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## **Algorithms Lab**

## Exercise - Planks

You have are n planks (i.e., one-dimensional pieces of wood), marked with numbers 1 through n. Each plank has a length  $\ell_i$ , which is a positive integer. Armed with these planks, you set about the task of building a square.

For example, using the six planks

whose lengths are  $(\ell_1,\ell_2,\ell_3,\ell_4,\ell_5,\ell_6)=(2,2,1,1,1,1)$ , you are able to build

$$\begin{array}{c|c}
2 \\
4 \\
5
\end{array}$$

As you start experimenting you see that starting from this square, it is easy to create other squares, by a combination of permuting the planks used within a given side and permuting the sides themselves. In this way, you build

and many other squares. This keeps you amused for hours, until you make the startling discovery that the square

$$\begin{array}{c|c}
2 \\
1 & 4 \\
\hline
5 & 3
\end{array}$$

can never be obtained from your starting square using only permutations of the sides or permutations of the planks withing a side. You thus decide to say that this this square is really different TM from the one you started with.

Now you start wondering: what is the largest collection of squares that you can build such that each square uses all the given planks and such that any two squares in the collection are really different<sup>TM</sup>?

**Input** The first line of the input contains the number  $t \le 30$  of test cases. Each of the t test cases is described as follows.

- The first line contains an integer n, the number of planks  $(1 \le n \le 20)$ .
- The following line contains n integers  $1_1 \dots 1_n$ , separated by a space, denoting the lengths of planks, and such that  $1 \le \ell_i \le 2^{31}$ , for all  $i \in \{1, \dots, n\}$ .

**Output** For each test case, output a separate line containing the size of the largest collection of squares that can be built using the given planks such that (1) each square uses all the given planks and (2) any two squares in the collection are really different<sup>TM</sup>.

Remark: You may assume that the answer is always at most  $2^{31}$ .

**Points** There are three groups of test sets, worth 100 points in total.

- 1. For the first group of test sets, worth 20 points, you may assume that  $n \leq 8$  and that the answer in each test case is either 0 or 1.
- 2. For the second group of test sets, worth 40 points, you may assume that the answer in each test case is either 0 or 1.

0

3. For the third group of test sets, worth 40 points, there are no additional assumptions.

Corresponding sample test sets are contained in test i. in/out, for  $i \in \{1, 2, 3\}$ .

## Sample Input

## **Sample Output**

2 7 6 6 2 2 2 3 3 4 1 2 2 3