HOUSEHOLDER REFLECTIONS

$x \in \mathbb{R}^n$, $\sigma = \pm \|x\|$, $\|x\|^2 = x^T x$

$$e_k = \begin{pmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{pmatrix} \; k\text{-esima}$$

$$\begin{cases} u = x + \sigma e_k \\ u^T u = x^T x + 2\sigma x_k + \sigma^2 = 2\sigma(\sigma + x_k) \\ u_k = u^T e_k = x_k + \sigma \end{cases}$$

$$\rho = \frac{2}{\|u\|^2} = \frac{2}{u^T u} = \frac{1}{\sigma u_k}$$

$$\boxed{H = \mathbb{1} - \rho u u^T}$$

$$Hx = x - \rho u u^T x = x - \rho u \sigma(\sigma + x_k) = x - \frac{u\sigma}{\sigma u_k} u_k = -\sigma e_k$$

$$= \mp \|x\| e_k$$