ETH Zurich
Institut für Theoretische Informatik
Prof. Dr. Angelika Steger
Prof. Dr. Emo Welzl
Prof. Dr. Peter Widmayer

Algorithms Lab

Exercise – *Light Pattern*

New Year is just a couple of months away, and the city officials are already starting to plan how to decorate Bahnhofstrasse. This year they want to prepare a big surprise!

It starts, of course, with a lot of light bulbs in a straight line. Here *a lot* means n, and the bulbs are enumerated from 0 up to $n-1$ according to their unique distance from the Hauptbahnhof.

At the beginning of the night, each bulb is either turned on or off (we refer to this as the *state* of a bulb). Exactly at the midnight, the city officials will start changing the state of the bulbs until for all i, $0 \leq i < n/k$, all the bulbs numbered from $i \cdot k$ to $(i+1) \cdot k - 1$, in that order, follow a fixed light pattern. A light pattern is a sequence of states, and an ordered set of bulbs follows the pattern if the sequence of the states of the bulbs (in the given order) matches the sequence of the pattern.

There are two ways to change the state of a bulb,

1. change the state of a single bulb, or
2. change the state of every bulb from 0 up to $t \cdot k - 1$, for some $t \leq n/k$.

Each operation takes one second to perform, and the operation can be started only after the previous one has finished. Given the initial states of the bulbs, the city officials wants to know what is the minimum amount of time needed to produce the desirable patterns.

Input The first line of the input contains the number of test cases $1 \leq t \leq 20$. Each of the t test cases is described as follows.

- It starts with one line containing the total number of light bulbs $1 \leq n \leq 100000$, the number of bulbs $1 \leq k \leq 16$ that form a light pattern, and an integer $x \geq 0$ which represents a light pattern as following: for every $1 \leq i \leq k$, the $(k-i+1)$-th bulb of the pattern should be turned off if the i-th least significant bit of x is 0, and turned on otherwise. It is guaranteed that n is a multiple of k and that $\log_2 x < k$.
- The following line contains n bits, separated by spaces, which represent the current state of bulbs (i-th bit is 0 if the bulb numbered with $i-1$ is initially turned off, and 1 if it is turned on).

Output For each test case output the smallest possible amount of time needed to reach the desired light patterns.

Points There are two test sets. The individual points are specified below; the total number of points is 100.

1. For the first test set, worth 50 points, you may assume $k = 1$ and $x = 0$.
2. For the second test set, worth 50 points, there are no additional assumptions.

Corresponding example test sets are contained in the files `test`i`.in/out` for $i \in \{1, 2\}$.

Sample Input

```
2
8 1 0
1 1 1 0 0 1 1 1
8 2 1
1 0 1 0 0 0 1 1
```

Sample Output

```
3
3
```

Explanation In the first test case, the optimal strategy would be to first change the state of bulbs numbered with 3 and 4, and then change all the states.

The initial state of the second test case is "on off on off off off on on" and the desired light pattern is "off on". Thus the optimal strategy is to change the state of the first 4 bulbs, and then of bulbs numbered with 5 and 7.