# Determination of $\alpha_{S}$ at 500 GeV from Event Shapes and Jet Rates

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#### Abstract

The potential of the TESLA linear e<sup>+</sup>e<sup>-</sup> collider to determine the strong coupling constant,  $\alpha_S$ , at 500 GeV is investigated. Experimental complications due to background from W- and Z-pairs, top-production, initial state photon radiation and from beamstrahlung are considered. The hadronic event selection procedures used by the experiments at LEP II are reviewed for the applicability at TESLA. An estimate of the various error contributions to the total uncertainty of an  $\alpha_S$  determination is presented. It confirms that hadronisation effects are diminished while the uncertainty from the choice of the renormalisation scale will dominate. Fits of the  $\ln(R)$ -matched second order  $(\mathcal{O}(\alpha_S^2))$  and resummed calculation (NLLA) to six observables are used to estimate the error contributions. This yields the expectation of the precision for  $\alpha_S(500 \text{ GeV})$  of  $\pm 0.0025$ .

#### 1 Introduction

The discovery of asymptotic freedom is an important corner stone of the foundation of Quantum Chromodynamics (QCD) as the theory of the strong interaction. Despite the substantial effort in testing the predictions of this theory at  $e^+e^-$  colliders like e.g. PETRA [1] and LEP [2], QCD is less precisely surveyed than the electroweak sector of the standard model of electroweak and strong interaction. The precision of the determination of the strong coupling constant,  $\alpha_S$ , at centre-of-mass energies of up to 200 GeV at LEP II is only at the level of a few percent (see e.g. [3]).

To test the details of the running of  $\alpha_S$ , which means the asymptotic freedom, one has to strive for a precision of better than one percent. This could be achieved by a determination of  $\alpha_S$  at a linear collider operating at a centre-of-mass energy of 500 GeV or above. It would provide a long lever arm for the test of the asymptotic freedom.

Besides the running of the coupling constant, QCD also predicts the value of the renormalised quark masses to depend on the scale of the interaction process [4]. First investigations of this running of the quark masses were performed by the LEP and SLC experiments [5], which suffered from large statistical uncertainties. Further experimental manifestation of the effect is, therefore, still required. This could be achieved at a linear collider due to its high luminosity and by using very high resolution silicon vertex detectors. Here the silicon micro vertex detectors allow to distinguish bottom and charm quark flavours created by the annihilation process from the other lighter quarks.

The identification of the primary quark flavour could furthermore be used to study the properties of quark and gluon jets. Such studies have already be successfully conducted at LEP I and SLC (see e.g. [6]). At the higher energies of LEP II, however, the data statistics is scarce for a significant investigation of the quark and gluon properties. Again, a linear collider would allow such studies because of its high specific luminosity.

Apart from the advantage of very high luminosity, QCD studies at a linear collider also gain from the high centre-of-mass energy. This is because the impact of hadronisation, which blurs the view at the quarks and gluons, diminishes with a reciprocal power of the scale of the process. Hence the distortion due to hadronisation effects at 500 GeV is only about 20% of that at the Z mass scale [7].

This article presents a scrutiny of the potential of the TESLA linear collider to perform a determination of the strong coupling constant at a centre-of-mass energy of 500 GeV.

#### 2 Initial considerations

The study is restricted to an investigation based on Monte Carlo generator simulations using the PYTHIA 5.722 [8], HERWIG 5.9 [9], and ARIADNE 4.8 [10] programs which, except for HERWIG, have been tuned by the OPAL collaboration at LEP [11]. These generators are used to simulate the production of quark-antiquark pairs from the annihilation. To account for the substantial production cross-section of massive gauge boson pairs (see e.g. [12]), background from hadronic decays of Z and W bosons is considered, where in addition also the decay  $W \rightarrow \tau \nu_{\tau}$  is included. All other leptonic decays of these massive bosons can easily be identified and, hence, rejected.

Likewise, background contributions from  $\gamma\gamma$  scattering and  $\tau$  pair production are not included. Their contribution can be removed very efficiently as is known from the experiments at LEP II (see e.g. [13]).

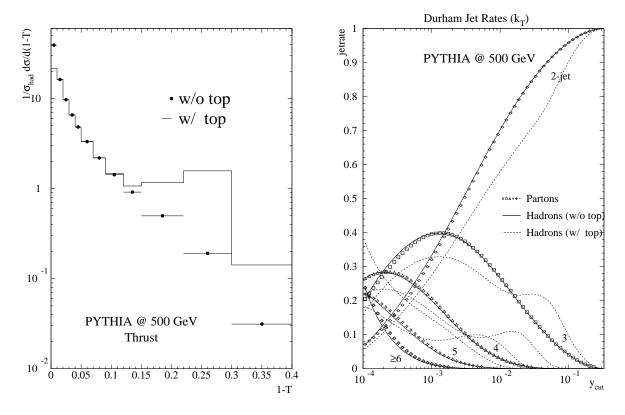


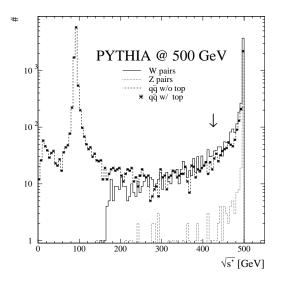
Figure 1: Distortions of thrust distribution (left) and of the jet rates (right) due to the top quark pair production. The negligible impact of hadronisation is made visible for the jet rates.

The production of top quarks leads to significant effects. This is due to its large mass and the appearance of typically three jets of particles from its decay. Top quark events, therefore, appear rather spherically in the detector, thus immitating multijet events. Figure 1 shows the impact of the pair-produced top quarks on the thrust observable and the jet rates, both of which are usually used to determine the value of the strong coupling constant. It is therefore indispensable to exclude the contribution from top quarks when performing an  $\alpha_S$  determination.

The impact of initial state radiation is accounted for using its next-to-leading order treatment in the PYTHIA generator. Radiative events constitute a large fraction of the hadronic final states, especially radiative returns to the Z pole. Most of these can easily be rejected by requiring the reconstructed mass of the whole event to be close to the centre-of-mass energy of the colliding beams. Such a cut on the effective centre-of-mass energy,  $\sqrt{s'}$ , is usually applied in the hadronic event selection at LEP II (e.g. [14]). It effectively takes out radiative Z return events as can be seen in Figure 2.

Beamstrahlung, which is of relevance at a linear collider only, is simulated using the program CIRCE 1.28<sup>1</sup> [15] which has been interfaced to the PYTHIA generator. When using the parameters proposed for the TESLA linear collider, only a negligible fraction of the events is lost due to beamstrahlung (see Figure 2) if the cut  $\sqrt{s'} > 0.85 \cdot \sqrt{s}$  is applied which is usually used by the LEP experiments.

<sup>&</sup>lt;sup>1</sup>Parameter set version 5, revision 1998 05 05



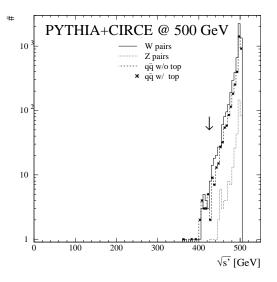


Figure 2: Distribution of the effective centre-of-mass energy  $\sqrt{s'}$  affected by initial state radiation (ISR, left) and by beamstrahlung (BS, right).

	$e^+e^- \rightarrow q\bar{q}$	$e^+e^-(\gamma) \to q\bar{q}$	$\rightarrow$ W <sup>+</sup> W <sup>-</sup>	$\rightarrow$ ZZ	$\rightarrow t \bar{t}$
$\sigma(500 \text{ GeV}) \text{ [pb]}$	2.8	11.6	4.8	0.3	0.3
# events	$\approx 900 \mathrm{k}$	$\approx 3.6 \mathrm{M}$	$\approx 1.5 \mathrm{M}$	$\approx 90 \mathrm{k}$	$\approx 90 \mathrm{k}$

Table 1: Approximate number of events for  $300 \text{ fb}^{-1}$  and the cross-sections taken from PYTHIA [8].

#### 3 Hadronic event selection

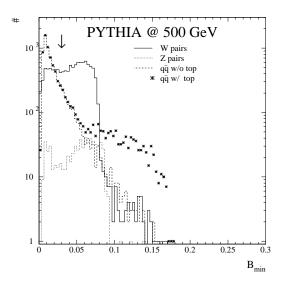
To study the prospects of a determination of  $\alpha_S$  at the TESLA linear collider, an integrated luminosity of 300 fb<sup>-1</sup> has been simulated. This corresponds to roughly one year of data taking at the design luminosity of the collider. Adopting the cross-sections from the PYTHIA program, this integrated luminosity translates into the number of events listed in Table 1.

The main issue of the hadronic event selection is to reject the contributions from W and Z pair, and from top-antitop production. The option of polarising the electron beam at the TESLA collider, which allows to strongly suppress the production of W pairs, is disregarded for this study.

Quite some experience in rejecting W pair events has been collected by the experiments at LEP II (see e.g. [13, 16]). The detailed examination showed, however, that cuts on 4-jet matrix element weight,  $W_4$ , and on the 3- to 4-jet flip  $y_{\text{cut}}$  value,  $y_{34}$ , both of which have a good separation power at LEP II energies, cannot discriminate between W pair and  $q\bar{q}$  events at 500 GeV.

The minimum value of the jet broadening,  $B_{\min} \equiv \min(B_1, B_2)$ , determined from the jet broadening in each hemisphere defined by the thrust axis of an event, proves to powerfully reject top-antitop events (see Figure 3). It also reduces the contribution of W and Z pair events.

These boson pair events can be eliminated more effectively by cuts on the mass of each thrust hemisphere. This is due to the high centre-of-mass energy giving both of the bosons a significant boost such that the thrust axis yields the vector bosons' direction of flight. Thus the



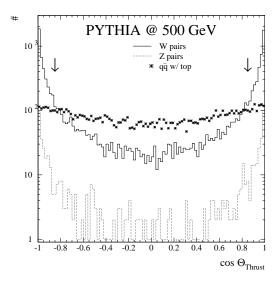


Figure 3: Effective suppression of W and Z pair, and of top-antitop events by cutting on the minimum value of the jet broadening ( $B_{\min} < 0.03$ , left) and on polar angle of the thrust axis ( $|\cos \theta_{\text{thrust}}| < 0.85$ , right).

hemisphere masses are close to the mass of the W or Z boson which allows to easily reject such events. Requiring a minimum hemisphere mass of 1% of the centre-of-mass energy eliminates events with a W $\to \tau \nu_{\tau}$  decay. Fully hadronic decays of the vector bosons are sorted out if either the smaller hemisphere mass is less than 65 GeV or the larger hemisphere mass is above 105 GeV.

Since beam polarisation has been disregarded, W and Z pairs can be further diminished due to the dominating t-channel production by cutting on the polar angle of the thrust axis,  $\theta_{\text{thrust}}$ . Figure 3 shows that such a cut, although it very effectively eliminates the boson pairs, reduces the selection efficiency for  $q\bar{q}$  events.

Applying all cuts mentioned, the selection efficiency for hadronic  $q\bar{q}$  final state is typically 65% while the purity is about 90%. The efficiency is less than the 85% usually achieved at LEP II, but can be improved by employing the beam polarisation option of TESLA or the advantages of high resolution vertex detectors.

#### 4 Observables and corrections

The experiments at LEP II routinely use thrust, heavy jetmass, total and wide jet broadening, C-parameter, and the differential 2-jet rate from the Durham algorithm to determine  $\alpha_S$ . The definition of these observables can be found in e.g. [14]. All these observables are known in second order  $(\mathcal{O}(\alpha_S^2))$  and, moreover, the leading and next-to-leading logarithms have been resummed (NLLA). Combining these calculations provides a better description of the data. This can be expected because the resummed logarithms dominate in the extreme 2-jet region while the fixed order calculation is applicable to the  $\geq$  3-jet region. The LEP II experiments employ different prescriptions to combine the two calculations. For the remainder of this study the  $\ln(R)$ -matching scheme is adopted (see e.g. [14]).

The value of the strong coupling constant is determined from the differential distributions of these observables. Since this study relies on Monte-Carlo event generators only, the effect of

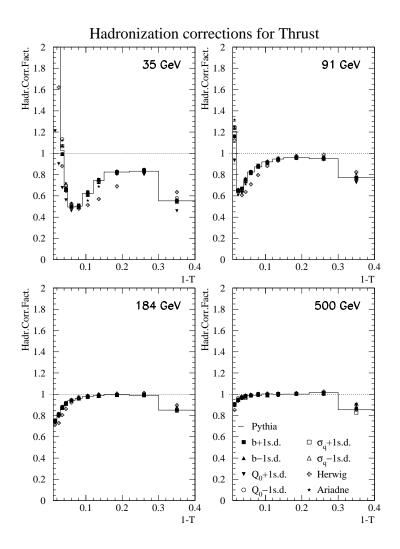


Figure 4: Hadronisation correction factors for the thrust observable at different centre-of-mass energies. The marker symbols indicate different choices of hadronisation parameters in the PYTHIA program and other shower and fragmentation models.

the finite resolution and the limited acceptance of a real detector has to be considered. In [17] such impacts were found to be small.

To assess hadronisation uncertainties, the approach of [13] is used for this investigation. Some parameters of the PYTHIA program are varied to find out the sensitivity of the determined value of  $\alpha_S$  on the hadronisation correction. The range of variation is given by the uncertainties of the parameter values found during the tune of the PYTHIA generator. In detail the b parameter of the Lund symmetric fragmentation is varied by  $\pm 0.04/\text{GeV}^2$  about the tuned value of  $0.52/\text{GeV}^2$ . The cut-off of the parton shower  $Q_0$  is changed from 1.9 GeV to 1.4 and 2.4 GeV. The width of the gaussian distribution of the additional transverse momentum,  $\sigma_q$ , acquired by the particles during the fragmentation is varied by  $\pm 0.03$  GeV about 0.40 GeV. Effects of the PYTHIA model of the parton shower development and of the fragmentation are investigated using HERWIG and ARIADNE alternatively to PYTHIA.

In general, the hadronisation corrections at a centre-of-mass energy of 500 GeV are much less than 5% in the  $\geq$  3-jet region. This can be seen from the comparison of the hadronisation

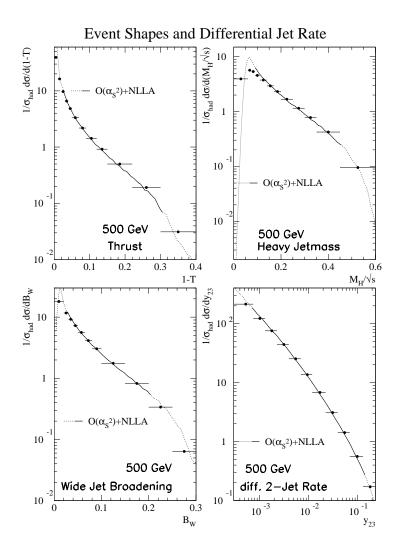


Figure 5: Event shape and differential 2-jet distributions fitted using the  $\ln(R)$ -matched second order and resummed calculations  $(\mathcal{O}(\alpha_S^2) + NLLA)$ . The extrapolation of the fit, whose range is indicated by the solid line, is shown by the dotted line.

corrections for the thrust observable in the range of 35 through 500 GeV in Figure 4. The reduced uncertainty due to the choice of the hadronisation parameters at increasing centre-of-mass energies is also noticeable from the Figure.

Apart from the shrinkage of uncertainties due to the higher centre-of-mass energy it should also be kept in mind that a tune of the event generators at the end of LEP using the full data statistics of all experiments will definitely improve the precision of the parameters. Thus the range of variation will be more constrained than it is for this investigation. In conclusion, hadronisation corrections and associated uncertainties will be much reduced at TESLA energies.

# 5 Estimation of the error budget of $\alpha_{S}$

The error budget of an  $\alpha_S$  determination is compiled from different sources. Since a perfect detector has been assumed, the corresponding experimental uncertainties are adopted from measurements at LEP II (e.g. [18]). Effects due to the roughly 10% background from boson

source	$\delta(\alpha_S(M_{ m Z}))$	comments
statistics	$\pm 0.0001$	
experimental	$\pm 0.002 \dots 0.004$	taken from LEP II
hadronisation	$\pm 0.0006 \dots 0.0015$	$Q_0$ dominates
HERWIG	$+0.017\ldots+0.025$	generator not tuned
ARIADNE	$-0.004\ldots+0.002$	
renormalisation scale	$\pm 0.001 \dots 0.004$	
total uncertainty	$\pm 0.004$	

Table 2: Compilation of the error budget of a determination of the strong coupling constant from six observables at a centre-of-mass energy of 500 GeV. The quoted ranges indicate the spread of the  $\alpha_S$  values obtained from the different observables.

pairs are neglected. The statistical uncertainty is estimated by considering a 65% selection efficiency and an expected integrated luminosity of 300 fb<sup>-1</sup> within one year. This corresponds to about 0.5 million hadronic events which have been generated using the PYTHIA generator employing the tuned set of parameters.

The simulated events are used to assess the remaining uncertainties due to hadronisation corrections and due to the choice of the hadronisation parameters by fitting the  $\ln(R)$ -matched second order  $(\mathcal{O}(\alpha_S^2))$  and resummed calculations (NLLA) to distributions of the observables in the hadronic final state. Distributions from the thrust, heavy jetmass, total and wide jet broadening, C-parameter, and the 2 to 3-jet flip  $y_{\text{cut}}$  value,  $y_{23}$ , obtained from the Durham jet algorithm are used. Hadronisation correction factors are applied to these distributions before fitting the theoretical expressions. The result of the fits is exemplified for thrust, heavy jetmass, wide jet broadening, and the jet flip in Figure 5. In general good fits are obtained. However, for the heavy jetmass and the wide jet broadening an excursion of the fitted curve from the simulated data is visible at small values of these observables. This could indicate a problem of either the theoretical formulae or of the event generator program. The fit must, therefore, not go too far into this 2-jet region, which represents the bulk of statistics indeed.

Finally, also the impact of a variation of the renormalisation scale factor,  $x_{\mu}$ , is considered. As has become usual  $x_{\mu}$  is varied from unity to 0.5 and 2, respectively, assigning the change in  $\alpha_S(\sqrt{s})$  as the uncertainty due to the choice of the renormalisation scale factor.

Table 2 lists the individual contributions to the total uncertainty of  $\alpha_S$  at the Z mass scale,  $M_{\rm Z}$ . To calculate a total uncertainty, the individual error contributions from all six observables are averaged for each systematic variation individually. The total error is then obtained by adding the individual contributions in quadrature. Excluding the uncertainty derived when choosing the HERWIG generator to correct for hadronisation effects, because it is not tuned to the LEP data yet, the total error becomes  $\delta(\alpha_S(M_{\rm Z})) = \pm 0.004$ . This error incorporates a significant contribution of about  $\pm 0.003$  due to the scale uncertainty. It should be noted that it is of similar size as the typical error obtained by the LEP and SLC experiments. The error  $\delta(\alpha_S(M_{\rm Z}))$  corresponds to  $\delta(\alpha_S(500~{\rm GeV})) = \pm 0.0025$ .

### 6 Conclusions

A feasibility study of the determination of the strong coupling constant,  $\alpha_S$ , at the TESLA linear collider has been presented starting from the selection of hadronic events. Neither initial state

radiation nor beamstrahlung constitute a substantial problem for such a QCD investigation at this collider.

The major background stems from the pair production of W and Z bosons. Both can be easily suppressed by cutting on the hemisphere masses and on the polar angle of the thrust axis. Top quarks significantly distort event shape and jet rate distributions and must, therefore, also be excluded from an  $\alpha_S$  determination based on such observables. It has been found that a cut on the minimum value of the jet broadening effectively removes the contribution from top quarks without affecting the selection efficiency.

The hadronisation corrections and the associated uncertainties are very small at TESLA energies. However, the contribution from different hadronisation models could become a dominating source of error.

Fits of the  $\ln(R)$ -matched second order and resummed calculations to six observables have been performed to assess the systematic uncertainties of the derived value of  $\alpha_S$  using data from Monte Carlo generators only. Exerimental uncertainties have been adopted from the LEP II experiments and are found to be small. A very significant contribution to the total uncertainty of  $\delta(\alpha_S(500~{\rm GeV})) = 0.0025$  stems from the choice of the renormalisation scale. Its absolute contribution will shrink with increasing centre-of-mass energy but the relative uncertainty is not diminished. To improve on the scale uncertainty theoretical calculations of third order in  $\alpha_S$  are absolutely required.

## Acknowledgements

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