

Electron =  $-1.602 \times 10^{-19} \text{ C}$  =  $9.11 \times 10^{-31} \text{ kg}$   
 Proton =  $1.602 \times 10^{-19} \text{ C}$  =  $1.67 \times 10^{-27} \text{ kg}$   
 Neutron =  $0 \text{ C}$  =  $1.67 \times 10^{-27} \text{ kg}$   
 $6.022 \times 10^{23}$  atoms in one atomic mass unit  
 $e$  is the elementary charge:  $1.602 \times 10^{-19} \text{ C}$

### Addition of Multiple Vectors:

$$\begin{aligned}\vec{R} &= \vec{A} + \vec{B} + \vec{C} & \text{Resultant = Sum of the vectors} \\ \vec{R}_x &= \vec{A}_x + \vec{B}_x + \vec{C}_x & \text{x-component} \quad A_x = A \cos \theta \\ \vec{R}_y &= \vec{A}_y + \vec{B}_y + \vec{C}_y & \text{y-component} \quad A_y = A \sin \theta \\ R &= \sqrt{R_x^2 + R_y^2} & \text{Magnitude (length) of } R \\ \theta_R &= \tan^{-1} \frac{R_y}{R_x} \quad \text{or} \quad \tan \theta_R = \frac{R_y}{R_x} & \text{Angle of the resultant}\end{aligned}$$

### ELECTRIC CHARGES AND FIELDS

Coulomb's Law: [Newtons N]

$$F = k \frac{|q_1||q_2|}{r^2}$$

where:  $F$  = force on one charge by the other [N]  
 $k = 8.99 \times 10^9 \text{ [N}\cdot\text{m}^2/\text{C}^2]$   
 $q_1$  = charge [C]  
 $q_2$  = charge [C]  
 $r$  = distance [m]

Electric Field: [Newtons/Coulomb or Volts/Meter]

$$E = k \frac{|q|}{r^2} = \frac{|F|}{|q|}$$

where:  $E$  = electric field [N/C or V/m]  
 $k = 8.99 \times 10^9 \text{ [N}\cdot\text{m}^2/\text{C}^2]$   
 $q$  = charge [C]  
 $r$  = distance [m]  
 $F$  = force

Electric Field inside a spherical shell: [N/C]

$$E = \frac{kqr}{R^3}$$

$E$  = electric field [N/C]  
 $q$  = charge [C]  
 $r$  = distance from center of sphere to the charge [m]  
 $R$  = radius of the sphere [m]

Electric Field outside a spherical shell: [N/C]

$$E = \frac{kq}{r^2}$$

$E$  = electric field [N/C]  
 $q$  = charge [C]  
 $r$  = distance from center of sphere to the charge [m]

Flux: the rate of flow (of an electric field) [N·m<sup>2</sup>/C]

$$\begin{aligned}\Phi &= \oint \vec{E} \cdot d\vec{A} & \Phi & \text{ is the rate of flow of an electric field [N}\cdot\text{m}^2/\text{C}] \\ &= \int E(\cos \theta) dA & \oint & \text{ integral over a closed surface} \\ & & \vec{E} & \text{ is the electric field vector [N/C]} \\ & & d\vec{A} & \text{ is the area vector [m}^2] \text{ pointing outward normal to the surface.}\end{aligned}$$

Gauss' law:

$$\Phi_E = \oint_S \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$\begin{aligned}\vec{F} &= q\vec{E} \\ \vec{E} &= k \int_Q \frac{dq}{r^2} \hat{r} = \frac{1}{4\pi\epsilon_0} \int_Q \frac{dq}{r^2} \hat{r}\end{aligned}$$

Electric potential:

$$V = k \int_Q \frac{dq}{r} = \frac{1}{4\pi\epsilon_0} \int_Q \frac{dq}{r} \quad \text{relative to } V = 0 \text{ at } r \rightarrow \infty$$

Relationship between  $\vec{E}$  and  $V$ :

$$V_a - V_b = \int_a^b \vec{E} \cdot d\vec{l} \quad E_i = - \frac{\partial V}{\partial x_i}$$

Work done by the field in moving a charge  $q$  from a to b:  $W_{ab} = U_a - U_b = q(V_a - V_b)$

Potential energy of a system of point charges:  $U = \sum_{i < j} \frac{kQ_i Q_j}{r_{ij}}$

### WORK AND POTENTIAL:

$$\Delta U = U_f - U_i = -W$$

$$U = -W_{\infty}$$

$$W = \mathbf{F} \cdot \mathbf{d} = Fd \cos \theta$$

$$W = q \int_i^f \mathbf{E} \cdot d\mathbf{s}$$

$$\Delta V = V_f - V_i = -\frac{W}{q}$$

$$V = -\int_i^f \mathbf{E} \cdot d\mathbf{s}$$

$U$  = electric potential energy [J]

$W$  = work done on a particle by a field [J]

$W_{\infty}$  = work done on a particle brought from infinity (zero potential) to its present location [J]

$\mathbf{F}$  = is the force vector [N]

$\mathbf{d}$  = is the distance vector over which the force is applied [m]

$F$  = is the force scalar [N]

$d$  = is the distance scalar [m]

$\theta$  = is the angle between the force and distance vectors

$d\mathbf{s}$  = differential displacement of the charge [m]

$V$  = volts [V]

$q$  = charge [C]

### Capacitors:

$$C = \frac{Q}{V}$$

Capacitance of different capacitors (for air or vacuum,  $K = 1$ ):

Capacitor	Capacitance
Parallel-plate capacitor with plate-area $A$ and thickness $d$	$K\epsilon_0 \frac{A}{d}$
Spherical capacitor of radii $a$ and $b$	$K\epsilon_0 \frac{4\pi ab}{b-a}$
Isolated sphere of radius $a$	$K\epsilon_0 4\pi a$
Cylindrical capacitor of radii $a$ and $b$ , and length $L$	$K\epsilon_0 \frac{2\pi L}{\ln(b/a)}$

### Capacitor combinations:

Series connection:  $\frac{1}{C_{eq}} = \sum_{i=1}^N \frac{1}{C_i}$

Parallel connection:  $C_{eq} = \sum_{i=1}^N C_i$

Energy stored:  $U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$

Energy density:  $u = \frac{1}{2}K\epsilon_0 E^2$

### Electric current and resistance:

$$I = \frac{dQ}{dt} = \int \vec{J} \cdot d\vec{A}$$

$$\vec{J} = nq\vec{v}_d$$

$$Q = \int I dt$$

$$V = IR \quad R = \rho \frac{L}{A}$$

$$\rho_T = \rho_0 \left[ 1 + \alpha (T - T_0) \right]$$

$$P = IV = I^2 R = \frac{V^2}{R}$$

### Resistor combinations:

Series connection:  $R_{eq} = \sum_{i=1}^N R_i$

Parallel connection:  $\frac{1}{R_{eq}} = \sum_{i=1}^N \frac{1}{R_i}$

## Kirchhoff's Rules

1. The sum of the currents entering a junctions is equal to the sum of the currents leaving the junction.
2. The sum of the potential differences across all the elements around a closed loop must be zero.

**Kirchhoff's junction rule**  
(valid at any junction):

The sum of the currents into any junction ...

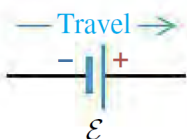
$$\sum I = 0 \quad \dots \text{equals zero.}$$

**Kirchhoff's loop rule**  
(valid for any closed loop):

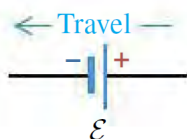
The sum of the potential differences around any loop ...

$$\sum V = 0 \quad \dots \text{equals zero.}$$

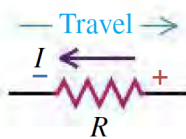
$+\mathcal{E}$ : Travel direction  
from  $-$  to  $+$ :



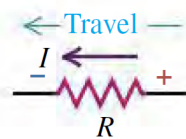
$-\mathcal{E}$ : Travel direction  
from  $+$  to  $-$ :



$+IR$ : Travel *opposite*  
to current direction:



$-IR$ : Travel *in*  
current direction:



$$F = |q|v_{\perp}B = |q|vB \sin \phi$$

Magnetic force on a moving charged particle  $\vec{F} = q\vec{v} \times \vec{B}$

Particle's charge  
Particle's velocity  
Magnetic field

1 tesla = 1 T = 1 N/A · m    Another unit of B, the **gauss** (1 G = 10<sup>-4</sup> T)

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad 1 \text{ Wb} = 1 \text{ T} \cdot \text{m}^2 = 1 \text{ N} \cdot \text{m/A}$$

Magnetic flux through a surface  $\Phi_B = \int B \cos \phi \, dA = \int B_{\perp} \, dA = \int \vec{B} \cdot d\vec{A}$

Magnitude of magnetic field  $\vec{B}$   
Angle between  $\vec{B}$  and normal to surface  
Component of  $\vec{B}$  perpendicular to surface  
Element of surface area  
Vector element of surface area

Gauss's law for magnetism:  $\oint \vec{B} \cdot d\vec{A} = 0$  ... equals zero.

The total magnetic flux through any closed surface ...

$$F = |q|vB = m \frac{v^2}{R} \quad v = R\omega \quad f = \omega/2\pi$$

$$v = \frac{E}{B} \quad \frac{1}{2}mv^2 = eV$$

Magnetic force on a straight wire segment  $\vec{F} = I\vec{l} \times \vec{B}$

Current  
Magnetic field  
Vector length of segment (points in current direction)

Magnetic force on an infinitesimal wire segment  $d\vec{F} = I d\vec{l} \times \vec{B}$

Current  
Magnetic field  
Vector length of segment (points in current direction)

Magnitude of magnetic torque on a current loop  $\tau = IBA \sin \phi$

Current  
Magnetic-field magnitude  
Angle between normal to loop plane and field direction  
Area of loop

$$\mu = IA \quad \tau = \mu B \sin \phi$$

Vector magnetic torque on a current loop  $\vec{\tau} = \vec{\mu} \times \vec{B}$

Magnetic dipole moment  
Magnetic field

Magnetic field due to a point charge with constant velocity  $\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$

Magnetic constant  
Charge  
Velocity  
Unit vector from point charge toward where field is measured  
Distance from point charge to where field is measured

Magnetic field due to an infinitesimal current element  $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2}$

Magnetic constant  
Current  
Vector length of element (points in current direction)  
Unit vector from element toward where field is measured  
Distance from element to where field is measured

Magnetic field near a long, straight, current-carrying conductor  $B = \frac{\mu_0 I}{2\pi r}$

Magnetic constant  
Current  
Distance from conductor

Magnetic force per unit length between two long, parallel, current-carrying conductors  $\frac{F}{L} = \frac{\mu_0 II'}{2\pi r}$

Magnetic constant  
Current in first conductor  
Current in second conductor  
Distance between conductors

Magnetic field on axis of a circular current-carrying loop

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$

Magnetic constant:  $\mu_0$   
Current:  $I$   
Radius of loop:  $a$   
Distance along axis from center of loop to field point:  $x$

Magnetic field at center of  $N$  circular current-carrying loops

$$B_x = \frac{\mu_0 N I}{2a}$$

Magnetic constant:  $\mu_0$   
Number of loops:  $N$   
Current:  $I$   
Radius of loop:  $a$

Ampere's law:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$$

Line integral around a closed path  
Magnetic constant:  $\mu_0$   
Net current enclosed by path:  $I_{\text{encl}}$   
Scalar product of magnetic field and vector segment of path

Faraday's law:

The induced emf in a closed loop ...

$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

... equals the negative of the time rate of change of magnetic flux through the loop.

$$\mathcal{E} = -N \frac{d\Phi_B}{dt}$$

The direction of any magnetic induction effect is such as to oppose the cause of the effect.

Motional emf,

conductor length and velocity perpendicular to uniform  $\vec{B}$

$$\mathcal{E} = vBL$$

Conductor speed:  $v$   
Conductor length:  $L$   
Magnitude of uniform magnetic field:  $B$

Motional emf, general case

$$\mathcal{E} = \oint (\vec{v} \times \vec{B}) \cdot d\vec{l}$$

Line integral over all elements of closed conducting loop  
Length vector of conductor element:  $d\vec{l}$   
Velocity of conductor element:  $\vec{v}$   
Magnetic field at position of element:  $\vec{B}$

Faraday's law for a stationary integration path:

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

Line integral of electric field around path  
Negative of the time rate of change of magnetic flux through path

Displacement current through an area

$$i_D = \epsilon \frac{d\Phi_E}{dt}$$

Time rate of change of electric flux through area:  $\frac{d\Phi_E}{dt}$   
Permittivity of material in area:  $\epsilon$

Gauss's law for  $\vec{E}$ :

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl}}}{\epsilon_0}$$

Flux of electric field through a closed surface:  $\oint \vec{E} \cdot d\vec{A}$   
Charge enclosed by surface:  $Q_{\text{encl}}$   
Electric constant:  $\epsilon_0$

Ampere's law for a stationary integration path:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (i_c + \epsilon_0 \frac{d\Phi_E}{dt})_{\text{encl}}$$

Line integral of magnetic field around path  
Magnetic constant:  $\mu_0$   
Conduction current through path:  $i_c$   
Displacement current through path:  $\epsilon_0 \frac{d\Phi_E}{dt}$   
Time rate of change of electric flux through path:  $\frac{d\Phi_E}{dt}$

Mutually induced emfs:

$$\mathcal{E}_2 = -M \frac{di_1}{dt} \quad \text{and} \quad \mathcal{E}_1 = -M \frac{di_2}{dt}$$

Rate of change of current in coil 1:  $\frac{di_1}{dt}$   
Rate of change of current in coil 2:  $\frac{di_2}{dt}$   
Mutual inductance of coils 1 and 2:  $M$

Mutual inductance of coils 1 and 2

$$M = \frac{N_2 \Phi_{B2}}{i_1} = \frac{N_1 \Phi_{B1}}{i_2}$$

Turns in coil 2:  $N_2$   
Magnetic flux through each turn of coil 2:  $\Phi_{B2}$   
Current in coil 1 (causes flux through coil 2):  $i_1$   
Turns in coil 1:  $N_1$   
Magnetic flux through each turn of coil 1:  $\Phi_{B1}$   
Current in coil 2 (causes flux through coil 1):  $i_2$

Self-inductance (or inductance) of a coil

$$L = \frac{N\Phi_B}{i}$$

Number of turns in coil:  $N$   
Flux due to current through each turn of coil:  $\Phi_B$   
Current in coil:  $i$

Self-induced emf in a circuit

$$\mathcal{E} = -L \frac{di}{dt}$$

Inductance of circuit:  $L$   
Rate of change of current in circuit:  $\frac{di}{dt}$

Energy stored in an inductor

$$U = L \int_0^I i di = \frac{1}{2} LI^2$$

Inductance:  $L$   
Final current:  $I$   
Integral from initial (zero) value of instantaneous current to final value

Time constant for an  $R$ - $L$  circuit

$$\tau = \frac{L}{R}$$

Inductance:  $L$   
Resistance:  $R$

$$i = \frac{\mathcal{E}}{R} (1 - e^{-(R/L)t})$$

(current in an  $R$ - $L$  circuit with emf)

$$i = I_0 e^{-(R/L)t}$$

Current Decay in an  $R$ - $L$  Circuit