

Electron = $-1.602 \times 10^{-19} \text{ C}$ = $9.11 \times 10^{-31} \text{ kg}$
 Proton = $1.602 \times 10^{-19} \text{ C}$ = $1.67 \times 10^{-27} \text{ kg}$
 Neutron = 0 C = $1.67 \times 10^{-27} \text{ kg}$
 6.022×10^{23} atoms in one atomic mass unit
 e is the elementary charge: $1.602 \times 10^{-19} \text{ C}$

Addition of Multiple Vectors:

$$\begin{aligned}\vec{R} &= \vec{A} + \vec{B} + \vec{C} & \text{Resultant = Sum of the vectors} \\ \vec{R}_x &= \vec{A}_x + \vec{B}_x + \vec{C}_x & \text{x-component} \quad A_x = A \cos \theta \\ \vec{R}_y &= \vec{A}_y + \vec{B}_y + \vec{C}_y & \text{y-component} \quad A_y = A \sin \theta \\ R &= \sqrt{R_x^2 + R_y^2} & \text{Magnitude (length) of } R \\ \theta_R &= \tan^{-1} \frac{R_y}{R_x} \quad \text{or} \quad \tan \theta_R = \frac{R_y}{R_x} & \text{Angle of the resultant}\end{aligned}$$

$$\vec{F} = q\vec{E}$$

$$\vec{E} = k \int_Q \frac{dq}{r^2} \hat{r} = \frac{1}{4\pi\epsilon_0} \int_Q \frac{dq}{r^2} \hat{r}$$

Gauss' law: $\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$

ELECTRIC CHARGES AND FIELDS

Coulomb's Law: [Newtons N]

$$F = k \frac{|q_1||q_2|}{r^2}$$

where: F = force on one charge by the other [N]
 $k = 8.99 \times 10^9 \text{ [N}\cdot\text{m}^2/\text{C}^2]$
 q_1 = charge [C]
 q_2 = charge [C]
 r = distance [m]

Electric Field: [Newtons/Coulomb or Volts/Meter]

$$E = k \frac{|q|}{r^2} = \frac{|F|}{|q|}$$

where: E = electric field [N/C or V/m]
 $k = 8.99 \times 10^9 \text{ [N}\cdot\text{m}^2/\text{C}^2]$
 q = charge [C]
 r = distance [m]
 F = force

Flux: the rate of flow (of an electric field) [$\text{N}\cdot\text{m}^2/\text{C}$]

$$\begin{aligned}\Phi &= \oint \vec{E} \cdot d\vec{A} & \Phi & \text{ is the rate of flow of an electric field } [\text{N}\cdot\text{m}^2/\text{C}] \\ &= \int E(\cos \theta) dA & \oint & \text{ integral over a closed surface}\end{aligned}$$

\vec{E} is the electric field vector [N/C]
 \vec{A} is the area vector [m^2] pointing outward normal to the surface.

Electric field calculations:

Charge distribution	Electric field E (magnitude)
Point charge	$E = \frac{k Q }{r^2}$
Charged ring of radius a (along its axis, which is taken to be the X -axis)	$E = \frac{k Qx }{(x^2 + a^2)^{3/2}}$
Charged disc of radius a (along its axis, which is taken to be the X -axis)	$E = \frac{ \sigma }{2\epsilon_0} \left[1 - \frac{ x }{\sqrt{x^2 + a^2}} \right]$
Infinite non-conducting charged sheet perpendicular to the X -axis	$E = \frac{ \sigma }{2\epsilon_0}$
Infinite line with uniform charge distribution	$E = \frac{ \lambda }{2\pi\epsilon_0 r} = \frac{2k \lambda }{r}$
Sphere of radius a with uniform charge distribution	$E = \frac{k Q }{r^2} \quad r \geq a$ $E = \frac{k Q r}{a^3} \quad r \leq a$

Electric potential: $V = k \int_Q \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \int_Q \frac{dq}{r}$ relative to $V = 0$ at $r \rightarrow \infty$

Relationship between \vec{E} and V : $V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l}$ $E_i = -\frac{\partial V}{\partial x_i}$

Work done by the field in moving a charge q from a to b: $W_{ab} = U_a - U_b = q(V_a - V_b)$

Potential energy of a system of point charges: $U = \sum_{i < j} \frac{kQ_i Q_j}{r_{ij}}$

Work and Potential:

$$\Delta U = U_f - U_i = -W$$

$$U = -W_\infty$$

$$W = \mathbf{F} \cdot \mathbf{d} = Fd \cos \theta$$

$$W = q \int_i^f \mathbf{E} \cdot d\mathbf{s}$$

$$\Delta V = V_f - V_i = -\frac{W}{q}$$

$$V = -\int_i^f \mathbf{E} \cdot d\mathbf{s}$$

U = electric potential energy [J]

W = work done on a particle by a field [J]

W_∞ = work done on a particle brought from infinity (zero potential) to its present location [J]

\mathbf{F} = is the force vector [N]

\mathbf{d} = is the distance vector over which the force is applied [m]

F = is the force scalar [N]

d = is the distance scalar [m]

θ = is the angle between the force and distance vectors

$d\mathbf{s}$ = differential displacement of the charge [m]

V = volts [V]

q = charge [C]

$$W_{a \rightarrow b} = \int_a^b \vec{F} \cdot d\vec{l} = \int_a^b F \cos \phi \, dl \quad (\text{work done by a force})$$

$$K_a + U_a = K_b + U_b$$

Electric potential calculations (Relative to $V = 0$ at ∞):

Charge distribution	Electric potential V
Point charge	$\frac{kQ}{r}$
Charged ring of radius a (along its axis, which is taken to be the X -axis)	$\frac{kQ}{\sqrt{x^2 + a^2}}$
Charged disc of radius a (along its axis, which is taken to be the X -axis)	$\frac{\sigma}{2\epsilon_0} [\sqrt{x^2 + a^2} - x]$
Sphere of radius a with uniform charge distribution	$\frac{kQ}{r} \quad r \geq a$
	$\frac{kQ}{2a} \left[3 - \frac{r^2}{a^2} \right] \quad r \leq a$