Task Discussion
Canteen

Andreas Bärtschi

ETH Zürich

December 2, 2015

Problem Statement

Input describing **menu demands and supplies** on each day:

- n days for which to plan ahead.
- $a_i =$ **a**mount of menus that can be supplied on day i.
- $s_i =$ number of **s**tudents who want to eat on day i.
- $v_i =$ freezer **v**olume to carry on menues to the following day.

Additionally we have **assigned costs** (c_i for production, e_i for freezing energy) and **student prices** (p_i).

Questions:

- How to compute the maximum number of students that can be served?
- How to optimize the canteen's profit (while still serving all of the students)?

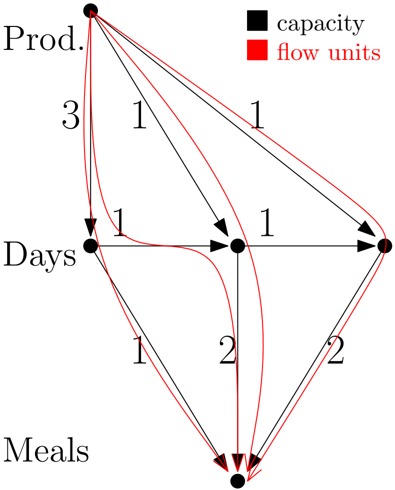
First Steps

Maximizing the number of students:

Greedy?

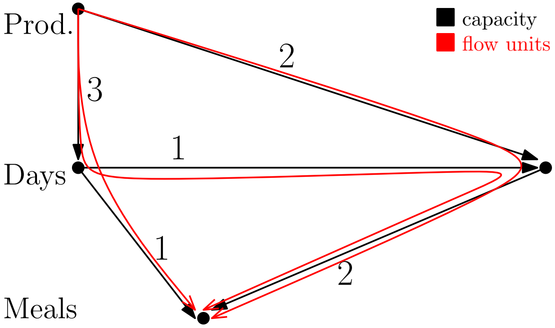
- Keep track of the partial sums of menu amounts and students:
  $\sum_{i=1}^{k} a_i \overset{?}{\lesseqqgtr} \sum_{i=1}^{k} s_i$
- Compare for each day the production + incoming freezer volume against the hungry students:
  $a_i + v_i \quad \text{vs.} \quad s_i?$
- Why should those (or even a combination of the two) work (they don't)?

For each day we have supplies and demands.
We can formulate it as a **flow problem**!



First Testset: Flow is enough

All production costs $c_i = c$, all student prices $p_i = p$, all freezing energy $e_i = 0$.

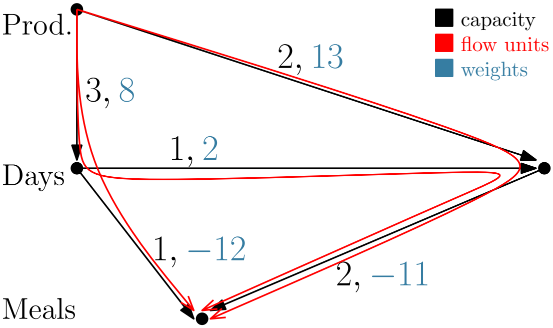


$$\begin{aligned}\textbf{Profit} &= \text{Total student prices} \\ &\quad - \text{Total production costs} \\ &= \text{Menus sold} \\ &\quad \times (\text{menu price} - \text{menu cost}) \\ &= |\text{flow}| \cdot (p - c).\end{aligned}$$

Second Testset: Mincost Maxflow

Model production costs c_i and freezing energy costs e_i as positive weights, student prices p_i as negative weights.

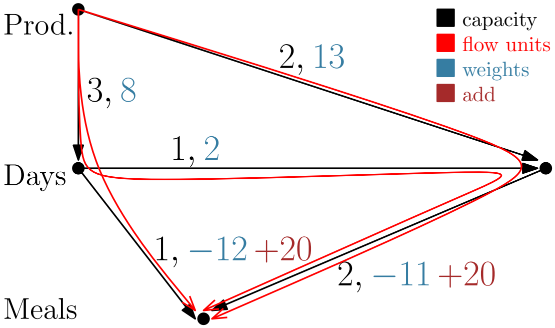
$$\begin{aligned}\textbf{Profit} &= \text{Total student prices}\\ &\quad - \text{Total production costs}\\ &= 12 + 2 \cdot 11 - 2 \cdot 8 - 2 - 13\\ &= 3 = -(-3)\\ &= -\ \text{flowcost.}\end{aligned}$$



Third Testset: Mincost Maxflow with nonnegative weights

Note: Each flow unit must go through *exactly* one edge of negative weight.
$\Rightarrow$ Add large additive weight (20) and compensate later.

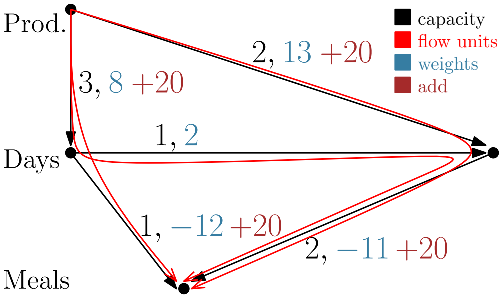
$$
\begin{aligned}
\textbf{Profit} &= \text{Total student prices} \\
&\quad - \text{Total production costs} \\
&= 12 + 2 \cdot 11 - 2 \cdot 8 - 2 - 13 \\
&= 3 = -(-3 + 3 \cdot 20) + 3 \cdot 20 \\
&= -\ (57) + 3 \cdot 20 \\
&= -\ \text{flowcost} + |\text{flow}| \cdot 20.
\end{aligned}
$$



Caveats

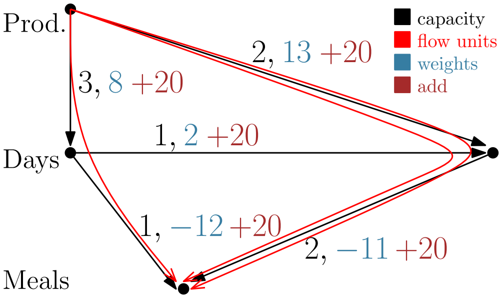
Rule of thumb: Each flow unit should go through the *same number* of edges with modified weights.

This works:



Profit $= -$ flowcost $+ |$flow$| \cdot 2 \cdot 20$

This doesn't work:



Weights make flow deviate from the optimal solution!