ETH Zurich
Institut für Theoretische Informatik
Prof. Dr. Angelika Steger
Prof. Dr. Emo Welzl
Prof. Dr. Peter Widmayer

Algorithms Lab

Exercise – *On Her Majesty's Secret Service*

James Bond is out there again, saving the world. This time his mission takes him into the Swiss Alps, to the *Schilthorn*. With 2'970 meters, this summit overlooks the valley of Lauterbrunnen in the Bernese Oberland and features a panoramic view of the Alps. But James is not here for the view: On top of *Piz Gloria*, the evil mastermind and head of *SPECTRE* Ernst Stavro Blofeld set up a secret allergy-research institute where he hypnotizes and brainwashes his patients to prepare them for bacteriological warfare.

Bond and his fellow agents surrounded the compound and spread throughout the mountain. But they are in great danger! Blofeld threatens to launch a huge avalanche that would bury the whole mountain resort instantly. Fortunately, all agents are on skis so that they can escape quickly and hide inside some high-security shelters nearby. Your task is to compute the *minimum advance warning time*, such that every agent can enter a shelter before the avalanche launch.

To reach a shelter the agents can use slopes and ski lifts. But moving around takes time. Time is also required to enter a shelter, which involves a protocol including a retina scan and voice recognition tests. At every shelter, the entering protocol can be performed by at most one agent at a time. If an agent wants to enter a shelter while another agent is still in protocol there, (s)he has to wait. Finally, the capacity of each shelter is limited. Therefore, careful coordination is required to decide who hides where…

Input The first line of the input contains the number $t \leq 30$ of test cases. Each of the t test cases is described as follows:

- It starts with a line that contains six integers `n m a s c d`, separated by a space. They denote
  - n, the number of relevant positions $p_0, \ldots, p_{n-1}$ around Schilthorn ($1 \leq n \leq 10^3$).
  - m, the number of slopes and ski lifts around Schilthorn ($0 \leq m \leq 5 \cdot 10^3$).
  - a, the number of secret agents ($1 \leq a \leq 10^2$).
  - s, the number of shelters ($1 \leq s \leq 10^2$).
  - c, the capacity of each shelter ($1 \leq c \leq 2$).
  - d, the time (in seconds) an agent needs to perform the entering protocol at a shelter ($1 \leq d \leq 10^3$).
- The following m lines describe the slopes and ski lifts around Schilthorn. Each line contains four entities `w x y z`, separated by a space, where $w \in \{\texttt{S}, \texttt{L}\}$ is a character and x, y, z are integers with $0 \leq x, y \leq n-1$ and $1 \leq z \leq 10^4$. If $\texttt{w} = \texttt{S}$, then the line describes a *slope* that an agent can use to ski from p_x to p_y in z seconds (in this direction only). Otherwise, $\texttt{w} = \texttt{L}$ and the line describes a *ski lift* between p_x and p_y. A lift can be used in both directions, each taking z seconds. Slopes and lifts can be used by arbitrarily many agents at the same time.

- The following line describes the starting positions of the agents. The line contains a integers $\alpha_0 \ \ldots \ \alpha_{a-1}$, separated by a space and such that $0 \leq \alpha_i \leq n-1$, for $i \in \{0, \ldots, a-1\}$. This means that agent i starts at p_{α_i}.
- The following line describes the positions of the shelters. The line contains s integers $\sigma_0 \ \ldots \ \sigma_{s-1}$, separated by a space and such that $0 \leq \sigma_i \leq n-1$, for $i \in \{0, \ldots, s-1\}$. This means that shelter i is located at p_{σ_i}.

Note: There may be multiple shelters and/or agents at the same position.

Output For each test case, output a line with one integer t, the minimum number of seconds needed until all agents safely entered a shelter. You may assume that there always exists a way such that all the agents can reach and enter a shelter.

Points There are six groups of test sets which are worth 100 points in total.

1. For the first group of test sets, worth 20 points, you may assume that James Bond is on his own ($a = 1$).
2. For the second group of test sets, worth 20 points, you may assume that only one agent fits into each shelter and it takes either 9 or 10 seconds for all agents to safely enter a shelter ($c = 1$ and $t \in \{9, 10\}$).
3. For the third group of test sets, worth 10 points, you may assume that only one agent fits into each shelter and no more than 10 seconds are needed for all agents to safely enter a shelter ($c = 1$ and $t \leq 10$).
4. For the forth group of test sets, worth 10 points, you may assume that only one agent fits into each shelter ($c = 1$).
5. For the fifth group of test sets, worth 20 points, there are no additional assumptions.
6. The sixth group of test sets, worth 20 points, was the hidden test set at the exam.
   You may assume that only one agent fits into each shelter ($c = 1$).

Hint: Note that for almost all the subtasks, only a single agents fits into each shelter ($c = 1$). Try to solve this case first and let the restrictions of the many subtasks guide you to the full solution!

Corresponding sample test sets are contained in `test`i`.in/out`, for $i \in \{1, 2, 3, 4, 5\}$.

Sample Input

```
5
3 3 1 1 1 1
S 0 1 2
L 0 2 6
S 1 2 3
0
2
6 6 3 3 1 1
S 0 3 8
S 0 4 9
S 1 4 8
S 1 5 8
S 2 3 8
S 2 5 9
0 1 2
3 4 5
6 5 3 4 1 1
S 0 1 1
S 1 2 2
L 0 2 4
S 3 4 5
S 5 3 3
0 3 5
3 2 4 4
6 5 3 4 2 1
S 0 1 1
S 1 2 2
L 0 2 4
S 3 4 5
S 5 3 3
0 3 5
3 2 4 4
6 5 3 4 2 4
S 0 1 1
S 1 2 2
L 0 2 4
S 3 4 5
S 5 3 3
0 3 5
3 2 4 4
```

Sample Output

```
6
10
6
4
8
```

Hint: Draw these sample graphs on a sheet of paper.