Proceedings of the 10-th IFAC Workshop on Time Delay Systems
The International Federation of Automatic Control
Northeastern University, Boston, USA. June 22-24, 2012

DC Motor Control Using the Lambert W Function Approach

Sun Yi * **Patrick W. Nelson** ** **A. Galip Ulsoy** ***

* *Sun Yi (corresponding author) is with the Department of Mechanical Engineering, NC A&T State University, Greensboro, NC 27411 (e-mail: syi@ncat.edu).*
** *Patrick W. Nelson is with the Center for Computational Medicine and Bioinformatics, University of Michigan, Ann Arbor, MI 48109 (e-mail: pwn@umich.edu).*
*** *A. Galip Ulsoy is with the Department of Mechanical Engineering, University of Michigan, Ann Arbor, MI 48109 (e-mail: ulsoy@umich.edu).*

Abstract: A new design method for proportional-plus integral (PI) and proportional-plus velocity (PV) control for DC motors to regulate angular velocity and position considering time-delays, is presented. Time-delays inherent in plants can arise from signal processing and actuation and can lead to undesired system performance including instability. Thus, time-delays should be considered in designing controllers. In this paper, a method based on the Lambert W function is used to address such problems caused by time-delays. The method enables one to find the rightmost (i.e., dominant) characteristic roots in the infinite eigenspectrum. Then, PI and PV control gains are obtained by assigning the rightmost eigenvalues to desired positions in the complex plane. The assignment can be achieved thanks to a novel property of the Lambert W function. System performance can be improved as well as successfully stabilized. Effectiveness of the presented method is verified through simulations and experiments. Also, sensitivity analysis of the rightmost eigenvalues with respect to delay mismatches is conducted to compare the proposed approach to a prediction-based method.

Keywords: Time delay systems, Lambert W function, Motor control, PI control.

1. INTRODUCTION

DC motors are used for applications, such as robot arm drives, machine tools, rolling mills, and aircraft control (Teeter et al. (1996)). In such applications, time-delays are inherent. For example, networks for factory automation have communications delays ($\leq$ 0.0862s, Tipsuwan et al. (2004)), or measurement delays are caused due to given sampling periods in encoders (0.5ms, Hong et al. (1998)). Use of vision systems require image processing, with resulting delays of up to 100ms (Magana et al. (1998)). Also, time-delays may be a consequence of the computer controlled implementation and/or a consequence of interconnection of sensors and actuators with controllers. Therefore, for precision control, it is quite important to develop DC motor control strategies where the effects of time-delays are properly considered.

Several research efforts have demonstrated the ability to handle time-delays in designing feedback controllers for motor systems. For example, stability analysis of delayed motor systems can be conducted via Padé approximation-based approaches (Guzzella et al. (2004), Tipsuwan et al. (2004)). Although the Padé approximation is not a state-of-the-art technique for time-delay systems, because it provides convenience and straightforward extension of various methods for non-delayed systems it is still being widely used. Design of controllers has been realized using bifurcation methods that substitute a purely imaginary number for the characteristic root in the Laplace domain (see, e.g., Silva et al. (2001) and the references therein). The control gains are chosen based on stability charts (Guzzella et al. (2005)), often in conjunction with the Nyquist method (Krajewski et al. (2004)). Alternatively, such problems have been addressed by using H_∞ feedback control with a discrete event based model to improve robustness to disturbances and modeling uncertainties (Mianzo et al. (2001)). Although discrete models can be used to avoid exponential terms in the characteristic equations (Kim et al. (2001), Powell et al. (1998)) robustness issues with regard to the delay value cannot be analyzed via discretization (Richard (2003)). The Smith predictor has been widely used, and successfully improved the performance in both simulations and experiments for motor systems (Liaw et al. (1994)), and in Nakagawa et al. (2002) the method was compared to an experimental tuning method (Ziegler-Nichols step response method).

As an alternative to the above existing methods, the Lambert W function-based method is used in this paper. The Lambert W function has been used to find analytical solutions to systems of delay differential equations (DDEs)

978-3-902823-04-5/12/$20.00 © 2012 IFAC

10.3182/20120622-3-US-4021.00014

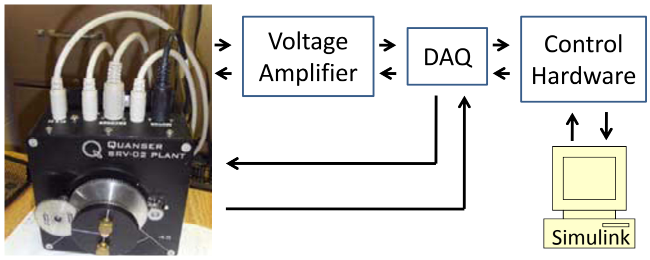

Fig. 1. Schematic of the experimental DC motor system (SRV 02 made by Quanser Quanser (2010))

(Corless et al. (1996)). Based on the solutions, a systematic approach to analysis and control of time-delay systems has been developed (Yi et al. (2010a)). By obtaining and assigning the rightmost (i.e., dominant) eigenvalues in the infinite eigenspectrum, time-delay systems can be analyzed and controlled systematically using the Lambert W function. In Yi et al. (2011), the approach has been used to design PI controllers for first-order plants with delay. Using the Lambert W function, PI controllers are designed by assigning the rightmost eigenvalues to desired positions in the complex plane. The designed controller improves performance as well as successfully stabilizing unstable systems. Also, sensitivity of the rightmost eigenvalue with respect to system parameters (including delays) was shown to compare favorably to Smith predictor-based controllers (Yi et al. (2011)).

In this paper, the Lambert W function-based approach for PI controller design is applied to regulate the speed of DC motors. Also, the method is extended to design PV controllers for position regulation. A case study is provided to illustrate the effectiveness of the design method in terms of convenience and robustness. Many existing techniques require a completely redesigned controller to handle time-delays. However, the approach presented here does not require change of controller structure. Also, the proposed method makes use of analytical solution forms, not depending on approximation or prediction, for improved accuracy and robustness. First, an open-loop model is combined with a PI (or PV) controller. The closed-loop model is converted into a system of DDEs (a matrix-vector form). Then, by using the Lambert W function approach, the rightmost eigenvalues are obtained. The PI (or PV) gains are calculated by assigning the desired rightmost eigenvalues. Subsequently, the designed controller is implemented on the DC motor system (see Fig. 1). The importance of the proposed design method is its ability to obtain control gains through eigenvalue assignment. Through the assignment of the rightmost (i.e., dominant) eigenvalues stability can be achieved and transient response can be improved as non-delayed systems. The paper is organized as follows. In Section 2, for speed regulation, PI controllers are designed and implemented on a DC motor system. The method is compared with the Smith predictor in terms of sensitivity. In Section 3, the design method is extended to PV controllers for position regulation. Lastly, Section 4 provides the conclusions from the study.

2. PI CONTROL FOR SPEED REGULATION

In this section, PI controllers are designed to regulate the shaft speeds according to a set of specifications. The design method is based on the Lambert W function presented in Yi et al. (2011). The controller is verified through simulations and experiments. For experiments, the setup in Fig. 1 of the DC motor control was used. It consists of the following parts: 1) a mechanical system composed of a shaft and a disc, 2) a feedback controller that is implemented on a PC using Matlab/Simulink, 3) sensors (a encoder and a tachometer) to obtain the states of the system, and 4) an actuator that consists of an DC servo-motor driven by a voltage amplifier. In Simulink, the time-delay can be readily introduced and removed to demonstrate and assess its effect on the control with different methods. Also, the results are compared with the Smith predictor in terms of sensitivity. Even if the Smith predictor is not an advanced control technique for time-delay systems, it is one of the most popular methods in literature along with Padé approximation as discussed in the Introduction. Comparison to the Padé approximation was provided in Yi et al. (2010a).

The voltage to speed transfer function of the DC motor system in Fig. 1 can be described using the first-order transfer function given by (Quanser (2010)):

$$P(s) = \frac{K}{\tau s + 1} = \frac{\Omega(s)}{V_m(s)} \tag{1}$$

When PI control is used, the control input is a function of the desired reference input and the measured speed of the shaft as

$$V_m(t) = k_P \left\{ b_{sp}\omega_d(t) - \omega(t) \right\} + k_I \int \left(\omega_d(t) - \omega(t) \right) dt \tag{2}$$

where k_P is the proportional control gain, k_I is the integral control gain, $\omega_d(t)$ is the setpoint or desired reference load angular speed, and $\omega(t)$ is the measured load shaft speed. The control input, $V_m(t)$, is the input voltage applied to the SRV 02 motor. In general, the set-point weight, b_{st}, is used for improved disturbance attenuation and/or transient responses. In this paper, the set-point weight is set to be zero to focus on stabilization via eigenvalue assignment. From Eqs. (1) and (2), the transfer function of the closed-system is given by

$$\frac{\Omega(s)}{\Omega_d(s)} = \frac{K\left(b_{sp}k_P s + k_I\right)}{\tau s^2 + (1 + Kk_P)\, s + Kk_I} \tag{3}$$

In reality, however, due to data analysis, state-observers, filters, teleoperation, data transmission, visual sensors, object recognitions, and tracking, and image processing, time-delays are introduced. Considering the time-delay, h, in the feedback loop, the input, $V_m(t)$, is the function of delayed output measurement, $\omega(t-h)$, instead of $\omega(t)$. Then, the closed-loop system in state-space form ignoring the reference input to focus on stability is given by

$$\dot{\mathbf{x}}(t) = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{1}{\tau} \end{bmatrix}}_{\equiv \mathbf{A}'} \mathbf{x}(t) + \underbrace{\begin{bmatrix} 0 & 0 \\ -\dfrac{Kk_I}{\tau} & -\dfrac{Kk_P}{\tau} \end{bmatrix}}_{\equiv \mathbf{A}'_d} \mathbf{x}(t-h) \tag{4}$$

by defining $x_1 \equiv \omega$ and $x_2 \equiv \dot{\omega}$. For DDEs as in Eq. (4), computing and controlling the entire infinite eigenspectrum is not as straightforward as for systems of ordinary differential equations (ODEs). Alternatively, it may be desired to locate the dominant eigenvalues, which are rightmost in the complex plane, and to assign them

10-th IFAC Workshop on Time Delay Systems
Boston, USA. June 22-24, 2012

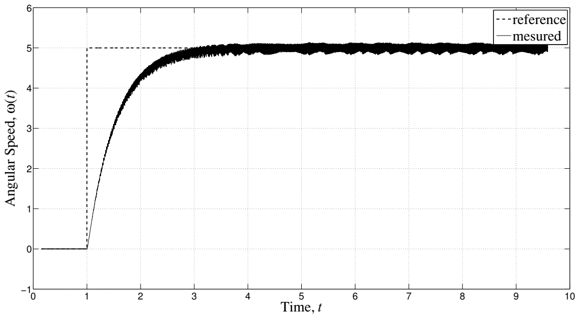

Fig. 2. Measured angular speed, ω, of the motor system in Fig. 1 with gains $\hat{k}_P = 2.75$ and $\hat{k}_I = 6.25$ for $h = 0$ (Case 2-1): the response is stable.

to desired positions. The Lambert W function approach is an efficient tool for conducting such a process. With the coefficient matrices, $\mathbf{A}'$ and $\mathbf{A}'_\mathbf{d}$, the solution matrix, $\mathbf{S}_0$ is computed. Then after setting an equation so that the desired positions for eigenvalues are equal to those of $\mathbf{S}_0$, the gains, k_P and k_I are obtained by solving the equation numerically. For a detailed explanation on the Lambert W function approach, including a method for eigenvalue assignment, refer to Yi et al. (2010a). Matlab-based software for use of the Lambert W function to calculate and assign the rightmost eigenvalues is available online (Duan (2010)).

Case 2-1: Assume that the desired time-constant of the closed loop is 0.4 (s). Then, the desired position for rightmost characteristic root is $\lambda_d = -2.5$. If there is no time-delay $(h = 0)$, the gains can be selected without difficulty as $\hat{k}_P = 2.75$ and $\hat{k}_I = 6.25$ for $\lambda = -2.5\ \&-5$ for the transfer function in Eq. (3) with the parameters given in Quanser (2010)

$$\tau = 0.0254 \quad \text{and} \quad K = 1.53 \tag{5}$$

In this paper, the hatted gain, $\hat{k}$, denotes one for non-delay systems or the Smith Predictor to differentiate from gains obtained by using the Lambert W function. Figure 2 shows the experimental response. The response is stable and non-oscillatory as desired and the designed controller successfully yields good performance. However, as delay increases, the response is different from the desired one. For example, for $h = 0.1$, the experimental response of the system is unstable as shown in Fig. 3. Introduction of the delay induces instability and the classical control method, which ignores delay, cannot be used to obtain the desired response. For safety, the maximum angular speed is saturated at ± 10 rad/s.

Using the Lambert W function explained above, the PI gains are obtained as $k_P = 0.4451$ and $k_I = 2.3046$ for $h = 0.1$ by assigning the rightmost eigenvalue to the desired position (-2.5). Then, the experimental result is shown in Fig. 4. The obtained gains successfully stabilized the plant even with the time-delay in the feedback loop (compare to the response in Fig. 3). As confirmed in Fig. 5, the rightmost eigenvalue is assigned exactly to -2.5 and others are placed behind it.

Case 2-2: Assume that $\omega_n = 2$ and $\zeta = 0.7$ are the desired natural frequency and damping ratio. Desired positions

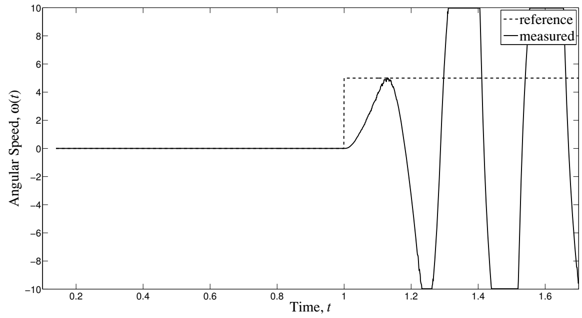

Fig. 3. Measured angular speed with gains $\hat{k}_P = 2.75$ and $\hat{k}_I = 6.25$ for $h = 0.1$ (Case 2-1): introduction of the delay induces instability.

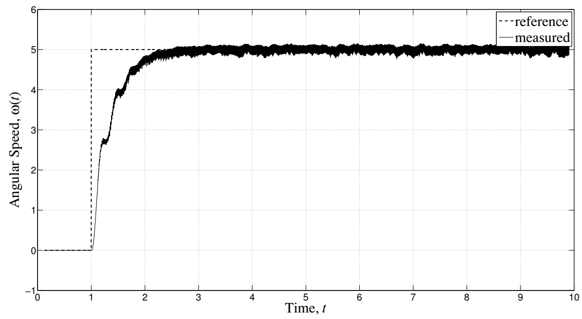

Fig. 4. Experiment result using the motor system in Fig. 1 with gains $k_P = 0.4451$ and $k_I = 2.3046$ for $h = 0.1s$ (Case 2-1).

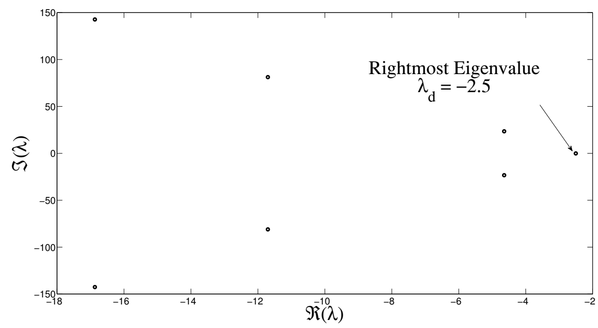

Fig. 5. Rightmost eigenvalue is assigned exactly to -2.5 and others are placed behind it with gains $k_P = 0.4451$ and $k_I = 2.3046$ for $h = 0.1s$ (Case 2-1).

for rightmost eigenvalues in assigning them can be chosen from the desired natural frequency and damping ratio using the relation in Eq. (6).

$$\lambda_d = -\omega_n\zeta \pm \omega_n\sqrt{1-\zeta^2}i = -\sigma \pm \omega_d i, \tag{6}$$

Then, the desired positions of the rightmost eigenvalues are $\lambda_d = -1.4 \pm 1.43i$. If time-delay does not exist (i.e., $h = 0$) then the PI gains are $\hat{k}_P = -0.6071$ and $\hat{k}_I = 0.0664$. However, as the delay increases, the rightmost roots move to the rights $(-0.2680 \pm 0.8754i$ when $h = 0.1)$. Because the roots are closer to the imaginary axis the transient response will have larger overshoot than expected due to a decreased damping ratio (see Fig. 6). To resolve this problem, using the Lambert W function, the gains,

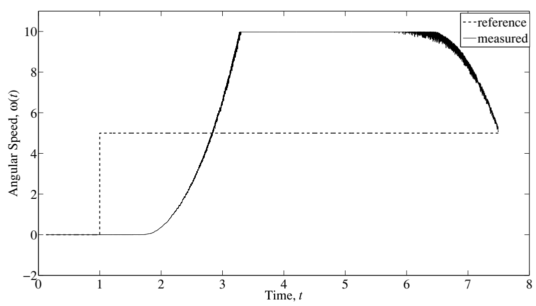

Fig. 6. Case 2-2: When time-delay is introduced, the transient response has a large maximum overshoots over the limit (10 rad/sec).

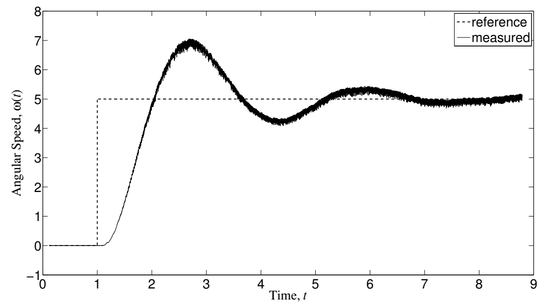

Fig. 7. Case 2-2: the gains ($k_P = -0.4430$ and $k_I = 0.2759$) obtained by considering the time-delay can improve the transient response (compared to Fig. 6).

$k_P = -0.4430$ and $k_I = 0.2759$, for $h = 0.1$ are obtained. The improved response is shown in Fig. 7.

The Smith predictor is also an effective tool for eigenvalue assignment and can be used for PI control of time-delay systems. With an additional block and feedback loop, it predicts the response of the plant one delay-time (h) ahead. Unlike the Lambert W function-based method, the Smith predictor, uses the same control gains ($\hat{k}_P = -0.6071$ and $\hat{k}_I = 0.0664$) as the non-delayed system. Thus, additional calculation of PI gains considering time-delays is not needed. The Smith predictor is based on pole-zero cancelation and, thus, cannot stabilize open-loop unstable plants (Furukawa et al. (1983), Guzzella et al. (1998)). Even for open-loop stable plants, stability of the closed-loop systems is vulnerable to uncertainty in system parameters (Michiels et al. (2003)).

In Yi et al. (2011), sensitivity analysis with respect to parameter uncertainty was conducted through eigenvalue sensitivity. Eigenvalue sensitivity can be derived by differentiating the characteristic equation as

$$\frac{\partial}{\partial h} Char.Eq.(\lambda) = 0 \Rightarrow \frac{\partial \lambda}{\partial h} = f(\lambda, \tau_M, K_M, \cdots) \quad (7)$$

Eigenvalue sensitivity represents an incremental change in the positions of the eigenvalues corresponding to an incremental change in the time-delay. Thus, Smaller sensitivity means more robustness against mismatch in parameters. For detailed derivation and discussion, refer to Yi et al. (2011). When sensitivity analysis of rightmost eigenvalues was applied to Case 2-2, the sensitivity of the rightmost eigenvalues is reduced ($S1 = 60.7225 + 6.2448i$ by the Smith predictor, $S2 = 12.1626 - 0.8678i$ by the Lambert W function approach). Thus, there is an 80% improvement $((\Re(S1) - \Re(S2))/\Re(S1) \times 100)$.

Figure 8 shows the simulated responses by using Smith predictor (dashed line) and the Lambert W function-based approach (straight line) when there is a delay mismatch. The real time-delay was 0.1 (s). When the time-delay increases to 0.131 (s) the Smith predictor fails to stabilize the system. On the other hand, the Lambert W function still stabilizes even with the 31% mismatch in h. Further discussion of sensitivity can be found in Yi et al. (2011).

3. PV CONTROL FOR POSITION REGULATION

Proportional-plus velocity feedback control is a simple controller common in angular position control of DC motors (Kelly et al. (2005)). In this section, PV controllers are designed using the Lambert W function. The open-loop voltage-to-load gear position transfer function is given by Quanser (2010)

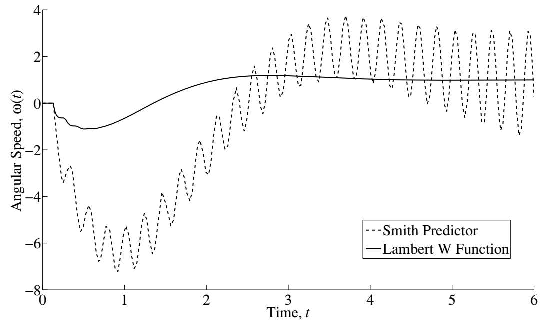

Fig. 8. Case 2-2: simulated responses by using Smith predictor (dashed line) and the Lambert W function-based approach (straight line) when there is a delay mismatch.

$$P(s) = \frac{\Theta(s)}{V_m(s)} = \frac{K}{(\tau s + 1)\, s} \quad (8)$$

The proportional-velocity (PV) controller used to regulate the position of the SRV 02 has the structure

$$V_m(s) = k_P(\Theta_d - \Theta) - k_V s\Theta = k_P\Theta_d - (k_P + k_V s)\,\Theta \quad (9)$$

where k_P is the proportional control gain, k_V is the velocity control gain. The angle Θ_d is the setpoint or desired reference load angle and Θ is the measured load shaft angle. Also, $V_m(t)$ is the input voltage. Thus, the transfer function of the closed-loop system is derived from Eqs. (8)-(9) as

$$\frac{\theta(s)}{\theta_d(s)} = \frac{Kk_P}{\tau s^2 + (1 + Kk_V)\, s + Kk_P} \quad (10)$$

If there is no time-delay in the system, the gains, k_P and k_V, can be obtained typically via coefficient comparison method using the denominator of the transfer function in Eq. (10). For example, for parameters given by the manufacturer

$$\tau = 0.0254 \;\; [s] \qquad k_P = 7.82 \;\; \left[\frac{V}{rad}\right]$$
$$K = 1.53 \;\; \left[\frac{rad}{V \times s}\right] \qquad k_V = -0.157 \;\; \left[\frac{V \times s}{rad}\right] \quad (11)$$

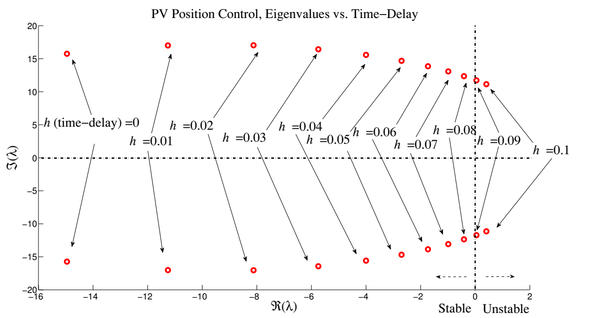

Fig. 9. Movement of Rightmost eigenvalues as time-delay, h, increases (Case 3-1).

Table 1. Gains, K_P and K_V, of PV controller: obtained by using the Lambert W function approach via rightmost eigenvalue assignment

Real parts of desired	Lambert W function	
rightmost eigenvalues, $\Re(\lambda_d)$	k_P	k_V
-5	5.9660	0.2989
-7	4.9215	0.2051
-9	4.3243	0.2064
-11	3.7911	0.1828

assuming the desired damping ratio is $\zeta = 0.690$ and the desired natural frequency is $\omega_n = 21.7[rad/s]$.

However, when the control input, $V_m(t)$, is a function of delayed state variables:

$$u(t) = V_m(t) = k_P\theta_d(t) - Kk_P\theta(t-h) - Kk_V\dot{\theta}(t-h) \quad (12)$$

Thus, the closed-loop system is given by ($\theta_d = 0$, for stability analysis)

$$\tau\ddot{\theta}(t) + \dot{\theta}(t) = u(t) = -Kk_P\theta(t-h) - Kk_V\dot{\theta}(t-h) \quad (13)$$

3.1 Using the Lambert W Function

The control gains, k_P and k_V, can be obtained using the Lambert W function approach (Yi et al. (2011)). By defining

$$\mathbf{x}(t) = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} \theta \\ \dot{\theta} \end{Bmatrix} \quad (14)$$

the system of DDEs representing the closed-loop system is given by

$$\dot{\mathbf{x}}(t) = \underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{1}{\tau} \end{bmatrix}}_{\mathbf{A}'} \mathbf{x}(t) + \underbrace{\begin{bmatrix} 0 & 0 \\ -\dfrac{Kk_P}{\tau} & -\dfrac{Kk_V}{\tau} \end{bmatrix}}_{\mathbf{A}'_\mathbf{d}} \mathbf{x}(t-h) \quad (15)$$

Case 3-1: When no delay, the system has stable eigenvalues ($\lambda = -14.9565 \pm 15.7274i$) with the parameters and gains in Eq. (11). However, as the delay increases, the rightmost eigenvalues move to other positions in the complex plane. Figure 9 shows the rightmost eigenvalues obtained by using the Lambert W function for various time-delays. Thus, with time-delays, the system response can be different from the desired response. For example, when $h = 0.1s$, the response is unstable as seen in Fig. 10.

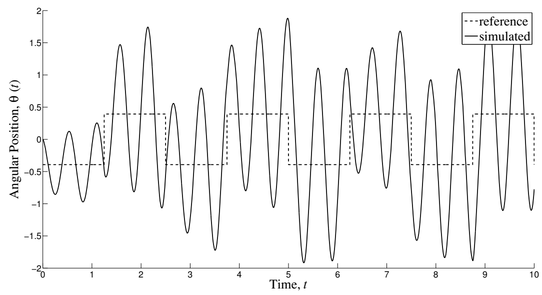

Fig. 10. Case 3-1: Simulation result with the gains provided in Eq. (11) (Quanser (2010)): Although the PV gains work to stabilize the non-delay system successfully, the system is destabilized when a time-delay, $h = 0.1s$, is introduced.

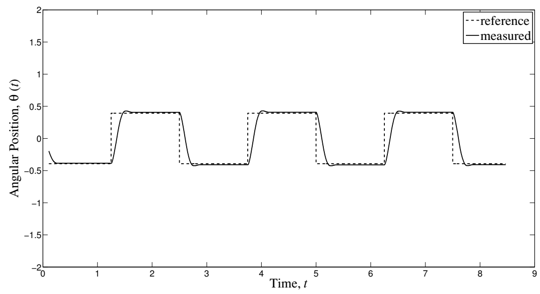

Fig. 11. Case 3-1: Measured angular position: the real part of the rightmost eigenvalues are assigned to -11 with $k_P = 3.7911$ and $k_V = 0.1828$

Table 2. Gains of the PV controller

Desired real & imag. parts	Damping	Lambert W function	
of rightmost eigenvalues, λ_d	ratio, ζ	k_P	k_V
$-11 \pm 14j$	0.6178	3.7928	0.1812
$-11 \pm 13j$	0.6459	3.7931	0.1712
$-11 \pm 12j$	0.6757	3.7575	0.1610
$-11 \pm 11j$	0.7071	3.6804	0.1504

The Lambert W function-based method can assign the rightmost eigenvalues of the delayed system to desired positions. The obtained gains through the process are in Table 1. Figure 11 shows the measured angular position, θ, with the gain in Table 1 for the real part (-11) of eigenvalues, respectively. As the rightmost eigenvalues move to the left in the complex plane, the settling time and overshoot decrease. Thus, one can adjust transient responses though eigenvalues assignment in a way similar to ODEs. The angular position was measured using an optical encoder, which measures the relative angle of the shaft at a resolution of 8192 counts per revolution (Quanser (2010)). As seen in Fig. 11, the experimental data has a very small non-zero steady-state error. This is due to un-modeled effects, such as the friction of the motor (Quanser (2010)). Using the Lambert W function can also assign the imaginary parts of the rightmost eigenvalues as well as the real parts. Table 2 shows the obtained gains for some desired eigenvalues. Thus, the damping ratio can be adjusted by using the proposed design method.

4. CONCLUSIONS AND FUTURE WORKS

A method for angular speed and position regulation using the Lambert W function has been developed and implemented for verification and comparison. The designed controllers successfully improve the performance of a DC motor system having time-delays. While the method requires use of numerical nonlinear solvers, it provides an effective and efficient approach in situations where effects of time-delays cannot be ignored. From a designer's point of view, because an additional loop for prediction is not needed, the designer can use the same structure as for control of non-delay systems. Thus, the Lambert W function is more convenient than prediction-based methods. A common control structure can be used whether there is a time-delay or not. The results presented here support the view that the Lambert W function-based approach has a promising future in the design and implementation of feedback control for time delay systems.

In the presented work, the authors assumed that the models of DC motor systems are deterministic. In real applications, more non-deterministic factors (e.g., modeling errors and disturbances) need be considered. An approach for robust stabilization against such uncertainties was developed using the Lambert W function method combined with 'stability radius' concept (Yi et al. (2010b)). Some simulation results were provided with numerical examples. As future work, the authors plan to apply the method presented in Yi et al. (2010b) for development of more realistic controllers for DC motor systems considering both uncertainty and time-delay.

Also, a sensitivity analysis shows that the Lambert W function-based method, compared with Smith Predictor, can reduce sensitivity with respect to uncertainty in the delay. The approach presented is being extended to PIV and PID control, and is now being implemented on mobile robots with vision sensors where effects of time-delays on system performance are significant.

REFERENCES

R. M. Corless, G. H. Gonnet, D. E. G. Hare, D. J. Jeffrey, and D. E. Knuth. On the Lambert W function. *Advances in Computational Mathematics*, 5:329-359, 1996.

S. Duan. Supplement for "Time-Delay Systems-Analysis and Control Using the Lambert W Function", by Sun Yi, P. W. Nelson, A. G. Ulsoy, World Scientific. http://www-personal.umich.edu/∼ulsoy/TDS_Supplement.htm.

T. Furukawa and E. Shimemura. Predictive control for systems with time-delay. *International Journal of Control*, 37:399-412, 1983.

L. Guzzella and C. H. Onder. *Introduction to Modeling and Control of Internal Combustion Engine Systems*. Springer, Berlin, 2004.

L. Guzzella and A. Amstutz. Control of diesel engines. *IEEE Control Systems Magazine*, 18:53-71, 1998.

K. Hong and K. Nam. A load torque compensation scheme under the speed measurement delay. *IEEE Transactions on Industrial Electronics*, 45:283-290, 1998.

R. Kelly, V. S. Davila, and A. Loria. Proportional Control plus Velocity Feedback and PD Control. *Control of Robot Manipulators in Joint Space*, pages 141–156 , 2005.

K. H. Kim and M. J. Youn. A simple and robust digital current control technique of a PM synchronous motor using time delay control approach. *IEEE Transactions on Power Electronics*, 16:72-82, 2001.

W. Krajewski, A. Lepschy, and U. Viaro. Designing PI controllers for robust stability and performance. *IEEE Transactions on Control Systems Technology*, 12:973-983, 2004.

G.M. Liaw and F.J. Lin. A robust speed controller for induction motor drives. *IEEE Transactions on Industrial Electronics*, 41:308-315, 1994.

M. E. Magana and F. Holzapfel. Fuzzy-logic control of an inverted pendulum with vision feedback. *IEEE Transactions on Education*, 41:165-170, 1998.

L. Mianzo, H. Peng, and I. Haskara. Transient air-fuel ratio H_∞ preview control of a drive-by-wire internal combustion engine. *Proc. 2001 American Control Conference*, 2867-2871, 2001.

W. Michiels and S. I. Niculescu. On the delay sensitivity of Smith Predictors. *International Journal of Systems Science*, 34:543-551, 2003.

W. Michiels, K. Engelborghs, P. Vansevenant, and D. Roose. Continuous pole placement for delay equations. *Automatica*, 38:747-761 , 2002.

S. Nakagawa, K. Katogi, and M. Oosuga. A new air-fuel ratio feed back control for ULEV/SULEV statndard. *SAE World Congress*, SAE Paper 2002-01-0194, 2002.

J. D. Powell, N. P. Fekete, and C. F. Chang. Observer-based air-fuel ratio control. *IEEE Control Systems Magazine*, 18:72-83, 1998.

Quanser. *SRV 02 User Manual, Quanser.* 2010.

J. P. Richard. Time-delay systems: an overview of some recent advances and open problems. *Automatica*, 39: 1667-1694, 2003.

G. J. Silva, A. Datta, and S. P. Bhattacharyya. *PID controllers for time-delay systems.* Birkhäuser, Boston, 2005.

G. J. Silva, A. Datta, and S. P. Bhattacharyya. PI stabilization of first-order systems with time delay. *Automatica*, 37:2025-2031, 2001.

J. T. Teeter, M. Y. Chow, and J. J. Brickley Jr. A novel fuzzy friction compensation approach to improve the performance of a DC motor control system. *IEEE Transactions on Industrial Electronics*, 43:113-120, 1996.

Y. Tipsuwan and M. Y. Chow. On the gain scheduling for networked PI controller over IP network. *IEEE-ASME Transactions on Mechatronics*, 9:491-498, 2004.

S. Yi, P. W. Nelson, and A. G. Ulsoy. PI Control of First Order Time-Delay Systems via Eigenvalue Assignment. *Proc. 2011 American Control Conference*, 4213-4218, 2011.

S. Yi, P. W. Nelson, and A. G. Ulsoy. *Time-Delay Systems: Analysis and Control Using the Lambert W Function.* World Scientific, 2010a.

S. Yi, P. W. Nelson, and A. G. Ulsoy. Robust Control and Time-Domain Specifications for Systems for Delay Differential Equations via Eigenvalue Assignment. *Journal of Dynamic Systems, Measurement and Control*, 132:7, 2010b.