ETH Zurich
Institut für Theoretische Informatik
Prof. Dr. Angelika Steger
Prof. Dr. Emo Welzl
Prof. Dr. Peter Widmayer

# Algorithms Lab

**Exercise** – *Deck of Cards*

You are given a deck of $n$ custom made cards, denoted by $0, \ldots, n-1$ going from the top of the deck to the bottom. Each card $i$ has a number $v_i$ which represents its value. You play a strange game with your younger brother, where he tells you his favorite number $k$ and you need to find cards $i$ and $j$ such that $i \leq j$ and $\sum_{\ell=i}^{j} v_\ell = k$. Since you are older than your brother, you know that finding such a subset of the deck won't always be possible. Thus, you want to write a program which finds two cards $i$ and $j$ such that the sum $\sum_{\ell=i}^{j} v_\ell$ is as close as possible to $k$. If there are multiple candidates for the solution, find the one which is lexicographically smallest.

**Input**  The first line of the input contains the number $1 \leq t \leq 80$ of test cases in the test set. Each of the $t$ test cases is described as follows:

- It starts with a line that contains two integers n and k, the number of cards and the number $k$, separated by space, where $1 \leq n \leq 10^5$ and $0 \leq k \leq 2^{30}$.

- The following line defines the values of the cards 0 to $n - 1$, in that order. It contains $n$ integers $v_0 \ldots v_{n-1}$, separated by spaces and such that $0 \leq v_i \leq 2^{30}$, for $i \in \{0, \ldots, n-1\}$.

It is guaranteed that $\sum_{i=0}^{n-1} v_i \leq 2^{30}$.

**Output**  A solution is a pair $i, j$ of cards with $i \leq j$. We define the value of the solution $i, j$ as

$$\mathrm{val}(i, j) := \left| k - \sum_{\ell=i}^{j} v_\ell \right|.$$

For each test case output a separate line containing two numbers i and j, separated by space, corresponding to the solution $i, j$ with the smallest value. If there are multiple such solutions, output the lexicographically smallest one.

*Note:* $(i, j)$ is lexicographically smaller than $(i', j')$ iff $i < i'$ or $i = i'$ and $j < j'$.

**Points**  There are four test sets, which amount to 100 points in total.

1. For the first test set, worth 30 points, you may assume $n \leq 200$

2. For the second test set, worth 40 points, you may assume $n \leq 3000$.

3. For the third test set, worth 10 points, there are no additional assumptions.

4. For the fourth (hidden) test set, worth 20 points, there are also no additional assumptions.

Corresponding sample test sets are contained in test$i$.in/out, for $i \in \{1, 2, 3\}$.

| Sample Input | Sample Output |
|---|---|
| 3 | 1 1 |
| 5 1 | 0 1 |
| 3 1 5 2 1 | 0 4 |
| 5 6 | |
| 3 2 5 2 3 | |
| 5 22 | |
| 2 3 4 6 2 | |