Bridge Principles for Probabilistic and Non-Probabilistic Notions of Belief

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The script in `ex.py` is an illustration of how to use `bridge_principles.py` to move from a probabilistic notion of belief to a notion of outright belief. For example, if Mary assigns the proposition p the subjective probability 0.6, is it appropriate to say that Mary believes p? The view taken here is based on the proposals of Leitgeb (2013) and Leitgeb (2015). In particular, I adopt the following bridge principle from Leitgeb (2015), where X, Y are variables ranging over propositions and $r \in \mathbb{R}$:

(1) *The Humean Thesis Explicated*: If *Bel* is a perfectly rational agent's class of believed propositions at a time, P is the same agent's subjective probability measure at the same time, $Poss(Y)$ if and only if not $Bel(\neg Y)$, and $\frac{1}{2} \leq r < 1$, then for all X, $Bel(X)$ if and only if for all Y, if $Poss(Y)$ and $P(Y) > 0$, then $P(X|Y) > r$.

The examples in `ex.py` illustrate two consequences of Leitgeb's proposal. First, for any rational agent with a subjective probability measure P, it is not possible, in general, to identify a unique set of propositions that the agent believes outright. At the same time, the sets of propositions that the agent may rationally believe are constrained. Second, a rational agent's beliefs are sensitive to the issues that he or she is considering or is attending to.

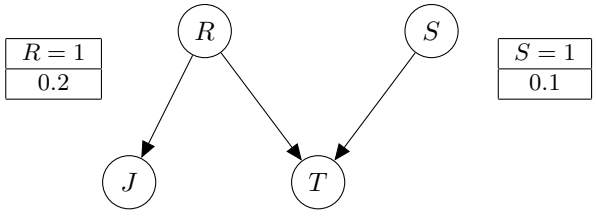
The first example is discussed in Leitgeb (2015), but was originally published in **?** and derives from an earlier example by Judea Pearl. Tracey is wondering whether her lawn is wet and is considering four propositions related to her lawn's wetness:

(2) a. T is the proposition that Tracey's lawn is wet.
b. J is the proposition that Jack's lawn is wet. (Jack is Tracey's next door neighbor.)

c. R is the proposition that it rained the previous night.

d. S is the proposition that Tracey's sprinkler was left on overnight.

Tracey's beliefs are described by the following Bayesian network:



$R = 1$
0.2

$S = 1$
0.1

$J = 1$	R
1	1
0.2	0

$T = 1$	R	S
1	1	1
1	1	0
0.9	0	1
0	0	0

The Bayesian network gives us Tracey's beliefs in probabilistic terms. We know, for example, that she takes the probability that it rained last night to be 0.2 and the probability that Jack's lawn is wet is 0.36.

Leitgeb's theory tells us what options Tracey may be taken to believe outright. It is somewhat simpler to describe these beliefs in terms of possible worlds, rather than the propositions given above. There are only eight worlds that have non-zero probability given the Bayesian network above. Below, I list these worlds, along with the subjective probability assigned to them by Tracey:

(3) a. $w_1 : \neg T \wedge \neg J \wedge \neg R \wedge \neg S$
$P(w_1) = 0.576$

b. $w_2 : \neg T \wedge J \wedge \neg R \wedge \neg S$
$P(w_2) = 0.144$

c. $w_3 : \neg T \wedge \neg J \wedge \neg R \wedge S$
$P(w_3) = 0.0064$

d. $w_4 : T \wedge \neg J \wedge \neg R \wedge S$
$P(w_4) = 0.0576$

e. $w_5 : \neg T \wedge J \wedge \neg R \wedge S$
$P(w_5) = 0.0016$

f. $w_6 : T \wedge J \wedge \neg R \wedge S$
$P(w_6) = 0.0144$

g. $w_7 : T \wedge J \wedge R \wedge \neg S$
$P(w_7) = 0.18$

h. $w_8 : T \wedge J \wedge R \wedge S$
$P(w_8) = 0.02$

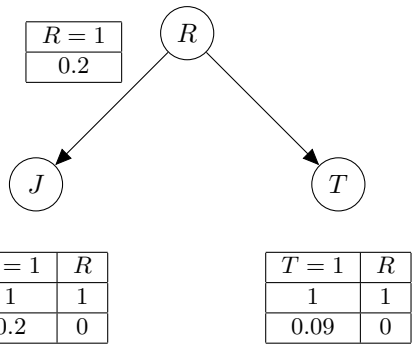
Leitgeb's proposal tells us that the following are rational sets of "live options" for Tracey to entertain:

(4) a. $\{w_1\}$
b. $\{w_1, w_2, w_7\}$
c. $\{w_1, w_2, w_4, w_7\}$
d. $\{w_1, w_2, w_4, w_6, w_7, w_8\}$
e. $\{w_1, w_2, w_3, w_4, w_6, w_7, w_8\}$
f. $\{w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8\}$

Option (4a) corresponds to the boldest stance that Tracey can take, in which she only considers the most likely world to be a live option. In this case, she believes it did not rain, the sprinkler was not on, and no one's lawn is wet. In contrast, (4f) reflects the most cautious stance she can take. Here, she only rules out contradictions, but believes that it might have rained, that the sprinkler might have been on, and that either her lawn or Jack's lawn might be wet. In intermediate cases, like (4b), she makes more commitments than in (4f), but fewer than in (4a). In (4b) in particular, Tracey believes that either no one's lawn is wet or that the sprinkler was not on and Jack's lawn is wet.

Importantly, the theory does not privelege anyone of these stances as the unique, rational set of beliefs for Tracey to have. Rather, any of these stances reflects a rational set of outright beliefs consistent with Tracey's probabilistic beliefs. But the theory does constrain which sets of worlds Tracey may take as live options. For example, it would be irrational for Tracey to believe w_1 and w_2 to be live options without also believing that w_7 is a live option.

The next example illustrates how Tracey's beliefs are sensitive to the issues that she is considering. For example, suppose Tracey is not considering the proposition of whether her sprinkler is on. Without modifying anything else, we can describe her beliefs with the following Bayesian network:



There are now only five possible worlds with non-zero probability. To avoid confusion with the worlds discussed above, each of these new worlds will receive a $\prime$ in its name:

(5) a. $w'_1 : \neg T \wedge \neg J \wedge \neg R$
$P(w'_1) = 0.5824$

b. $w'_2 : T \wedge \neg J \wedge \neg R$
$P(w'_2) = 0.0576$

c. $w'_3 : \neg T \wedge J \wedge \neg R$
$P(w'_3) = 0.1456$

d. $w'_4 : T \wedge J \wedge \neg R$
$P(w'_4) = 0.0144$

e. $w'_5 : T \wedge J \wedge R$
$P(w'_5) = 0.2$

Each world in this new model corresponds to some (possibly singleton) set of worlds on the old model. For example, w'_1 corresponds to worlds w_1 and w_3, w'_2 corresponds to world w_4, ... On the new model, here are the possible sets of live options that Tracey may entertain:

(6) a. $\{w'_1\}$

b. $\{w'_1, w'_3, w'_5\}$

c. $\{w'_1, w'_2, w'_3, w'_5\}$

d. $\{w'_1, w'_2, w'_3, w'_4, w'_5\}$

Interestingly, by narrowing her view, the possible stances that Tracey may take with respect to her outright beliefs have shifted. Suppose Tracey takes the bold stance now, taking w'_1 to be the only possibility. As with the bold stance before, she beliefs that neither lawn is wet and that it did not rain last night. But since w'_1 corresponds to w_1 and w_3, she also leaves open the

possibility that the sprinkler was left on (although she does not recognize this possibility explicitly). Note also that the belief in w_1 and w_3 as the only two live options was not a possibility on the original model. Thus, if Tracey had take the bold stance of believing only w_1' and were then to explicitly consider the proposition S, she would either have to move to a much more cautious position in terms of her outright beliefs, such as (4e), or reject w_3 as one of her outright beliefs.

All of which is to say that an agent's outright beliefs are sensitive to the issues that the agent is considering.

References

Leitgeb, H. (2013). Reducing belief simpliciter to degrees of belief. *Annals of Pure and Applied Logic*, 164(12):1338–1389.

Leitgeb, H. (2015). The Humean thesis on belief. *Aristotelian Society Supplementary Volume*, 89(1):143–185.