

## On-line Appendix

On-line appendix to accompany *It might not make a big DIF: Improved Differential Test Functioning statistics that account for sampling variability*. The tables contain simulation results pertaining to Type I error rates ( $sDTF$ ) and cut-off values at the 95% percentile ( $uDTF$ ) when varying sample size (500, 1000, 3000), DIF size (0.5 and 1.0), test size (30, 40, and 50), parameters containing DIF ( $d$ ,  $a$ ,  $d$  and  $a$ ), and number of items containing DIF (4, 8, and 12 in the 3PLM design, and 4, 6, and 8 in the GRM design). Finally, Type I error rates for a simulation with 2PLM items is included.

DIF Parameters	Test Size	DIF Size	# DIF Items	<i>sDTF</i>			<i>uDTF</i> <sub>%</sub> ( $\alpha = .95$ )				
				$p < .10$	$p < .05$	$p < .01$	> 2	> 2.5	> 3	> 3.5	> 4
<i>a &amp; d</i>	30	0.5	4	.178	.106	.026	1.000	.969	.665	.341	.142
			8	.419	.306	.132	1.000	.997	.948	.769	.530
			12	.734	.623	.362	1.000	1.000	.996	.975	.907
		1	4	.699	.584	.373	1.000	1.000	.995	.969	.897
			8	.984	.975	.931	1.000	1.000	1.000	1.000	1.000
			12	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000
		0.5	4	.145	.081	.019	1.000	.931	.509	.190	.054
			8	.331	.198	.062	1.000	.991	.818	.507	.249
			12	.534	.393	.178	1.000	.999	.964	.836	.605
		1	4	.517	.394	.184	1.000	1.000	.976	.845	.650
			8	.928	.873	.733	1.000	1.000	1.000	1.000	.997
			12	.999	.992	.967	1.000	1.000	1.000	1.000	1.000
	40	0.5	4	.096	.047	.010	1.000	.934	.486	.148	.038
			8	.211	.121	.027	1.000	.988	.747	.362	.146
			12	.380	.250	.093	1.000	.997	.919	.683	.398
		1	4	.354	.244	.092	1.000	.998	.916	.700	.403
			8	.840	.758	.529	1.000	1.000	1.000	.999	.984
			12	.986	.965	.893	1.000	1.000	1.000	1.000	1.000
	50	0.5	4	.079	.042	.008	1.000	.909	.447	.156	.050
			8	.089	.044	.010	1.000	.927	.486	.166	.052
			12	.165	.088	.024	1.000	.943	.579	.229	.075
		1	4	.103	.052	.012	1.000	.931	.485	.170	.047
			8	.264	.162	.048	1.000	.964	.609	.275	.082
			12	.514	.375	.161	1.000	.984	.780	.442	.172
<i>d</i>	30	0.5	4	.124	.066	.008	1.000	.889	.506	.203	.077
			8	.227	.133	.028	1.000	.935	.573	.252	.000
			12	.411	.287	.107	.999	.961	.708	.353	.140
		1	4	.237	.154	.033	1.000	.929	.598	.229	.079
			8	.626	.493	.245	1.000	.992	.824	.530	.252
			12	.922	.851	.629	1.000	1.000	.973	.856	.570
		0.5	4	.081	.038	.010	1.000	.863	.420	.148	.034
			8	.136	.084	.022	1.000	.892	.465	.203	.075
			12	.246	.153	.053	1.000	.922	.559	.234	.081
		1	4	.148	.077	.016	1.000	.894	.466	.173	.046
			8	.402	.277	.104	1.000	.949	.673	.328	.104
			12	.727	.591	.348	1.000	.992	.865	.567	.257
	40	0.5	4	.079	.042	.008	1.000	.909	.447	.156	.050
			8	.089	.044	.010	1.000	.927	.486	.166	.052
			12	.165	.088	.024	1.000	.943	.579	.229	.075
		1	4	.103	.052	.012	1.000	.931	.485	.170	.047
			8	.264	.162	.048	1.000	.964	.609	.275	.082
			12	.514	.375	.161	1.000	.984	.780	.442	.172
	50	0.5	4	.079	.042	.008	1.000	.909	.447	.156	.050
			8	.089	.044	.010	1.000	.927	.486	.166	.052
			12	.165	.088	.024	1.000	.943	.579	.229	.075
		1	4	.103	.052	.012	1.000	.931	.485	.170	.047
			8	.264	.162	.048	1.000	.964	.609	.275	.082
			12	.514	.375	.161	1.000	.984	.780	.442	.172
<i>a</i>	30	0.5	4	.084	.045	.011	1.000	.938	.584	.266	.097
			8	.102	.053	.012	1.000	.995	.875	.657	.411
			12	.127	.065	.016	1.000	1.000	.980	.919	.801
		1	4	.144	.079	.029	1.000	.999	.986	.927	.818
			8	.214	.136	.041	1.000	1.000	1.000	.999	.999
			12	.284	.191	.090	1.000	1.000	1.000	1.000	1.000
		0.5	4	.072	.037	.008	1.000	.920	.446	.153	.041
			8	.088	.041	.012	1.000	.973	.739	.436	.195
			12	.100	.050	.008	1.000	.996	.915	.736	.495
		1	4	.126	.064	.010	1.000	.998	.935	.768	.531
			8	.190	.107	.032	1.000	1.000	1.000	.999	.999
			12	.164	.091	.032	1.000	1.000	1.000	1.000	1.000
	40	0.5	4	.079	.034	.004	1.000	.937	.461	.136	.041
			8	.066	.034	.004	1.000	.970	.624	.294	.101
			12	.081	.036	.005	1.000	.988	.829	.574	.312
		1	4	.084	.037	.006	1.000	.999	.855	.578	.305
			8	.111	.052	.012	1.000	1.000	.998	.984	.946
			12	.125	.068	.021	1.000	1.000	1.000	1.000	.999

Table 1: DTF statistics for the 3PLM when the simulated parameter DIF are unidirectional and  $N = 500$ .

DIF Parameters	Test Size	DIF Size	# DIF Items	<i>sDTF</i>			<i>uDTF</i> <sub>%</sub> ( $\alpha = .95$ )				
				$p < .10$	$p < .05$	$p < .01$	$> 2$	$> 2.25$	$> 2.5$	$> 2.75$	$> 3$
<i>a &amp; d</i>	30	0.5	4	.355	.260	.112	.695	.480	.306	.163	.071
			8	.718	.621	.425	.988	.952	.882	.771	.640
			12	.922	.889	.756	1.000	1.000	.995	.982	.969
		1	4	.907	.868	.758	1.000	1.000	.999	.990	.971
			8	.996	.994	.987	1.000	1.000	1.000	1.000	1.000
			12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		40	4	.257	.167	.059	.486	.264	.112	.039	.012
			8	.551	.448	.243	.911	.767	.592	.413	.246
			12	.820	.739	.545	.997	.978	.936	.867	.736
		1	4	.812	.745	.534	.994	.982	.958	.898	.784
			8	.991	.987	.973	1.000	1.000	1.000	1.000	1.000
			12	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000
	50	0.5	4	.188	.116	.036	.319	.124	.046	.013	.006
			8	.437	.314	.144	.765	.568	.364	.188	.080
			12	.691	.564	.370	.967	.896	.777	.597	.412
		1	4	.682	.582	.351	.976	.920	.813	.665	.480
			8	.989	.974	.930	1.000	1.000	1.000	1.000	.998
			12	.999	.999	.997	1.000	1.000	1.000	1.000	1.000
<i>d</i>	30	0.5	4	.196	.102	.024	.288	.126	.038	.015	.005
			8	.474	.356	.167	.525	.290	.126	.051	.013
			12	.736	.616	.389	.751	.508	.284	.147	.063
		1	4	.474	.348	.145	.531	.291	.139	.053	.022
			8	.918	.862	.683	.919	.767	.544	.347	.171
			12	.999	.998	.974	.999	.994	.960	.852	.703
		40	4	.145	.083	.023	.208	.082	.024	.007	.002
			8	.332	.240	.095	.346	.159	.063	.025	.007
			12	.499	.389	.183	.438	.239	.096	.031	.003
		1	4	.306	.205	.086	.322	.138	.046	.015	.002
			8	.766	.660	.417	.715	.456	.261	.113	.045
			12	.966	.939	.820	.960	.863	.682	.447	.237
	50	0.5	4	.127	.071	.017	.165	.061	.015	.005	.001
			8	.272	.168	.053	.240	.087	.028	.008	.003
			12	.409	.286	.126	.346	.176	.064	.025	.003
		1	4	.226	.136	.057	.227	.082	.026	.006	.002
			8	.598	.465	.256	.498	.261	.131	.054	.016
			12	.870	.804	.587	.816	.601	.381	.215	.092
<i>a</i>	30	0.5	4	.125	.086	.021	.583	.393	.216	.115	.049
			8	.170	.100	.028	.951	.878	.756	.627	.470
			12	.208	.132	.051	.999	.994	.977	.939	.883
		1	4	.293	.227	.102	.995	.990	.984	.972	.931
			8	.381	.288	.164	1.000	1.000	1.000	1.000	1.000
			12	.421	.326	.217	1.000	1.000	1.000	1.000	1.000
		40	4	.134	.075	.016	.387	.209	.100	.040	.019
			8	.146	.084	.022	.800	.639	.469	.292	.147
			12	.151	.085	.024	.976	.930	.836	.702	.556
		1	4	.231	.151	.051	.986	.956	.907	.810	.676
			8	.306	.224	.112	1.000	.999	.999	.999	.999
			12	.378	.287	.157	1.000	1.000	1.000	1.000	1.000
	50	0.5	4	.114	.051	.009	.270	.112	.044	.012	.003
			8	.122	.077	.026	.634	.404	.229	.128	.053
			12	.152	.089	.024	.914	.784	.632	.449	.279
		1	4	.172	.114	.034	.937	.859	.710	.534	.362
			8	.231	.154	.063	1.000	1.000	1.000	1.000	.989
			12	.295	.209	.095	1.000	1.000	1.000	1.000	1.000

Table 2: DTF statistics for the 3PLM when the simulated parameter DIF are unidirectional and  $N = 1000$ .

DIF Parameters	Test Size	DIF Size	# DIF Items	<i>sDTF</i>			<i>uDTF</i> <sub>%</sub> ( $\alpha = .95$ )				
				$p < .10$	$p < .05$	$p < .01$	$> 2$	$> 2.5$	$> 3$	$> 3.5$	$> 4$
<i>a &amp; d</i>	30	0.5	4	.137	.072	.021	.302	.051	.007	.001	.000
			8	.223	.137	.049	.484	.129	.024	.003	.000
			12	.326	.220	.116	.688	.298	.089	.016	.003
		1	4	.419	.309	.176	.868	.545	.232	.071	.014
			8	.782	.703	.541	.995	.968	.882	.671	.410
			12	.922	.902	.809	1.000	.999	.995	.978	.917
	40	0.5	4	.142	.081	.018	.216	.024	.000	.000	.000
			8	.160	.092	.023	.271	.047	.004	.000	.000
			12	.225	.153	.064	.463	.132	.018	.006	.000
		1	4	.348	.249	.100	.692	.250	.053	.007	.001
			8	.628	.541	.365	.966	.866	.590	.256	.067
			12	.844	.798	.667	.999	.988	.955	.820	.554
	50	0.5	4	.122	.063	.015	.164	.013	.000	.000	.000
			8	.150	.083	.029	.205	.025	.001	.001	.000
			12	.196	.117	.041	.312	.051	.005	.000	.000
		1	4	.223	.151	.053	.506	.123	.015	.000	.000
			8	.544	.441	.267	.937	.685	.304	.065	.005
			12	.755	.680	.518	.998	.961	.798	.476	.161
<i>d</i>	30	0.5	4	.099	.046	.011	.234	.031	.001	.000	.000
			8	.101	.045	.007	.253	.052	.004	.001	.000
			12	.098	.052	.010	.228	.037	.001	.000	.000
		1	4	.108	.053	.013	.269	.046	.006	.000	.000
			8	.093	.046	.011	.287	.055	.011	.001	.000
			12	.085	.044	.008	.379	.100	.015	.004	.001
	40	0.5	4	.112	.048	.010	.179	.017	.001	.000	.000
			8	.102	.050	.010	.172	.025	.002	.000	.000
			12	.095	.042	.006	.162	.018	.003	.000	.000
		1	4	.112	.059	.016	.183	.022	.002	.000	.000
			8	.122	.063	.012	.210	.021	.001	.000	.000
			12	.105	.047	.009	.261	.035	.004	.000	.000
	50	0.5	4	.104	.055	.010	.153	.014	.000	.000	.000
			8	.104	.046	.013	.136	.011	.000	.000	.000
			12	.110	.055	.009	.148	.012	.001	.000	.000
		1	4	.092	.051	.005	.143	.008	.001	.000	.000
			8	.112	.065	.026	.159	.020	.002	.000	.000
			12	.104	.055	.014	.175	.016	.001	.000	.000
<i>a</i>	30	0.5	4	.129	.067	.017	.290	.042	.003	.000	.000
			8	.156	.095	.016	.403	.078	.008	.000	.000
			12	.166	.108	.028	.582	.225	.052	.004	.001
		1	4	.209	.123	.039	.790	.446	.161	.039	.004
			8	.293	.212	.109	.991	.944	.804	.543	.280
			12	.339	.268	.147	.999	.997	.973	.920	.782
	40	0.5	4	.104	.061	.008	.169	.021	.003	.000	.000
			8	.138	.081	.021	.289	.041	.004	.000	.000
			12	.142	.079	.017	.380	.078	.007	.001	.000
		1	4	.143	.091	.027	.596	.202	.039	.002	.001
			8	.225	.142	.049	.947	.741	.400	.138	.023
			12	.268	.185	.086	.997	.967	.859	.636	.349
	50	0.5	4	.129	.074	.017	.159	.014	.002	.000	.000
			8	.119	.066	.021	.186	.018	.000	.000	.000
			12	.141	.070	.018	.272	.044	.006	.000	.000
		1	4	.162	.102	.029	.439	.095	.009	.000	.000
			8	.191	.122	.039	.838	.503	.181	.036	.003
			12	.202	.134	.060	.988	.903	.616	.312	.089

Table 5: DTF statistics for the 3PLM when the simulated parameter DIF are bidirectional and  $N = 1000$ .

DIF Parameters	Test Size	DIF Size	# DIF Items	<i>sDTF</i>			<i>uDTF</i> <sub>%</sub> ( $\alpha = .95$ )				
				$p < .10$	$p < .05$	$p < .01$	> 2	> 2.5	> 3	> 3.5	> 4
<i>a &amp; d</i>	30	0.5	4	.639	.553	.381	.115	.007	.000	.000	.000
			8	.959	.940	.877	.918	.549	.153	.022	.001
			12	.993	.989	.979	1.000	.987	.870	.516	.196
		1	4	.985	.983	.964	.999	.988	.919	.675	.341
			8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		0.5	4	.548	.436	.258	.015	.000	.000	.000	.000
			8	.895	.856	.741	.564	.123	.010	.001	.000
			12	.983	.972	.935	.982	.753	.292	.044	.002
		1	4	.973	.957	.928	.978	.820	.416	.112	.010
			8	.999	.999	.998	1.000	1.000	1.000	.998	.984
			12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	40	0.5	4	.416	.311	.170	.003	.000	.000	.000	.000
			8	.818	.751	.574	.180	.009	.000	.000	.000
			12	.965	.944	.883	.816	.270	.022	.001	.000
		1	4	.934	.911	.843	.859	.402	.062	.003	.000
			8	.999	.999	.998	1.000	1.000	.987	.936	.716
			12	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	50	0.5	4	.235	.150	.049	.000	.000	.000	.000	.000
			8	.538	.450	.219	.000	.000	.000	.000	.000
			12	.805	.713	.519	.003	.000	.000	.000	.000
		1	4	.536	.416	.199	.000	.000	.000	.000	.000
			8	.951	.910	.802	.016	.000	.000	.000	.000
			12	1.000	1.000	.994	.282	.006	.000	.000	.000
<i>d</i>	30	0.5	4	.403	.290	.127	.000	.000	.000	.000	.000
			8	.854	.795	.570	.004	.000	.000	.000	.000
			12	.991	.985	.937	.104	.004	.000	.000	.000
		1	4	.855	.772	.577	.011	.001	.000	.000	.000
			8	.999	.998	.995	.497	.059	.001	.000	.000
			12	1.000	1.000	.999	.996	.800	.230	.007	.000
	40	0.5	4	.312	.215	.067	.000	.000	.000	.000	.000
			8	.658	.553	.351	.001	.000	.000	.000	.000
			12	.933	.895	.732	.005	.000	.000	.000	.000
		1	4	.677	.567	.349	.000	.000	.000	.000	.000
			8	.995	.987	.938	.081	.000	.000	.000	.000
			12	1.000	1.000	.999	.747	.126	.000	.000	.000
	50	0.5	4	.235	.150	.049	.000	.000	.000	.000	.000
			8	.538	.450	.219	.000	.000	.000	.000	.000
			12	.805	.713	.519	.003	.000	.000	.000	.000
		1	4	.536	.416	.199	.000	.000	.000	.000	.000
			8	.951	.910	.802	.016	.000	.000	.000	.000
			12	1.000	1.000	.994	.282	.006	.000	.000	.000
<i>a</i>	30	0.5	4	.224	.144	.054	.062	.002	.000	.000	.000
			8	.314	.236	.113	.793	.312	.064	.002	.000
			12	.404	.304	.164	.995	.941	.665	.243	.042
		1	4	.492	.420	.293	.994	.943	.773	.483	.213
			8	.608	.551	.432	1.000	1.000	1.000	1.000	1.000
			12	.666	.599	.493	1.000	1.000	1.000	1.000	1.000
	40	0.5	4	.201	.125	.046	.007	.000	.000	.000	.000
			8	.282	.181	.071	.313	.024	.001	.000	.000
			12	.318	.225	.123	.887	.491	.095	.007	.001
		1	4	.416	.336	.214	.925	.666	.280	.061	.002
			8	.524	.441	.308	1.000	1.000	.997	.973	.888
			12	.591	.525	.407	1.000	1.000	1.000	1.000	1.000
	50	0.5	4	.174	.109	.037	.001	.000	.000	.000	.000
			8	.238	.165	.063	.088	.002	.000	.000	.000
			12	.266	.177	.074	.593	.113	.006	.000	.000
		1	4	.377	.284	.163	.735	.280	.038	.000	.000
			8	.457	.379	.249	1.000	.998	.972	.817	.460
			12	.546	.475	.333	1.000	1.000	1.000	1.000	.994

Table 3: DTF statistics for the 3PLM when the simulated parameter DIF are unidirectional and  $N = 3000$ .

DIF Parameters	Test Size	DIF Size	# DIF Items	<i>sDTF</i>			<i>uDTF</i> <sub>%</sub> ( $\alpha = .95$ )				
				$p < .10$	$p < .05$	$p < .01$	$> 2$	$> 2.5$	$> 3$	$> 3.5$	$> 4$
<i>a &amp; d</i>	30	0.5	4	.087	.033	.003	.999	.912	.470	.167	.044
			8	.132	.066	.016	1.000	.935	.568	.245	.096
			12	.181	.098	.039	1.000	.961	.680	.323	.138
		1	4	.220	.132	.044	1.000	.984	.779	.467	.212
			8	.519	.390	.185	1.000	1.000	.982	.922	.770
			12	.766	.671	.476	1.000	1.000	1.000	.996	.986
		40	4	.079	.035	.002	1.000	.855	.377	.129	.034
			8	.091	.044	.006	1.000	.891	.420	.145	.036
			12	.115	.064	.017	1.000	.931	.482	.168	.058
		1	4	.146	.086	.017	1.000	.958	.601	.233	.066
			8	.337	.234	.092	1.000	.999	.922	.708	.442
			12	.589	.482	.262	1.000	1.000	.987	.950	.855
	50	0.5	4	.063	.028	.004	1.000	.915	.437	.151	.038
			8	.078	.036	.003	1.000	.929	.458	.144	.035
			12	.092	.042	.010	1.000	.944	.493	.160	.047
		1	4	.094	.046	.007	1.000	.954	.519	.172	.041
			8	.242	.141	.036	1.000	.997	.832	.513	.255
			12	.422	.285	.099	1.000	1.000	.971	.839	.596
<i>d</i>	30	0.5	4	.047	.025	.003	1.000	.857	.403	.144	.049
			8	.065	.034	.006	.999	.874	.447	.179	.054
			12	.064	.031	.006	1.000	.883	.443	.175	.046
		1	4	.076	.046	.007	1.000	.896	.481	.174	.068
			8	.072	.031	.003	1.000	.922	.504	.192	.061
			12	.072	.033	.004	1.000	.924	.542	.207	.069
	40	0.5	4	.071	.023	.003	1.000	.875	.426	.165	.042
			8	.058	.023	.006	.999	.868	.392	.134	.038
			12	.053	.029	.002	1.000	.870	.405	.138	.042
		1	4	.065	.026	.004	.999	.872	.409	.146	.028
			8	.075	.033	.003	1.000	.898	.423	.145	.042
			12	.084	.045	.007	1.000	.911	.472	.161	.041
	50	0.5	4	.060	.025	.005	1.000	.900	.431	.144	.036
			8	.083	.035	.006	1.000	.910	.470	.157	.051
			12	.057	.024	.001	1.000	.924	.468	.152	.042
		1	4	.060	.024	.005	1.000	.911	.402	.137	.041
			8	.062	.027	.003	1.000	.917	.453	.134	.040
			12	.061	.022	.002	1.000	.936	.457	.146	.045
<i>a</i>	30	0.5	4	.082	.039	.008	1.000	.887	.448	.183	.048
			8	.100	.052	.009	1.000	.909	.473	.172	.065
			12	.096	.053	.012	1.000	.947	.543	.246	.096
		1	4	.115	.064	.011	1.000	.987	.728	.405	.170
			8	.156	.094	.032	1.000	.998	.969	.837	.612
			12	.242	.139	.048	1.000	1.000	.995	.970	.920
	40	0.5	4	.078	.038	.006	1.000	.878	.413	.129	.026
			8	.085	.039	.004	1.000	.894	.408	.141	.041
			12	.078	.038	.004	1.000	.902	.433	.144	.038
		1	4	.093	.041	.009	1.000	.935	.531	.197	.049
			8	.129	.067	.018	1.000	.992	.863	.624	.352
			12	.165	.093	.018	1.000	.998	.979	.887	.719
	50	0.5	4	.047	.018	.004	1.000	.914	.426	.139	.039
			8	.070	.024	.004	1.000	.900	.403	.136	.029
			12	.073	.032	.007	1.000	.943	.456	.138	.033
		1	4	.069	.027	.003	1.000	.944	.477	.152	.045
			8	.086	.029	.010	1.000	.982	.749	.393	.149
			12	.107	.056	.011	1.000	1.000	.939	.742	.476

Table 4: DTF statistics for the 3PLM when the simulated parameter DIF are bidirectional and  $N = 500$ .

DIF Parameters	Test Size	DIF Size	# DIF Items	<i>sDTF</i>			<i>uDTF</i> <sub>%</sub> ( $\alpha = .95$ )				
				$p < .10$	$p < .05$	$p < .01$	> 2	> 2.5	> 3	> 3.5	> 4
<i>a &amp; d</i>	20	0.5	4	.672	.587	.381	.919	.627	.271	.078	.010
			6	.889	.828	.708	.999	.959	.801	.517	.203
			8	.975	.956	.902	1.000	.999	.977	.905	.693
		1	4	.991	.981	.965	1.000	1.000	.999	.996	.987
			6	.999	.999	.998	1.000	1.000	1.000	1.000	1.000
			8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		25	4	.551	.446	.243	.800	.386	.106	.013	.003
			6	.781	.694	.519	.978	.804	.471	.159	.033
			8	.936	.895	.780	1.000	.987	.876	.601	.279
		1	4	.985	.977	.938	1.000	1.000	.999	.977	.852
			6	1.000	.999	.995	1.000	1.000	1.000	1.000	1.000
			8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	30	0.5	4	.421	.321	.135	.616	.242	.042	.004	.002
			6	.710	.612	.364	.932	.635	.258	.058	.007
			8	.844	.791	.608	.993	.910	.649	.271	.080
		1	4	.967	.948	.877	1.000	.998	.966	.844	.590
			6	.998	.996	.986	1.000	1.000	1.000	1.000	.994
			8	1.000	1.000	.999	1.000	1.000	1.000	1.000	1.000
<i>d</i>	20	0.5	4	.396	.252	.079	.063	.005	.000	.000	.000
			6	.743	.616	.345	.178	.010	.000	.000	.000
			8	.935	.877	.673	.368	.024	.000	.000	.000
		1	4	.931	.868	.654	.418	.039	.002	.000	.000
			6	.999	.998	.986	.928	.406	.038	.002	.000
			8	1.000	1.000	1.000	1.000	.915	.420	.064	.003
		25	4	.269	.165	.036	.048	.003	.000	.000	.000
			6	.526	.393	.159	.080	.003	.000	.000	.000
			8	.805	.671	.360	.152	.009	.000	.000	.000
		1	4	.788	.668	.368	.204	.009	.001	.000	.000
			6	.984	.953	.854	.644	.090	.002	.000	.000
			8	1.000	.998	.988	.955	.518	.056	.003	.000
	30	0.5	4	.194	.108	.022	.031	.000	.000	.000	.000
			6	.404	.273	.084	.052	.001	.000	.000	.000
			8	.626	.484	.234	.118	.004	.000	.000	.000
		1	4	.650	.469	.210	.122	.009	.000	.000	.000
			6	.941	.880	.677	.408	.028	.000	.000	.000
			8	.995	.990	.944	.798	.186	.010	.001	.000
<i>a</i>	20	0.5	4	.161	.101	.025	.873	.568	.239	.064	.014
			6	.192	.126	.041	.991	.895	.688	.386	.148
			8	.240	.153	.051	.998	.993	.958	.803	.533
		1	4	.405	.331	.185	1.000	.999	.996	.986	.958
			6	.457	.373	.250	1.000	1.000	1.000	1.000	1.000
			8	.535	.457	.337	1.000	1.000	1.000	1.000	1.000
		25	4	.106	.055	.007	.717	.335	.072	.008	.000
			6	.161	.094	.030	.959	.759	.413	.119	.035
			8	.199	.130	.048	.997	.936	.754	.459	.184
		1	4	.305	.218	.098	.999	.997	.984	.923	.788
			6	.406	.318	.198	1.000	1.000	1.000	1.000	1.000
			8	.475	.393	.255	1.000	1.000	1.000	1.000	1.000
	30	0.5	4	.084	.04	.009	.573	.209	.040	.003	.000
			6	.107	.055	.012	.886	.557	.206	.037	.003
			8	.139	.075	.024	.972	.841	.548	.218	.052
		1	4	.255	.177	.080	.999	.992	.949	.777	.509
			6	.354	.265	.148	1.000	1.000	1.000	.999	.974
			8	.406	.318	.176	1.000	1.000	1.000	1.000	1.000

Table 8: DTF statistics for the GRM when the simulated parameter DIF are unidirectional and  $N = 1000$ .

DIF Parameters	Test Size	DIF Size	# DIF Items	<i>sDTF</i>			<i>uDTF</i> <sub>%</sub> ( $\alpha = .95$ )				
				$p < .10$	$p < .05$	$p < .01$	$> 2$	$> 2.5$	$> 3$	$> 3.5$	$> 4$
<i>a &amp; d</i>	30	0.5	4	.229	.154	.068	.002	.000	.000	.000	.000
			8	.425	.331	.198	.025	.001	.000	.000	.000
			12	.609	.520	.359	.136	.015	.001	.000	.000
		1	4	.729	.656	.527	.472	.105	.013	.001	.000
			8	.943	.922	.877	.984	.883	.621	.295	.082
			12	.993	.990	.974	1.000	.998	.979	.916	.763
		0.5	4	.193	.122	.043	.000	.000	.000	.000	.000
			8	.330	.245	.132	.004	.000	.000	.000	.000
			12	.455	.369	.222	.027	.001	.000	.000	.000
		1	4	.599	.508	.368	.157	.009	.000	.000	.000
			8	.907	.871	.795	.857	.533	.179	.021	.003
			12	.970	.962	.931	.994	.959	.796	.460	.138
	40	0.5	4	.203	.124	.040	.000	.000	.000	.000	.000
			8	.282	.195	.082	.000	.000	.000	.000	.000
			12	.382	.284	.146	.007	.000	.000	.000	.000
		1	4	.527	.446	.262	.024	.000	.000	.000	.000
			8	.842	.801	.677	.627	.146	.012	.002	.000
			12	.944	.927	.875	.973	.785	.370	.083	.008
	50	0.5	4	.117	.063	.010	.001	.000	.000	.000	.000
			8	.113	.065	.012	.000	.000	.000	.000	.000
			12	.126	.071	.021	.000	.000	.000	.000	.000
		1	4	.104	.063	.018	.001	.000	.000	.000	.000
			8	.127	.069	.017	.000	.000	.000	.000	.000
			12	.149	.087	.025	.010	.001	.000	.000	.000
<i>d</i>	30	0.5	4	.123	.074	.013	.000	.000	.000	.000	.000
			8	.124	.069	.015	.000	.000	.000	.000	.000
			12	.136	.073	.021	.000	.000	.000	.000	.000
		1	4	.137	.075	.017	.000	.000	.000	.000	.000
			8	.135	.077	.017	.001	.000	.000	.000	.000
			12	.148	.072	.019	.001	.000	.000	.000	.000
	40	0.5	4	.140	.069	.013	.000	.000	.000	.000	.000
			8	.148	.090	.027	.000	.000	.000	.000	.000
			12	.145	.081	.021	.000	.000	.000	.000	.000
		1	4	.144	.087	.017	.000	.000	.000	.000	.000
			8	.150	.086	.018	.000	.000	.000	.000	.000
			12	.139	.081	.022	.000	.000	.000	.000	.000
	50	0.5	4	.177	.105	.035	.000	.000	.000	.000	.000
			8	.258	.190	.075	.006	.000	.000	.000	.000
			12	.321	.220	.093	.056	.001	.000	.000	.000
		1	4	.388	.307	.186	.364	.075	.003	.000	.000
			8	.502	.431	.303	.953	.786	.459	.173	.044
			12	.572	.511	.378	.997	.984	.930	.791	.534
<i>a</i>	30	0.5	4	.165	.094	.028	.000	.000	.000	.000	.000
			8	.210	.124	.043	.000	.000	.000	.000	.000
			12	.276	.174	.083	.006	.000	.000	.000	.000
		1	4	.312	.232	.109	.087	.001	.000	.000	.000
			8	.434	.345	.206	.763	.380	.104	.014	.000
			12	.461	.386	.255	.991	.910	.630	.304	.088
	40	0.5	4	.161	.093	.025	.000	.000	.000	.000	.000
			8	.194	.136	.048	.000	.000	.000	.000	.000
			12	.224	.151	.047	.001	.000	.000	.000	.000
		1	4	.272	.196	.093	.027	.000	.000	.000	.000
			8	.385	.288	.151	.514	.111	.004	.000	.000
			12	.462	.373	.247	.927	.622	.237	.022	.002
	50	0.5	4	.161	.093	.025	.000	.000	.000	.000	.000
			8	.194	.136	.048	.000	.000	.000	.000	.000
			12	.224	.151	.047	.001	.000	.000	.000	.000
		1	4	.272	.196	.093	.027	.000	.000	.000	.000
			8	.385	.288	.151	.514	.111	.004	.000	.000
			12	.462	.373	.247	.927	.622	.237	.022	.002

Table 6: DTF statistics for the 3PLM when the simulated parameter DIF are bidirectional and  $N = 3000$ .



DIF Parameters	Test Size	DIF Size	# DIF Items	<i>sDTF</i>			<i>uDTF</i> <sub>%</sub> ( $\alpha = .95$ )				
				$p < .10$	$p < .05$	$p < .01$	> 2	> 2.5	> 3	> 3.5	> 4
<i>a &amp; d</i>	20	0.5	4	.367	.259	.087	.997	.936	.749	.483	.257
			6	.661	.564	.324	1.000	.998	.969	.878	.674
			8	.830	.741	.544	1.000	1.000	.998	.986	.921
		1	4	.945	.920	.831	1.000	1.000	1.000	1.000	.999
			6	.997	.993	.978	1.000	1.000	1.000	1.000	1.000
			8	1.000	.999	.996	1.000	1.000	1.000	1.000	1.000
		25	4	.226	.132	.027	.998	.906	.687	.392	.170
			6	.471	.346	.135	1.000	.991	.901	.727	.474
			8	.676	.555	.304	1.000	1.000	.986	.927	.787
		1	4	.884	.816	.644	1.000	1.000	1.000	.999	.988
			6	.988	.977	.931	1.000	1.000	1.000	1.000	1.000
			8	1.000	1.000	.997	1.000	1.000	1.000	1.000	1.000
	30	0.5	4	.110	.047	.007	.999	.915	.664	.359	.160
			6	.320	.197	.044	.999	.987	.873	.657	.405
			8	.495	.364	.133	1.000	.999	.979	.900	.696
		1	4	.775	.666	.414	1.000	1.000	.998	.989	.937
			6	.963	.935	.828	1.000	1.000	1.000	1.000	1.000
			8	.993	.992	.971	1.000	1.000	1.000	1.000	1.000
<i>d</i>	20	0.5	4	.151	.074	.010	.864	.343	.075	.014	.004
			6	.359	.210	.046	.954	.466	.096	.017	.003
			8	.616	.452	.167	.965	.618	.168	.016	.004
		1	4	.601	.422	.148	.975	.627	.188	.031	.004
			6	.942	.855	.599	1.000	.908	.503	.133	.019
			8	.997	.985	.927	1.000	.993	.871	.505	.153
		25	4	.092	.038	.003	.936	.404	.108	.015	.003
			6	.168	.081	.017	.951	.468	.114	.021	.001
			8	.374	.230	.050	.969	.583	.144	.024	.001
		1	4	.345	.218	.054	.979	.588	.156	.029	.005
			6	.751	.595	.293	.997	.842	.356	.077	.007
			8	.955	.896	.670	1.000	.959	.685	.234	.032
	30	0.5	4	.037	.013	.002	.990	.570	.150	.031	.006
			6	.099	.039	.002	.991	.604	.180	.040	.007
			8	.187	.093	.011	.987	.648	.180	.028	.006
		1	4	.191	.088	.009	.993	.702	.204	.052	.005
			6	.507	.307	.072	.999	.829	.349	.078	.013
			8	.801	.657	.293	.999	.948	.569	.173	.035
<i>a</i>	20	0.5	4	.066	.030	.008	.992	.923	.730	.475	.270
			6	.103	.051	.014	1.000	.990	.954	.827	.621
			8	.100	.050	.013	1.000	.997	.993	.955	.876
		1	4	.196	.119	.039	1.000	1.000	.998	.996	.989
			6	.289	.205	.092	1.000	1.000	1.000	1.000	1.000
			8	.339	.259	.137	1.000	1.000	1.000	1.000	1.000
		25	4	.039	.014	.001	.994	.870	.638	.349	.148
			6	.057	.023	.002	.999	.978	.890	.693	.440
			8	.084	.039	.006	1.000	.997	.980	.898	.749
		1	4	.131	.061	.013	1.000	1.000	.998	.986	.952
			6	.179	.106	.031	1.000	1.000	1.000	1.000	.999
			8	.239	.160	.052	1.000	1.000	1.000	1.000	1.000
	30	0.5	4	.018	.005	.000	.999	.912	.620	.343	.149
			6	.035	.009	.000	1.000	.981	.879	.676	.392
			8	.044	.016	.000	1.000	.997	.971	.881	.659
		1	4	.079	.031	.004	1.000	1.000	.999	.986	.897
			6	.129	.060	.015	1.000	1.000	1.000	1.000	.999
			8	.139	.071	.021	1.000	1.000	1.000	1.000	1.000

Table 7: DTF statistics for the GRM when the simulated parameter DIF are unidirectional and  $N = 500$ .

DIF Parameters	Test Size	DIF Size	# DIF Items	<i>sDTF</i>			<i>uDTF</i> <sub>%</sub> ( $\alpha = .95$ )				
				$p < .10$	$p < .05$	$p < .01$	$> 2$	$> 2.5$	$> 3$	$> 3.5$	$> 4$
<i>a &amp; d</i>	20	0.5	4	.170	.100	.030	.199	.038	.005	.001	.000
			6	.276	.185	.072	.355	.095	.017	.002	.000
			8	.355	.273	.133	.554	.204	.054	.012	.002
		1	4	.675	.603	.446	.941	.764	.479	.175	.043
			6	.851	.816	.722	.998	.965	.860	.662	.400
			8	.927	.904	.839	1.000	.996	.983	.923	.819
	25	0.5	4	.146	.089	.023	.146	.018	.001	.000	.000
			6	.195	.117	.034	.268	.047	.004	.001	.000
			8	.288	.195	.071	.405	.118	.019	.003	.000
		1	4	.559	.474	.315	.863	.533	.211	.032	.002
			6	.777	.708	.567	.979	.907	.678	.347	.108
			8	.901	.878	.793	.999	.990	.930	.750	.464
	30	0.5	4	.099	.049	.003	.108	.012	.000	.000	.000
			6	.152	.074	.021	.179	.028	.000	.000	.000
			8	.218	.138	.033	.283	.064	.005	.001	.001
		1	4	.478	.362	.181	.793	.374	.098	.014	.003
			6	.725	.631	.431	.957	.796	.476	.159	.026
			8	.856	.809	.685	.998	.954	.810	.489	.197
<i>d</i>	20	0.5	4	.048	.019	.003	.041	.003	.000	.000	.000
			6	.051	.023	.002	.040	.001	.000	.000	.000
			8	.055	.020	.002	.058	.003	.000	.000	.000
		1	4	.040	.020	.005	.106	.010	.000	.000	.000
			6	.063	.027	.004	.134	.021	.003	.000	.000
			8	.070	.034	.005	.187	.040	.003	.000	.000
	25	0.5	4	.045	.021	.004	.026	.001	.000	.000	.000
			6	.041	.022	.007	.034	.001	.000	.000	.000
			8	.051	.021	.007	.046	.004	.000	.000	.000
		1	4	.062	.025	.004	.075	.003	.000	.000	.000
			6	.049	.018	.001	.090	.007	.000	.000	.000
			8	.072	.033	.004	.115	.024	.001	.000	.000
	30	0.5	4	.047	.018	.005	.039	.002	.000	.000	.000
			6	.045	.017	.001	.045	.003	.000	.000	.000
			8	.044	.019	.000	.040	.002	.000	.000	.000
		1	4	.034	.016	.003	.056	.004	.000	.000	.000
			6	.052	.020	.000	.074	.006	.000	.000	.000
			8	.030	.017	.001	.110	.011	.002	.000	.000
<i>a</i>	20	0.5	4	.112	.062	.019	.201	.040	.003	.000	.000
			6	.154	.085	.028	.305	.068	.011	.002	.000
			8	.174	.114	.037	.477	.159	.036	.001	.000
		1	4	.325	.239	.116	.920	.719	.421	.141	.042
			6	.410	.307	.186	.981	.922	.776	.570	.327
			8	.436	.368	.229	1.000	.991	.963	.893	.739
	25	0.5	4	.104	.040	.008	.129	.019	.001	.000	.000
			6	.104	.054	.009	.192	.046	.001	.001	.000
			8	.167	.083	.022	.346	.071	.008	.001	.000
		1	4	.256	.162	.062	.822	.478	.181	.043	.003
			6	.317	.23	.107	.972	.852	.574	.287	.090
			8	.352	.267	.148	.995	.976	.892	.685	.421
	30	0.5	4	.089	.037	.002	.094	.009	.000	.000	.000
			6	.105	.050	.010	.168	.017	.001	.000	.000
			8	.103	.061	.021	.267	.046	.005	.000	.000
		1	4	.173	.095	.028	.697	.332	.081	.006	.000
			6	.251	.164	.076	.931	.687	.358	.115	.018
			8	.292	.210	.100	.990	.932	.707	.381	.126

Table 11: DTF statistics for the GRM when the simulated parameter DIF are bidirectional and  $N = 1000$ .

DIF Parameters	Test Size	DIF Size	# DIF Items	<i>sDTF</i>			<i>uDTF</i> <sub>%</sub> ( $\alpha = .95$ )				
				$p < .10$	$p < .05$	$p < .01$	> 2	> 2.5	> 3	> 3.5	> 4
<i>a &amp; d</i>	20	0.5	4	.916	.878	.801	.585	.169	.010	.001	.000
			6	.989	.978	.955	.985	.783	.362	.094	.014
			8	.995	.994	.985	1.000	.994	.912	.619	.251
		1	4	.997	.996	.990	1.000	1.000	.999	.989	.940
			6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
			8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
		0.5	4	.870	.834	.690	.272	.026	.000	.000	.000
			6	.978	.960	.915	.859	.402	.066	.002	.000
			8	.991	.985	.967	.999	.893	.513	.124	.009
		1	4	.997	.997	.994	1.000	.999	.988	.895	.636
			6	1.000	1.000	.999	1.000	1.000	1.000	1.000	.999
			8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	30	0.5	4	.806	.720	.561	.073	.004	.000	.000	.000
			6	.953	.928	.871	.567	.097	.004	.000	.000
			8	.988	.981	.956	.954	.560	.131	.014	.000
		1	4	.994	.992	.984	1.000	.974	.845	.516	.155
			6	1.000	1.000	.999	1.000	1.000	.999	.995	.962
			8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
<i>d</i>	20	0.5	4	.874	.782	.546	.000	.000	.000	.000	.000
			6	.995	.991	.937	.000	.000	.000	.000	.000
			8	.999	.999	.999	.010	.000	.000	.000	.000
		1	4	1.000	1.000	1.000	.009	.000	.000	.000	.000
			6	1.000	1.000	1.000	.593	.019	.000	.000	.000
			8	1.000	1.000	1.000	.998	.651	.041	.000	.000
		0.5	4	.738	.606	.351	.000	.000	.000	.000	.000
			6	.956	.917	.768	.000	.000	.000	.000	.000
			8	.998	.993	.969	.000	.000	.000	.000	.000
		1	4	.997	.993	.973	.002	.000	.000	.000	.000
			6	1.000	1.000	1.000	.075	.000	.000	.000	.000
			8	1.000	1.000	1.000	.759	.038	.000	.000	.000
	30	0.5	4	.566	.422	.186	.000	.000	.000	.000	.000
			6	.898	.836	.615	.000	.000	.000	.000	.000
			8	.988	.971	.900	.000	.000	.000	.000	.000
		1	4	.978	.959	.881	.000	.000	.000	.000	.000
			6	1.000	1.000	1.000	.005	.000	.000	.000	.000
			8	1.000	1.000	1.000	.243	.002	.000	.000	.000
<i>a</i>	20	0.5	4	.358	.274	.151	.478	.105	.010	.000	.000
			6	.445	.361	.216	.943	.671	.240	.053	.007
			8	.465	.368	.236	.999	.972	.761	.378	.100
		1	4	.655	.594	.475	1.000	1.000	.992	.971	.882
			6	.680	.632	.548	1.000	1.000	1.000	1.000	1.000
			8	.731	.687	.589	1.000	1.000	1.000	1.000	1.000
		0.5	4	.287	.212	.096	.174	.011	.000	.000	.000
			6	.345	.259	.150	.716	.233	.017	.000	.000
			8	.411	.317	.208	.988	.771	.315	.048	.002
		1	4	.590	.522	.38	1.000	.995	.938	.752	.449
			6	.649	.580	.468	1.000	1.000	1.000	.999	.981
			8	.697	.642	.534	1.000	1.000	1.000	1.000	1.000
	30	0.5	4	.265	.177	.071	.044	.000	.000	.000	.000
			6	.295	.211	.101	.445	.050	.001	.000	.000
			8	.334	.249	.126	.871	.418	.045	.001	.000
		1	4	.529	.455	.319	.997	.959	.773	.401	.109
			6	.585	.521	.384	1.000	1.000	.999	.989	.911
			8	.639	.565	.461	1.000	1.000	1.000	1.000	1.000

Table 9: DTF statistics for the GRM when the simulated parameter DIF are unidirectional and  $N = 3000$ .

DIF Parameters	Test Size	DIF Size	# DIF Items	<i>sDTF</i>			<i>uDTF</i> <sub>%</sub> ( $\alpha = .95$ )				
				$p < .10$	$p < .05$	$p < .01$	$> 2$	$> 2.5$	$> 3$	$> 3.5$	$> 4$
<i>a &amp; d</i>	20	0.5	4	.070	.025	.002	.928	.497	.209	.079	.023
			6	.131	.055	.013	.968	.628	.310	.136	.041
			8	.185	.110	.028	.975	.725	.443	.228	.083
		1	4	.422	.312	.124	.997	.955	.823	.606	.380
			6	.678	.572	.391	1.000	.998	.969	.917	.786
			8	.826	.757	.594	1.000	1.000	.997	.985	.953
		0.5	4	.052	.018	.002	.961	.532	.219	.074	.015
			6	.057	.021	.003	.971	.632	.313	.120	.033
			8	.087	.042	.010	.978	.715	.387	.159	.034
		1	4	.258	.146	.051	.999	.939	.770	.494	.232
			6	.512	.389	.203	1.000	.990	.935	.810	.583
			8	.703	.588	.368	1.000	1.000	.990	.960	.854
	30	0.5	4	.030	.014	.003	.994	.661	.245	.065	.015
			6	.023	.007	.000	.993	.721	.333	.102	.029
			8	.047	.022	.001	.997	.798	.436	.181	.060
		1	4	.150	.068	.010	1.000	.947	.724	.427	.185
			6	.331	.217	.065	1.000	.990	.918	.761	.490
			8	.537	.393	.176	1.000	.997	.978	.918	.764
<i>d</i>	20	0.5	4	.027	.004	.001	.845	.313	.082	.017	.002
			6	.021	.009	.000	.861	.333	.104	.016	.004
			8	.025	.008	.000	.858	.341	.083	.021	.006
		1	4	.025	.007	.000	.879	.377	.108	.029	.004
			6	.026	.008	.000	.880	.403	.145	.048	.006
			8	.039	.012	.000	.913	.506	.211	.070	.022
	25	0.5	4	.018	.008	.000	.935	.412	.100	.022	.001
			6	.023	.005	.000	.921	.393	.099	.021	.003
			8	.027	.006	.000	.926	.420	.097	.017	.003
		1	4	.021	.007	.001	.928	.413	.138	.031	.006
			6	.021	.009	.002	.947	.513	.166	.039	.010
			8	.021	.006	.000	.957	.568	.218	.070	.012
	30	0.5	4	.011	.002	.000	.987	.572	.167	.038	.006
			6	.009	.002	.000	.985	.589	.160	.028	.003
			8	.015	.003	.000	.988	.589	.182	.041	.011
		1	4	.010	.004	.000	.993	.617	.190	.051	.007
			6	.009	.002	.000	.991	.646	.241	.066	.015
			8	.013	.003	.000	.991	.653	.246	.072	.017
<i>a</i>	20	0.5	4	.041	.017	.001	.920	.492	.185	.065	.014
			6	.061	.029	.003	.941	.606	.305	.117	.034
			8	.084	.031	.009	.954	.660	.370	.190	.072
		1	4	.152	.089	.025	.993	.928	.763	.552	.296
			6	.182	.118	.046	1.000	.990	.959	.872	.691
			8	.241	.183	.086	1.000	1.000	.992	.971	.912
	25	0.5	4	.028	.008	.001	.958	.484	.168	.048	.014
			6	.038	.011	.001	.971	.635	.271	.088	.020
			8	.049	.021	.001	.981	.673	.329	.128	.038
		1	4	.082	.035	.008	.996	.917	.699	.421	.194
			6	.148	.070	.015	1.000	.989	.925	.765	.546
			8	.155	.089	.026	1.000	.997	.985	.906	.763
	30	0.5	4	.013	.003	.000	.994	.659	.232	.081	.017
			6	.019	.004	.000	.996	.731	.316	.113	.031
			8	.031	.010	.000	.994	.754	.347	.123	.032
		1	4	.047	.017	.002	1.000	.927	.669	.359	.148
			6	.061	.031	.003	1.000	.985	.876	.663	.382
			8	.097	.046	.011	1.000	.998	.979	.878	.688

Table 10: DTF statistics for the GRM when the simulated parameter DIF are bidirectional and  $N = 500$ .

DIF Parameters	Test Size	DIF Size	# DIF Items	<i>sDTF</i>			<i>uDTF</i> <sub>%</sub> ( $\alpha = .95$ )				
				$p < .10$	$p < .05$	$p < .01$	$> 2$	$> 2.5$	$> 3$	$> 3.5$	$> 4$
<i>a &amp; d</i>	20	0.5	4	.404	.320	.186	.004	.000	.000	.000	.000
			6	.552	.473	.337	.035	.002	.000	.000	.000
			8	.636	.581	.426	.102	.006	.000	.000	.000
		1	4	.870	.840	.776	.764	.417	.119	.012	.000
			6	.943	.931	.903	.972	.884	.639	.348	.108
			8	.974	.970	.962	.999	.988	.932	.796	.591
		0.5	4	.321	.237	.124	.000	.000	.000	.000	.000
			6	.488	.400	.257	.002	.000	.000	.000	.000
			8	.576	.505	.359	.021	.001	.000	.000	.000
		1	4	.842	.797	.700	.495	.113	.008	.000	.000
			6	.918	.896	.844	.892	.618	.255	.056	.003
			8	.960	.943	.925	.996	.932	.749	.403	.135
	30	0.5	4	.275	.182	.086	.000	.000	.000	.000	.000
			6	.367	.298	.159	.002	.000	.000	.000	.000
			8	.497	.408	.258	.000	.000	.000	.000	.000
		1	4	.789	.741	.636	.245	.020	.000	.000	.000
			6	.913	.881	.835	.768	.350	.065	.000	.000
			8	.962	.947	.911	.966	.771	.441	.114	.014
<i>d</i>	20	0.5	4	.076	.031	.007	.000	.000	.000	.000	.000
			6	.075	.037	.009	.000	.000	.000	.000	.000
			8	.088	.033	.008	.000	.000	.000	.000	.000
		1	4	.095	.058	.010	.000	.000	.000	.000	.000
			6	.118	.062	.014	.000	.000	.000	.000	.000
			8	.127	.075	.023	.004	.000	.000	.000	.000
	25	0.5	4	.070	.038	.008	.000	.000	.000	.000	.000
			6	.064	.025	.004	.000	.000	.000	.000	.000
			8	.087	.036	.006	.000	.000	.000	.000	.000
		1	4	.095	.040	.012	.000	.000	.000	.000	.000
			6	.092	.044	.007	.000	.000	.000	.000	.000
			8	.114	.067	.019	.003	.000	.000	.000	.000
	30	0.5	4	.065	.026	.004	.000	.000	.000	.000	.000
			6	.074	.040	.004	.000	.000	.000	.000	.000
			8	.073	.032	.003	.000	.000	.000	.000	.000
		1	4	.096	.056	.010	.000	.000	.000	.000	.000
			6	.083	.038	.008	.000	.000	.000	.000	.000
			8	.105	.040	.006	.000	.000	.000	.000	.000
<i>a</i>	20	0.5	4	.291	.221	.092	.001	.000	.000	.000	.000
			6	.392	.299	.159	.012	.000	.000	.000	.000
			8	.413	.328	.188	.067	.003	.000	.000	.000
		1	4	.576	.507	.382	.674	.339	.099	.017	.001
			6	.628	.570	.457	.952	.822	.576	.251	.074
			8	.670	.610	.497	.993	.968	.878	.706	.446
	25	0.5	4	.226	.152	.060	.000	.000	.000	.000	.000
			6	.286	.201	.096	.003	.000	.000	.000	.000
			8	.357	.261	.132	.013	.000	.000	.000	.000
		1	4	.490	.407	.265	.416	.094	.005	.000	.000
			6	.582	.517	.380	.839	.537	.212	.038	.002
			8	.601	.546	.439	.975	.871	.628	.307	.104
	30	0.5	4	.195	.114	.038	.000	.000	.000	.000	.000
			6	.252	.175	.078	.000	.000	.000	.000	.000
			8	.271	.200	.093	.001	.000	.000	.000	.000
		1	4	.421	.313	.169	.191	.014	.000	.000	.000
			6	.483	.412	.287	.676	.276	.048	.004	.000
			8	.529	.459	.313	.933	.704	.317	.067	.006

Table 12: DTF statistics for the GRM when the simulated parameter DIF are bidirectional and  $N = 3000$ .

test_size	sample_size	sDTF_0.1	sDTF_0.05	sDTF_0.01	uDTF_2	uDTF_2.5	uDTF_3	uDTF_3.5	uDTF_4
30	500	0.115	0.063	0.019	0.963	0.636	0.312	0.124	0.047
40	500	0.099	0.047	0.010	0.979	0.662	0.328	0.150	0.055
50	500	0.061	0.025	0.001	0.994	0.757	0.376	0.153	0.047
30	1000	0.145	0.083	0.021	0.250	0.074	0.017	0.002	0.001
40	1000	0.137	0.084	0.014	0.239	0.060	0.013	0.001	0
50	1000	0.118	0.071	0.018	0.255	0.057	0.007	0.001	0
30	3000	0.172	0.099	0.034	0.003	0.001	0	0	0
40	3000	0.158	0.089	0.029	0	0	0	0	0
50	3000	0.147	0.096	0.029	0	0	0	0	0
30	5000	0.150	0.094	0.021	0	0	0	0	0
40	5000	0.165	0.095	0.025	0	0	0	0	0
50	5000	0.164	0.100	0.028	0	0	0	0	0

Table 13: Type I error rates for 2PLM.