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Bài 1: Tìm khai triển Taylor tại $x_0 = 2$ đến cấp 3 của hàm

$$f(x) = \frac{2x-1}{x-1} \quad \text{đKxđ} = x \neq 1$$

→ Áp dụng CT Taylor:

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + o[(x-x_0)^3]$$

. Xét: $f(x_0) = \frac{2x_0-1}{x_0-1} \Rightarrow f(2) = \frac{3}{1} = 3.$

$$f'(x_0) = \frac{-1}{(x_0-1)^2} \Rightarrow f'(2) = \frac{-1}{1} = -1$$

$$f''(x_0) = \frac{2}{(x-1)^3} \Rightarrow f''(2) = 2$$

$$f'''(x_0) = \frac{-6}{(x-1)^4} \Rightarrow f'''(2) = -6.$$

Vậy $f(x) = 3 - (x-2) + (x-2)^2 - (x-2)^3 + o[(x-2)^3]$

Câu 2: Tìm khai triển Taylor tại $x_0 = 2$ đến cấp 3 của hàm số

$$f(x) = \frac{x-1}{x^2-5x+6} = \frac{x-1}{(x-2)(x-3)} \quad (\text{đKxđ}: x \neq 2)$$

→ Áp dụng CT Taylor:

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + o[(x-x_0)^3]$$

$x_0 = 2 \Rightarrow$ Không tìm được

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Câu 3. Tìm khai triển Taylor tại $x_0 = 1$ đến cấp 3 của hàm số

$f(x) = \ln(2+3x)$ ĐKXĐ: $(2+3x) > 0 \Leftrightarrow x > \frac{-2}{3}$

$\Rightarrow f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + o[(x-x_0)^3]$

Xét: $f(1) = \ln 5$.

$f'(x_0) = \frac{3}{2+3x_0} \Rightarrow f'(1) = \frac{3}{5}$; $f''(1) = \frac{-9}{25}$; $f'''(1) = \frac{54}{125}$

$\Rightarrow f(x) = \ln 5 + \frac{3}{5}(x - \cancel{x_0}1) - \frac{9}{50}(x-1)^2 + \frac{6}{125}(x-1)^3 + o[(x-1)^3]$

Câu 5. Tìm khai triển Maclaurint đến cấp n.

a) $f(x) = y = \frac{x^2 + 3e^x}{e^{2x}}$; $n = 3$.

Ta có:

$y = f(x_0) + \frac{f'(x_0)}{1}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 + \frac{f'''(x_0)}{3!}(x-x_0)^3 + o[(x-x_0)^3]$

• Xét: $f(0) = 3$.

$f(x) = \left(\frac{x}{e^x}\right)^2 + \frac{3}{e^x} \Rightarrow f(0) = 3$; $f'(0) = -3$;

$f''(x) = \frac{4e^{6x}x^2 - 8e^{6x}x + 3e^{7x} + 2e^{6x}}{e^{8x}} \Rightarrow f''(0) = 5$

$f'''(x) = \frac{24e^{14x}x - 8e^{14x}x^2 - 3e^{15x} - 12e^{14x}}{e^{16x}} \Rightarrow f'''(0) = -15$

$\Rightarrow y = 3 - 3x + \frac{5}{2}x^2 - \frac{15}{6}x^3 + o x^3$

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b) $f(x) = \ln\left(\dfrac{2-3x}{3+2x}\right)$; $n = 3$. đk: $-\dfrac{3}{2} < x < \dfrac{2}{3}$

→ Áp dụng CT Maclaurint:

$$f(x) = f(0) + f'(0)x + \frac{f''(0).x^2}{2!} + \frac{f'''(0).x^3}{3!} + o[(x)^3]$$

. Xđ: $f'(x) = -\dfrac{13}{(2-3x)(3+2x)}$; $f''(x) = (-12x-5).\left(\dfrac{13}{(2-3x)^2(3+2x)^2}\right)$

$$f'''(x) = (156x+65)\left[4(2-3x)^2.(3+2x) - 6.(3+2x)^2(2-3x)\right]$$

$$\times\left[\frac{1}{(2-3x)^4(3+2x)^4}\right] - \frac{156}{(2-3x)^2(3+2x)^2}.$$

$$\Rightarrow f(x) = \ln\left(\frac{2}{3}\right) - \frac{13x}{6} - \frac{65}{72}x^2 - \frac{793}{648}x^3 + o(x^3)$$

c) $f(x) = \ln(x^2+3x+2)$; $n = 4$

→ Áp dụng CT Maclaurint:

$$f(x) = f(0) + f'(0)x + \frac{f''(0).x^2}{2!} + \frac{f'''(0).x^3}{3!} + \frac{f^{(4)}(0).x^4}{4!} + o(x^4)$$

Ta có: $f'(x) = \dfrac{2x+3}{x^2+3x+2.}$; $f''(x) = \dfrac{-(2x^2+6x+5)}{(x^2+3x+2)^2}$

$f'''(x) = \dfrac{4x^3+18x^2+30x+18}{(x^2+3x+2)^3.}$; ~~$f^{(4)}(x) = -12x^4+72x^3+180x^2$~~

$$f^{(4)}(x) = -\frac{12x^4+72x^3+180x^2+216x+102.}{(x^2+3x+2)^4}$$

$$\Rightarrow f(x) = \ln 2 + \frac{3x}{2} - \frac{5x^2}{8} + \frac{3x^3}{8} - \frac{17}{64}x^4 + o(x^4).$$

Câu 3: (1,5 điểm) Khai triển Maclaurint tới số hạng chứa x^2 của hàm số $f(x) = \ln(1 + 2x + 3x^2)$.

→ Đặt $u = 2x + 3x^2$

→ $x = 0$ thì $u = 0$

→ Áp dụng công thức khai triển Maclaurint:

$$\ln(1+u) = u - \frac{u^2}{2} + \frac{u^3}{3} + o(x^2)$$

$$= (2x + 3x^2) - \frac{(2x + 3x^2)^2}{2} + \frac{(2x + 3x^2)^3}{3} + o(x^2)$$

$$= 2x + 3x^2 - \frac{4x^2 + 12x^3 + 9x^4}{2} + o(x^2)$$

$$= 2x + x^2 + o(x^2)$$

Câu 3. Hãy sử dụng khai triển Maclaurint để tìm $g^{(2016)}(0)$ của hàm số sau: $g(x) = (1 + x^3)e^{x^3}$.

→ Áp dụng CT Leibnitz:

$$g^{(2016)}(x) = \sum_{k=0}^{2016} \left[C_{2016}^{k} \cdot (1+x^3)^{(k)} \cdot (e^{x^3})^{(2016-k)} \right]$$

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$$\Rightarrow g^{(2016)}(x) = C_{2016}^{0}(1+x^3)\left(e^{x^3}\right)^{(2016)} + C_{2016}^{1}(1+x^3)^{(1)}.\left(e^{x^3}\right)^{(2015)} + \dots + C_{2016}^{2016}(1+x^3)^{(2016)}.e^{x^3}.$$

Mà: $(1+x^3)^{(1)} = 2x^3$; $(1+x^3)^{(2)} = 6x$; $(1+x^3)^{(3)} = 6$.

$(1+x^3)^{(k)} = 0 \ \forall k \geq 4$

$$\Rightarrow g^{(2016)}(x) = C_{2016}^{0}.(1+x^3)\left(e^{x^3}\right)^{(2016)} + 3.C_{2016}^{1}.x^2.\left(e^{x^3}\right)^{(2015)} + 6.C_{2016}^{2}.x\left(e^{x^3}\right)^{(2014)} + 6C_{2016}^{3}.\left(e^{x^3}\right)^{(2013)}$$

Sdụng khai triển Maclaurint, ta có:

$$e^{x^3} = 1 + \frac{x^3}{2!} + \frac{x^6}{3!} + \frac{x^9}{4!} + \dots + \frac{x^{3n}}{(n+1)!} + o(x^{3n})$$

$$\Rightarrow \left(e^{x^3}\right)^{(n)}(0) = \begin{cases} \dfrac{n!}{\left(\frac{n}{3}+1\right)!} & \text{với } n = 3k \in \mathbb{N}^* \\ 0 & \text{với } n \neq 3k \in \mathbb{N}^* \end{cases}$$

$$\Rightarrow g^{(2016)}(0) = C_{2016}^{0}.(1+0^3).\frac{2016!}{673!} + 3C_{2016}^{1}.0^2.0 + 6.C_{2016}^{2}.0.0 + 6.C_{2016}^{3}.\frac{2013!}{672!}$$

$$\Rightarrow \cancel{A} \ g^{(2016)}(0) = A_{2016}^{1343} + 6C_{2016}^{3}.A_{2013}^{1341}.$$

Bài 7. Tìm các giới hạn sau bằng khai triển Maclaurint nếu có thể

a) $\lim\limits_{x \to 0} \dfrac{\cos x - 1 + \frac{x^2}{2}}{x^4}$

Ta có: $\cos x = 1 - \dfrac{x^2}{2} + \dfrac{x^4}{4!} + o(x^4)$

$\Leftrightarrow \cos x - 1 + \dfrac{x^2}{2} = \dfrac{x^4}{4!} + o(x^4)$

$$\Rightarrow \lim_{x \to 0} \frac{\frac{x^4}{4!} + o(x^4)}{x^4} = \frac{1}{24} + 0 = \frac{1}{24}.$$

b) $\lim\limits_{x\to 0} \dfrac{\arctan x - \arcsin x}{\tan x - \sin x} = \lim\limits_{x\to 0} \dfrac{\arctan x - \arcsin x}{\sin(1-\cos x)} \cdot \cos x$

$= \lim\limits_{x\to 0} \dfrac{\arctan x - \arcsin x}{\sin x(1-\cos x)}$

Nhân $\sin^2 x$

$= \lim\limits_{x\to 0} \dfrac{\arctan x - \arcsin x\,(1+\cos x)}{\sin^3 x}$

$[\sin^2 x = 1-\cos^2 x = (1+\cos x)(1-\cos x)]$

$= 2\lim\limits_{x\to 0} \dfrac{\arctan x - \arcsin x}{x^3} \cdot \dfrac{x^3}{\sin x}$

$= 2\lim\limits_{x\to 0} \dfrac{\arctan x - \arctan\frac{x}{\sqrt{1-x^2}}}{x^3}$

$= 2\lim\limits_{x\to 0} \dfrac{\arctan\left(\dfrac{x - \frac{x}{\sqrt{1-x^2}}}{1+\frac{x^2}{\sqrt{1-x^2}}}\right)}{x^3}$

$= 2\lim\limits_{x\to 0} \dfrac{\arctan\left[\dfrac{x(\sqrt{1-x^2}-1)}{\sqrt{1-x^2}+x^2}\right]}{x(\sqrt{1-x^2}-1)} \cdot \dfrac{\frac{x(\sqrt{1-x^2}-1)}{\sqrt{1-x^2}+x^2}}{x^3}$

$= 2\lim\limits_{x\to 0} \dfrac{\sqrt{1-x^2}-1}{x^2(\sqrt{1-x^2}+x^2)} = 2\lim\limits_{x\to 0} \dfrac{\sqrt{1-x^2}-1}{x^2}$

$= 2\lim\limits_{x\to 0} \dfrac{-1}{\sqrt{1-x^2}+1} = -2\cdot\dfrac{1}{2}$

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c) $\lim\limits_{x \to 0} \dfrac{1 + x\cos x - \sqrt{1+2x}}{\ln(1+x) - x}$

$\left\{ \dfrac{1 + x.\cos x - \sqrt{1+2x}}{\ln(1+x) - x} \overset{(L)}{=} \dfrac{\cos x - x.\sin x - \dfrac{1}{\sqrt{1+2x}}}{\dfrac{1}{1+x} - 1} \right.$

$\overset{(L)}{=} \dfrac{-\sin x - (\sin x + x.\cos x) + \dfrac{\frac{1}{\sqrt{1+2x}}}{1+2x}}{\dfrac{-1}{(1+x)^2}}$

$= \dfrac{-2\sin x - x.\cos x + \sqrt{1+2x}}{\dfrac{-1}{(1+x)^2}}$

$\Rightarrow \lim\limits_{x \to 0} \dfrac{1 + x\cos x - \sqrt{1+2x}}{\ln(1+x) - x} = \lim\limits_{x \to 0} \dfrac{-2\sin x - x.\cos x + \sqrt{1+2x}}{\dfrac{-1}{(1+x)^2}}$

$= \dfrac{1}{-1} = -1.$