Dataset	0% CS									KMEANS
	PCCC	PCCC-N2	PCCC-N2-S	PCCC-N3-S	PCCC-N5-S	COPKM	CSC	DILS	LCC	
n500-k2	0.3	0.6	0.3	0.4	0.4	**0.1**	2.7	2,718.1	–	0.3
n500-k5	0.6	0.5	0.5	0.6	0.7	**0.1**	2.9	2,459.8	–	0.2
n500–k10	1.5	0.5	0.5	0.7	1.0	**0.3**	3.2	2,414.0	–	0.5
n500-k20	4.3	0.8	0.9	1.1	1.6	**0.6**	3.0	2,484.1	–	0.3
n500-k50	7.7	**0.6**	**0.6**	1.0	1.1	2.6	11.0	2,313.2	–	0.2
n500-k100	11.0	1.2	**0.6**	0.7	1.1	8.7	4.2	2,307.5	–	0.5
n1000-k2	0.8	0.7	0.8	0.7	0.8	**0.1**	25.0	3,639.8	–	0.2
n1000-k5	2.0	1.3	1.2	1.7	2.3	**0.2**	27.8	3,614.3	–	0.2
n1000-k10	3.4	0.9	1.1	1.6	2.0	**0.6**	27.1	3,614.5	–	0.2
n1000-k20	8.3	1.7	**1.5**	2.0	2.5	1.7	31.5	3,618.3	–	0.2
n1000-k50	24.4	1.9	**1.8**	2.6	3.6	5.7	32.1	3,614.9	–	0.2
n1000-k100	26.8	1.8	**1.4**	1.6	2.1	18.3	34.1	3,617.5	–	0.6
n2000-k2	1.3	1.5	1.4	1.4	1.3	**0.2**	243.1	3,682.0	–	0.2
n2000-k5	4.0	2.5	2.3	3.0	4.2	**0.4**	258.7	3,675.7	–	0.4
n2000-k10	8.4	2.2	2.4	3.2	4.6	**1.2**	259.6	3,651.8	–	0.4
n2000-k20	18.0	3.2	**2.9**	4.3	5.8	3.8	262.5	3,609.1	–	0.5
n2000-k50	59.4	**3.8**	4.0	5.4	7.6	11.9	268.8	3,663.2	–	0.5
n2000-k100	77.7	**2.6**	2.8	3.8	5.1	39.9	271.6	3,631.1	–	0.6
n5000-k2	3.6	2.7	3.0	3.0	3.0	**0.7**	3,269.0	3,869.5	–	0.5
n5000-k5	9.9	4.3	4.8	6.1	9.2	**1.4**	3,462.1	3,771.3	–	0.5
n5000-k10	14.8	3.7	4.1	5.3	8.0	**3.3**	3,682.9	3,746.6	–	0.4
n5000-k20	81.2	10.9	11.6	17.3	25.4	**9.8**	3,633.0	3,685.8	–	0.9
n5000-k50	200.9	**11.4**	12.2	17.7	25.9	41.3	3,499.3	3,718.3	–	0.9
n5000-k100	458.2	**11.9**	13.1	19.0	28.1	105.8	3,609.3	3,731.3	–	1.1
Sum	1,028.7	**73.2**	75.9	104.2	147.6	258.5	22,924.5	80,851.8	86,400.0*	10.6

*Nan values (–) are replaced with 3,600 before computing the sum.

Table W26: Average running times (in seconds) of five versions of the PCCC algorithm and the four state-of-the-art algorithms (COPKM, CSC, DILS, LCC) for the constraint sets of size 0% CS. The lowest values are stated in bold. The column KMEANS reports the average running time of the unconstrained k-means algorithm. The hyphen indicates that the respective algorithm returned no solution within the time limit of 3,600 seconds. We noticed that the LCC algorithm stops with a runtime error when the constraint set is empty. This is why the LCC algorithm did not return any solutions for the constraint sets of size 0% CS.

| Dataset | 5% CS | | | | | | | | | |
	PCCC	PCCC-N2	PCCC-N2-S	PCCC-N3-S	PCCC-N5-S	COPKM	CSC	DILS	LCC	KMEANS
n500-k2	**0.4**	0.5	0.5	0.5	0.7	–	2.9	3,039.7	8.2	0.3
n500-k5	1.4	1.4	0.7	0.9	1.3	**0.1**	2.9	2,506.3	20.2	0.1
n500–k10	3.3	2.0	0.6	0.8	1.1	**0.3**	2.7	2,549.4	28.7	0.1
n500-k20	8.1	2.2	1.1	1.3	1.8	**0.6**	3.0	2,441.9	47.1	0.1
n500-k50	10.3	1.4	**0.7**	0.8	1.3	2.6	3.7	2,507.1	108.4	0.1
n500-k100	17.6	1.1	0.8	**0.7**	1.0	8.6	2.7	2,411.4	210.6	0.2
n1000-k2	0.5	**0.4**	0.8	0.8	0.9	4.2	23.4	3,612.9	165.3	0.1
n1000-k5	2.8	2.2	**1.3**	1.6	2.2	–	22.9	3,617.4	37.3	0.1
n1000-k10	5.6	5.3	1.1	1.5	2.4	**0.5**	24.4	3,606.4	63.3	0.2
n1000-k20	20.9	19.1	2.2	2.5	3.7	**1.8**	22.9	3,620.8	104.2	0.1
n1000-k50	53.2	8.3	**1.9**	2.4	3.6	6.0	24.9	3,611.6	232.5	0.2
n1000-k100	73.1	5.5	**1.5**	1.9	2.7	18.6	25.5	3,630.2	–	0.2
n2000-k2	**0.6**	0.7	0.7	**0.6**	0.7	503.5	263.6	3,716.9	3,669.7	0.3
n2000-k5	5.5	5.3	**1.6**	2.4	4.5	3.0	267.3	3,661.7	403.9	0.2
n2000-k10	12.2	10.7	2.6	3.7	6.5	**1.2**	270.6	3,651.1	136.2	0.4
n2000-k20	109.0	105.6	5.1	6.8	13.2	**3.6**	280.1	3,646.3	254.1	0.2
n2000-k50	182.6	81.7	**3.2**	4.9	7.9	14.3	277.3	3,615.8	537.3	0.7
n2000-k100	414.7	56.3	**3.9**	4.5	7.0	41.4	270.1	3,627.7	1,006.2	0.4
n5000-k2	0.8	0.7	0.7	0.7	**0.5**	–	2,764.0	4,006.1	3,793.6	0.4
n5000-k5	6.3	6.0	**1.7**	2.0	3.9	5,929.3	2,770.6	3,815.8	3,802.6	0.3
n5000-k10	75.5	70.1	**2.7**	4.2	10.2	–	2,696.5	3,860.4	3,863.2	0.5
n5000-k20	443.5	433.5	**10.2**	15.6	36.6	–	2,733.4	3,772.5	2,310.8	0.4
n5000-k50	2,600.7	2,604.5	**14.8**	22.4	33.5	45.0	2,730.1	3,813.6	2,283.4	0.6
n5000-k100	3,607.3	3,318.0	**14.0**	16.9	30.3	116.9	2,639.6	3,759.0	3,608.7	0.6
Sum	7,656.0	6,742.7	**74.3**	100.6	177.4	24,701.6[*]	18,125.0	82,102.0	30,295.4[*]	6.9

[*]Nan values (–) are replaced with 3,600 before computing the sum.

Table W27: Average running times (in seconds) of five versions of the PCCC algorithm and the four state-of-the-art algorithms (COPKM, CSC, DILS, LCC) for the constraint sets of size 5% CS. The lowest values are stated in bold. The column KMEANS reports the average running time of the unconstrained k-means algorithm. The hyphen indicates that the respective algorithm returned no solution within the time limit of 3,600 seconds.

	10% CS									
Dataset	PCCC	PCCC-N2	PCCC-N2-S	PCCC-N3-S	PCCC-N5-S	COPKM	CSC	DILS	LCC	KMEANS
n500-k2	**0.3**	0.6	**0.3**	0.5	0.5	7.8	4.2	3,370.1	197.6	0.1
n500-k5	1.3	1.1	**0.8**	1.0	1.7	–	6.0	3,088.7	40.0	0.1
n500–k10	3.4	3.6	**0.6**	1.0	1.7	4.3	4.4	2,884.1	45.9	0.1
n500-k20	14.8	14.1	**1.3**	1.5	2.5	3.0	6.3	3,003.3	79.4	0.1
n500-k50	17.9	6.6	**0.8**	1.0	1.2	2.9	4.6	2,765.2	157.5	0.2
n500-k100	39.7	3.5	**0.8**	1.0	1.4	19.6	3.6	2,892.1	306.4	0.2
n1000-k2	**0.3**	0.5	0.4	0.5	0.4	201.5	24.6	3,649.8	2,158.6	0.2
n1000-k5	1.3	1.3	0.9	**0.8**	1.3	28.9	25.8	3,623.0	1,160.5	0.1
n1000-k10	8.7	8.4	**0.9**	1.5	3.2	–	27.6	3,629.8	299.7	0.2
n1000-k20	67.9	66.2	**2.7**	4.4	7.2	–	27.6	3,638.4	270.8	0.1
n1000-k50	144.7	139.8	**2.4**	3.1	4.6	7.7	27.5	3,645.3	381.8	0.4
n1000-k100	212.4	59.5	**1.9**	1.9	2.9	39.0	31.4	3,628.5	708.9	0.2
n2000-k2	0.7	0.4	0.5	0.5	**0.3**	3,742.6	238.8	3,701.4	35.8	0.3
n2000-k5	0.9	0.9	0.6	**0.5**	0.9	870.3	232.5	3,755.0	3,434.3	0.1
n2000-k10	9.7	8.8	**1.1**	1.4	3.1	–	218.2	3,656.9	3,792.7	0.3
n2000-k20	298.7	293.9	**2.9**	4.0	11.2	–	218.8	3,639.1	3,547.9	0.4
n2000-k50	596.2	591.1	6.1	**5.8**	13.3	35.2	222.8	3,661.9	1,238.0	0.3
n2000-k100	1,114.9	1,133.9	**4.7**	4.9	7.9	157.9	219.8	3,651.1	1,916.3	0.5
n5000-k2	1.5	1.3	**0.8**	0.9	1.0	–	2,832.7	3,763.4	–	0.4
n5000-k5	1.1	1.2	**0.9**	0.9	1.0	–	2,854.0	3,877.6	–	0.3
n5000-k10	5.1	4.9	**1.1**	1.3	1.9	–	2,682.4	3,888.9	–	0.4
n5000-k20	219.2	212.9	**2.6**	4.5	13.7	–	2,619.7	3,886.3	–	0.5
n5000-k50	3,188.1	3,159.7	**11.4**	20.7	61.8	–	2,663.6	4,056.7	4,121.8	0.5
n5000-k100	3,668.6	3,680.5	**17.5**	20.4	51.5	417.0	2,598.4	3,995.0	3,705.7	0.7
Sum	9,617.4	9,394.7	**63.9**	84.0	196.5	41,537.8[*]	17,795.1	85,351.6	41,999.4[*]	6.8

[*]Nan values (–) are replaced with 3,600 before computing the sum.

Table W28: Average running times (in seconds) of five versions of the PCCC algorithm and the four state-of-the-art algorithms (COPKM, CSC, DILS, LCC) for the constraint sets of size 10% CS. The lowest values are stated in bold. The column KMEANS reports the average running time of the unconstrained k-means algorithm. The hyphen indicates that the respective algorithm returned no solution within the time limit of 3,600 seconds.

Dataset	15% CS									
	PCCC	PCCC-N2	PCCC-N2-S	PCCC-N3-S	PCCC-N5-S	COPKM	CSC	DILS	LCC	KMEANS
n500-k2	0.5	**0.3**	0.4	0.5	0.4	27.1	2.9	3,616.4	8.7	0.1
n500-k5	0.8	0.8	**0.5**	**0.5**	0.6	4.3	3.1	3,623.7	256.4	0.1
n500–k10	5.0	5.6	**0.8**	1.2	2.0	–	2.8	3,557.5	95.7	0.3
n500-k20	27.6	28.5	**1.5**	1.6	3.2	9.1	3.5	3,549.5	119.9	0.1
n500-k50	50.7	51.8	**1.2**	1.5	2.1	9.3	4.1	3,530.5	220.3	0.2
n500-k100	94.4	37.3	**1.2**	**1.2**	1.7	10.2	3.5	3,529.4	390.7	0.2
n1000-k2	0.6	0.5	**0.4**	0.5	0.5	538.4	26.3	3,651.7	–	0.2
n1000-k5	0.8	**0.6**	**0.6**	**0.6**	0.7	125.4	23.0	3,648.1	1,564.4	0.1
n1000-k10	3.8	3.7	**0.7**	0.8	1.3	19.7	26.1	3,640.2	3,082.7	0.2
n1000-k20	90.2	90.8	**1.8**	2.4	5.5	–	27.9	3,631.4	1,068.1	0.3
n1000-k50	317.1	314.8	**2.3**	2.8	6.5	37.0	26.0	3,617.4	593.4	0.1
n1000-k100	434.5	432.4	**2.0**	2.2	3.6	40.8	27.6	3,638.6	993.7	0.2
n2000-k2	0.8	0.9	**0.4**	0.6	0.7	–	236.3	3,720.4	–	0.4
n2000-k5	0.9	0.7	0.7	**0.6**	**0.6**	2,432.7	234.7	3,713.4	515.0	0.3
n2000-k10	2.9	2.8	**0.6**	0.8	1.3	561.9	222.1	3,732.1	–	0.5
n2000-k20	51.6	51.8	**1.6**	1.9	3.9	116.6	232.1	3,715.0	4,564.0	0.4
n2000-k50	783.1	787.6	**4.1**	6.0	15.3	–	219.4	3,711.7	3,621.9	0.6
n2000-k100	1,517.3	1,523.4	**6.0**	7.7	14.7	161.0	222.5	3,804.0	3,621.1	0.4
n5000-k2	1.8	1.9	**1.5**	**1.5**	1.8	–	2,930.6	3,888.4	–	0.3
n5000-k5	1.8	1.7	1.5	1.6	**1.2**	–	2,927.7	4,376.8	–	0.3
n5000-k10	1.6	1.5	**1.2**	1.3	1.5	–	2,833.4	4,394.7	–	0.3
n5000-k20	10.7	11.3	**1.7**	2.0	2.8	6,542.6	2,671.3	4,293.2	–	0.3
n5000-k50	1,820.6	1,828.7	**3.8**	9.6	170.7	–	2,656.8	4,159.0	–	0.5
n5000-k100	3,669.2	3,682.6	**23.7**	69.1	299.8	–	2,646.7	4,268.2	4,494.3	0.7
Sum	8,888.4	8,862.0	**60.2**	118.6	542.3	43,036.1*	18,210.3	91,011.3	54,010.3*	7.1

*Nan values (–) are replaced with 3,600 before computing the sum.

Table W29: Average running times (in seconds) of five versions of the PCCC algorithm and the four state-of-the-art algorithms (COPKM, CSC, DILS, LCC) for the constraint sets of size 15% CS. The lowest values are stated in bold. The column KMEANS reports the average running time of the unconstrained k-means algorithm. The hyphen indicates that the respective algorithm returned no solution within the time limit of 3,600 seconds.

	20% CS									
Dataset	PCCC	PCCC-N2	PCCC-N2-S	PCCC-N3-S	PCCC-N5-S	COPKM	CSC	DILS	LCC	KMEANS
n500-k2	**0.4**	0.7	**0.4**	**0.4**	**0.4**	54.5	3.5	3,638.6	6.2	0.1
n500-k5	0.6	0.6	**0.4**	0.5	0.5	12.4	3.5	3,621.5	1,076.1	0.1
n500–k10	2.2	2.1	**0.6**	0.8	1.1	7.3	2.8	3,613.8	1,239.7	0.1
n500-k20	21.3	23.9	**0.9**	1.2	2.4	–	4.4	3,622.3	303.5	0.1
n500-k50	103.0	105.6	**1.5**	1.8	2.3	13.1	4.6	3,615.3	325.3	0.1
n500-k100	150.1	150.2	**1.1**	1.3	2.0	40.3	3.2	3,619.9	461.7	0.2
n1000-k2	0.6	0.8	0.6	**0.5**	0.5	956.0	27.6	3,691.6	–	0.2
n1000-k5	0.6	0.5	**0.4**	0.5	**0.4**	252.6	25.4	3,696.6	44.8	0.2
n1000-k10	1.5	1.5	**0.6**	0.7	0.9	55.7	24.4	3,659.6	2,649.5	0.1
n1000-k20	43.4	42.4	**1.0**	1.7	3.2	–	24.2	3,646.8	3,670.0	0.2
n1000-k50	859.2	882.2	**2.6**	3.2	5.9	–	27.1	3,671.4	1,929.3	0.3
n1000-k100	1,250.3	1,257.3	**2.1**	3.1	4.1	80.2	28.4	3,655.6	1,585.6	0.5
n2000-k2	0.9	1.0	**0.7**	0.8	0.8	–	264.8	3,941.0	–	0.3
n2000-k5	0.8	0.9	**0.7**	**0.7**	**0.7**	4,395.5	260.7	3,877.4	–	0.4
n2000-k10	0.9	1.1	**0.6**	0.7	0.7	1,207.8	252.8	3,839.2	3,073.3	0.3
n2000-k20	10.7	11.9	**1.0**	1.1	2.2	288.6	250.9	3,768.9	4,291.7	0.3
n2000-k50	561.2	574.9	**2.9**	9.3	37.8	–	255.3	3,715.4	3,875.9	0.3
n2000-k100	3,750.8	3,802.3	**6.3**	7.8	17.3	168.7	248.8	3,696.2	3,666.4	0.6
n5000-k2	3.0	2.7	**2.3**	2.6	2.9	–	2,981.8	5,503.9	–	0.2
n5000-k5	2.0	2.3	2.0	**1.9**	2.1	–	2,939.3	4,871.0	–	0.2
n5000-k10	2.2	2.1	**1.8**	2.0	2.1	–	2,926.3	4,733.6	–	0.3
n5000-k20	2.8	2.9	1.9	**1.8**	2.2	–	2,752.5	4,511.0	–	0.3
n5000-k50	390.3	396.3	**3.0**	4.6	37.3	1,924.0	2,750.3	4,570.2	–	0.4
n5000-k100	3,905.5	3,908.7	**9.1**	47.8	333.6	–	2,649.0	4,514.9	–	0.5
Sum	11,064.1	11,175.0	**44.3**	96.7	463.4	45,456.7*	18,711.7	95,295.8	60,599.0*	6.2

*Nan values (–) are replaced with 3,600 before computing the sum.

Table W30: Average running times (in seconds) of five versions of the PCCC algorithm and the four state-of-the-art algorithms (COPKM, CSC, DILS, LCC) for the constraint sets of size 20% CS. The lowest values are stated in bold. The column KMEANS reports the average running time of the unconstrained k-means algorithm. The hyphen indicates that the respective algorithm returned no solution within the time limit of 3,600 seconds.