1. **A particle (charge = 40 μC) moves directly toward a second particle (charge = 80 μC) which is held in a fixed position. At an instant when the distance between the two particles is 2.0m, the kinetic energy of the moving particle is 16J. Determine the distance separating the two particles when the moving particle is momentarily stopped.**

$q_1 = 40\ \mu C$ $\qquad$ $q_2 = 80\ \mu C$

$r_0 = 2m$
$K_0 = 16\ J$
$k = 9 \times 10^9\ N \cdot m^2/C^2$

By applying the law of Conservation of Energy:

$$K_0 + \frac{kq_1q_2}{r_0} = \frac{kq_1q_2}{r}$$

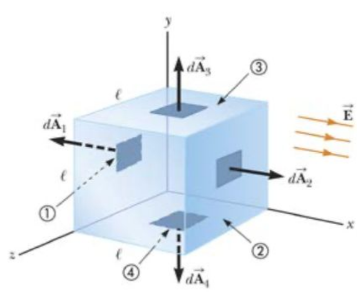
$$16 + \frac{(9\times 10^9\ N\cdot m^2/C^2)\cdot(40\times 10^{-6}\ C)\cdot(80\times 10^{-6}\ C)}{r_0} = \frac{(9\times 10^9\ N\cdot m^2/C^2)\cdot(40\times 10^{-6}\ C)\cdot(80\times 10^{-6}\ C)}{r}$$

$$16 + 14.4 = \frac{28.8}{r}$$

$$r = \frac{28.8}{30.4}$$

$$\boxed{= 0.95m}$$

2. ***N/A***
3. **Consider a uniform electric field $\overline{E}$ oriented in the x direction in empty space. A cube of edge length l, is placed in the field, oriented as shown. Find the net electric flux through the surface of the cube.**



The flux through four of the faces is zero because $\vec{E}$ is parallel to the four faces and therefore perpendicular to $d\vec{A}$ on these faces.

Integrate net flux through ① and ②

$$\phi_E = \int_1 \vec{E} \cdot d\vec{A} + \int_2 \vec{E} \cdot d\vec{A}$$

① $\vec{E}$ is constant and inward direction. $d\vec{A}_1$ has outward direction. $(\theta = 180°)$

$$\int_1 \vec{E} \cdot d\vec{A} = \int_1 E(cos\ 180°)\ dA = -E\int_1 dA = -EA = -El^2$$

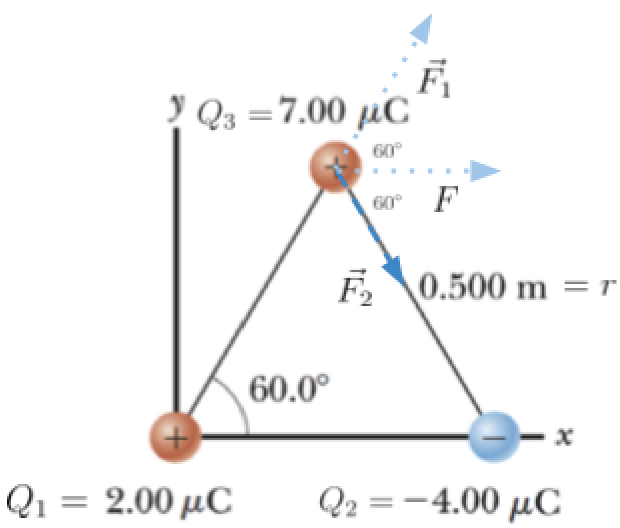
② $\vec{E}$ is constant and outward direction. $d\vec{A}_2$ has the same direction. $(\theta = 0°)$

$$\int_2 \vec{E} \cdot d\vec{A} = \int_2 E(cos\ 0°)\ dA = E\int_2 dA = +EA = El^2$$

By adding the flux for all six faces, we get:

$$\phi_E = -El^2 + El^2 + 0 + 0 + 0 + 0 = \boxed{0}$$

4. Three charged particles are located at the corners of an equilateral triangle as shown. Calculate the total electric force on the 7.00 μC charge.



$$k = 9 \times 10^9\ N \cdot m^2/C^2$$
$$Q_1 = 2 \times 10^{-6}\ C$$
$$Q_2 = -4 \times 10^{-6}\ C$$
$$Q_3 = 7 \times 10^{-6}\ C$$
$$r = 0.500\ m$$

Coulomb's Law

$$F = \frac{kQ_1Q_2}{r^2}$$

$$|\vec{F}_1| = \frac{kQ_1Q_3}{r^2} = \frac{(9 \times 10^9\ N \cdot m^2/C^2) \cdot (2 \times 10^{-6}\ C) \cdot (7 \times 10^{-6}\ C)}{(0.500m)^2}$$

$$|\vec{F}_1| = 0.5040\ N$$

$$|\vec{F}_2| = \frac{kQ_2Q_3}{r^2} = \frac{(9 \times 10^9\ N \cdot m^2/C^2) \cdot (-4 \times 10^{-6}\ C) \cdot (7 \times 10^{-6}\ C)}{(0.500m)^2}$$

$$|\vec{F}_2| = 1.0080\ N$$

$$\text{Resultant Force}$$

$$\vec{F} = F_1\ cos(60°)\hat{x} + F_1\ sin(60°)\hat{y} + F_2\ cos(60°)\hat{x} + F_2\ sin(60°)-\hat{y}$$

$$\vec{F} = (F_1 + F_2) \cdot cos(60°\hat{x}) + (F_1 - F_2) \cdot sin(60°\hat{y})$$

$$\vec{F} = (0.5040 + 1.0080) \cdot \frac{1}{2}\hat{x} + (0.5040 - 1.0080) \cdot \frac{\sqrt{3}}{2}\hat{y}$$

$$\vec{F} = 0.7560\hat{x} + (-0.4365)$$

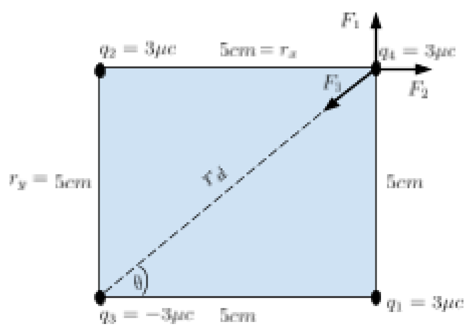
$$\vec{F} = (0.7560)\hat{x} + (-0.4364)\hat{y}$$

$$\text{Magnitude of Resultant on } Q_3$$

$$|\vec{F}| = \sqrt{(0.7560)^2 + (-0.4364)^2}\ at\ \theta = tan^{-1}(\frac{-0.4364}{0.7560}) = -30°$$

$$|\vec{F}| = \boxed{0.873\ N\ at\ \theta = -30°}$$

5. **Three point charades, each of magnitude 3.00 μC, are at separate corners of a square of edge length 5.00cm. The two point charges at opposite corners are positive, and the third point charge is negative. Find the electric force exerted by these point charges on a fourth point charge q_4 = +3.00 μC at the remaining corner.**



$$k = 9 \times 10^9\ N \cdot m^2/C^2$$

$$q_1 = 3 \times 10^{-9}$$

$$q_2 = 3 \times 10^{-9}$$

$$q_3 = -3 \times 10^{-9}$$

$$q_4 = 3 \times 10^{-9}$$

$$\theta \cdot tan^{-1}(\frac{5}{5}) = 45°$$

$r_d = \sqrt{5^2 + 5^2} = 7.0711cm = 0.07011m$, $r_x = 5cm = 0.05m$, $r_y = 5cm = 0.05m$

$$F_x = \frac{k|q_2||q_4|}{{r_x}^2} - \frac{k|q_3||q_4|}{{r_d}^2}cos(45°)$$

$$F_x = (9 \times 10^9) \cdot (3 \times 10^{-9})^2[\frac{1}{(0.05m)^2} - \frac{cos(45°)}{(0.07011m)^2}]$$

$$F_x = 2.0944 \times 10^{-5}\ N$$

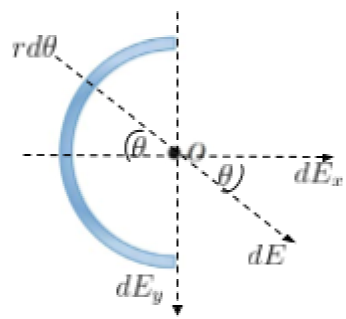
$$F_y = \frac{k|q_1||q_4|}{{r_y}^2} - \frac{k|q_3||q_4|}{{r_d}^2}sin(45°)$$

$$F_y = (9 \times 10^9) \cdot (3 \times 10^{-9})^2[\frac{1}{(0.05m)^2} - \frac{sin(45°)}{(0.07011m)^2}]$$

$$F_y = 2.0944 \times 10^{-5}\ N$$

$$\vec{F} = \boxed{(2.0944 \times 10^{-5})\hat{i} + (2.0944 \times 10^{-5})\hat{j}}$$

6. **A uniformly charged insulating rod of length 14.0cm is bent into the shape of a semicircle as shown. The rod has a total charge of -7.50 μC. Find (a) the magnitude and the direction of the electric field, and (b) the electric potential at O, the center of the semicircle.**



From symmetry, $dE_y = 0$

$$dE = \frac{kq}{r^2}$$

$$q = \frac{Q}{\pi r} \cdot (rd\theta) = \frac{Qd\theta}{\pi}$$

$$dE_x = dEcos(\theta) = \frac{k(\frac{Qd\theta}{\pi})}{r^2} \cdot cos(\theta)$$

$$E_x = \frac{kQ}{\pi r^2} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} cos(\theta) d\theta$$

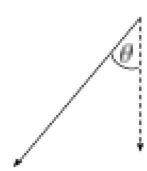
$$= \frac{kQ}{\pi r^2}[sin(\frac{\pi}{2}) - sin(-\frac{\pi}{2})]$$

$$\pi \cdot r^2 = \pi \cdot (0.140)^2$$
$$r = 0.0446$$

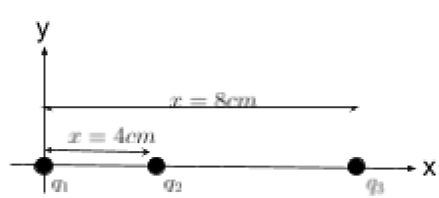
$$E_x = \frac{2 \cdot (9 \times 10^9 \ N \cdot m^2/C^2) \cdot (7.5 \times 10^{-6} \ C)}{\pi \cdot 0.0446 m^2}[sin(\frac{\pi}{2}) - sin(\frac{-\pi}{2})]$$

a) $E_x = \boxed{2.1603 \times 10^7 \ N/C}$

b) $\boxed{\text{To the left}}$



7. **A point particle that has a charge of 5.00 μC is located at x = 0, y = 0, and a point particle that has a charge q is located x = 4.00cm, y = 0. The electric force on a point particle that has a charge of 2.00 μC at x = 8.00cm, y = 0 is -(19.7N) $\hat{i}$. When this 2.00 μC charge is repositioned at x = 17.8cm, y = 0, the electric force on it is zero. Determine the charge q.**



$$k = 9 \times 10^9 \ N \cdot m^2/C^2$$
$$q_1 = 5 \times 10^{-6} \ C$$
$$q_2 = q$$

$$q_3 = 2 \times 10^{-6}\ C$$

$$x_1 = 4cm = 0.04m,\ x_2 = 8cm = 0.08m,\ x_3 = 17.8cm = 0.178m$$

$$F_{31} = \frac{kq_3q_1}{{x_2}^2} = \frac{9 \times 10^9\ N \cdot m^2/C^2 \cdot (2 \times 10^{-6}\ C) \cdot (5 \times 10^{-6}\ C)}{(0.08m)^2} = 14.0625\ N$$

$$F_{32} = \frac{kq_3q_2}{{x_1}^2} = \frac{9 \times 10^9\ N \cdot m^2/C^2 \cdot (2 \times 10^{-6}\ C) \cdot (q)}{(0.04m)^2} = 1.125 \times 10^7 \cdot q$$

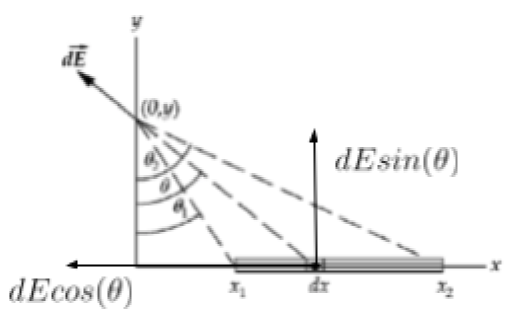
$$\begin{aligned} \vec{F} &= \vec{F_{31}} + \vec{F_{32}} \\ -19.7\ N\hat{i} &= 14.0625\ N\hat{i} + 1.125 \times 10^7 \cdot q\ \hat{i} \\ -33.7625 &= 1.125 \times 10^7 \cdot q\ \hat{i} \\ q &= \frac{-33.7625}{1.125 \times 10^7} = -3.0011 \times 10^{-6} \\ q &= -3\mu C \end{aligned}$$

$$F_{31} = \frac{kq_3q_1}{{x_3}^2} = \frac{9 \times 10^9\ N \cdot m^2/C^2 \cdot (2 \times 10^{-6}) \cdot (5 \times 10^{-6})}{(0.178)^2} = 2.8406\ N$$

$$F_{32} = \frac{9 \times 10^9\ N \cdot m^2/C^2 \cdot (2 \times 10^{-6}) \cdot q}{(x_3 - x_1)^2} = 9.4518 \times 10^5\ q$$

$$\begin{aligned} &F_{31} + F_{32} = 0 \\ &2.8406 + 9.4518 \times 10^5\ q = 0 \\ &q = \frac{-2.8406}{9.4518 \times 10^5} = 3.0054 \times 10^{-6}\ C = \boxed{-3\mu C} \end{aligned}$$

8. A line charge that has a uniform linear charge density λ lies along the x-axis from $x = x_1$ to $x = x_2$ where $x_1 < x_2$. Show that the x component of the electric field at a point on the y-axis is given by $E_x = \frac{k\lambda}{y}(cos\theta_2 - cos\theta_1)$ where
$\theta_1 = tan^{-1}(\frac{x_1}{y}), \theta_2 = tan^{-1}(\frac{x_2}{y})$ $and\ y \neq 0.$



$$\begin{aligned} dE &= \frac{kdq}{x^2 + y^2} \\ &= \frac{k\lambda dx}{x^2 + y^2} \end{aligned}$$

$$
\begin{aligned}
dE_x &= dEcos(\theta) \\
&= \frac{k\lambda dx}{x^2+y^2} \cdot \frac{x}{\sqrt{x^2+y^2}} \\
&= \frac{k\lambda x dx}{(x^2+y^2)^{\frac{3}{2}}}
\end{aligned}
$$

$$
\begin{aligned}
E_x &= \int_{x-x_1}^{x-x_2} \frac{xdx}{(x^2+y^2)^{\frac{3}{2}}} \\
&= k\lambda \int_{x-x_1}^{x-x_2} \frac{xdx}{(x^2+y^2)^{\frac{3}{2}}}
\end{aligned}
$$

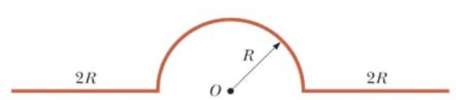
Let $x = ytan(\theta)$

$dx = ysec^2\theta d\theta$,

$x = \theta, x_1 = \theta_1, x_2 = \theta_2$

$$
\begin{aligned}
E_x &= k\lambda \int_{x-x_1}^{\theta-\theta_2} \frac{y\ tan(\theta) \cdot y\ sec^2\theta d\theta}{y^3\ sec^3\theta} \\
&= \frac{k\lambda}{y} \int_{x-x_1}^{\theta-\theta_2} sin(\theta) d\theta \\
&= \boxed{\frac{k\lambda}{y}(cos(\theta_2) - cos(\theta_1))}
\end{aligned}
$$

9. **A wire having a uniform linear charge density λ is bent into the shape shown. Find the electric potential at point O.**



$$V = k_e \int \frac{dq}{r}$$

$$V_{left} = k_e \int_{-R}^{-3R} \frac{\lambda dx}{|x|} = k_e\lambda \int_{-R}^{-3R} \frac{dx}{|x|} = k_e\lambda ln(|x|)|_{-R}^{-3R} = k_e\lambda ln(3)$$

$$V_{right} = k_e \int_{R}^{3R} \frac{\lambda dx}{x} = k_e\lambda \int_{R}^{3R} \frac{dx}{x} = k_e\lambda ln(x)|_{R}^{3R} = k_e\lambda ln(3)$$

$$V_{semi} = k_e \int \frac{dq}{r} = k_e \int_0^{\pi} \frac{\lambda R d\theta}{R} = k_e\lambda \int_0^{\pi} d\theta = \pi \cdot k_e\lambda$$

$$V_0 = V_{semi} + V_{left} + V_{right} = \pi \cdot k_e\lambda + 2k_e\lambda ln(3) = \boxed{k_e\lambda(\pi + 2ln(3)}$$