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Algorithms Lab

Exercise – *Light Pattern*

New Year is just a couple of months away, and the city officials are already starting to plan how to decorate Bahnhofstrasse. This year they want to prepare a big surprise!

It starts, of course, with a lot of light bulbs in a straight line. Here *a lot* means n, and the bulbs are enumerated from 0 up to $n-1$ according to their unique distance from Hauptbahnhof.

At the beginning of the night, each bulb is either turned on or off (we refer to this as the *state* of a bulb). Exactly at midnight, the city officials start changing the state of the bulbs until for all i, $0 \leq i < n/k$, all the bulbs numbered from $i \cdot k$ to $(i+1) \cdot k - 1$, in that order, follow a fixed light pattern. A light pattern is a sequence of states, and an ordered set of bulbs follows the pattern if the sequence of the states of the bulbs (in the given order) matches the sequence of the pattern.

There are two ways to change the state of a bulb,

1. change the state of a single bulb, or
2. change the state of every bulb from 0 up to $t \cdot k - 1$, for some $t \leq n/k$.

Each operation takes one second to perform, and the operation can be started only after the previous one has finished. Given the initial states of the bulbs, the city officials want to know what is the minimum amount of time needed to produce the desirable pattern.

Input The first line of the input contains the number $t \leq 20$ of test cases. Each of the t test cases is described as follows.

- It starts with a line that contains three integers `n k x`, separated by a space. They denote
  - n, the total number of light bulbs ($1 \leq n \leq 10^5$ and n is a multiple of k);
  - k, the number of bulbs that form a light pattern ($1 \leq k \leq 16$);
  - x, represents the light pattern as follows: for every $1 \leq i \leq k$, the $(k-i+1)$-th bulb of the pattern should be turned off if the i-th least significant bit of x is 0, and turned on otherwise ($0 \leq x$ and $\log_2 x < k$).
- The following line contains n bits, separated by a space, which represent the current state of bulbs: i-th bit is 0 if the bulb numbered with $i-1$ is initially turned off, and 1 if it is turned on.

Output For each test case output the smallest possible amount of time needed to reach the desired light pattern.

Points There are two groups of test sets, worth 100 points in total.

1. For the first group of test sets, worth 50 points, you may assume $k = 1$ and $x = 0$.
2. For the second group of test sets, worth 50 points, there are no additional assumptions.

Corresponding sample test sets are contained in `test`i`.in/out` for $i \in \{1, 2\}$.

Sample Input

```
2
8 1 0
1 1 1 0 0 1 1 1
8 2 1
1 0 1 0 0 0 1 1
```

Sample Output

```
3
3
```

Explanation In the first test case, an optimal strategy would be to first change the state of bulbs numbered with 3 and 4, and then change all the states.

The initial state of the second test case is `on off on off off off on on` and the desired light pattern is `off on`. Thus, an optimal strategy is to change the state of the first 4 bulbs, and then of bulbs numbered with 5 and 7.