BT SBT :

20. Ánh làm ok

1. $y = x\sqrt{1+x^2}$

$\Rightarrow y' = \sqrt{1+x^2} + x\dfrac{2x}{2\sqrt{1+x^2}} = \dfrac{1+2x^2}{\sqrt{1+x^2}}$

$\Rightarrow y'' = \dfrac{4x\sqrt{1+x^2} - \frac{x}{\sqrt{1+x^2}}\cdot(1+2x^2)}{1+x^2}$

$= \dfrac{x(3+2x^2)}{(1+x^2)^{3/2}}$

2. $y = \dfrac{x}{\sqrt{1-x^2}} = x\left(\sqrt{1-x^2}\right)^{-1} = x(1-x^2)^{-1/2}$

$y' = (1-x^2)^{1/2} + x\left(-\frac{1}{2}\right)(-2x)(1-x^2)^{-3/2}$

$= (1-x^2)^{1/2} + x^2(1-x^2)^{-3/2} = \dfrac{1}{(1-x^2)^{3/2}}$

$\boxed{y'' = \dfrac{3x}{(1-x^2)^{5/2}}}$

3, $y = e^{-x^2} \Rightarrow y' = -2x.e^{-x^2}$ sao nhanh thế

$\boxed{\Rightarrow y'' = 2e^{-x^2}(2x^2-1)}$ cần viết cẩn thận hơn

4, $y = \ln f(x) \Rightarrow y' = \dfrac{f'(x)}{f(x)}$.

$\Rightarrow y'' = \dfrac{f''(x).f(x) - f'(x).f'(x)}{f^2(x)}$, $f(x) > 0$.

Bài 21 Tìm y'_x, y''_{xx} của hls

1, $\begin{cases} x = 2t - t^2 \\ y = 3t - t^3 \end{cases} \Rightarrow \begin{cases} x'(t) = 2 - 2t \\ y'(t) = 3 - 3t^2 \end{cases}$

$y'_x = \dfrac{dy}{d} = \dfrac{y'(t)}{x'(t)} = \dfrac{3-3t^2}{2-2t}$

$\Rightarrow y''_{xx} = \dfrac{-6t(2-2t) + 2(3-3t^2)}{(2-2t)^2} = \dfrac{3t^2-6t+3}{4(1-t)^2} = \dfrac{3}{4(1-t)}$

2. $dx = -a\sin t\, dt$, $t'_x = \dfrac{-1}{a\sin t}$

$dy = a\cos t\, dt$.

$y'_x = -\cot t$

$$\Rightarrow y''_{xx} = -(\cot t)'_t \cdot t'_x = \frac{1}{\sin^2 t}\left(-\frac{1}{a\sin t}\right) = \frac{-1}{a\sin^3 t}.$$

3. $dx = a(1-\cos t)\,dt$

$$t'_x = \frac{1}{a(1-\cos t)} = \frac{1}{2a\sin^2\frac{t}{2}}$$

$$dy = a\sin t\,dt = 2a\sin\frac{t}{2}\cdot\cos\frac{t}{2}\,dt$$

$$y'_x = \frac{dy}{dx} = \frac{2a\sin\frac{t}{2}\cos\frac{t}{2}\,dt}{2a\sin^2\frac{t}{2}\,dt} = \cot\frac{t}{2}$$

$$y''_{xx} = \frac{d^2y}{dx^2} = \left(\cot\frac{t}{2}\right)'\cdot t'_x = -\frac{1}{2\sin^2\frac{t}{2}}\cdot\frac{1}{2a\sin^2\frac{t}{2}}$$

$$= -\frac{1}{4a\sin^4\frac{t}{2}}, \quad t \neq 2k\pi \quad k\in\mathbb{Z}$$

22. 1, $y = \dfrac{x^2}{1-x}$, $y^{(8)}$

$$y = \frac{x^2}{1-x} = \frac{1-(1-x^2)}{1-x} = \frac{1}{1-x} - (1+x)$$

$$\Rightarrow \quad y^{(8)} = \frac{8!}{(1-x)^9}$$

2. $y = \dfrac{1+x}{\sqrt{1-x}} = (1+x)(1-x)^{-1/2}$

$$y^{(100)} = \left[(1-x)^{-1/2}\right]^{(100)}(1+x) + 100\left[(1-x)^{1/2}\right]^{(99)}(1+x)'$$

mà $\left[(1-x)^{-1/2}\right]^{(n)} = \dfrac{(2n-1)!!}{2^n}\cdot\dfrac{1}{(1-x)^n\sqrt{1-x}}$

Thay ll $n=99$, $n=100$ vào $y^{(100)}$

$$y^{(100)} = \frac{199!!}{2^{100}}\cdot\frac{1+x}{(1-x)^{100}\sqrt{1-x}} + \frac{197!!}{2^{99}}\cdot\frac{100}{(1-x)^{99}\cdot\sqrt{1-x}}$$

$$= \frac{197!!}{2^{100}(1-x)^{100}\sqrt{1-x}}(399-x)$$

3. $y = e^{2x} x^2.$ $\qquad (uv)^n = \sum_{k=0}^{n} C_n^k u^{n-k} v^k$

$u = e^{2x}$ | $v = x^2$

$v' = 2x \quad , \quad v'' = 2.$

$u^{(n)} = \cancel{e^{2x}} \quad 2^n e^{2x}$

$\Rightarrow y^{(n)} = C_n^0 \, 2^n e^{2x} \cdot x^2 + C_n^1 \, 2^{n-1} e^{2x} \cdot 2x + C_n^2 \, 2^{n-2} e^{2x} \cdot 2.$

thay $n = 20$ ta có

$y^{(20)} = 2^{20} \cdot e^{2x} \cdot (x^2 + 20x + 95$

4. $y = (\sin 2x) \; x^2$

$u = \sin 2x \qquad v = x^2. \Rightarrow v' = 2x, \; v'' = 2$

$u^{(n)} = 2^n \sin(2x + \frac{n\pi}{2})$

$\Rightarrow y^{(n)} = C_n^0 \, 2^n \cdot \sin(2x + \frac{n\pi}{2}) \cdot x^2 + C_n^1 \, 2^{n-1} \sin\left(2x + \frac{(n-1)\pi}{2}\right) \cdot 2x$

$+ \; C_n^2 \cdot 2^{n-2} \cdot \sin\left(2x + \frac{(n-2)\pi}{2}\right) \cdot 2.$

thay $n = 50$ ta có $y^{(50)} = 2^{50}\left(-x^2 \cdot \sin 2x + 50x \cdot \cos 2x + \frac{1225}{2} \sin 2x\right)$

23. Tìm $y^{(n)}$

1. $y = \frac{1}{x(1-x)} = \frac{x + 1 - x}{x(1-x)} = \frac{1}{1-x} + \frac{1}{x}.$

$y^{(n)} = \left(\frac{1}{1-x}\right)^{(n)} + \left(\frac{1}{x}\right)^{(n)} = n! \left[\frac{1}{(1-x)^{n+1}} + \frac{(-1)^n}{x^{n+1}}\right]$

2. $y = \frac{1}{x^2 - 3x + 2} = \frac{1}{(1-x)(2-x)} = \frac{1}{1-x} - \frac{1}{2-x}$

$\Rightarrow y^{(n)} = n! \left[\frac{1}{(1-x)^{n+1}} - \frac{1}{(2-x)^{n+1}}\right]$

3. $y = \dfrac{x}{\sqrt[3]{1+x}} = x(1+x)^{-1/3}$

$$y^{(n)} = \left[(1+x)^{-1/3}\, x\right]^{(n)} = \left[(1+x)^{-1/3}\right]^{(n)} x + n\left((1+x)^{-1/3}\right)^{(n-1)}$$

$$\Rightarrow y^{(n)} = \frac{(-1)^{n-1}}{3^n}\left(1.4\ldots(3n-5)\right)\frac{3n+2x}{(1+x)^{n+1/3}}, \quad n \geq 2.$$

4. $y = e^{ax}.\sin bx.$