Chapter 21

Electric Charge and Electric Field

PowerPoint® Lectures for
University Physics, 14th Edition
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Lectures by Jason Harlow

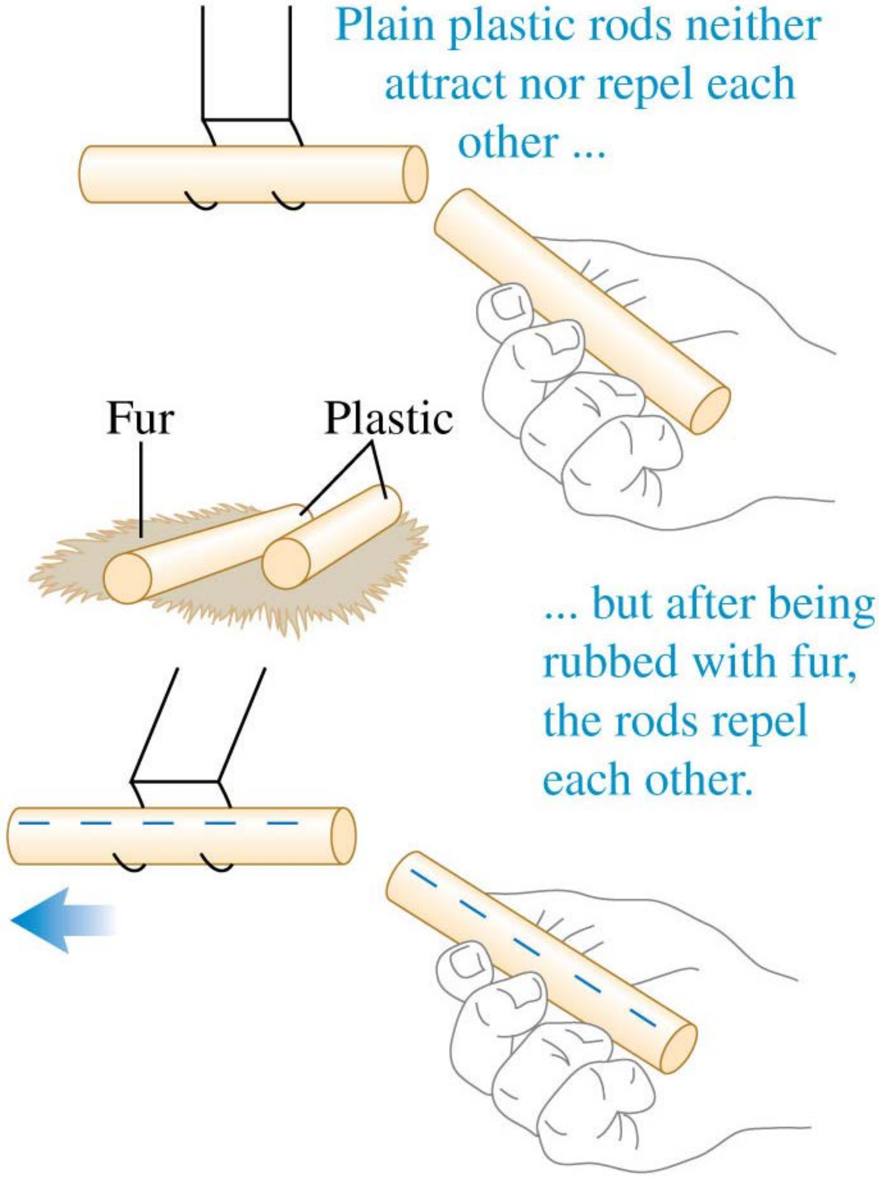
Learning Goals for Chapter 21

Looking forward at …

- how objects become electrically charged, and how we know that electric charge is conserved.
- how to use Coulomb's law to calculate the electric force between charges.
- the distinction between electric force and electric field.
- how to use the idea of electric field lines to visualize and interpret electric fields.
- how to calculate the properties of electric charge distributions, including dipoles.

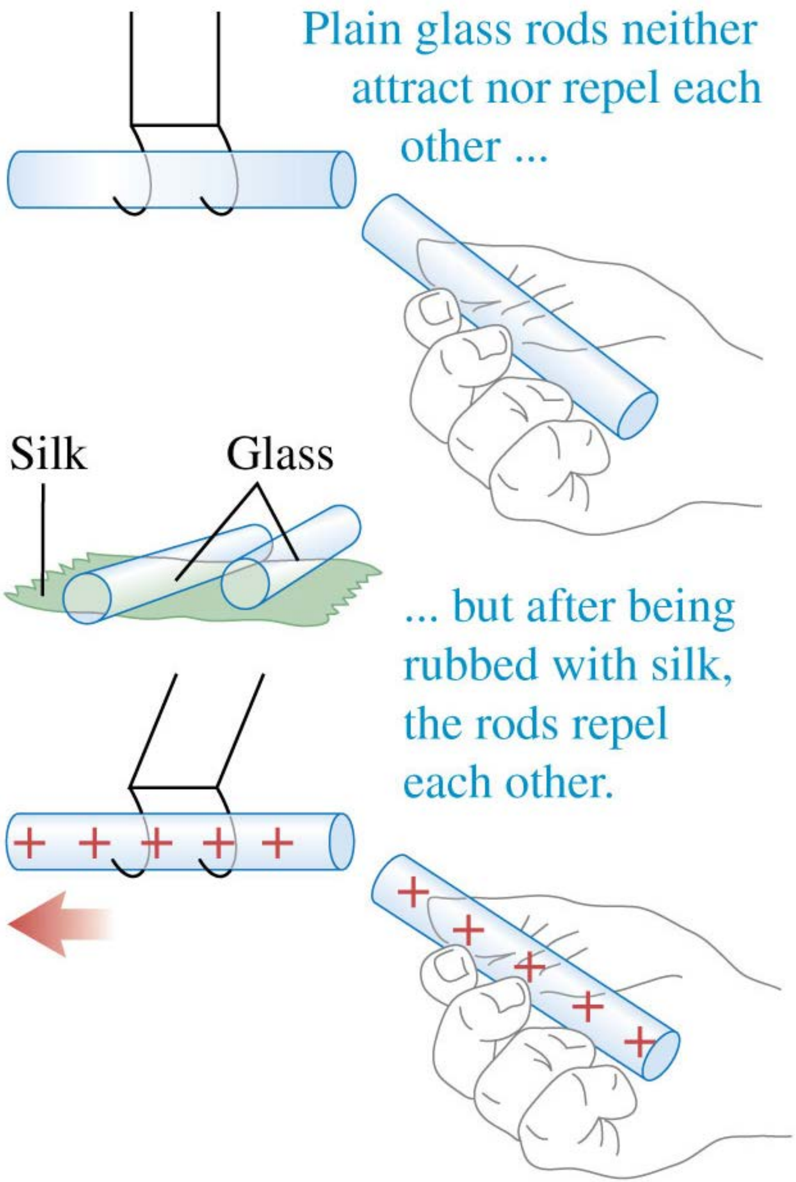
Electric charge

- Plastic rods and fur (real or fake) are particularly good for demonstrating **electrostatics**, the interactions between electric charges that are at rest (or nearly so).
- After we charge both plastic rods by rubbing them with the piece of fur, we find that the rods *repel* each other.



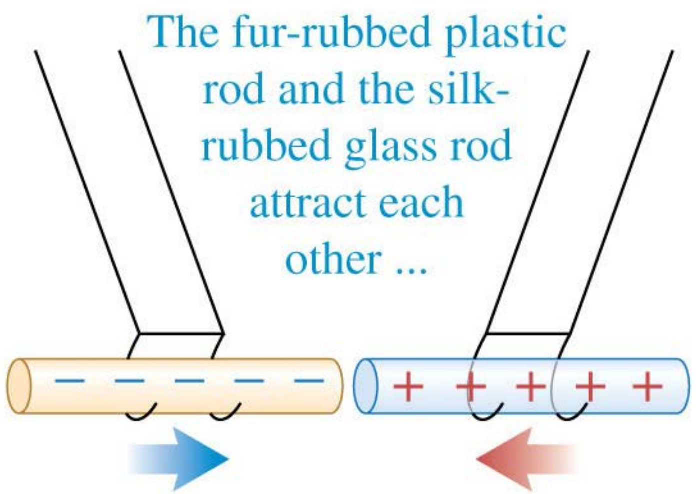
Electric charge

- When we rub glass rods with silk, the glass rods also become charged and repel each other.

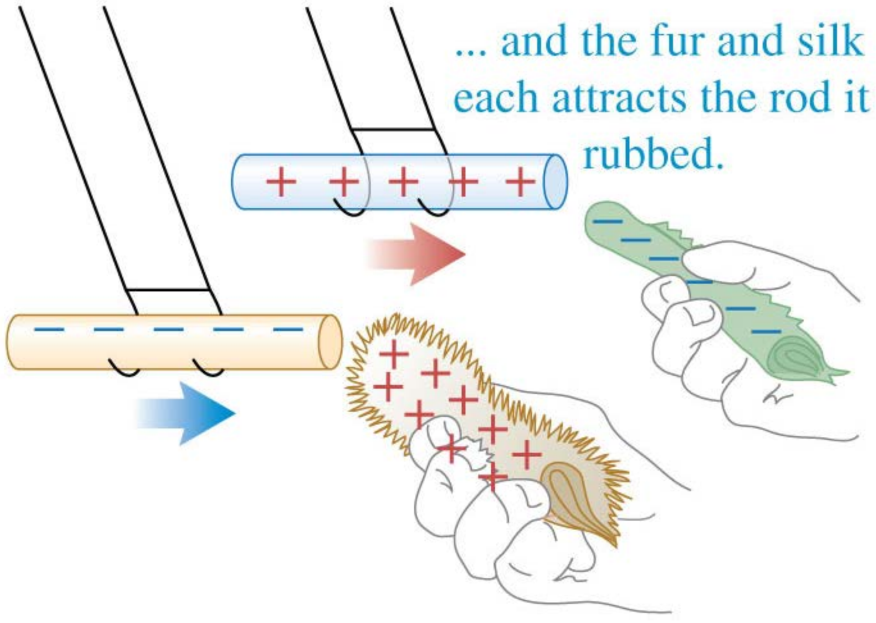


Electric charge

- A charged plastic rod *attracts* a charged glass rod; furthermore, the plastic rod and the fur *attract* each other, and the glass rod and the silk *attract* each other.



- These experiments and many others like them have shown that there are exactly two kinds of electric charge: The kind on the plastic rod rubbed with fur (*negative*) and the kind on the glass rod rubbed with silk (*positive*).



Electric charge

Charge on the electron:

$$e = 1.602 \times 10^{-19}\ \text{C}.$$

Electric charge is quantized in units of the electron charge.

Electric Charge and Its Conservation

The Law of Conservation of Electric Charge

Electric charge is conserved – the arithmetic sum of the total charge cannot change in any interaction.

No net electric charge can be created or destroyed

Example 21.1

- **A glass rod is charged to +5.0 nC by rubbing.**

  a. **Have electrons been removed from the rod or protons added?**

  In the process of charging by rubbing, electrons are removed from one material and transferred to the other because they are relatively free to move. Protons, on the other hand, are tightly bound in nuclei. So, electrons have been removed from the glass rod to make it positively charged.

  b. **How many electrons have been removed or protons added?**

  - Number of electrons removed is $$\frac{5\times10^{-9}\ \text{C}}{1.6\times10^{-19}\ \text{C}} = 3.1\times10^{10}$$

Example 21.2

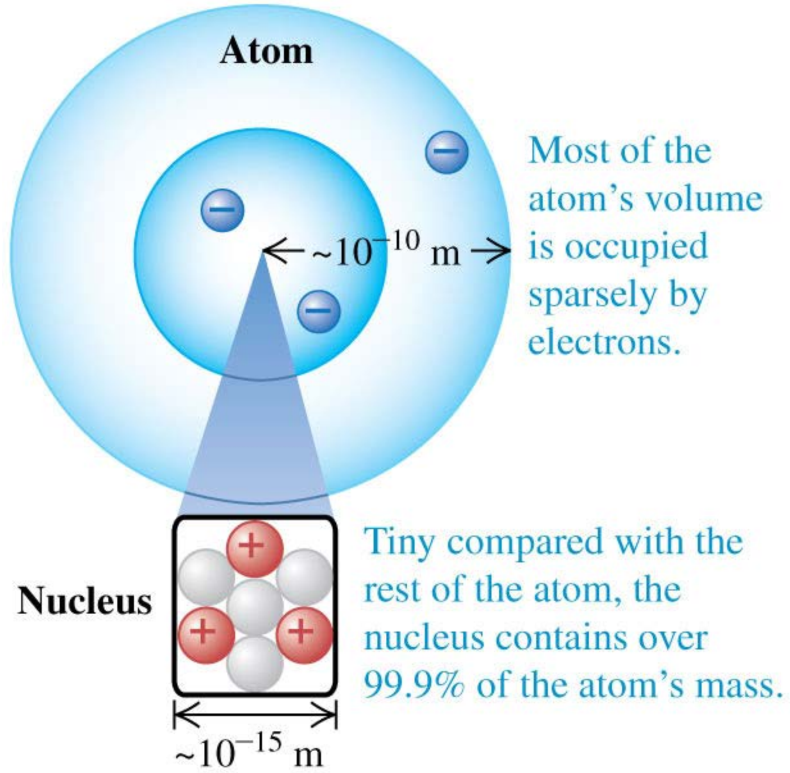
- **Suppose you have 1.0 mol of 0_2 gas. How many coulombs of positive charge are contained in the atomic nuclei of this gas?**

Each oxygen molecule has 16 protons (8 per atom), and there are 6.02 x 10^{23} oxygen molecules in 1.0 mole of oxygen. The proton has a positive charge of magnitude 1.6 x 10^{-19} C. The amount of positive charge in 1.0 mole of oxygen is

$$6.02 \times 10^{23} \times (16 \times 1.6 \times 10^{-19}\ \mathrm{C}) = 1.5 \times 10^{6}\ \mathrm{C}$$

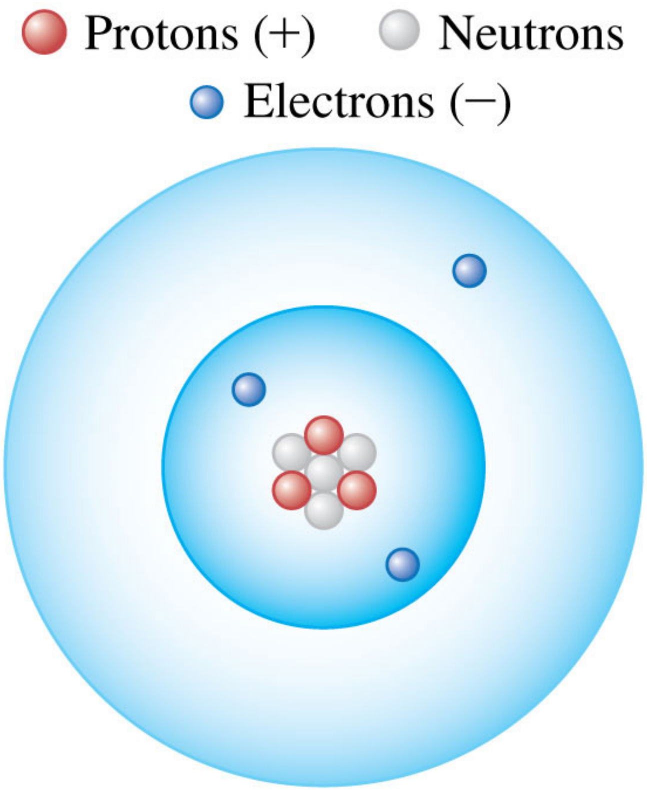
Electric charge and the structure of matter

- The particles of the atom are the negative *electrons* (dark blue spheres in this figure), the positive *protons* (red spheres), and the uncharged *neutrons* (gray spheres).
- Protons and neutrons make up the tiny dense nucleus, which is surrounded by electrons.



Atoms and ions

- A neutral atom has the same number of protons as electrons.
- The electron "shells" are a schematic representation of the actual electron distribution, a diffuse cloud many times larger than the nucleus.



Neutral lithium atom (Li):

3 protons (3+)

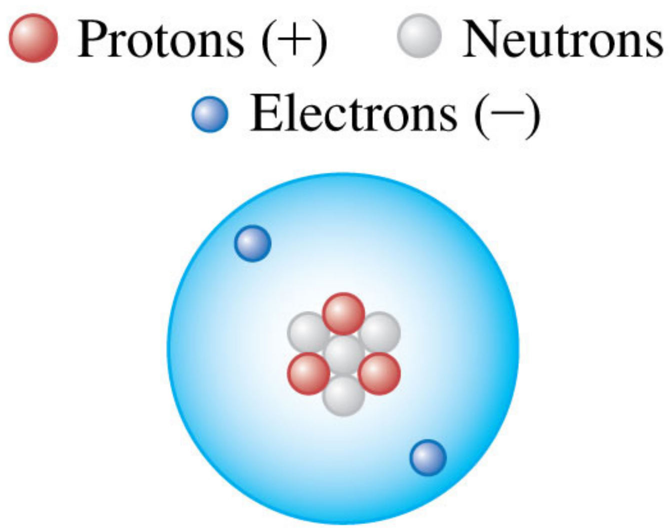
4 neutrons

3 electrons (3−)

Electrons equal protons:
Zero net charge

Atoms and ions

- A *positive ion* is an atom with one or more electrons removed.



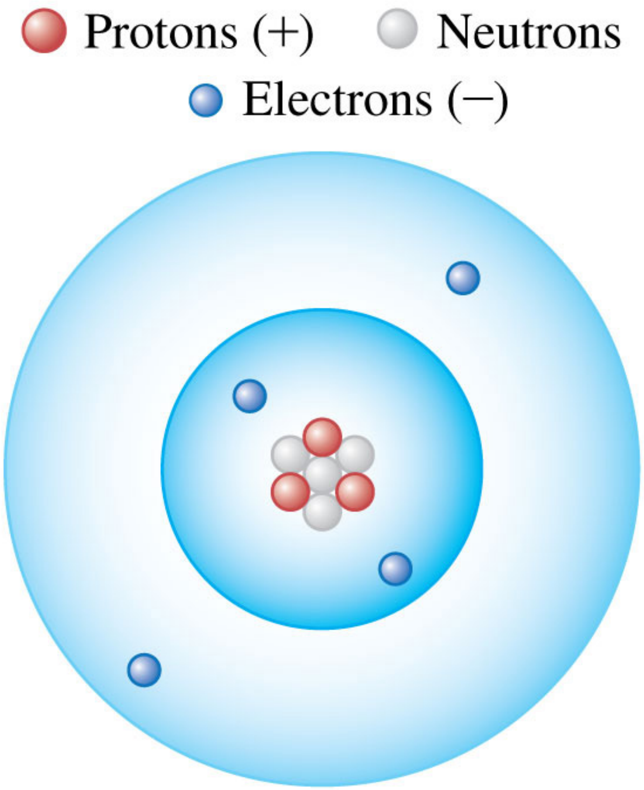
Positive lithium ion (Li^+):

3 protons (3+)
4 neutrons
2 electrons (2−)

Fewer electrons than protons:
Positive net charge

Atoms and ions

- A *negative ion* is an atom with an excess of electrons.



Negative lithium ion (Li^-):

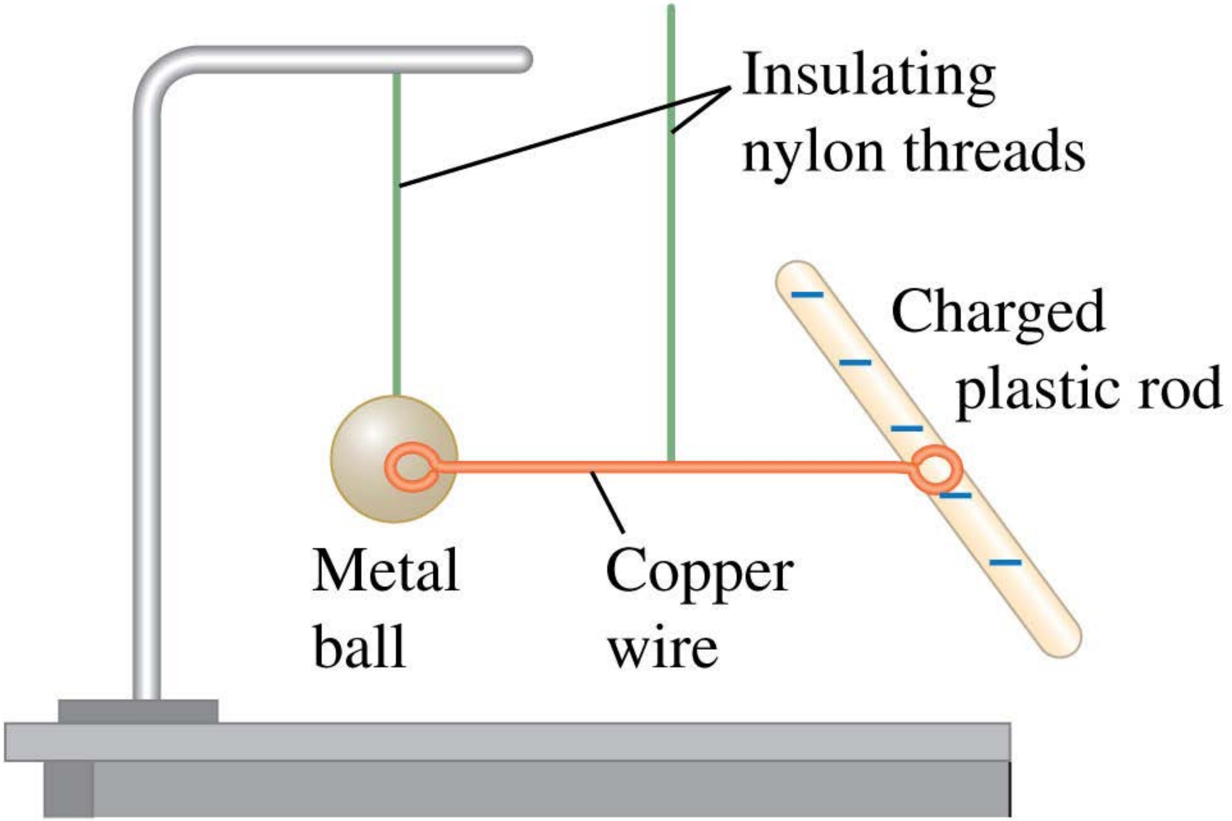
3 protons (3+)
4 neutrons
4 electrons (4−)

More electrons than protons: Negative net charge

Conservation of charge

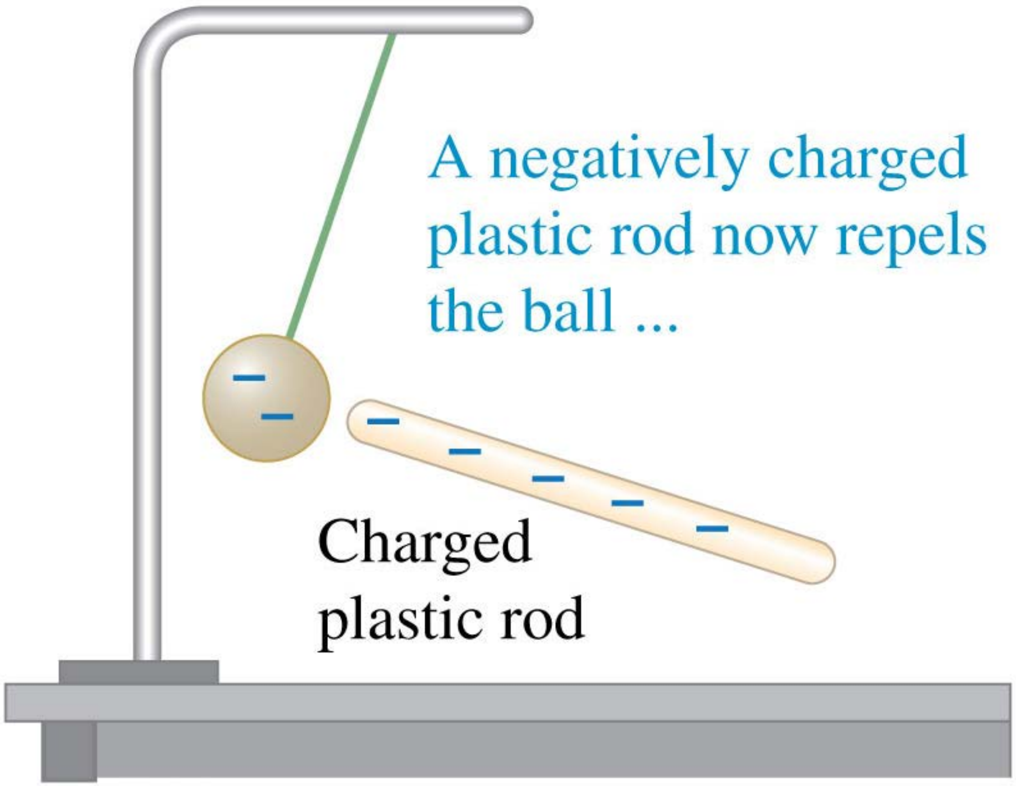
- The proton and electron have the same magnitude charge.
- The magnitude of charge of the electron or proton is a natural unit of charge. All observable charge is *quantized* in this unit.
- The universal **principle of charge conservation** states that the algebraic sum of all the electric charges in any closed system is constant.

Conductors and insulators



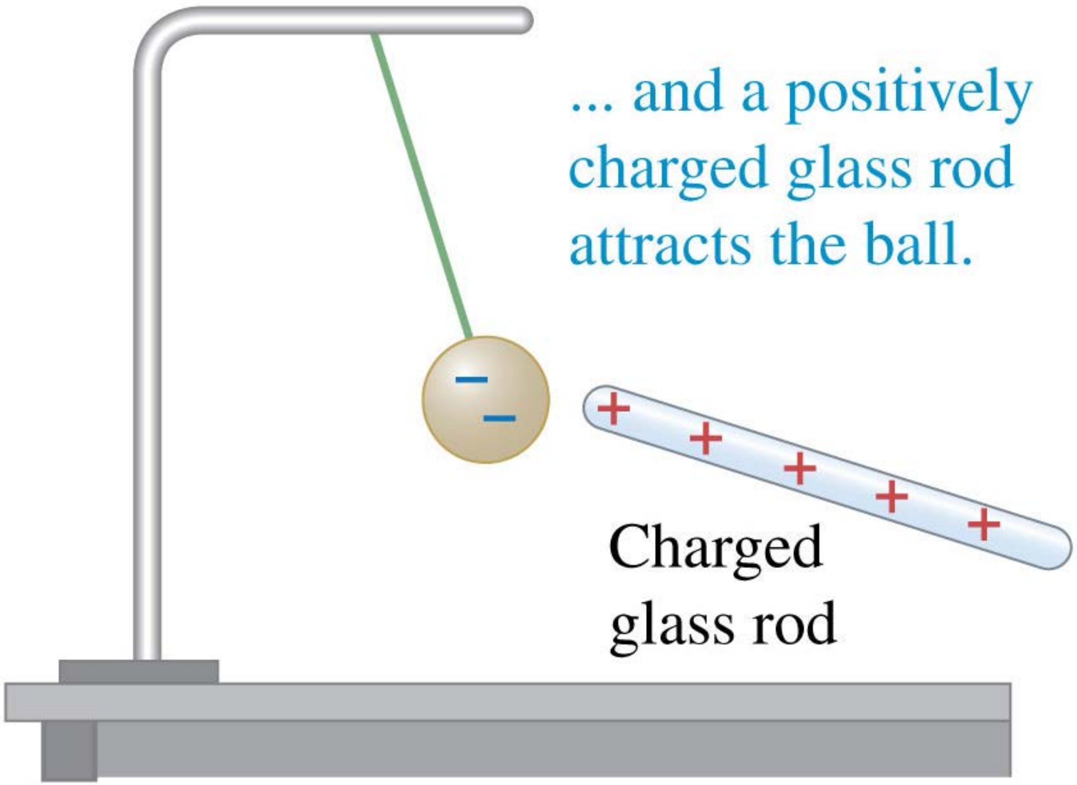
- Copper is a good conductor of electricity; nylon is a good insulator. The copper wire shown conducts charge between the metal ball and the charged plastic rod to charge the ball negatively.

Conductors and insulators



- After it is negatively charged, the metal ball is repelled by a negatively charged plastic rod.

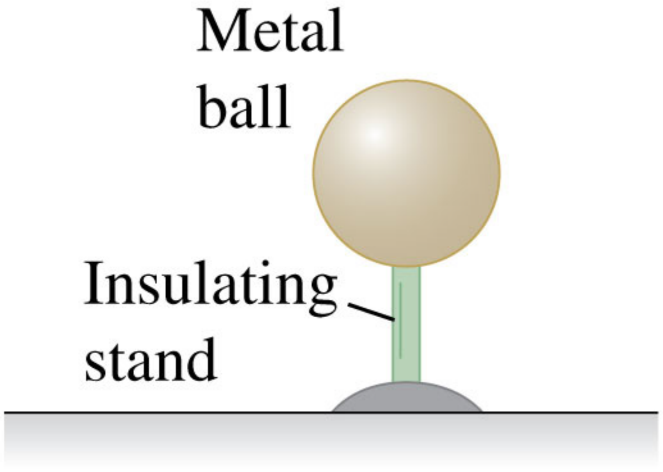
Conductors and insulators



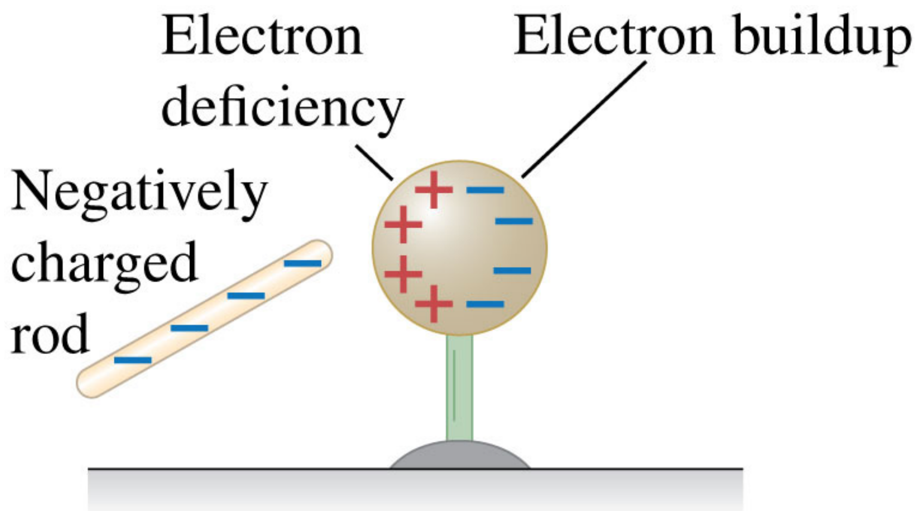
- After it is negatively charged, the metal ball is attracted by a positively charged glass rod.

Charging by induction in 4 steps: Steps 1 and 2

1. Start with an uncharged metal ball supported by an insulating stand.

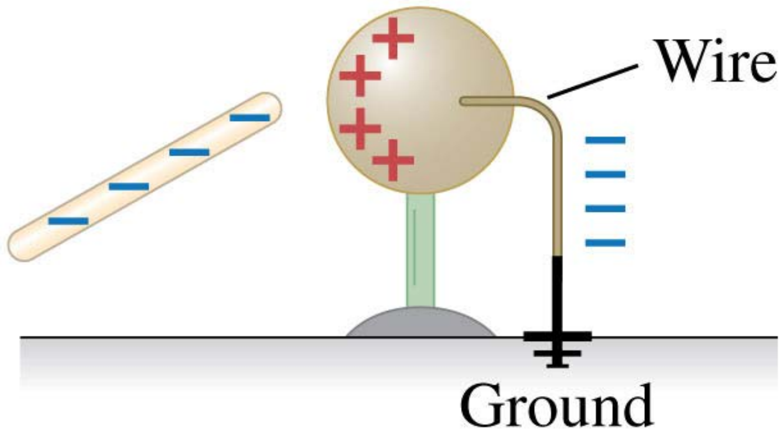


2. When you bring a negatively charged rod near it, without actually touching it, the free electrons in the metal ball are repelled by the excess electrons on the rod, and they shift toward the right, away from the rod.

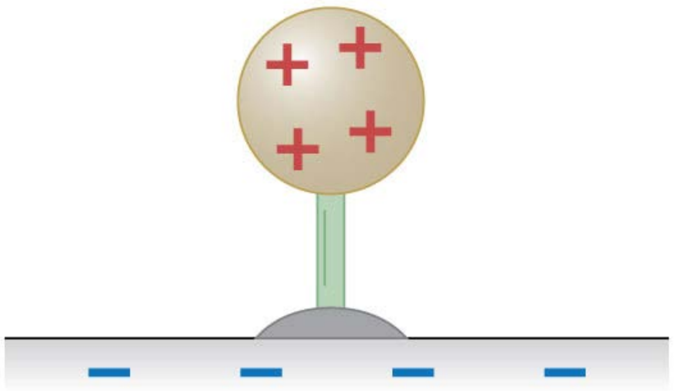


Charging by induction in 4 steps: Steps 3 and 4

3. While the plastic rod is nearby, you touch one end of a conducting wire to the right surface of the ball and the other end to the ground.



4. Now disconnect the wire, and then remove the rod. A net positive charge is left on the ball. The earth acquires a negative charge that is equal in magnitude to the induced positive charge remaining on the ball.

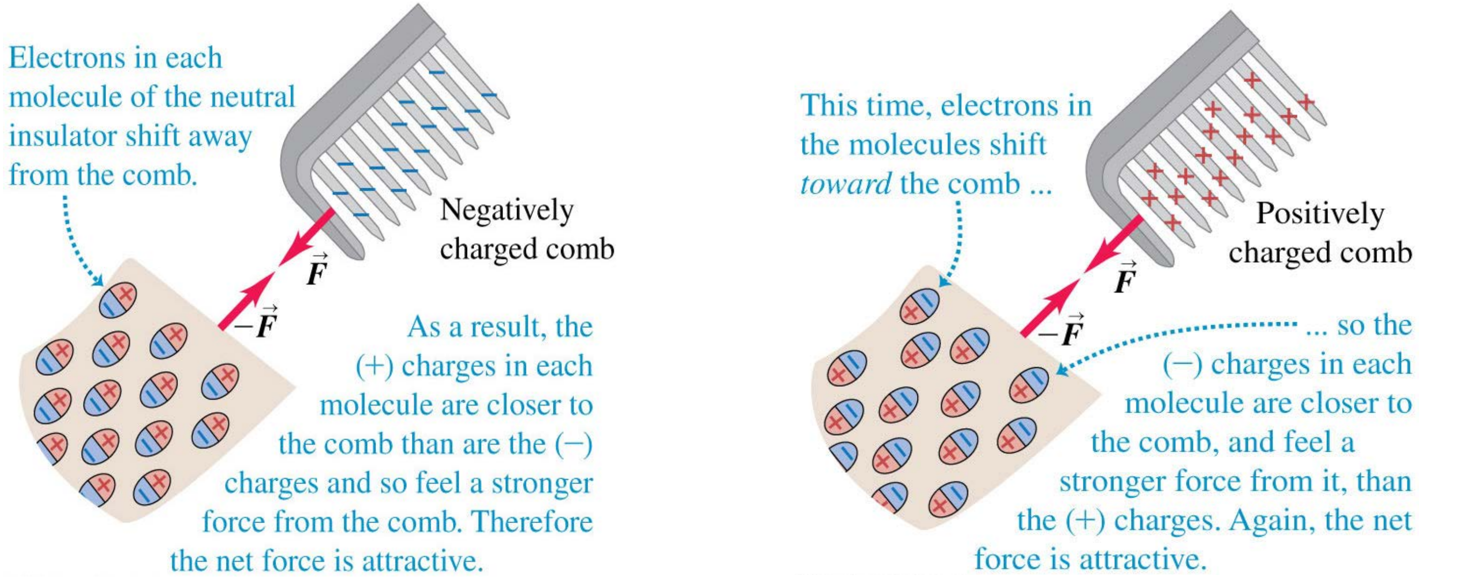


Electric forces on uncharged objects

- A charged body can exert forces even on objects that are not charged themselves.
- If you rub a balloon on the rug and then hold the balloon against the ceiling, it sticks, even though the ceiling has no net electric charge.
- After you electrify a comb by running it through your hair, you can pick up uncharged bits of paper or plastic with it.
- How is this possible?

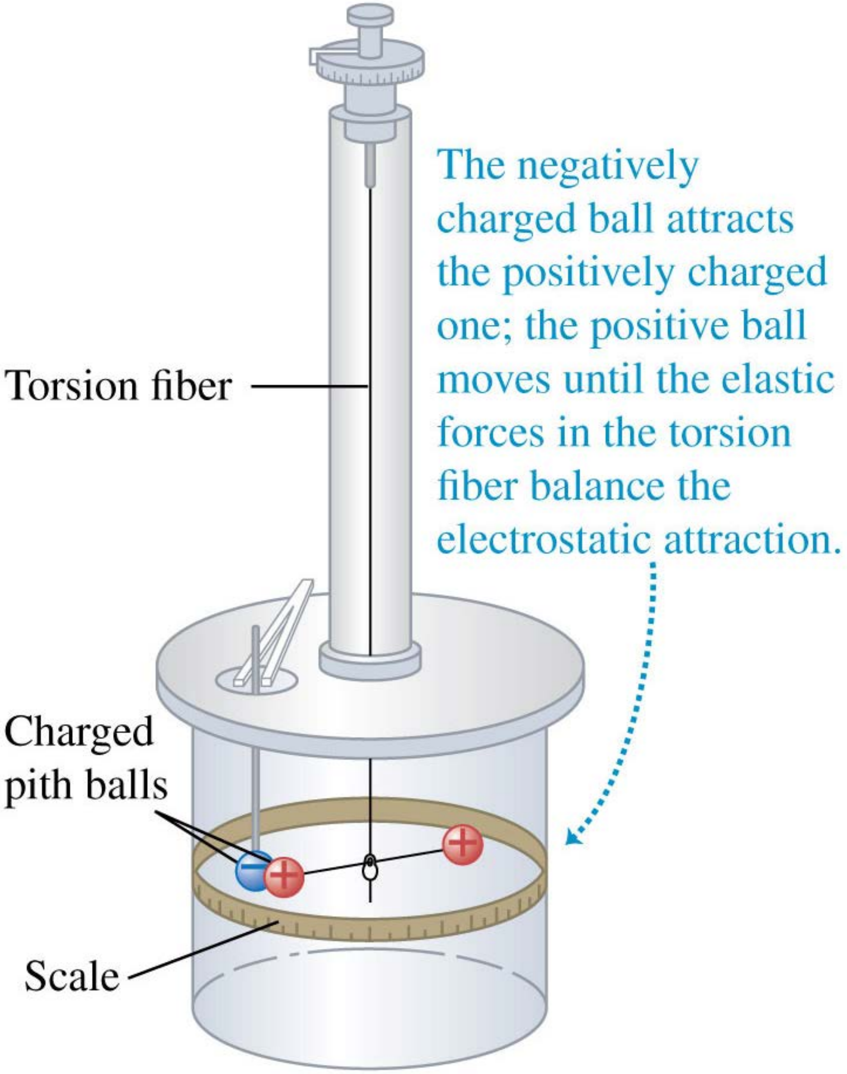
Electric forces on uncharged objects

- The negatively charged plastic comb causes a slight shifting of charge within the molecules of the neutral insulator, an effect called **polarization**.
- Note that a neutral insulator is also attracted to a *positively* charged comb.
- A charged object of *either* sign exerts an *attractive* force on an uncharged insulator.



Measuring the electric force between point charges

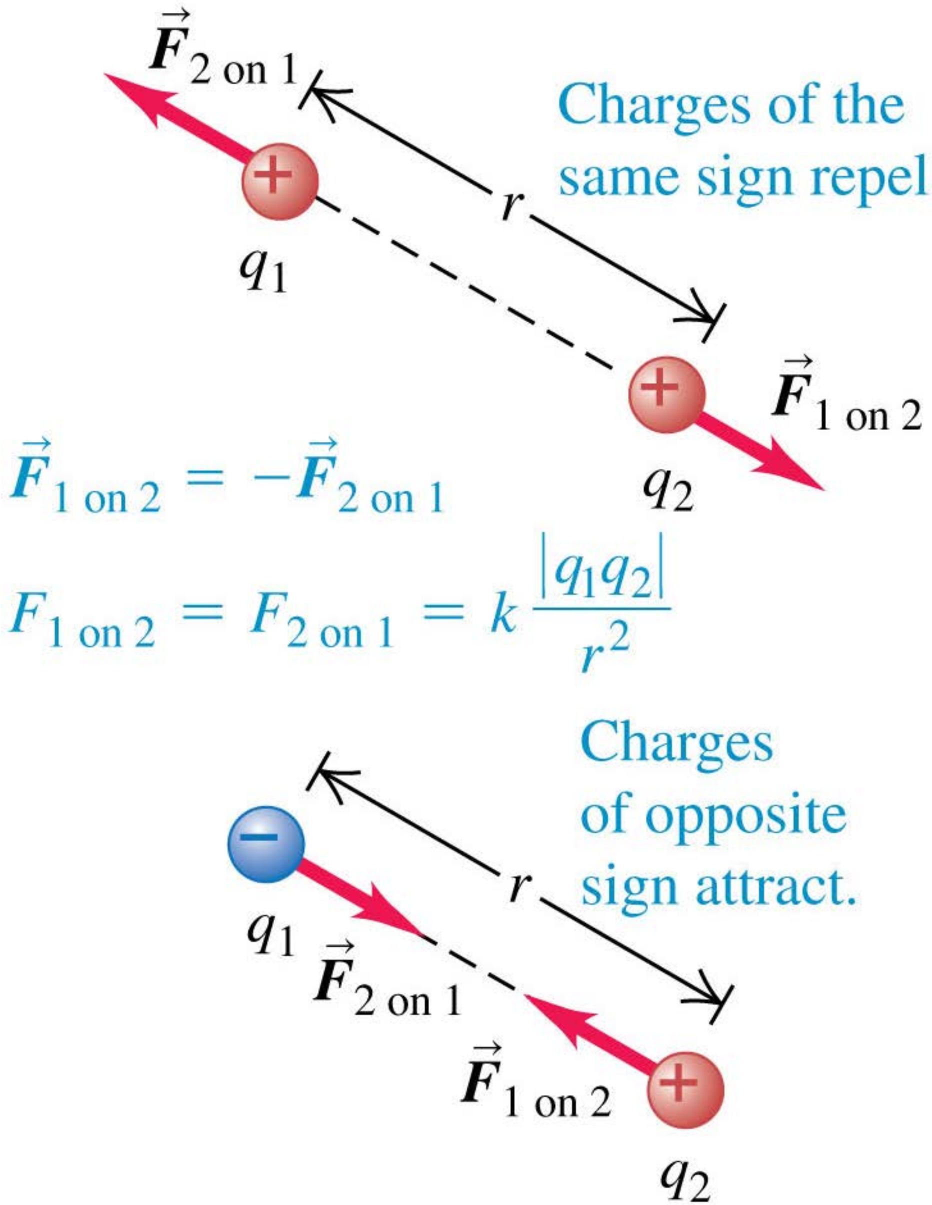
- Coulomb studied the interaction forces of charged particles in detail in 1784.
- He used a torsion balance similar to the one used 13 years later by Cavendish to study the much weaker gravitational interaction.
- For point charges, Coulomb found that the magnitude of the electric force is inversely proportional to the square of the distance between the charges.



Coulomb's Law

- **Coulomb's Law**: The magnitude of the electric force between two point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.

$$F = k\frac{|q_1 q_2|}{r^2}$$

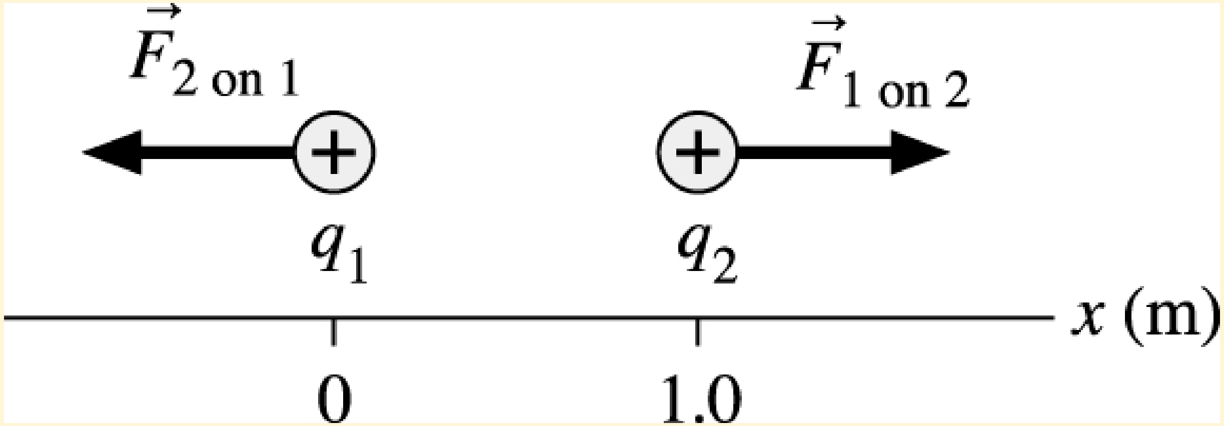


Example 21.3

Two 1.0 kg masses are 1.0 m apart on a frictionless table. Each has +1.0 μC of charge.

a. **What is the magnitude of the electric force on one of the masses?**

b. **What is the initial acceleration of each mass if they are released and allowed to move?**



Example 21.3 (Solution)

- a.

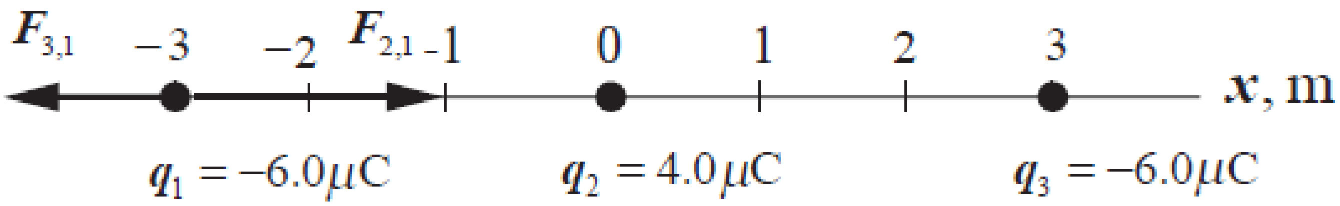
$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{K|q_1||q_2|}{r_{12}^2} = \frac{(9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.0\times10^{-6}\ \text{C})\,(1.0\times10^{-6}\ \text{C})}{(1.0\ \text{m})^2} = 9.0\times10^{-3}\ \text{N}$$

- b.

$$a_1 = \frac{9.0\times10^{-3}\ \text{N}}{1.0\ \text{kg}} = 9.0\times10^{-3}\ \text{m/s}^2$$

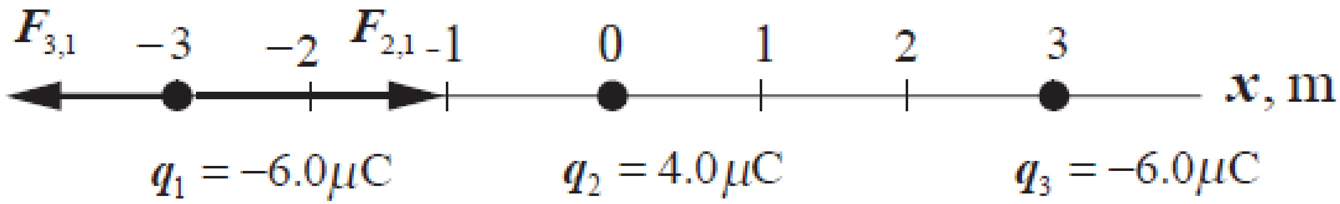
Example 21.4: Three Point Charges

Three point charges are on the *x*-axis: q_1 = –6.0 μC is at *x* = –3.0 m, q_2 = 4.0 μC is at the origin, and q_3 = –6.0 μC is at *x* = 3.0 m. Find the electric force on q_1.



Picture the Problem q_2 exerts an attractive electric force $\boldsymbol{F_{2,1}}$ on point charge q_1 and q_3 exerts a repulsive electric force $\boldsymbol{F_{3,1}}$ on point charge q_1. We can find the net electric force on q_1 by adding these forces (that is, by using the superposition principle).

Example 21.4: Three Point Charges (Solution)



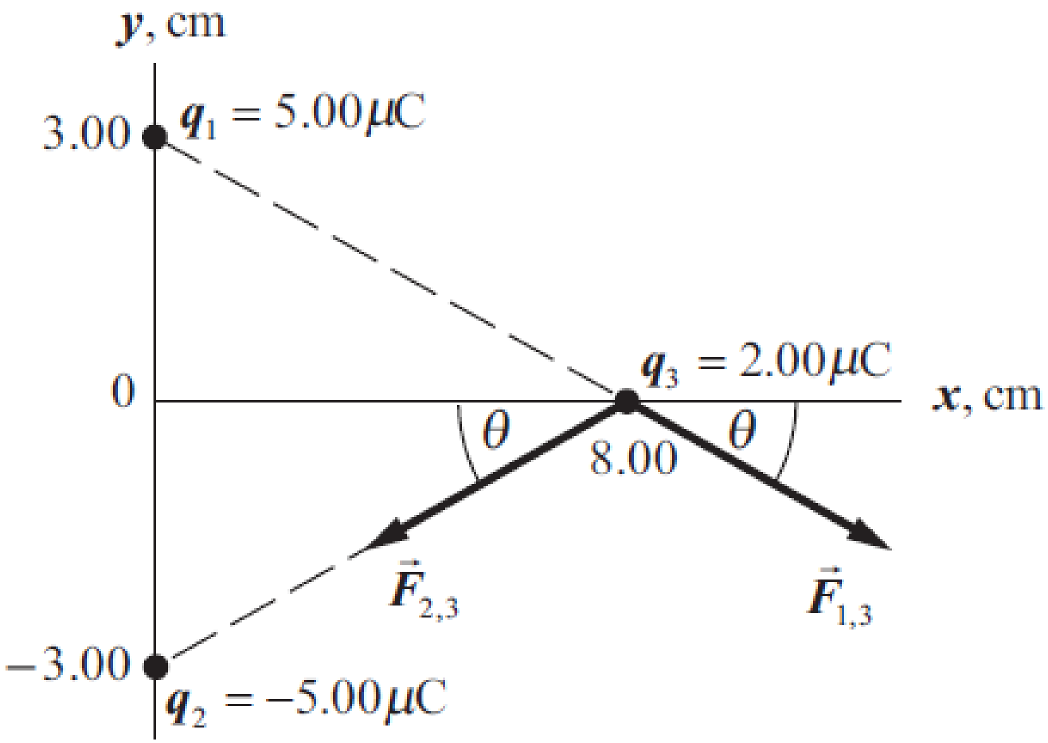
$\vec{F}_1 = \vec{F}_{2,1} + \vec{F}_{3,1}$ where $\vec{F}_{2,1} = \dfrac{k|q_1||q_2|}{r_{2,1}^2}\hat{i}$ $\quad \vec{F}_{3,1} = \dfrac{k|q_1||q_3|}{r_{3,1}^2}\left(-\hat{i}\right)$

$$\vec{F}_1 = \frac{k|q_1||q_2|}{r_{2,1}^2}\hat{i} - \frac{k|q_1||q_3|}{r_{3,1}^2}\hat{i} = k|q_1|\left(\frac{|q_2|}{r_{2,1}^2} - \frac{|q_3|}{r_{3,1}^2}\right)\hat{i}$$

$$\vec{F}_1 = \left(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(6.0\,\mu\text{C}\right)\left(\frac{4.0\,\mu\text{C}}{(3.0\,\text{m})^2} - \frac{6.0\,\mu\text{C}}{(6.0\,\text{m})^2}\right)\hat{i} = \boxed{\left(1.5\times10^{-2}\ \text{N}\right)\hat{i}}$$

Example 21.5

A point charge of 5.00 μC is on the *y* axis at *y* = 3.00 cm, and a second point charge of –5.00 μC is on the *y* axis at *y* = –3.00 cm. Find the electric force on a point charge of 2.00 μC on the *x* axis at *x* = 8.00 cm.



Example 21.5 (Solution)

- **Picture the Problem** The configuration of the point charges and the forces on point charge q_3 are shown in the figure … as is a coordinate system. From the geometry of the charge distribution it is evident that the net force on the 2.00 μC point charge is in the negative y direction.

$$\vec{F}_3 = \vec{F}_{1,3} + \vec{F}_{2,3} \qquad \vec{F}_{1,3} = F\cos\theta\,\hat{i} - F\sin\theta\,\hat{j}$$

The force that point charge q_1 exerts on point charge q_3 is:

$$F = \frac{kq_1q_3}{r^2} = \frac{\left(8.988\times10^9\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)(5.00\,\mu\mathrm{C})(2.00\,\mu\mathrm{C})}{(0.0300\,\mathrm{m})^2+(0.0800\,\mathrm{m})^2} = 12.32\,\mathrm{N}$$

$$\theta = \tan^{-1}\left(\frac{3.00\,\mathrm{cm}}{8.00\,\mathrm{cm}}\right) = 20.56^\circ$$

Example 21.5 (Solution)

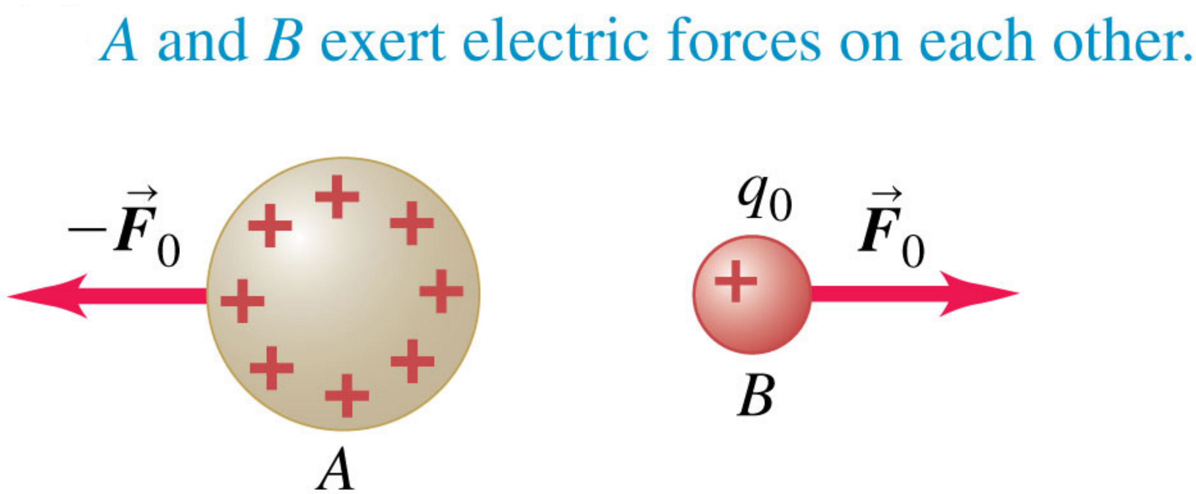
- The force that point charge q_2 exerts on point charge q_3 is:

$$\vec{F}_{2,3} = -F\cos\theta\,\hat{\boldsymbol{i}} - F\sin\theta\,\hat{\boldsymbol{j}}$$

$$\begin{aligned}\vec{F}_3 &= F\cos\theta\,\hat{\boldsymbol{i}} - F\sin\theta\,\hat{\boldsymbol{j}} - F\cos\theta\,\hat{\boldsymbol{i}} \\ &\quad - F\sin\theta\,\hat{\boldsymbol{j}} \\ &= -2F\sin\theta\,\hat{\boldsymbol{j}}\end{aligned}$$

$$\begin{aligned}\vec{F}_3 &= -2(12.32\,\mathrm{N})\sin 20.56^\circ\,\hat{\boldsymbol{j}} \\ &= \boxed{-(8.65\,\mathrm{N})\hat{\boldsymbol{j}}}\end{aligned}$$

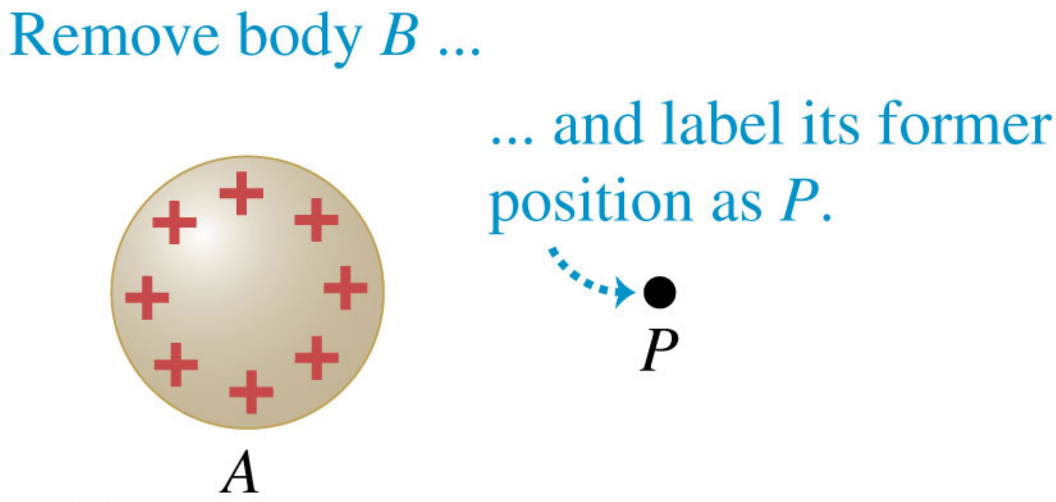
Electric field: Introduction Slide 1 of 3



- To introduce the concept of **electric field**, first consider the mutual repulsion of two positively charged bodies *A* and *B*.

Electric field: Introduction Slide 2 of 3

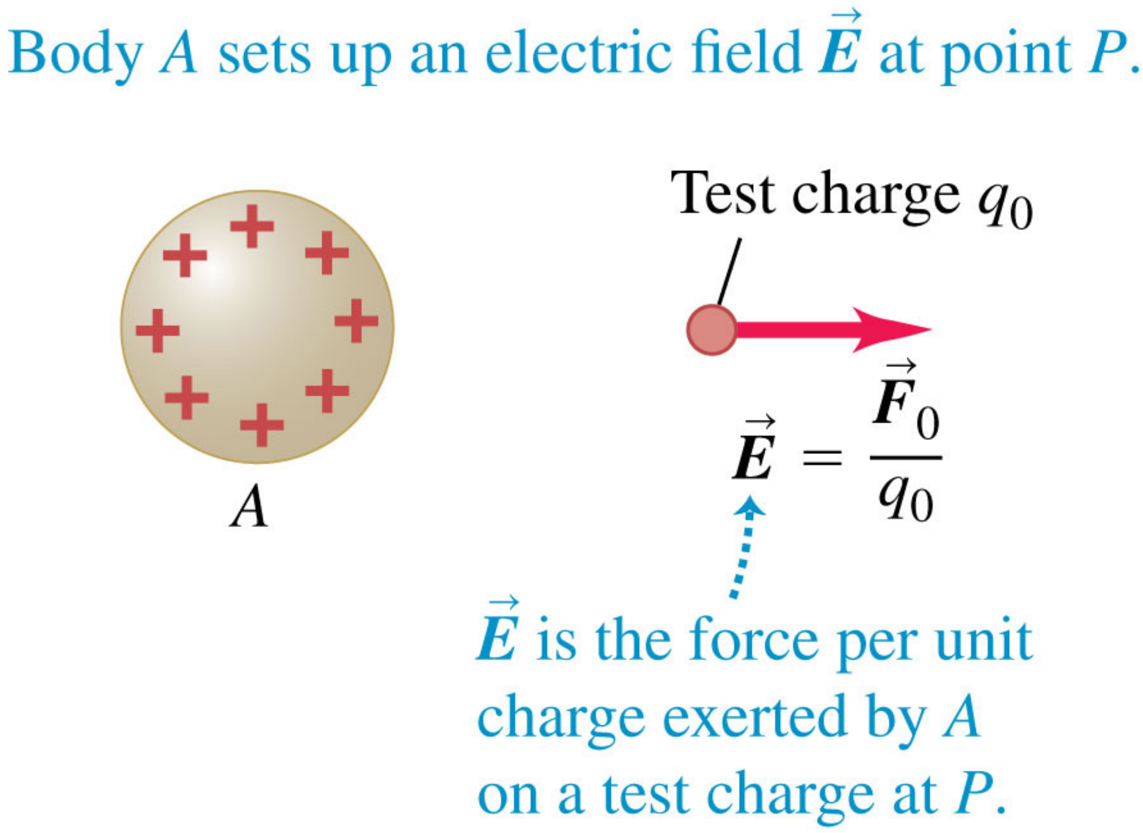
- Next consider body *A* on its own.



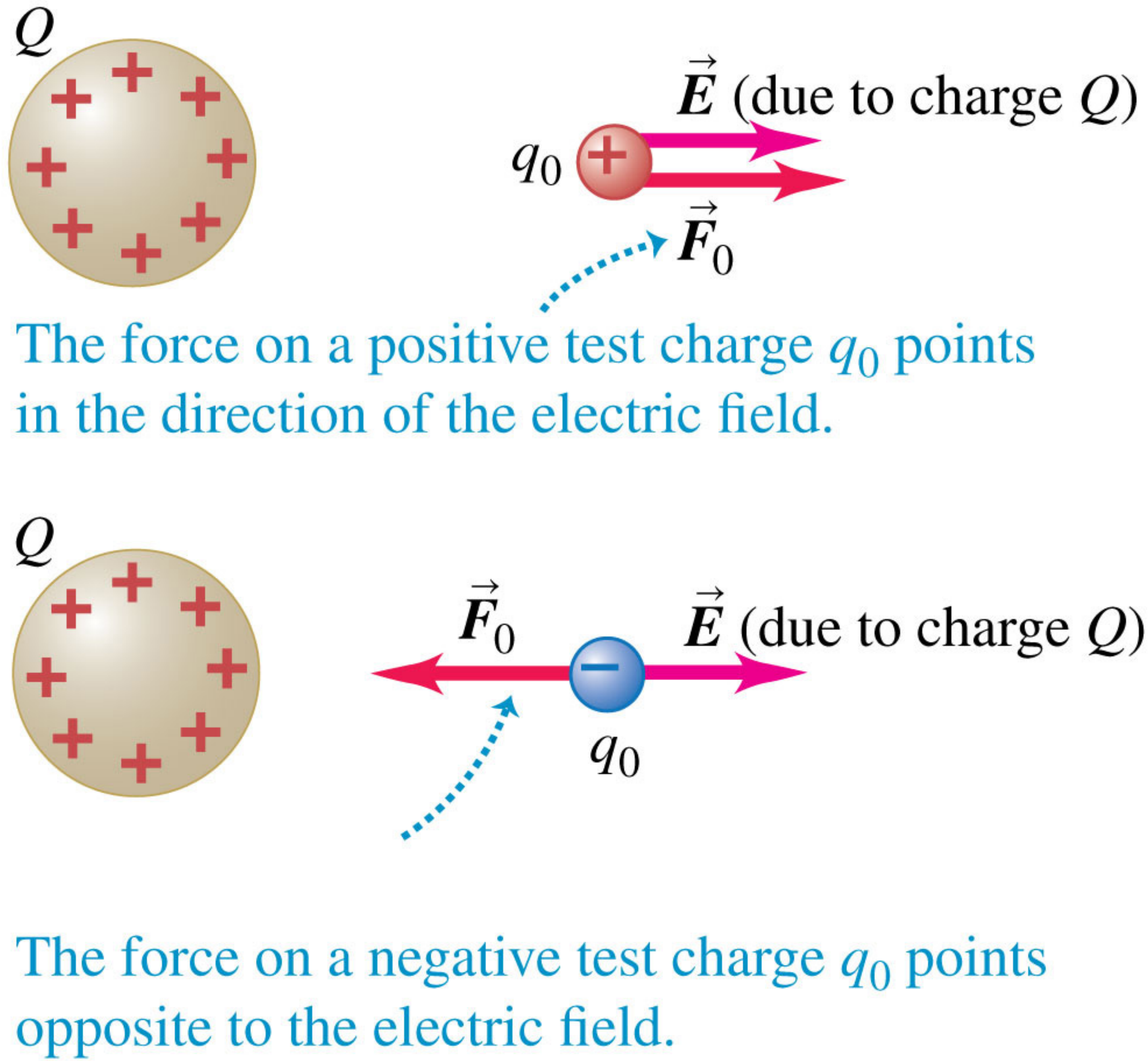
- We can say that body *A* somehow *modifies the properties of the space* at point *P*.

Electric field: Introduction Slide 3 of 3

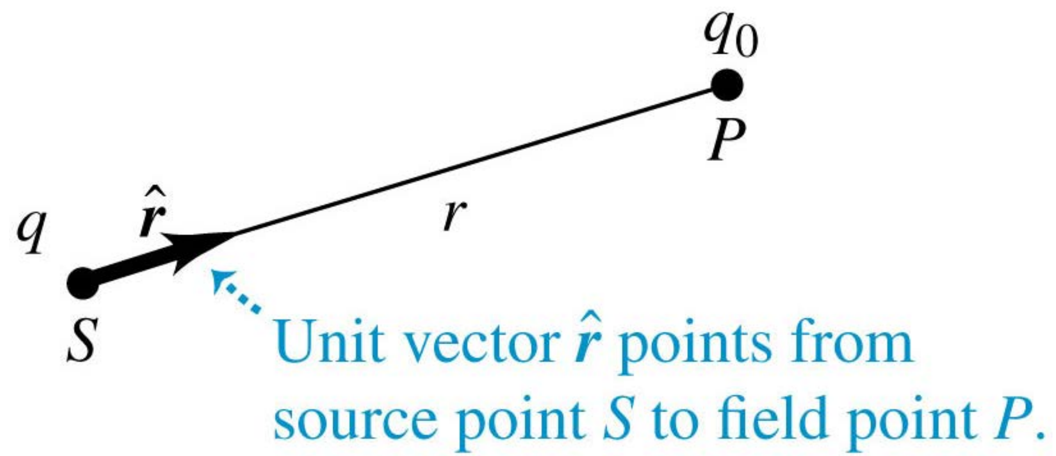
- We can measure the electric field produced by *A* with a test charge.

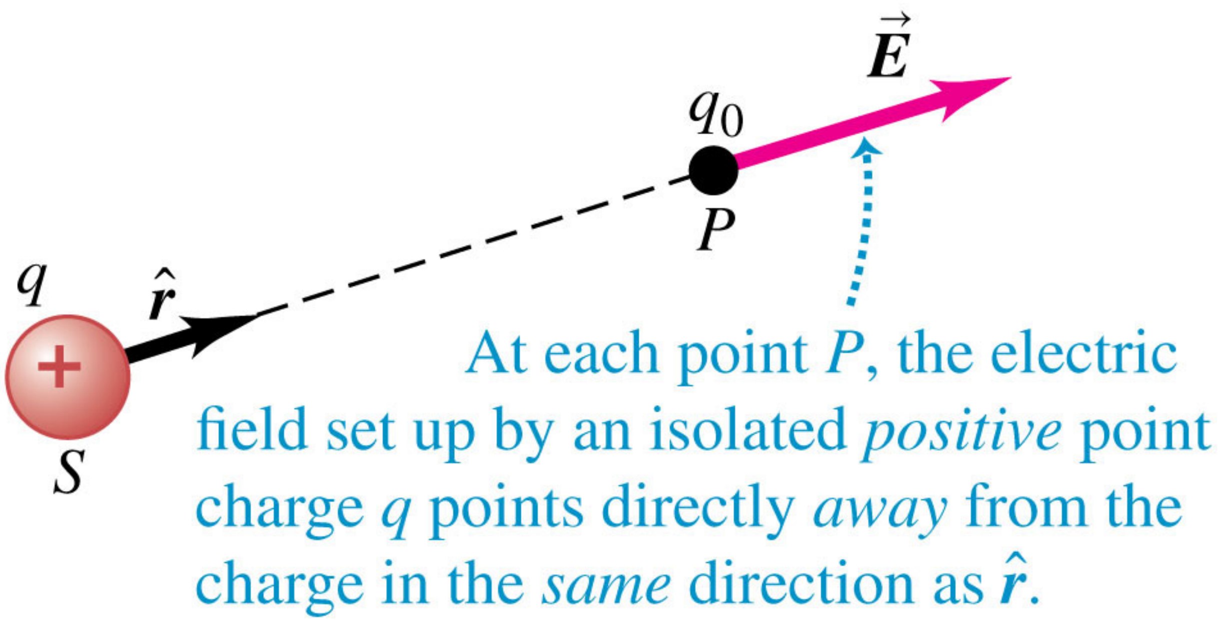


Electric force produced by an electric field



The electric field of a point charge





The electric field of a point charge

- Using a unit vector that points away from the origin, we can write a vector equation that gives both the magnitude and the direction of the electric field:

Value of point charge

Electric field due to a point charge

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

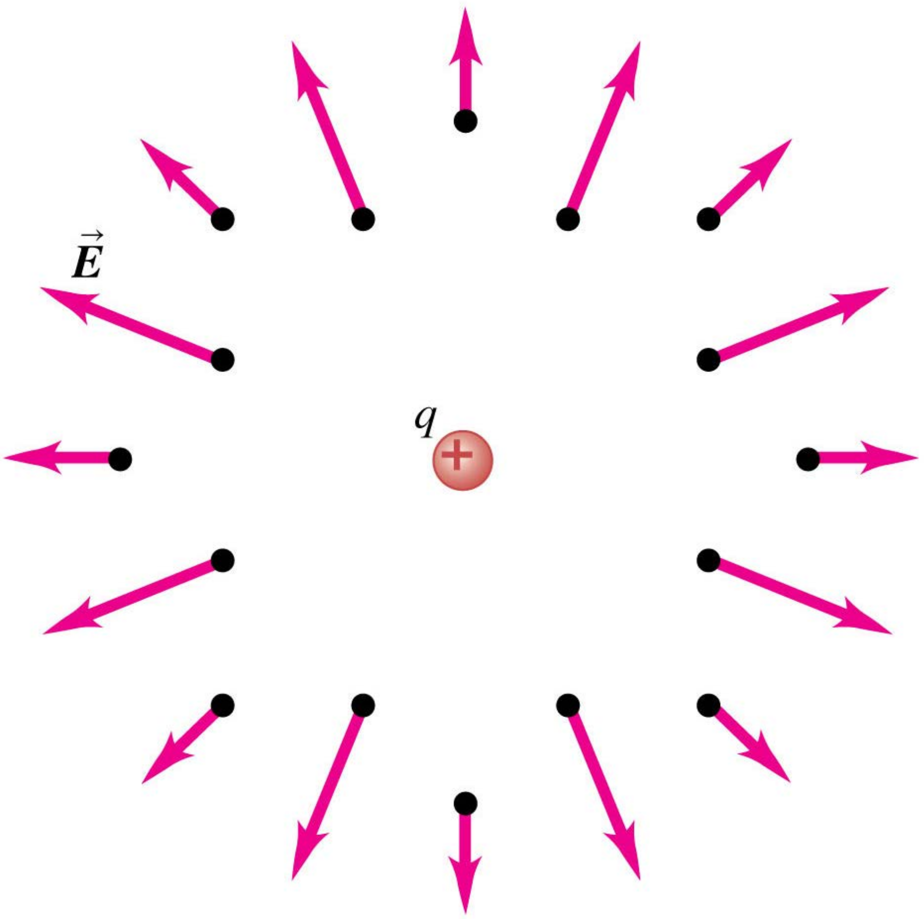
Unit vector from point charge toward where field is measured

Distance from point charge to where field is measured

Electric constant

Electric field of a point charge

- A point charge q produces an electric field at *all* points in space.
- The field strength decreases with increasing distance.
- The field produced by a positive point charge points *away from* the charge.



Electric field of a point charge

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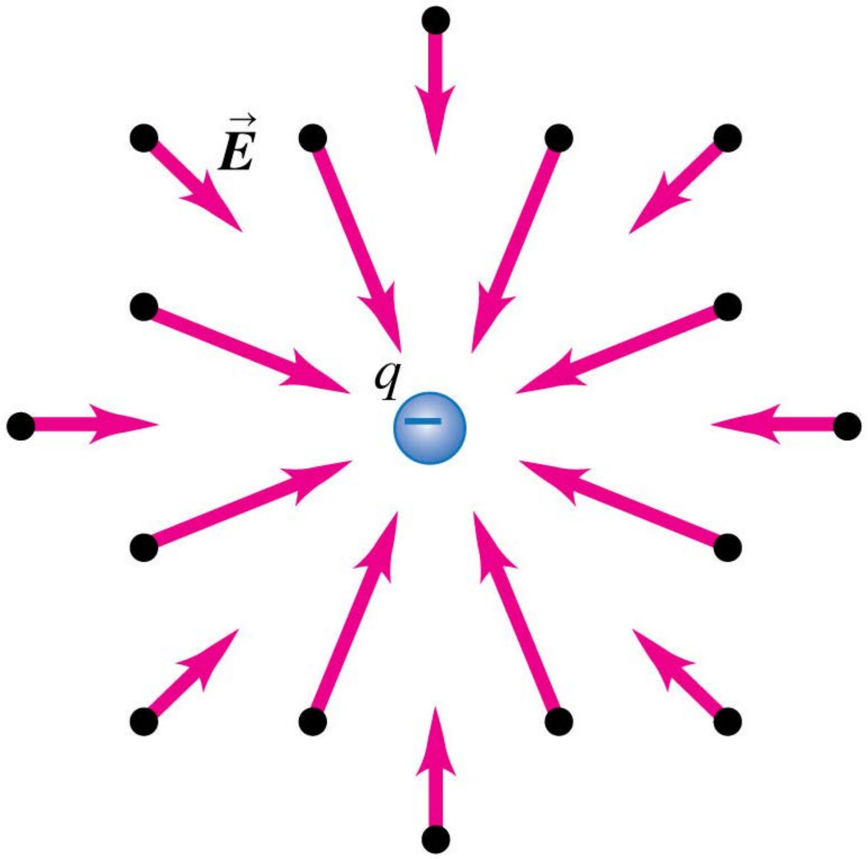
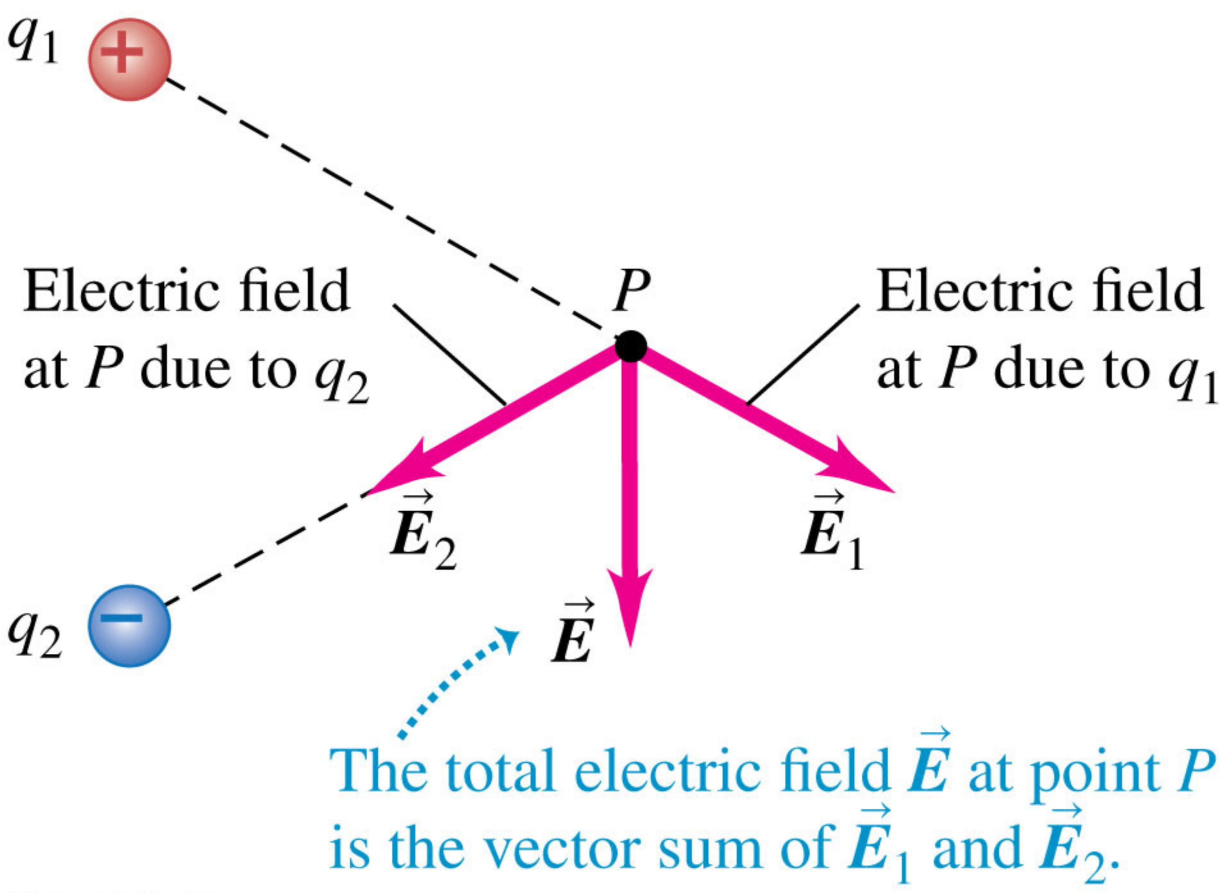


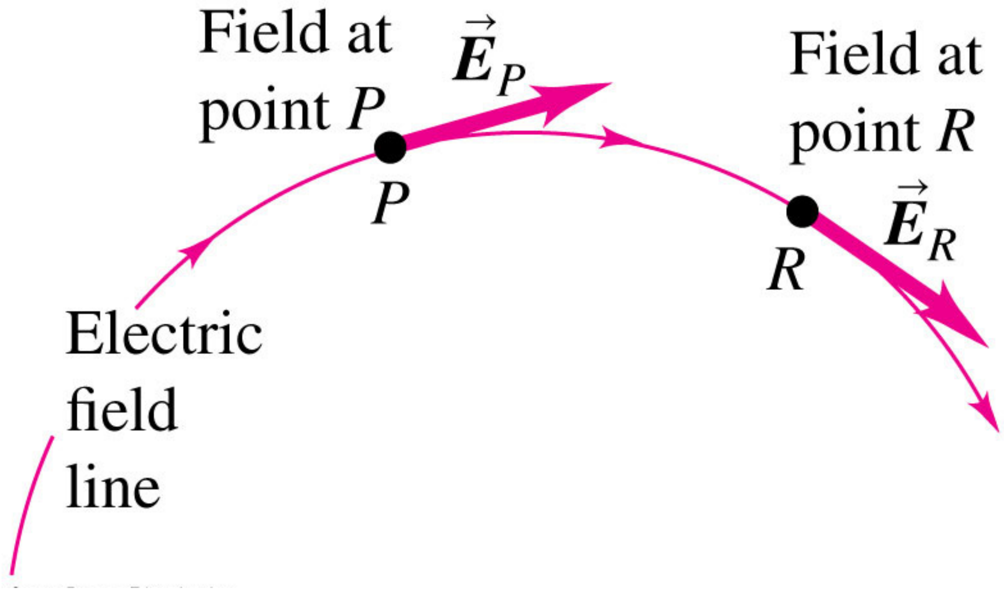
Superposition of electric fields

- The total electric field at a point is the vector sum of the fields due to all the charges present.



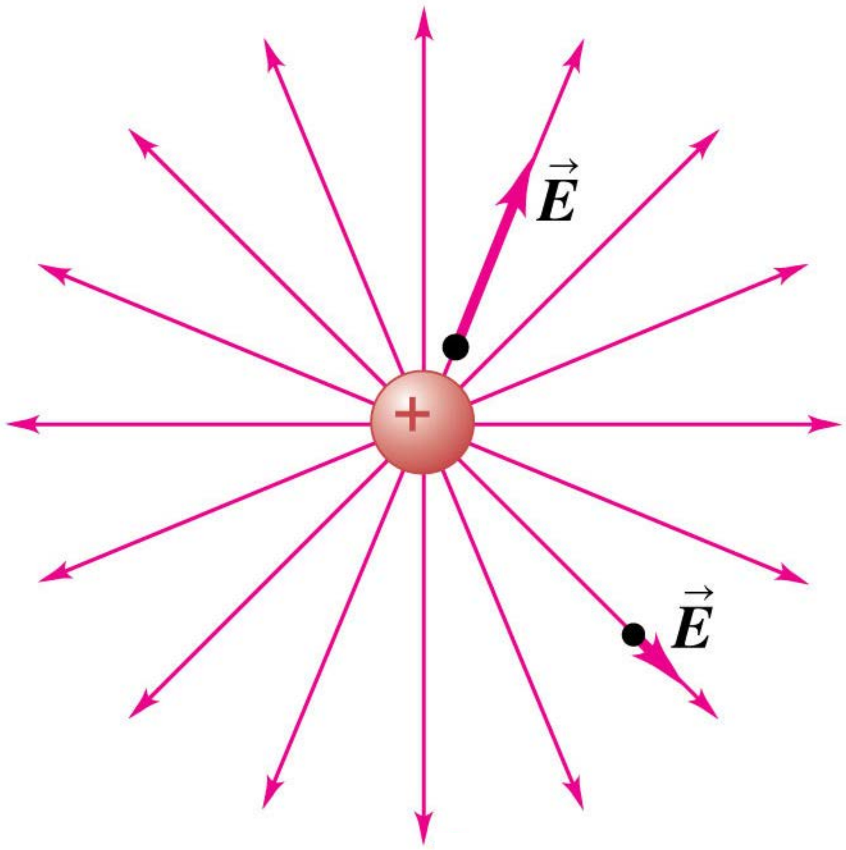
Electric field lines

- An **electric field line** is an imaginary line or curve whose tangent at any point is the direction of the electric field vector at that point.



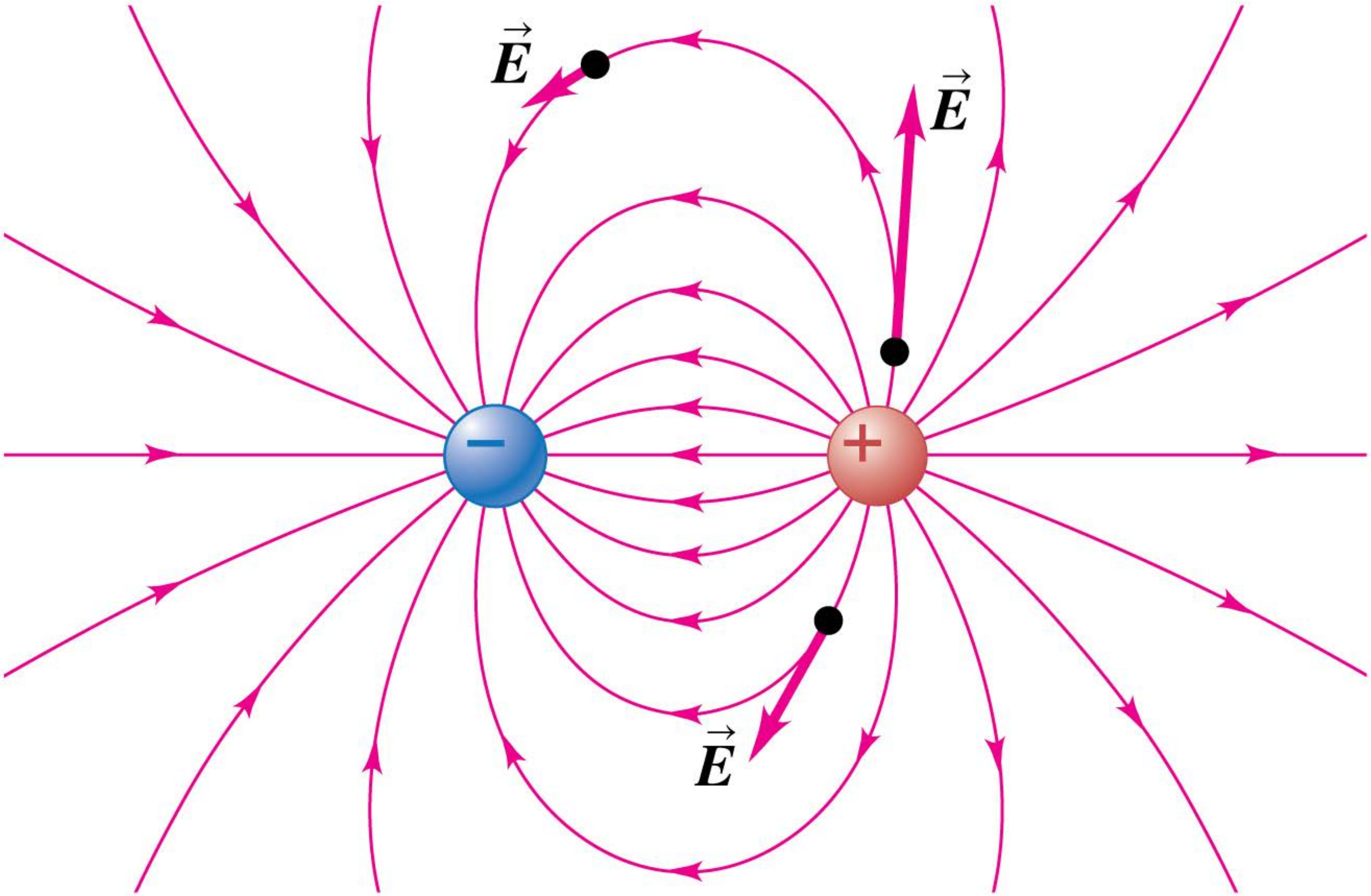
Electric field lines of a point charge

- Electric field lines show the *direction* of the electric field at each point.
- The spacing of field lines gives a general idea of the *magnitude* of the electric field at each point.



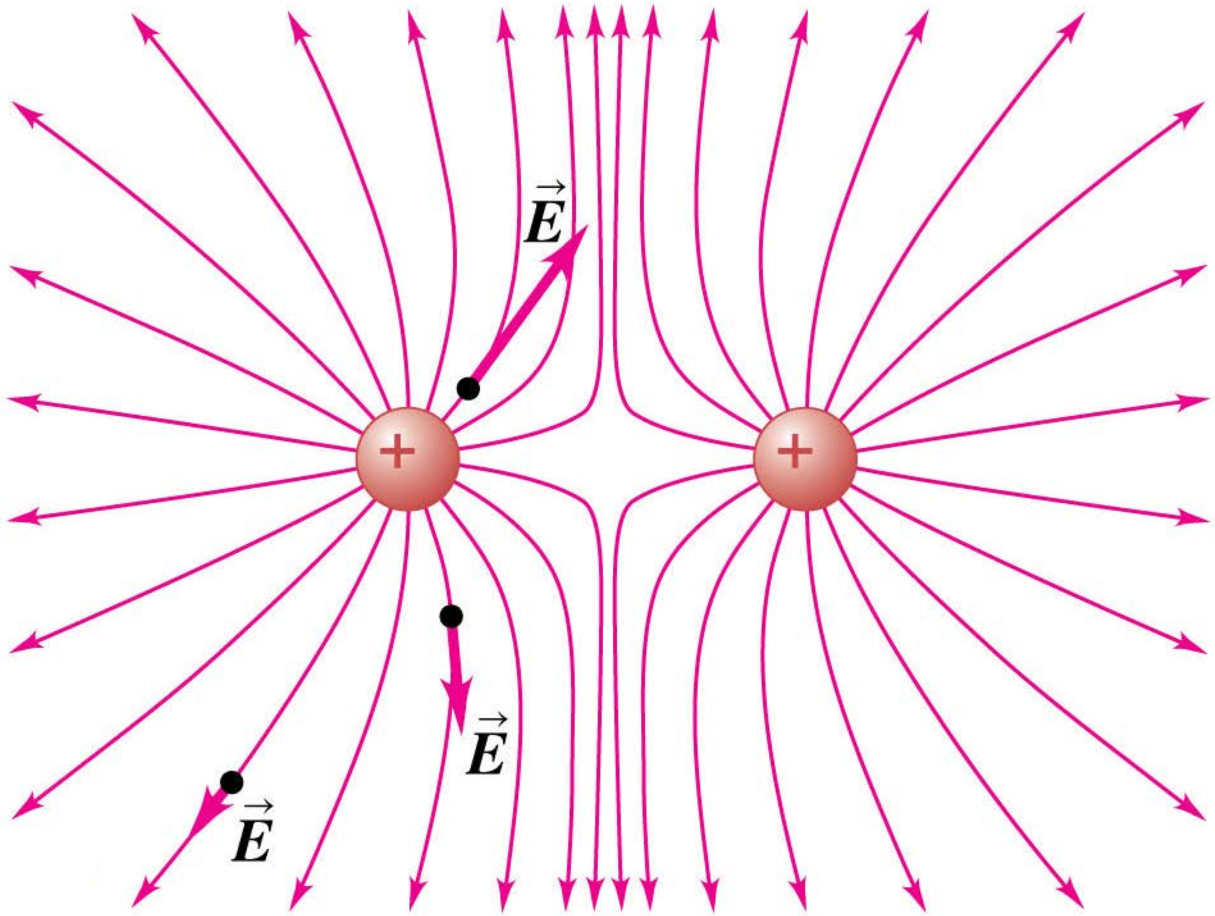
Electric field lines of a dipole

- Field lines point away from + charges and toward – charges.



Electric field lines of two equal positive charges

- At any point, the electric field has a unique direction, so *field lines never intersect*.

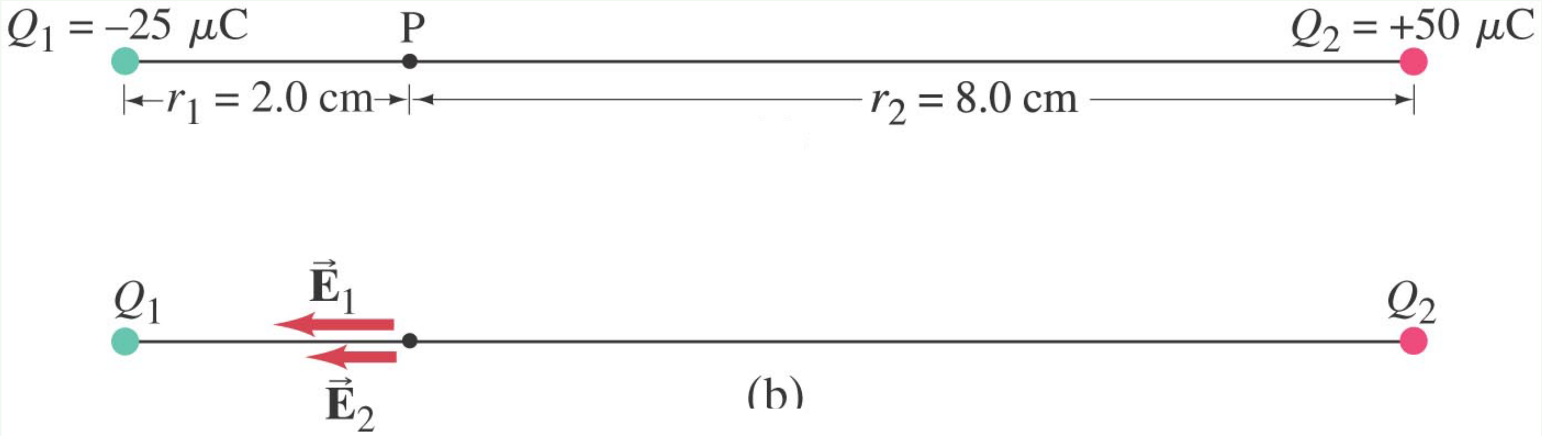


Example 21.6: E at a point between two charges

Two point charges are separated by a distance of 10.0 cm. One has a charge of -25 μC and the other +50 μC.

(a) Determine the direction and magnitude of the electric field at a point P between the two charges that is 2.0 cm from the negative charge.

(b) If an electron (mass = 9.11 x 10^{-31} kg) is placed at rest at P and then released, what will be its initial acceleration (direction and magnitude)?



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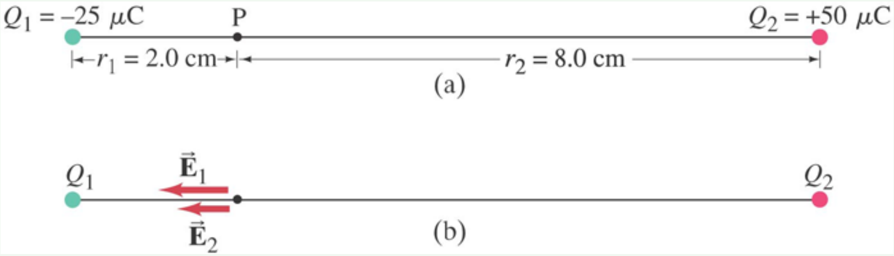


The electric fields add in magnitude, as both are directed towards the negative charge.

$$E = \frac{kQ_1}{r_1^2} + \frac{kQ_2}{r_2^2} = k\left(\frac{Q_1}{r_1^2} + \frac{Q_2}{r_2^2}\right)$$

$$E = (8.99 \times 10^9\ {}^{Nm^2}/_{C^2})(\frac{25 \times 10^{-6} C}{(2.0 \times 10^{-2} m)^2} + \frac{50 \times 10^{-6} C}{(8.0 \times 10^{-2} m)^2} = 6.3 \times 10^8 N/C$$

Example 21.6: E at a point between two charges

(b) If an electron (mass = 9.11 x 10^{-31} kg) is placed at rest at P and then released, what will be its initial acceleration (direction and magnitude)?

$$a = \frac{F}{m} = \frac{F_e}{m} = \frac{qE}{m}$$

$$a = \frac{(1.60 \times 10^{-19} C)(6.3 \times 10^{8} \frac{N}{C})}{9.11 \times 10^{-31} kg} = 1.1 \times 10^{20}\ m/s^2$$

The acceleration is the force will be opposite to the direction of the field (due to the negative charge of the electron)

Example 21.7

An electron is released from rest in a weak electric field $\vec{E} = (-1.50 \times 10^{-10} \frac{N}{C})\hat{j}$. After the electron has traveled a vertical distance of *1.0 μm*, what is its speed? (Do not neglect the gravitational force on the electron.)

Picture the Problem Because the electric field is in the −*y* direction, the force it exerts on the electron is in the +*y* direction. Applying Newton's 2nd law to the electron will yield an expression for the acceleration of the electron in the *y* direction. We can then use a constant-acceleration equation to relate its speed to its acceleration and the distance it has traveled.

Example 21.7 (Solutions)

Apply $\sum F_y = ma_y$ $F_E - F_g = ma_y$ $eE - mg = ma_y$

$$a_y = \frac{eE}{m} - g$$

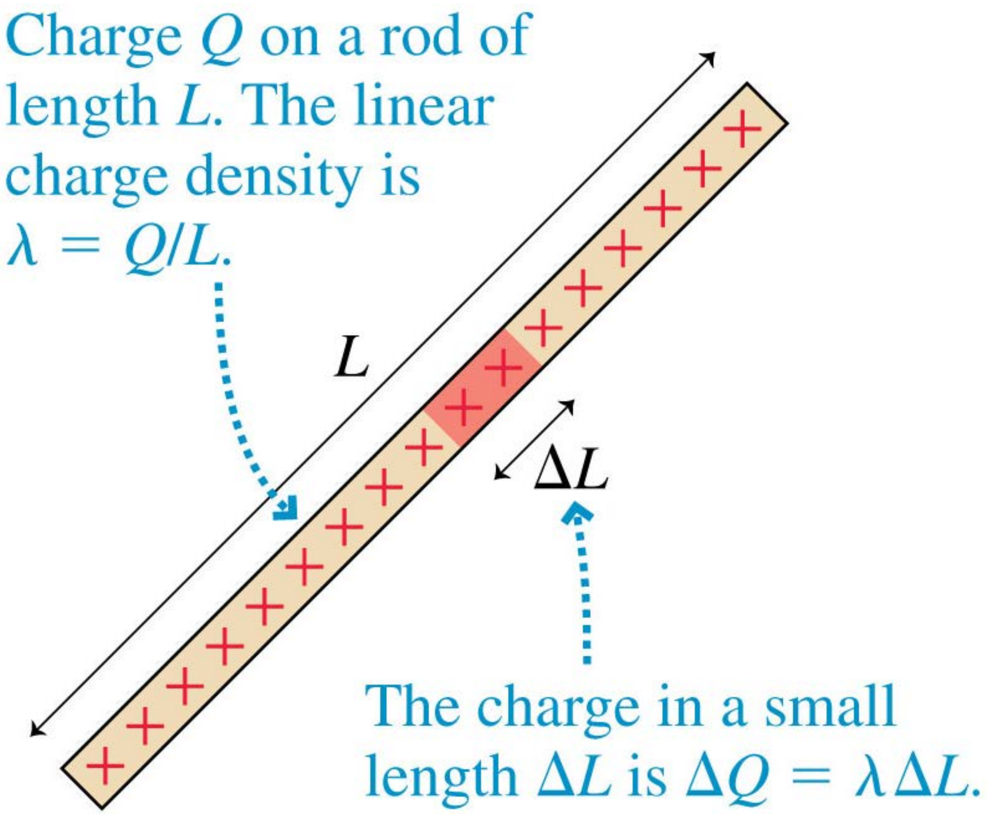
We use a constant-acceleration equation to relate the speed of the electron to its acceleration and the distance it travels (remember kinematic equations?):

$$v_y^2 = v_0^2 + 2a_y\Delta y \qquad v_y^2 = 2a_y\Delta y \Rightarrow v_y = \sqrt{2a_y\Delta y} \qquad v_y = \sqrt{2\left(\frac{eE}{m} - g\right)\Delta y}$$

$$v_y = \sqrt{2\left[\frac{(1.602\times10^{-19}\ \text{C})(1.50\times10^{-10}\ \text{N/C})}{9.109\times10^{-31}\ \text{kg}} - 9.81\,\text{m/s}^2\right](1.0\times10^{-6}\ \text{m})}$$
$$= \boxed{5.8\ \text{mm/s}}$$

Continuous Charge Distributions

The linear charge density of an object of length L and charge Q is defined as



Linear charge density, which has units of $\mathrm{C/m}$, is the amount of charge *per meter* of length.

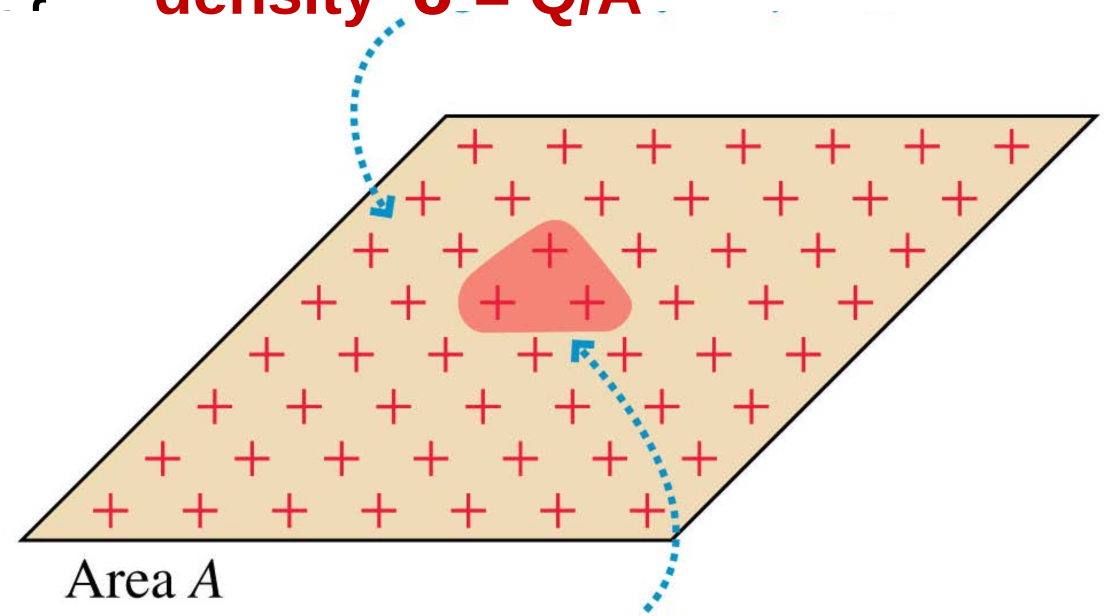
Continuous Charge Distributions

The surface charge density of a two-dimensional distribution of charge across a surface area A is defined as:

$$\sigma = \frac{Q}{A}$$

Surface charge density, with units C/m^2, is the amount of charge *per square meter*.

Charge Q on a surface of area A. The surface charge density σ = Q/A

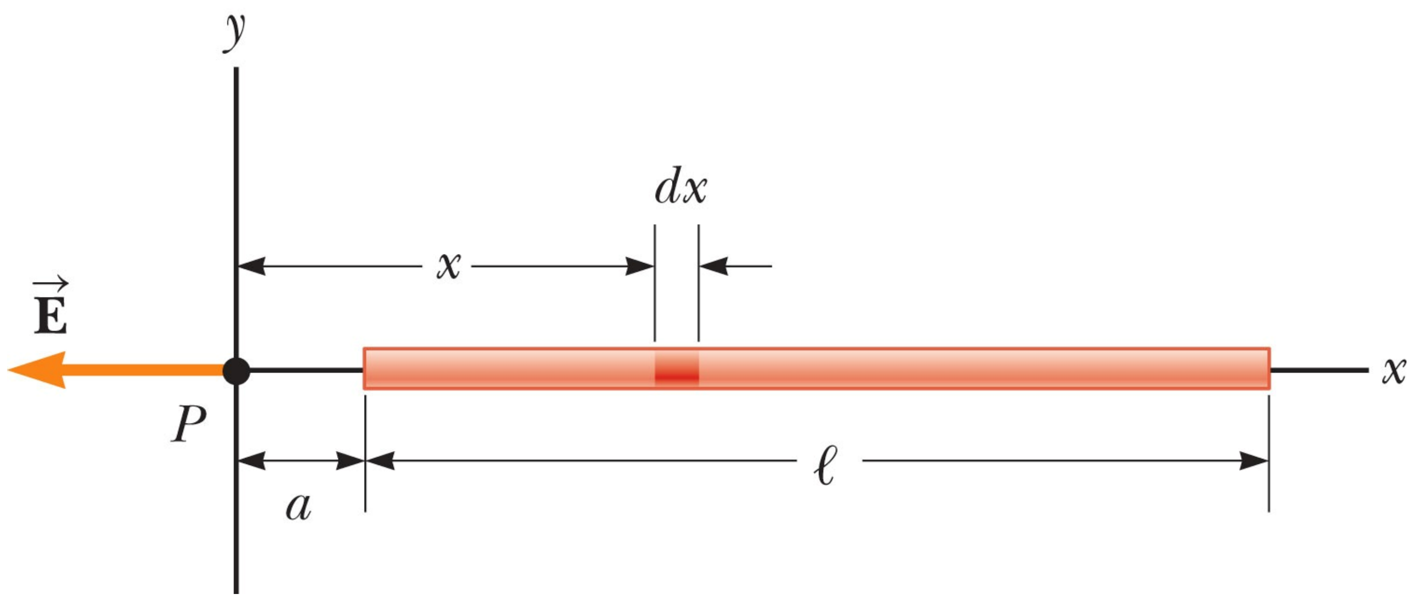


The charge in a small are ΔA is ΔQ = σΔA

$$\Rightarrow \rho = \frac{Q}{V} \; for\ volume\ charge\ density$$

Example 21.8 The Electric Field Due to a Charged Rod

A rod of length , has a uniform positive charge per unit length and a total charge *Q*. Calculate the electric field at a point *P* that is located along the long axis of the rod and a distance *a* from one end.



Example 21.8 The Electric Field Due to a Charged Rod

- Find the magnitude of the electric field at *P* due to one segment of the rod having a charge *dq:*

$$E = k_e \lambda \int_a^{\ell + a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x} \right]_a^{\ell + a}$$

$$(1) \quad E = k_e \frac{Q}{\ell}\left(\frac{1}{a} - \frac{1}{\ell + a}\right) = \boxed{\frac{k_e Q}{a(\ell + a)}}$$

- Find the total field at *P*:

$$dE = k_e \frac{dq}{x^2} = k_e \frac{\lambda \, dx}{x^2}$$

- Evaluate the integral:

$$E = \int_a^{\ell + a} k_e \lambda \frac{dx}{x^2}$$

Example 21.8 The Electric Field Due to a Charged Rod

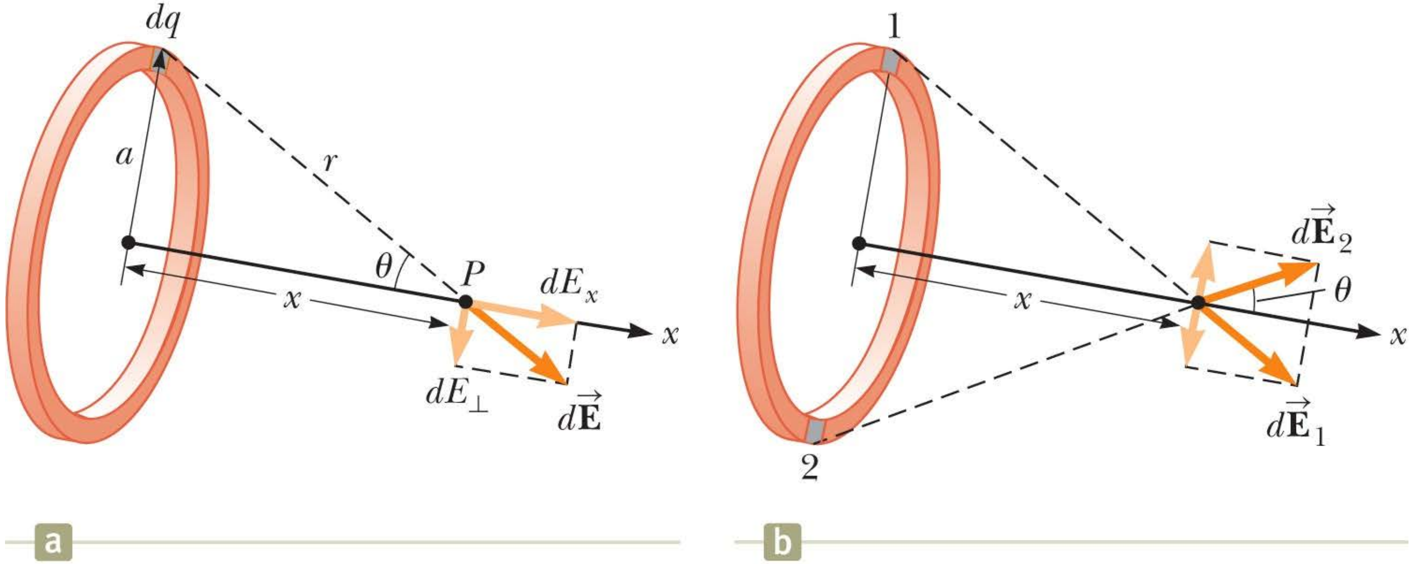
Noting that k_e and $\lambda = Q/\ell$, are constants and can be removed from the integral, evaluate the integral:

$$E = k_e \lambda \int_a^{\ell + a} \frac{dx}{x^2} = k_e \lambda \left[-\frac{1}{x} \right]_a^{\ell + a}$$

$$(1) \quad E = k_e \frac{Q}{\ell} \left(\frac{1}{a} - \frac{1}{\ell + a} \right) = \boxed{\frac{k_e Q}{a(\ell + a)}}$$

Example 21.9 The Electric Field of a Uniform Ring of Charge

A ring of radius a carries a uniformly distributed positive total charge Q. Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring.



Example 21.9 The Electric Field of a Uniform Ring of Charge

- Evaluate the parallel component of an electric field contribution from a segment of charge *dq* on the ring:
- Ev

$$(1) \quad dE_x = k_e \frac{dq}{r^2} \cos\theta = k_e \frac{dq}{a^2 + x^2} \cos\theta$$

$$(2) \quad \cos\theta = \frac{x}{r} = \frac{x}{(a^2 + x^2)^{1/2}}$$

Example 21.9 The Electric Field of a Uniform Ring of Charge

- Substitute Equation (2) into Equation (1):

$$dE_x = k_e \frac{dq}{a^2 + x^2} \frac{x}{(a^2 + x^2)^{1/2}} = \frac{k_e x}{(a^2 + x^2)^{3/2}} dq$$

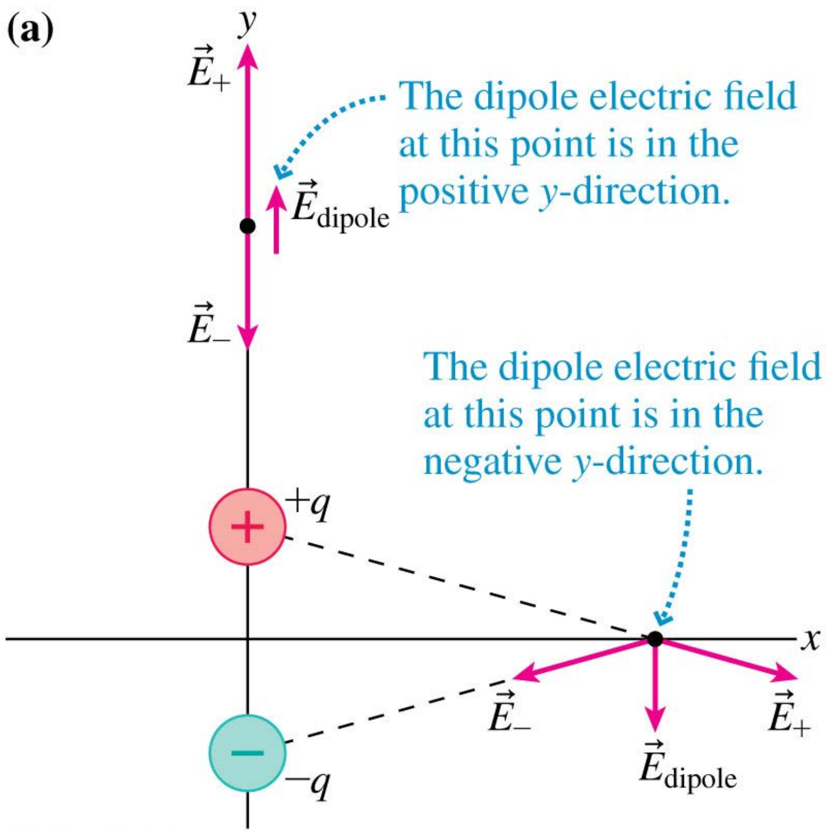
- Integrate to obtain the total field at *P*:

$$E_x = \int \frac{k_e x}{(a^2 + x^2)^{3/2}} dq = \frac{k_e x}{(a^2 + x^2)^{3/2}} \int dq$$

$$(3) \quad E = \boxed{\frac{k_e x}{(a^2 + x^2)^{3/2}} Q}$$

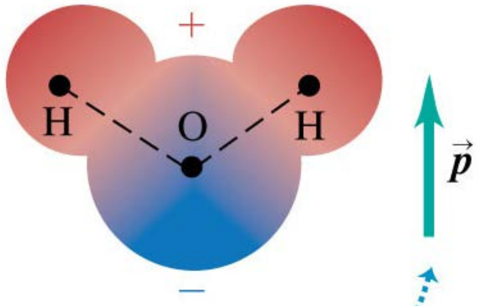
Applications of the Electric Field

- For an electric dipole, we can find the the electric field at any point by a vector addition of the fields of the two charges.



The water molecule is an electric dipole

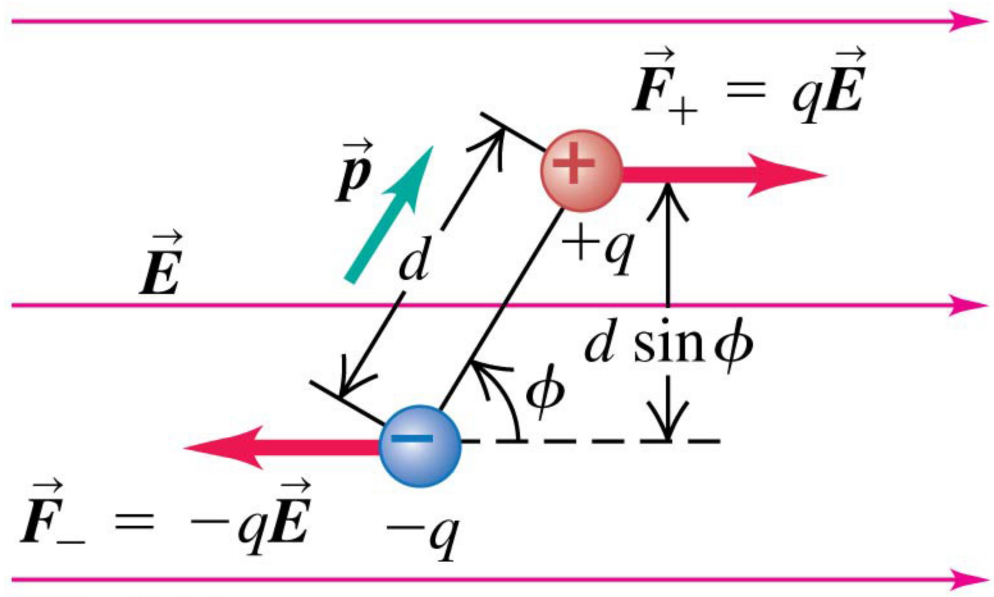
- The water molecule as a whole is electrically neutral, but the chemical bonds within the molecule cause a displacement of charge.
- The result is a net negative charge on the oxygen end of the molecule and a net positive charge on the hydrogen end, forming an electric dipole.



The electric dipole moment $\vec{p}$ is directed from the negative end to the positive end of the molecule.

Force and torque on a dipole

- When a dipole is placed in a uniform electric field, the net *force* is always zero, but there can be a net *torque* on the dipole.



$$p = qd \qquad \text{(magnitude of electric dipole moment)}$$

Vector torque on an electric dipole

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Electric dipole moment

Electric field

$$\tau = pE \sin\phi \text{ (Magnitude of torque on an electric dipole)}$$

- **Extra Examples**

Example 1

A small plastic sphere with a charge of -5.0 nC is near another plastic sphere with a charge of -12.0 nC. If the spheres repel one another with a force of magnitude 8.2×10^{-4} N, what is the distance between the spheres?

$$r^2 = K\frac{|q_1||q_2|}{F}$$

$$r = \sqrt{K\frac{|q_1||q_2|}{F}} = \sqrt{(9.0\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{|-0.5\times10^{-9}\ \mathrm{C}||-12\times10^{-9}\ \mathrm{C}|}{8.2\times10^{-4}\ \mathrm{N}}} = 0.026\ \mathrm{m} = 2.6\ \mathrm{cm}$$

Example 1

A –2.0 μC point charge and a 4.0 μC point charge are a distance *L* apart. Where should a third point charge be placed so that the electric force on that third charge is zero?

- **Picture the Problem** The third point charge should be placed at the location at which the forces on the third point charge due to each of the other two point charges cancel. There can be no such place between the two point charges. Beyond the 4.0 μC point charge, and on the line containing the two point charges, the force due to the 4.0 μC point charge overwhelms the force due to the –2.0 μC point charge. Beyond the –2.0 μC point charge, and on the line containing the two point charges, however, we can find a place where these forces cancel because they are equal in magnitude and oppositely directed.

Example 1 (Solution)

We denote the −2.0 μC and 4.0 μC point charges by the numerals 2 and 4, respectively, and the third point charge by the numeral 3. Let the +*x* direction be to the right with the origin at the position of the −2.0 μC point charge and the 4.0 μC point charge be located at $x = L$.

$$\vec{F}_{4,3} + \vec{F}_{2,3} = 0 \qquad F_{4,3} = F_{2,3} \qquad F_{4,3} = \frac{kq_3q_4}{(L+x)^2} \qquad F_{2,3} = \frac{kq_3q_2}{x^2}$$

$$\frac{kq_3q_4}{(L+x)^2} = \frac{kq_3q_2}{x^2} \Rightarrow \frac{q_4}{(L+x)^2} = \frac{q_2}{x^2}$$

$$x^2 - 2Lx - L^2 = 0 \qquad x = \frac{2L \pm \sqrt{4L^2 + 4L^2}}{2} = L \pm \sqrt{2}L$$

Hence the third point charge should be placed a distance equal to 0.41*L* from the –2.0-μC charge on the side away from the 4.0-μC charge.

Example 2

A housefly walking across a clean surface can accumulate a significant positive or negative charge. In one experiment, the largest positive charge observed was +73 pC. A typical housefly has a mass of 12 mg. What magnitude and direction of an electric field would be necessary to "levitate" a housefly with the maximum charge? Could such a field exist in air?

$$\text{The electric force would need to exceed}$$

$$W = mg = \left(12 \times 10^{-6}\ \text{kg}\right)\left(9.8\,{}^{m}\!/_{s^2}\right) = 0.0001\ N$$

$$E = \frac{F_e}{q} = \frac{0.0001\ N}{(73 \times 10^{-12}\ C)} = 1.4 \times 10^{6}\ {}^{N}\!/_{C}$$

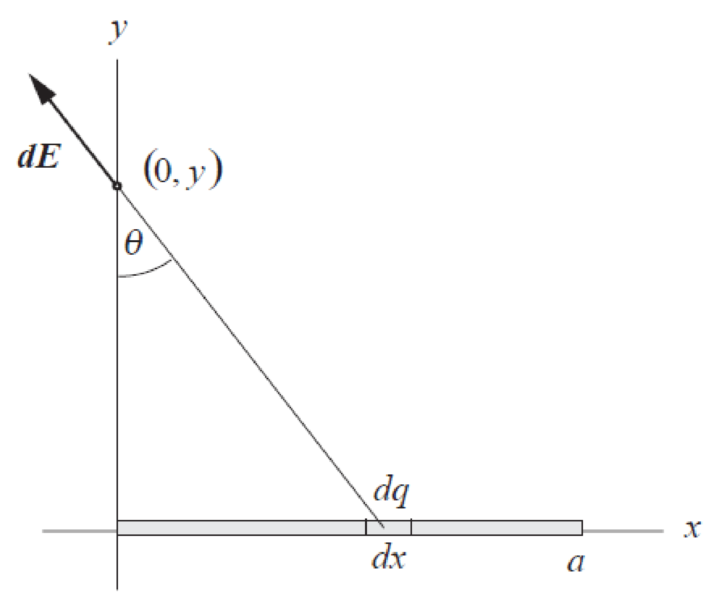
This is about the field strength resulting in a discharge through air, so it is unlikely the field would be strong enough to levitate a fly: the field would be reduced by a discharge before reaching the strength necessary to levitate the fly.

Example 3

A line of charge that has uniform linear charge density λ lies on the *x* axis from *x* = 0 to *x* = *a*. Show that the *y* component of the electric field at a point on the *y* axis is given by

$$\boldsymbol{E_y = \frac{k\lambda}{y}\frac{a}{\sqrt{y^2 + a^2}}, y \neq 0.}$$

Picture the Problem The line of charge and the point whose coordinates are (0, y) are shown in the diagram. Also shown is a segment of the line of length dx and charge dq. The field due to this charge at (0, y) is $d\vec{E}$. We can find dE_y from $d\vec{E}$ and then integrate from $x = 0$ to $x = a$ to find the y component of the electric field at a point on the y axis.



Example 3 (Solution)

(a) Express the magnitude of the field $d\vec{\mathbf{E}}$ due to charge dq of the element of length dx:

$$dE = \frac{kdq}{r^2}$$

where $r^2 = x^2 + y^2$

Because $dq = \lambda dx$:

$$dE = \frac{k\lambda dx}{x^2 + y^2}$$

Express the y component of dE:

$$dE_y = \frac{k\lambda}{x^2 + y^2}\cos\theta \, dx$$

Refer to the diagram to express $\cos\theta$ in terms of x and y:

$$\cos\theta = \frac{y}{\sqrt{x^2 + y^2}}$$

Substitute for $\cos\theta$ in the expression for dE_y to obtain:

$$dE_y = \frac{k\lambda y}{\left(x^2 + y^2\right)^{3/2}} dx$$

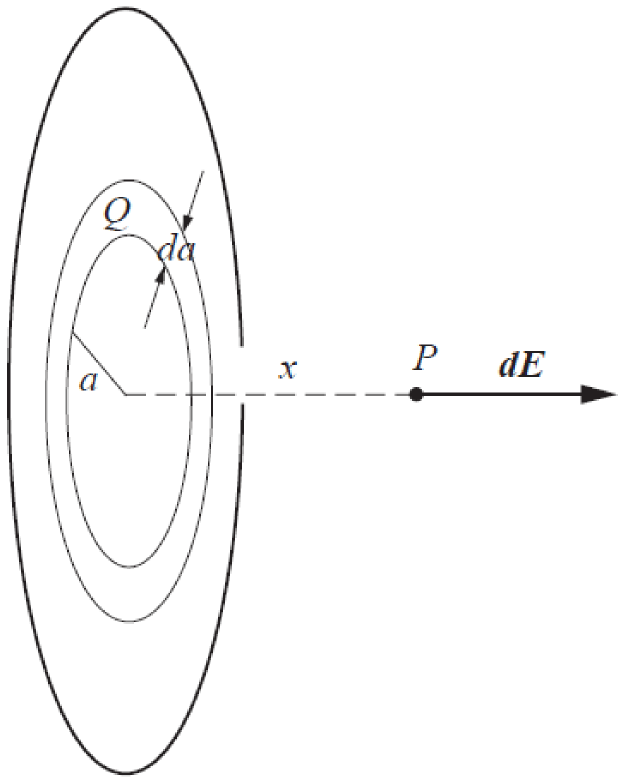
Example 3 (Solution)

Integrate from $x = 0$ to $x = a$ and simplify to obtain:

$$E_y = k\lambda y \int_0^a \frac{1}{\left(x^2 + y^2\right)^{3/2}} dx = k\lambda y \left[\frac{x}{y^2 \sqrt{x^2 + y^2}} \right]_0^a = \boxed{\frac{k\lambda}{y} \frac{a}{\sqrt{a^2 + y^2}}}$$

Example 4

Calculate the electric field a distance *x* from a uniformly charged infinite flat non-conducting sheet by modeling the sheet as a continuum of infinite circular rings of charge.



Picture the Problem The field at a point on the axis of a uniformly charged ring lies along the axis. The diagram shows one ring of the continuum of circular rings of charge. The radius of the ring is *a* and the distance from its center to the field point *P* is *x.* The ring has a uniformly distributed charge *Q*. The resultant electric field at *P* is the sum of the fields due to the continuum of circular rings.

Example 4 (Solution)

Express the field of a single uniformly charged ring with charge Q and radius a on the axis of the ring at a distance x away from the plane of the ring:

$$\vec{E} = E_x \hat{i}\text{, where } E_x = \frac{kQx}{\left(x^2 + a^2\right)^{3/2}}$$

Substitute dq for Q and dE_x for E_x to obtain:

$$dE_x = \frac{kx\,dq}{\left(x^2 + a^2\right)^{3/2}}$$

The resultant electric field at P is the sum of the fields due to all the circular rings. Integrate both sides to calculate the resultant field for the entire plane. The field point remains fixed, so x is constant:

$$E = \int \frac{kx\,dq}{\left(x^2 + a^2\right)^{3/2}} = kx \int \frac{dq}{\left(x^2 + a^2\right)^{3/2}}$$

Example 4 (Solution)

To evaluate this integral we change integration variables from q to a. The charge $dq = \sigma\, dA$ where $dA = 2\pi\, a\, da$ is the area of a ring of radius a and width da:

$$dq = 2\pi\sigma\, a\, da$$

so

$$E = kx\int_0^{\infty} \frac{2\pi\, \sigma\, a\, da}{\left(x^2 + a^2\right)^{3/2}}$$

$$= 2\pi\, \sigma\, kx\int_0^{\infty} \frac{a\, da}{\left(x^2 + a^2\right)^{3/2}}$$

To integrate this expression, let $u = \sqrt{x^2 + a^2}$. Then:

$$du = \tfrac{1}{2}\frac{1}{\sqrt{x^2 + a^2}}\left(2ada\right) = \frac{a}{u}da$$

or

$$ada = udu$$

Noting that when $a = 0$, $u = x$, substitute and simplify to obtain:

$$E = 2\pi\sigma\, kx\int_x^{\infty} \frac{u}{u^3}du = 2\pi\sigma\, kx\int_x^{\infty} u^{-2}du$$

Evaluating the integral yields:

$$E = 2\pi\sigma\, kx\left(-\frac{1}{u}\right)\Bigg|_x^{\infty} = 2\pi k\sigma = \boxed{\frac{\sigma}{2\,\epsilon_0}}$$