c}$$

and as $t \to \infty$, we have

$$\lim_{t\to\infty} \|x(t)\|^2 \leqslant \frac{V_0\zeta}{c} \prod_{i=0}^{\infty} R_i = 0$$

Following from the Lyapunov stability theory, the conclusion of the theorem obviously holds.

Theorem 1 provides a sufficient criterion for asymptotical stability of linear time-varying systems. Accordingly, the global stability of system (4) can be easily guaranteed by applying additional restrictions (17) and (18), which, following from Lemma 2, come down to the requirements on the eigenvalues s_j and free vectors f_j^i, $j = 1, 2, \dots, n$.

To sum up, the gain-scheduled controller for the linear time-varying system (1), which asymptotically stabilizes the system and at the same time guarantees desired performance, can be obtained by properly choosing the closed-loop eigenvalues s_j and the free vectors f_j^i, $j = 1, 2, \dots n$, $i = 0, 1, \dots, r_0$. In the following, the proposed method is used to design the autopilot system of a BTT missile to further illustrate its effect.

4. Application on BTT missile

BTT steering has been developed rapidly and has recently received considerable attention. Since in BTT steering, the kinematics and inertial coupling of the roll and yaw systems during combined pitch and roll maneuvers are significant, the autopilot system of the BTT missile is a complicated multi-variable time-varying system.

The mathematic model of the pitch/yaw channel of a BTT missile is given as[10]

$$\begin{cases} \dot{x} = A(t)x + B(t)u \\ y = C(t)x + D(t)u \end{cases} \tag{20}$$

where

$$A(t)=\begin{bmatrix} -a_1-e_1 & e_1a_4-a_2 & \dfrac{(J_x-J_y)\omega_x}{57.3J_z} & \dfrac{e_1\omega_x}{57.3} \\ 1 & -a_4 & 0 & \dfrac{-\omega_x}{57.3} \\ \dfrac{(J_z-J_x)\omega_x}{57.3J_y} & \dfrac{-e_2\omega_x}{57.3} & -b_1-e_2 & e_2b_4-b_2 \\ 0 & \dfrac{\omega_x}{57.3} & 1 & -b_4 \end{bmatrix}$$

$$B(t)=\begin{bmatrix} -e_1a_5-a_3 & 0 \\ -a_5 & 0 \\ 0 & e_2b_5-b_3 \\ 0 & -b_5 \end{bmatrix}$$

$$C(t)=\begin{bmatrix} 0 & 0 & 0 & \dfrac{-b_4v_t}{57.3g} \\ 0 & \dfrac{-a_4v_t}{57.3g} & 0 & 0 \end{bmatrix}$$

$$D(t)=\begin{bmatrix} 0 & \dfrac{-b_5V_t}{57.3g} \\ \dfrac{a_5V_t}{57.3g} & 0 \end{bmatrix}$$

$x = [\omega_z\ \alpha\ \omega_y\ \beta]^{\mathrm{T}}$, $u = [\delta_z\ \delta_y]^{\mathrm{T}}$ and $y = [n_z\ n_y]^{\mathrm{T}}$ are respectively the state vector, the input vector, and the output vector. ω_z, ω_y, α and β are respectively the pitch rate, the yaw rate, the attack angle, and the sideslip angle. δ_z, δ_y stand for the actuator deflections and n_z, n_y are the overloads on the normal and side direction. J_x, J_y, J_z are the rotary inertia of the missile corresponding to the body coordinate; v_t, g are respectively the instantaneous speed and the gravity acceleration.

During the flight course of the missile, the parameters a_i, b_i, $i = 1, 2, \ldots, 6$ and e_1, e_2 vary continuously as the height and velocity of the missile change, and therefore, the system is time-varying. Moreover, the changing pattern of these parameters are complicated and it is difficult to give the analytical forms.

Here we use the gain-scheduled method proposed in Section 3 to design a global stabilizing controller for the system. In this article, the flight at the time interval [4.4 s, 11.9 s] is considered; the case on the whole trajectory can be treated similarly.

Choose two characteristic operating points $t_1 = 4.4$ s and $t_2 = 11.9$ s, and then the global stabilizing controller $K(t)$ can be built according to the method proposed in Section 3. Further, in order to realize the tracking of given overload signals, we employ a feedforward tracking controller based on the model-reference theory[11]

$$G(t) = H(t) - K(t)Z(t) \tag{21}$$

where the coefficient matrices $H(t)$ and $Z(t)$ can be calculated by

$$\begin{bmatrix} Z(t) \\ H(t) \end{bmatrix} = \begin{bmatrix} A(t) & B(t) \\ C(t) & D(t) \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix} \tag{22}$$

Thus the controller for the original time-varying system (20) is given in the following form

$$u(t) = K(t)x + G(t)y_r \tag{23}$$

where y_r is the overload signal to be followed.

The closed-loop eigenvalues s_j and the free vectors f_j^1, f_j^2, $j = 1, 2, \ldots, 4$ give all the design freedom, which can be determined by solving the following optimization problem

$$\min_{s_j, f_j^1, f_j^2, j=1,2,\ldots,4} J$$

$$\text{s. t.} \begin{cases} \underline{c}_j \leqslant \mathrm{Re}(s_j) \leqslant \bar{c}_j < 0 \\ \underline{d}_j \leqslant \mathrm{Im}(s_j) \leqslant \bar{d}_j \\ f_i^{1,2} = \bar{f}_j^{1,2} \text{ if } s_i = \bar{s}_j \\ \displaystyle\int_{t_1}^{t} \alpha_1(\tau)\mathrm{d}\tau < 0, t_1 \leqslant t \leqslant t_2 \\ R_1 < 0 \end{cases} \tag{24}$$

where $\underline{c}_j$, $\bar{c}_j$, $\underline{d}_j$, and $\bar{d}_j$ specify the desired areas of the closed-loop eigenvalues and $f_j^{1,2}$ are the free vectors for the local characteristic systems Σ_1 and Σ_2.

For the BTT autopilot system, the output of the system should follow the given overload signal accurately and rapidly, and accordingly, a tracking performance index is chosen as follows

$$J = \sum_{k=1}^{N} \|y(k) - y_r(k)\|_2^2 \tag{25}$$

where $y(k)$ and $y_r(k)$ are respectively the output of the resulted system and the reference signal at every sampling time position, and N is the number of sampling points.

Suppose the initial state vector is $x_0 = [4\ \ 4\ \ -27\ \ 0]^{\mathrm{T}}$, and the rolling rate of the missile is $\omega_x = 400°/\mathrm{s}$. In order to test the traceability of the system, we give a time-varying overload signal as

$$y_r(t) = \begin{bmatrix} 0 \\ n_{yc}(t) \end{bmatrix}$$

where n_{yc} is a square wave with period 6 s as shown in Fig. 1.

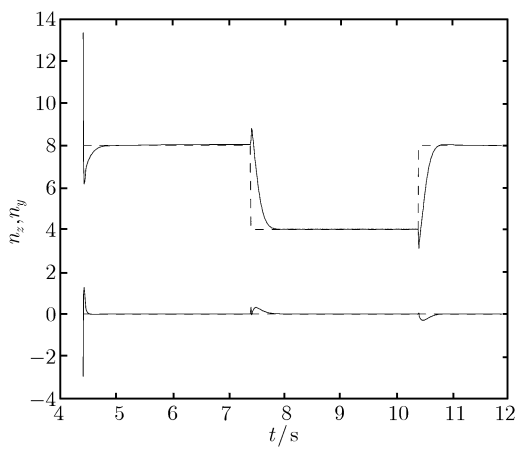

Fig. 1 Output response

According to the method proposed in this article, we choose

$$s_{1,2} = -11.300\,0 \pm 4.000\,0i$$

$$s_{3,4} = -61.000\,0 + 5.000\,0i$$

$$f_{1,2}^{1} = [-0.011\,3\ \ 0.051\,1]^{\mathrm{T}}$$

$$f_{3,4}^{1} = [0.461\,7\ \ -0.885\,5]^{\mathrm{T}}$$

$$f_{1,2}^{2} = [0.020\,0\ \ 0.999\,8]^{\mathrm{T}}$$

$$f_{3,4}^{2} = [-0.000\,4\ \ 0.000\,7]^{\mathrm{T}}$$

Then, the global stabilizing gain-scheduled controller $K(t)$ can be established according to (11) and (12). The feedforward gain matrix $G(t)$ can be calculated by (21) and (22).

The simulation results are given in Fig.1. The continuous line and the broken line are respectively the output response of the closed-loop system and the given overload signals. For compare, we also give the output response curves of the system designed using the traditional gain-scheduled method, according to which, the global controller $K(t)$ is constructed by linear interpolation between the feedback gain matrices of local subsystems, that is,

$$K(t) = \frac{t_2 - t}{t_2 - t_1} K_1 + \frac{t - t_1}{t_2 - t_1} K_2,\ t \in (t_1,\, t_2)$$

Obviously, the system outputs in the two cases can both track the given overload signals; however, the system designed using the gain-scheduled method proposed in this article has considerably better transient behavior than the traditional one. Since according our method, the eigenvalues and eigenvectors of the system are, to some extent, slow-varying, the performance between adjacent characteristic points is guaranteed, which can be seen from Fig. 1. However, for the one designed using the traditional method, as shown in Fig. 2, the performance declines between characteristic points.

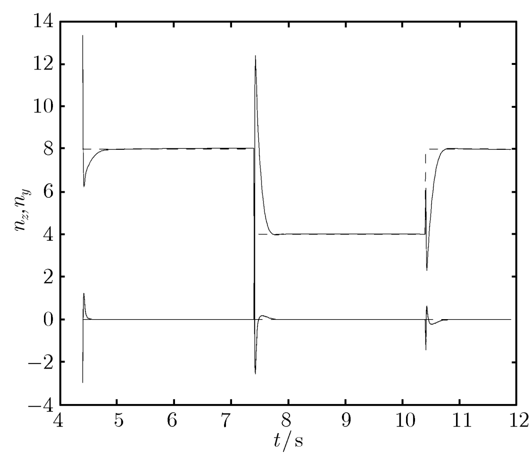

Fig. 2 Output response (traditional)

The superiority of the proposed gain-scheduled method can also be validated from the stability point of view. The stability of the resulted time-varying system can be verified using the sufficient criterion in Theorem 1. For the system designed employing the proposed method, we have

$$R_1 = 2.032\ 8e - 022 < 1$$

while for the traditional one, it derives

$$R_1' = 1.132\ 3e + 003 > 1$$

Accordingly, the controller designed in this article globally asymptotically stabilizes the system. However, the stability cannot be guaranteed for the traditional one.

5. Conclusion

The stabilizing controller design problem of linear time-varying systems is considered. A new scheduling method based on the eigenstructure assignment theory is proposed. Since our approach takes account of the internal characteristics of the system, it is more reasonable than the traditional gain-scheduled methods and is superior to maintain satisfied performance between adjacent characteristic points. In order to access global stability of the resulted system, a sufficient stability criterion is given for linear time-varying systems. Accordingly, the stabilizing gain-scheduled controller can be obtained by properly selecting the eigenvalues and free vectors of the local subsystems. Both the desired performance and the asymptotic stability of the resulted time-varying system are guaranteed. This is an instructive case, which solves the design problem of linear time-varying systems employing linear time-invariant theory.

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