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## Exercise – *Hong Kong*

Tricked by the relentless Detective Fix, Passepartout gets wasted in a Hong Kong opium den. He barely manages to carry himself onto the steamboat heading for Japan in time, but forgets to inform Phileas about its early departure. As a result, Phileas is stuck in Hong Kong and in desperate need of transportation.

In lack of a suitable transport option on water, he decides to try his luck through the air and go by balloon. But as balloon rides are considered to be extremely dangerous, they are forbidden by local authorities. Therefore, the preparations for the trip have to be done well hidden in a forest.

Phileas has a map of the forest, along with a number of available balloons, each specified by location and size. He may need to move the balloon to a spot suitable for takeoff, which is any location with a sufficient takeoff clearance (sufficiently far away from all trees). As balloons are rather heavy, Phileas can move a balloon only when it is inflated. In addition, the balloon must not get punctured by a tree. Therefore, careful planning is required.

On the map we consider the situation in a two-dimensional model, as seen from above. Trees appear as open<sup>‡</sup> disks. For simplicity we assume that all trees have the same size. Initially all balloons are deflated. In order to move a balloon, it must be inflated first. After inflation, a balloon appears as an open disk that is centered at its initial position. It may be impossible to inflate a balloon if some tree is too close. An inflated balloon can be *moved* around freely, as long as it remains disjoint from all trees. As we use open disks in our model, a balloon and a tree may touch but they must not properly overlap. Other balloons do not impede the movement of an inflated balloon because they can be left deflated so that they are "flat".

Which of the balloons can be moved to a spot suitable for takeoff?

**Input** The first line of the input contains the number  $t \leq 30$  of test cases. Each of the t test cases is described as follows.

- The first line contains three integers n m r, separated by a space. They denote
  - n, the number of trees  $(1 \le n \le 4 \cdot 10^4)$ ;
  - m, the number of available balloons  $(1 \le m \le 9 \cdot 10^4)$ ;
  - r, the radius of the trees  $(1 \leqslant r < 2^{50})$ .
- The following n lines define the positions  $a_0, \ldots, a_{n-1}$  of the trees. Each position is described by two integer coordinates x y, separated by a space and such that  $|x|, |y| < 2^{50}$ . You may assume that these positions are pairwise distinct. However, trees (as disks) may intersect.
- The next m lines define the available balloons  $b_0, \ldots, b_{m-1}$ . Each balloon is described by three integers x y s, separated by a space and such that  $1 \le s$  and  $|x|, |y|, s < 2^{50}$ . Here x and y define the initial position of the center of the balloon, and s defines its radius.

<sup>&</sup>lt;sup>‡</sup>that is, the boundary circle is not part of the disk

Output For each test case output one line with a single character "y" or "n" for each balloon, that is, a string  $u_0u_1\cdots u_{m-1}$  of m characters. For each  $i\in\{0,\ldots,m-1\}$ , the character  $u_i$  is "y" if and only if there exists a position  $t_i\in\mathbb{R}^2$  that the balloon  $b_i$  with radius  $s_i$  can be moved to from its starting position so that

- at any point during this movement the balloon is disjoint from all trees and
- if centered at  $t_i$ , the balloon has takeoff clearance at least  $r + s_i$ , that is, all trees have distance at least  $r + s_i$  to the balloon.

**Points** There are five groups of test sets, each of which is worth 20 points.

- 1. For the first group of test sets, you may assume that the balloons need not be moved before takeoff. That is, either a takeoff is possible from the starting position or it is impossible altogether.
- 2. For the second group of test sets, you may assume  $m \leq 10^2$  and that a balloon can be moved to a suitable spot for takeoff if and only if it can be moved arbitrarily far away from all trees.
- 3. For the third group of test sets, you may assume that a balloon can be moved to a suitable spot for takeoff if and only if it can be moved arbitrarily far away from all trees.
- 4. For the fourth group of test sets, there are no additional assumptions.
- 5. For the fifth group of test sets, which is hidden, there are no additional assumptions.

Corresponding sample test sets are contained in testi.in/out, for  $i \in \{1, 2, 3, 4\}$ .

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<sup>\*</sup>Recall that the distance between two geometric objects is the distance between a pair of closest points. For instance, the distance between a unit disk centered at (0,0) and a unit disk centered at (3,0) is one.