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Switched nonlinear singular systems with time-delay: Stability analysis

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SUMMARY

This paper presents an approach to the stability analysis of a class of nonlinear interconnected continuous-time singular systems with arbitrary switching signals. This class of interconnected subsystems consists of unknown but bounded state delay and nonlinear terms, and each subsystem can be globally stable, unstable, or locally stable. By constructing a new Lyapunov-like Krasovskii functional, sufficient conditions are derived and formulated to check the asymptotic (exponential) stability of such systems with arbitrary switching signals. Then, some new general criteria for asymptotic (exponential) stability with average dwell-time switching signals are also established. The theoretical developments are demonstrated by two numerical simulations. Copyright © 2014 John Wiley & Sons, Ltd.

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KEY WORDS: hybrid systems; nonlinear singular systems; stability analysis; time-delay

1. INTRODUCTION

Switched systems are composed of several subsystems and use a switching signal to specify which subsystem is activated at each instant of time. Switched systems are considered as an important class of hybrid dynamical systems. Some examples for switched systems are automated highway systems, automotive engine control system, chemical processes, constrained robotics, power systems and power electronics, robot manufacture, and stepper motors. For more examples of switched systems and their applications, we refer the readers to [1] and the references cited therein. As a crucial factor, switching signals determine the dynamic behavior of a switched system in the most cases. Among the large variety of problems encountered in practice, the existence of a switching signal that ensures stability of the switched system was studied in [2–5]. One can assume that the switching sequence is not known a priori and look for stability results under arbitrary switching sequences [5, 6]. Finally, one can also study the stability of the switched system under particular classes of switching signals as in [7, 8].

When partial or total subsystems of a switched system are singular systems, the switched system becomes a class of hybrid switched singular systems. Singular systems, also referred to as implicit systems, descriptor systems, or generalized state-space systems, have extensive applications in many practical situations, for example, circuit boundary control systems, chemical processes, electrical networks, economical systems, and other areas [9]. Owing to switches among multiple singular

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subsystems, it is inevitably difficult to analyze and synthesize such systems. The stability problem for hybrid switched nonlinear singular systems with time-delay has not been fully investigated yet and it is challenging because of the difficult extension of the existing stability results.

Besides switching properties, many engineering systems involve time-delay phenomenon because of various reasons such as the finite speed of information processing and inherent phenomena. Also, it has been well recognized that the presence of time-delay has complex effects on stability [10]. In addition, regular time-delay systems with algebraic constraint conditions and time-delay composite control systems are all singular time-delay systems; for retarded problem, the reader can refer, for instance, to papers such as [11, 12]. When the time-delay is very small, it can be ignored for control design purposes. If it is large, however, it should be taken into account when designing the controller; otherwise, the closed-loop may become unstable. In general, the dynamic behavior of continuous-time singular systems with delays is more complicated than that of system without any time-delay because the continuous singular time-delay system is infinite dimensional. Thus, the stability analysis of singular time-delay systems has practical significance [11–13].

In this note, the class of singular subsystems considered is subject to time-delays, nonlinear terms, and switching. The approach followed in this note looks at the existence of new Lyapunov-like Krasovskii functional to check asymptotic (exponential) stability of the switched system under consideration through LMIs. Firstly, results for arbitrary switching signal are provided. Then, some sufficient conditions are derived to guarantee that the state trajectories of the system are asymptotically (exponentially) stable with average dwell-time switching signals. In the proposed approach, each subsystem can be global stable, global unstable, or local stable. Thus, notice that not all the subsystems have to be stable and globally stable, unstable, or locally stable subsystems may compose the complete switched system. However, in order to prove stability, there must be at least one globally/locally stable system taking part of the overall switched system.

This note is organized as follows. In Section 2, we give the problem formulation. Section 3 is dedicated to the stability analysis of switched nonlinear singular time-delay systems by mean of new Lyapunov-like Krasovskii functional. Some numerical evaluation examples are given in Section 4. Finally, the paper is concluded in Section 5.

2. PROBLEM FORMULATION

Let the dynamics of a class of hybrid switched nonlinear singular systems with time-delay be described by the following equation:

$$\begin{cases} E\dot{x}(t) = A_{\sigma(t)}x(t) + A_{d\sigma(t)}x(t - d(t)) + f_{\sigma(t)}(t, x(t)) \\ x(\theta) = \phi(\theta) \quad \forall\theta \in [-h_2, 0], \sigma(t) \in P \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ denotes the state and satisfies $x(\theta) = \phi(\theta), \forall\theta \in [-h_2, 0]$, in which $\phi(\theta)$ is an initial vector-valued continuous function and $\|\phi(\theta)\|_c$ is defined as $\|\phi(\theta)\|_c = sup_{-h_2 \leqslant s \leqslant 0}\|\phi(s)\|$, which stands for the norm of initial condition $\phi(\theta)$. $(E, A_i, A_{di}), i \in \mathcal{P}$, with $\mathcal{P}$ finite: $\mathcal{P} = \{1, 2, \cdots, m\}$ are known constant matrices with appropriate dimensions, where $rank(E) = r \leqslant n$ (i.e. E may be singular), $f_i(t, x(t)), i \in \mathcal{P}$ are nonlinear functions satisfying certain conditions stated in the succeeding text, and $d(t)$ is a time-varying continuous function that satisfies $0 < h_1 \leqslant d(t) \leqslant h_2$, $\dot{d}(t) \leqslant \overline{d} < 1$ in which h_2 and h_1 are scalars representing the upper and lower bound of delay, respectively. The piecewise right continuous (and constant) function $\sigma(t) : [0, \infty) \to \mathcal{P}$ is a switching signal to specify, at each time instant t, the index $\sigma(t) \in \mathcal{P}$, of the active subsystem, that is, $\sigma(t) = i$ means that the ith subsystem is activated at time t. Let $t_0 = 0 < \tau_1 < \tau_2 < \cdots < \tau_k < t, k = 1, 2, \cdots$, denote the switching points within $[0, t)$. Constructing switching sequence $\mathfrak{a} = \{(\sigma(t_0), t_0), (\sigma(\tau_1), \tau_1), \cdots, (\sigma(\tau_k), \tau_k), \cdots | k = 1, 2, \cdots\}$ means that the $\sigma(\tau_k)$th subsystem is activated during $[\tau_k, \tau_{k+1})$. Also, $N_\sigma(t_0, t)$ denotes the switching number during (t_0, t). In other words, $N_\sigma(t_0, t)$ is the number of discontinuities of $\sigma(t)$ within the open interval (t_0, t). For an illustration, a simple switching signal with $\mathcal{P} = \{1, 2\}$ has been shown in Figure 1. Without loss of generality, we assume that $E = diag(I_r, \mathbf{0}_{n-r})$(i.e. $0 < r \leqslant n$) and then the switched system can be rewritten as follows:

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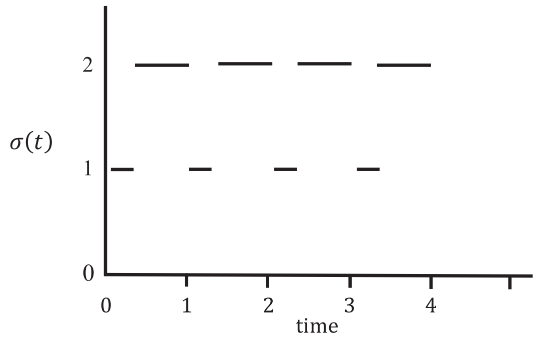

Figure 1. Example of a simple switching signal.

$$\begin{cases} \dot{x}_1(t) = h_{\sigma(t)_1}(t, x(t), x(t-d(t))) \\ 0 = h_{\sigma(t)_2}(t, x(t), x(t-d(t))) \end{cases} \tag{2}$$

where $x(t) = \left(x_1(t)^T, x_2(t)^T\right)^T$ and

$$\begin{aligned} \begin{pmatrix} h_{\sigma(t)_1}(t, x(t), x(t-d(t))) \\ h_{\sigma(t)_2}(t, x(t), x(t-d(t))) \end{pmatrix} &= \begin{pmatrix} A_{\sigma(t)_{11}} & A_{\sigma(t)_{12}} \\ A_{\sigma(t)_{21}} & A_{\sigma(t)_{22}} \end{pmatrix} \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} \\ &+ \begin{pmatrix} A_{d\sigma(t)_{11}} & A_{d\sigma(t)_{12}} \\ A_{d\sigma(t)_{21}} & A_{d\sigma(t)_{22}} \end{pmatrix} \begin{pmatrix} x_1(t-d(t)) \\ x_2(t-d(t)) \end{pmatrix} + \begin{pmatrix} f_{\sigma(t)_1}(t, x(t)) \\ f_{\sigma(t)_2}(t, x(t)) \end{pmatrix} \end{aligned}$$

The following assumptions are made through the paper:

Assumption 1
It is assumed that the nonlinear functions $f_i(t, x(t))$ satisfy

$$f_i(t, x(t))^T \Lambda_i f_i(t, x(t)) \leqslant \phi_i(t) x(t)^T \Lambda_i x(t), t \geqslant 0, \forall i (i \in \mathcal{P})$$

in which Λ_i is a positive-definite matrix and $\varphi_i(t)$ is a continuous positive function.

Remark 1
It is easy to see that Assumption 1 implies that the nonlinear function $f_i(t, x(t))$ satisfies a Lipschitz condition, which is usually assumed in literature; see, for instance, [6, 14, 15]. Also, Assumption 1 guarantees that $f_i(t, \mathbf{0}) = \mathbf{0}$, which implies the origin ($x(t) \equiv \mathbf{0}$) is the trivial solution of each subsystem.

Assumption 2
Each subsystem has a unique real-valued smooth solution for each compatible initial condition.

Remark 2
Similar to the discussion in [16], compatible initial condition is referred to an initial condition that satisfies $h_{\sigma(0)_2}(0, x(0), x(-d(0))) = \mathbf{0}$. As mentioned in [16], for ordinary singular time-delay systems with incompatible initial conditions, jump can occur in the algebraic variables ($x_2(t)$) or in the derivatives of the differential variables ($\dot{x}_1(t)$), that is, the differential variables are always continuous. Therefore, the switched singular time-delay system (1) might have discontinuities at switching points because of potentially incompatible initial condition and some impulses should be considered in analysis. For more information about impulsive system, see [17, 18]. The conditions $h_{\sigma(0)_2}(0) = \mathbf{0}$ and $h_{\sigma(\tau_k^-)_2}\left(\tau_k^-\right) = h_{\sigma\left(\tau_k^+\right)_2}\left(\tau_k^+\right)$ (for abbreviation, from now on, we use $h_{\sigma(t)_2}(t)$ instead of $h_{\sigma(t)_2}(t, x(t), x(t-d(t)))$) must be satisfied at switching points to guarantee the continuity of states. As an example of systems satisfying these compatibility conditions, we can consider nonlinear singular time-delay subsystems with a common algebraic equation or consider a switched nonlinear singular time-delay system whose switching points occur when the trajectories intersect

the compatibility space of the new subsystems. Therefore, if all subsystems have a unique real-valued smooth solution for each compatible initial condition, and $h_{\sigma(\tau_k^-)_2}(\tau_k^-) = h_{\sigma(\tau_k^+)_2}(\tau_k^+)$, then the trajectories of the switched case system are continuous everywhere. So, Assumption 2 provides a necessary condition to guarantee the continuity of the trajectories everywhere.

Remark 3
There are many applications for systems of type (1) in the real-world such as excitation control of power system connected with on-load tap-changer service and constant power load [19], micro machined-switch microelectromechanical systems device as nonlinear descriptor [20], Markovian jump systems with time-delay [21], impulse detection in power electronics systems [22], anti-windup control [23], evaporator vessel [22], rigid bodies [22], pulse-width modulator boost converter [20], and see [24] for more applications. These theoretical and practical significances have motivated us to carry out the present study.

Now, the following lemma is given to prove the main results in Section 3.

Lemma 1
Suppose that a non-negative continuous function $f(t)$ satisfies

$$f(t) \leqslant \xi_1 sup_{t-\tau\leqslant s\leqslant t} f(s) + \xi_2 e^{\psi(t_0,t)} \qquad \forall t \geqslant t_0(a)$$

where $0 < \xi_1 < 1, \xi_2 > 0$, $\psi(t_0,t)$ is a continuous function with $\psi(t_0,t_0) = 0$ and $\tau > 0$. Then

$$f(t) \leqslant \left(sup_{t_0-\tau\leqslant s\leqslant t_0} f(s) + \frac{\xi_2}{1-\xi_1 e^{\xi_0\tau}} \right) e^{\psi(t_0,t)}(b)$$

where $\xi_1 e^{\xi_0 \tau} \leqslant 1$

Proof
The proof is given in Appendix. □

It should be noted that in Lemma 1, the restriction on the exponential term is only less than $1(0 < \xi_1 < 1)$. If it is equal and less than 1 $(0 < \xi_1 \leqslant 1)$, there will be no sense for the inequality and as it can be seen in the proof of the lemma (Appendix), $f(t) \leqslant \left(sup_{t_0-\tau\leqslant s\leqslant t_0} f(s) + \frac{\xi_2}{1-\xi_1 e^{\xi_0\tau}}\right) e^{\psi(t_0,t)}$ cannot be satisfied anywhere. Also, Lemma 1 can be applied when $f(t)$ is right-continuous $\left(f\left(t_i^+\right) = f(t_i) < \infty\right)$ and the same result in Lemma 1 is obtained.

3. STABILITY ANALYSIS

In this section, the main results are given. Under Assumptions 1 and 2 and Lemma 1, sufficient conditions for the asymptotic (exponential) stability of the switched nonlinear singular time-delay system are given by using a new Lyapunov-like Krasovskii functional. Firstly, the results for arbitrary switching signal are provided, then the average dwell-time approach is considered.

Let $T_i(t_0,t)$ denote the union of time intervals where the ith subsystem is activated within (t_0,t), which yields $\bigcup_{i=1}^m T_i(t_0,t) = t_0 - t$. If T_i is an interval, its union must be an interval as well:(t_0,t) rather than $t_0 - t$. By this definition, the following theorem allows us to analyze stability problem of the switched nonlinear singular time-delay system (1).

Theorem 1
For given, $0 < h_1 \leqslant d(t) \leqslant h_2, \dot{d}(t) \leqslant \overline{d} < 1$, assume that there exists symmetric and positive-definite matrices P_{i_1}, Q_i with $r \times r$ and $n \times n$ dimension, respectively, non-singular matrices P_{i_2} with $(n-r)\times(n-r)$ dimension, scalars $\mathcal{J}_i$, positive scalars $\eta_{i_j}(j = 1,2,3)$ and ϵ_i, and scalar $\mu \geqslant 1$, such that the following inequalities hold:

$$P_{s_1} \leqslant \mu P_{l_1}, Q_s \leqslant \mu Q_l \tag{3a}$$

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$$\int_0^t (\lambda_l(r) - \lambda_s(r))dr \leqslant ln(g(t)), \forall t \geqslant 0 \tag{3b}$$

$$\begin{pmatrix} \mathcal{L}_i(t) & P_i^T A_{di} \\ * & -e^{\mathcal{J}_i}(1-\overline{d})Q_i \end{pmatrix} - diag(\lambda_i(t)E^T P_i, \mathbf{0}_{(n-r)\times(n-r)}) < 0, \forall t \geqslant 0 \tag{3c}$$

$$\begin{pmatrix} -(1-\eta_{i_1})Q_{i_{22}} & \left(\sum_{j=2}^{3} \eta_{i_j}\right) P_{i_2}^T \\ * & -\left(\sum_{j=2}^{3} \eta_{i_j}\right) I_{(n-r)\times(n-r)} \end{pmatrix} < 0 \tag{3d}$$

$$ln\left(\prod_{j=1}^{k} \mu\left(1+(g(\tau_j))^2\right)\right) + \sum_{i=1}^{m} \int_{T_i(t_0,t)} \lambda_i(r)dr \leqslant 2\psi(t_0,t), \forall t \geqslant 0 \tag{3e}$$

where $\mathcal{L}_i(t) = A_i^T P_i + P_i^T A_i + Q_i + \frac{1}{\epsilon_i} P_i^T P_i + \epsilon_i \varphi_i(t) I_{n\times n}$, $P_i = diag(P_{i_1}, P_{i_2})$, $g(t)$ is a non-decreasing positive continuous function for $t \geqslant 0$, $\lambda_i(t)$ is a locally integrable function that satisfies $\int_{t-d(t)}^{t} \lambda_i(r)dr \geqslant \mathcal{J}_i > -\infty$, $\tau_k (k = 1, 2, \cdots)$ are switching instants, and $i, s, l \in \mathcal{P}$. Then, the states of switched nonlinear singular time-delay system (1) under Assumptions 1 and 2 and switching condition $h_{\sigma(\tau_k^-)_2}\left(\tau_k^-\right) = h_{\sigma\left(\tau_k^+\right)_2}\left(\tau_k^+\right)$, can be upper-bounded by $e^{\psi(t_0,t)}$ as $\|x(t)\| \leqslant \mathcal{D}e^{\psi(t_0,t)}\|\phi(t)\|_c$ in which $\mathcal{D}$ is a sufficiently large scalar and $\psi(t_0,t)$ is a continuous function. This implies that, the switched nonlinear singular time-delay system (1) is asymptotically stable if $\lim_{t\to\infty} \psi(t_0,t) = -\infty$ and $\psi(t_0,t_0) = 0$. Also, if $\psi(t_0,t) \leqslant -c(t-t_0)$ with $c > 0$, then, the switched nonlinear singular time-delay system (1) is exponentially stable. Here, '$*$' denotes the matrix entries implied by the symmetry of a matrix.

Proof

The proof is divided into two parts: (1) to show the asymptotic (exponential) stability of differential variables of the switched nonlinear singular time-delay system; (2) to show the asymptotic (exponential) stability of algebraic variables of the switched nonlinear singular time-delay system.

Parts: (1) Asymptotic (exponential) stability of differential variables of the switched nonlinear singular time-delay system; for the $\sigma(t)$th subsystem, the following new Lyapunov-like Krasovskii functional is selected to estimate the behavior of the system.

$$V_{\sigma(t)}(t, x(t)) = x(t)^T E^T P_{\sigma(t)} x(t) + e^{\mathcal{K}(t)} \int_{t-d(t)}^{t} x(s)^T e^{-\mathcal{K}(s)} Q_{\sigma(t)} x(s)ds \tag{4}$$

where $\mathcal{K}(t) = \int_0^t \lambda_{\sigma(t)}(r)dr$. Taking the time derivative of $V_{\sigma(t)}(t, x(t))$ at time $t(t \neq \tau_k)$ yields

$$\begin{aligned}\dot{V}_{\sigma(t)}(t, x(t)) &= 2\dot{x}(t)^T E^T P_{\sigma(t)} x(t) + \lambda_{\sigma(t)}(t) e^{\int_0^t \lambda_{\sigma(t)}(r)dr} \int_{t-d(t)}^{t} x(s)^T e^{-\mathcal{K}(s)} Q_{\sigma(t)} x(s)ds \\ &+ x(t)^T Q_{\sigma(t)} x(t) - e^{\int_{t-d(t)}^{t} \lambda_{\sigma(t)}(r)dr} \left(1-\dot{d}(t)\right) x(t-d(t))^T Q_{\sigma(t)} x(t-d(t))\end{aligned} \tag{5}$$

By using the known inequality for two vectors x and y [25]; $2x^T y \leqslant \epsilon x^T x + \epsilon^{-1} y^T y$, yields $2x(t)^T P_{\sigma(t)}^T f_{\sigma(t)}(t, x(t)) \leqslant \frac{1}{\epsilon_{\sigma(t)}} x(t)^T P_{\sigma(t)}^T P_{\sigma(t)} x(t) + \epsilon_{\sigma(t)} f_{\sigma(t)}(t, x(t))^T f_{\sigma(t)}(t, x(t))$ in which

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ϵ and $\epsilon_{\sigma(t)}$ are positive scalars. Therefore, by considering Assumption 1, $-(1-\dot{d}(t)) \leqslant -(1-\overline{d})$, and (5), we have

$$\begin{aligned}
&\dot{V}_{\sigma(t)}(t,x(t)) - \lambda_{\sigma(t)}(t)V_{\sigma(t)}(t,x(t)) \\
&\quad \leqslant x(t)^T \Bigg(A_{\sigma(t)}^T P_{\sigma(t)} + P_{\sigma(t)}^T A_{\sigma(t)} + Q_{\sigma(t)} + \frac{1}{\epsilon_{\sigma(t)}} P_{\sigma(t)}^T P_{\sigma(t)} \\
&\qquad\qquad + \epsilon_{\sigma(t)}\phi_{\sigma(t)}(t) I_{n\times n} - \lambda_{\sigma(t)}(t) E^T P_{\sigma(t)} \Bigg) x(t) \\
&\quad + 2x(t)^T P_{\sigma(t)}^T A_{d\sigma(t)} x(t-d(t)) - e^{\mathcal{J}_i}(1-\overline{d}) x(t-d(t))^T Q_{\sigma(t)} x(t-d(t)) \\
&\quad = \begin{pmatrix} x(t)^T & x(t-d(t))^T \end{pmatrix} \begin{pmatrix} \mathcal{L}_{\sigma(t)}(t) & P_{\sigma(t)}^T A_{d\sigma(t)} \\ * & -e^{\mathcal{J}_i}(1-\overline{d}) Q_{\sigma(t)} \end{pmatrix} \begin{pmatrix} x(t) \\ x(t-d(t)) \end{pmatrix}
\end{aligned} \tag{6}$$

then, (3c) implies that $V_{\sigma(t)}(t,x(t)) \leqslant V_{\sigma(\tau_k)}(\tau_k, x(\tau_k)) e^{\int_{\tau_k}^{t} \lambda_{\sigma(\tau_k)}(s)ds}$. From (3a) and (3b), and considering (4), at switching instants τ_k, we have $V_{\sigma(\tau_k)}(\tau_k, x(\tau_k)) \leqslant \mu(1 + (g(\tau_k))^2) V_{\sigma(\tau_{k-1})}\left(\tau_k^-, x(\tau_k^-)\right)$. Using (5)–(6) successively for each subinterval leads to the results for $t \in [\tau_k, \tau_{k+1})$, we have $V_{\sigma(t)}(t,x(t)) \leqslant V_{\sigma(\tau_k)}\left(\tau_k^+, x\left(\tau_k^+\right)\right) e^{\int_{\tau_k}^{t} \lambda_{\sigma(\tau_k)}(s)ds}$ and

$$\begin{aligned}
V_{\sigma(\tau_k)}\left(\tau_k^+, x\left(\tau_k^+\right)\right) &\leqslant \mu(1+(g(\tau_k))^2) V_{\sigma(\tau_{k-1})}\left(\tau_k^-, x\left(\tau_k^-\right)\right) \\
V_{\sigma(\tau_{k-1})}\left(\tau_k^-, x\left(\tau_k^-\right)\right) &\leqslant V_{\sigma(\tau_{k-1})}\left(\tau_{k-1}^+, x\left(\tau_{k-1}^+\right)\right) e^{\int_{\tau_{k-1}}^{\tau_k} \lambda_{\sigma(\tau_{k-1})}(s)ds} \\
V_{\sigma(\tau_{k-1})}\left(\tau_{k-1}^+, x\left(\tau_{k-1}^+\right)\right) &\leqslant \mu(1+(g(\tau_{k-1}))^2) V_{\sigma(\tau_{k-2})}\left(\tau_{k-1}^-, x\left(\tau_{k-1}^-\right)\right) \\
V_{\sigma(\tau_{k-2})}\left(\tau_{k-1}^-, x\left(\tau_{k-1}^-\right)\right) &\leqslant V_{\sigma(\tau_{k-2})}\left(\tau_{k-2}^+, x\left(\tau_{k-2}^+\right)\right) e^{\int_{\tau_{k-2}}^{\tau_{k-1}} \lambda_{\sigma(\tau_{k-2})}(s)ds} \\
&\vdots \\
V_{\sigma(t_0)}\left(\tau_1^-, x\left(\tau_1^-\right)\right) &\leqslant V_{\sigma(t_0)}(t_0, x(t_0)) e^{\int_{t_0}^{\tau_1} \lambda_{\sigma(t_0)}(s)ds}
\end{aligned} \tag{7}$$

Finally, we can get

$$\begin{aligned}
V_{\sigma(t)}(t,x(t)) &\leqslant \mu^k \left(1+(g(\tau_1))^2\right) \cdots \left(1+(g(\tau_k))^2\right) e^{\int_{t_0}^{\tau_1} \lambda_{\sigma(t_0)}(s)ds + \cdots + \int_{\tau_k}^{t} \lambda_{\sigma(\tau_k)}(s)ds} V_{\sigma(t_0)}(t_0, x(t_0)) \\
&= \prod_{j=1}^{k} \mu \left(1+(g(\tau_j))^2\right) e^{\sum_{i=1}^{m} \int_{T_i(t_0,t)} \lambda_i(r)dr} V_{\sigma(t_0)}(t_0, x(t_0))
\end{aligned} \tag{8}$$

Because $V_{\sigma(t)}(t,x(t))$ is a bounded quadratic function, we can find scalar $\lambda_{\sigma(t)_1}$ and a sufficiently large scalar $\lambda_{\sigma(t)_2}$ such that $\lambda_{\sigma(t)_1}\|x_1(t)\|^2 \leqslant V_{\sigma(t)}(t,x(t)) \leqslant \lambda_{\sigma(t)_2}\|x(t)\|^2$, which leads to

$$\|x_1(t)\| \leqslant \mathcal{M} e^{\frac{1}{2} ln\left(\prod_{j=1}^{k} \mu\left(1+(g(\tau_j))^2\right)\right) + \frac{1}{2}\sum_{i=1}^{m} \int_{T_i(t_0,t)} \lambda_i(r)dr} \|\phi(t)\|_c \leqslant \mathcal{M} e^{\psi(t_0,t)} \|\phi(t)\|_c \tag{9}$$

in which $\mathcal{M}$ is a sufficiently large scalar. From (3e), if $lim_{t\to\infty}\psi(t_0,t) = -\infty$ (9) leads the asymptotic stability of $x_1(t)$. Also, $\psi(t_0,t) \leqslant -c(t-t_0)$ implies the exponential stability of $x_1(t)$.

Until now, we have proved the asymptotic (exponential) stability of differential variables. Our next step is to prove the asymptotic (exponential) stability of algebraic variables.

Parts: (2) Asymptotic (exponential) stability of algebraic variables of the switched nonlinear singular time-delay system: Here, we are interested in the asymptotic (exponential) stability of algebraic variables by using Lemma 1. From (2), we have $0 = h_{\sigma(t)_2}(t, x(t), x(t-d(t)))$. By multiplying $2x_2(t)^T P_{\sigma(t)_2}$, we can get $0 = 2x_2(t)^T P_{\sigma(t)_2} h_{\sigma(t)_2}(t, x(t), x(t-d(t)))$, which implies

$$\begin{aligned}
0 &= 2x_2(t)^T P_{\sigma(t)_2} A_{\sigma(t)_{21}} x_1(t) + 2x_2(t)^T P_{\sigma(t)_2} A_{\sigma(t)_{22}} x_2(t) + 2x_2(t)^T P_{\sigma(t)_2} A_{d\sigma(t)_{21}} x_1(t-d(t)) \\
&\quad + 2x_2(t)^T P_{\sigma(t)_2} A_{d\sigma(t)_{22}} x_2(t-d(t)) + 2x_2(t)^T P_{\sigma(t)_2} f_{\sigma(t)_2}(t, x(t))
\end{aligned} \tag{10}$$

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Let $J_{\sigma(t)}(t) = x_2(t)^T Q_{\sigma(t)_{22}} x_2(t)$, then we can conclude that $J_{\sigma(t)}(t) = J_{\sigma(t)}(t) + 2x_2(t)^T P_{\sigma(t)_2}$ $h_{\sigma(t)_2}(t, x(t), x(t-d(t)))$. Consider that

$$\lambda_{\sigma(t)}(t) E^T P_{\sigma(t)} = diag(\lambda_{\sigma(t)}(t) P_{\sigma(t)_1}, \mathbf{0}), \quad Q_{\sigma(t)} = \begin{pmatrix} Q_{\sigma(t)_{11}} & Q_{\sigma(t)_{12}} \\ Q_{\sigma(t)_{21}} & Q_{\sigma(t)_{22}} \end{pmatrix},$$

$$P_{\sigma(t)}^T P_{\sigma(t)} = diag\left(P_{\sigma(t)_1}^T P_{\sigma(t)_1}, P_{\sigma(t)_2}^T P_{\sigma(t)_2}\right), P_{\sigma(t)}^T A_{d\sigma(t)} = \begin{pmatrix} P_{\sigma(t)_1}^T A_{d\sigma(t)_{11}} & P_{\sigma(t)_1}^T A_{d\sigma(t)_{12}} \\ P_{\sigma(t)_2}^T A_{d\sigma(t)_{21}} & P_{\sigma(t)_2}^T A_{d\sigma(t)_{22}} \end{pmatrix}$$

$$A_{\sigma(t)}^T P_{\sigma(t)} + P_{\sigma(t)}^T A_{\sigma(t)} = \begin{pmatrix} P_{\sigma(t)_1}^T A_{\sigma(t)_{11}} + A_{\sigma(t)_{11}}^T P_{\sigma(t)_1} & * \\ P_{\sigma(t)_2}^T A_{\sigma(t)_{21}} + A_{\sigma(t)_{12}}^T P_{\sigma(t)_1} & P_{\sigma(t)_2}^T A_{\sigma(t)_{22}} + A_{\sigma(t)_{22}}^T P_{\sigma(t)_2} \end{pmatrix} \tag{11}$$

By using the known inequality for two vectors x and$y$ [25];$2x^T y \leqslant \epsilon x^T x + \epsilon^{-1} y^T y$, yields

$$\begin{aligned} J_{\sigma(t)}(t) \leqslant\ & x_2(t)^T Q_{\sigma(t)_{22}} x_2(t) + 2x_2(t)^T A_{\sigma(t)_{22}}^T P_{\sigma(t)_2} x_2(t) + \frac{1}{\epsilon_{\sigma(t)}} x_2(t)^T P_{\sigma(t)_2}^T P_{\sigma(t)_2} x_2(t) \\ & + \epsilon_{\sigma(t)} \phi_{\sigma(t)}(t) x_2(t)^T x_2(t) + 2x_2(t)^T P_{\sigma(t)_2} A_{d\sigma(t)_{22}} x_2(t-d(t)) \\ & + e^{\mathcal{J}_{\sigma(t)}}(1-\overline{d}) x_2(t-d(t))^T Q_{\sigma(t)_{22}} x_2(t-d(t)) \\ & - e^{\mathcal{J}_{\sigma(t)}}(1-\overline{d}) x_2(t-d(t))^T Q_{\sigma(t)_{22}} x_2(t-d(t)) + \eta_{\sigma(t)_2} x_2(t)^T P_{\sigma(t)_2}^T P_{\sigma(t)_2} x_2(t) \\ & + \eta_{\sigma(t)_3} x_2(t)^T P_{\sigma(t)_2}^T P_{\sigma(t)_2} x_2(t) + \frac{1}{\eta_{\sigma(t)_2}} x_1(t)^T A_{\sigma(t)_{21}}^T A_{\sigma(t)_{21}} x_1(t) \\ & + \frac{1}{\eta_{\sigma(t)_3}} x_1(t-d(t))^T A_{d\sigma(t)_{21}}^T A_{d\sigma(t)_{21}} x_1(t-d(t)) \end{aligned} \tag{12}$$

Similar to (6), from (3c), we have $\begin{pmatrix} x_2(t)^T & x_2(t-d(t))^T \end{pmatrix} \begin{pmatrix} \mathcal{L}_{\sigma(t)_{22}}(t) & P_{\sigma(t)_2}^T A_{d\sigma(t)_{22}} \\ * & -e^{\mathcal{J}_{\sigma(t)}}(1-\overline{d}) Q_{\sigma(t)_{22}} \end{pmatrix}$ $\begin{pmatrix} x_2(t) \\ x_2(t-d(t)) \end{pmatrix} \leqslant 0$ in which $\mathcal{L}_{\sigma(t)_{22}}(t) = Q_{\sigma(t)_{22}} + P_{\sigma(t)_2}^T A_{\sigma(t)_{22}} + A_{\sigma(t)_{22}}^T P_{\sigma(t)_2} + \frac{1}{\epsilon_{\sigma(t)}} P_{\sigma(t)_2}^T$ $P_{\sigma(t)_2} + \epsilon_{\sigma(t)} \phi_{\sigma(t)}(t) I_{(n-r)\times(n-r)}$. So, from (12), we have

$$\begin{aligned} J_{\sigma(t)} - \left(\eta_{\sigma(t)_2} + \eta_{\sigma(t)_3}\right) x_2(t)^T P_{\sigma(t)_2}^T P_{\sigma(t)_2} x_2(t) \leqslant\ & e^{\mathcal{J}_{\sigma(t)}}(1-\overline{d}) x_2(t-d(t))^T Q_{\sigma(t)_{22}} x_2(t-d(t)) \\ & + \frac{1}{\eta_{\sigma(t)_2}} x_1(t)^T A_{\sigma(t)_{21}}^T A_{\sigma(t)_{21}} x_1(t) + \frac{1}{\eta_{\sigma(t)_3}} x_1(t-d(t))^T A_{d\sigma(t)_{21}}^T A_{d\sigma(t)_{21}} x_1(t-d(t)) \end{aligned} \tag{13}$$

From (9), it is easy to find a scalar $\mathcal{N}_{\sigma(t)}^2$ such that

$$\begin{aligned} & \frac{1}{\eta_{\sigma(t)_2}} x_1(t)^T A_{\sigma(t)_{21}}^T A_{\sigma(t)_{21}} x_1(t) + \frac{1}{\eta_{\sigma(t)_3}} x_1(t-d(t))^T A_{d\sigma(t)_{21}}^T A_{d\sigma(t)_{21}} x_1(t-d(t)) \\ & \leqslant \frac{1}{\eta_{\sigma(t)_2}} \lambda_{max}\left(A_{\sigma(t)_{21}}^T A_{\sigma(t)_{21}}\right) \|x_1(t)\|^2 + \frac{1}{\eta_{\sigma(t)_3}} \lambda_{max}\left(A_{d\sigma(t)_{21}}^T A_{d\sigma(t)_{21}}\right) \|x_1(t-d(t))\|^2 \\ & \leqslant \mathcal{N}_{\sigma(t)}^2 e^{\psi(t_0,t)} \|\phi(t)\|_c^2 \end{aligned} \tag{14}$$

From (3d) and considering (14), we have $Q_{\sigma(t)_{22}} - \left(\eta_{\sigma(t)_2} + \eta_{\sigma(t)_3}\right) P_{\sigma(t)_2}^T P_{\sigma(t)_2} \geqslant \eta_{\sigma(t)_1} Q_{\sigma(t)_{22}}$, which yields $0 < \eta_{\sigma(t)_1} < 1$ and

$$x_2(t)^T Q_{\sigma(t)_{22}} x_2(t) \leqslant \frac{e^{\mathcal{J}_{\sigma(t)}}(1-\overline{d})}{\eta_{\sigma(t)_1}} x_2(t-d(t))^T Q_{\sigma(t)_{22}} x_2(t-d(t)) + \frac{\mathcal{N}_{\sigma(t)}^2}{\eta_{\sigma(t)_1}} e^{\psi(t_0,t)} \|\phi(t)\|_c^2 \tag{15}$$

Now, we have obtained an inequality in respect of $x_2(t)$ and Lemma 1 can be used to show the asymptotic (exponential) stability of $x_2(t)$. If $x_2(t)^2 \leqslant \frac{\lambda_{max}(Q_{\sigma(t)22})}{\lambda_{min}(Q_{\sigma(t)22})} \frac{e^{\mathcal{J}_{\sigma(t)}(1-\overline{d})}}{\eta_{\sigma(t)1}} \|x_2(t-d(t))\|^2 + \frac{\mathcal{N}^2_{\sigma(t)}}{\lambda_{min}(Q_{\sigma(t)22})\eta_{\sigma(t)1}} e^{\psi(t_0,t)} \|\phi(t)\|_c^2$, by using Lemma 1 and letting $f(t) = x_2(t)^T x_2(t)$, we have

$$\|x_2(t)\|^2 \leqslant \left(\sup_{\tau_k - h_2 \leqslant s \leqslant \tau_k} \|x_2(s)\|^2 + m_{\sigma(t)} \right) e^{\psi(t_0,t)} \tag{16}$$

where $m_{\sigma(t)} = \frac{\mathcal{N}^2_{\sigma(t)}}{\lambda_{min}(Q_{\sigma(t)22})\eta_{\sigma(t)1}\left(1-\xi_{1\sigma(t)}e^{\xi_{0\sigma(t)}h_2}\right)}$ and $\xi_{\sigma(t)1} = \frac{\lambda_{max}(Q_{\sigma(t)22})}{\lambda_{min}(Q_{\sigma(t)22})} \frac{e^{\mathcal{J}_{\sigma(t)}(1-\overline{d})}}{\eta_{\sigma(t)1}} < 1$. Now, assume $t \in [\tau_k, \tau_{k+1})$, by an iterative method, we show the asymptotic (exponential) stability of the algebraic variables. From (16), we have

$$\sup_{\tau_k - h_2 \leqslant s \leqslant \tau_k} \|x_2(s)\|^2 \leqslant \left(\sup_{\tau_{k-1} - h_2 \leqslant s \leqslant \tau_{k-1}} \|x_2(s)\|^2 + m_{k-1} \right) e^{\psi\left(t_0, t^*_{k-1}\right)} \tag{17}$$

where $\psi\left(t_0, t^*_{k-1}\right)$ is the maximum amount of $\psi(t_0,t)$ in $[\tau_{k-1}, \tau_k)$. By substituting (17) in (16), for $t \in [\tau_k, \tau_{k+1})$, we have

$$x_2(t)^2 \leqslant \left(\left(\sup_{\tau_{k-1} - h_2 \leqslant s \leqslant \tau_{k-1}} \|x_2(s)\|^2 + m_{k-1} \right) e^{\psi(t_0, t^*_{k-1})} + m_k \right) e^{\psi(t_0,t)} \tag{18}$$

by induction and repeating (17) for the previous subsystems, we can conclude that

$$\begin{aligned} \|x_2(t)\|^2 \leqslant \Bigg(& \|\phi(t)\|_c^2 e^{\overbrace{\psi(t_0,t_1^*) + \psi\left(t_0,t_1^*\right) + \cdots + \psi(t_0,t_k^*)}^{N_\sigma(t_0,t)}} \\ & + m_1 e^{\psi \overbrace{\left(t_0,t_1^*\right) + \psi(t_0,t_1^*) + \cdots + \psi\left(t_0,t^*_{k-1}\right)}^{N_\sigma(t_0,t)-1}} + \cdots + m_{k-1} e^{\psi\left(t_0,t^*_{k-1}\right)} + m_k \Bigg) e^{\psi(t_0,t)} \end{aligned} \tag{19}$$

Because $\psi(t_0,t)$ is continuous and $\psi(t_0,\infty) = -\infty$, so $e^{\psi(t_0,t)}$ is positive and continuous with $e^{\psi(t_0,\infty)} = 0$. Certainly, after $\overline{n}$ switching, the amount of $e^{\psi(t_0,t)}$ becomes less that $q(q<1)$. In other words, there exist $\overline{t}$ such that for $t \leqslant \overline{t}$, $e^{\psi(t_0,\overline{t})} \leqslant q$. So,

$$\begin{aligned} \|x_2(t)\|^2 \leqslant \Bigg(& \|\phi(t)\|_c^2 e^{\sum_{j=1}^{\overline{n}} \psi\left(t_0,t_j^*\right)} q^{N_\sigma(t_0,t)-\overline{n}} + \overline{m} e^{\sum_{j=1}^{\overline{n}-1} \psi\left(t_0,t_j^*\right)} q^{N_\sigma(t_0,t)-\overline{n}} + \cdots \\ & + \overline{m} q^{N_\sigma(t_0,t)-\overline{n}} e^{\psi(t_0,\overline{t})} + \overline{m} q^{N_\sigma(t_0,t)-\overline{n}-1} + \overline{m} q^{N_\sigma(t_0,t)-\overline{n}-2} + \cdots + \overline{m} \Bigg) e^{\psi(t_0,t)} \end{aligned} \tag{20}$$

in which $\overline{m} = max_i\{m_i\}$. Therefore,

$$\|x_2(t)\|^2 \leqslant \overline{\mathcal{M}} \left(\overline{n} e^{\sum_{j=1}^{\overline{n}} \psi\left(t_0,t_j^*\right)} q^{N_\sigma(t_0,t)-\overline{n}} + \frac{1}{1-q} \right) e^{\psi(t_0,t)} \tag{21}$$

where $\overline{\mathcal{M}} = max_i \left\{ \|\phi(t)\|_c^2, \overline{m} \right\}$. It should be noted that $\frac{1}{1-q} = 1 + q + q^2 + \cdots \geqslant q^{N_\sigma(t_0,t)-\overline{n}-2} + \cdots + 1$ where $\sum_{j=1}^{\infty} q^j$ is a convergent geometric series. So we can find a positive scalar $\hat{\mathcal{M}}$ such that $\|x_2(t)\| \leqslant \hat{\mathcal{M}} e^{\psi(t_0,t)} \|\phi(t)\|_c$. Therefore, $lim_{t\to\infty} \psi(t_0,t) = -\infty$ leads the asymptotic stability of $x_2(t)$. Also, $\psi(t_0,t) \leqslant -c(t-t_0)$ implies the exponential stability of $x_2(t)$. Finally, it can be concluded that

$$\|x(t)\| \leqslant \|x_1(t)\| + \|x_2(t)\| \leqslant \mathcal{M} e^{\psi(t_0,t)} \|\phi(t)\|_c + \hat{\mathcal{M}} e^{\psi(t_0,t)} \|\phi(t)\|_c \leqslant \mathcal{D} e^{\psi(t_0,t} \|\phi(t)\|_c$$

in which $\mathcal{D} \geqslant \left(\mathcal{M} + \hat{\mathcal{M}}\right)$. This completes the proof. □

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Remark 4
From the proof of Theorem 1 stated earlier, it is easy to conclude that

(i) From (16) and noting that $\frac{\lambda_{max}(Q_{i_{22}})}{\lambda_{min}(Q_{i_{22}})}\frac{e^{\mathcal{J}_i}(1-\overline{d})}{\eta_{i_1}} < 1$, the maximum amount of $\mathcal{J}_i$ is obtained as $\mathcal{J}_i < ln\left(\frac{\eta_{i_1}}{(1-\overline{d})}\right)$ where $\kappa_i = \frac{\lambda_{max}(Q_{i_{22}})}{\lambda_{min}(Q_{i_{22}})}$ is the condition number of $Q_{i_{22}}$.
(ii) The parameters $\lambda_i(t), \forall i \in \mathcal{P}$ may be positive, negative, or sign varying, which implies that globally stable, unstable or locally stable subsystems may compose the switched system. Thus, there is not the requirement of all the subsystems to be stable. However, at least the states of one subsystem must be upper-bounded by a decreasing function. This implies that, at least one of subsystems activated during (t_0, t) must be globally or locally stable. In case of local stability, trajectories of the locally stable subsystem must be in the domain of attraction of the subsystem. In other words, all activated subsystems during (t_0, t) cannot be globally unstable. This is a necessary condition to guarantee (3e) in Theorem 1.
(iii) In Theorem 1, 'sufficiently large' means that $\mathcal{D} \geqslant (\mathcal{M} + \hat{\mathcal{M}})$ in which $\mathcal{M}$ and $\hat{\mathcal{M}}$ were defined in Parts 1 and 2 of the proof of Theorem 1.
(iv) With a pre-defined η_{i_1} and considering $\mathcal{L}_i(t)$ as $\begin{pmatrix} A_i^T P_i + P_i^T A_i + Q_i + \epsilon_i \varphi_i(t) I_{n\times n} & P_i^T \\ P_i & -\epsilon_i I_{n\times n} \end{pmatrix}$ in (3c), (3c) is changed to a convex problem and some efficient methods such as LMI solvers can be used to solve (3c) and (3d) simultaneously.
(v) The extensively augmented Lyapunov-like Krasovskii functional proposed in (5) is in fact a considerably more generalized form of the augmented type of Lyapunov functional presented in the literature. Because this generalized extensively augmented Lyapunov-like Krasovskii functional approach presents additional design parameters, it has the potential advantage of yielding further improvements in the stability results. For example, if a subsystem is global exponentially stable or global unstable, we can use $\lambda_{\sigma(t)}(t) = -\alpha(\alpha > 0)$ and $\lambda_{\sigma(t)}(t) = \beta(\beta > 0)$, respectively, which has been used in the recent literature [26] for non-singular switched time-delay systems .
(vi) In Theorem 1, conditions (3a) and (3b) have been obtained from stability analysis at switching points, (3c) and (3d) have been obtained from stability analysis of each subsystem, and (3e) has been obtained to guarantee the whole stability of each composing. Also, it is not difficult to find a scalar $\mathcal{J}_i$ such that $\int_{t-d(t)}^{t} \lambda_i(r)dr \geqslant \mathcal{J}_i > -\infty$, so it is not a conservative condition.

In the following discussion, the concept of 'average dwell-time' introduced by [27] will be used. For more results of stability of switched systems with average dwell-time switching signal, we refer the readers to [28, 29] and the references cited therein.

Definition 1
For any $T_2 > T_1 \geqslant 0$, let $N_\sigma(T_1, T_2)$ denote the number of switching of $\sigma(t)$ over (T_1, T_2). If $N_\sigma(T_1, T_2) \leqslant N_0 + \frac{T_2-T_1}{T_a}$ holds for $T_a > 0$, $N_0 \geqslant 0$, then T_a is called the average dwell-time [30].

Remark 5
Let $S_a[T_a; N_0]$ denote the set of all switching signals satisfying $N_\sigma(T_1, T_2) \leqslant N_0 + \frac{T_2-T_1}{T_a}$. This implies that, for $\sigma(t) \in S_a[T_a; N_0]$ the average time interval between consecutive switching is at least T_a. $S_a[T_a; N_0]$ may contain signals that occasionally have consecutive discontinuities separated by less than a constant T_a. Therefore, the average dwell-time switching signal has been recognized to be a flexible and efficient in system stability analysis.

Theorem 2
For given, $0 < h_1 \leqslant d(t) \leqslant h_2, \dot{d}(t) \leqslant \overline{d} < 1$, if there exist symmetric and positive-definite matrices P_{i_1}, Q_i with $r \times r$ and $n \times n$ dimension, respectively, non-singular matrices P_{i_2} with $(n-r) \times (n-r)$ dimension, scalars $\mathcal{J}_i$, positive scalars η_{i_j} $(j = 1, 2, 3)$ and ϵ_i, and scalar $\mu \geqslant 1$, such that (3a), (3b), (3c), (3d), and the following inequality hold

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$$\frac{t-t_0}{T_a}ln(\mu(1+(g(t))^2))+\sum_{i=1}^{m}\int_{T_i(t_0,t)}\lambda_i(r)dr \leqslant 2\psi(t_0,t), \forall t \geqslant 0 \tag{22}$$

then, the switched nonlinear singular time-delay system (1) under Assumptions 1 and 2 and switching condition $h_{\sigma(\tau_k^-)_2}(\tau_k^-) = h_{\sigma(\tau_k^+)_2}\left(\tau_k^+\right)$, is asymptotically stable for every switching signal $\sigma(t) \in S_a[T_a; N_0]$ provided that $\lim_{t\to\infty}\psi(t_0,t) = -\infty$ and $\psi(t_0,t_0) = 0$. Also, if $\psi(t_0,t) \leqslant -c(t-t_0)$ with $c > 0$, then, the switched nonlinear singular time-delay system (1) under Assumptions 1 and 2 and switching condition $h_{\sigma(\tau_k^-)_2}\left(\tau_k^-\right) = h_{\sigma(\tau_k^+)_2}\left(\tau_k^+\right)$, is exponentially stable for every switching signal $\sigma(t) \in S_a[T_a; N_0]$.

Proof
The proof is similar to the proof of Theorem 1. The only variation is that because $g(t)$ is non-decreasing function, we can conclude that $ln\left(\prod_{j=1}^{k}\mu(1+(g(\tau_j))^2)\right) \leqslant N_\sigma(t_0,t)ln(\mu(1+(g(t))^2))$. Therefore,

$$e^{ln\left(\prod_{j=1}^{k}\mu(1+(g(\tau_j))^2)\right)+\sum_{i=1}^{m}\int_{T_i(t_0,t)}\lambda_i(r)dr} \leqslant e^{\frac{t-t_0}{Ta}ln(\mu(1+(g(t))^2))+\sum_{i=1}^{m}\int_{T_i(t_0,t)}\lambda_i(r)dr}$$

And this completes the proof. □

Remark 6
In Theorems 1 and 2, there is no need to choose $\psi(t_0,t)$ exactly. Indeed, the function $\psi(t_0,t)$ is only used to over-estimate the behavior of functions $ln\left(\prod_{j=1}^{k}\mu(1+(g(\tau_j))^2)\right) + \sum_{i=1}^{m}\int_{T_i(t_0,t)}\lambda_i(r)dr$ and $\frac{t-t_0}{T_a}ln(\mu(1+(g(t))^2)) + \sum_{i=1}^{m}\int_{T_i(t_0,t)}\lambda_i(r)dr$ in (3e) and (22), respectively. For example, if all subsystems are globally exponentially stable, we can use $\lambda_{\sigma(t)}(t) = -\alpha(\alpha > 0)$, and (22) is changed to $\frac{t-t_0}{T_a}ln(\mu(1+(g(t))^2)) - \alpha(t-t_0) \leqslant 2\psi(t_0,t)$. In order to achieve asymptotic stability, the condition $lim_{t\to\infty}\left(\frac{1}{T_a}ln(\mu(1+(g(t))^2)) - \alpha\right)(t-t_0) = -\infty$ has to be satisfied. We use the sandwich rule to check the aforementioned condition. Therefore, if we want to achieve $lim_{t\to\infty}\left(\frac{1}{T_a}ln(\mu(1+(g(t))^2)) - \alpha\right)(t-t_0) = -\infty$, the following condition has alternatively to be satisfied:

$$-\infty < \lim_{t\to\infty}\left(\frac{1}{T_a}ln\left(\mu(1+(g(t))^2)\right) - \alpha\right) \leqslant 0$$

which yields to $lim_{t\to\infty}ln(\mu(1+(g(t))^2)) \leqslant \alpha T_a$ and $(1+g(\infty)) \leqslant \frac{e^{\alpha T_a}}{\mu}$. If we choose $g(t) = \overline{c} = constant$, then $(1+\overline{c}^2) \leqslant \frac{e^{\alpha T_a}}{\mu}$. The function $\psi(t_0,t)$ is, thus, any function upper-bounding $\frac{t-t_0}{T_a}ln(\mu(1+(g(t))^2)) + \sum_{i=1}^{m}\int_{T_i(t_0,t)}\lambda_i(r)dr$, which exists as it has been shown. To satisfy the exponential stability, we obtain the condition $\frac{t-t_0}{T_a}ln(\mu(1+(g(t))^2)) + \sum_{i=1}^{m}\int_{T_i(t_0,t)}\lambda_i(r)dr \leqslant -c(t-t_0)$, then $\left(\frac{1}{T_a}ln(\mu(1+(g(t))^2)) - \alpha\right)(t-t_0) \leqslant -c(t-t_0)$, which implies $(1+g(\infty)) \leqslant \frac{e^{T_a(\alpha-c)}}{\mu}$. These conditions can be extended when some subsystems are global unstable or local stable. So there are many functions that satisfy $lim_{t\to\infty}\psi(t_0,t) = -\infty$ and $\psi(t_0,t_0) = 0$ and (22). Thus, we can choose $\psi(t_0,t)$ easily according to the behavior of each subsystem. Here, by choosing function $\psi(t_0,t)$, we consider a more general case, which includes many of previous research results. In other words, the presented method includes, as particular cases, the methods that have been used till now by researchers.

Remark 7
Subsystem (2) is said to have index one if $x_2(t) = \overline{h}_i(t, x_1(t), x_1(t-d(t)))$ is the unique solution of $\mathbf{0} = h_{i_2}(t, x(t), x(t-d(t)))$. Because any particular index is not assumed in the proof of Theorem 1, we can see that this theorem can be applicable for subsystems with higher index.

It is worth pointing out that for the case $f_i(t, x(t)) = 0$ (i.e., for the linear case), the following corollary is obtained.

Corollary 1
For given, $0 < h_1 \leqslant d(t) \leqslant h_2, \dot{d}(t) \leqslant \overline{d} < 1$, if there exist non-singular matrices P_{i_2} with $(n-r) \times (n-r)$ dimension, symmetric and positive-definite matrices ,P_{i_1}, Q_i with $r \times r$ and $n \times n$ dimension, respectively, scalars $\mathcal{J}_i$, positive scalars $\eta_{i_j}(j = 1, 2, 3)$ and ϵ_i, and scalar $\mu \geqslant 1$, such that (3a),(3b),(3d),(22), and the following inequality hold for $t \geqslant 0$:

$$\begin{pmatrix} \mathcal{L}_i(t) & P_i^T A_{di} \\ * & -e^{\mathcal{J}_i}(1-\overline{d})Q_i \end{pmatrix} - diag\left(\lambda_i(t) E^T P_i, \mathbf{0}_{(n-r)\times(n-r)}\right) < 0 \tag{23}$$

in which $\mathcal{L}_i(t) = A_i^T P_i + P_i^T A_i + Q_i$, then, the switched singular time-delay system (1) under Assumption 2 and switching condition $h_{\sigma(\tau_k^-)_2}\left(\tau_k^-\right) = h_{\sigma\left(\tau_k^+\right)_2}\left(\tau_k^+\right)$, is asymptotically stable for every switching signal $\sigma(t) \in S_a[T_a; N_0]$, provided that $\lim_{t\to\infty} \psi(t_0, t) = -\infty$ and $\psi(t_0, t_0) = 0$. Also, if $\psi(t_0, t) \leqslant -c(t - t_0)$ with $c > 0$, then, the switched singular time-delay system (1) is exponentially stable for every switching signal $\sigma(t) \in S_a[T_a; N_0]$.

Proof
By setting $f_i(t, x(t)) = \mathbf{0}$ in (5) and (10), we can prove Corollary 1 directly. So, it is omitted here □

Now, we want to define an average dwell-time switching signal by supposing that all subsystems are globally exponentially stable. Switching signals determine the dynamic behavior of a switched system in the most cases and may cause the instability of the switched case if all subsystems are stable. For stability analysis of non-singular (and nonlinear) switched system by consisting of stable subsystems, the reader can refer, for instance, to papers such as [31]. The following corollary gives some sufficient conditions such that the exponential stability of the switched nonlinear singular time-delay system under Assumptions 1 and 2 is guaranteed.

Corollary 2
For given, $0 < h_1 \leqslant d(t) \leqslant h_2, \dot{d}(t) \leqslant \overline{d} < 1$, if there exist symmetric and positive-definite matrices P_{i_1}, Q_i with $r \times r$ and $n \times n$ dimension, respectively, non-singular matrices P_{i_2} with $(n-r) \times (n-r)$ dimension, scalars $\mathcal{J}_i$, positive scalars $\eta_{i_j}(j = 1, 2, 3)$ and ϵ_i, and scalar $\mu \geqslant 1$, such that (3a), (3c), and (3d) are satisfied, then the switched nonlinear singular time-delay system (1) under Assumptions 1 and 2 and switching condition $h_{\sigma(\tau_k^-)_2}\left(\tau_k^-\right) = h_{\sigma(\tau_k^+)_2}\left(\tau_k^+\right)$, is exponentially stable for every average dwell-time switching signal with $T_a \geqslant (\ln \mu/2\alpha)$.

Proof
The proof is similar to the proof of Theorem 2. By letting $g(t) = 0$ in (6) and $\lambda_i(r) = -\alpha \forall i \in \mathcal{P}$, which implies that all the subsystems are stable, we can get $V_{\sigma(\tau_k)}(\tau_k, x(\tau_k)) \leqslant \mu V_{\sigma(\tau_{k-1})}\left(\tau_k^-, x\left(\tau_k^-\right)\right)$. Therefore, if $\frac{t-t_0}{2T_a}(ln(\mu) - \alpha) \leqslant -c(t - t_0)$, then the switched nonlinear singular time-delay system (1) is exponentially stable with $T_a \geqslant (\ln \mu/2\alpha)$ Thus, the proof is completed. □

Remark 8
In a singular time-delay system, if the pair matrices (E, A_p) are regular and impulse-free, it can still have finite discontinuities because of incompatible initial condition. Furthermore, in switched singular time-delay systems, continuity at switching points may not always be satisfied. Therefore, unlike standard switched time-delay systems, discontinuities in switched singular time-delay systems can propagate between different times because of the existence of delayed solution terms and discontinuities of previous switching points. Moreover, the stability analysis based on the average dwell-time of switched singular systems is closely related to the exponential stability of algebraic

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equations, and there are some features in switched nonlinear singular systems with time-delay that are neither found in singular nor in switched time-delay systems. Concerning the comparison with other well-known results in the literature [32–36], all these results have been presented for singular systems without considering simultaneously average dwell-time, time-delay, and nonlinear terms. As an example, in [32], stability analysis of discrete-time switched singular time-delay systems is presented, but unfortunately, exponential stability of algebraic states has not been given correctly and the authors could not investigate a proper discussion on exponential stability. Also, by concerning the comparison with the papers, which have been published in the current year by the authors of the paper [37, 38], exponentially stability for switched nonlinear singular systems with time-delay without considering nonlinear terms and stability analysis of switched nonlinear singular systems with time delay by considering stable subsystems have been investigated. However, the delay range-dependent stability problem for switched nonlinear singular time-delay systems by considering exponential stability, average dwell-time switching signal, time-delay, and nonlinear terms has not been fully investigated yet, which will be challenging because of the difficult extension of the existing stability.

Remark 9

A trend in the development of stability theory of switched nonlinear systems is the study of stability within a finite region. In the case where global absolute stability does not hold, we need to restrict our attention to a finite region in the state space. In the finite region, a guaranteed domain of attraction can be then obtained by using some invariant level set of a quadratic type Lyapunov function. But, in nonlinear singular systems, because the domain of attraction have not been fully definite and investigated, it seems stability analysis by considering domain of attraction will be challenging because of the difficult extension of the existing stability results. In this paper, by considering a locally integrable function $\lambda_i(t)$, we can envelop the behavior of the systems without estimating domain of attraction.

Remark 10

As mentioned in Remark 8, the stability analysis based on the average dwell-time of switched singular time-delay systems are closely related to the exponential stability of algebraic equations and there are some features in switched singular systems with time-delay are found in neither singular systems nor switched time-delay systems. When the considered system is non-singular time delay system, our results can be reduced as those in [3, 39] as special cases. For non-singular switched systems, one can also study the stability of the switched system under particular classes of switching signals as in [3, 39]. Of course, [3, 39] presents a method for switched time-delay systems without considering nonlinear terms and singularity. So, it is impossible to apply the method presented in [3, 39] for switched nonlinear singular systems. On the other hand, L_2 gains are also considered in stability analysis in [3]. However, sufficient conditions for the asymptotic (exponential) estimation of Lyapunov-like Krasovskii functional for all subsystems are presented as Theorems 1 and 2. Our results are delay dependent, which is less conservative than the delay-dependent results when time delay is small. Also, our results can be applicable to three cases: one is that interval time-varying and the other is constant time-delay. Because some additional matrix variables have been added in stability analysis, this method can be also applicable to continuous and bounded time-varying delay without considering a bound on derivative of time-delay. Finally, this method can be applicable to singular and nonlinear systems. Thus, our results are less conservative.

4. NUMERICAL EXAMPLE

In this section, two numerical examples contained in two subsections are presented to verify the results of the proposed procedure in switched nonlinear singular time-delay systems.

(4.a) Consider the following switched nonlinear singular time-delay system, composed of two subsystems described by (In the examples in the succeeding text $E_1 = E_2 = E$)

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- **Subsystem** $i = 1$:

$$E = diag(1,1,0), A_1 = \begin{pmatrix} -1.1 & 0.2 & 0.1 \\ -0.3 & -2.3 & 0 \\ -0.5 & 1 & 1 \end{pmatrix}, A_{d1} = \begin{pmatrix} -0.7 & -0.5 & 0.6 \\ -0.6 & 0.3 & 0.1 \\ -0.1 & 0.3 & 0.5 \end{pmatrix},$$

$$f_1(t, x(t)) = \begin{pmatrix} x_1(t)sin(x_1(t)) \\ x_2(t)cos(x_2(t))^2 \\ x_1(t)e^{-|x_2(t)|} \end{pmatrix}$$

- **Subsystem** $i = 2$:

$$A_2 = \begin{pmatrix} -1.4 & -0.2 & 0.2 \\ 0.2 & -1.5 & 0 \\ -0.5 & 1 & 1 \end{pmatrix}, A_{d2} = \begin{pmatrix} 0.6 & 0.3 & 0.9 \\ 0.5 & 0.2 & 0.1 \\ -0.1 & 0.3 & 0.5 \end{pmatrix}, f_2(t, x(t)) = \begin{pmatrix} x_1(t)sin(x_1(t)) \\ 0 \\ x_1(t)e^{-|x_2(t)|} \end{pmatrix}$$

where $d(t) = 0.1 + 0.01sin(t)$. It is found that both subsystems are stable for $\alpha = 1.15$. Hence, given $\mu = 4.46$, it follows that the proposed system is exponentially stable with average dwell-time $T_a \geqslant 0.6450$s. Furthermore, the corresponding variables can be obtained by using MATLAB software as follows:

$$P_1 = 1.0e-004 \times \begin{pmatrix} 0.0371 & 0.0087 & 0 \\ 0.0087 & 0.1222 & 0 \\ 0 & 0 & -0.0142 \end{pmatrix}, Q_1 = 1.0e-004 \times \begin{pmatrix} 0.0373 & 0.0141 & 0 \\ 0.0141 & 0.1790 & 0 \\ 0 & 0 & 0.0364 \end{pmatrix}$$

$$P_2 = 1.0e{-}005 \times \begin{pmatrix} 0.4426 & -0.1148 & 0 \\ -0.1148 & 0.8449 & 0 \\ 0 & 0 & -0.2894 \end{pmatrix}, Q_2 = 1.0e{-}005 \times \begin{pmatrix} 0.4105 & 0.0018 & 0 \\ 0.0018 & 0.4134 & 0 \\ 0 & 0 & 0.4090 \end{pmatrix}$$

$$\eta_{1_1} = 0.8700, \sum_{j=2}^{3} \eta_{1_j} = 2.0377e-009, \eta_{2_1} = 0.7530, \sum_{j=2}^{3} \eta_{2_j} = 4.1249e-009,$$

$$\epsilon_1 = 3.5851e-006, \epsilon_2 = 4.0888e-006, \mathcal{J}_1 = \mathcal{J}_2 = -0.1265.$$

The complete simulation results are shown in Figure 2. Figure 2(a) shows the states trajectories of the switched system, Figure 2(b) shows state space and Figure 2(c) shows the switching signal.

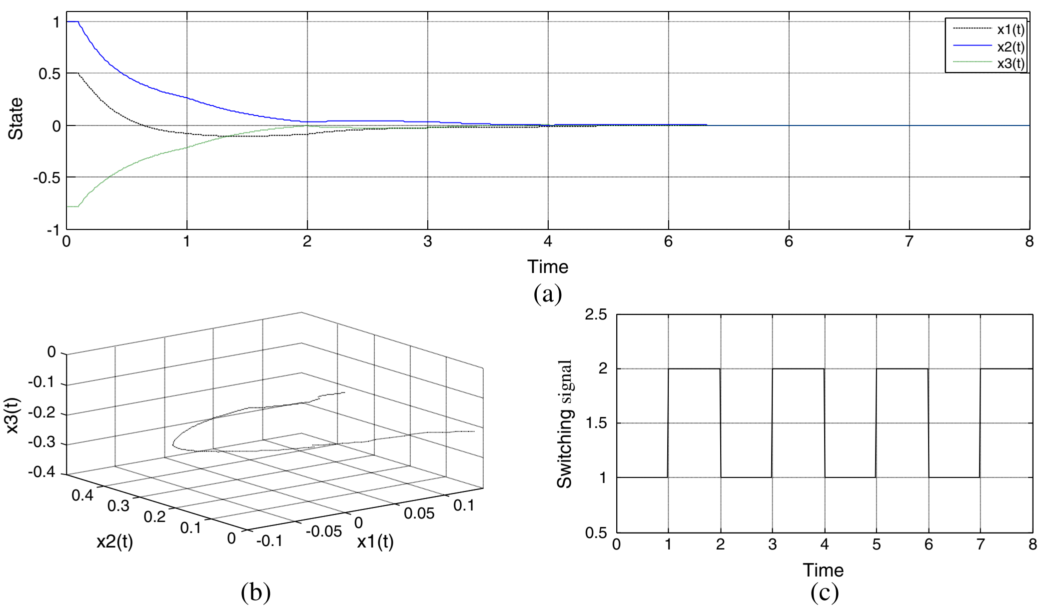

Figure 2. Responses of the switched nonlinear singular time-delay system; Example 4.a.

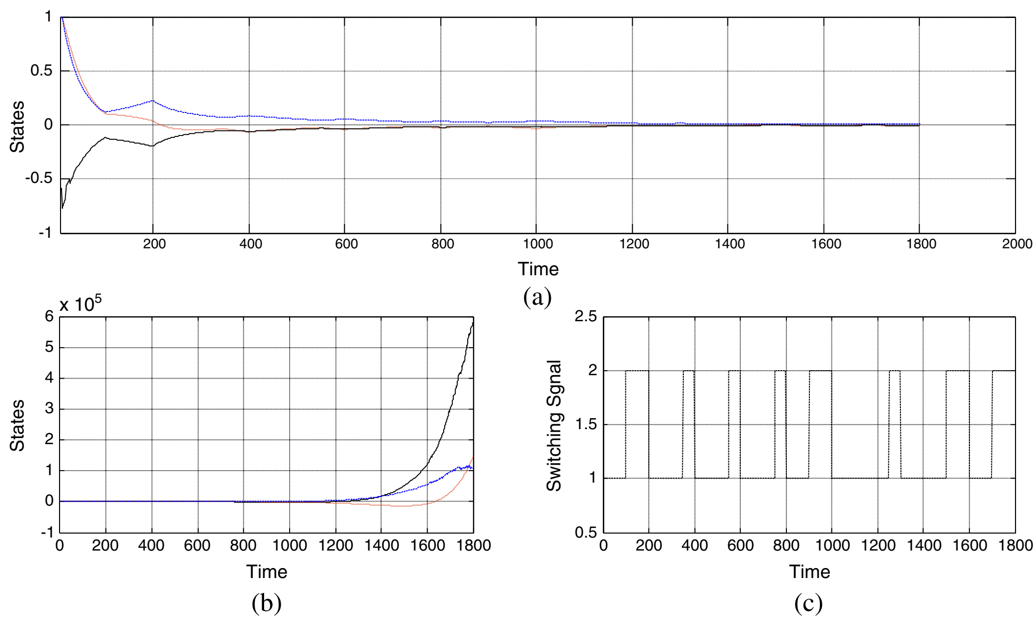

Figure 3. Responses of the switched nonlinear singular time-delay system; Example 4.b.

4.b) Now, consider the same subsystem 1 in 4.a with the second subsystem as follows, which is unstable:

- **Subsystem** $i = 1$: The same subsystem $i = 1$ in 4.a.
- **Subsystem** $i = 2$

$$A_2 = \begin{pmatrix} 1 & -0.1 & 0 \\ 0.2 & 0.6 & 0 \\ 0.05 & 0.01 & -1 \end{pmatrix}, A_{d2} = \begin{pmatrix} -0.6 & 0.3 & 0.9 \\ 0.05 & 0 & 0.03 \\ -0.1 & 0.3 & 0.5 \end{pmatrix}, f_2(t, x(t)) = \begin{pmatrix} x_1(t)sin(x_1(t)) \\ 0 \\ x_1(t)e^{-|x_2(t)|} \end{pmatrix}$$

where $d(t) = 0.1 + 0.01sin(t)$. It is found that the second subsystem is unstable for $\beta = 0.01$. It follows that the proposed switched system is asymptotically stable for the switching law depicted in Figure 3(c). The corresponding variables can be obtained as follows:

$$P_2 = 1.0e{-}005 \times \begin{pmatrix} 0.1858 & -0.0061 & 0 \\ -0.0061 & 0.2219 & 0 \\ 0 & 0 & -0.2894 \end{pmatrix}, Q_2 = 1.0e{-}005 \times \begin{pmatrix} 0.1878 & -0.0080 & 0 \\ -0.0080 & 0.2522 & 0 \\ 0 & 0 & 0.7651 \end{pmatrix}$$

$$\eta_{2_1} = 0.5300, \epsilon_2 = 1.8828e - 006.$$

The complete simulation results are shown in Figure 3. Figure 3(a) shows the states trajectories of the switched system, Figure 3(b) shows trajectories of the unstable subsystem and Figure 3(c) shows the switching signal.

5. CONCLUSION

We have given a set of sufficient conditions for stability analysis of switched singular systems, where each subsystem includes time-delay and nonlinear terms. First, a set of asymptotic (exponential) stability conditions have been derived for switched nonlinear singular time-delay system. It has been shown that the stability can be analyzed by solving a set of inequalities. Second, this approach has been used to find an average dwell-time switching signal such that the exponential stability of the switched nonlinear singular time-delay system is guaranteed. Such conditions for the problem to

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be solvable have been derived by using a new kind of Lyapunov-like Krasovskii functional. In the propose approach, it was not supposed that all the subsystems have to be stable which is of great significance.

APPENDIX

Proof of Lemma 1

In Lemma 1, from (a) we want to show (b). We know that $f(t) \leqslant \xi_1 sup_{t-\tau\leqslant s\leqslant t} f(s) + \xi_2 e^{\psi(t_0,t)}$. Next, we want to prove that for any $\epsilon_0 > 0$,

$$f(t) < sup_{t_0-\tau\leqslant s\leqslant t_0} f(s) e^{\psi(t_0,t)} + \frac{\xi_2}{1-\xi_1 e^{\xi_0\tau}} e^{\psi(t_0,t)} + \epsilon_0 \tag{A.1}$$

From (a), it can be concluded that $f(t_0) \leqslant \xi_1 sup_{t_0-\tau\leqslant s\leqslant t_0} f(s) + \frac{\xi_2}{1-\xi_1 e^{\xi_0\tau}} + \epsilon_0$. If (b) in Lemma 1 is not true, then $\bar{t}$ exists such that

$$f(\bar{t}) = sup_{t_0-\tau\leqslant s\leqslant t_0} f(s) e^{\psi(t_0,\bar{t})} + \frac{\xi_2}{1-\xi_1 e^{\xi_0\tau}} e^{\psi(t_0,\bar{t})} + \epsilon_0 \tag{A.2}$$

And $f(t) < sup_{t_0-\tau\leqslant s\leqslant t_0} f(s) e^{\psi(t_0,t)} + \frac{\xi_2}{1-\xi_1 e^{\xi_0\tau}} e^{\psi(t_0,t)} + \epsilon_0$, $\forall t \leqslant \bar{t}$. In fact, for $t \in [\bar{t}-\tau, \bar{t}]$, we have

$$f(t) \leqslant sup_{\bar{t}-\tau\leqslant s\leqslant \bar{t}} f(s) \leqslant sup_{t_0-\tau\leqslant s\leqslant t_0} f(s) e^{\psi(t_0,\bar{t})} + \frac{\xi_2}{1-\xi_1 e^{\xi_0\tau}} e^{\psi(t_0,\bar{t})} + \epsilon_0 \tag{A.3}$$

So,

$$\begin{aligned} f(\bar{t}) &\leqslant \xi_1 \sup_{\bar{t}-\tau\leqslant s\leqslant \bar{t}} f(s) e^{\psi(t_0,\bar{t})} + \xi_2 e^{\psi(t_0,\bar{t})} \\ &\leqslant \xi_1 \sup_{t_0-\tau\leqslant s\leqslant t_0} f(s) e^{\psi(t_0,\bar{t})} + \frac{\xi_1\xi_2 e^{\xi_0\tau}}{1-\xi_1 e^{\xi_0\tau}} e^{\psi(t_0,\bar{t})} + \xi_1\epsilon_0 + \xi_2 e^{\psi(t_0,\bar{t})} \\ &< \xi_1 sup_{t_0-\tau\leqslant s\leqslant t_0} f(s) e^{\psi(t_0,\bar{t})} + \frac{\xi_2}{1-\xi_1 e^{\xi_0\tau}} e^{\psi(t_0,\bar{t})} + \epsilon_0 \end{aligned} \tag{A.4}$$

which is contradicts (A.2). By letting $\epsilon_0 \to 0$ in (A.1), we obtain Lemma 1. This completes the proof. □

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