Practical Exercise – Continuous Probability Distributions

The Actuarial team requires a library of probability distributions for use in their models.

Some initial analysis shows that a common interface can be built for all continuous probability distributions. The initial design of the interface has the following properties and methods:

Properties

1. Mean
2. Variance

Methods

- Probability Density Function (PDF)

The Normal and Log-normal probability distributions have been chosen for implementation in the first release of the library.

The implementation detail of each distribution is given below:

1. **Normal**

   Parameters: μ (mu) and σ^2 (sigma squared) where $\sigma^2 > 0$

   Mean: μ

   Variance: σ^2

   PDF(x) = $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

2. **Log-normal**

   Parameters: μ and σ^2 where $\sigma^2 > 0$

   Mean: $e^{\mu+\sigma^2/2}$

   Variance: $\left(e^{\sigma^2} - 1\right)e^{2\mu+\sigma^2}$

   PDF(x) = $\frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}}$ where $x > 0$

The ability to change the parameters of a distribution after creation is not required.

Step 1

Create a new solution comprising of the following projects:

1. Distributions.Console (C# Console Application Project);
2. Distributions (C# Class Library Project); and
3. Distributions.Test (C# Test Project).

Step 2

Implement the common interface and distribution classes as defined above in project Distributions.

Add test classes and methods to project Distributions.Test to test for the following behaviour, that:

1. creating a **Normal** or **Log-normal** distribution with invalid parameters throws an argument exception;
2. calling the probability density function of a Log-normal distribution with a value less than 0 throws an argument exception;
3. creating a **Normal** distribution with parameters $\mu = 3$ and $\sigma^2 = 1.5$ gives a:
    a. Mean value of 3;
    b. Variance of 1.5; and
    c. that calling PDF(3.6) returns 0.2889.
4. creating a **Log-normal** distribution with parameters $\mu = 3$ and $\sigma^2 = 1.5$ gives a:
    a. Mean value of 42.5211;
    b. Variance value of 6295.0415; and
    c. that calling PDF(3.6) returns 0.0338.

All numerical comparisons in the above tests should be performed to **4 decimal places**.

Step 3

Create a command line console application that takes three parameters:

1. μ (mu);
2. σ^2 (sigma squared); and
3. x.

Output to the console the Mean, Variance and the Probability Density Function (PDF) of x for both the Normal and Log-Normal distributions in the following format:

> Normal(<x>;<mu>,<sigma squared>) = <PDF(x)> (Mean : <mean>, Variance : <variance>)
>
> Log-normal(<x>;<mu>,<sigma squared>) = <PDF(x)> (Mean : <mean>, Variance : <variance>)

The console application should display any errors that occur in evaluating values for either distribution.