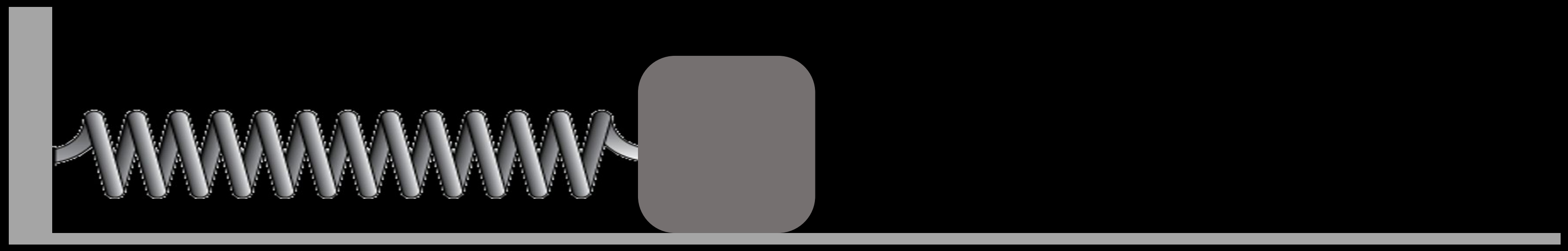
Part I: Hooke’s Law

the physics of springs

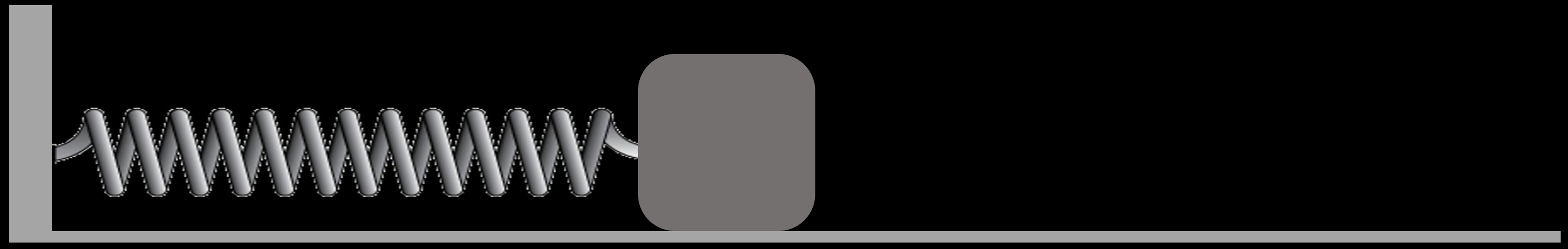


Hooke’s Law: physics of springs

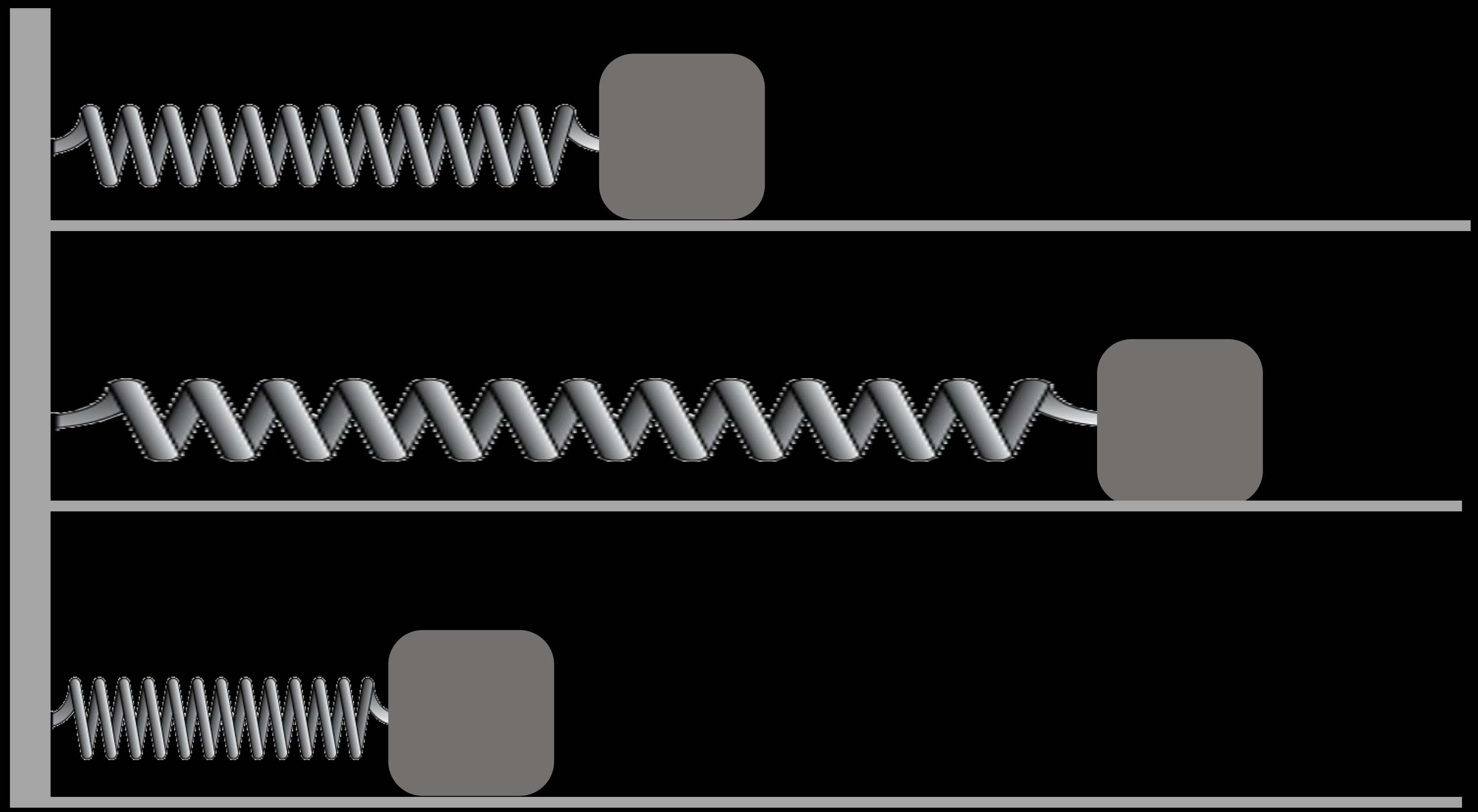
What do you know about the behavior of springs? What are the important properties of springs? (open discussion)

Consider a spring, one end firmly attached to a wall, and the other of object of mass m that is free to move.

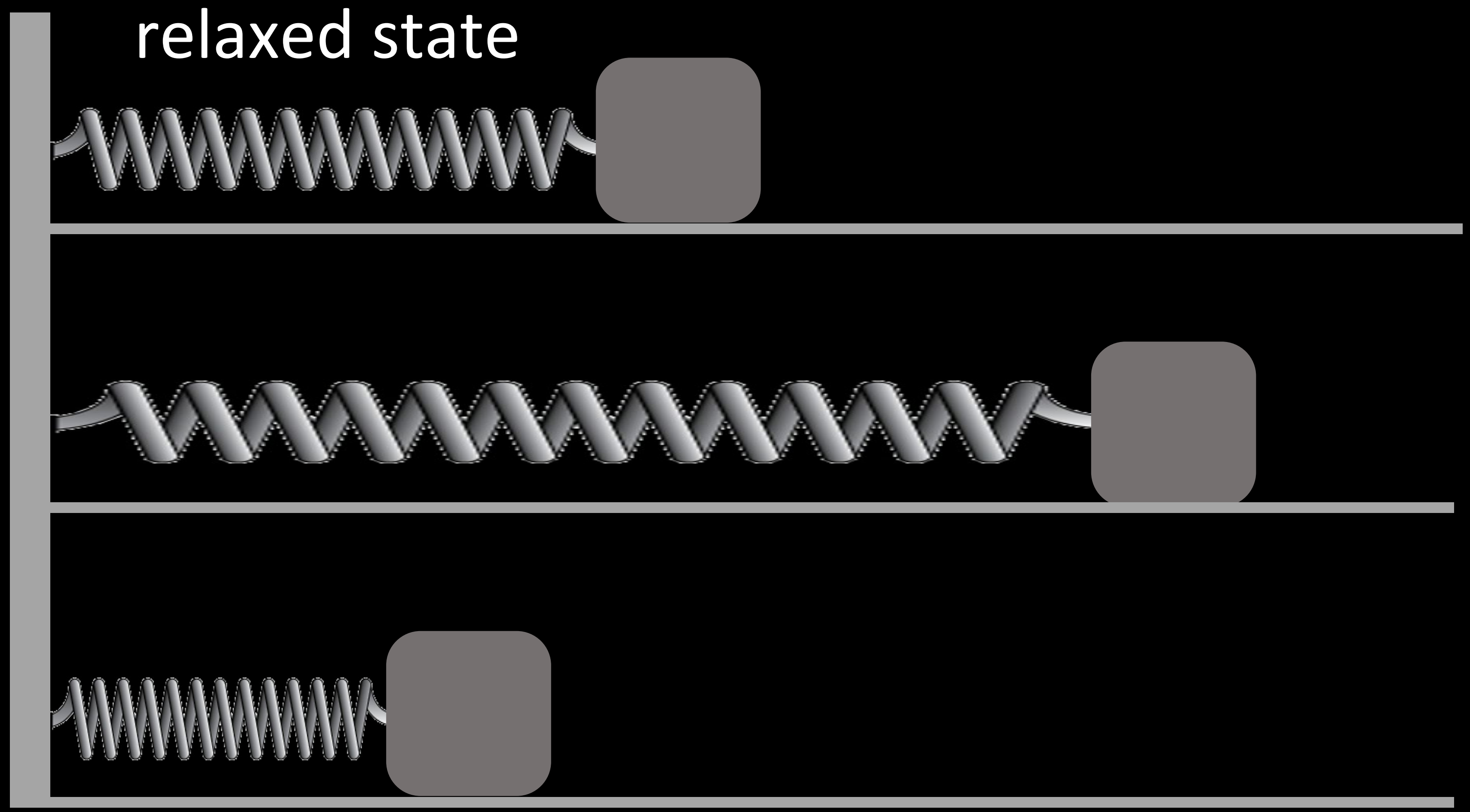
Under what conditions will the spring act on the mass?



Hooke’s Law: physics of springs

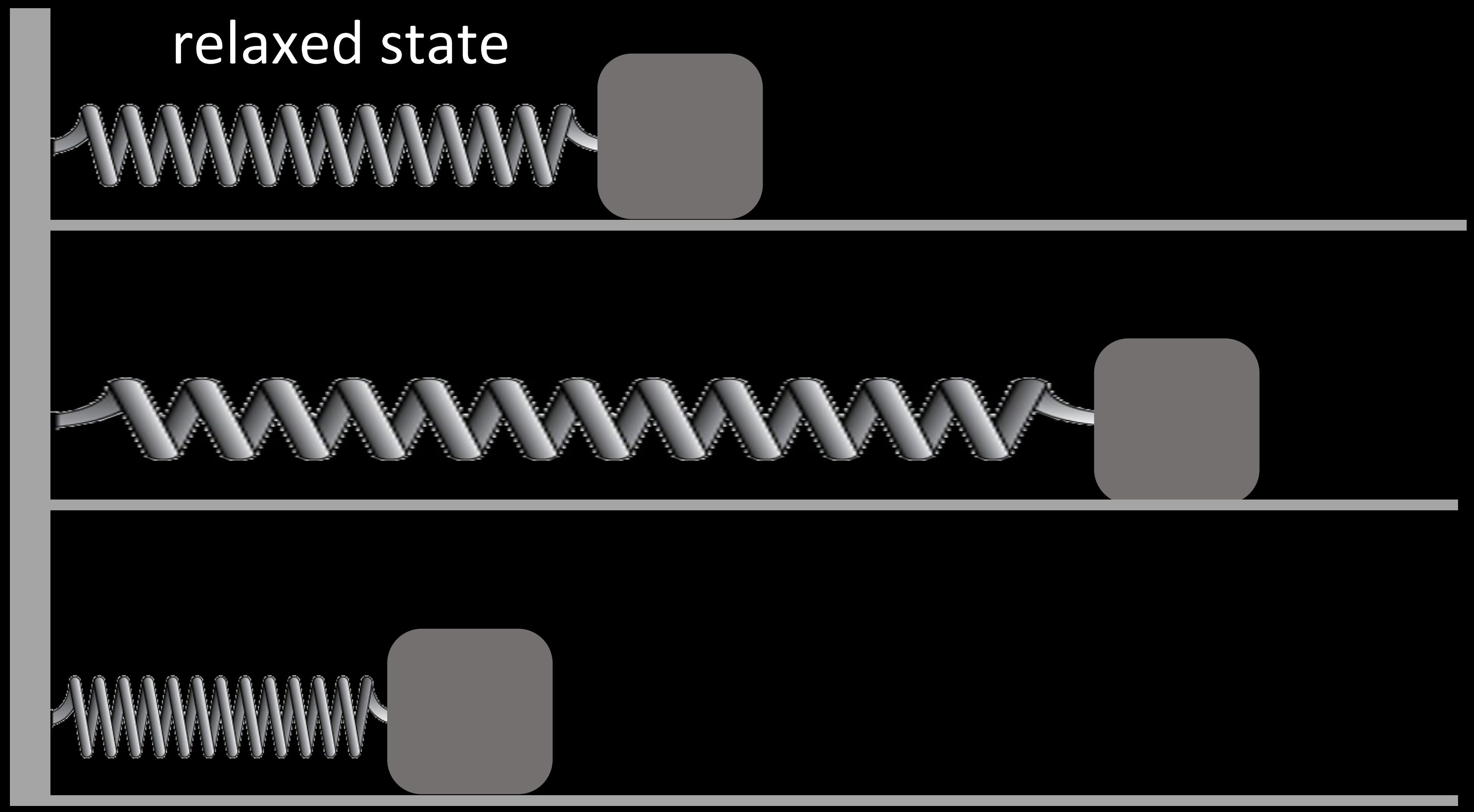


Hooke's Law: physics of springs



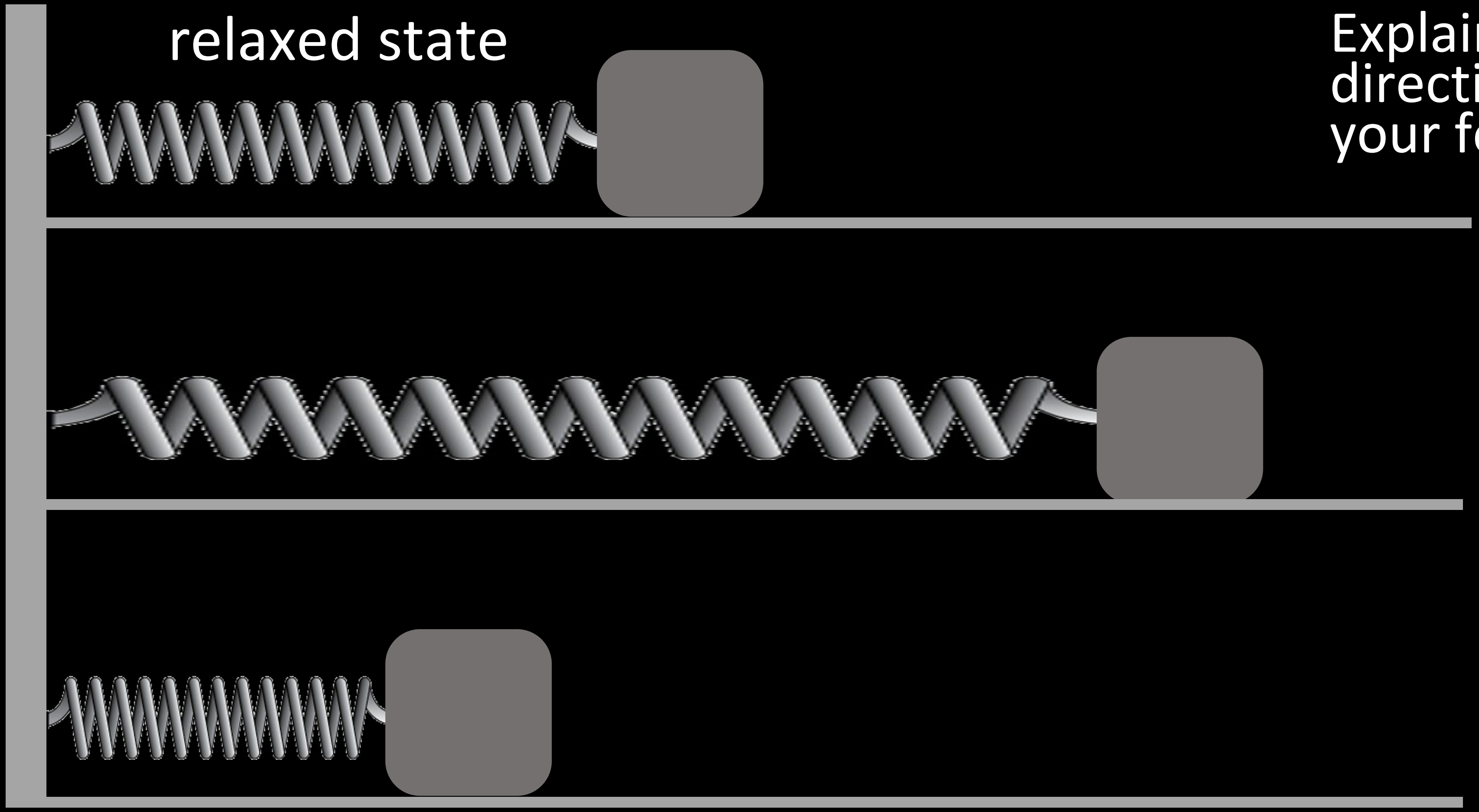
Hooke’s Law: physics of springs

For the 3 situations below, where applicable, draw vector arrows to show the force on the mass from the spring.



Hooke's Law: physics of springs

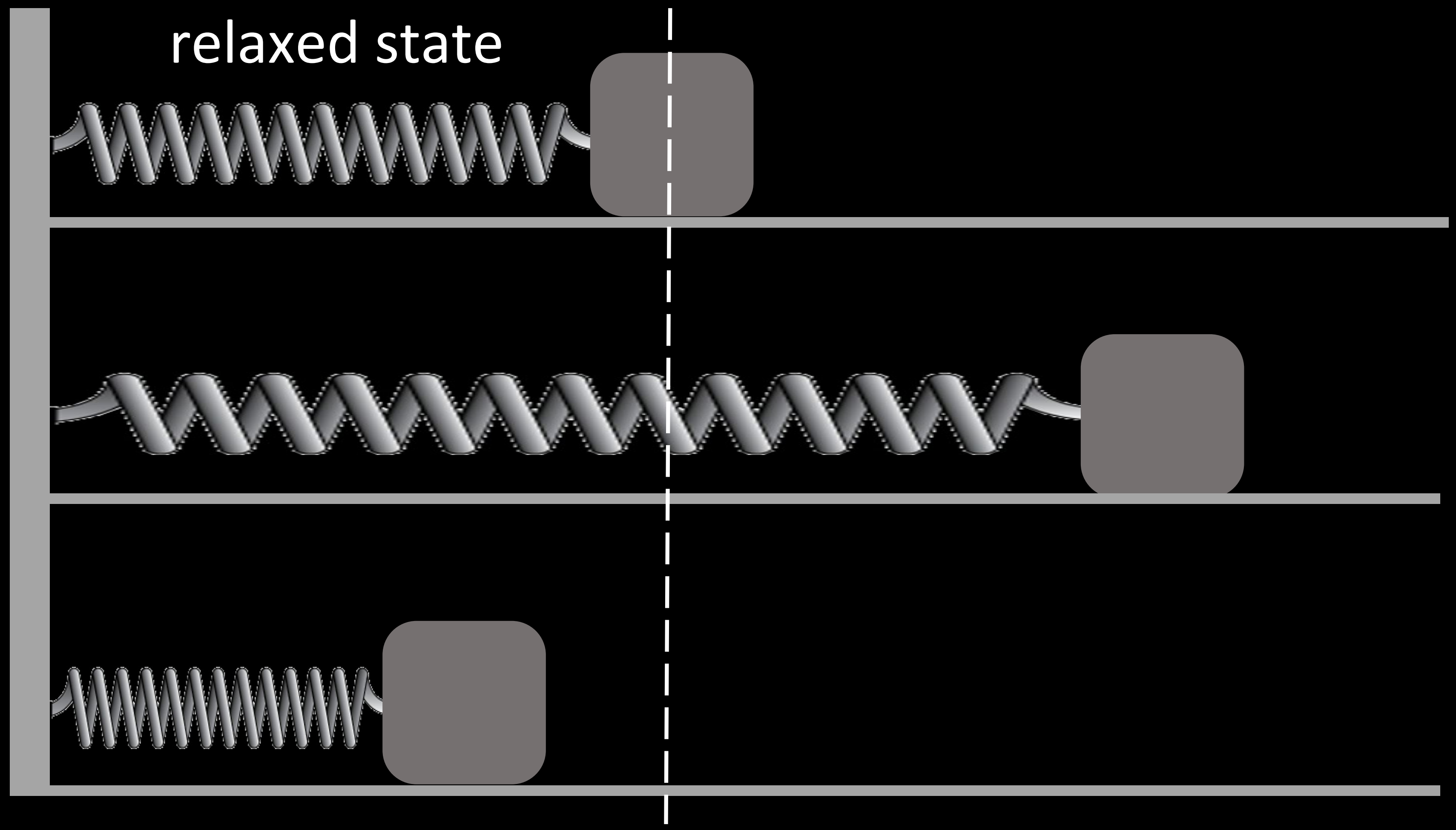
For the 3 situations below, where applicable, draw vector arrows to show the force on the mass from the spring.



Explain how you chose the direction, and relative sizes of your force vectors.

Hooke's Law: physics of springs

Now draw the displacement vectors for each mass.



Hooke’s Law: physics of springs

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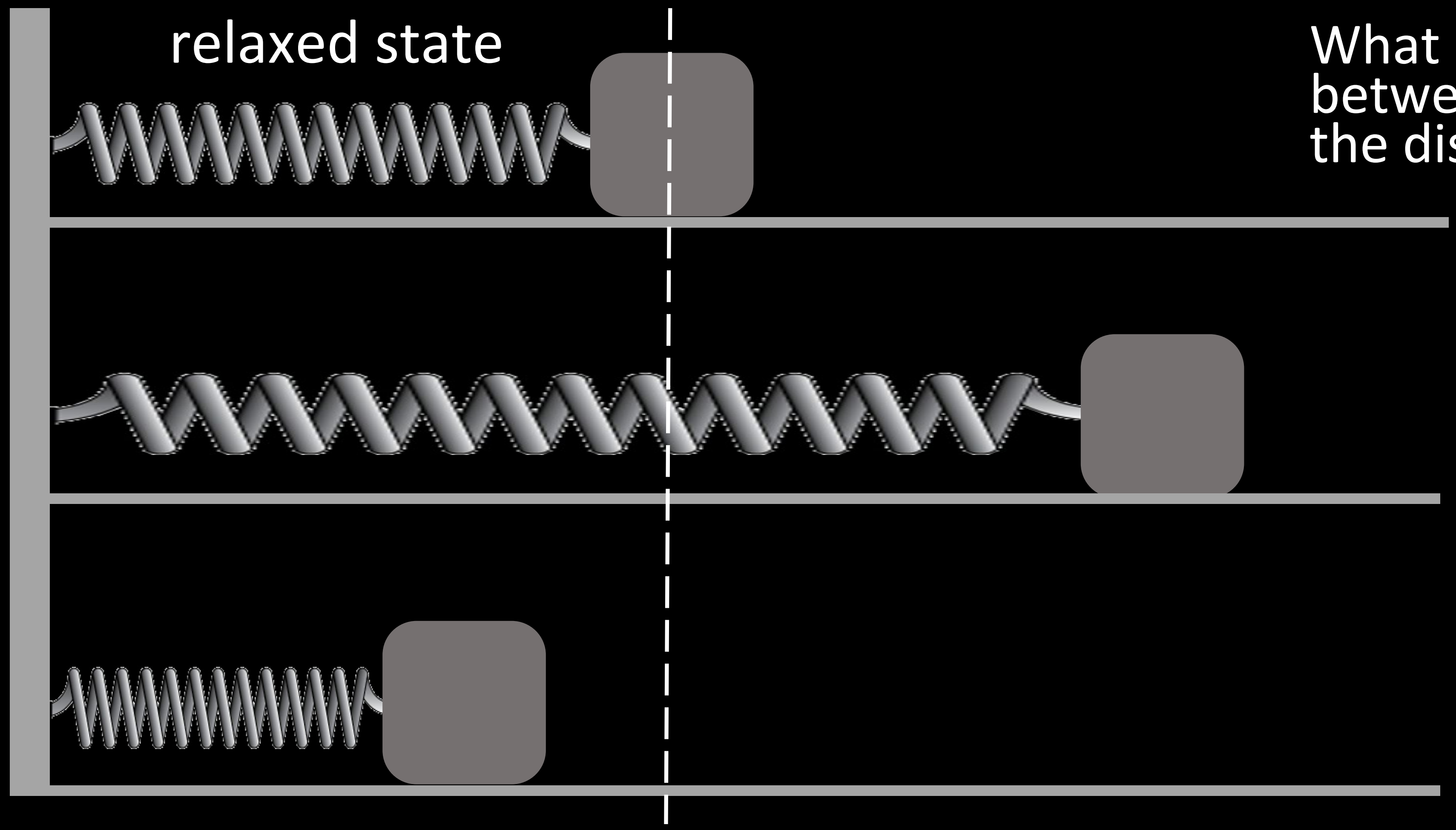
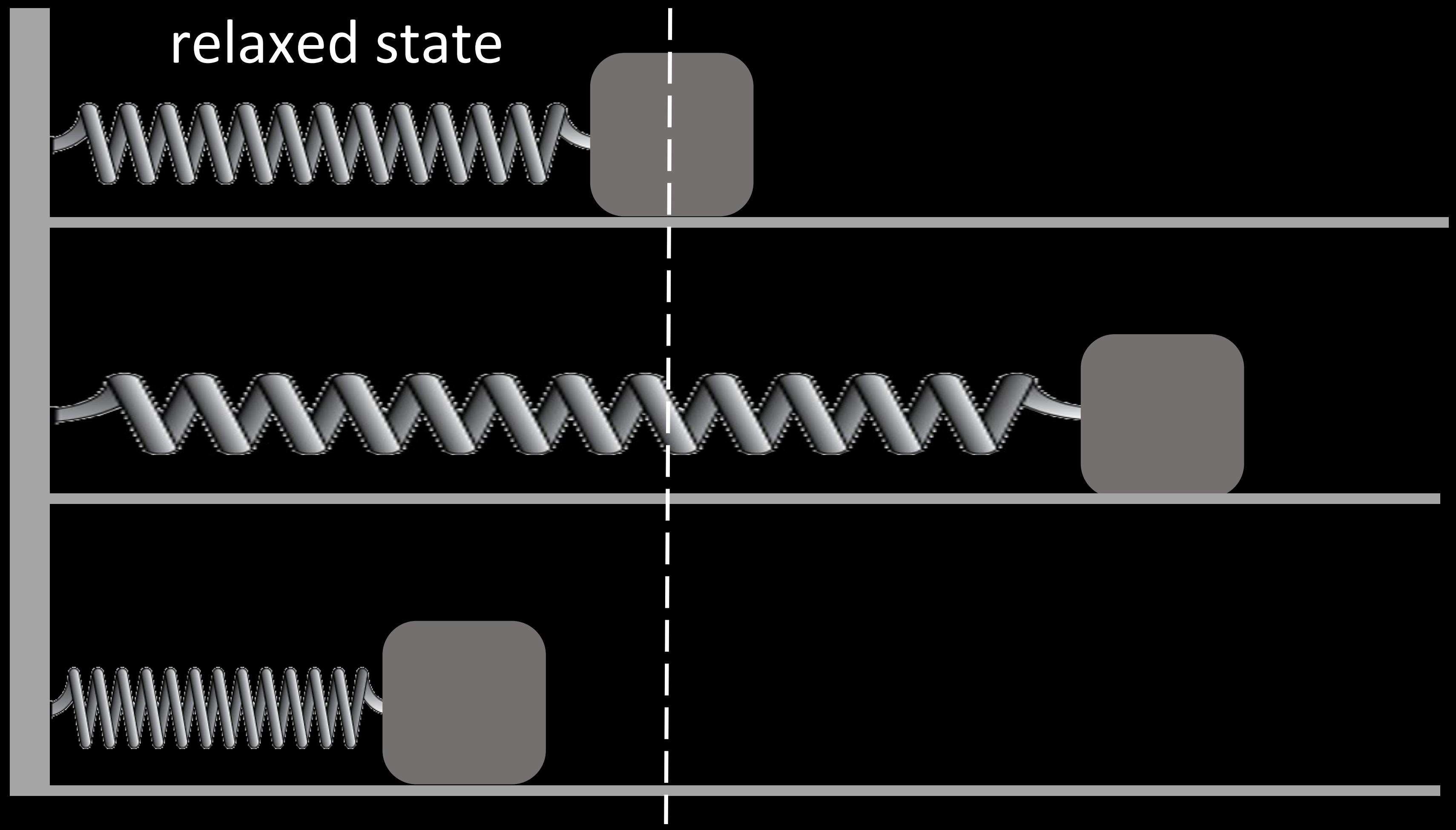


What is the relationship between the force vector and the displacement vector?

Hooke's Law: physics of springs

Now draw the displacement vectors for each mass.



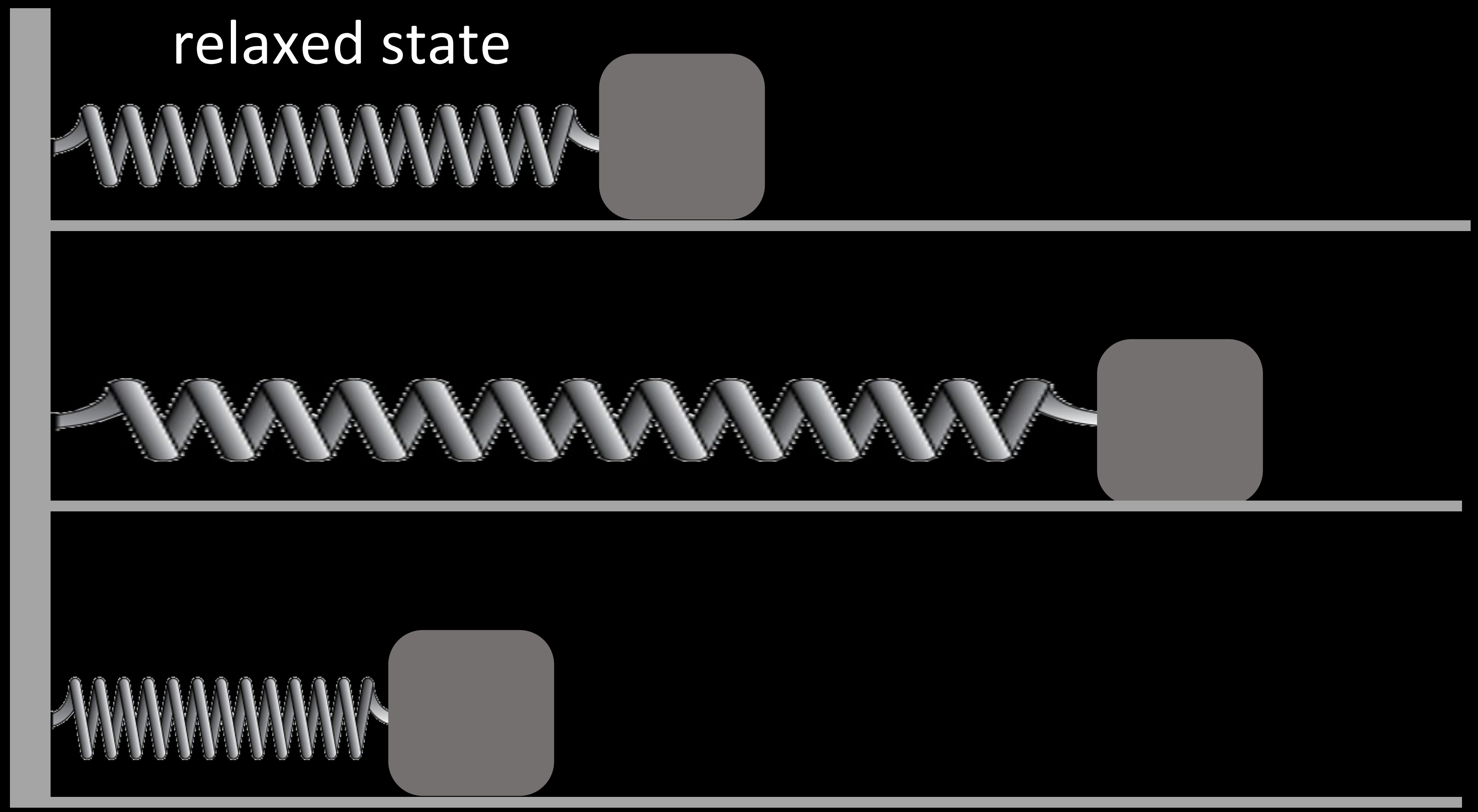
Examine a mass on a spring on the workbench.

Set the system into motion and describe what you see.

Hooke’s Law: physics of springs

Enable the spring force vector arrows for the mass.

Move the mass and check to see if the force arrows you drew match the simulation.



Hooke's Law: physics of springs

Create a graph of force magnitude v. displacement magnitude.

Move the mass along the horizontal direction and take a few data points.

Once you have a few data points, describe a mathematical relationship for Force F as a function of displacement x. F(x)= ?

Hooke's Law: physics of springs

$F(x) = -kx$

This equation is linear, taking the form, $y=mx+b$

with k as the slope and an intercept of $b=0$

$F(x) = -kx$

k = spring constant. What are the units on k?

A. kg/m
B. N/m2
C. N/m
D. Nm
E. m/N

$F(x) = -kx$

k = spring constant. What are the units on k?

Quick Discussion:

In your own words- what property of the spring does k describe?

$F(x) = -kx$

A spring is measured to have a very large spring constant, $k = 10{,}000$ N/m. Which statement best describes this spring?

A. the spring is easy to stretch like a slinky toy
B. the spring could be used like a bungee cable
C. the spring is very stiff and hard to compress
D. the spring is flexible like a noodle

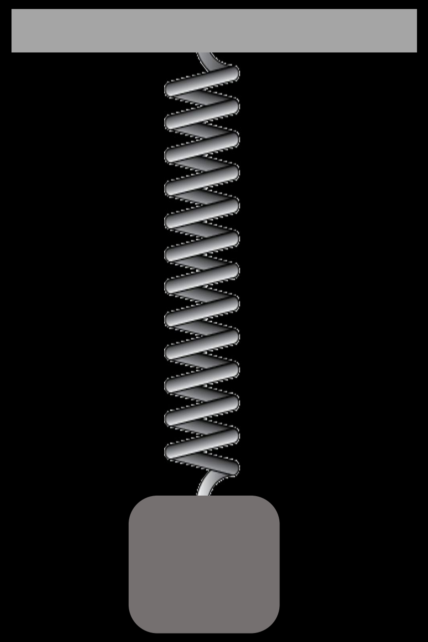
a vector relationship

$$\vec{F}(x) = -k\vec{x}$$

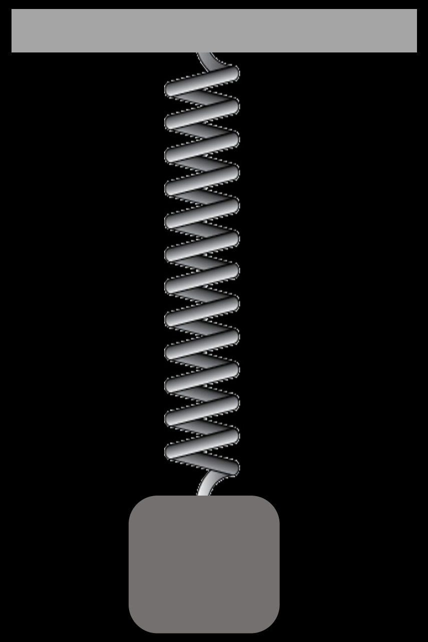
The full description of Hooke’s Law is a vector equation.

What does the minus sign tell us about the relationship between the spring force and the displacement of the mass?

Part II: springs and simple harmonic motion



Part II: springs and simple harmonic motion



Set a mass on a spring into motion. What are the patterns of movement?

What quantities could describe the main features of this motion?

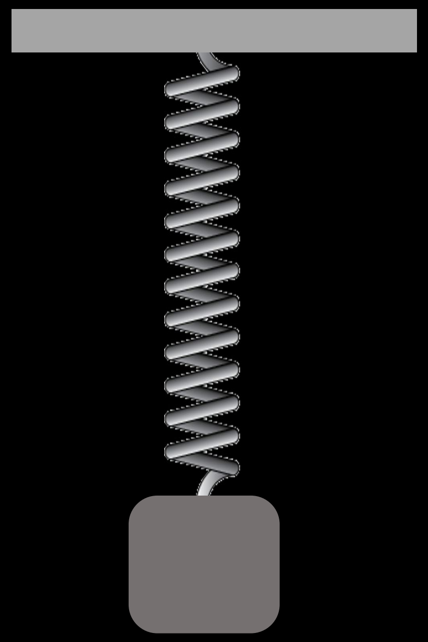
Make a list of these most important quantities, and describe how one would measure each.

"birds eye" view of spring-mass system on a table top

springs and simple harmonic motion

Apply Newton's second law to this system:

$$\sum F = F_s = ma$$



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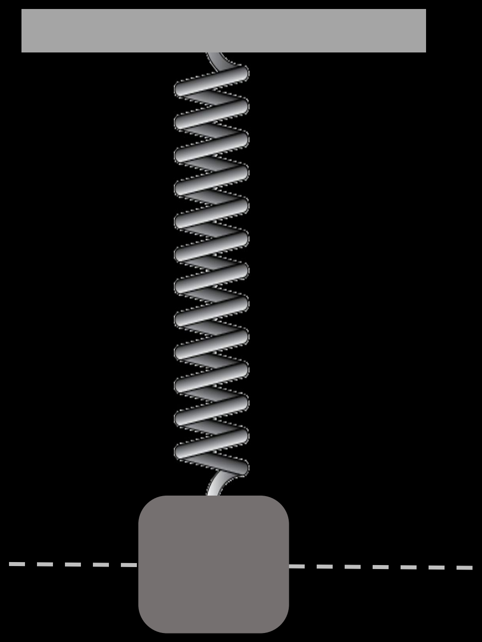

springs and simple harmonic motion

Apply Newton's second law to this system:

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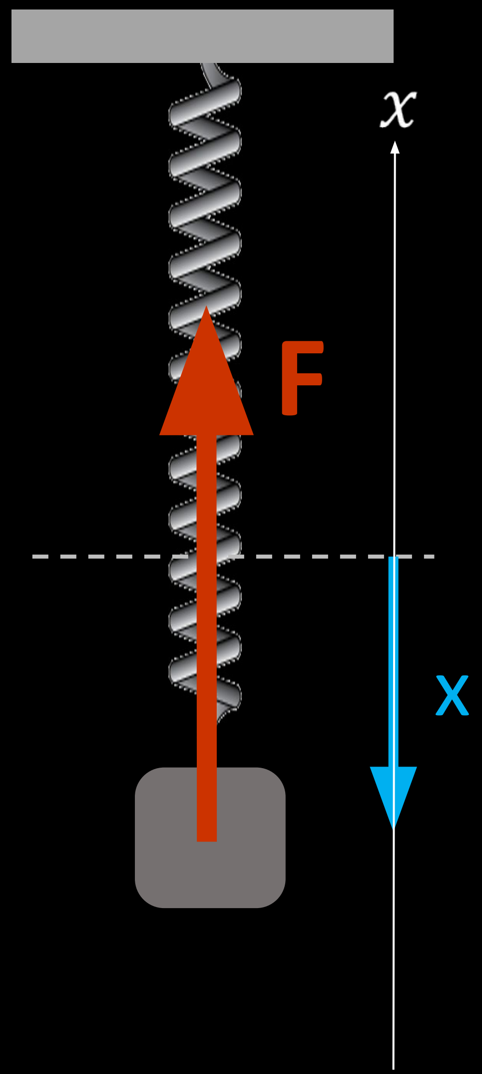

Concept review:

Which is a correct statement concerning acceleration?

A. $a = \frac{dx}{dt}$

B. $a = \frac{d^2x}{dt^2}$

C. $a = \frac{d^2v}{dt^2}$

D. $a = \frac{d^2x}{dy^2}$

springs and simple harmonic motion

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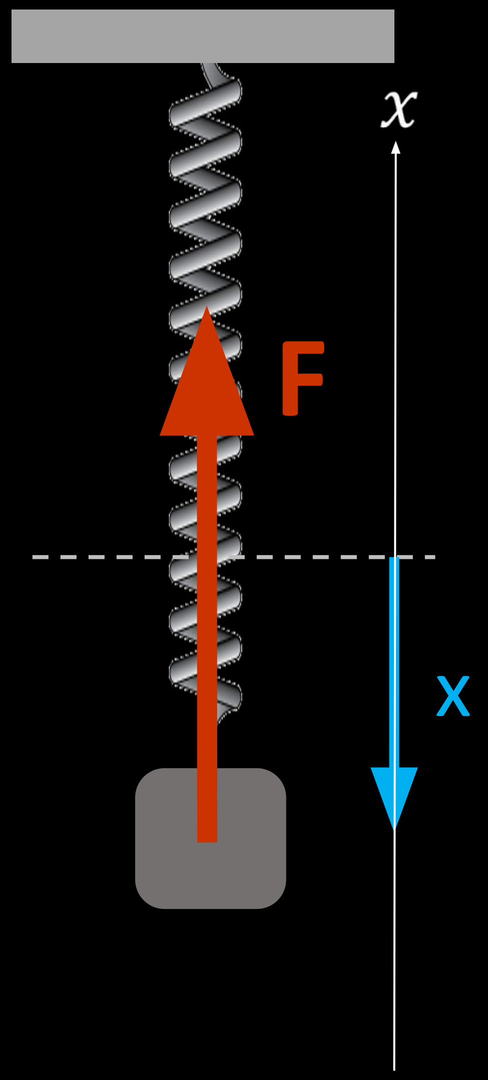
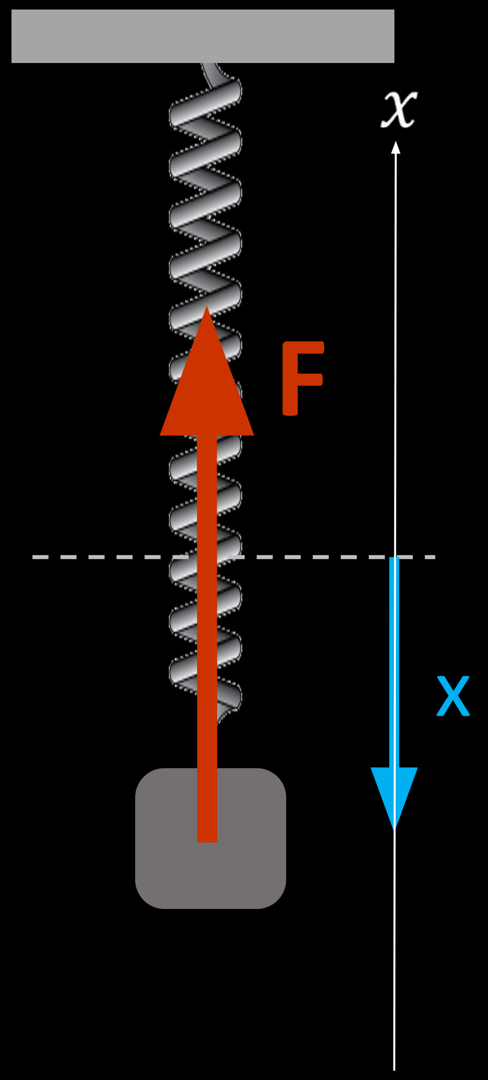


springs and simple harmonic motion

Apply Newton's second law to this system:

$$\sum F = F_s = ma \qquad F_s = -kx$$

$$-kx = m\frac{d^2x}{dt^2}$$



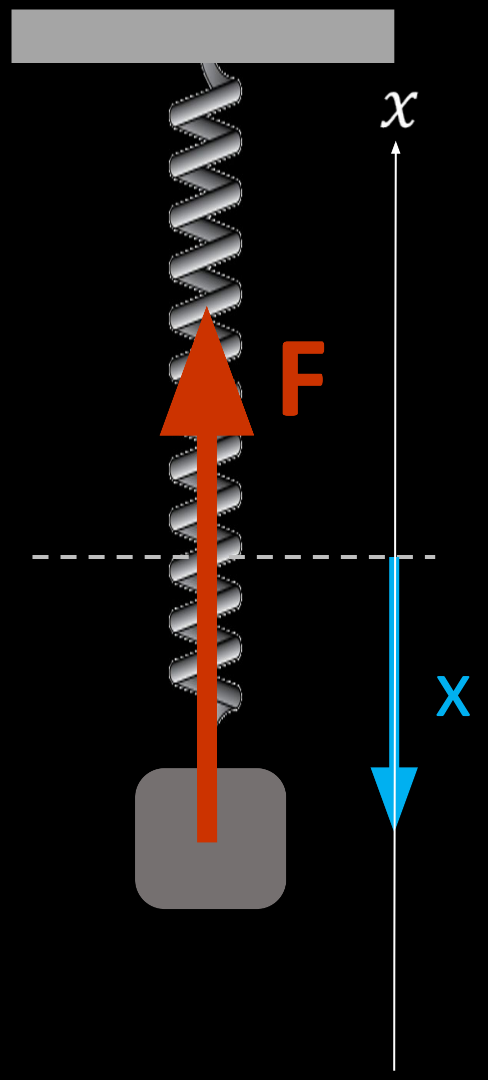
springs and simple harmonic motion

Apply Newton's second law to this system:

$$\sum F = F_s = ma \qquad F_s = -kx$$

$$-kx = m\frac{d^2x}{dt^2}$$

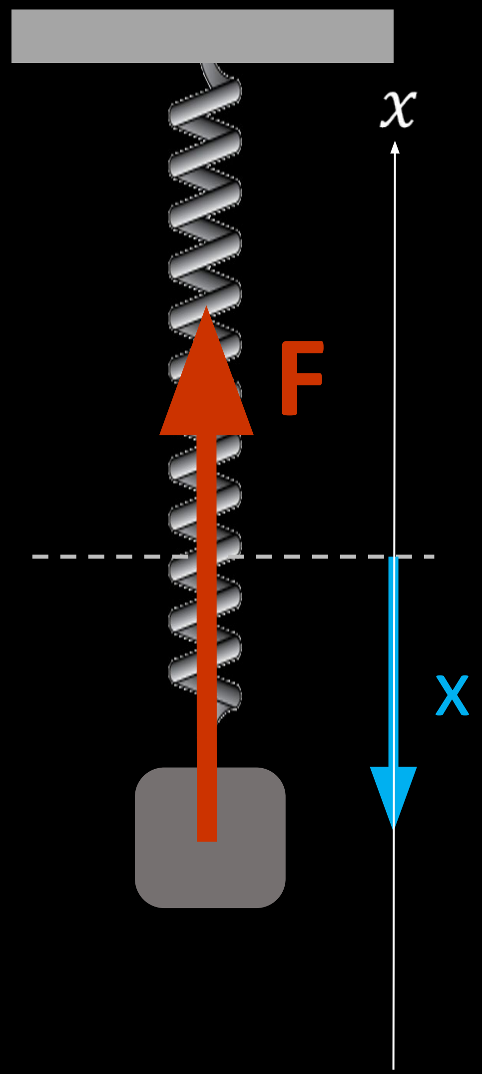
$$\Rightarrow \frac{d^2x}{dt^2} = -\frac{k}{m}x$$



simple harmonic motion

Newton's second law + Hooke's Law:

$$\frac{d^2\mathrm{x}}{dt^2} = -\frac{\mathrm{k}}{\mathrm{m}}\mathrm{x}$$

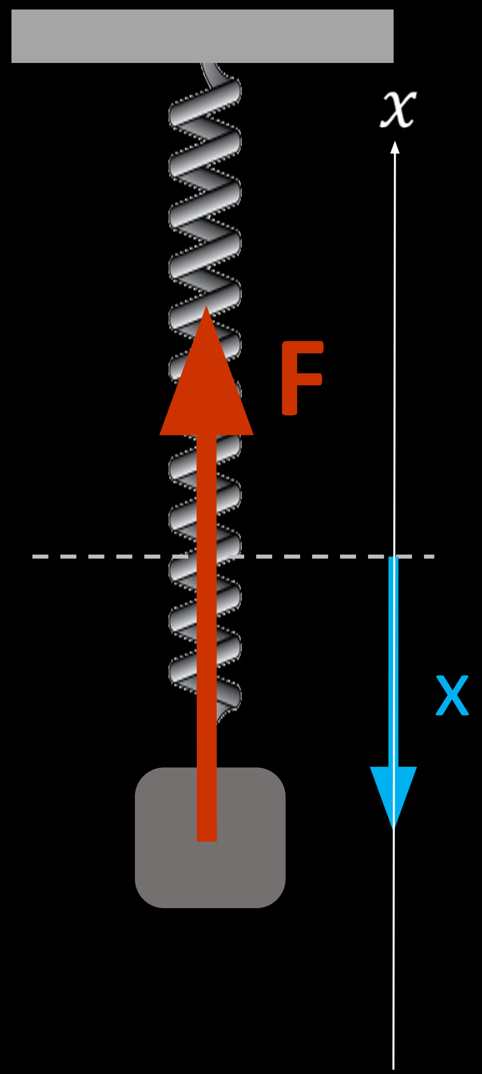


simple harmonic motion

Newton's second law + Hooke's Law:

$$\frac{d^2 x(t)}{dt^2} = -\frac{k}{m}x(t)$$

$x(t) = ?$



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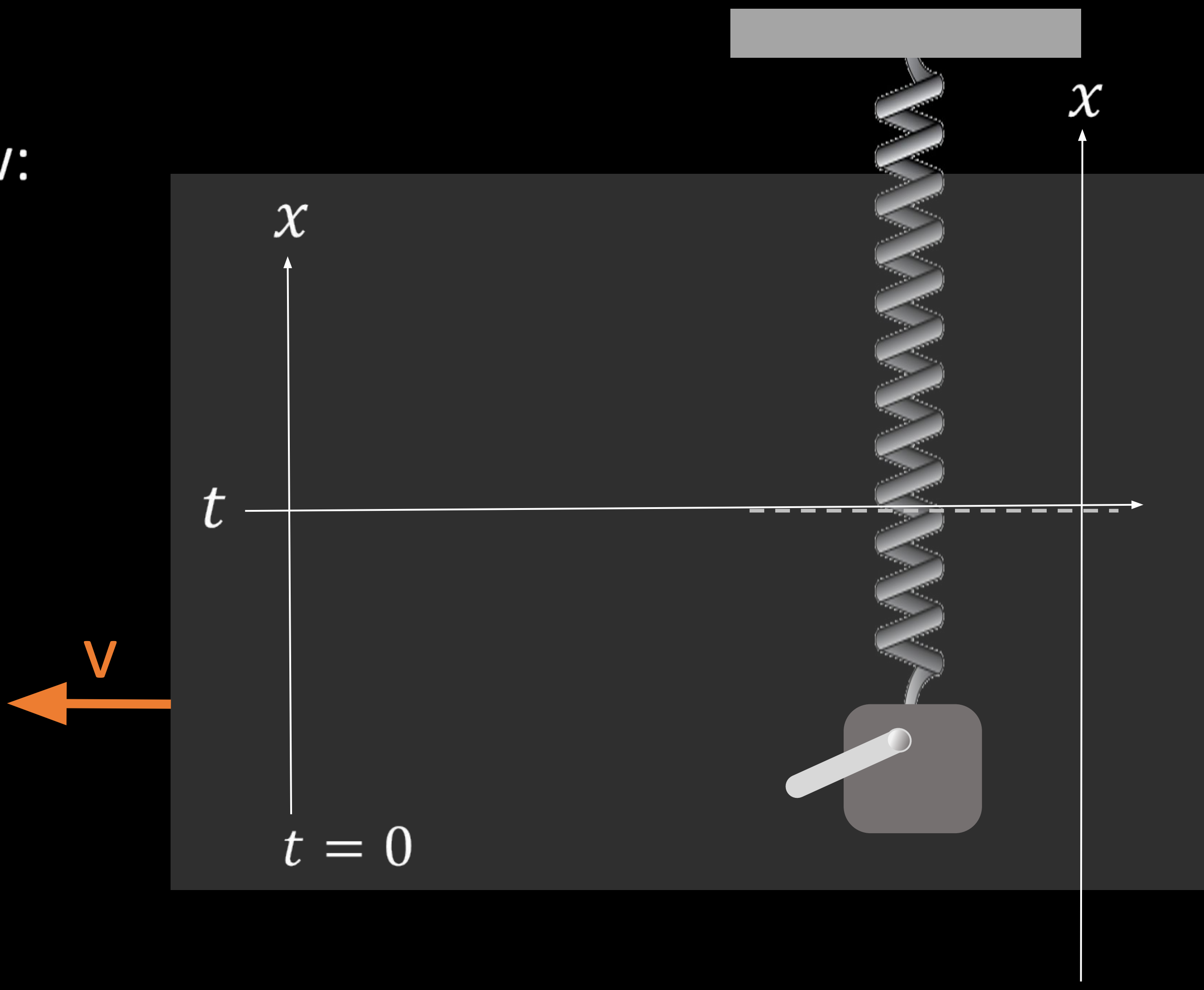

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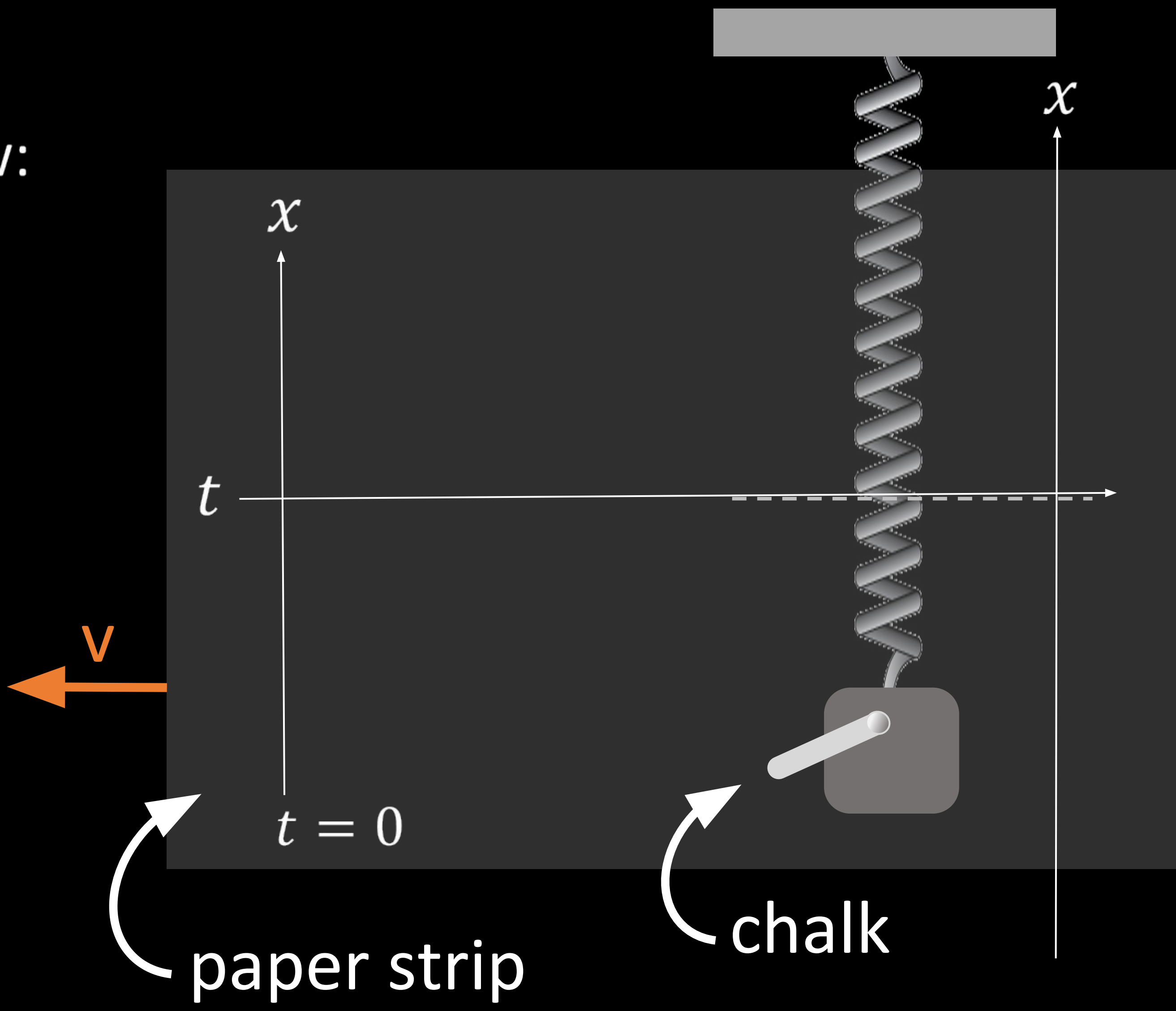
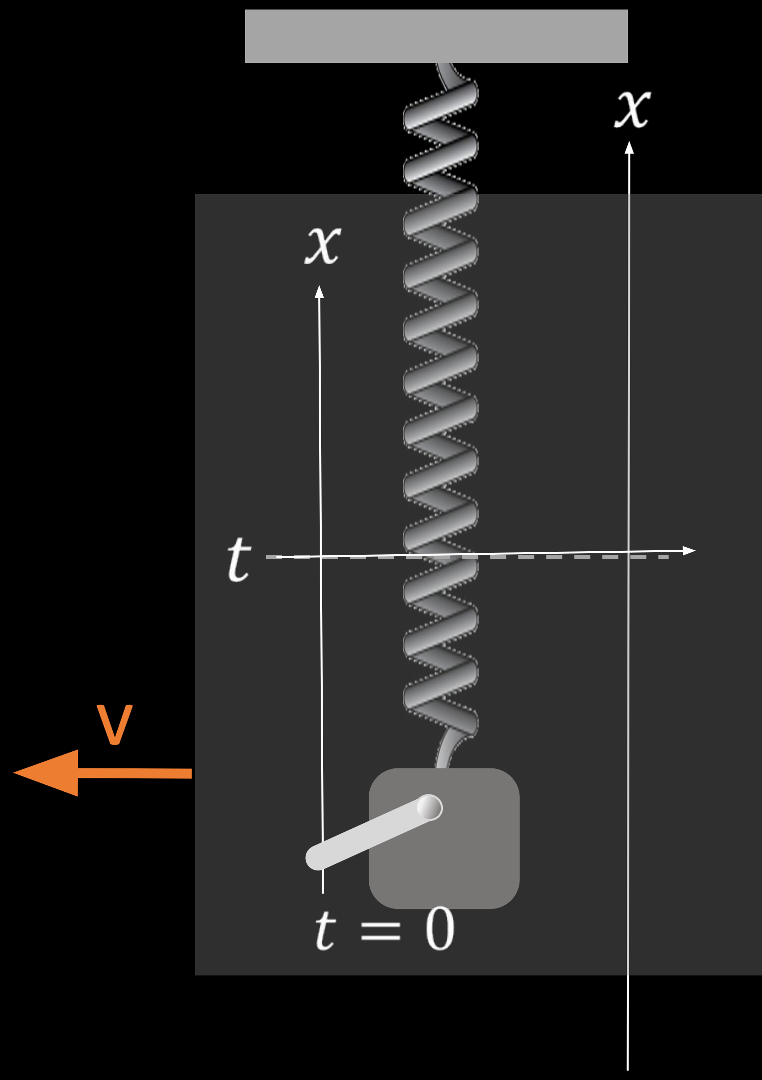
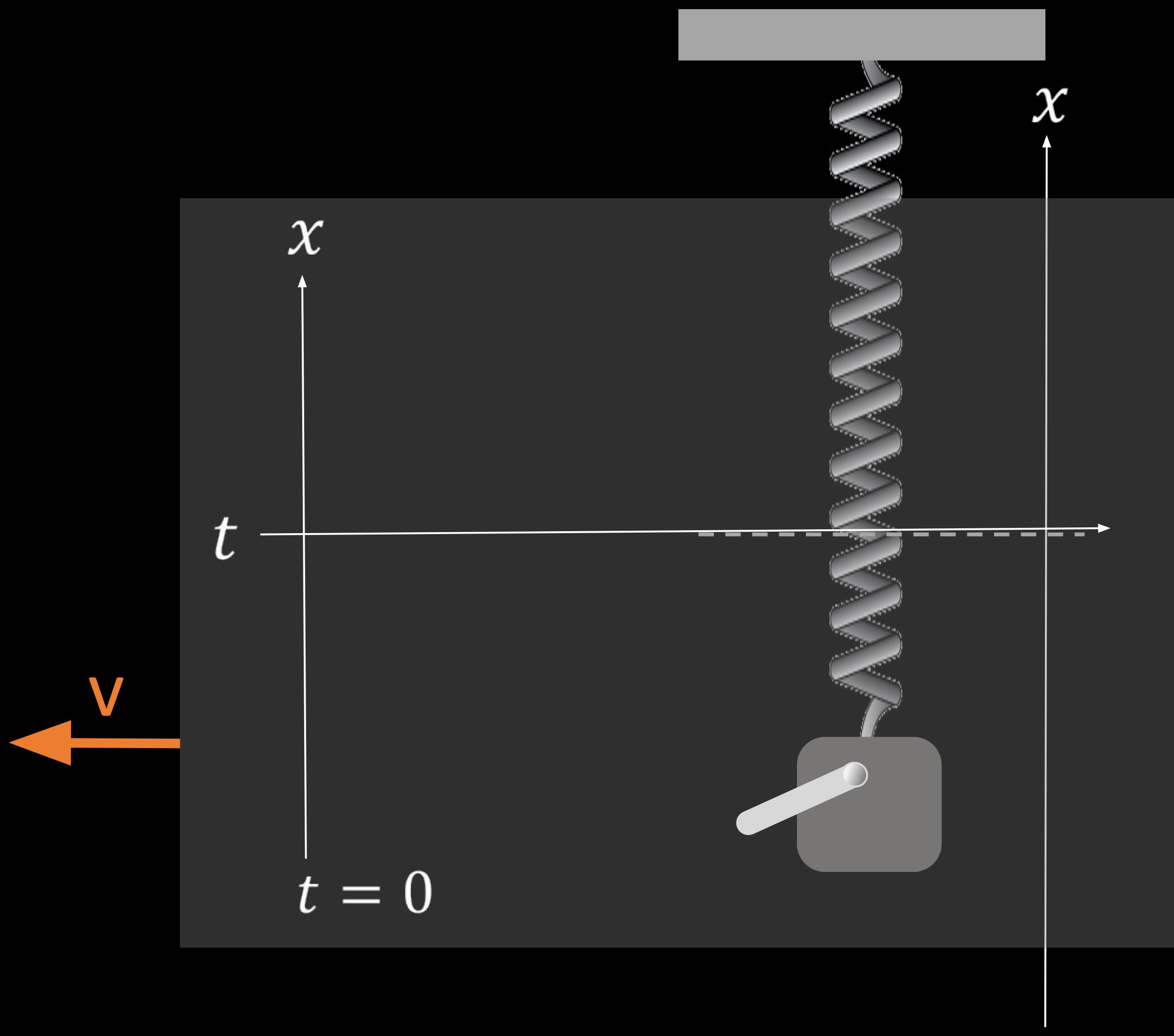


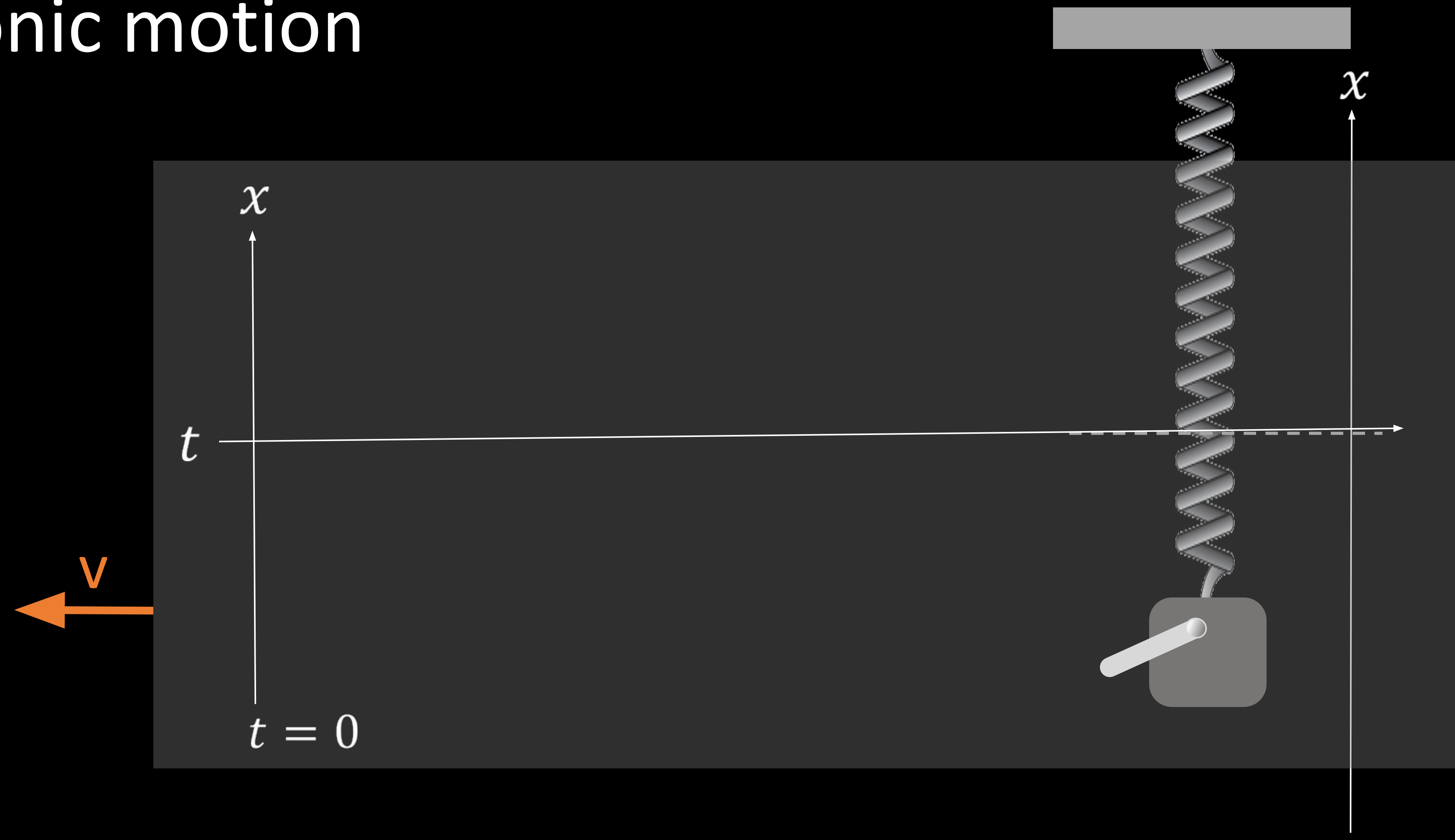
simple harmonic motion



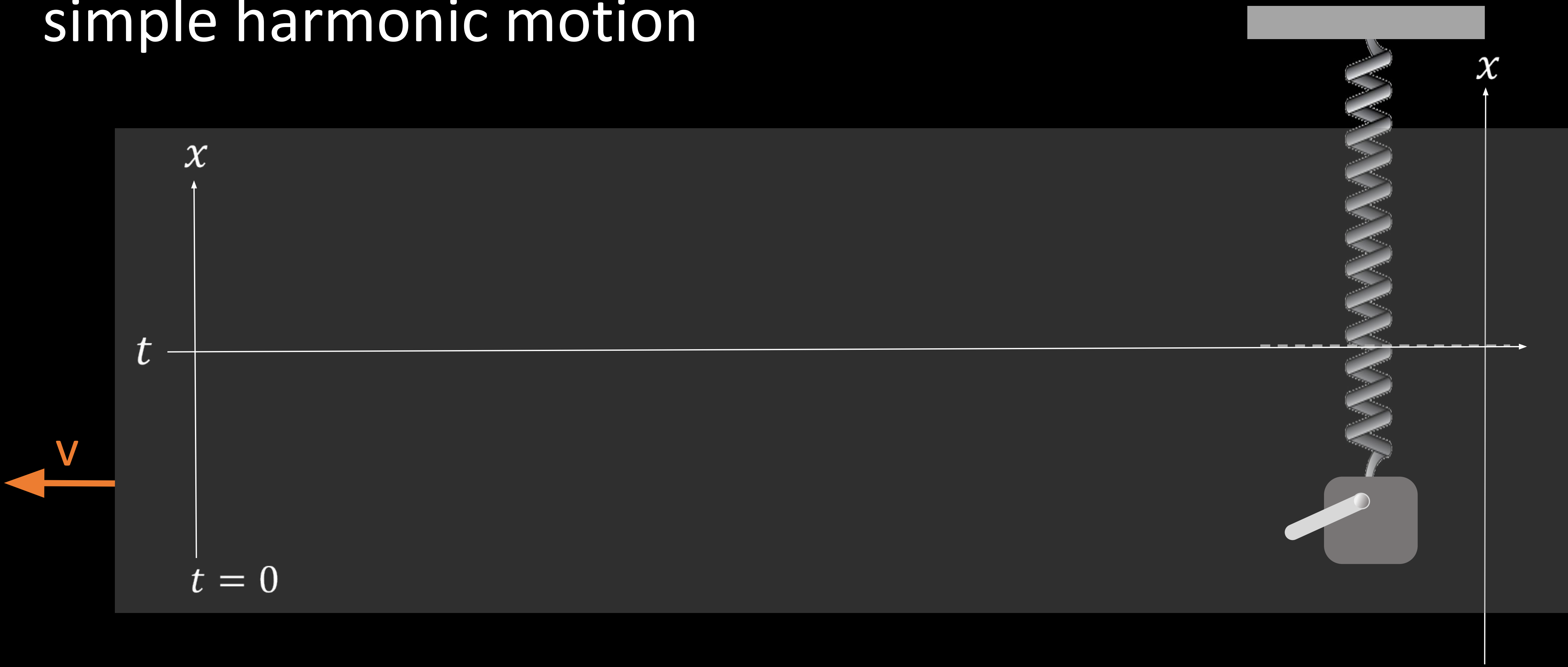
simple harmonic motion



simple harmonic motion



simple harmonic motion



$x(t)$=?   What would the curve traced by the chalk look like?

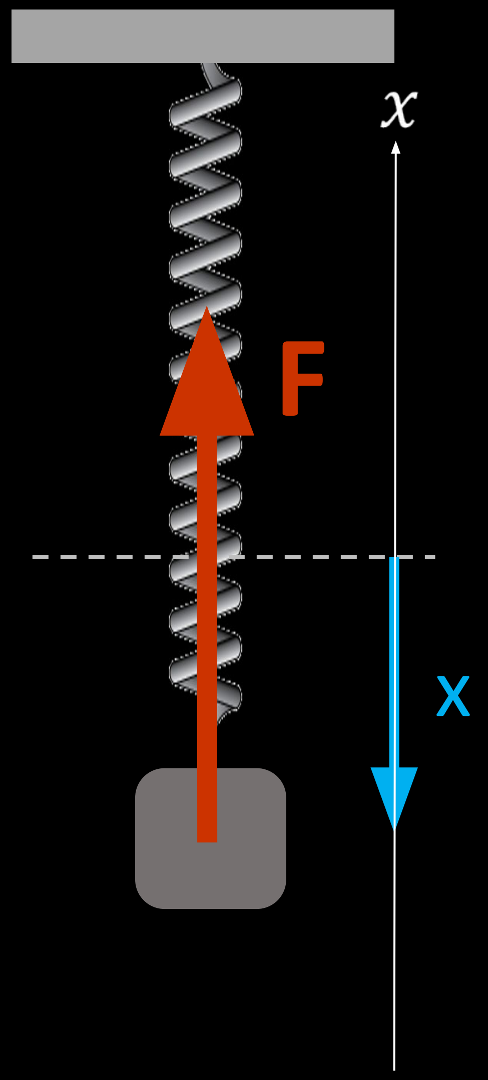
springs and simple harmonic motion

Apply Newton's second law to this system:

$$\sum F = F_s = ma \qquad F_s = -kx$$

$$-kx = m\frac{d^2x}{dt^2}$$

$$\Rightarrow \quad \frac{d^2x}{dt^2} = -\frac{k}{m}x$$



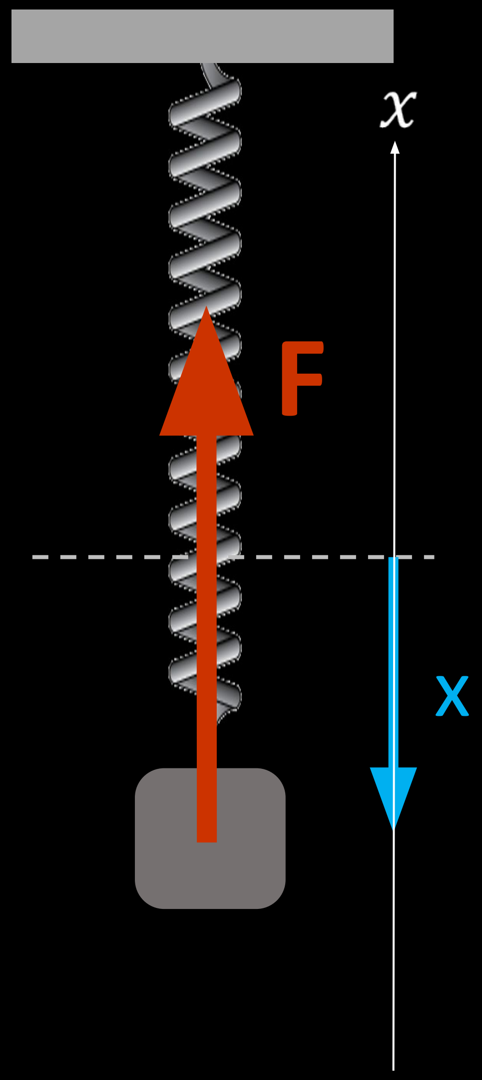
simple harmonic motion

Newton’s second law + Hooke’s Law:

$$\frac{d^2x(t)}{dt^2} = -\frac{k}{m}x(t)$$

$$x(t) = A\cos(\omega t)$$

$$\frac{dx(t)}{dt} = ?$$



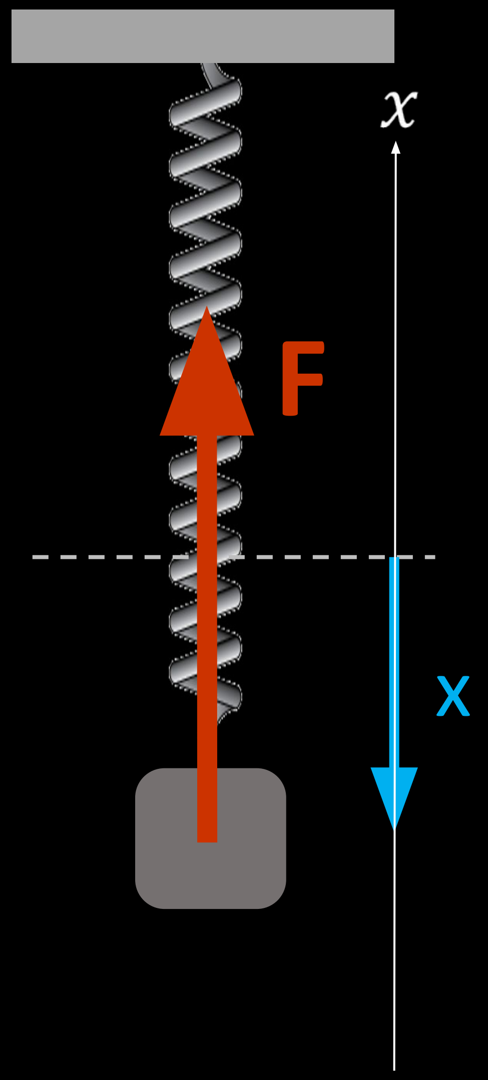
simple harmonic motion

Newton's second law + Hooke's Law:

$$\frac{d^2 x(t)}{dt^2} = -\frac{k}{m} x(t)$$

$$x(t) = A\cos(\omega t)$$

$$\frac{dx(t)}{dt} = \omega A \sin(\omega t)$$



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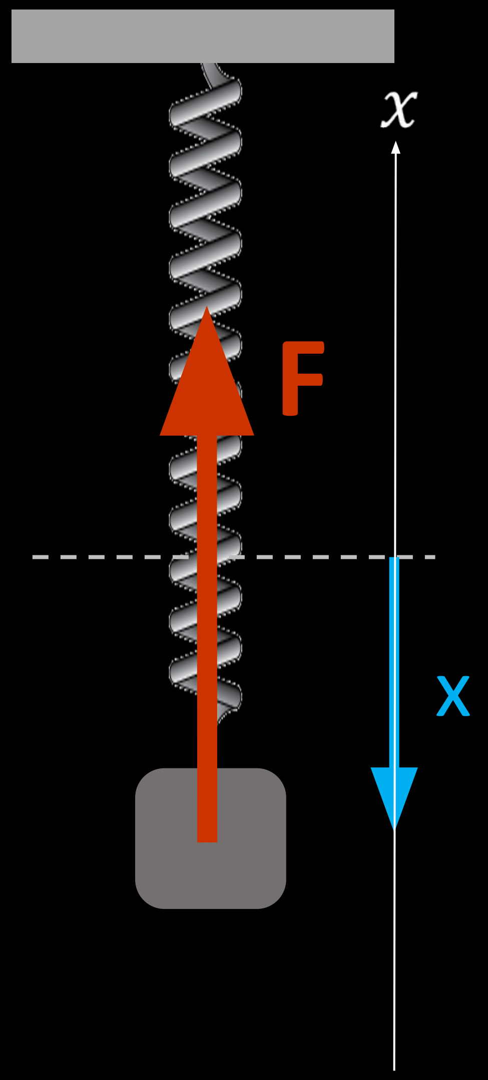

simple harmonic motion

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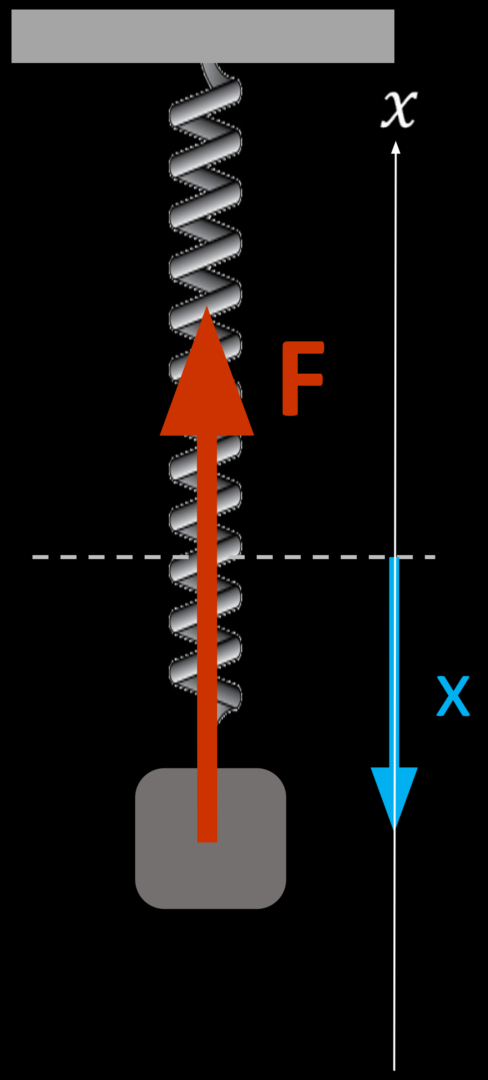


simple harmonic motion

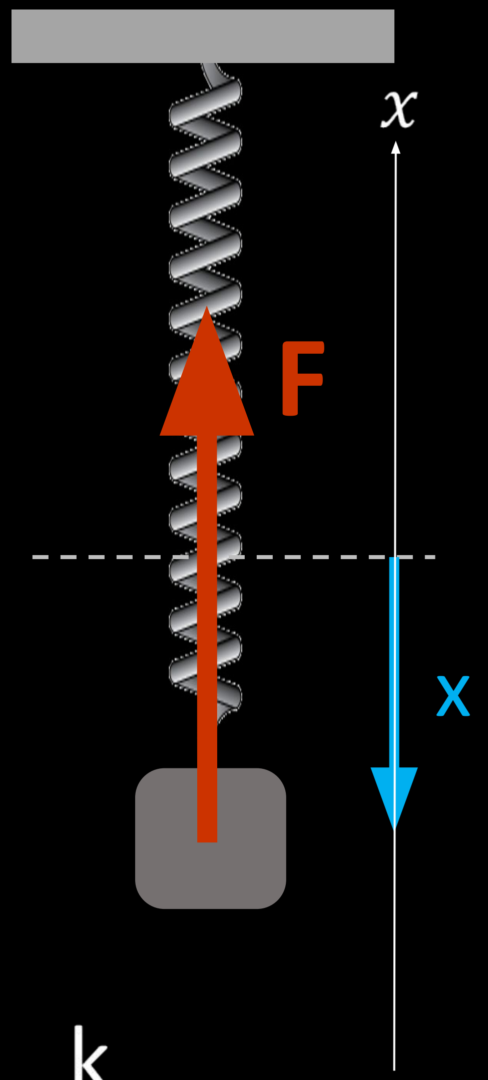
Newton's second law + Hooke's Law:

$$\frac{d^2x(t)}{dt^2} = -\frac{k}{m}x(t)$$

$$x(t) = A\cos(\omega t)$$

$$\frac{dx(t)}{dt} = \omega A\sin(\omega t)$$

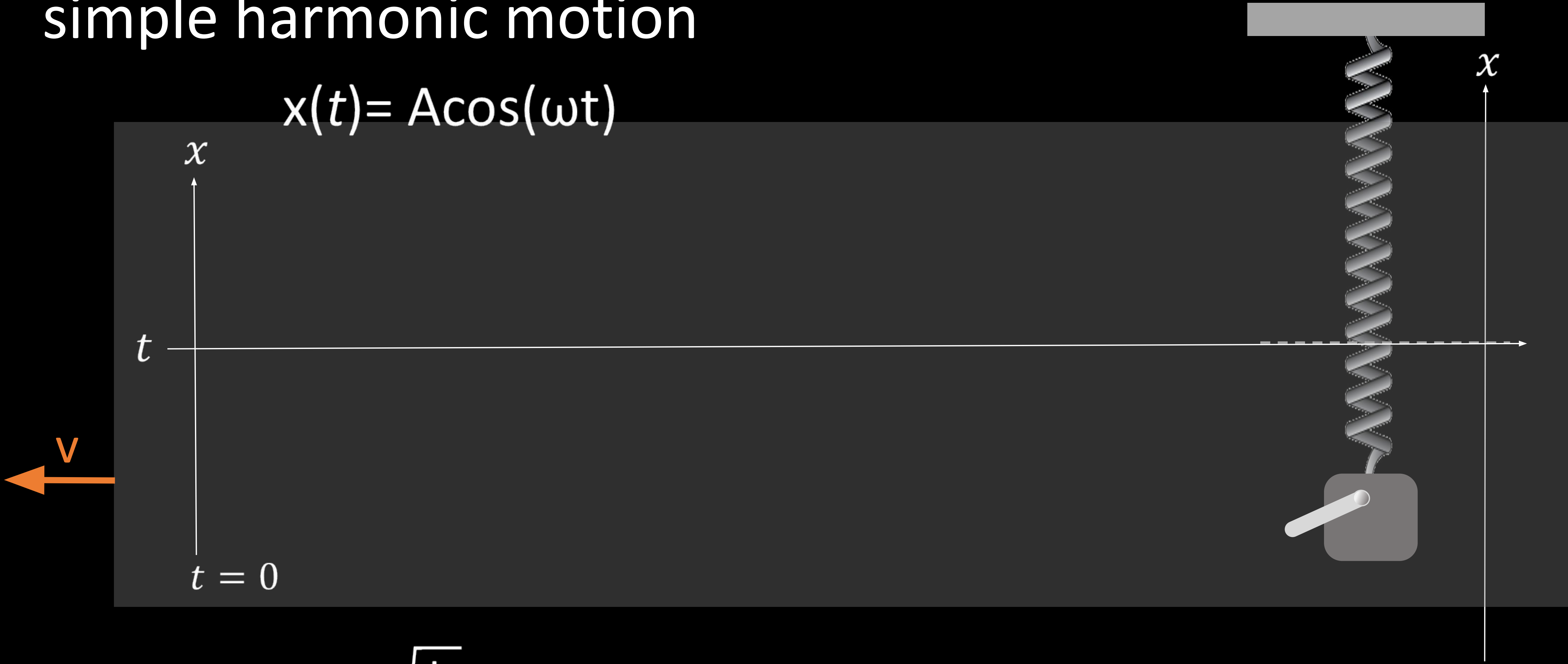
$$\frac{d^2x(t)}{dt^2} = -\omega^2 A\cos(\omega t)$$



$$\Rightarrow -\omega^2 A\cos(\omega t) = -\frac{k}{m}A\cos(\omega t) \qquad \text{but only if:} \quad \omega^2 = \frac{k}{m}$$

simple harmonic motion

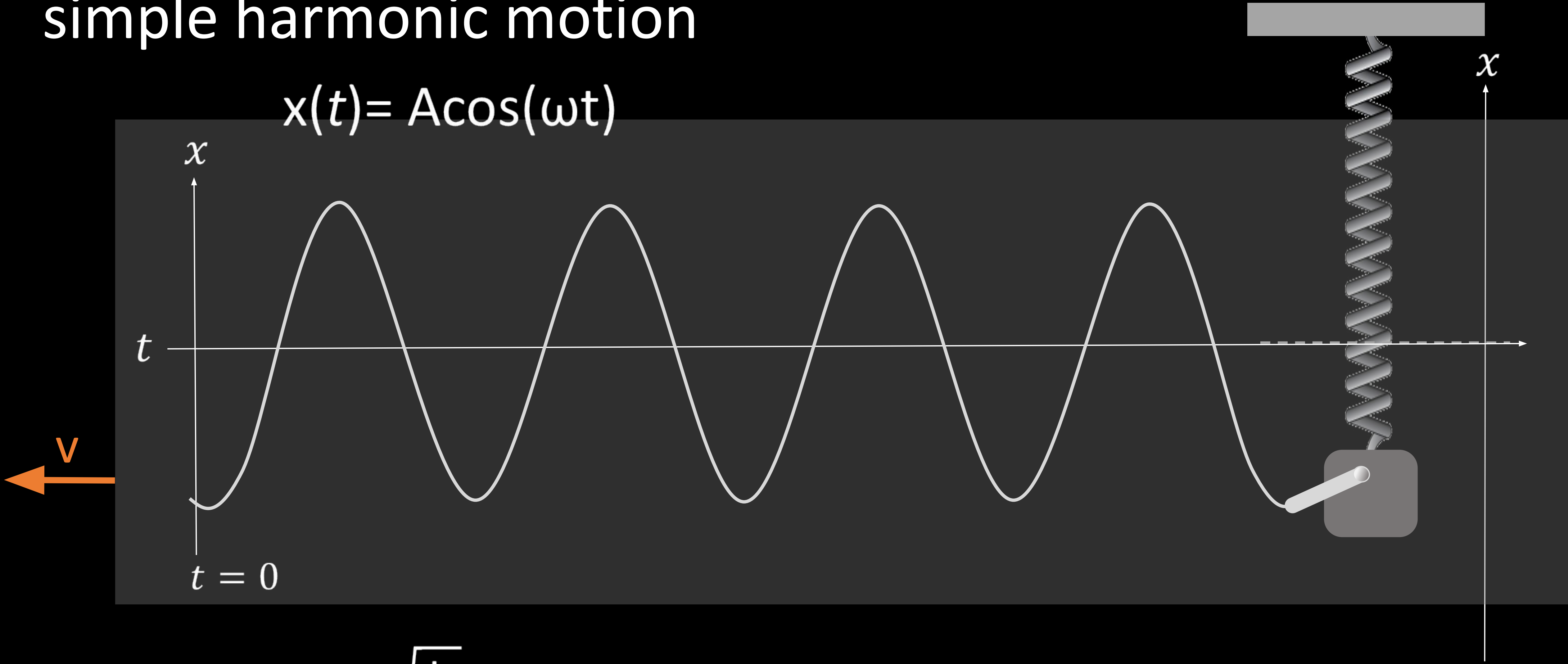
$$x(t) = A\cos(\omega t)$$



$$\omega = \sqrt{\frac{k}{m}}$$

simple harmonic motion

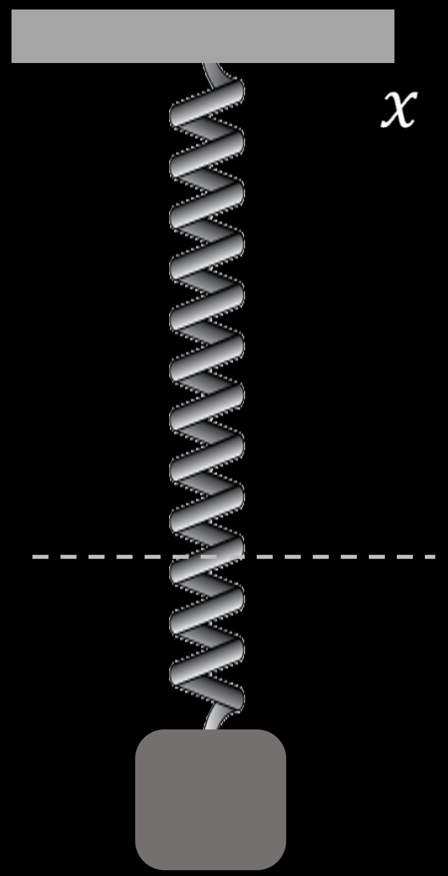
$x(t) = A\cos(\omega t)$



$$\omega = \sqrt{\frac{k}{m}}$$

simple harmonic motion

$$x(t) = A\cos(\omega t) \qquad \omega = \sqrt{\frac{k}{m}}$$

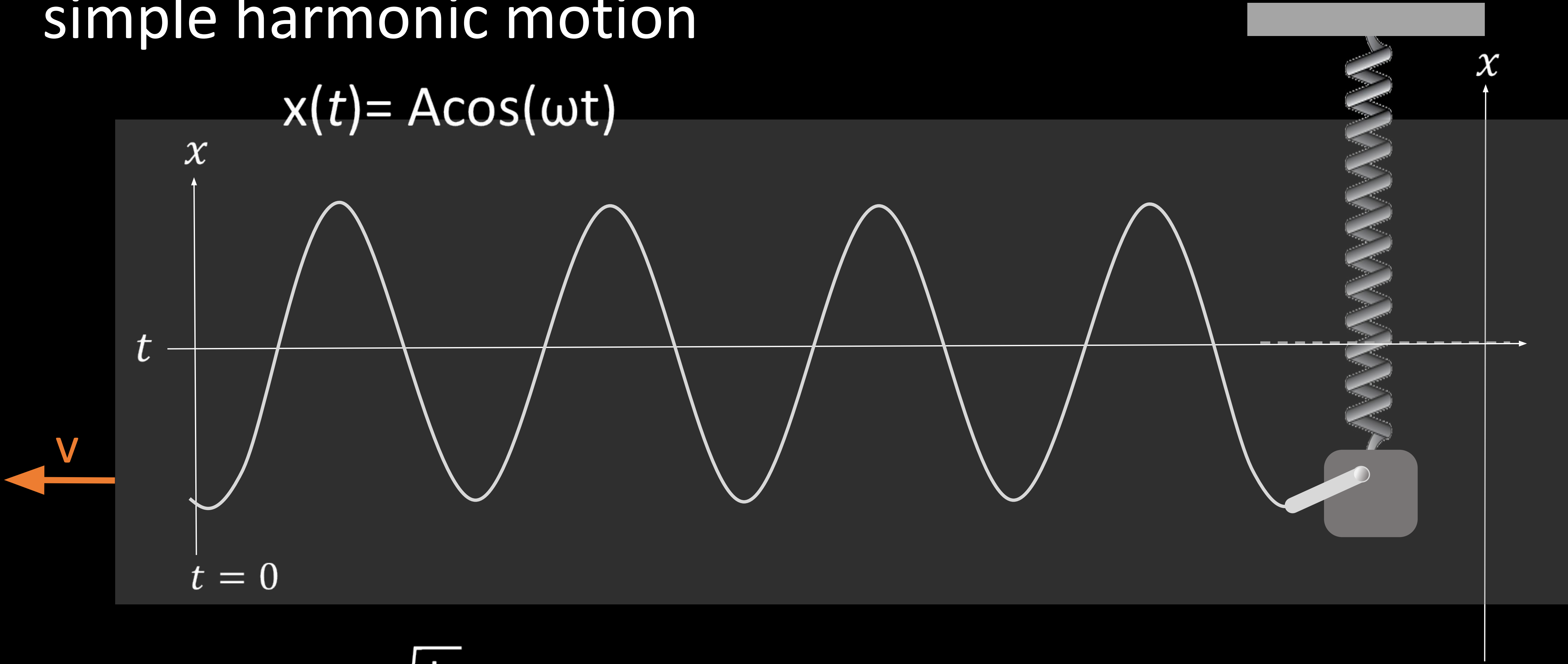


What are the units on ω?

A. m/s
B. Hz
C. s
D. rad/s
E. kg/s

simple harmonic motion

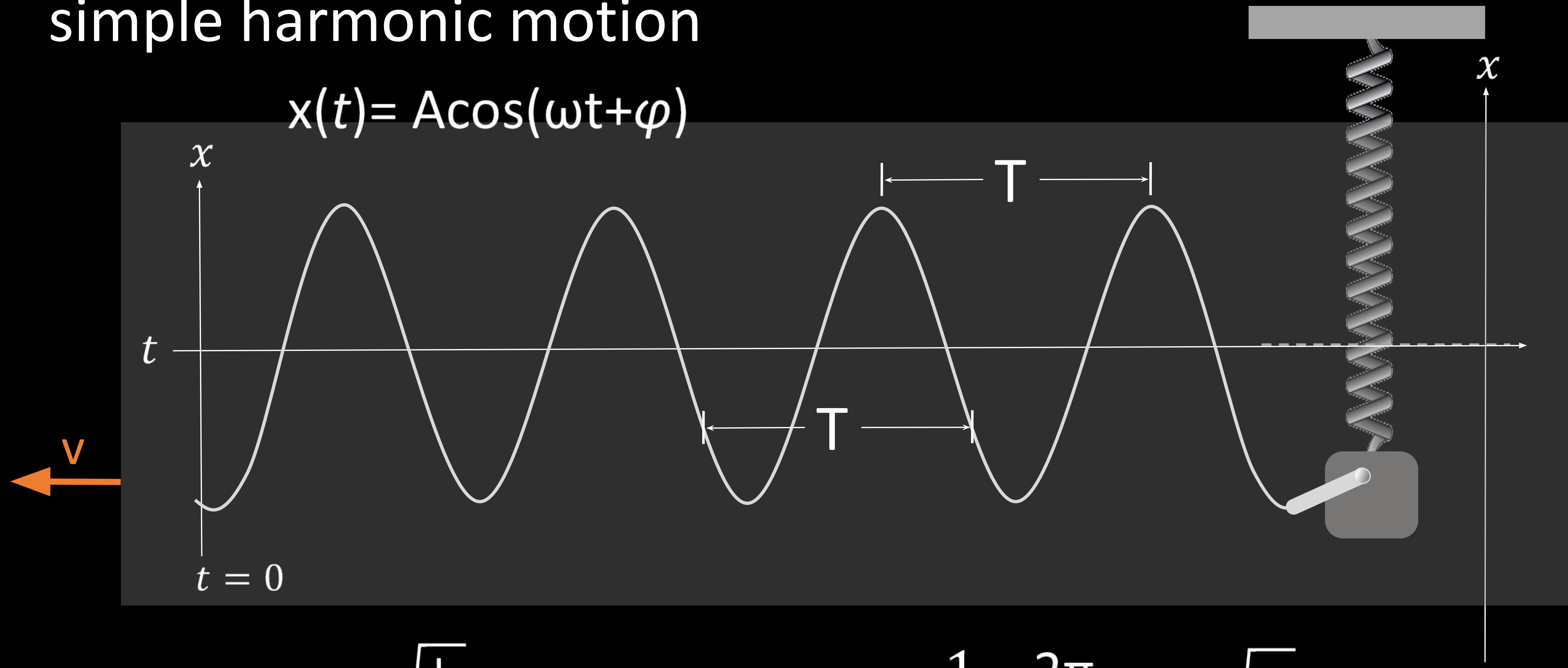
$$x(t) = A\cos(\omega t)$$



$$\omega = \sqrt{\frac{k}{m}}$$

simple harmonic motion

$$x(t) = A\cos(\omega t + \varphi)$$



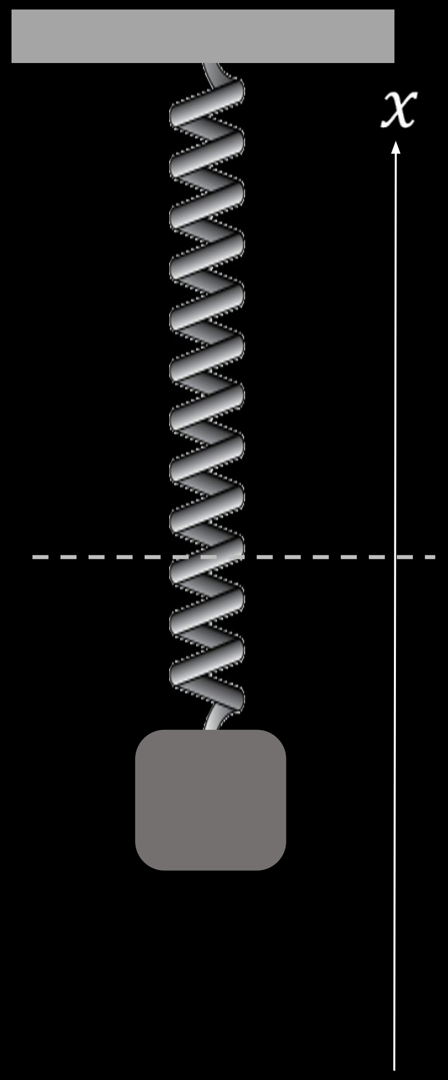
$$\omega = \sqrt{\frac{k}{m}} \qquad \omega = 2\pi f \qquad T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

simple harmonic motion

$$x(t) = A\cos(\omega t + \varphi)$$

Set a mass-spring system in to motion

⇨ How does the motion depend on the mass, m?



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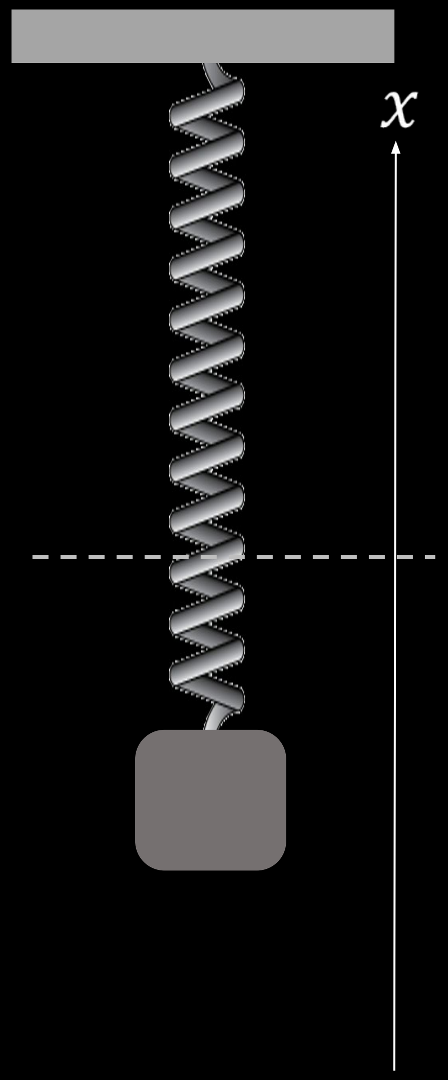
simple harmonic motion

$$x(t) = A\cos(\omega t + \varphi)$$

Set a mass-spring system in to motion

⇨ How does the motion depend on the mass, m?

⇨ How does the motion depend on the spring constant, k?



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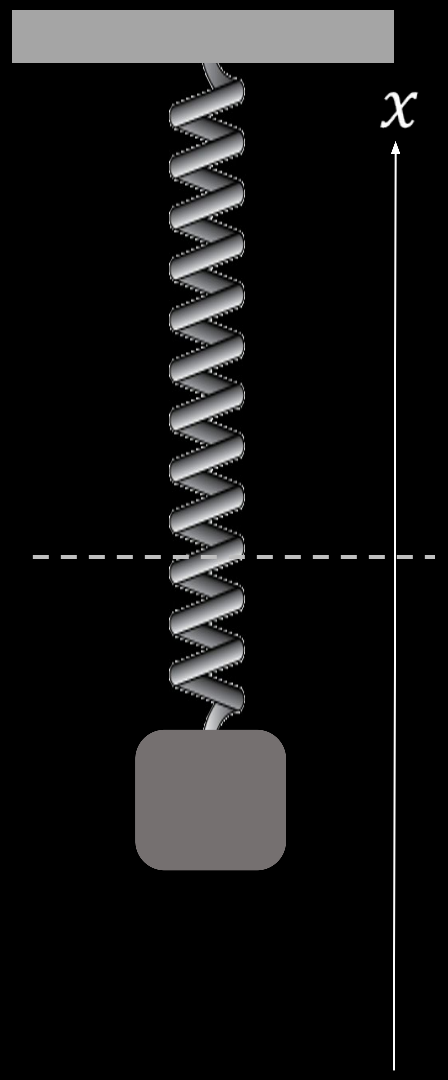
simple harmonic motion

$$x(t) = A\cos(\omega t + \varphi)$$



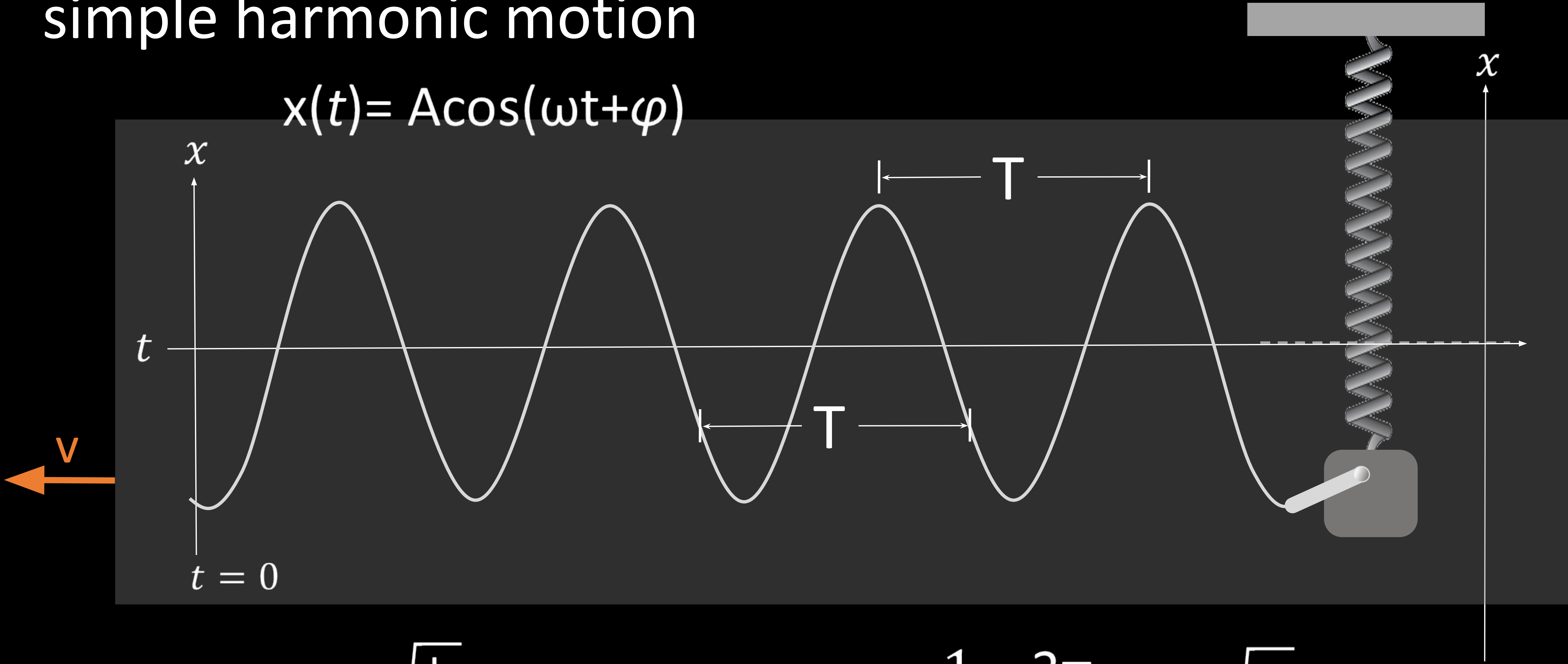
A system has a period T for a mass m. If the mass is made 4 times larger, what is the new period?

A. still T
B. 2T
C. 4T
D. ½ T

$$\omega = \sqrt{\frac{k}{m}} \qquad \omega = 2\pi f \qquad T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

simple harmonic motion

$$x(t) = A\cos(\omega t + \varphi)$$



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simple harmonic motion

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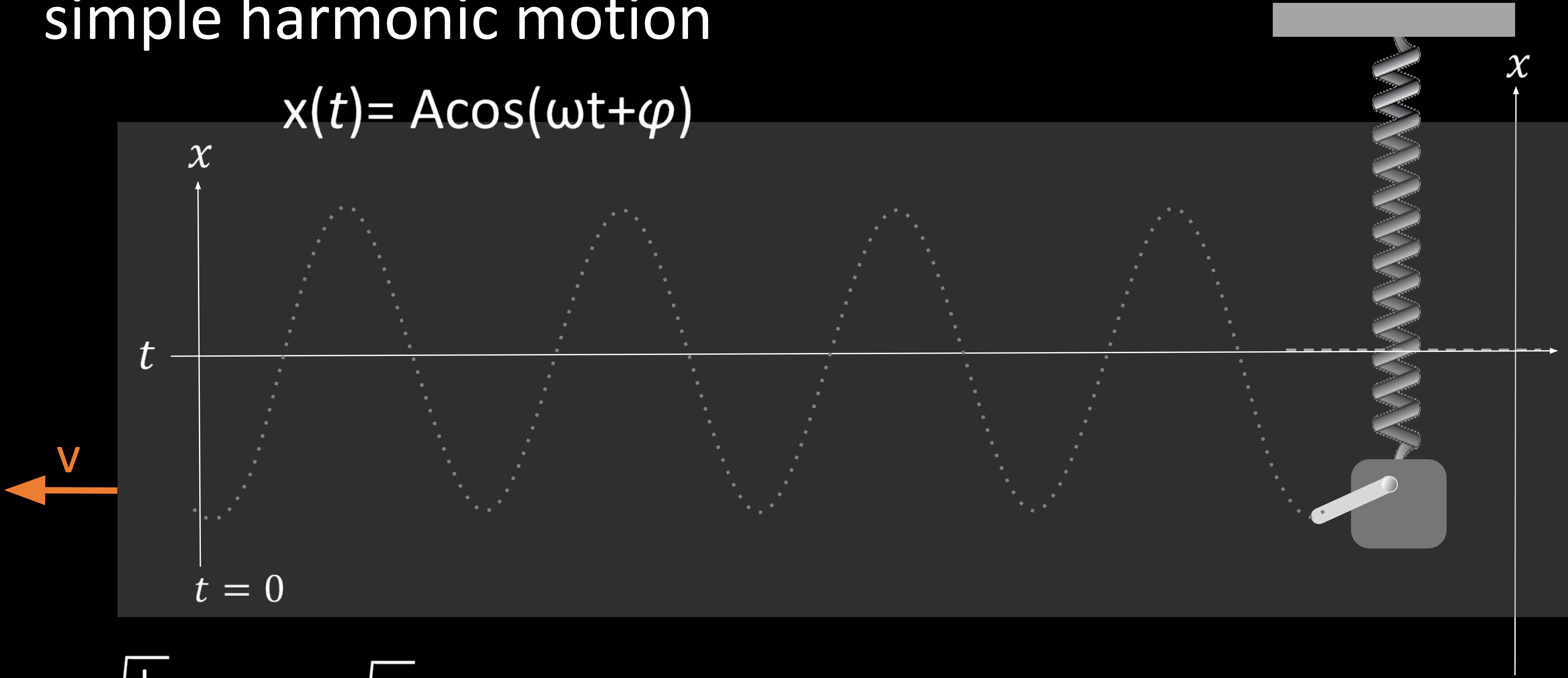
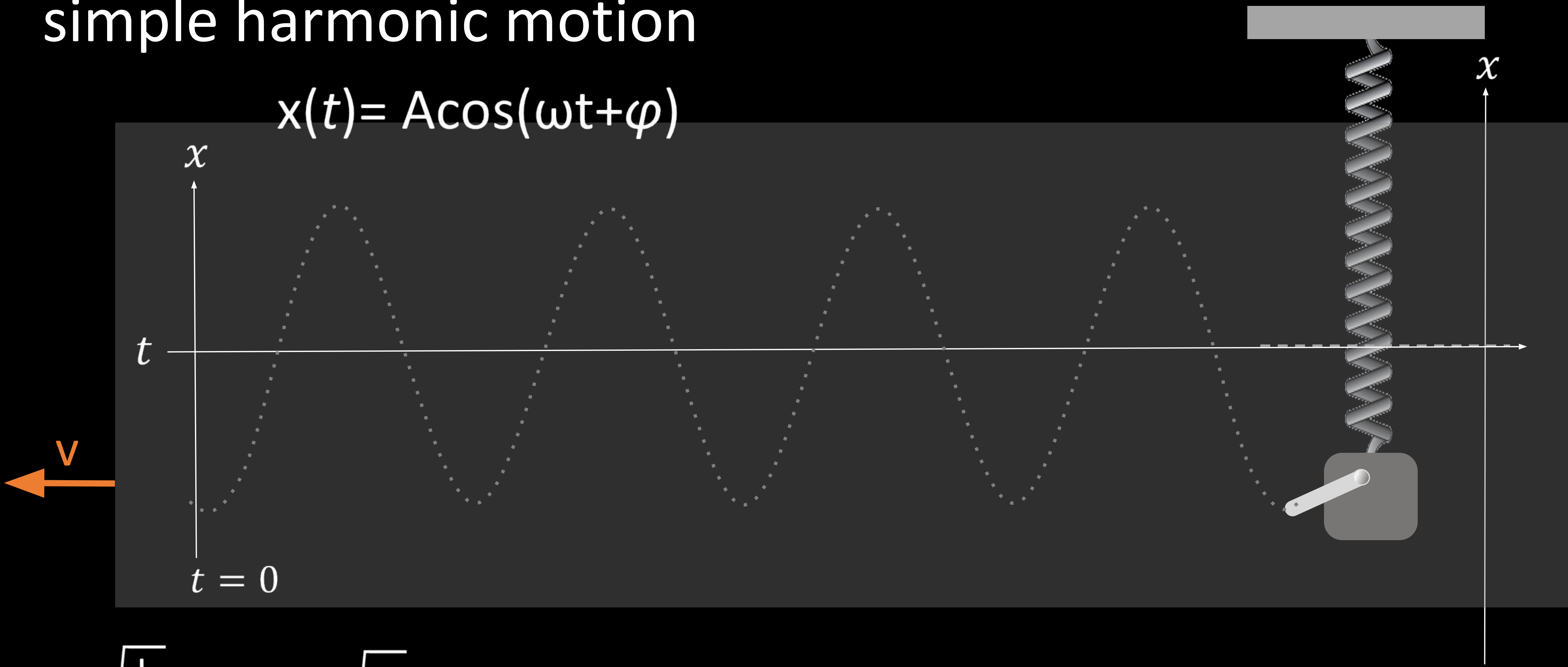


$\omega = \sqrt{\frac{k}{m}}$ $T = 2\pi\sqrt{\frac{m}{k}}$ ⇨ sketch the curve for the 4m mass

simple harmonic motion

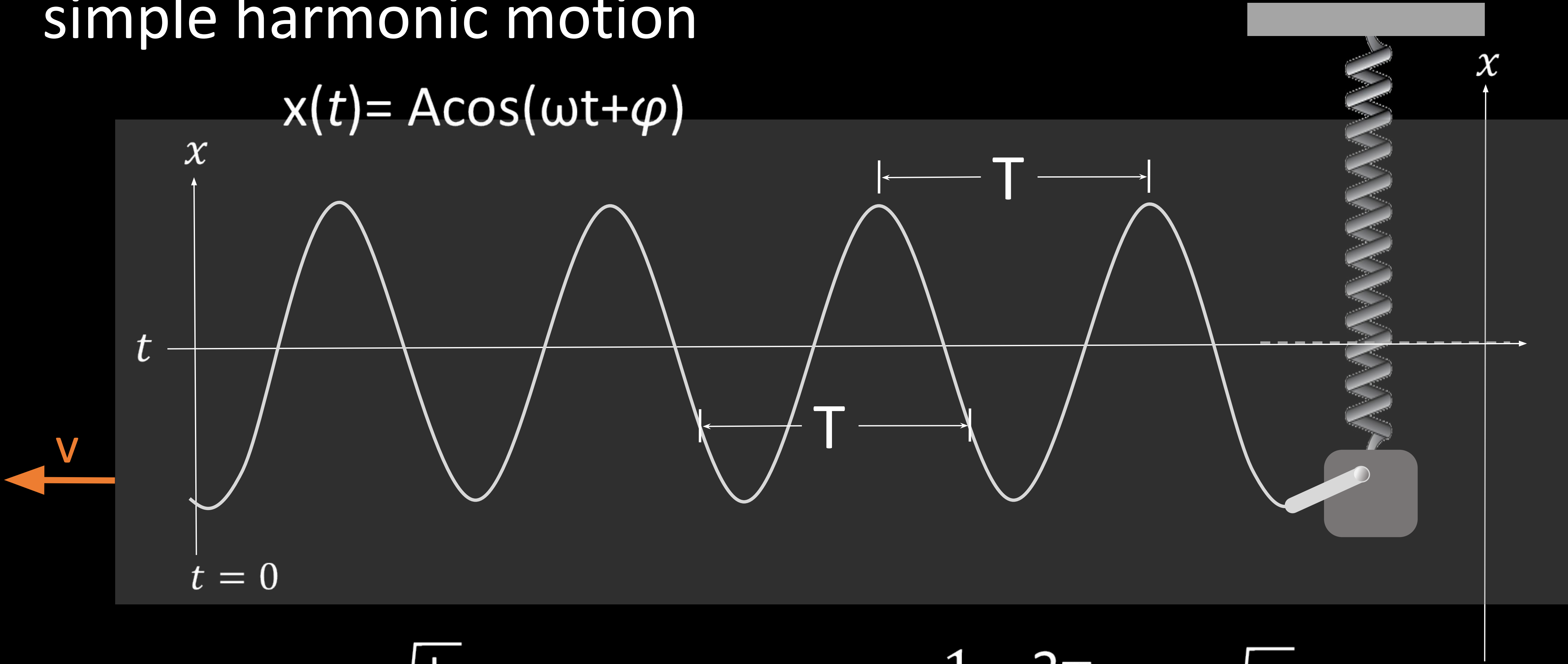
$x(t) = A\cos(\omega t + \varphi)$



$\omega = \sqrt{\frac{k}{m}}$ $T = 2\pi\sqrt{\frac{m}{k}}$ ⇨ sketch the curve for a ¼m mass

simple harmonic motion

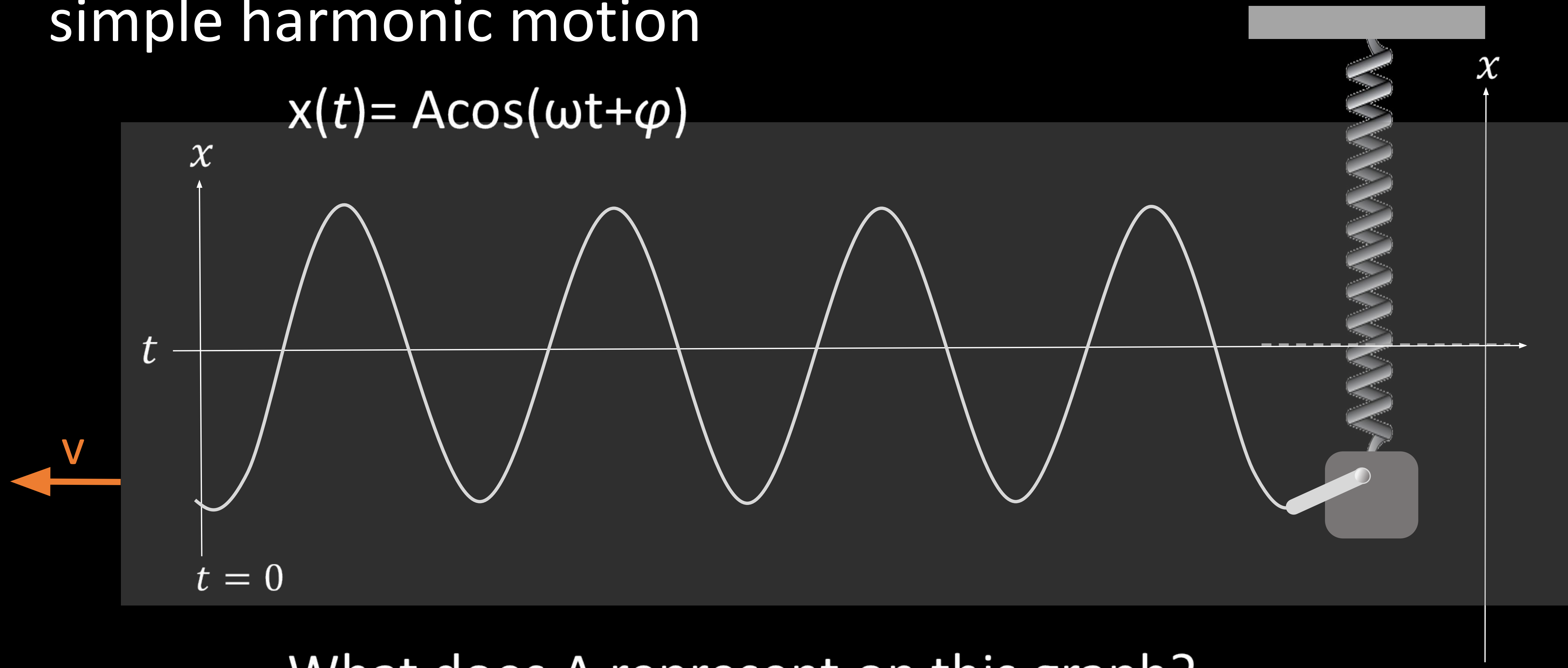
$$x(t) = A\cos(\omega t + \varphi)$$



$$\omega = \sqrt{\frac{k}{m}} \qquad \omega = 2\pi f \qquad T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

simple harmonic motion

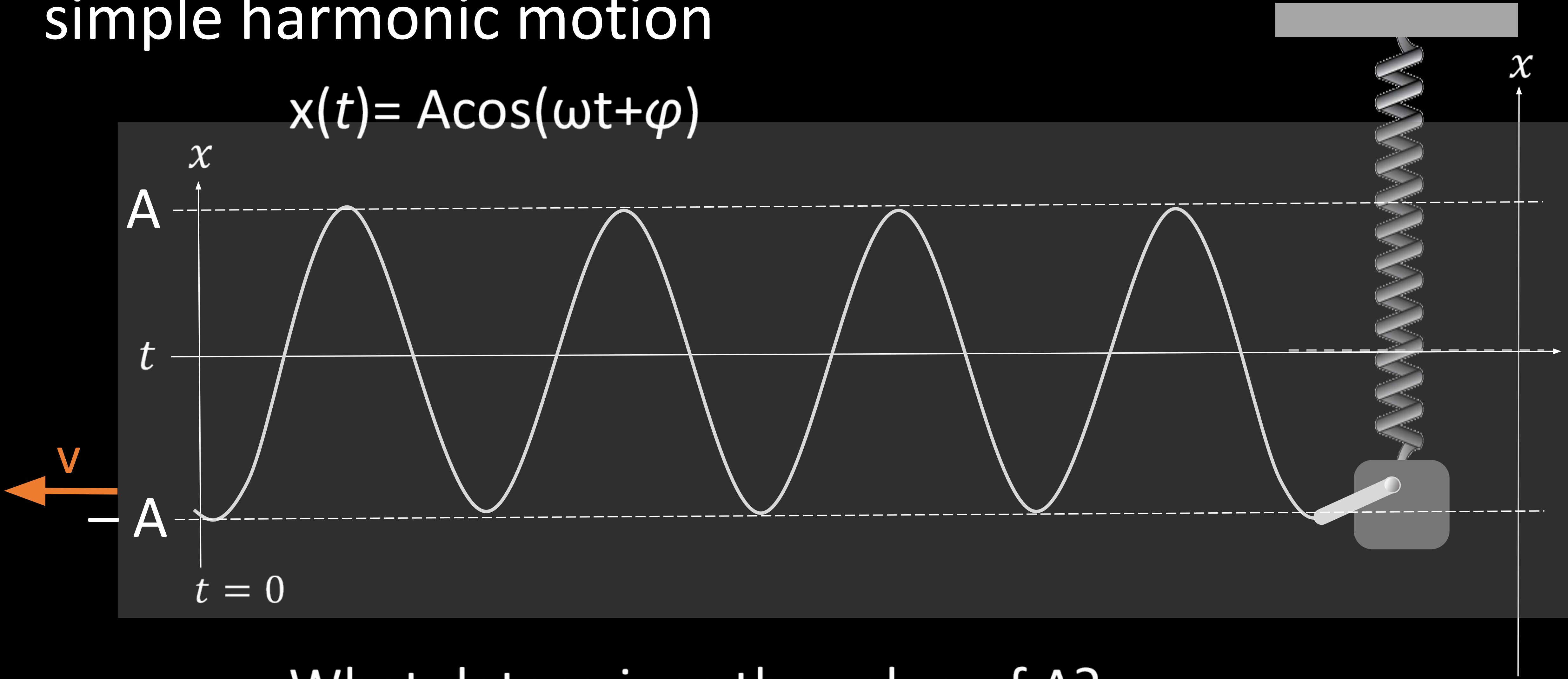
$$x(t) = A\cos(\omega t + \varphi)$$



What does A represent on this graph?

simple harmonic motion

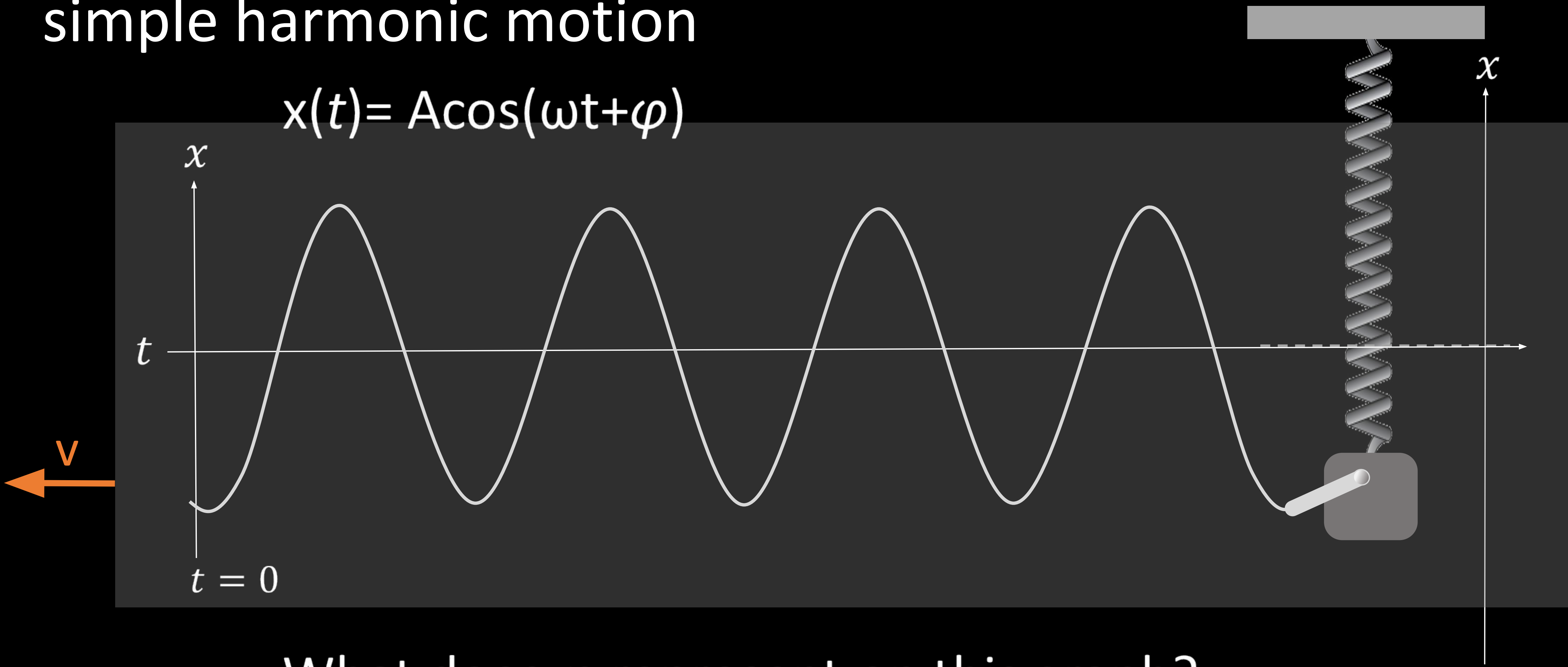
$$x(t) = A\cos(\omega t + \varphi)$$



What determines the value of A?

simple harmonic motion

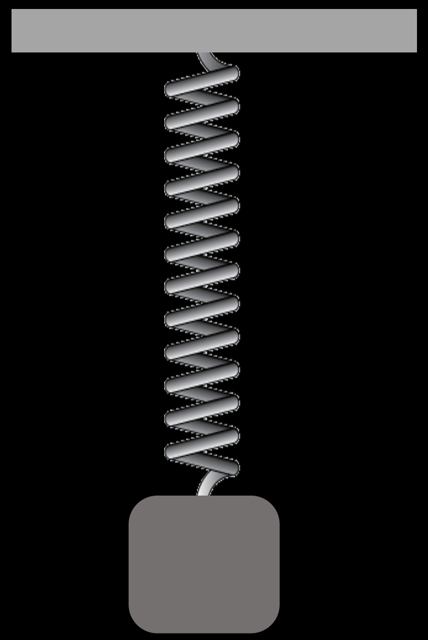
$x(t) = A\cos(\omega t + \varphi)$



What does φ represent on this graph?

What value of φ describes the curve shown?

Part III: Resonance Phenomena



To be continued –