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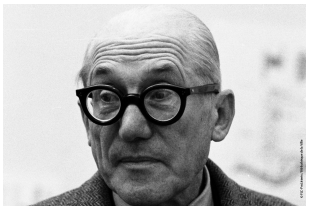
Algorithms Lab

Exercise – *Corbusier's Modulor*

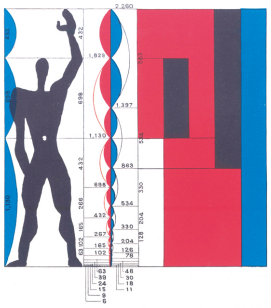


Charles-Édouard Jeanneret-Gris, who was better known as *Le Corbusier* (October 6, 1887–August 27, 1965), was a Swiss-French architect, designer, painter, urban planner, writer, and one of the pioneers of what is now called modern architecture. He was born in Switzerland and became a French citizen in 1930. His career spanned five decades, with his buildings constructed throughout Europe, India, and the Americas.

Le Corbusier invented a system for the scale of architectural proportion called *Modulor*. He described it as a 'range of harmonious measurements to suit the human scale, universally applicable to architecture and to mechanical things'.

One of your friends studies architecture and has to apply Le Corbusiers Modulor in a project where she has to design a tower. Their version of the Modulor is simple: she gets two positive integers i and k and the height of the tower needs to be congruent to i modulo k. Your friend wants to build the tower by stacking up disks of the same radius, but different (integer) heights. She already designed a couple of individual disks. Now she wants to pile up some of these disks (each disk can only be used at most once) to a tower which obeys the rules of the Modulor, that is its height, which is the sum of its individual disk heights, is congruent to i modulo k. The deadline of the project is coming closer and she asks you to help her deciding whether or not it is even possible to find a suitable *non-empty* subset of the disks which satisfies the rules of the Modulor.

(a) Centre Le Corbusier Zürich



(b) Modulor

Input The first line of the input contains the number $t \leq 100$ of test cases Each test case is described as follows.

- It starts with a line that contains three integers `n i k`, separated by a space, where n is the number of disks ($1 \leq n \leq 10^3$) and i and k are the parameters of the Modulor ($0 \leq i < k \leq 10^3$).
- The following n lines each describe a disk. They contain one integer each `h`, which is the height of the disk ($1 \leq h \leq 10^9$).

Output For every test case, your program should output, on a separate line, whether it is possible to find a non-empty subset of disks such that the sum of their heights is congruent to i modulo k. Output `yes` if it is possible and `no` otherwise.

Points There are three groups of test sets, worth 100 points in total.

1. For the first group of test sets, worth 30 points, you may assume that $n \leq 20$.
2. For the second group of test sets, worth 30 points, you may assume that $n \leq 30$.
3. For the third group of test sets, worth 40 points, there are no additional assumptions.

Corresponding sample test sets are contained in `test`i`.in/out`, for $i \in \{1, 2, 3\}$.

Sample Input

```
2
3 0 5
4
12
7
3 0 5
4
11
7
```

Sample Output

```
no
yes
```