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Algorithms Lab

Exercise – *Even matrices*

You are still part of a team to develop a pseudorandom number generator. Your generator has already passed some simple statistical tests, but now it is time to get serious. You arrange a list of bits produced by the generator in an n by n matrix. If

$$M = \begin{pmatrix} x_{1,1} & x_{1,2} & & x_{1,n} \\ x_{2,1} & x_{2,2} & \cdots & x_{2,n} \\ & \vdots & \ddots & \\ x_{n,1} & x_{n,2} & & x_{n,n} \end{pmatrix}$$

was a truly random matrix of bits, then it would have the property that the sum

$$\sum_{i'=i_1}^{i_2} \sum_{j'=j_1}^{j_2} x_{i',j'}$$

is even for about half of the quadruples (i_1, i_2, j_1, j_2), $1 \leq i_1 \leq i_2 \leq n$ and $1 \leq j_1 \leq j_2 \leq n$.

To check whether this is the case for your generator, you need to be able to count the number of such quadruples.

Input The first line of the input contains the number $t \leq 15$ of test cases. Each of the t test cases is described as follows.

- It starts with a line that contains an integer `n` such that $1 \leq n \leq 200$.
- This is followed by n lines, where the i-th line contains the n integers $\texttt{x}_{i,1} \ \ldots \ \texttt{x}_{i,n}$, separated by a space, such that $x_{i,j} \in \{0, 1\}$, for all $i \in \{1, \ldots, n\}$ and $j \in \{1, \ldots, n\}$.

Output For each test case output a single line that contains the number of quadruples (i_1, i_2, j_1, j_2) where $1 \leq i_1 \leq i_2 \leq n$ and $1 \leq j_1 \leq j_2 \leq n$ and for which the sum

$$\sum_{i'=i_1}^{i_2} \sum_{j'=j_1}^{j_2} x_{i',j'}$$

is even.

Points There are three groups of test sets, worth 100 points in total.

1. For the first group of test sets, worth 20 points, you may assume that $1 \leq n \leq 10$.
2. For the second group of test sets, worth 50 points, you may assume that $1 \leq n \leq 50$.
3. For the third group of test set, worth 30 points, there are no additional assumptions.

Corresponding sample test sets are contained in `test`i`.in/out` for $i \in \{1, 2, 3\}$.

Sample Input

```
3
2
1 1
1 1
3
1 0 1
0 1 0
0 0 1
4
1 1 0 0
0 0 1 1
1 0 1 0
0 1 0 1
```

Sample Output

```
5
15
52
```