ETH Zurich
Institut für Theoretische Informatik
Prof. Dr. Angelika Steger
Prof. Dr. Emo Welzl
Prof. Dr. Peter Widmayer

Algorithms Lab

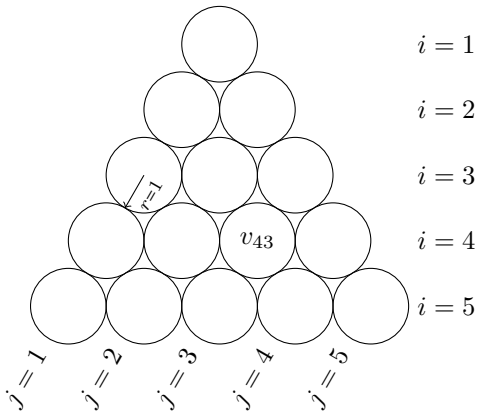
Exercise – *Bob's Burden*

Main Story Al Gore-Riethmer and his wife Ada prepared a special treat for Christmas this year.

For each of their kids they have prepared a pile of presents. Each present is wrapped in a ball-shaped package, and the balls form three Christmas trees, one for each child. Of course, Al and Ada want their children to learn something on the way. Hence they have written an integer value on each of the balls. Your task is to help Alice, Bob and Carol to get the most out of their Christmas tree.

For simplicity, we consider only two-dimensional piles of presents. Each of the three piles has the following form: At the top (first layer), there is one ball. It rests on two balls (second layer), which in turn are put on top of three balls. This goes on all the way to the bottom (layer k), where we have k balls on the ground.

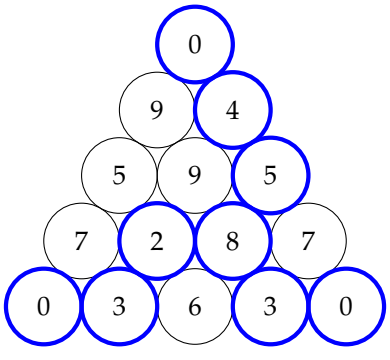
The jth ball in the ith level, denoted B_{ij}, is labeled with an integer value v_{ij}. Each of the overall $\frac{k(k+1)}{2}$ many balls in the Christmas tree has radius 1 and touches all of its surrounding balls (up to 6, depending on the position).



Bob's Burden Bob is the oldest of the three siblings and a freshman Computer Science student at PolyALG. Therefore his parents decided to give him only three presents this year, and they conveniently wrapped those in the corner packages B_{11}, B_{k1}, B_{kk} of Bob's Christmas tree. The other balls are empty, but they are necessary to support the tree construction. Depending on the strength and thickness of the cardboard it is made of, each ball has a particular weight. Luckily for Bob, Al and Ada wrote down on each ball its corresponding weight in grams.

Problem To guarantee structural soundness of Bob's present, he can not simply pick the three packages B_{11}, B_{k1}, B_{kk}. Instead he is required to choose a connected subset of balls in his Christmas tree, i.e. a subset of balls that connects the three apices of the tree.

For each ball B_{ij}, Bob is given its weight v_{ij}. Bob has to switch trains several times on his way home to his flat share. Naturally, he wants to minimize the sum of all weights v_{ij} in his subset of packages. Since Bob certainly picks his presents, we may assume they have weight $v_{11} = v_{k1} = v_{kk} = 0$, while all other balls have weight $v_{ij} > 0$. Help Bob carry the *minimum burden*.



Input The first line contains $1 \leq t \leq 30$, the number of testcases. Each of the t testcases is described as follows:

- It starts with a line that contains $\mathtt{k}$, the number of layers in Bob's tree ($2 \leq k \leq 2'000$).
- k lines follow. The ith of these lines contains the values of the i balls in layer i, given as a list of i integers $v_{i1}\ v_{i2}\ \ldots\ v_{ii}$, separated by a space (where $v_{11} = v_{k1} = v_{kk} = 0$ and $v_{ij} \in [1, 1'000]$ otherwise).

Output For each test case output one line containing the smallest possible burden in grams. A burden is the total weight of a subset S of balls that connects the three apex packages. (S connects the three apex packages, if for each pair $B_a, B_b \in \{B_{11}, B_{k1}, B_{kk}\}$ there is a sequence of balls $B_0, B_1, \ldots, B_j \in S$, such that $B_0 = B_a$, $B_j = B_b$ and for all $i,\ 0 \leq i < j$, B_i touches B_{i+1}.)

Points There are two groups of test sets which are worth 100 points in total.

1. For the first group of test sets, worth 40 points, you may assume that $k \leq 50$.
2. For the second group of test sets, worth 60 points, there are no further assumptions.

Sample Input

```
3
3
0
8 6
0 2 0
4
0
9 8
5 7 5
0 6 3 0
5
0
9 4
5 9 5
7 2 8 7
0 3 6 3 0
```

Sample Output

```
8
22
25
```