PROOF

Question

Write down the numbers $1, 2, \ldots, 2n$ where n is an odd integer. Pick any two numbers j, k and replace them with $|j - k|$. Continue this process until only one number remains. Prove that this integer must be odd.

Answer

Consider the possible combinations for j and k:

1. Without loss of generality, j is odd and k is even. $|j - k|$ is, therefore, odd; reducing the number of evens in the list by one.
2. In the case where j and k are both even, $|j - k|$ is even. This reduces the number of even numbers in the list by one.
3. In the case where j and k are both odd, $|j - k|$ is even. This reduces the number of odd numbers in the list by two, and increases the number of even numbers by one.

Since case 3 is the only one in which the number of odd integers is reduced in the list, and since that number is reduced by 2 in each application, it must be applied l times for $n = 2l + 1$. Leaving one odd integer. Any application of cases 1 or 2 will not result in the reduction of odd numbers. Thus, the remaining integer must be odd, completing the proof.