

COMP 256 - FA2020

Philip Kim

September 5, 2020

Prove 0.1. Write down the numbers $1, 2, \dots, 2n$, where n is an odd integer. Pick any two numbers j, k and replace them with $|j - k|$. Continue this process until only one number remains. Prove that this integer must be odd.

Proof. Let n be an odd integer.

$$1 = 2 \cdot 0,$$

$$2 = 2 \cdot 1,$$

$$4 = 2 \cdot 2,$$

$$6 = 2 \cdot 3,$$

$$8 = 2 \cdot 4,$$

$$10 = 2 \cdot 5,$$

\dots

$$2n = 2 \cdot n$$

All the terms are even except for 1. The remaining integer is odd since the sequence always contains an odd value and the final sequence of one value also contains an odd value. \square