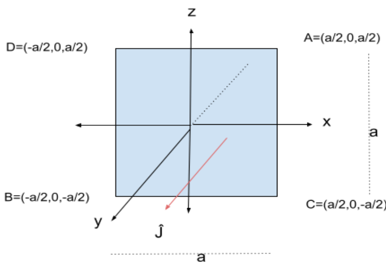
1) A region of space contains an electric field $\overrightarrow{E} = E_1\widehat{i} + E_2\widehat{j}$ where E_1 and E_2 are positive constants. A frame whose corners are located at (x, y, z) = (a/2,0,a/2),(-a/2,0,-a/2),(a/2,0,-a/2) and (-a/2,0,a/2). What is the magnitude of the electric flux through the frame?



Frame is on xz plane, so the direction of vector area is towards $\bar{j}$ and the area is a^2, so area = $\bar{j} \cdot a^2$.

Frame flux: $\phi = \vec{E} \cdot \vec{A}$

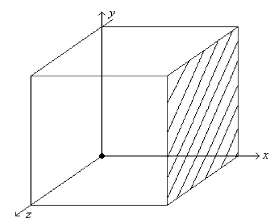
$$\phi = (E_1 \cdot \ \hat{i} + E_2 \cdot \hat{j}) \cdot (a^2 j)$$

$$\phi = \boxed{E_2 \cdot a^2}$$

2) A uniform electric field with a magnitude of 6×10^6 N/C is applied to a cube of edge length 0.1m as shown in Fig. 22-2. If the direction of the E-field is along the +x-axis, what is the electric flux passing through the shaded face of the cube?

$$E = \ 6.0 \times 10^6 \ N/C$$

$$L = \ 0.1m$$



$$\phi = E \times A$$

$$\phi = 6.0 \times 10^6 \ N/C \cdot (0.1)^2$$

$$\phi = \boxed{6.0 \times 10^4 \ N \cdot m^2/C}$$

3) A spherical, non-conducting shell of inner radius $r_1 = 10$ cm and outer radius $r_2 = 15$ cm carries a total charge Q = 15 μC distributed uniformly throughout its volume. What is the electric field at a distance r = 12 cm from the center of the shell?

$$V = \frac{4\pi r^3}{3}, \quad \rho = \frac{Q}{V}$$

$$V_{outer} = \frac{4\pi(15^3 - 10^3)}{3}, \quad \rho = \frac{15.0 \times 10^{-6}}{9.9484 \times 10^3}$$

$$V_{outer} = 9948cm^3, \quad \rho = 1.5078 \times 10^{-9}/m^2$$

$$V_{inner} = \frac{4\pi(12^3 - 10^3)}{3}, \quad \rho = \frac{Q_{in}}{3049 \cdot cm^3}$$

$$V_{inner} = 3049 \cdot cm^3, \quad Q_{in} = (1.5078 \times 10^{-9} \cdot C/cm^3) \cdot (3049 \cdot cm^3)$$

$$Q_{in} = 4.59 \times 10^{-6} \cdot C$$

$$E = \frac{kQ_{in}^2}{{r_1}^2}, \quad E = \frac{(9.0 \times 10^9 \ N \cdot m^2/C^2) \cdot (4.59 \times 10^{-6} \ C)}{(0.12)^2}$$

$$E = \boxed{2.87 \times 10^6 \ N/C}$$

4) A nonconducting spherical shell of inner radius R_1 and outer radius R_2 contains a uniform volume charge density E throughout the shell. Use Gauss's law to derive an equation for the magnitude of the electric field at the following radial distances r from the center of the sphere. Your answers should be in terms of ρ, R_1, R_2, r, ϵ_0 , and π (SHOW ALL YOUR WORK, DERIVE THE EXPRESSION)

a) $r < R_1$,

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$E \cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$q = 0$$

$$E = \boxed{0\frac{N}{C}}$$

b) $R_1 < r < R_2$,

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$E \cdot 4\pi r^2 = \frac{\rho(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi R_1^3)}{\epsilon_0}$$

$$E = \frac{\rho(r^3 - R_1^3)}{3\epsilon_0 r}$$

$$E = \boxed{\frac{\rho}{3\epsilon_0}(r - \frac{R_1^3}{r^2})}$$

c) $r > R_2$

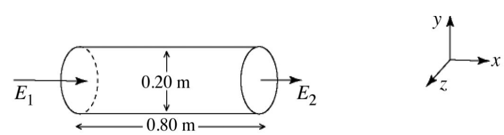
$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$E \cdot 4\pi r^2 = \frac{\rho(\frac{4}{3}\pi R_2^3 - \frac{4}{3}\pi R_1^3)}{\epsilon_0}$$

$$E = \frac{\rho(R_2^3 - R_1^3)}{3\epsilon_0 r^2}$$

$$E = \boxed{\frac{\rho}{3\epsilon_0 r^2} \cdot (R_2^3 - R_1^3)}$$

5) A nonuniform electric field is directed along the x-axis at all points in space. This magnitude of the field varies with x, but not with respect to y or z. The axis of a cylindrical surface, 0.80 m long and 0.20 m in diameter, is aligned parallel to the x-axis, as shown in the figure. The electric fields E_1 and E_2, at the ends of the cylindrical surface, have magnitudes of 9000 N/C and 5000 N/C respectively, and are directed as shown. (ϵ_0 = 8.85 × 10-12 $C^2/N \cdot m^2$) The charge enclosed by the cylindrical surface is closest to



$$D = 0.2m,\ r = 0.1m,\ A = \pi \cdot (0.1)^2$$

$\Phi_1 = \overline{E_1} \cdot \overline{A_1}$,

Φ_1 = 9000 $\cdot \pi \cdot 10^{-2}$

Φ_1 = 282.74 N/C $\cdot m^2$

$\Phi_2 = \overline{E_2} \cdot \overline{A_2}$,

Φ_2 = 5000 $\cdot \pi \cdot 10^{-2}$

Φ_2 = 157.08 N/C $\cdot m^2$

$\Phi_{net} = \Phi_2 - \Phi_1$

$\Phi_{net} = 157.08 - 282.74$

$\Phi_{net} = -125.67$

$$\cong \boxed{-1.1\ nC}$$