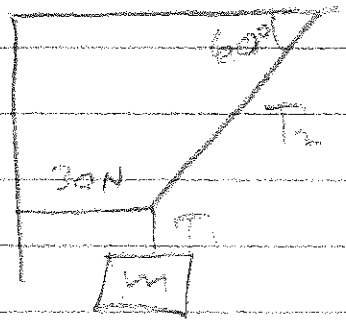
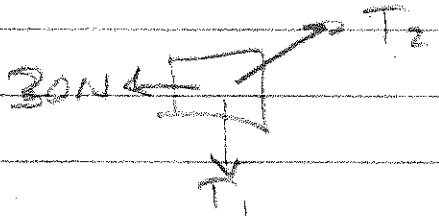
Phys 209 PS4 Solutions

1. a)



Look at point where 3 wires meet.

FBD:



$$\Sigma F_x = T_2 \cos 60° - 30N = ma_x = 0 \quad (1)$$

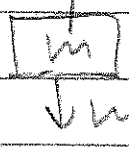
$$\Sigma F_y = T_2 \sin 60° - T_1 = ma_y = 0 \quad (2)$$

$$(1) \Rightarrow T_2 \cos 60° = 30N$$

$$\Rightarrow T_2 = \frac{30N}{\cos 60°} = \boxed{60N}$$

$$(2) \Rightarrow T_1 = T_2 \sin 60° = (60N)\sin 60° = \boxed{52N}$$

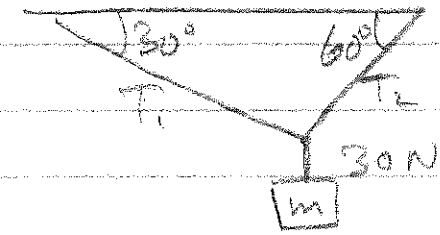
Now look at mass:



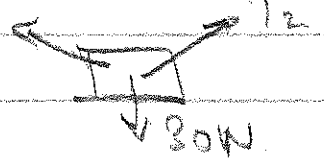
$$\Sigma F_y = T_1 - mg = ma_y = 0 \Rightarrow T_1 = mg$$

$$\Rightarrow m = \frac{T_1}{g} = \frac{52N}{9.8 m/s^2} = \boxed{5.3 kg}$$

b)



Look at point where 3 wires meet!

FBD: 

$\Sigma F_x = T_2 \cos 60° - T_1 \cos 30° = ma_x = 0$ (1)

$\Sigma F_y = T_1 \sin 30° + T_2 \sin 60° - 30N = ma_y = 0$ (2)

(1) $\Rightarrow T_2 \cos 60° = T_1 \cos 30°$

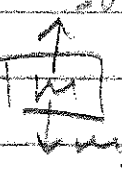
$\Rightarrow T_2 = T_1 \dfrac{\cos 30°}{\cos 60°}$ (3)

Sub into (2): $T_1 \sin 30° + T_1 \tan 60° \cos 30° = 30N$

$\Rightarrow T_1 = \dfrac{30N}{\sin 30° + \tan 60° \cos 30°} = \boxed{15N}$

Sub into (3): $T_2 = 26N$

Now look at hanging mass:



$\Sigma F_y = 30N - mg = ma_y = 0$

$\Rightarrow 30N = mg \Rightarrow m = \dfrac{30N}{g} = \dfrac{30N}{9.8 m/s^2} = \boxed{3.1kg}$

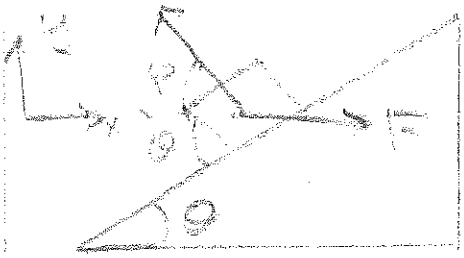
2.

$\boxed{10\,kg} \longrightarrow F$ $\qquad F = 70\,N$

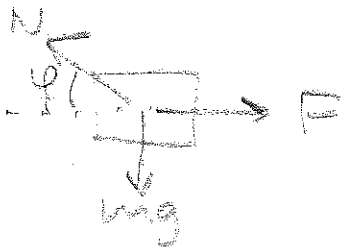
$\Sigma F_x = F = ma$

$\Rightarrow a = \frac{F}{m} = \frac{70\,N}{10\,kg} \Rightarrow \boxed{7\,m/s^2}$

3.



F.B.D.



$\varphi = 90° - \theta$

$\Sigma F_x = F - N\cos\varphi = ma_x = 0$ (1)

$\Sigma F_y = N\sin\theta - mg = ma_y = 0$ (2)

(1) $\Rightarrow N\cos\varphi = F$, (2) $\Rightarrow N\sin\varphi = mg$

Using $\sin(90° - \theta) = \cos(\theta)$

$\Rightarrow \sin(\varphi) = \cos(\theta)$

And $\cos(90° - \theta) = \cos(\varphi) = \sin(\theta)$

So, (1) $\Rightarrow N\sin\theta = F$

(2) $\Rightarrow N\cos\theta = mg$

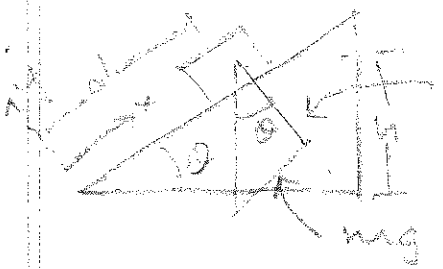
$\frac{(1)}{(2)} = \frac{N\sin\theta}{N\cos\theta} = \frac{F}{mg}$

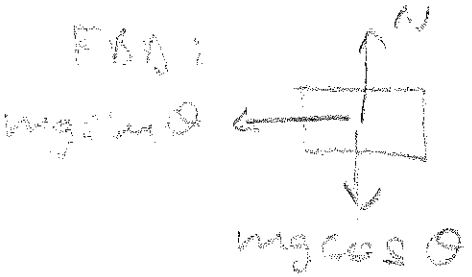
$$\Rightarrow \tan\theta = \frac{F}{mg}$$

$$\Rightarrow \theta = \arctan\left(\frac{F}{mg}\right)$$

$$= \arctan\left(\frac{20\text{ N}}{(10\text{ kg})(9.8\text{ m/s}^2)}\right) = \boxed{11.3^\circ}$$

4.





$$\Sigma F_x = -mg\sin\theta = ma_x$$

$$\Rightarrow a_x = -g\sin\theta$$

$$v_f^2 = v_0^2 + 2ad$$

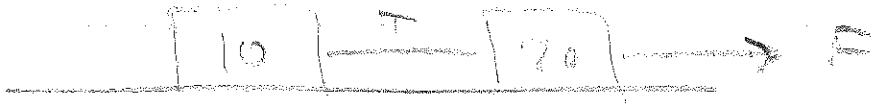
where d = distance along incline

$$v_f = 0 \Rightarrow d = \frac{-v_0^2}{2a_x} = \frac{-v_0^2}{2(-g\sin\theta)}$$

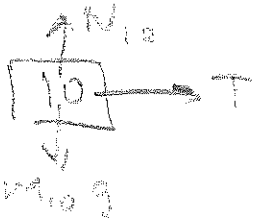
Now $d\sin\theta = h$

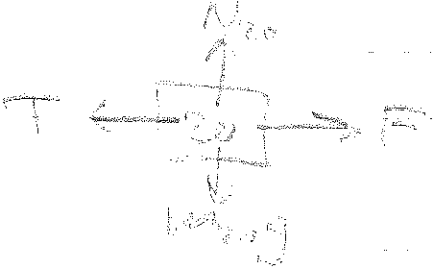
$$h = \frac{-v_0^2}{2(-g\sin\theta)}\sin\theta = \frac{-v_0^2}{2(-g)} = \boxed{\frac{v_0^2}{2g}}$$

2.



FBD:





a) $\Sigma F_{10x} = T = m_{10} a$ (1)

$\Sigma F_{20x} = F - T = m_{20} a$ (2)

Add (1) + (2):

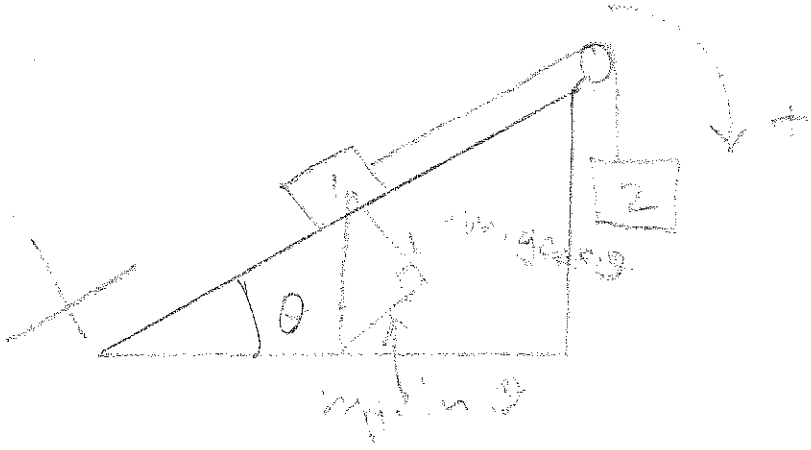
$F = (m_{10} + m_{20}) a = (10\,kg + 20\,kg)(2\,m/s^2)$

$= \boxed{60\,N}$

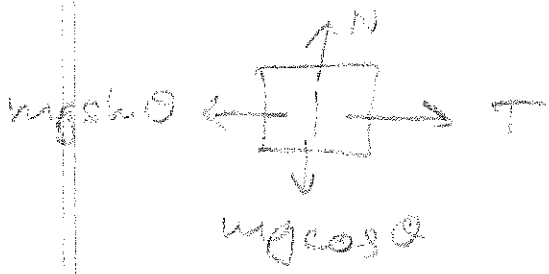
b) Eq (1): $T = m_{10} a = (10\,kg)(2\,m/s^2)$

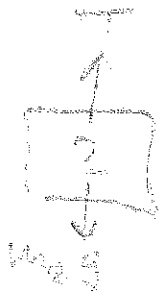
$= \boxed{20\,N}$

6.



FBD:





a) $\sum F_{1x} = T - m_1 g \sin\theta = m_1 a$ (1)

$\sum F_2 = m_2 g - T = m_2 a$ (2)

Add together!

$\Rightarrow m_2 g - m_1 g \sin\theta = (m_1 + m_2) a$

$\Rightarrow a = \dfrac{g(m_2 - m_1 \sin\theta)}{m_1 + m_2}$

Into (2): $T = m_2 (g - a)$

$\Rightarrow T = m_2 \left(g - \dfrac{g(m_2 - m_1 \sin\theta)}{m_1 + m_2} \right)$

$= m_2 g \left(1 - \dfrac{m_2 - m_1 \sin\theta}{m_1 + m_2} \right)$

$= \dfrac{m_2 g}{m_1 + m_2} \left(m_1 + \cancel{m_2} - \cancel{m_2} + m_1 \sin\theta \right)$

$$= \frac{m_1 m_2 g}{m_1 + m_2}\left(1 + \sin\theta\right)$$

b) For $m_1 = m_2 = 5\,kg$, $\theta = 30°$,

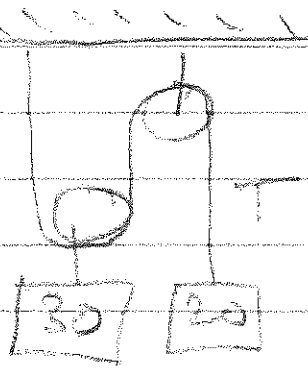
$$a = \frac{(9.8\,m/s^2)(5\,kg - (5\,kg)\sin 30°)}{5\,kg + 5\,kg}$$

$$= 2.45\ m/s^2$$

$$T = \frac{(5\,kg)(5\,kg)(9.8\,m/s^2)}{5\,kg + 5\,kg}\left(1 + \sin 30°\right)$$

$$= 36.75\,N$$

7.



Assume 20 kg mass will go up, 30 kg mass will go down

FBD:

Because the (magnitude of) the displacement of the 30kg block is half that of the 20 kg block, yet they move in the same amount of time, $a_{30} = 2a_{20}$

(1) $\sum F_{30} = m_{30}g - 2T = m_{30}a_{30} = m_{30}a$

(2) $\sum F_{20} = T - m_{20}g = m_{20}a_{20} = m_{20}(2a_{30}) = 2m_{20}a$

(1) + 2·(2):

$m_{30}g - 2T + 2T - 2m_{20}g = m_{30}a + 4m_{20}a$

$\Rightarrow a = \dfrac{g(m_{30} - 2m_{20})}{m_{30} + 4m_{20}}$

$= \dfrac{(9.8\ m/s^2)(30kg - 2(20kg))}{30kg + 4(20kg)}$

$= -0.89\ m/s^2 \Rightarrow a_{20} = 2a_{30} = \boxed{-1.8\ m/s^2}$

(Negative because 20kg mass actually goes down.)