evolution operator (*cf.* complement $F_{III}$), which simply expresses the conservation over time of the total probability of finding the particle somewhere on the $Ox$ axis (norm of the wave function).

## 2. Transmission and reflection coefficients

To calculate the reflection and transmission coefficients for a particle encountering the potential $V(x)$, one should (as in complement $J_I$) construct a wave packet with the eigenfunctions of $H$ which we have just studied. Consider, for example, an incident particle of energy $E_i$ coming from the left. The corresponding wave packet is obtained by superposing functions $\varphi_k(x)$, for which we set $\tilde{A}' = 0$, with coefficients given by a function $g(k)$ which has a marked peak in the neighborhood of $k = k_i = \sqrt{2mE_i/\hbar^2}$. We shall not go into these calculations in detail here; they are analogous in every way to those of complement $J_I$. They show that the reflection and transmission coefficients are equal, respectively, to $|A'(k_i)/A(k_i)|^2$ and $|\tilde{A}(k_i)/A(k_i)|^2$.

Since $\tilde{A}' = 0$, relations (22) and (26) yield:

$$\begin{aligned} \tilde{A}(k) &= \frac{1}{F^*(k)} A(k) \\ A'(k) &= -\frac{G(k)}{F^*(k)} A(k) \end{aligned} \tag{28}$$

The reflection and transmission coefficients are therefore equal to:

$$R_1(k_i) = \left| \frac{A'(k_i)}{A(k_i)} \right|^2 = \left| \frac{G(k_i)}{F(k_i)} \right|^2 \tag{29-a}$$

$$T_1(k_i) = \left| \frac{\tilde{A}(k_i)}{A(k_i)} \right|^2 = \frac{1}{|F(k_i)|^2} \tag{29-b}$$

[it is easy to verify that condition (21) insures that $R_1(k_i) + T_1(k_i) = 1$].

If we now consider a particle coming from the right, we must take $A = 0$, which gives:

$$\begin{aligned} \tilde{A}(k) &= \frac{G^*(k)}{F^*(k)} \tilde{A}'(k) \\ A'(k) &= \frac{1}{F^*(k)} \tilde{A}'(k) \end{aligned} \tag{30}$$

The transmission and reflection coefficients are now equal to:

$$T_2(k) = \left| \frac{A'(k)}{\tilde{A}'(k)} \right|^2 = \frac{1}{|F(k)|^2} \tag{31-a}$$

and:

$$R_2(k) = \left| \frac{\tilde{A}(k)}{\tilde{A}'(k)} \right|^2 = \left| \frac{G(k)}{F(k)} \right|^2 \tag{31-b}$$

Comparison of (29) and (31) shows that $T_1(k) = T_2(k)$ and that $R_1(k) = R_2(k)$: for a given energy, the transparency of a barrier (whether symmetrical or not) is therefore always the same for particles coming from the right and from the left.

In addition, from (21) we have:

$$|F(k)| \geqslant 1 \tag{32}$$

When the equality is realized, the reflection coefficient is zero and the transmission coefficient is equal to 1 (resonance). On the other hand, the inverse situation is not possible : since (21) imposes that $|F(k)| > |G(k)|$, one can never have $T = 0$ and $R = 1$ [except in the case where $F$ and $G$ tend simultaneously towards infinity]. Actually, such a situation can only occur for $k = 0$. To see this, divide the function $v_k(x)$ defined in (3) by $F(k)$. If $F(k)$ goes to infinity, the wave function will be identically zero on the left hand side, and hence necessarily, by extension, zero on the right hand side. However, this is impossible unless $k = 0$ and $F = -G$.

## 3. Example

Let us return to the square potentials studied in § 2-b of complement $\text{H}_\text{I}$: in the region $-l/2 < x < +l/2$, $V(x)$ is equal to a constant $V_0$* (see figure 2, where $V_0$ has been chosen to be positive).

First, let us assume that $E$ is smaller than $V_0$, and set:

$$\rho = \sqrt{2m(V_0 - E)/\hbar^2} \tag{33}$$

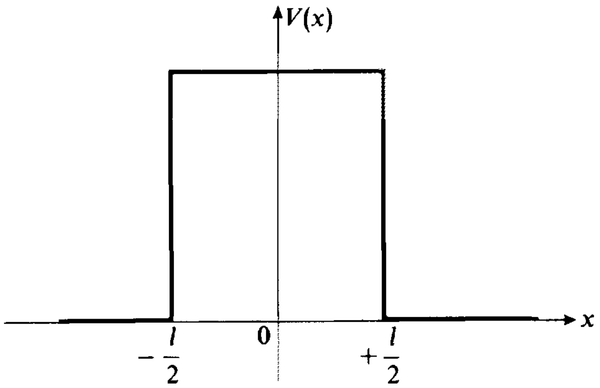


FIGURE 2

**Square potential barrier.**

An elementary calculation analogous to the one in complement $\text{H}_\text{I}$ yields:

$$M(k) = \begin{pmatrix} \left[\cosh \rho l + i\dfrac{k^2-\rho^2}{2k\rho}\sinh \rho l\right] e^{-ikl} & -i\dfrac{k_0^2}{2k\rho}\sinh \rho l \\ i\dfrac{k_0^2}{2k\rho}\sinh \rho l & \left[\cosh \rho l - i\dfrac{k^2-\rho^2}{2k\rho}\sinh \rho l\right] e^{ikl} \end{pmatrix} \tag{34}$$

* In fact, we are considering here a barrier which is displaced relative to that of complement $\text{H}_\text{I}$, since we are assuming it to be situated between $x = -l/2$ and $x = +l/2$, instead of between $x = 0$ and $x = l$.

with:

$$k_0 = \sqrt{\frac{2mV_0}{\hbar^2}} \tag{35}$$

($V_0$ is necessarily positive here, since we have assumed $E < V_0$).

If now we assume that $E > V_0$, we set:

$$k' = \sqrt{\frac{2m}{\hbar^2}(E - V_0)} \tag{36}$$

and:

$$k_0 = \sqrt{\varepsilon \frac{2mV_0}{\hbar^2}} \tag{37}$$

(where $\varepsilon = +1$ if $V_0 > 0$ and $-1$ if $V_0 < 0$). We thus obtain:

$$M(k) = \begin{pmatrix} \left[\cos k'l + i\dfrac{k^2 + k'^2}{2kk'} \sin k'l\right] e^{-ikl} & -i\varepsilon\dfrac{k_0^{\,2}}{2kk'} \sin k'l \\ i\varepsilon\dfrac{k_0^{\,2}}{2kk'} \sin k'l & \left[\cos k'l - i\dfrac{k^2 + k'^2}{2kk'} \sin k'l\right] e^{ikl} \end{pmatrix} \tag{38}$$

It is easy to verify that the matrices $M(k)$ written in (34) and (38) satisfy relations (16), (17) and (21).

**References and suggestions for further reading:**

Merzbacher (1.16), chap. 6, §§5, 6 and 8; see also the references of complement $M_{III}$.