Câu 1:

$f(x) = \frac{2x-1}{x-1} = 2 + \frac{1}{x-1} \Rightarrow f(2) = 3.$

$f'(x) = \frac{-1}{(x-1)^2} \Rightarrow f'(2) = -1.$

$f''(x) = \frac{2}{(x-1)^3} \Rightarrow f''(2) = 2.$

$f'''(x) = \frac{-6}{(x-1)^4} \Rightarrow f'''(2) = -6.$

Khai triển Taylor tại $x_0 = 2$ đến cấp 3 của hàm $f(x) = \frac{2x-1}{x-1}$ là:

$f(x) = 3 + (-1)(x-2) + \frac{2(x-2)^2}{2!} + \frac{(-6)(x-2)^3}{3!} + o((x-2)^3)$

$= 3 - (x-2) + (x-2)^2 - (x-2)^3 + o((x-2)^3).$

Câu 2:

$f(x) = \frac{x-1}{x^2-5x+6} = \frac{x-1}{(x-3)(x-2)} = \frac{2}{x-3} - \frac{1}{x-2}$

$= \frac{-2}{3-x} + \frac{1}{2-x} = \frac{-2}{3} \cdot \frac{1}{1-\frac{x}{3}} + \frac{1}{2} \cdot \frac{1}{1-\frac{x}{2}}.$

$= \frac{-2}{3}\left(1 + \frac{x}{3} + \frac{x^2}{9} + \frac{x^3}{27} + o(x^3)\right) + \frac{1}{2}\left(1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8} + o(x^3)\right).$

$= \frac{-1}{6} + \frac{1}{36}x + \frac{11}{216}x^2 + \frac{49}{1296}x^3 + o(x^3).$

Câu 3:

$f(x) = \ln(2+3x) \Rightarrow f(1) = \ln 5.$

$f'(x) = \frac{3}{x+2} \Rightarrow f'(1) = \frac{3}{5}.$

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$f''(x) = \frac{-9}{(3x+2)^2} \Rightarrow f''(1) = \frac{-9}{25}$

$f'''(x) = \frac{54}{(3x+2)^3} \Rightarrow f'''(1) = \frac{54}{125}$

Khai triển Taylor tới $x = 1$ đến cấp 3 của hàm $f(x) = \ln(2+3x)$.

$f(x) = \ln 5 + \frac{3}{5}(x-1) - \frac{9}{50}(x-1)^2 + \frac{9}{125}(x-1)^3 + o((x-1)^3)$

Câu 5:

1) $f(x) = \frac{x^2 + 3e^x}{e^{2x}} \Rightarrow f(0) = 3.$

$f'(x) = \frac{2e^{2x}.x - 2x^2.e^{2x} - 3.e^{3x}}{e^{4x}} \Rightarrow f'(0) = -3.$

$f''(x) = \frac{4x^2e^{6x} - 8x.e^{6x} + 3e^{7x} + 2e^{6x}}{e^{8x}} \Rightarrow f''(0) = 5.$

$f'''(x) = \frac{24e^{14x}.x - 8x^2e^{14x} - 3e^{15x} - 12e^{14x}}{e^{16x}} \Rightarrow f'''(0) = -15.$

Khai triển Maclaurin của hàm $y = \frac{x^2 + 3e^x}{e^{2x}}$ đến cấp 3 là:

$f(x) = 3 - 3x + \frac{5}{2}x^2 - \frac{5}{2}x^3 + o(x^3).$

2) $f(x) = \ln\frac{2-3x}{3+2x} \Rightarrow f(0) = \ln\frac{2}{3}.$

$f'(x) = \frac{-13}{(2-3x)(3+2x)} \Rightarrow f'(0) = \frac{-13}{6}.$

$f''(x) = \frac{-13(12x+5)}{(2-3x)^2(3+2x)^2} \Rightarrow f''(0) = \frac{-65}{36}$

$f'''(x) = \frac{-54}{(2-3x)^3} + \frac{-16}{(3+2x)^3} \Rightarrow f'''(0) = \frac{-793}{108}$

Khai triển Maclaurin của hàm $f(x) = \ln\frac{2-3x}{3+2x}$ đến cấp 3 là:

$$f(x) = \ln\frac{2}{3} - \frac{13x}{6} - \frac{65x^2}{36\times 2} - \frac{793}{108}\frac{x^3}{3!} + o(x^3)$$

$$= \ln\frac{2}{3} - \frac{13x}{6} - \frac{65x^2}{72} - \frac{793}{324}x^3 + o(x^3)$$

3). $f(x) = \ln(x^2+3x+2) = \ln(x+1) + \ln(x+2) \Rightarrow f(0) = \ln 2$.

$$f'(x) = \frac{2x+3}{x^2+3x+2} \Rightarrow f'(0) = \frac{3}{2}.$$

$$f''(x) = \frac{-1}{(x+1)^2} + \frac{-1}{(x+2)^2} \Rightarrow f''(0) = \frac{-5}{4}.$$

$$f'''(x) = \frac{2}{(x+1)^3} + \frac{2}{(x+2)^3} \Rightarrow f'''(0) = \frac{9}{4}.$$

$$f^{(4)}(x) = \frac{-6}{(x+1)^4} + \frac{-6}{(x+2)^4} \Rightarrow f^{(4)}(0) = \frac{-51}{8}.$$

Khai triển Maclaurin của hàm $f(x) = \ln(x^2+3x+2)$ đến cấp 4 là:

$$f(x) = \ln 2 + \frac{3}{2}x + \frac{-5}{4}\frac{x^2}{2} + \frac{9}{4}\cdot\frac{x^3}{3!} + \frac{-51}{8}\cdot\frac{x^4}{4!} + o(x^4)$$

$$= \ln 2 + \frac{3}{2}x - \frac{5}{8}x^2 + \frac{3}{8}x^3 - \frac{17}{64}x^4 + o(x^4)$$

Câu 7:

1) $\lim\limits_{x\to 0}\frac{\cos x - 1 + \frac{x^2}{2}}{x^4} \overset{L'H}{=} \lim\limits_{x\to 0}\frac{-\sin x + x}{4x^3}$

$\overset{L'H}{=} \lim\limits_{x\to 0}\frac{-\cos x + 1}{12x^2} \overset{L'H}{=} \lim\limits_{x\to 0}\frac{\sin x}{24x}$.

$\overset{L'H}{=} \lim\limits_{x\to 0}\frac{\cos x}{24} = \frac{1}{24}$.

2) $\lim\limits_{x\to 0} \dfrac{\arctan x - \arcsin x}{\tan x - \sin x}$.

Theo khai triển Maclaurin ta có:

$\arctan x = x - \dfrac{x^3}{3} + o(x^3)$

$\arcsin x = x + \dfrac{x^3}{6} + o(x^3)$

$\Rightarrow \arctan x - \arcsin x = \dfrac{-x^3}{2} + o(x^3)$

$\tan x = x + \dfrac{x^3}{3} + o(x^3)$

$\sin x = x - \dfrac{x^3}{6} + o(x^3)$

$\Rightarrow \tan x - \sin x = \dfrac{x^3}{2} + o(x^3)$

$\Rightarrow \lim\limits_{x\to 0} \dfrac{\arctan x - \arcsin x}{\tan x - \sin x} = \dfrac{\frac{-x^3}{2} + o(x^3)}{\frac{x^3}{2} + o(x^3)} = -1$.

3) $\lim\limits_{x\to 0} \dfrac{1 + x\cos x - \sqrt{1+2x}}{\ln(1+x) - x}$.

$\overset{L'H}{=} \lim\limits_{x\to 0} \dfrac{\cos x - x\sin x + \frac{-1}{\sqrt{1+2x}}}{\frac{1}{1+x} - 1}$.

$\overset{L'H}{=} \lim\limits_{x\to 0} \dfrac{-\sin x - \sin x - x\cos x + \frac{1}{\sqrt{(1+2x)^3}}}{\frac{-1}{(1+x)^2}}$

$= -1$.

ẢI TIẾN

Một số bài tập trong đề thi các năm.

Câu 3:

$f(x) = \ln(1+2x+3x^2) \Rightarrow f(0) = 0.$

$f'(x) = \dfrac{2+6x}{1+2x+3x^2} \Rightarrow f'(0) = 2.$

$f''(x) = \dfrac{-12x-18x^2+2}{(1+2x+3x^2)} \Rightarrow f''(0) = 2.$

Khai triển Maclaurin tới số hạng chứa x^2 của hàm $f(x) = \ln(1+2x+3x^2)$

$f(x) = 0 + 2x + \dfrac{2x^2}{2} = 2x + x^2.$

Câu 4:

$g(x) = (1+x^3).e^{x^3} \Rightarrow g(0) = 1.$

$g'(x) = 6e^{x^3}.x^2 + 3e^{x^3}.x^5 \Rightarrow g'(0) = 0.$

$g''(x) = 12e^{x^3}.x + 33x^4.e^{x^3} + 9e^{x^3}.x^7 \Rightarrow g''(0) = 0$

$g'''(x) = 168e^{x^3}.x^3 + 162x^6.e^{x^3} + 27e^{x^3}.x^9 + 12e^{x^3}$

$\Rightarrow g'''(0) = 12.$

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Khai triển Maclaurin của $g(x) = (1+x^3)e^{x^3}$ tới cấp $n = 2016$ là:

$g(x) = 1 + 0 \times x + 0 \times x^2 + \dfrac{0}{3!}x^3 + \ldots + \dfrac{g^{(2016)}(0)}{2016!}.x^{2016}$

$+ 0(x^{2016}).$

$\Rightarrow g(0) = 0.$