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Algorithms Lab

Exercise – *Deck of Cards*

You are given a deck of n custom made cards, denoted by $0,\ldots,n-1$ going from the top of the deck to the bottom. Each card i has a number v_i which represents its value. You play a strange game with your younger brother, where he tells you his favorite number k and you need to find cards i and j such that $i \leq j$ and $\sum_{\ell=i}^{j} v_{\ell} = k$. Since you are older than your brother, you know that finding such a subset of the deck won't always be possible. Thus, you want to write a program which finds two cards i and j such that the sum $\sum_{\ell=i}^{j} v_{\ell}$ is as close as possible to k. If there are multiple candidates for the solution, find the one which is lexicographically smallest.

Input The first line of the input contains the number $1 \le t \le 80$ of test cases in the test set. Each of the t test cases is described as follows:

- It starts with a line that contains two integers n and k, the number of cards and the number k, separated by space, where $1 \le n \le 10^5$ and $0 \le k \le 2^{30}$.
- The following line defines the values of the cards 0 to n-1, in that order. It contains n integers v_0, \ldots, v_{n-1} , separated by spaces and such that $0 \le v_i \le 2^{30}$, for $i \in \{0, \ldots, n-1\}$.

It is guaranteed that $\sum_{i=0}^{n-1} v_i \leq 2^{30}$.

Output A solution is a pair i, j of cards with $i \le j$. We define the value of the solution i, j as

$$\operatorname{val}(i,j) := \left| k - \sum_{\ell=i}^{j} v_{\ell} \right|.$$

For each test case output a separate line containing two numbers i and j, separated by space, corresponding to the solution i, j with the smallest value. If there are multiple such solutions, output the lexicographically smallest one.

Note: (i, j) is lexicographically smaller than (i', j') iff i < i' or i = i' and j < j'.

Points There are four test sets, which amount to 100 points in total.

- 1. For the first test set, worth 30 points, you may assume $n \le 200$
- 2. For the second test set, worth 40 points, you may assume $n \leq 3000$.
- 3. For the third test set, worth 10 points, there are no additional assumptions.
- 4. For the fourth (hidden) test set, worth 20 points, there are also no additional assumptions.

Corresponding sample test sets are contained in test i. in/out, for $i \in \{1, 2, 3\}$.

Sample Input

Sample Output