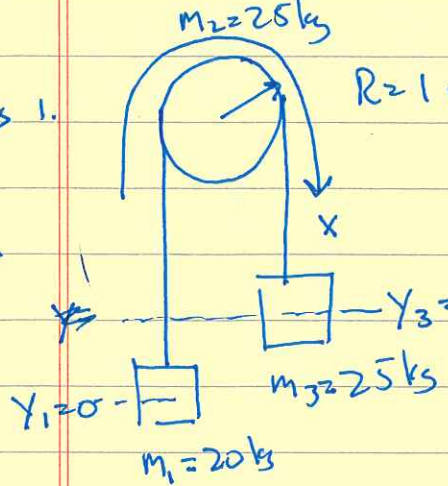
PS #9

20 pts 1.



$E_{mech} = KE + PE$

$E_{mech} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} I_2 \omega_2^2 + \frac{1}{2} m_3 v_3^2 + m_1 g y_1 + m_3 g y_3$

i: $E_{mech,i} = 0 + 0 + 0 + m_1 g(0) + m_3 g(0) = 0$

f: $y_1 = 2m$ $\quad y_3 = -2m$

f: $E_{mech,f} = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_3 v_{3f}^2 + \frac{1}{2} I_2 \omega_{2f}^2 + m_1 g y_{1f} + m_3 g y_{3f}$

$I_2 = \frac{1}{2} m_2 R^2$ $\quad \omega_{2f} \rightarrow v_{1f} = v_{3f} = +R\omega_f = v_f$

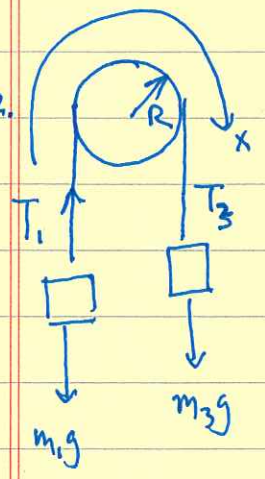
$$E_{mech,f} = \left(\frac{1}{2} m_1 + \frac{1}{2} m_3 + \frac{1}{4} m_2\right) v_f^2 + m_1 g(2) - m_3 g(2)$$

$$= \left(\frac{1}{2}20 + \frac{1}{2}25 + \frac{1}{4}25\right) v_f^2 + 20(9.8)(2) - 25(9.8)(2)$$

$$28.75 v_f^2 - 98 = E_{mech,i} = 0$$

$E_{mech,f} = 0 \Rightarrow v_f^2 = \frac{98}{28.75} \Rightarrow$ **$v_f = 1.8$ m/s**

20 pts. 2.



For 1: $\sum F = T_1 - m_1 g = m_1 a$

3: $\sum F = m_3 g - T_3 = m_3 a$

2: $\sum \tau = R T_3 - R T_1 = I_2 \alpha$

$\alpha = \frac{a}{R}$ $\quad I_2 = \frac{1}{2} m_2 R^2$

$\Rightarrow T_3 - T_1 = \frac{1}{2} m_2 a$

$\Rightarrow \quad T_1 = m_1 g + m_1 a$

$\Rightarrow \quad T_3 = m_3 g - m_3 a$

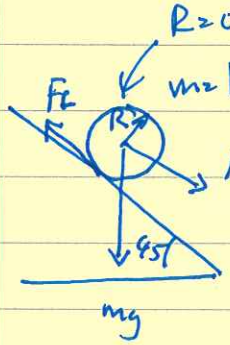
plug in: $T_3 - T_1 = \frac{1}{2} m_2 a$

$\Rightarrow \quad m_3 g - m_3 a - m_1 g - m_1 a = \frac{1}{2} m_2 a$

$\Rightarrow \quad (m_3 - m_1) g = \left(m_1 + m_3 + \frac{1}{2} m_2\right) a$

$\Rightarrow \quad a = \frac{m_3 - m_1}{m_1 + m_3 + \frac{1}{2} m_2} g = \frac{5}{57.5} g = \boxed{0.85 \ m/s^2}$

20 pts. 3.



$\sum F_x = mg \sin 45 - F_f = ma$

$\sum \tau = -R F_f = -I_z \alpha$

$\alpha = \frac{a}{R}$ & $I_z = \frac{1}{2} m R^2$

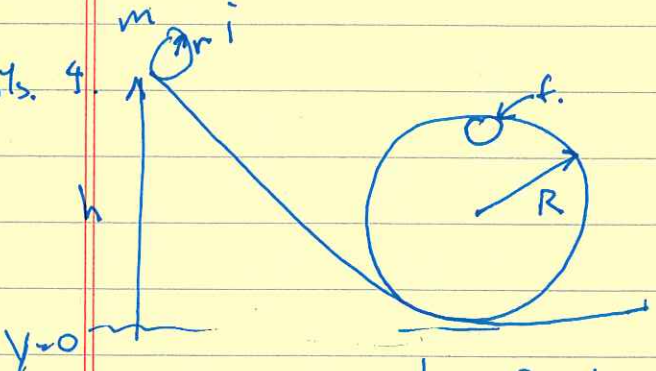
$\Rightarrow \quad -R F_f = -\frac{1}{2} m R^2 \frac{a}{R} \Rightarrow F_f = \frac{1}{2} ma$

$\Rightarrow \quad mg \sin 45 - \frac{1}{2} ma = ma$

$\Rightarrow \quad \frac{3}{2} ma = mg \sin 45 \Rightarrow \quad \boxed{a = \frac{2}{3} g \sin 45 = 4.6 \ m/s^2}$

$F_f = \frac{1}{2} ma = \frac{1}{2}(1)(4.6) = \boxed{2.3 \ N}$

20 pts. 4.



i: $E_{mech,i} = \frac{1}{2} m v_{x_i}^2 + \frac{1}{2} I \omega_i^2 + mgh$

f: $E_{mech,f} = \frac{1}{2} m v_f^2 + \frac{1}{2} I \omega_f^2 + mg 2R$

$E_{mech,i} = mgh$

$E_{mech,f} = \frac{1}{2} m v_f^2 + \frac{1}{2} \frac{2}{5} m r^2 \left(\frac{v_f}{r}\right)^2 + mg 2R$

$$E_{mech,f} = \cancel{m} V_f^2 \left(\frac{1}{2} + \frac{1}{5}\right) + \cancel{m} g\, 2R = E_{mech,i} = \cancel{m} g h$$

$$\Rightarrow V_f^2 \left(\frac{5}{10} + \frac{2}{10}\right) = g(h - 2R)$$

$$\Rightarrow V_f^2 = \frac{10}{7} g (h - 2R)$$

but I also know $\frac{V_f^2}{R} = g$

$$\Rightarrow \frac{V_f^2}{R} = g = \frac{10}{7} \frac{g}{R} (h - 2R) \Rightarrow 1 = \frac{10}{7}\left(\frac{h}{R} - 2\right)$$

$$\Rightarrow \frac{7}{10} + 2 = \frac{h}{R} \Rightarrow \boxed{h = 2.7R}$$

20 pts 5) (a)

	$\hat{i}$	$\hat{j}$	$\hat{k}$
A	4	3	0
B	0	6	5

$$\Rightarrow \boxed{\vec{\ell} = 15\hat{i} - 20\hat{j}}$$

(b)

	$\hat{i}$	$\hat{j}$	$\hat{k}$
A	2	3	-4
B	2	6	-8

$$\Rightarrow \vec{\ell} = (-24 + 24)\hat{i} + (-8 + 16)\hat{j} + (12 - 6)\hat{k}$$

$$\boxed{\vec{\ell} = 8\hat{j} + 6\hat{k}}$$

(c)

	$\hat{i}$	$\hat{j}$	$\hat{k}$
	1	2	3
	4	5	-6

$$\vec{\ell} = (-12 - 15)\hat{i} + (12 + 6)\hat{j} + (5 - 8)\hat{k}$$

$$\boxed{\vec{\ell} = -27\hat{i} + 18\hat{j} - 3\hat{k}}$$