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# Input-to-state stability for discrete-time non-linear switched singular systems

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**Abstract:** This study investigates the input-to-state stability (ISS) for a class of discrete-time non-linear switched singular systems. Two novel ISS criteria are proposed based on multiple Lyapunov functions method. In one case that all subsystems are input-to-state stable (ISS) but the system stability may be destroyed by the switching, the average dwell time switching algorithm is applied to guarantee the ISS of the whole system. In other case that all subsystems can be unstable, the state-dependent switching rule is designed to stabilise the system and ensure the ISS of the switched singular system. Finally, simulation examples are provided to illustrate the feasibility of the obtained results.

## 1 Introduction

Switched systems are a class of important hybrid systems, which consist of several subsystems and a switching rule orchestrating the switch among them. In the past decades, switched systems have received great attention due to their applications in many areas, such as power systems [1], economic systems [2], and flight control systems [3]. For this reason, many works focused on such systems have emerged [4–10]. Among many famous methods to cope with the stability of switched systems, the average dwell time (ADT) method is widely applied. In [11], the ADT of switching signal is given to guarantee the finite-time stability/boundedness of switched linear systems. In [12], the stability analysis and robust control for a special class of switched systems characterised by non-linear functions based on ADT are presented. Besides the time-dependent switching, state-dependent switching method is also considered as a useful tool to deal with the stability problem, which does not require the switching instants to be known in advance. In [13], the stabilisation problem of non-linear systems with distributed delays under state-dependent switching control and impulsive control is investigated. A mixed state-dependent and time-driven switching law is designed in [14] to solve the composite anti-disturbance control problem of switched systems.

On another research front, singular systems, which are also referred to as descriptor systems, generalised systems, have drawn a great amount of interests because it can better describe the behaviours of many practical systems than state-space systems. So far, such systems are widely used in mechanical systems, constraint robots and so on due to their great advantages. In recent years, the switching problem of singular systems is studied and many excellent results [15–20] are obtained. The solution theory and Lyapunov functions for general switched non-linear singular systems are addressed in [15]. By describing the two stages of state jumps at switching instants, the authors in [16] give some stability conditions on switched linear singular systems. Such a problem is considered in [17] for switched singular systems with impulsive effects. As to discrete-time switched linear singular systems, the dwell-time-based stabilisation is studied in [18] for the case of all unstable subsystems, and the robust sliding mode control is investigated in [19] with time-varying delays and external disturbances taking into account. Considering the time-varying state delays under asynchronous switching, the state feedback control problem is solved in [20].

In reality, the external disturbance inputs are inevitable, so input-to-state stability (ISS) plays an important role in the system analysis, which aims to investigate how the disturbance affect the system stability. ISS indicates that the system responses remain bounded when the disturbance is bounded, and tend to the origin with disturbance decays to zero. The research on ISS of systems has become a hot topic in control areas due to its practical significance. In [21], the ISS for non-linear impulsive systems with external input affecting both the continuous dynamics and the discrete dynamics is concerned. The ISS for a class of switched non-linear systems with time delays under asynchronous switching is studied in [22]. The ISS of switched systems is dealt with in [23] by designing a generalised switching signal that allows the number of switches to grow faster than that with ADT. In [24], some ISS criteria are proposed based on ADT approach and an iterative algorithm for discrete-time switched singular systems. However, to the best of our knowledge, the ISS of such a system with sector non-linearities and state-dependent switching laws has not been addressed yet.

Motivated by the above discussion, in this paper, we investigate the ISS problem for discrete-time non-linear switched singular systems with sector non-linear functions (less conservative than the Lipschitz ones). When all subsystems are stable, but the switching may deteriorate the system stability, the time intervals between two successive switching instants are required to be confined by an lower bound. The time-dependent switching rule is designed to guarantee the ISS of the system. When all subsystems are unstable, the state-dependent switching rule is designed to ensure the ISS. It is worth mentioning that the state-space division is not based on the full space. This treatment is efficient to avoid the description of the state jumps at switching instants, which is part of the main contributions of our results. As a potential application, we employ a DC motor controlling inverted pendulum systems to validate the proposed results.

The rest of the paper is organised as follows. Some definitions/assumptions and the problem description are formulated in Section 2. Two results based on ADT switching and state-dependent switching accordingly are presented in Section 3. Simulation examples are given in Section 4 to illustrate the effectiveness of the obtained results. At last, we conclude the paper in Section 5.

*Notation*: For a real-symmetric matrix $M$, $M > 0$ and $M < 0$ denote $M$ positive definite and negative definite, respectively, $M^{\mathrm{T}}$ denotes the transpose of $M$. $R$ is the set of all real numbers, $R^+$ is the subset of non-negative elements of $R$ and $R^n$ is the $n$-dimensional real Euclidean space. $Z^+$ denotes the set of all non-negative integers. $\| \cdot \|$ stands for the Euclidean norm for a given matrix or vector. The symbol $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of matrix $P$, respectively.

diag$\{\cdot\}$ indicates a block-diagonal matrix. In addition, $*$ in the matrix denotes the term which is induced by symmetry. Recall that a function $\alpha: R^+ \to R^+$ is said to be of class $\mathscr{K}$ if it is continuous, strictly increasing and satisfies $\alpha(0) = 0$, and we write it as $\alpha \in \mathscr{K}$. A function $\alpha$ is of class $\mathscr{K}_\infty$ if $\alpha \in \mathscr{K}$ and also satisfies $\alpha(t) \to \infty$ as $t \to \infty$, and we write it as $\alpha \in \mathscr{K}_\infty$. A function $\beta: R^+ \times R^+ \to R^+$ is said to be of class $\mathscr{KL}$ if $\beta(s,t)$ is of class $\mathscr{K}$ on the first argument $s$ and is decreasing on the second argument $t$, called $\beta \in \mathscr{KL}$.

## 2 Preliminaries and problem formulation

In this paper, we consider the following discrete-time non-linear switched singular system:

$$Ex(k+1) = A_{\sigma(k)}x(k) + D_{\sigma(k)}f(x(k)) + H_{\sigma(k)}\omega(k) \tag{1}$$

where $x(k) \in R^n$ is the state vector, the non-linear vector function is denoted as $f(x(k)) = [f_1^{\mathrm{T}}(x_1(k)), f_2^{\mathrm{T}}(x_2(k)), \ldots, f_n^{\mathrm{T}}(x_n(k))]^{\mathrm{T}}$, and $f_l(x_l(k))$ denotes the $l$th component of $f(x(k))$. $\omega(k)$ denotes the external input which cannot decay to zero and satisfies $\| \omega(k) \| \le \bar{\omega}, k = 0, 1, 2, \ldots$, where $\bar{\omega} > 0$ is a known upper bound. $E$ is a constant matrix with $\mathrm{rank}(E) = r (r < n)$. $\sigma: [0, \infty) \to J = \{1, 2, \ldots, m\}$ denotes the switching signal, which is a right continuous function. Denote $S = \{k^1, k^2, \ldots, k^n, \ldots\}$ as the switching sequence. Obviously, $S$ is the subsequence of the discrete time sequence $\{k\}$. When $k \in [k^n, k^{n+1})$, $\sigma(k) = i \in J$ implies the $i$th subsystem is activated. $A_i, D_i, H_i$ are constant matrices with appropriate dimensions. For simplicity, we use $(E, A_i)$ to denote the $i$th subsystem.

In this paper, the ISS problem of non-linear switched singular system (1) is considered. Before giving the main results, we review and introduce some definitions and assumptions.

*Definition 1 [25]:* For any $k^v \ge k^s \ge 0$, and a switching signal $\sigma(k)$, let $N_\sigma(k^v, k^s)$ denote the switching numbers of $\sigma(k)$ on interval $[k^s, k^v)$. we say that $\sigma(k)$ has an ADT $\tau_a$ if there exist two positive constants $\tau_a$ and $N_0$, such that

$$N_\sigma(k^v, k^s) \le N_0 + \frac{k^v - k^s}{\tau_a} \tag{2}$$

where $N_0$ is the chattering bound.

*Definition 2 [26]:* For each $i \in J$, the singular system $(E, A_i)$ is said to be

(1) regular, if det $(zE - A_i) \not\equiv 0$;
(2) impulse-free, if deg( det $(zE - A_i)) = \mathrm{rank}(E)$.

*Definition 3 [24]:* System (1) is said to be ISS under a certain signal $\sigma(k)$, if there exist functions $\beta \in \mathscr{KL}$ and $\gamma \in \mathscr{K}$, such that for each initial state $x_0$ and input disturbance $\omega(k)$, the solution of (1) satisfies

$$\| x(k) \| \le \beta(\| Ex_0 \|, k) + \gamma \sup (\| \omega(s) \|), \quad \forall s \in [k_0, k]$$

for each $k \in Z^+$.

*Assumption 1:* For $\forall x_l \in R, a_l, b_l \in R, a_l < b_l, l = 1, \ldots, n$, the non-linear function $f_l(x_l)$ satisfies

$$(f_l(x_l) - a_l x_l)(f_l(x_l) - b_l x_l) \le 0, \quad f_l(0) = 0. \tag{3}$$

Note that the non-linear functions are assumed to belong to sector sets with arbitrary slopes for the sector boundaries. Define $\eta^{\mathrm{T}}(k) = [x(k)\ f(x(k))]$. From Assumption 1, the following inequality:

$$\eta^{\mathrm{T}}(k)\begin{bmatrix} F_i D_a D_b & -\frac{1}{2}F_i(D_a + D_b) \\ * & F_i \end{bmatrix}\eta(k) \le 0, \quad i \in J \tag{4}$$

holds, where $F_i > 0$, $D_a = \mathrm{diag}(a_l)$, $D_b = \mathrm{diag}(b_l)$.

*Assumption 2:* For each $i \in J$, the singular system $(E, A_i)$ is regular and impulsive-free.

Since $\mathrm{rank}(E) = r\,(r < n)$, there exist non-singular matrices $M$ and $N$, such that $(E, A_i)$ takes the following dynamics decomposition form [27]:

$$MEN = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \quad MA_iN = \begin{bmatrix} A_{1i} & A_{2i} \\ A_{3i} & A_{4i} \end{bmatrix}. \tag{5}$$

Furthermore, $A_{4i}$ is non-singular under Assumption 2. Based on (5), system (1) can be decomposed into a slow subsystem and a fast subsystem as follows:

$$\begin{cases} \bar{x}_1(k+1) = A_{1i}\bar{x}_1(k) + A_{2i}\bar{x}_2(k) + D_{1i}f(x(k)) + H_{1i}\omega(k) \\ 0 = A_{3i}\bar{x}_1(k) + A_{4i}\bar{x}_2(k) + H_{2i}\omega(k) \end{cases} \tag{6}$$

where

$$\bar{x}(k) = \begin{bmatrix} \bar{x}_1(k) \\ \bar{x}_2(k) \end{bmatrix} = N^{-1}x(k), \quad MD_i = \begin{bmatrix} D_{1i} \\ 0 \end{bmatrix}, \quad MH_i = \begin{bmatrix} H_{1i} \\ H_{2i} \end{bmatrix}.$$

Rewrite system (6) as

$$\begin{cases} \bar{x}_1(k+1) = \tilde{A}_i\bar{x}_1(k) + D_{1i}f(x(k)) + \tilde{H}_i\omega(k) \\ \bar{x}_2(k) = -A_{4i}^{-1}(A_{3i}\bar{x}_1(k) + H_{2i}\omega(k)) \end{cases} \tag{7}$$

where $\tilde{A}_i = A_{1i} - A_{2i}A_{4i}^{-1}A_{3i}$, $\tilde{H}_i = H_{1i} - A_{2i}A_{4i}^{-1}H_{2i}$.

Based on the above discussion, we have

$$x(k) = N\bar{x}(k) = \Pi_i\begin{bmatrix} \bar{x}_1(k) \\ \omega(k) \end{bmatrix}$$

where

$$\Pi_i = N\begin{bmatrix} I & 0 \\ -A_{4i}^{-1}A_{3i} & -A_{4i}^{-1}H_{2i} \end{bmatrix}.$$

Define $\bar{\eta}^{\mathrm{T}}(k) = [\bar{x}_1(k)\ \omega(k)\ f(x(k))]$. It obviously has that

$$\eta(k) = \begin{bmatrix} \Pi_i & 0 \\ 0 & I \end{bmatrix}\bar{\eta}(k).$$

Combining with (4), it follows that $\bar{\eta}^{\mathrm{T}}(k)\Delta_i\bar{\eta}(k) \le 0$, where

$$\Delta_i = \begin{bmatrix} \Pi_i^{\mathrm{T}}F_iD_aD_b\Pi_i & -\frac{1}{2}\Pi_iF_i(D_a + D_b) \\ * & F_i \end{bmatrix}. \tag{8}$$

Thus, the relationship between $x(k)$ and $f(x(k))$ is converted to the relationship among $\bar{x}_1(k)$, $f(x(k))$ and $\omega(k)$.

*Remark 1:* It should be noted that the decomposition (5) is obtained via a singular decomposition on $E$ and is not unique.

## 3 Main results

In this section, two ISS theorems for system (1) are derived based on multiple Lyapunov functions method. Our first result is concerned with the ISS property of system (1), when all the subsystems governing the continuous dynamics are ISS, however the switching are destabilising. Thus, based on the ADT method, a switching rule is designed to deal with the problem.

*Theorem 1:* Consider switched singular system (1) with Assumptions 1 and 2 satisfied. If there exist positive definite matrices $P_{1i}$, $P_{1j}$, $S_i$, $T_i$, $Q_i$ and $F_i$, $i \neq j \in J$, such that

$$\Phi_{ii} - \Delta_i < 0 \tag{9}$$

$$\Phi_{ij} - \Delta_i < 0 \tag{10}$$

where

$$\Phi_{ii} = \begin{bmatrix} \tilde{A}_i^{\mathrm{T}} P_{1i} \tilde{A}_i + S_i - P_{1i} & \tilde{A}_i^{\mathrm{T}} P_{1i} \tilde{H}_i & \tilde{A}_i^{\mathrm{T}} P_{1i} D_{1i} \\ * & \tilde{H}_i^{\mathrm{T}} P_{1i} \tilde{H}_i - T_i & \tilde{H}_i^{\mathrm{T}} P_{1i} D_{1i} \\ * & * & D_{1i}^{\mathrm{T}} P_{1i} D_{1i} \end{bmatrix}$$

$$\Phi_{ij} = \begin{bmatrix} \tilde{A}_i^{\mathrm{T}} P_{1j} \tilde{A}_i - Q_i - P_{1i} & \tilde{A}_i^{\mathrm{T}} P_{1j} \tilde{H}_i & \tilde{A}_i^{\mathrm{T}} P_{1j} D_{1i} \\ * & \tilde{H}_i^{\mathrm{T}} P_{1j} \tilde{H}_i - T_i & \tilde{H}_i^{\mathrm{T}} P_{1j} D_{1i} \\ * & * & D_{1i}^{\mathrm{T}} P_{1j} D_{1i} \end{bmatrix}$$

and $\Delta_i$ is defined in (8), then system (1) is ISS under the switching law satisfying

$$\tau_a > \tau_a^* = 1 - \frac{\ln\lambda_2}{\ln\lambda_1} \tag{11}$$

where $0 < \lambda_1 = \max_{i \in J} \{1 - \lambda_{\min}(S_i)/\lambda_{\max}(P_{1i})\} < 1$, $\lambda_2 = \max_{i \in J} \{1 + \lambda_{\max}(Q_i)/\lambda_{\min}(P_{1i})\} > 1$.

*Proof:* Consider the Lyapunov function

$$V(k) := V_{\sigma(k)}(k) = x^{\mathrm{T}}(k) E^{\mathrm{T}} P_{\sigma(k)} x(k)$$

where $P_{\sigma(k)}$ is in the form of

$$P_{\sigma(k)} = M^{\mathrm{T}} \begin{bmatrix} P_{1\sigma(k)} & 0 \\ P_{2\sigma(k)} & P_{3\sigma(k)} \end{bmatrix} N^{-1}.$$

Using (5) yields

$$V(k) = \bar{x}_1^{\mathrm{T}}(k) P_{1\sigma(k)} \bar{x}_1(k). \tag{12}$$

The following proof is carried out in two cases.

i. When $\sigma(k+1) = \sigma(k) = i$, $k \in [k^{n-1}, k^n - 1)$, which implies that no switching occurs, it follows from (9) and (12) that

$$\begin{aligned} V\,(k+1) - V(k) &\le \bar{x}_1^{\mathrm{T}}(k+1) P_{1i} \bar{x}_1(k+1) - \bar{x}_1^{\mathrm{T}}(k) P_{1i} \bar{x}_1(k) \\ &\le \bar{\eta}^{\mathrm{T}}(k) \Phi_{ii} \bar{\eta}(k) - \bar{x}_1^{\mathrm{T}}(k) S_i \bar{x}_1(k) + \omega^{\mathrm{T}}(k) T_i \omega(k) \\ &\le -\bar{x}_1^{\mathrm{T}}(k) S_i \bar{x}_1(k) + \omega^{\mathrm{T}}(k) T_i \omega(k), \end{aligned} \tag{13}$$

and thus

$$V(k+1) \le \lambda_1 V(k) + \omega^{\mathrm{T}}(k) T_i \omega(k). \tag{14}$$

ii. When $k = k^n - 1$, $i = \sigma(k) \neq \sigma(k+1) = j$, which means there exits a switching. It follows from (9) and (12) that

$$\begin{aligned} V\,(k+1) - V(k) &= \bar{x}_1^{\mathrm{T}}(k+1) P_{1j} \bar{x}_1(k+1) - \bar{x}_1^{\mathrm{T}}(k) P_{1i} \bar{x}_1(k) \\ &\le \bar{\eta}^{\mathrm{T}}(k) \Phi_{ij} \bar{\eta}(k) + \bar{x}_1^{\mathrm{T}}(k) Q_i \bar{x}_1(k) + \omega^{\mathrm{T}}(k) T_i \omega(k) \\ &\le \bar{x}_1^{\mathrm{T}}(k) Q_i \bar{x}_1(k) + \omega^{\mathrm{T}}(k) T_i \omega(k) \end{aligned}$$

and thus

$$V(k+1) \le \lambda_2 V(k) + \omega^{\mathrm{T}}(k) T_i \omega(k). \tag{15}$$

The switchings are bad factors to system stability. Consider the worst-case scenario, the number of switchings on $[k_0, k]$ is $N_\sigma(k, k_0)$. Assume that the switching sequence is $k_0, k_1, \ldots, k_{N_\sigma(k,k_0)}$. We use $N_\sigma$ to denote $N_\sigma(k, k_0)$ for simplicity. Define $\tilde{\lambda} = \max\{\lambda(T_{\sigma(s)}) : s \in [k_0, k_{N_\sigma}]\}$. Then when $k = k_{N_\sigma}$, based on (15), it has

$$\begin{aligned} V(k) &< \lambda_2 V(k-1) + \lambda_{\max}(T_{\sigma(k-1)}) \|\, \omega(k-1) \,\| \\ &< \lambda_2^2 V(k-2) + \lambda_2 \lambda_{\max}(T_{\sigma(k-2)}) \|\, \omega(k-2) \,\| \\ &\quad + \lambda_{\max}(T_{\sigma(k-1)}) \|\, \omega(k-1) \,\| \\ &< \cdots < \lambda_2^{N_\sigma} V(k_0) + \sum_{\upsilon=0}^{N_\sigma - 1} \lambda_2^{\upsilon} \tilde{\lambda} \sup_{k_0 \le s \le k} \|\, \omega(s) \,\|. \end{aligned} \tag{16}$$

When $k > k_{N_\sigma}$, there is no switching occur, from (13) and (16), it has

$$\begin{cases} V(k_{N_\sigma} + 1) \le \lambda_1 \lambda_2^{N_\sigma} V(k_0) + \left(1 + \sum_{\upsilon=0}^{N_\sigma - 1} \lambda_2^{\upsilon}\right) \\ \times \tilde{\lambda} \sup_{k_0 \le s \le k} \|\, \omega(s) \,\| \\ V(k_{N_\sigma} + 2) \le \lambda_1^2 \lambda_2^{N_\sigma} V(k_0) + \left( \sum_{\upsilon=0}^{N_\sigma - 1} \lambda_2^{\upsilon} \lambda_1^2 + \lambda_1 + 1 \right) \\ \times \tilde{\lambda} \sup_{k_0 \le s \le k} \|\, \omega(s) \,\| \\ \cdots \\ V(k) \le \lambda_1^{k - N_\sigma} \lambda_2^{N_\sigma} V(k_0) + \left( \sum_{\upsilon=0}^{N_\sigma - 1} \lambda_2^{\upsilon} \lambda_1^{k - N_\sigma} \right. \\ \left. + \sum_{\upsilon=0}^{k - N_\sigma - 1} \lambda_1^{\upsilon} \right) \tilde{\lambda} \sup_{k_0 \le s \le k} \|\, \omega(s) \,\|. \end{cases} \tag{17}$$

Because $0 < \lambda_1 < 1 < \lambda_2$, it can be seen that the switching lead to instability. The number of switchings should be constrained, and the instability can be compensated by the operation of the stable subsystems. Using (2) and (17) yields that (see (18)) where $c_1 = (\lambda_2/\lambda_1)^{N_0}$, $c_2 = (\lambda_2 \lambda_1^{-1})^{N_0} \lambda_{\max}(T_i)/(\lambda_2 - 1)$, $c_3 = \lambda_{\max}(T_i)/(1 - \lambda_1)$, $\lambda = \lambda_1^{1 - 1/\tau_a} \lambda_2^{1/\tau_a}$.

Based on (12) and (18), it can be obtained that

$$\begin{aligned} \|\, \bar{x}_1(k) \,\| &\le \frac{V(k)}{\lambda_{\min}(P_{1i})} \\ &\le \frac{c_1 \lambda_{\max}(P_{1i})}{\lambda_{\min}(P_{1i})} e^{-\lambda(k - k_0)} \|\, M \,\| \|\, Ex(k_0) \,\| \\ &\quad + \frac{c_2 e^{-\lambda(k - k_0)} + c_3}{\lambda_{\min}(P_{1i})} \sup_{k_0 \le s \le k} \|\, \omega(s) \,\|. \end{aligned} \tag{19}$$

According to (7) and (19), we conclude that

$$\begin{aligned} \|\, \bar{x}(k) \,\| &\le \|\, \bar{x}_1(k) \,\| + \|\, \bar{x}_2(k) \,\| \\ &\le (1 + \left\| A_{4i}^{-1} A_{3i} \right\|) \|\, \bar{x}_1(k) \,\| \\ &\quad + \left\| A_{4i}^{-1} H_{2i} \right\| \|\, \omega(k) \,\| \\ &\le \frac{c_1 (1 + \left\| A_{4i}^{-1} A_{3i} \right\|) \lambda_{\max}(P_{1i}) \|\, M \,\|}{\lambda_{\min}(P_{1i})} \\ &\quad \times e^{-\lambda(k - k_0)} \|\, Ex(k_0) \,\| + \sup_{k_0 \le s \le k} \|\, \omega(s) \,\| \\ &\quad \times \left( \frac{c_2 e^{-\lambda(k - k_0)} + c_3}{\lambda_{\min}(P_{1i})} + \left\| A_{4i}^{-1} H_{2i} \right\| \right). \end{aligned} \tag{20}$$

Since $x(k) = N \bar{x}(k)$, based on (20), it obviously has

$$\begin{aligned} \| x(k) \| &\le \| N \| \| \bar{x}(k) \| \\ &\le c_4 e^{-\lambda(k-k_0)} \| Ex(k_0) \| \\ &\quad + c_5 \sup_{k_0 \le s \le k} \| \omega(s) \| \end{aligned} \tag{21}$$

where

$$c_4 = \frac{c_1(1 + \| A_{4i}^{-1} A_{3i} \|) \| M \| \| N \| \lambda_{\max}(P_{1i})}{\lambda_{\min}(P_{1i})}$$

$$c_5 = \| A_{4i}^{-1} H_{2i} \| \| N \| + \frac{(c_2 e^{-\lambda(k-k_0)} + c_3) \| N \|}{\lambda_{\min}(P_{1i})}.$$

Based on above discussion and Definition 3, (21) implies that system (1) is ISS. The proof is completed.□

*Remark 2:* As a special case, if there has no external inputs, (21) reduces to $\| x(k) \| \le c_4 e^{-\lambda(k-k_0)} \| Ex(k_0) \|$, which implies the exponential stability of system (1).

For the case that all subsystems of system (1) are unstable, but the switching are considered as a kind of control strategy to stabilise the system. The state-dependent switching rule is designed to guarantee the ISS of the switched system.

*Theorem 2:* Consider switched singular system (1) with Assumptions 1–2 satisfied. If there exist positive definite matrices $P_{1i}$, $P_{1j}$, $T_i$, $F_i$, negative definite matrices $W_{ij}$ and scalars $\rho_{ij} > 0$, $\kappa_{ij} > 0$, $i \ne j \in J$, such that

$$\Psi_{ij} - \Delta_i < 0 \tag{22}$$

where

$$\Psi_{ij} = \begin{bmatrix} \Psi_{11} & \tilde{A}_i^{\mathrm{T}} P_{1i} \tilde{H}_i & \tilde{A}_i^{\mathrm{T}} P_{1i} D_{1i} \\ * & \tilde{H}_i^{\mathrm{T}} P_{1i} \tilde{H}_i - T_i & \tilde{H}_i^{\mathrm{T}} P_{1i} D_{1i} \\ * & * & D_{1i}^{\mathrm{T}} P_{1i} D_{1i} \end{bmatrix}$$

$$\Psi_{11} = \tilde{A}_i^{\mathrm{T}} P_{1i} \tilde{A}_i + R_i - P_{1i} - \kappa_{ij}(P_{1i} - P_{1j} + \rho_{ij} W_{ij})$$

and $\Delta_i$ is given in (8). Then, system (1) is ISS under the following switching rule:

$$\sigma(k) = \begin{cases} \arg\min \{\Omega_i | \bar{x}_1(k_0) \in \Omega_i\} & k = k_0 \\ i & \bar{x}_1(k) \in \Omega_i,\ \sigma(k-1) = i \\ \arg\min \{\Omega_j | \bar{x}_1(k) \in \Omega_j\} & \bar{x}_1(k) \notin \Omega_i, \sigma(k-1) = i \end{cases} \tag{23}$$

where $\Omega_i = \{\bar{x}_1(k) | \bar{x}_1^{\mathrm{T}}(k)(P_{1i} - P_{1j} + \rho_{ij} W_{ij}) \bar{x}_1(k) \le 0\}$ denotes the switching region and satisfies $\bigcup \Omega_i = R^n \setminus \{0\}$.

*Proof:* Choose the same Lyapunov function of that in Theorem 1

$$V(k) = x^{\mathrm{T}}(k) E^{\mathrm{T}} P_{\sigma(k)} x(k) = \bar{x}_1^{\mathrm{T}}(k) P_{1\sigma(k)} \bar{x}_1(k).$$

Similarly, the proof is unfolded in two cases.

i. In this case that no switching occurs, namely $k \in [k^{n-1}, k^n - 1)$, $\sigma(k+1) = \sigma(k) = i$. It follows form (22) that

$$\begin{aligned} &V(k+1) - V(k) \\ &= \bar{x}_1^{\mathrm{T}}(k+1) P_{1i} \bar{x}_1(k+1) - \bar{x}_1^{\mathrm{T}}(k) P_{1i} \bar{x}_1(k) \\ &\le \bar{\eta}^{\mathrm{T}}(k) \Psi_{ij} \bar{\eta}(k) - \bar{x}_1^{\mathrm{T}}(k) R_i \bar{x}_1(k) + \omega^{\mathrm{T}}(k) T_i \omega(k) \\ &\le -\bar{x}_1^{\mathrm{T}}(k) R_i \bar{x}_1(k) + \omega^{\mathrm{T}}(k) T_i \omega(k). \end{aligned}$$

ii. When $k = k^n - 1$, $i = \sigma(k) \ne \sigma(k+1) = j$. At switching instants, according to switching rule (23), it is clear that $x_1^{\mathrm{T}}(k)(P_{1j} - P_{1i}) \bar{x}_1(k) \ge 0$, thus (19) still holds.

Above all, for each $k \ge k_0$, it has

$$V(k+1) \le \lambda_1 V(k) + \omega^{\mathrm{T}}(k) T_i \omega(k) \tag{24}$$

where $0 < \lambda_1 = \max_{i \in J} \{1 - \lambda_{\min}(R_i)/\lambda_{\max}(P_{1i})\} < 1$.
By iteration, it can be obtained for all $s \in [k_0, k]$ that

$$\begin{aligned} V(k) &\le \lambda_1 V(k-1) + \lambda_{\max}(T_{\sigma(k-1)}) \| \omega(k-1) \| \\ &\le \lambda_1^2 V(k-2) + \lambda_1 \lambda_{\max}(T_{\sigma(k-2)}) \| \omega(k-2) \| \\ &\quad + \lambda_{\max}(T_{\sigma(k-1)}) \| \omega(k-1) \| \\ &\le \cdots \le \lambda_1^{k-k_0} V(k_0) + \sum_{\upsilon=0}^{k-1} \lambda_1^{\upsilon} \lambda_{\max}(T_{\sigma(s)}) \sup (\| \omega(s) \|) \\ &\le \lambda_1^{k-k_0} V(k_0) + \frac{1}{1-\lambda_1} \lambda_{\max}(T_{\sigma(s)}) \sup (\| \omega(s) \|). \end{aligned}$$

Similar to the proof of Theorem 1, we have for all $s \in [k_0, k]$ that

$$\begin{aligned} \| \bar{x}_1(k) \| &\le \frac{\lambda_{\max}(P_{1i})}{\lambda_{\min}(P_{1i})} e^{(k-k_0)\ln\lambda_1} \| M \| \| Ex(k_0) \| \\ &\quad + \frac{\lambda_{\max}(T_i)}{(1-\lambda_1)\lambda_{\min}(P_{1i})} \sup (\| \omega(s) \|). \end{aligned}$$

Furthermore, it can be derived for all $s \in [k_0, k]$ that

$$\| x(k) \| \le c_6 e^{(k-k_0)\ln\lambda_1} \| Ex(k_0) \| + c_7 \sup (\| \omega(s) \|) \tag{25}$$

where

$$c_6 = \frac{(1 + \| A_{4i}^{-1} A_{3i} \|) \lambda_{\max}(P_{1i}) \| M \| \| N \|}{\lambda_{\min}(P_{1i})} e^{(k-k_0)\ln\lambda_1}$$

$$c_7 = \left( \frac{(1 + \| A_{4i}^{-1} A_{3i} \|) \lambda_{\max}(T_i)}{(1-\lambda_1)\lambda_{\min}(P_{1i})} + \| A_{4i}^{-1} H_{2i} \| \right) \| N \|.$$

Obviously, Definition 3 and (25) imply that system (1) is ISS. The proof is completed. □

$$\begin{aligned} V(k) &\le (\lambda_2 \lambda_1^{-1})^{N_\sigma} \lambda_1^k V(k_0) + \left( \frac{(\lambda_2 \lambda_1^{-1})^{N_\sigma} \lambda_1^k}{(\lambda_2 - 1)} \right. \\ &\quad \left. + \frac{1 - \lambda_1^{k-N_\sigma}}{(1-\lambda_1)} \right) \tilde{\lambda} \sup_{k_0 \le s \le k} \| \omega(s) \| \\ &\le (\lambda_2 \lambda_1^{-1})^{N_0 + ((k-k_0)/\tau_a)} \lambda_1^k V(k_0) + \left( \frac{(\lambda_2 \lambda_1^{-1})^{N_0 + ((k-k_0)/\tau_a)} \lambda_1^k}{(\lambda_2 - 1)} \right. \\ &\quad \left. + \frac{1}{(1-\lambda_1)} \right) \tilde{\lambda} \sup_{k_0 \le s \le k} \| \omega(s) \| \\ &\le c_1 e^{-\lambda(k-k_0)} V(k_0) + \left( c_2 e^{-\lambda(k-k_0)} + c_3 \right) \\ &\quad \times \sup_{k_0 \le s \le k} \| \omega(s) \| \end{aligned} \tag{18}$$

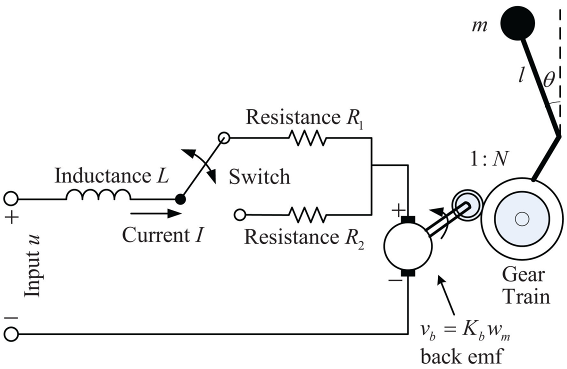


**Fig. 1** *Diagram of an inverted pendulum controlled by a DC motor*

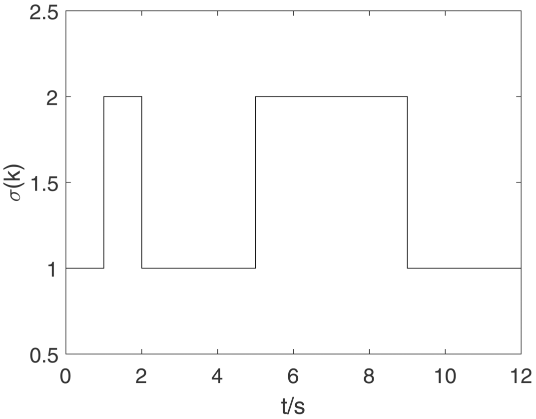


**Fig. 2** *Switching signal with ADT $\tau_a = 2\,\text{s}$ and $N_0 = 2$*

*Remark 3:* It is worth mentioning that due to the algebraic constraint given in (6), there may exist state jumps at switching instants. To describe the state regions of the full state space is not a trivial work, thus the state-space partitioning is only considered on $\bar{x}_1(k)$.

*Remark 4:* Here we list the state-dependent switching algorithm:

Step 1: Choose the initial mode by applying the minimum rule to $\bar{x}_1(k_0)$.
Step 2: Stay at the $i$th mode as long as

$$\bar{x}_1(k_0) \in \Omega_i = \{\bar{x}_1(k) | \bar{x}_1^{\mathrm{T}}(k)(P_{1i} - P_{1j} + \rho_{ij} W_{ij})\bar{x}_1(k) \le 0\}$$

for some $\rho_{ij}$ chosen beforehand.
Step 3: If $\bar{x}_1(k_0)$ hits the boundary of at $\Omega_i$, use the minimum rule to determine the next mode to be activated.

*Remark 5:* Compared with the existing results [8–10, 24], one of the main contributions of our paper lies in the design of state-dependent switching rule. This rule can also be used to the situation of that all or partial subsystems are stable since there always exists a subsystem with the lowest energy. So our method has a wide application range. Moreover, sector-bounded non-linearity is considered in our work, which is more general than Lipschitz non-linearity.

## 4 Simulation examples

In this section, we give two examples to demonstrate the effectiveness and applicability of the proposed methods.

*Example 1:* Consider an inverted pendulum system controlled by a DC motor (see Fig. 1 for the diagram), which has been modelled in [28] as

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = \dfrac{g}{l}\sin x_1(t) + \dfrac{NK_m}{ml^2}x_3(t) \\ L_a\dot{x}_3(t) = K_bNx_2(t) - R_{\sigma(t)}x_3(t) + u(t) \end{cases} \tag{26}$$

where $x_1(t) = \theta(t)$, $x_2(t) = \dot{\theta}(t)$, $x_3(t) = I(t)$, $\sigma(t) \in J = \{1, 2\}$. In [28], the switching between two resistances $R_1$ and $R_2$ subjects to a Markovian transition rate matrix while the ADT constraint is applied here. Let $N = 10$, $g = 9.8\,\text{m/s}^2$, $m = 1\,\text{kg}$, $l = 1\,\text{m}$, $R_1 = 1\,\Omega$, $R_2 = 0.5\,\Omega$, the motor torque constant be $K_m = 0.1\,\text{N}\cdot\text{m/A}$, the back emf constant be $K_b = 0.1\,\text{V}\cdot\text{s/rad}$, and the control input be $u(t) = -20x_1(t) - 5x_2(t)$. If we neglect the DC motor inductance, that is $L = \varepsilon H$ with $\varepsilon = 0$, then, (26) becomes the following non-linear switched singular system:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = x_3(t) + 9.8\sin x_1(t) \\ 0 = -20x_1(t) - 4x_2(t) - R_\sigma(t)x_3(t). \end{cases} \tag{27}$$

By using the zero-order-hold sampling (set sampling period $h = 1\,\text{s}$), the discrete-time form of (27) is obtained and fits with system (1) containing the following parameters:

$$A_1 = \begin{bmatrix} -0.1397 & -0.0256 & 0 \\ 0.5121 & -0.0373 & 0 \\ -20 & -4 & -1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 & 0.5584 & 0 \\ 0 & -0.2509 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.0113 & -0.0037 & 0 \\ 0.1470 & 0.0181 & 0 \\ -20 & -4 & -0.5 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0 & 0.2478 & 0 \\ 0 & -0.0360 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad H_1 = H_2 = \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix}, \quad f(x(k)) = \begin{bmatrix} 0 \\ \sin x_1(k) \\ 0 \end{bmatrix}.$$

Suppose that an bounded external disturbance is imposed on the system and set $\omega(k) = 0.01\sin(k)$. Note that the non-linear functions in this example have the form of $f_1(\cdot) = f_3(\cdot) \equiv 0$ and $f_2(x_1) = \sin x_1(k)$. They do not fit with Assumption 1 directly. However, if we choose $a_l = -1$ and $b_l = 1$ for $l = 1, 2, 3$, conditions (4) and (8) still hold which mean that Theorem 1 is applicable to this example. In fact, by writing $f_1(x_2) = f_3(x_3) = 0$, we have

$$\begin{aligned} (f_1(x_2) - a_2x_2)(f_1(x_2) - b_2x_2) &= -x_2^2 \le 0 \\ (f_2(x_1) - a_1x_1)(f_2(x_1) - b_1x_1) &= \sin^2 x_1 - x_1^2 \le 0 \\ (f_3(x_3) - a_3x_3)(f_3(x_3) - b_3x_3) &= -x_3^2 \le 0. \end{aligned} \tag{28}$$

With $D_a + D_b = 0$ in mind, (28) can guarantee (4) and (8) since the off-diagonal blocks of them are zero.
Solving the linear matrix inequalities (LMIs) given in Theorem 1, the feasible solutions are calculated as

$$P_{11} = \begin{bmatrix} 1.9245 & -0.1798 \\ -0.1798 & 1.9851 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} 1.9103 & -0.2226 \\ -0.2226 & 2.2222 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 1.1374 & 0.0808 \\ 0.0808 & 0.9809 \end{bmatrix}, \quad S_2 = \begin{bmatrix} 1.1498 & 0.0230 \\ 0.0230 & 1.0997 \end{bmatrix}$$

$$Q_1 = \begin{bmatrix} 1.2278 & -0.0829 \\ -0.0829 & 1.3507 \end{bmatrix}, \quad Q_2 = \begin{bmatrix} 1.1791 & -0.0233 \\ -0.0233 & 1.2318 \end{bmatrix}.$$

The lower bound of ADT is then computed from (11) that $\tau_a^* = 1.9909\,\text{s}$. Choose $\tau_a = 2\,\text{s}$ and $N_0 = 2$ here.
Choose the initial value $x(0) = [-2\ 3\ 4]^{\mathrm{T}}$. The switching signal is depicted in Fig. 2. Fig. 3 shows that the state trajectory of the system is ultimately bounded with the disturbance $\omega(k)$.

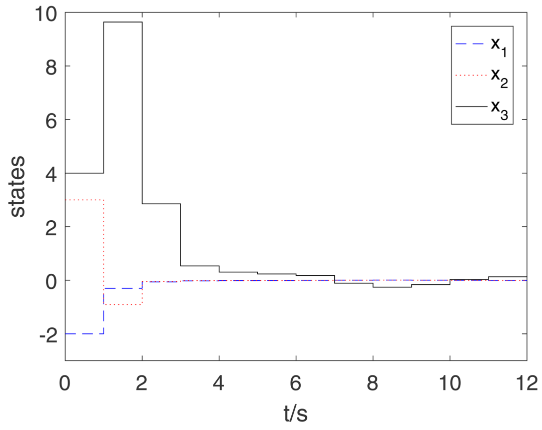


**Fig. 3** *State response with* $x(0) = [-2\ 3\ 4]^{\mathrm{T}}$

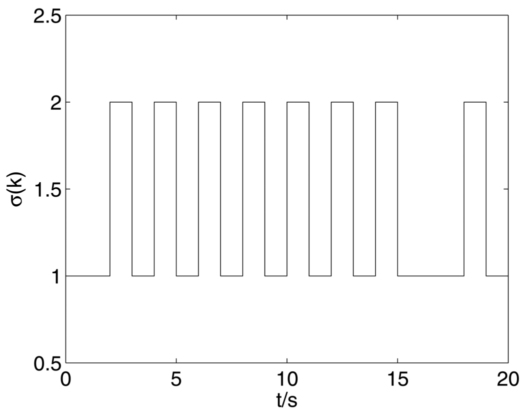


**Fig. 4** *Switching signal under state-dependent switching*

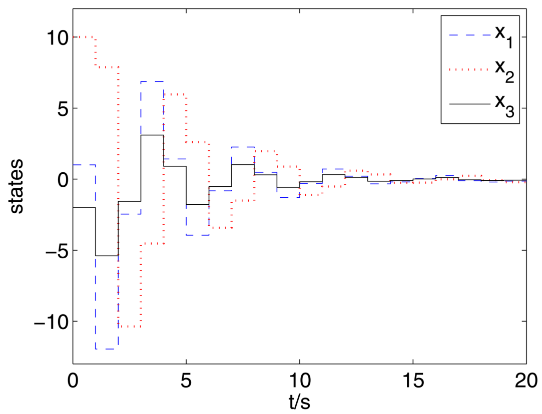


**Fig. 5** *State response with* $x(0) = [1\ 10\ -2]^{\mathrm{T}}$

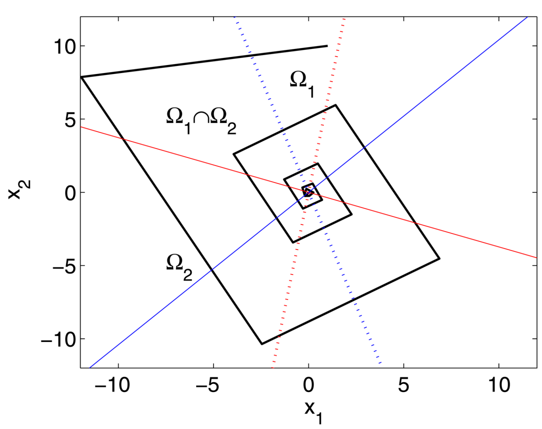


**Fig. 6** *State region partition*

*Example 2:* Consider system (1) composed of two subsystems with parameters

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad H_1 = \begin{bmatrix} -0.2 \\ 1 \\ 1 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 1 \\ 0.1 \\ -0.5 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0.9 & -1 & -3 \\ 1 & 0.2 & 0 \\ 0 & 0.12 & 2.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2.4286 & -0.7857 & -3 \\ 1 & 0.2 & 0 \\ 0 & 0.12 & 2.1 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} 1.2 & -0.2 & 0.2 \\ -0.5 & 1 & 1 \\ -0.5 & 1 & 1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} -0.2 & 0.5 & 0.1 \\ -1.2 & 0.2 & 0.2 \\ -1.2 & 0.2 & 0.2 \end{bmatrix}.$$

The sector non-linear function is defined as

$$f_1(x_1(k)) = \begin{cases} 0.55x_1(k) & x_1(k) \geq 0 \\ 0.45x_1(k) & x_1(k) < 0 \end{cases}$$

$$f_2(x_2(k)) = \begin{cases} 0.6x_2(k) & x_2(k) \geq 0 \\ 0.5x_2(k) & x_2(k) < 0 \end{cases}$$

$$f_3(x_3(k)) = \begin{cases} |\sin x_3(k)| & -\pi < x_3 < \pi \\ 0 & \text{otherwise.} \end{cases}$$

Suppose that the bounded external disturbance input is $\omega(k) = 0.01\sin(k)$. Choose $\kappa_{12} = 0.8$, $\kappa_{21} = 0.5$, $\rho_{12} = 1.2$, $\rho_{21} = 1.0$. Solving LMIs in Theorem 2, the following feasible solutions are obtained:

$$M = \begin{bmatrix} 1 & -1.4286 & 1.4286 \\ 0 & 1 & 0 \\ 0 & -0.4762 & 0.4762 \end{bmatrix}, \quad N = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0.4762 & 0.0381 & 1 \end{bmatrix}$$

$$P_{11} = \begin{bmatrix} 0.1095 & 0.0435 \\ 0.0435 & 0.0683 \end{bmatrix}, \quad P_{12} = \begin{bmatrix} 0.5276 & -0.1775 \\ -0.1775 & 0.3949 \end{bmatrix}.$$

The switching signal, states response and state region partition are plotted in Figs. 4–6, respectively. It can be seen from Figs. 5 and 6 that the considered system is ISS.

*Remark 6:* The state space is partitioned into two subregions as shown in Fig. 6. $\Omega_1 \cap \Omega_2$ represents the overlapped regions. At the initial time, $\sigma(k_0) = 1$ according to switching rule (23). When the states hit the boundary, subsystem 2 is activated, i.e. $\sigma(k) = 2$, and so on. It can be seen that the states remain bounded.

## 5 Conclusion

This paper has considered the ISS problem for discrete-time non-linear switched singular systems. Based on the multiple Lyapunov functions method, switching rules have been designed. In the case that all the subsystems are ISS, but the switchings may destroy the system stability, therefore we need an ADT to guarantee the ISS of the entire switched system. In addition, if all subsystems are not ISS, then the state-dependent switching rule has been designed to guarantee the ISS of the switched system. Finally, two simulation examples have been given to verify the effectiveness of the proposed approaches.

## 6 Acknowledgments

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## 7 References

[1] Esparza, M., Segundo-Ramírez, J., Kwon, J., et al.: 'Modeling of VSC-based power systems in the extended harmonic domain', *IEEE Trans. Power Electron.*, 2017, **32**, (8), pp. 5907–5916

[2] Cantó, B., Coll, C., Sánchez, E.: 'Positive n-periodic descriptor control systems', *Syst. Control Lett.*, 2004, **53**, (5), pp. 407–414
[3] Lian, J., Li, C., Xia, B.: 'Sampled-data control of switched linear systems with application to an F-18 aircraft', *IEEE Trans. Ind. Electron.*, 2017, **64**, (2), pp. 1332–1340
[4] Xiang, W., Xiao, J.: 'Stabilization of switched continuous-time systems with all modes unstable via dwell time switching', *Automatica*, 2014, **50**, (3), pp. 940–945
[5] Zhao, X., Zhang, L., Shi, P., et al.: 'Stability and stabilization of switched linear systems with mode-dependent average dwell time', *IEEE Trans. Autom. Control*, 2012, **57**, (7), pp. 1809–1815
[6] Geromel, J.C., Colaneri, P.: 'Stability and stabilization of continuous-time switched linear systems', *Siam J. Control Optim.*, 2006, **45**, (5), pp. 1915–1930
[7] Sun, X.M., Zhao, J., Hill, D.J.: 'Stability and $L_2$-gain analysis for switched delay systems: a delay-dependent method', *Automatica*, 2006, **42**, (10), pp. 1769–1774
[8] Wu, L., Su, X., Shi, P.: 'Output feedback control of Markovian jump repeated scalar nonlinear systems', *IEEE Trans. Autom. Control*, 2016, **59**, (1), pp. 199–204
[9] Su, X., Liu, X., Shi, P., et al.: 'Sliding mode control of discrete-time switched systems with repeated scalar nonlinearity', *IEEE Trans. Autom. Control*, 2016, DOI 10.1109/TAC.2016.2626398
[10] Su, X., Wu, L., Shi, P., et al.: 'Model approximation for fuzzy switched systems with stochastic perturbation', *IEEE Trans. Fuzzy Syst.*, 2014, **23**, (5), pp. 1458–1473
[11] Lin, X., Du, H., Li, S., et al.: 'Finite-time stability and finite-time weighted $L_2$-gain analysis for switched systems with time-varying delay', *IET Control Theory Appl.*, 2013, **7**, (7), pp. 1058–1069
[12] Hajiahmadi, M., Schutter, B.D., Hellendoorn, H.: 'Stabilization and robust $H_\infty$ control for sector-bounded switched nonlinear systems', *Bul. Inst. Politeh. 'Gheorghe Gheorghiu-Dej' Bucur. Ser. Autom.*, 2014, **50**, (10), pp. 2726–2731
[13] Liu, X., Stechlinski, P.: 'Hybrid control of impulsive systems with distributed delays', *Nonlinear Anal-Hybri.*, 2014, **11**, pp. 57–70
[14] Dong, Y., Zhao, J.: 'Composite anti-disturbance control for switched systems via mixed state-dependent and time-driven switching', *IET Control Theory Appl.*, 2016, **10**, (16), pp. 1981–1990
[15] Liberzon, D., Trenn, S.: 'Switched nonlinear differential algebraic equations: solution theory, Lyapunov functions, and stability', *Automatica*, 2012, **48**, (5), pp. 954–963
[16] Zhou, L., Ho, W.C.D., Zhai, G.: 'Stability analysis of switched linear singular systems', *Automatica*, 2013, **49**, (5), pp. 1481–1487
[17] Feng, G., Cao, J.: 'Stability analysis of impulsive switched singular systems', *IET Control Theory Appl.*, 2015, **9**, (6), pp. 863–870
[18] Li, J., Ma, R., Dimirovski, G.M.: 'Dwell-time-based stabilization of switched linear singular systems with all unstable-mode subsystems', *J. Frankl. Inst.*, 2017, **354**, (7), pp. 2712–2724
[19] Han, Y., Kao, Y., Gao, C.: 'Robust sliding mode control for uncertain discrete singular systems with time-varying delays and external disturbances', *Automatica*, 2017, **75**, pp. 210–216
[20] Lin, J., Fei, S., Gao, Z.: 'Control discrete-time switched singular systems with state delays under asynchronous switching', *Int. J. Syst. Sci.*, 2013, **44**, (6), pp. 1–13
[21] Chen, W.H., Zheng, W.X.: 'Input-to-state stability and integral input-to-state stability of nonlinear impulsive systems with delays', *Automatica*, 2009, **45**, (6), pp. 1481–1488
[22] Wang, Y.E., Sun, X.M., Shi, P., et al.: 'Input-to-state stability of switched nonlinear systems with time delays under asynchronous switching', *IEEE Trans. Cybern.*, 2013, **43**, (6), pp. 2261–2265
[23] Kundu, A., Chatterjee, D., Liberzon, D.: 'Generalized switching signals for input-to-state stability of switched systems', *Automatica*, 2016, **64**, pp. 270–277
[24] Liu, Y., Kao, Y., Karimi, H.R., et al.: 'Input-to-state stability for discrete-time nonlinear switched singular systems', *Inf. Sci.*, 2016, **358**, pp. 18–28
[25] Hespanha, J.P., Morse, A.S.: 'Stability of switched systems with average dwell-time'. Proc. Conf. Decision and Control, 1999, vol. **3**, pp. 2655–2660
[26] Dai, L.: '*Singular control systems*' (Springer-Verlag, Berlin, Germany, 1989)
[27] Xu, S.Y., Lam, J.: '*Robust control and filtering of singular systems*' (Springer-Verlag, Berlin, Germany, 2006)
[28] Wang, G., Bo, H.: 'Stabilization of singular Markovian jump systems by generally observer-based controllers', *Asian J. Control*, 2016, **18**, (1), pp. 328–339