Chapter 23

Electric Potential

PowerPoint® Lectures for
University Physics, 14th Edition
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Learning Goals for Chapter 23

Looking forward at …

- how to calculate the electric potential energy of a collection of charges.
- the meaning and significance of electric potential.
- how to calculate the electric potential that a collection of charges produces at a point in space.
- how to use equipotential surfaces to visualize how the electric potential varies in space.
- how to use electric potential to calculate the electric field.

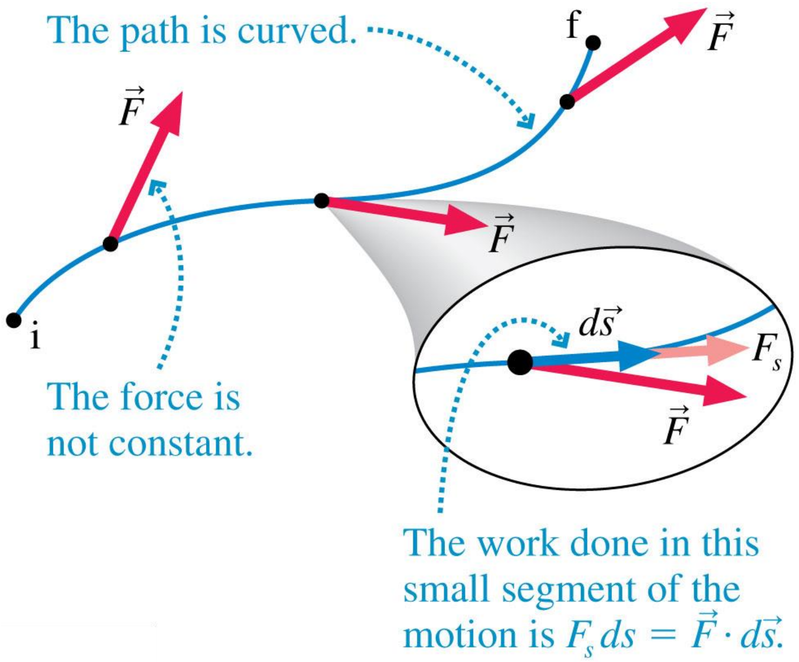
Introduction

- In one type of welding, electric charge flows between the welding tool and the metal pieces that are to be joined.
- This produces a glowing arc whose high temperature fuses the pieces together.

- The tool must be held close to the metal pieces in order to maximize the electric field.
- Electric potential energy is an integral part of our technological society.

Work

When a force $\vec{F}$ acts on a particle that moves from point s_i to point s_f, the work done by the force is given by a line integral



$$W = \sum_j (F_s)_j \Delta s_j \rightarrow \int_{s_i}^{s_f} F_s\,ds = \int_i^f \vec{F} \cdot d\vec{s}$$

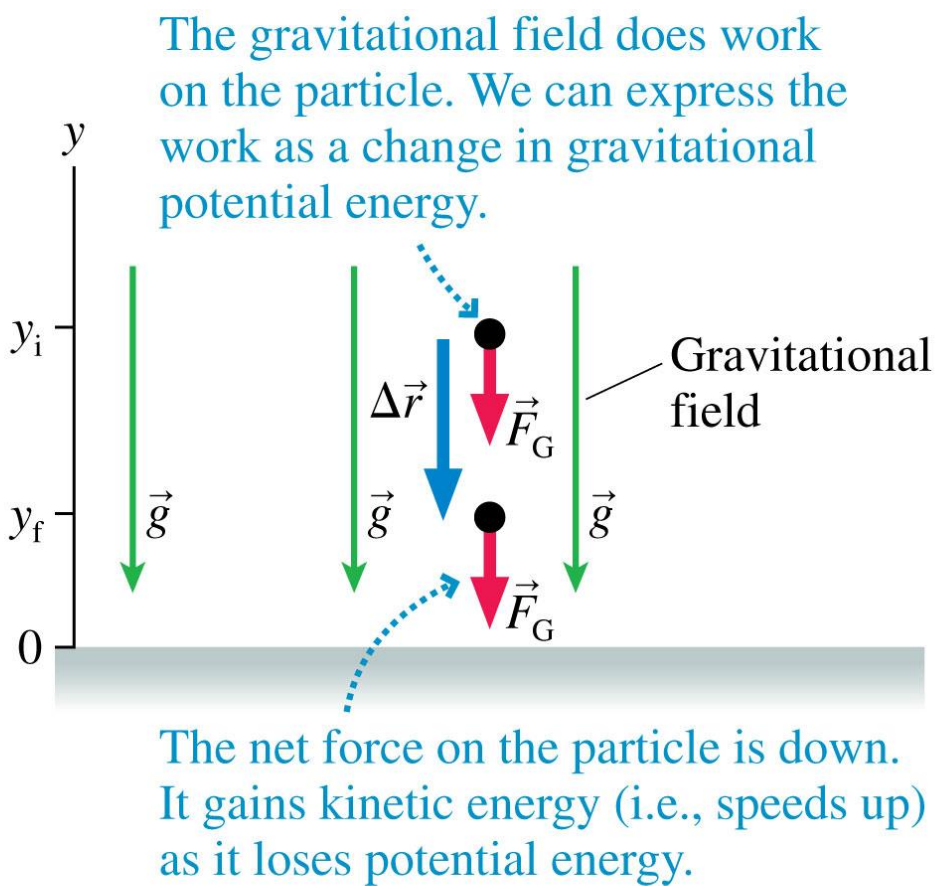
Gravitational Potential Energy

- Every conservative force is associated with a potential energy.
- In the case of gravity, the work done is:

$$W_{\mathbf{grav}} = mgy_{\mathbf{i}} - mgy_{\mathbf{f}}$$

- The change in gravitational potential energy is:

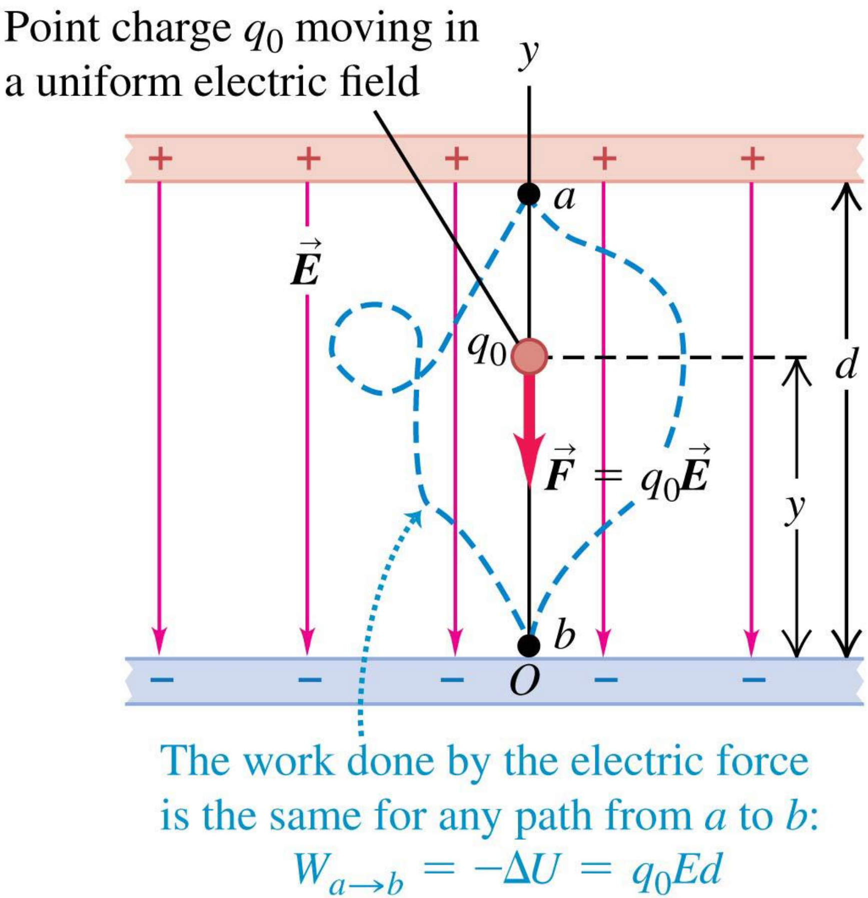
$$\mathbf{W_{grav} = -\Delta U_{grav}}$$



$$\mathbf{W_{grav}} = U_{\mathbf{i}} - U_f$$

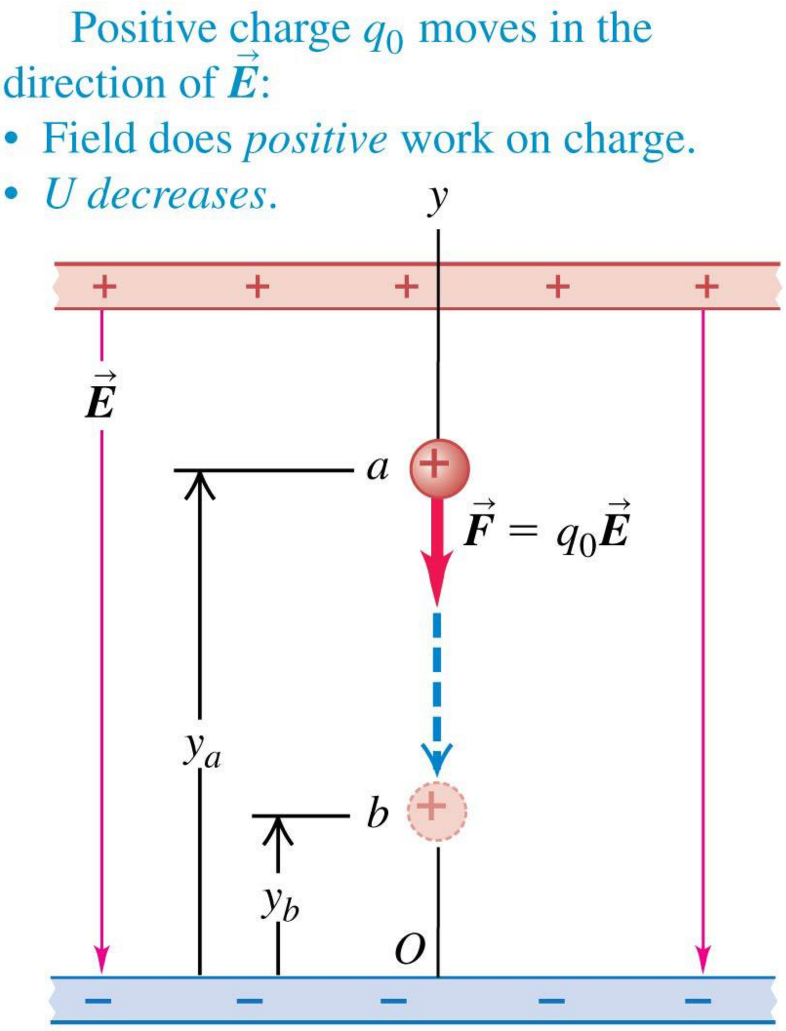
Electric potential energy in a uniform field

- In the figure, a pair of charged parallel metal plates sets up a uniform, downward electric field.
- The field exerts a downward force on a positive test charge.
- As the charge moves downward from point *a* to point *b*, the work done by the field is *independent* of the path the particle takes.



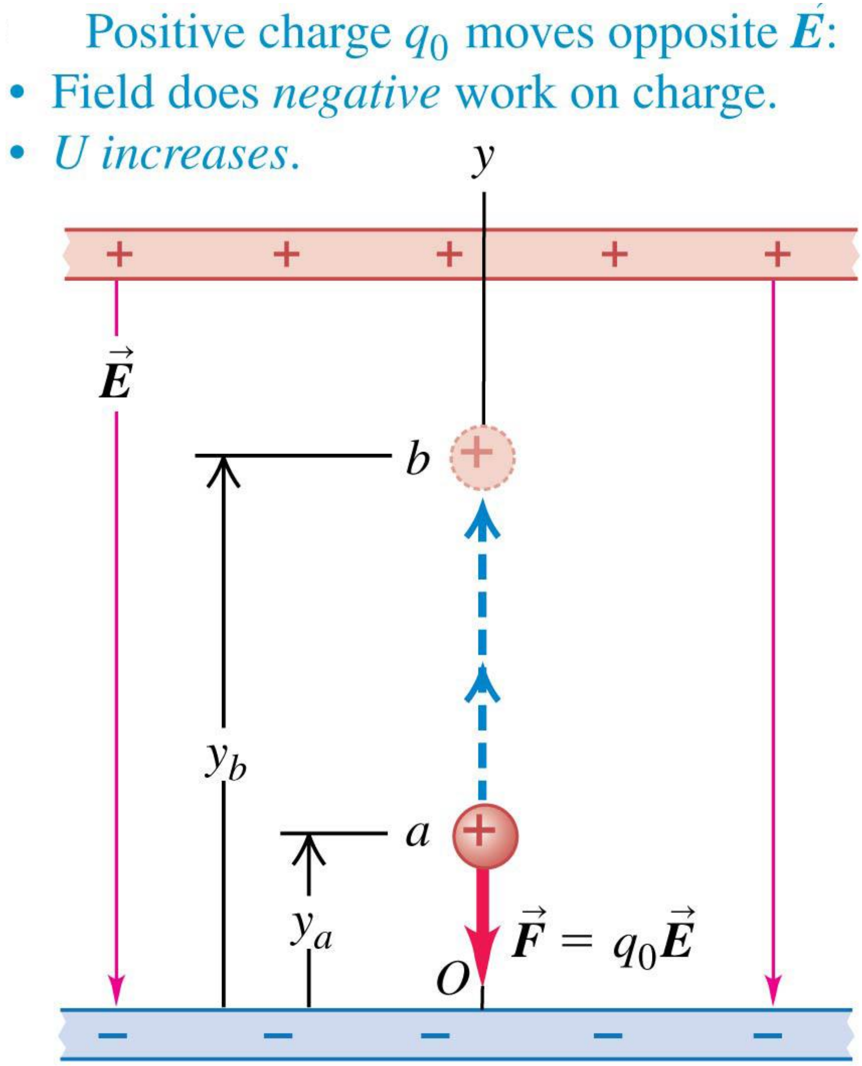
A positive charge moving in a uniform field

- If the positive charge moves in the direction of the field, the field does *positive* work on the charge.
- The potential energy *decreases*.



A positive charge moving in a uniform field

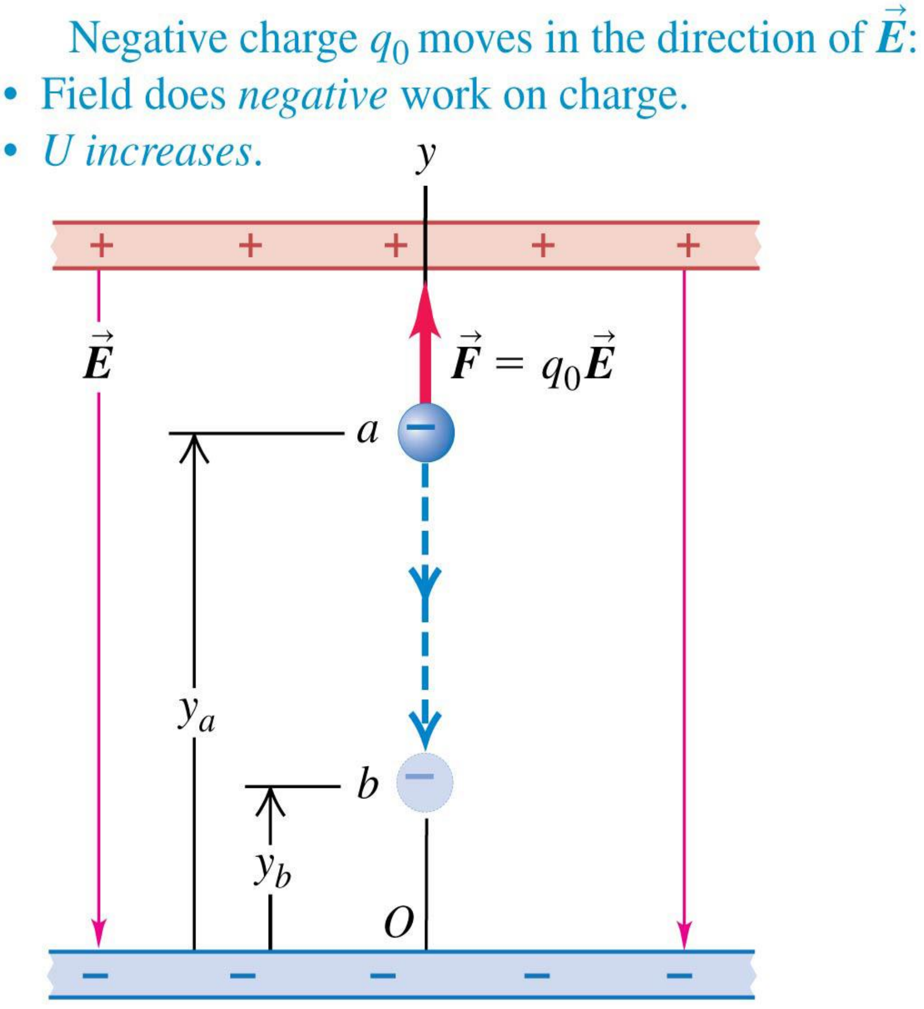
- If the positive charge moves opposite the direction of the field, the field does *negative* work on the charge.
- The potential energy *increases*.



A negative charge moving in a uniform field

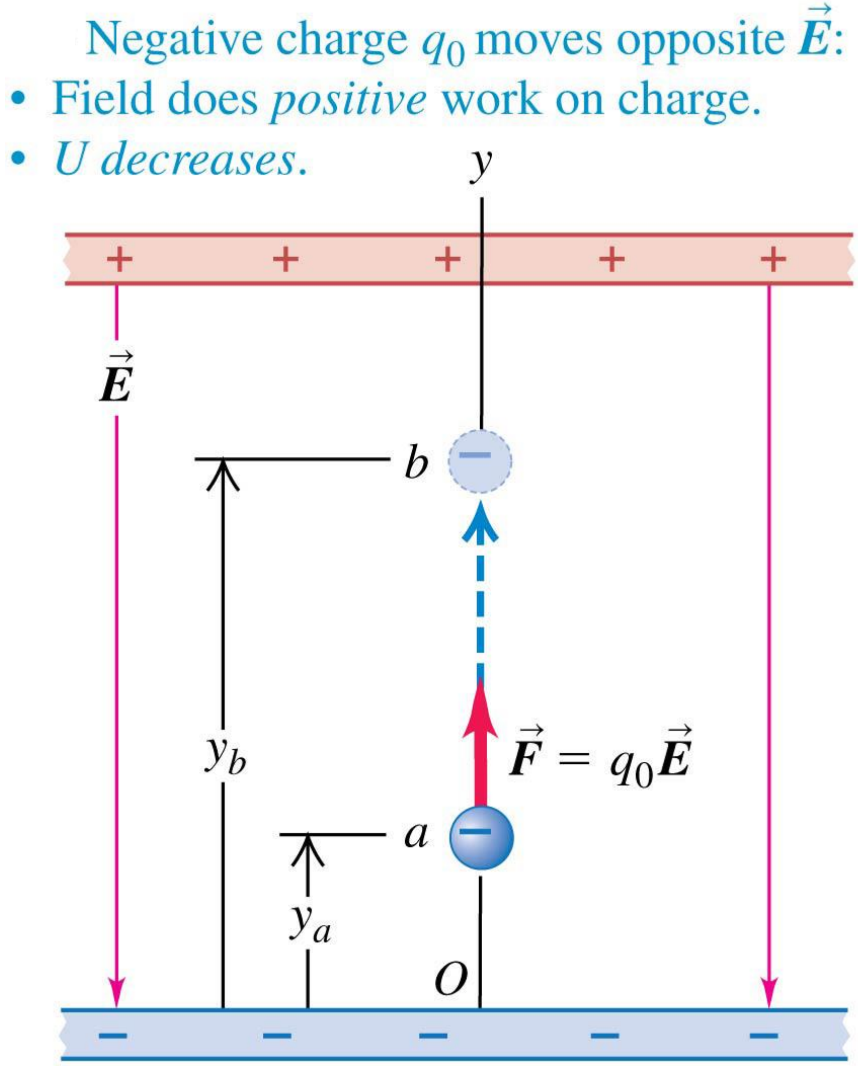
- If the negative charge moves in the direction of the field, the field does *negative* work on the charge.

- The potential energy *increases*.



A negative charge moving in a uniform field

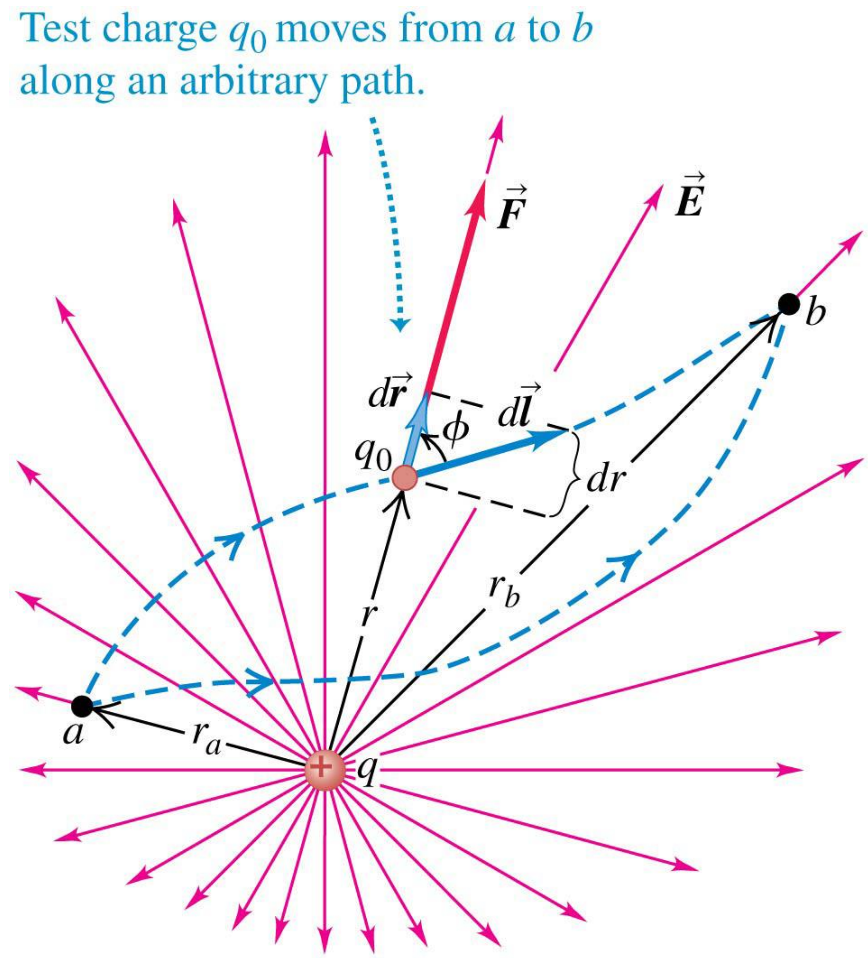
- If the negative charge moves opposite the direction of the field, the field does *positive* work on the charge.
- The potential energy *decreases*.



Electric potential energy of two point charges

- **The work done by the electric field of one point charge on another does not depend on the path taken.**

- **Therefore, the electric potential energy only depends on the distance between the charges.**



Electric potential energy of two point charges

- The electric potential energy of two point charges only depends on the distance between the charges.

Electric potential energy of two point charges

$$U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}$$

Values of two charges

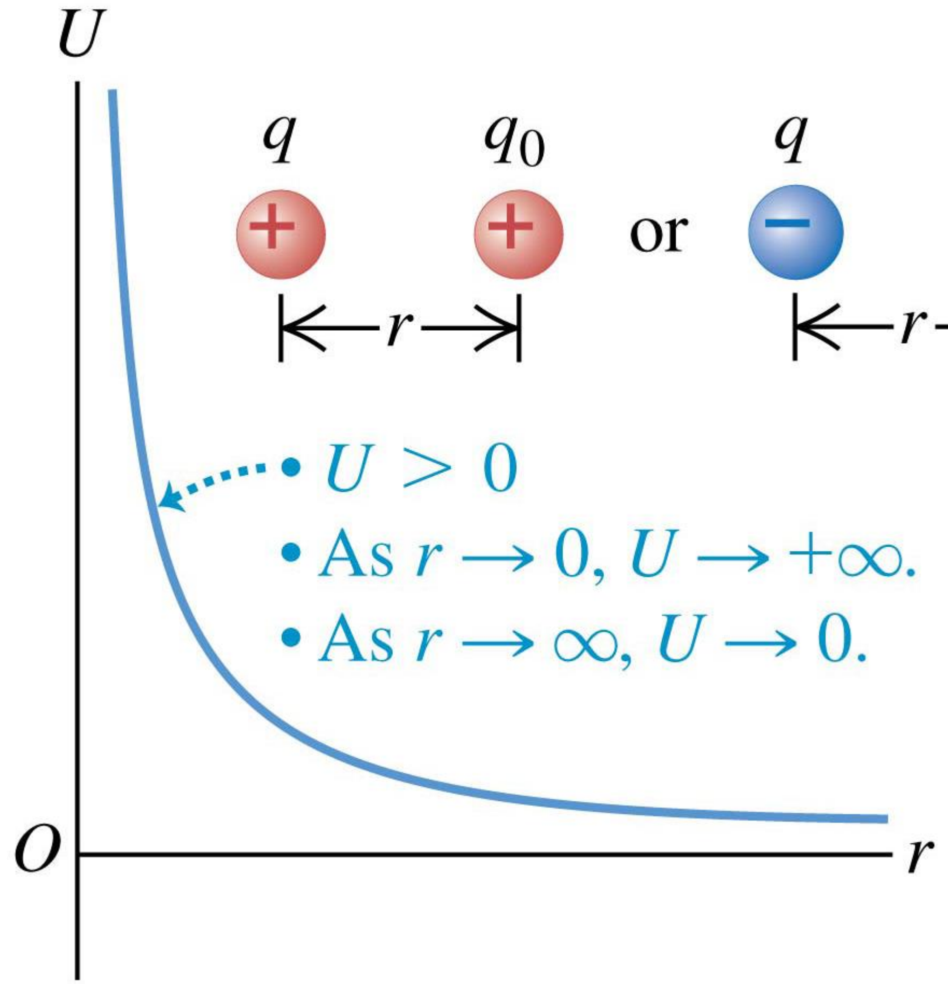
Distance between two charges

Electric constant

- This equation is valid no matter what the signs of the charges are.
- Potential energy is defined to be zero when the charges are infinitely far apart.

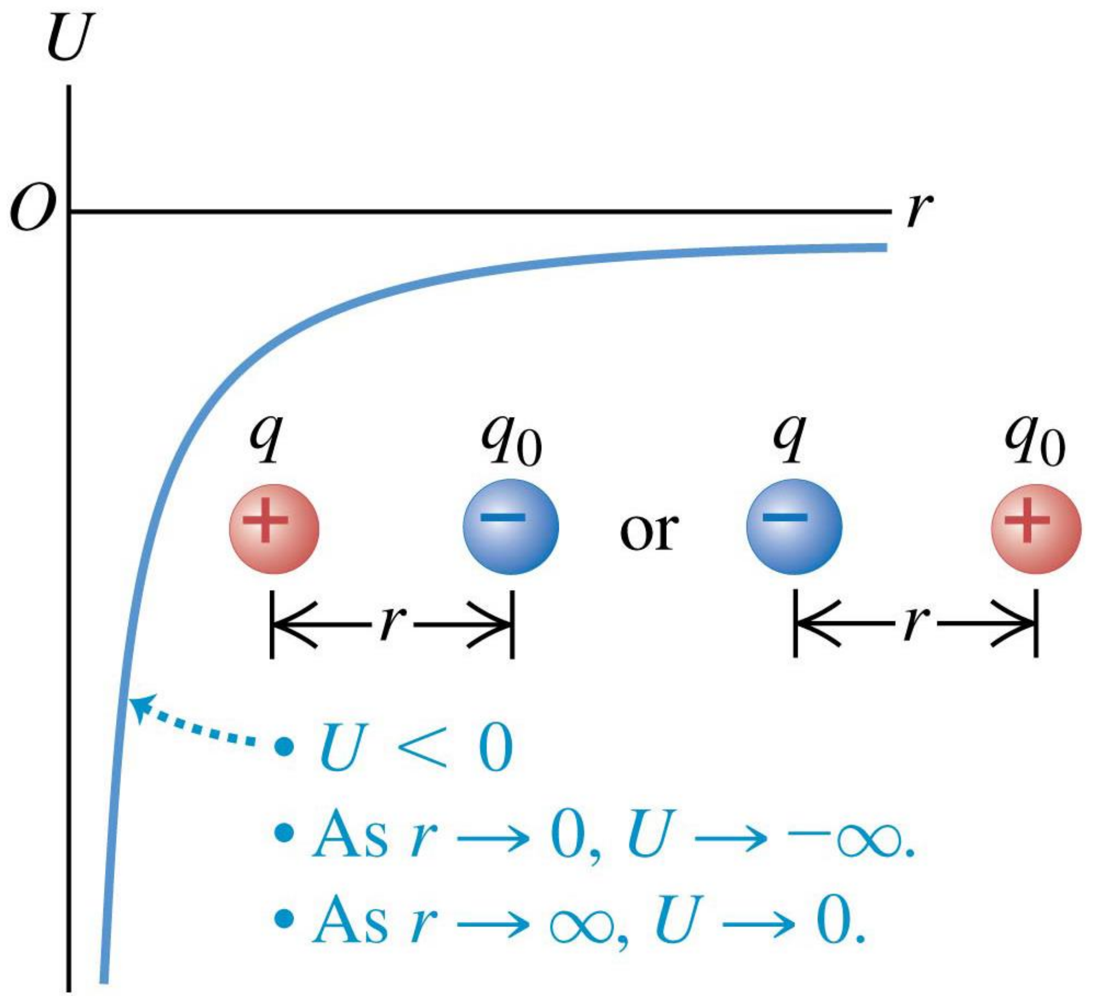
Graphs of the potential energy

- If two charges have the same sign, the interaction is *repulsive*, and the electric potential energy is *positive*.



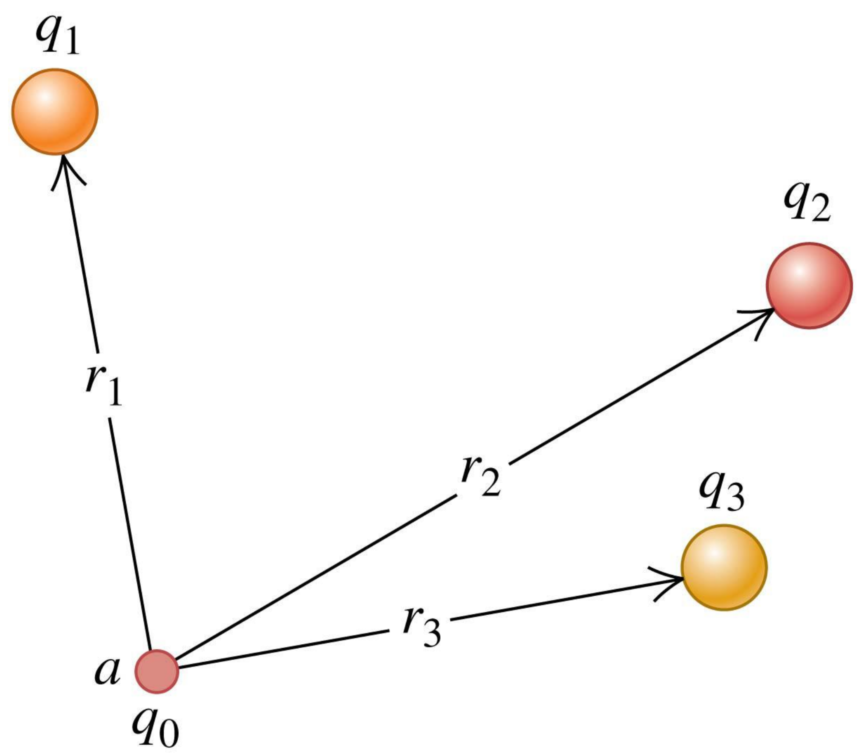
Graphs of the potential energy

- If two charges have opposite signs, the interaction is *attractive*, and the electric potential energy is *negative*.



Electrical potential with several point charges

- The potential energy associated with q_0 depends on the other charges and their distances from q_0.
- The electric potential energy is the *algebraic* sum:



Electric potential energy of point charge q_0 and collection of charges $q_1, q_2, q_3, \ldots$

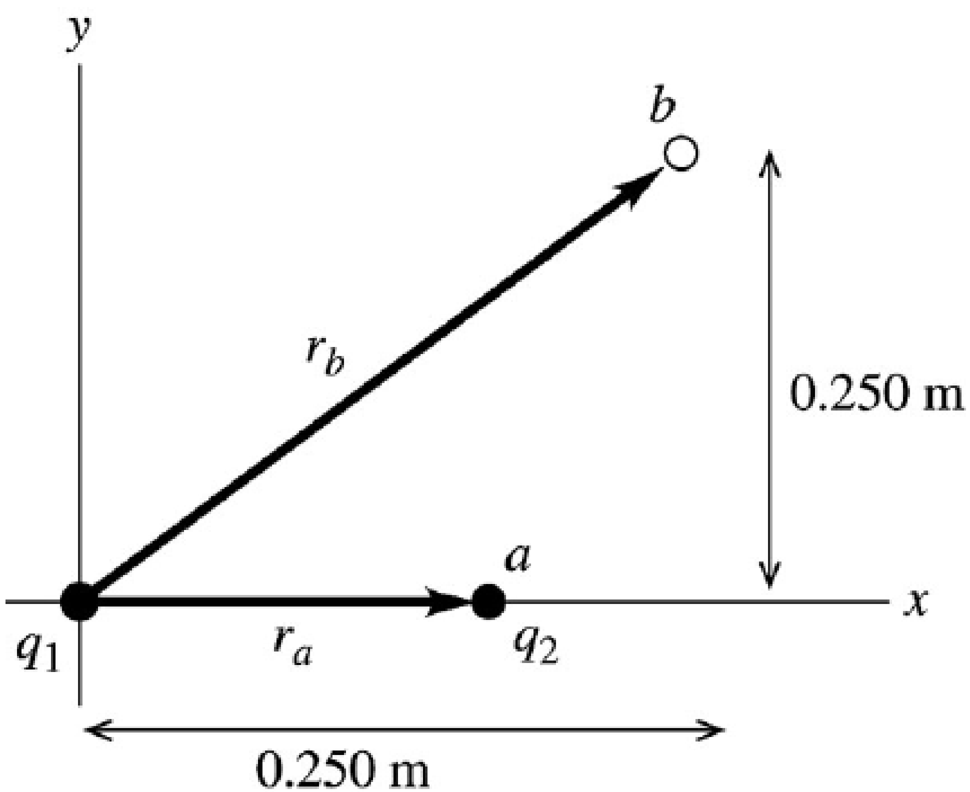
$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

Electric constant

Distances from q_0 to $q_1, q_2, q_3, \ldots$

Example 23.1

A point charge q_1 = +2.40 μC is held stationary at the origin. A second point charge q_2 = -4.30 μC moves from the point x = 0.150 m, y = 0 to the point x = 0.250 m, y = 0.250 m. How much work is done by the electric force on q_2?



Example 23.1 (Solution)

- Apply $W_{a \to b} = U_a - U_b$ to calculate the work. The electric potential energy of a pair of point charges is given by

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}.$$

- Let the initial position of q_2 be point a and the final position be point b

$$r_a = 0.150 \text{ m}.$$

$$r_b = \sqrt{(0.250 \text{ m})^2 + (0.250 \text{ m})^2}.$$

$$r_b = 0.3536 \text{ m}.$$

Example 23.1 (Solution)

$$W_{a\to b} = U_a - U_b.$$

$$U_a = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_a} = (8.988\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{(+2.40\times10^{-6}\ \mathrm{C})(-4.30\times10^{-6}\ \mathrm{C})}{0.150\ \mathrm{m}}.$$

$$U_a = -0.6184\ \mathrm{J}.$$

$$U_b = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_b} = (8.988\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{(+2.40\times10^{-6}\ \mathrm{C})(-4.30\times10^{-6}\ \mathrm{C})}{0.3536\ \mathrm{m}}.$$

$$U_b = -0.2623\ \mathrm{J}.$$

$$W_{a\to b} = U_a - U_b = -0.6184\ \mathrm{J} - (-0.2623\ \mathrm{J}) = -0.356\ \mathrm{J}.$$

Example 23.2: Energy of the Nucleus

How much work is needed to assemble an atomic nucleus containing three protons (such as Li) if we model it as an equilateral triangle of side 2.00×10^{-15} m with a proton at each vertex? Assume the protons started from very far away.

- The work needed to assemble the nucleus is the sum of the electrical potential energies of the protons in the nucleus, relative to infinity.
- The total potential energy is the scalar sum of all the individual potential energies, where each potential energy is $U = (1/4\pi\varepsilon_0)(qq_0/r)$
- Each charge is e and the charges are equidistant from each other, so the total potential energy is

$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{e^2}{r} + \frac{e^2}{r} + \frac{e^2}{r}\right) = \frac{3e^2}{4\pi\varepsilon_0 r}.$$

Example 23.2: Energy of the Nucleus

- The total potential energy is the scalar sum of all the individual potential energies, where each potential energy is

$$U = (1/4\pi\varepsilon_0)(qq_0/r)$$

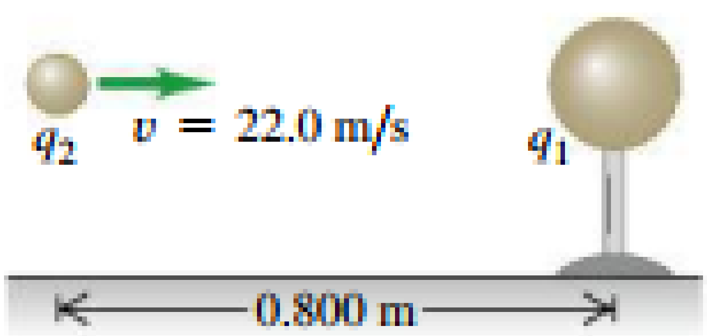
- Each charge is *e* and the charges are equidistant from each other, so the total potential energy is

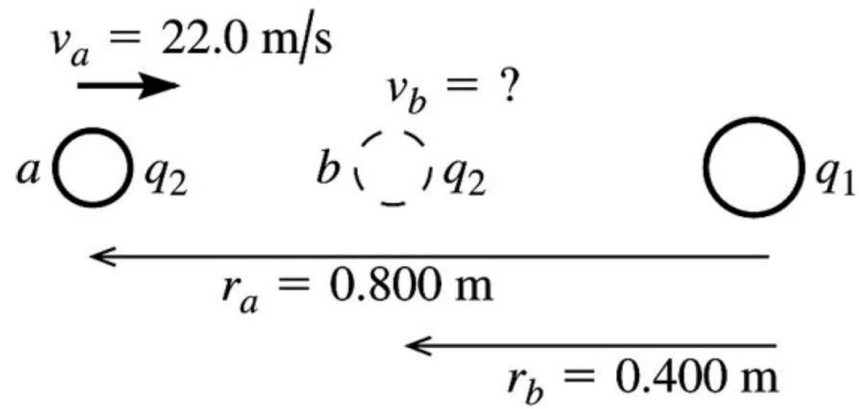
$$U = \frac{1}{4\pi\varepsilon_0}\left(\frac{e^2}{r} + \frac{e^2}{r} + \frac{e^2}{r}\right) = \frac{3e^2}{4\pi\varepsilon_0 r}.$$

$$U = \frac{3e^2}{4\pi\varepsilon_0 r} = \frac{3(1.60\times10^{-19}\ \mathrm{C})^2(9.00\times10^9\ \mathrm{N\cdot m^2/C^2})}{2.00\times10^{-15}\ \mathrm{m}} = 3.46\times10^{-13}\ \mathrm{J} = 2.16\ \mathrm{MeV}.$$

Example 23.3

A small metal sphere, carrying a net charge of q_1 = -2.80 mC, is held in a stationary position by insulating supports. A second smallmetal sphere, with a net charge of q_2 = -7.80 mC and mass 1.50 g, is projected toward q_1. When the two spheres are 0.800 m apart, q_2, is moving toward q_2 with speed 22.0 m/s. Assume that the two spheres can be treated as point charges. You can ignore the force of gravity. (a) What is the speed of q_2 when the spheres are 0.400 m apart? (b) How close does q_2 get to q_1?





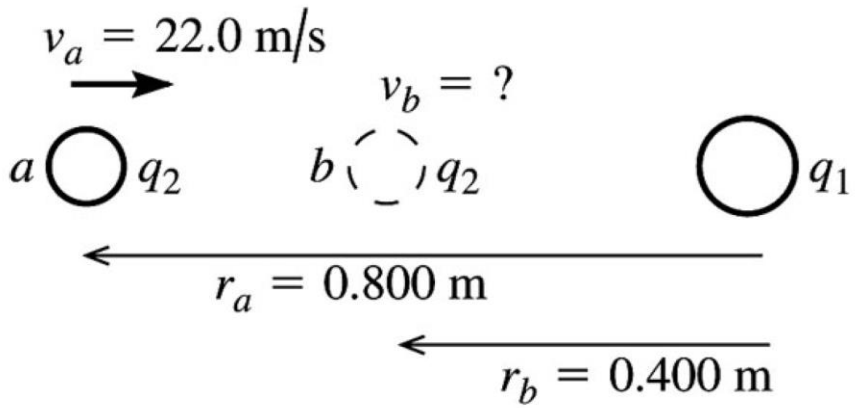
Example 23.3

(a) IDENTIFY: Use conservation of energy:

$$K_a + U_a + W_{\text{other}} = K_b + U_b$$

U for the pair of point charges is given by

$$U = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}.$$



Let point *a* be where q_2 is 0.800 m from q_1 and point *b* be where q_2 is 0.400 m from q_2, as shown.

Example 23.3

EXECUTE: Only the electric force does work, so $W_{\text{other}} = 0$ and $U = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r}$.

$$K_a = \tfrac{1}{2}mv_a^2 = \tfrac{1}{2}(1.50\times10^{-3}\ \text{kg})(22.0\ \text{m/s})^2 = 0.3630\ \text{J}.$$

$$U_a = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_a} = (8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\ \text{C})(-7.80\times10^{-6}\ \text{C})}{0.800\ \text{m}} = +0.2454\ \text{J}.$$

$$K_b = \tfrac{1}{2}mv_b^2.$$

$$U_b = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_b} = (8.988\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times10^{-6}\ \text{C})(-7.80\times10^{-6}\ \text{C})}{0.400\ \text{m}} = +0.4907\ \text{J}.$$

The conservation of energy equation then gives $K_b = K_a + (U_a - U_b)$.

$$\tfrac{1}{2}mv_b^2 = +0.3630\ \text{J} + (0.2454\ \text{J} - 0.4907\ \text{J}) = 0.1177\ \text{J}.$$

$$v_b = \sqrt{\frac{2(0.1177\ \text{J})}{1.50\times10^{-3}\ \text{kg}}} = 12.5\ \text{m/s}.$$

EVALUATE: The potential energy increases when the two positively charged spheres get closer togeth so the kinetic energy and speed decrease.

Example 23.3

(b) IDENTIFY: Let point c be where q_2 has its speed momentarily reduced to zero. Apply conservation of energy to points a and c: $K_a + U_a + W_{\text{other}} = K_c + U_c$.

SET UP: Points a and c are shown in Figure 23.5b.

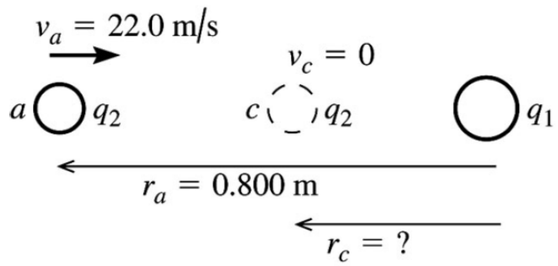


Figure 23.5b

EXECUTE: $K_a = +0.3630$ J (from part (a)).

$U_a = +0.2454$ J (from part (a)).

$K_c = 0$ (at distance of closest approach the speed is zero).

$$U_c = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_c}.$$

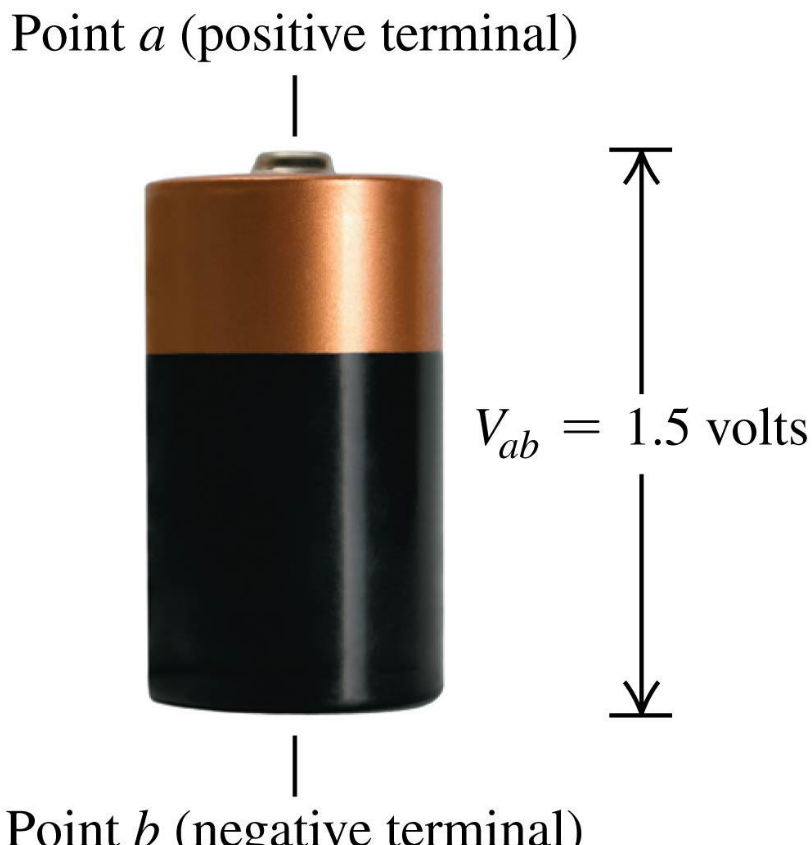
Thus conservation of energy $K_a + U_a = U_c$ gives $\dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1 q_2}{r_c} = +0.3630\text{ J} + 0.2454\text{ J} = 0.6084\text{ J}.$

$$r_c = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{0.6084\text{ J}} = (8.988\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\frac{(-2.80\times 10^{-6}\text{ C})(-7.80\times 10^{-6}\text{ C})}{+0.6084\text{ J}} = 0.323\text{ m}.$$

EVALUATE: $U \to \infty$ as $r \to 0$ so q_2 will stop no matter what its initial speed is.

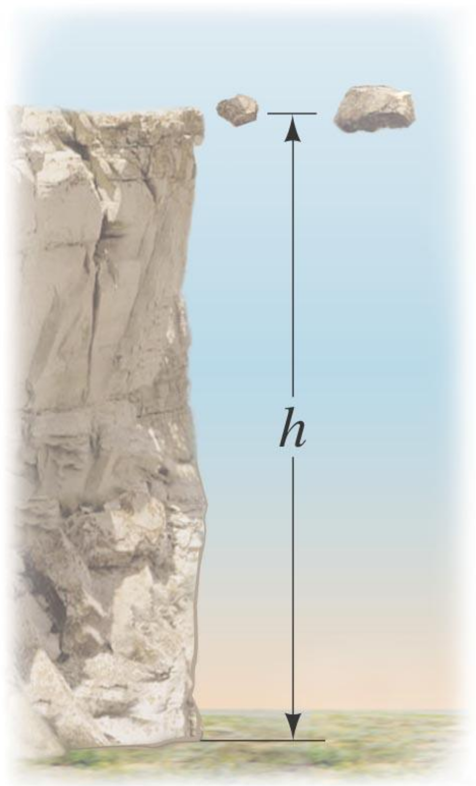
Electric potential

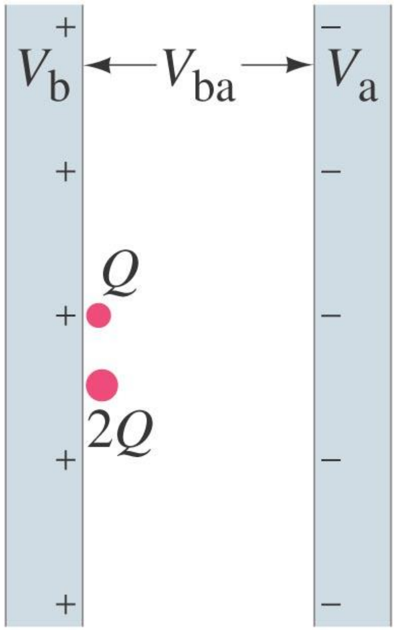
- **Potential** is *potential energy per unit charge*.
- The potential of a with respect to b ($V_{ab} = V_a - V_b$) equals the work done by the electric force when a *unit* charge moves from a to b.



Electrostatic Potential Energy and Potential Difference

Analogy between gravitational and electrical potential energy:





Electric potential

- The potential due to a single point charge is:

Electric potential due to a point charge
$$V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$
Value of point charge
Distance from point charge to where potential is measured
Electric constant

- Like electric field, potential is independent of the test charge that we use to define it.

- For a collection of point charges:

Electric potential due to a collection of point charges
$$V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$
Value of ith point charge
Distance from ith point charge to where potential is measured
Electric constant

Example 23.4

A point particle has a charge equal to +2.00 μC and is fixed at the origin. (*a*) What is the electric potential *V* at a point 4.00 m from the origin assuming that *V* = 0 at infinity? (*b*) How much work must be done to bring a second point particle that has a charge of +3.00 μC from infinity to a distance of 4.00 m from the +2.00-μC charge?

- The Coulomb potential at a distance r from the origin relative to $V = 0$ at infinity is given by $\boldsymbol{V = kq/r}$ where q is the charge at the origin. The work that must be done by an outside agent to bring a charge from infinity to a position a distance r from the origin is the product of the magnitude of the charge and the potential difference due to the charge at the origin.

Example 23.4 (Solution)

(a) The Coulomb potential of the charge is given by:

$$V = \frac{kq}{r}$$

Substitute numerical values and evaluate V:

$$V = \frac{\left(8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2}\right)\left(2.00\ \mu\mathrm{C}\right)}{4.00\ \mathrm{m}}$$

$$= 4.494\ \mathrm{kV} = \boxed{4.49\ \mathrm{kV}}$$

(b) The work that must be done is given by:

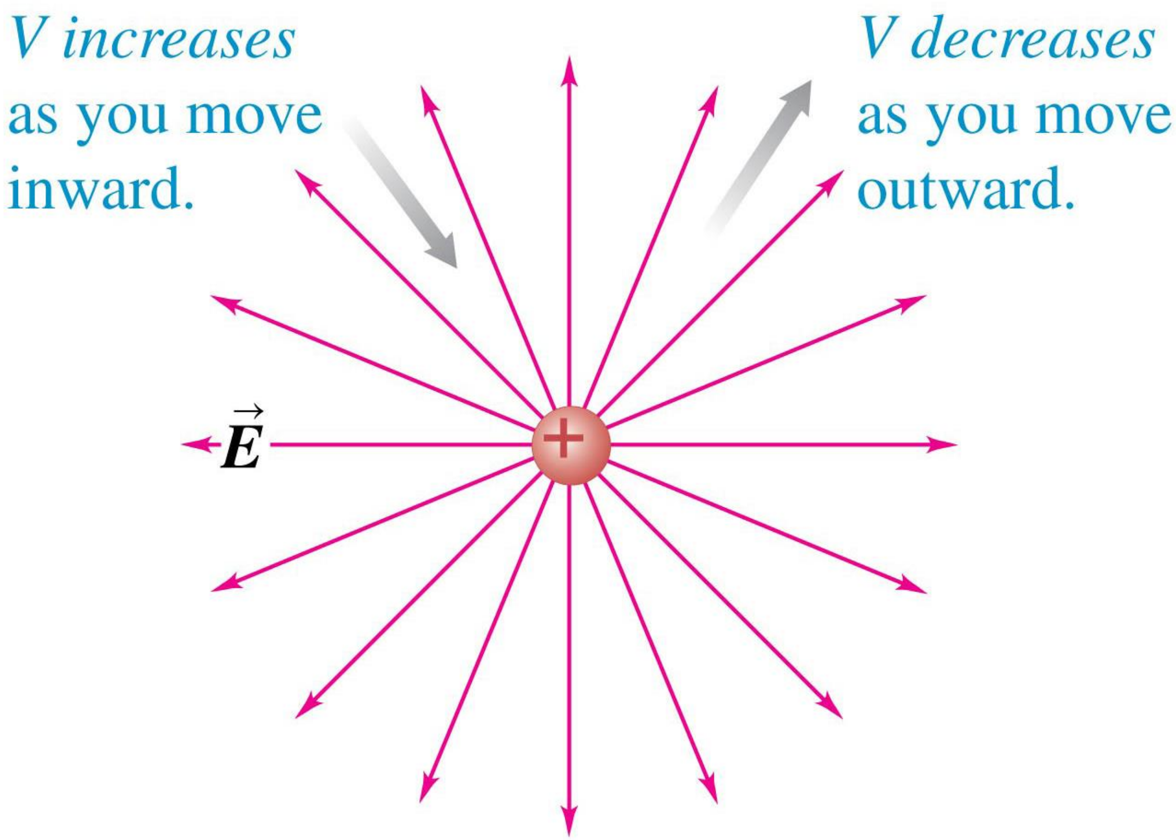
$$W = q\Delta V$$

Substitute numerical values and evaluate W:

$$W = \left(3.00\ \mu\mathrm{C}\right)\left(4.494\ \mathrm{kV}\right) = \boxed{13.5\ \mathrm{mJ}}$$

Finding electric potential from the electric field

- If you move in the direction of the electric field, the electric potential *decreases*, but if you move opposite the field, the potential *increases*.



Electric potential and electric field

- Moving with the direction of the electric field means moving in the direction of decreasing V, and vice versa.
- To move a unit charge slowly against the electric force, we must apply an external force per unit charge equal and opposite to the electric force per unit charge.
- The electric force per unit charge is the electric field.
- The potential difference $V_a - V_b$ equals the work done per unit charge by this external force to move a unit charge from b to a:

$$V_a - V_b = -\int_b^a \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{l}}$$

- The unit of electric field can be expressed as 1 N/C = 1 V/m.

The electron volt

- When a particle with charge q moves from a point where the potential is V_b to a point where it is V_a, the change in the potential energy U is

$$U_a - \mathrm{U}_b = q(V_a - V_b)$$

- If charge q equals the magnitude e of the electron charge, and the potential difference is 1 V, the change in energy is defined as one electron volt (eV):

$$1\ \mathrm{eV} = 1.602 \times 10^{-19}\ \mathrm{J}$$

Electron volts and cancer radiotherapy

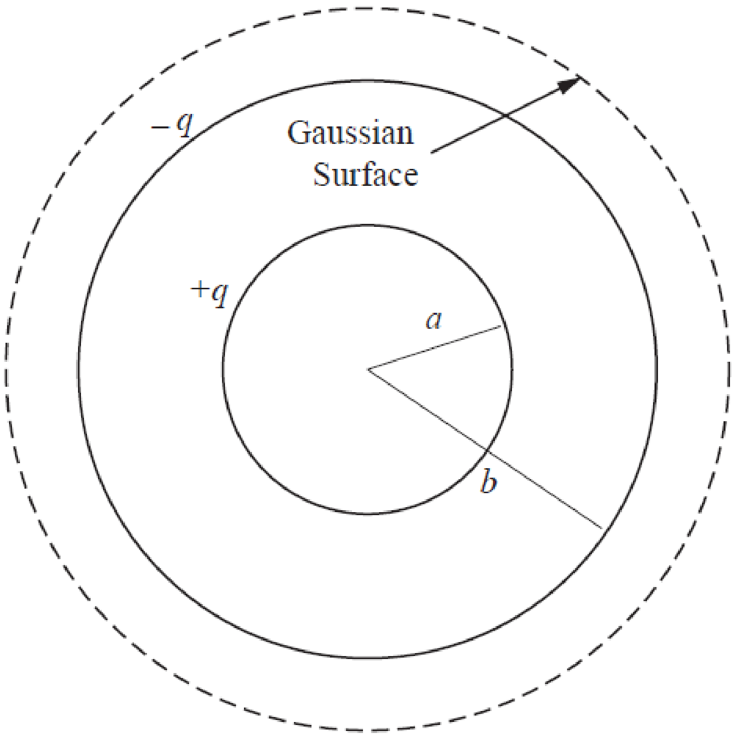
- One way to destroy a cancerous tumor is to aim high-energy electrons directly at it.
- Each electron has a kinetic energy of 4 to 20 MeV (1 MeV = 10^6 eV), and transfers its energy to the tumor through collisions with the tumor's atoms.
- Electrons in this energy range can penetrate only a few centimeters into a patient, which makes them useful for treating superficial tumors, such as those on the skin or lips.

Example 23.5

Two concentric conducting spherical shells have equal and opposite charges. The inner shell has outer radius *a* and charge +*q*; the outer shell has inner radius *b* and charge –*q*. Find the potential difference, $V_a - V_b$ between the shells.

- **Picture the Problem** The diagram is a cross-sectional view showing the charges on the concentric spherical shells. The Gaussian surface over which we'll integrate *E* in order to find *V* in the region $r \geq b$ is also shown. We'll also find *E* in the region for which $a < r < b$. We can then use the relationship $V = -\int E dr$ to find *a V* and *b V* and their difference.

Example 23.5 (Solution)



$$V_b = -\int_{\infty}^{b} E_{r \geq b}\, dr$$

Apply Gauss's law for $r \geq b$:

$$\oint_S \vec{\boldsymbol{E}}_r \cdot \hat{\boldsymbol{n}}\, dA = \frac{Q_{\text{enclosed}}}{\epsilon_0} = 0$$

and $E_{r \geq b} = 0$ because $\boldsymbol{Q}_{\text{enclosed}} = 0$ for $r \geq b$.

Example 23.5 (Solution)

Substitute for $E_{r\geq b}$ to obtain:

$$V_b = -\int_{\infty}^{b} (0)\,dr = 0$$

Express V_a :

$$V_a = -\int_{b}^{a} E_{r\geq a}\,dr$$

Apply Gauss's law for $r \geq a$:

$$E_{r\geq a}\left(4\pi r^2\right) = \frac{q}{\epsilon_0}$$

and

$$E_{r\geq a} = \frac{q}{4\pi \epsilon_0 r^2} = \frac{kq}{r^2}$$

Substitute for $E_{r\geq a}$ to obtain:

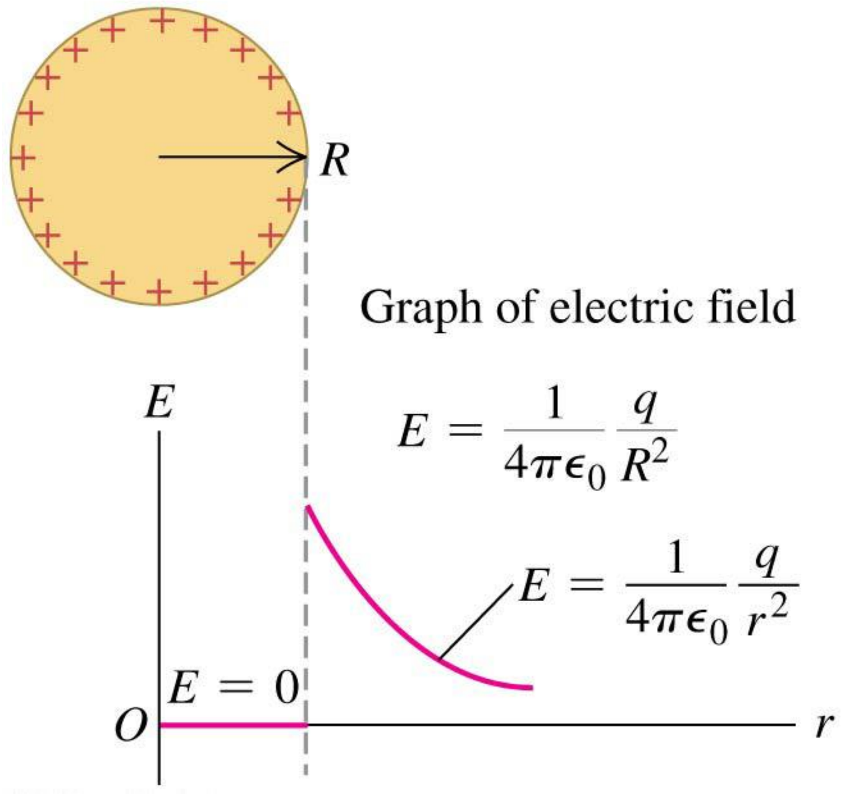
$$V_a = -kq\int_{b}^{a} \frac{dr}{r^2} = \frac{kq}{a} - \frac{kq}{b}$$

The potential difference between the shells is given by:

$$V_a - V_b = V_a = \boxed{kq\left(\frac{1}{a} - \frac{1}{b}\right)}$$

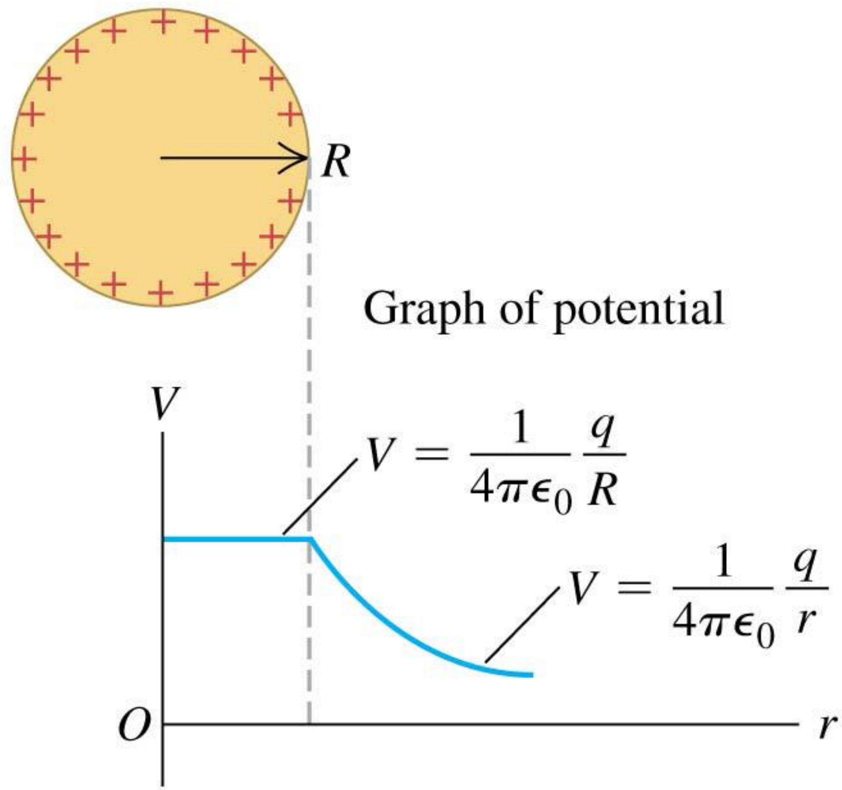
Electric potential and field of a charged conductor

- A solid conducting sphere of radius R has a total charge q.
- The electric field *inside* the sphere is zero everywhere.



Electric potential and field of a charged conductor

- The potential is the *same* at every point inside the sphere and is equal to its value at the surface.



Example 23.6

The electric potential at the surface of a uniformly charged sphere is 450 V. At a point outside the sphere at a (radial) distance of 20.0 cm from its surface, the electric potential is 150 V. (The potential is zero very far from the sphere.) What is the radius of the sphere, and what is the charge of the sphere?

- **Picture the Problem** Let R be the radius of the sphere and Q its charge. We can express the potential at the two locations given and solve the resulting equations simultaneously for R and Q.

Example 23.6 (Solution)

Relate the potential of the sphere at its surface to its radius:

$$\frac{kQ}{R} = 450\,\text{V} \qquad (1)$$

Express the potential at a distance of 20.0 cm from its surface:

$$\frac{kQ}{R + 0.200\,\text{m}} = 150\,\text{V} \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{\dfrac{kQ}{R}}{\dfrac{kQ}{R + 0.200\,\text{m}}} = \frac{450\,\text{V}}{150\,\text{V}}$$

or

$$\frac{R + 0.200\,\text{m}}{R} = 3 \Rightarrow R = \boxed{10.0\,\text{cm}}$$

Solving equation (1) for Q yields:

$$Q = (450\,\text{V})\frac{R}{k}$$

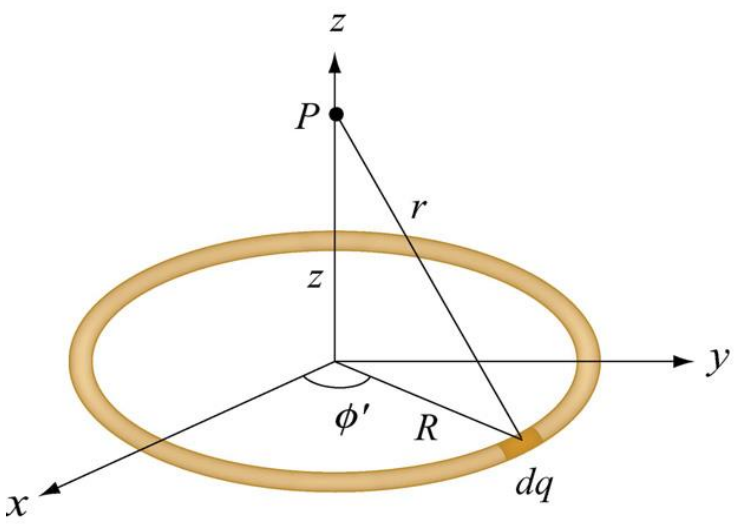
Substitute numerical values and evaluate Q:

$$Q = (450\,\text{V})\frac{(0.100\,\text{m})}{(8.988 \times 10^{9}\,\text{N} \cdot \text{m}^2/\text{C}^2)}$$

$$= \boxed{5.01\,\text{nC}}$$

Example 23.7: Uniformly Charged Ring

Consider a uniformly charged ring of radius R and charge density λ. What is the electric potential at a distance z from the central axis?



Example 23.7: Uniformly Charged Ring

Consider a small differential element $d\ell = R\,d\phi'$ on the ring. The element carries a charge $dq = \lambda\,d\ell = \lambda R\,d\phi'$, and its contribution to the electric potential at P is

$$dV = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R\,d\phi'}{\sqrt{R^2+z^2}}$$

The electric potential at P due to the entire ring is

$$V = \int dV = \frac{1}{4\pi\varepsilon_0}\frac{\lambda R}{\sqrt{R^2+z^2}}\oint d\phi' = \frac{1}{4\pi\varepsilon_0}\frac{2\pi\lambda R}{\sqrt{R^2+z^2}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{R^2+z^2}}$$

where we have substituted $Q = 2\pi R\lambda$ for the total charge on the ring. In the limit $z \gg R$, the potential approaches its "point-charge" limit:

$$V \approx \frac{1}{4\pi\varepsilon_0}\frac{Q}{z}$$

Example 23.8

A uniformly charged, thin ring has radius 15.0 cm and total charge +24.0 nC. An electron is placed on the ring's axis a distance 30.0 cm from the center of the ring and is constrained to stay on the axis of the ring. The electron is then released from rest. (a) Describe the subsequent motion of the electron. (b) Find the speed of the electron when it reaches the center of the ring.

Example 23.8

(a) IDENTIFY and SET UP: The electric field on the ring's axis is given by $E_x = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Qx}{(x^2+a^2)^{3/2}}$. The magnitude of the force on the electron exerted by this field is given by $F = eE$.

EXECUTE: When the electron is on either side of the center of the ring, the ring exerts an attractive force directed toward the center of the ring. This restoring force produces oscillatory motion of the electron along the axis of the ring, with amplitude 30.0 cm. The force on the electron is *not* of the form $F = -kx$ so the oscillatory motion is not simple harmonic motion.

(b) IDENTIFY: Apply conservation of energy to the motion of the electron.

SET UP: $K_a + U_a = K_b + U_b$ with a at the initial position of the electron and b at the center of the ring.

From Example 23.11, $V = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{\sqrt{x^2+a^2}}$, where a is the radius of the ring.

EXECUTE: $x_a = 30.0\text{ cm}$, $x_b = 0$.

$K_a = 0$ (released from rest), $K_b = \frac{1}{2}mv^2$.

Thus $\frac{1}{2}mv^2 = U_a - U_b$.

And $U = qV = -eV$ so $v = \sqrt{\dfrac{2e(V_b - V_a)}{m}}$.

Example 23.8

$$V_a = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x_a^2 + a^2}} = (8.988\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{24.0\times10^{-9}\ \mathrm{C}}{\sqrt{(0.300\ \mathrm{m})^2 + (0.150\ \mathrm{m})^2}}.$$

$$V_a = 643\ \mathrm{V}.$$

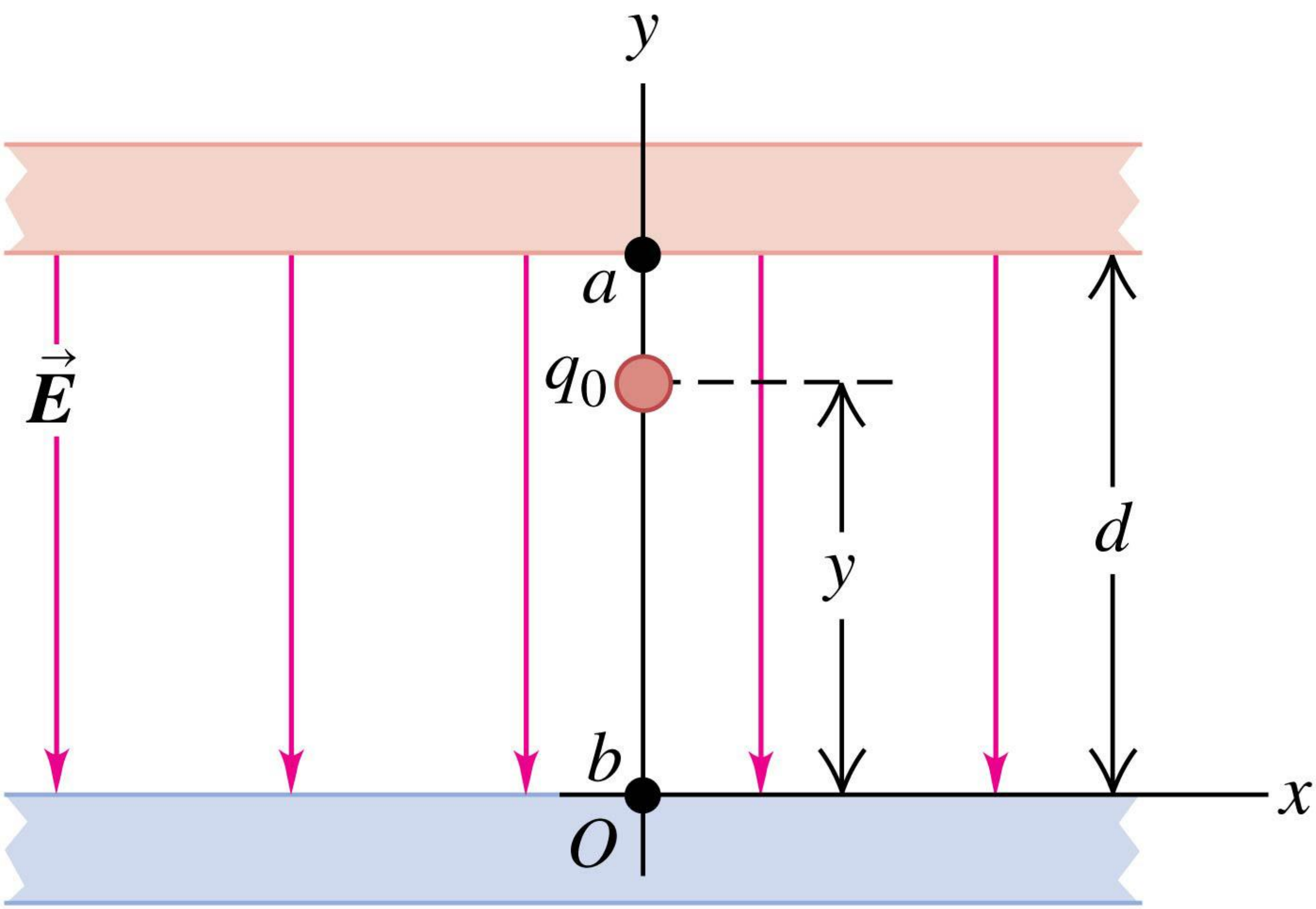
$$V_b = \frac{1}{4\pi\varepsilon_0}\frac{Q}{\sqrt{x_b^2 + a^2}} = (8.988\times10^9\ \mathrm{N\cdot m^2/C^2})\frac{24.0\times10^{-9}\ \mathrm{C}}{0.150\ \mathrm{m}} = 1438\ \mathrm{V}.$$

$$v = \sqrt{\frac{2e(V_b - V_a)}{m}} = \sqrt{\frac{2(1.602\times10^{-19}\ \mathrm{C})(1438\ \mathrm{V} - 643\ \mathrm{V})}{9.109\times10^{-31}\ \mathrm{kg}}} = 1.67\times10^7\ \mathrm{m/s}.$$

EVALUATE: The positively charged ring attracts the negatively charged electron and accelerates it. The electron has its maximum speed at this point. When the electron moves past the center of the ring the force on it is opposite to its motion and it slows down.

Oppositely charged parallel plates

- The potential at any height y between the two large oppositely charged parallel plates is $V = Ey$.



Example 23.9

The facing surfaces of two large parallel conducting plates separated by 10.0 cm have uniform surface charge densities that are equal in magnitude but opposite in sign. The difference in potential between the plates is 500 V.

(a) Is the positive or the negative plate at the higher potential?

(b) What is the magnitude of the electric field between the plates?

(c) An electron is released from rest next to the negatively charged surface. Find the work done by the electric field on the electron as the electron moves from the release point to the positive plate. Express your answer in both electron volts and joules.

(d) (*d*) What is the change in potential energy of the electron when it moves from the release point plate to the positive plate?

(e) What is its kinetic energy when it reaches the positive plate?

Example 23.9 (Solution)

- Because the electric field is uniform, we can find its magnitude from $E = \Delta V/\Delta x$. We can find the work done by the electric field on the electron from the difference in potential between the plates and the charge of the electron and find the change in potential energy of the electron from the work done on it by the electric field. We can use conservation of energy to find the kinetic energy of the electron when it reaches the positive plate.
- (*a*) Because the electric force on a test charge is away from the positive plate and toward the negative plate, the positive plate is at the higher potential.
- (b)

$$E = \frac{\Delta V}{\Delta x} = \frac{500\,\mathrm{V}}{0.100\,\mathrm{m}} = \boxed{5.00\,\mathrm{kV/m}}$$

Example 23.9 (Solution)

- *(c)*

$$W = q\Delta V = \left(1.602\times10^{-19}\,\mathrm{C}\right)\left(500\,\mathrm{V}\right)$$
$$= \boxed{8.01\times10^{-17}\,\mathrm{J}}$$

$$W = \left(8.01\times10^{-17}\,\mathrm{J}\right)\left(\frac{1\,\mathrm{eV}}{1.602\times10^{-19}\,\mathrm{J}}\right)$$
$$= \boxed{500\,\mathrm{eV}}$$

- *(d)*

$$\Delta U = -W = \boxed{-500\,\mathrm{eV}}$$

- *(e)*

$$\Delta K = -\Delta U = \boxed{500\,\mathrm{eV}}$$

Example 23.10

- **A uniform electric field that has a magnitude 2.00 kV/m points in the +*x* direction.**
- (*a*) What is the electric potential difference between the $x = 0.00$ m plane and the $x = 4.00$ m plane? A point particle that has a charge of +3.00 μC is released from rest at the origin.
- (*b*) What is the change in the electric potential energy of the particle as it travels from the $x = 0.00$ m plane to the $x = 4.00$ m plane?
- (*c*) What is the kinetic energy of the particle when it arrives at the $x = 4.00$ m plane?
- (*d*) Find the expression for the electric potential $V(x)$ if its value is chosen to be zero at $x = 0$.

Example 23.10 (Solution)

- (a)

$$V(4.00\,\mathrm{m})-V(0)=-\int_a^b \vec{\boldsymbol{E}}\cdot d\vec{\ell}=-\int_0^{4.00\,\mathrm{m}} E\,d\ell=-(2.00\,\mathrm{kN/C})(4.00\,\mathrm{m})=\boxed{-8.00\,\mathrm{kV}}$$

- (b)

$$\Delta U=q\Delta V=(3.00\,\mu\mathrm{C})(-8.00\,\mathrm{kV})$$
$$=\boxed{-24.0\,\mathrm{mJ}}$$

(c) Use conservation of energy to relate ΔU and ΔK:

$$\Delta K+\Delta U=0$$

or

$$K_{4\,\mathrm{m}}-K_0+\Delta U=0$$

Because $K_0 = 0$:

$$K_{4\,\mathrm{m}}=-\Delta U=\boxed{24.0\,\mathrm{mJ}}$$

Use the definition of finite potential difference to obtain:

$$V(x)-V(x_0)=-E_x(x-x_0)$$
$$=-(2.00\,\mathrm{kV/m})(x-x_0)$$

Example 23.10 (Solution)

(d) For $V(0) = 0$:

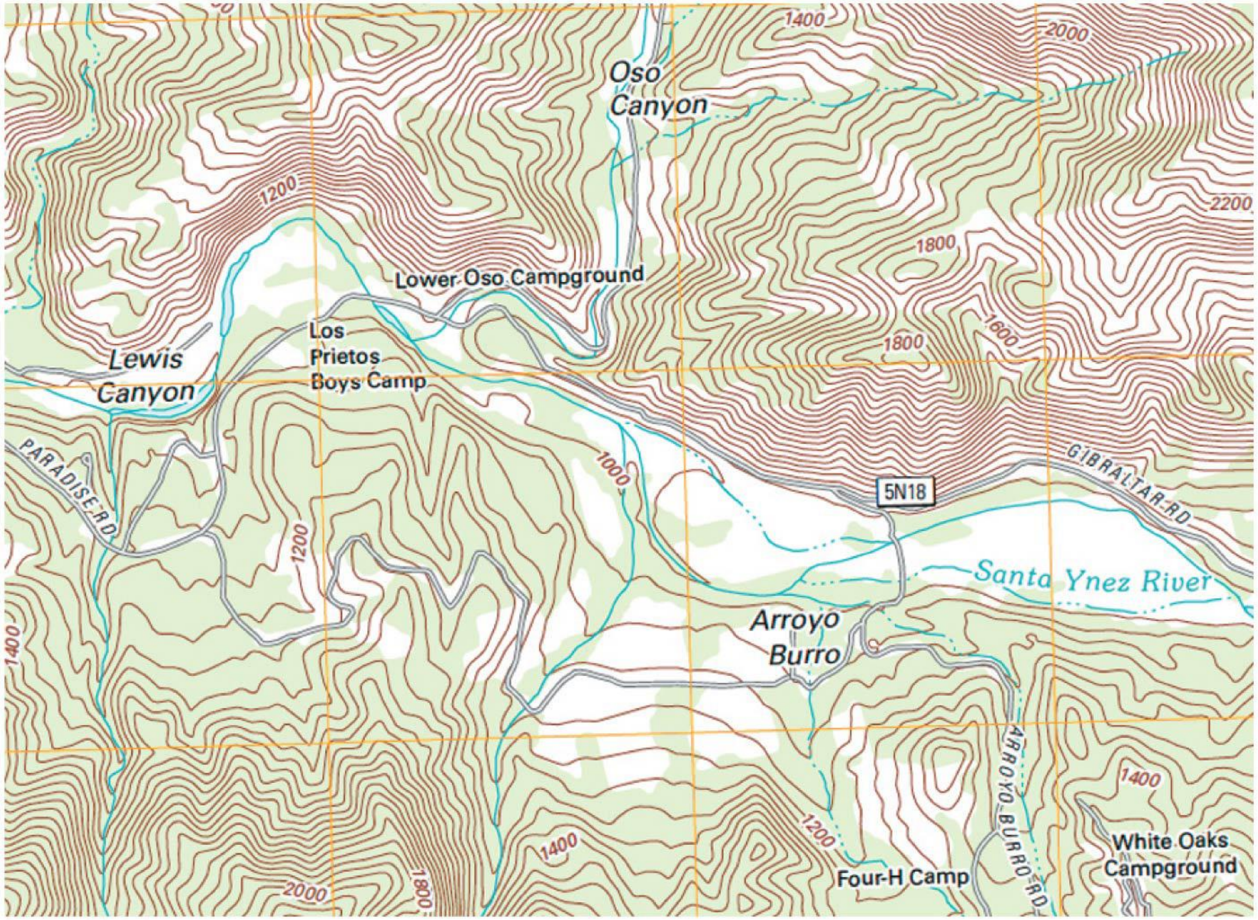
$$V(x) - 0 = -(2.00\,\mathrm{kV/m})(x - 0)$$

or

$$V(x) = \boxed{-(2.00\,\mathrm{kV/m})x}$$

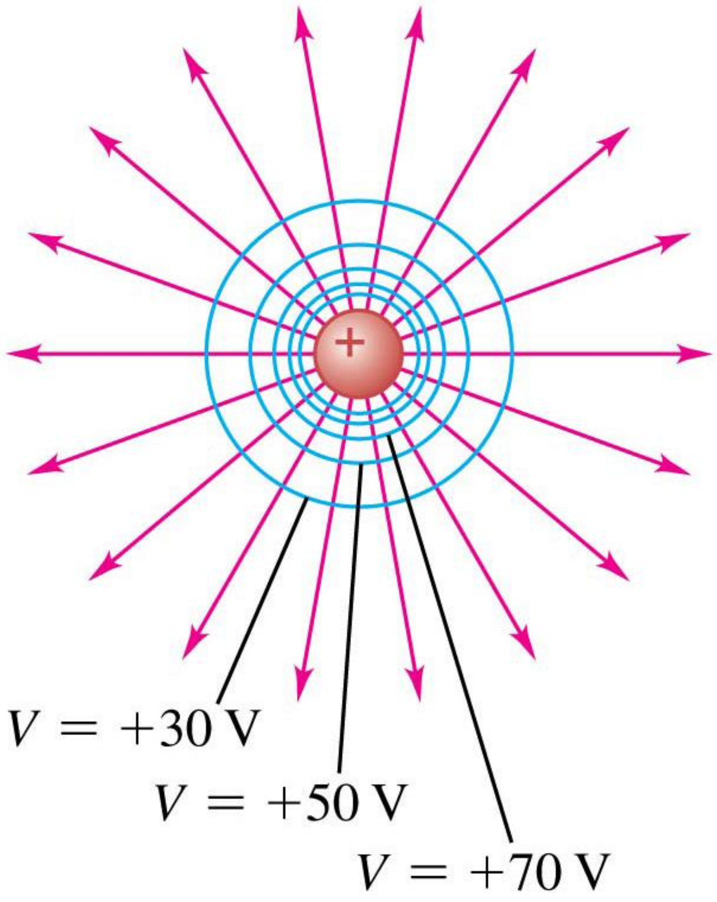
Equipotential surfaces

- Contour lines on a topographic map are curves of constant elevation and hence of constant gravitational potential energy.

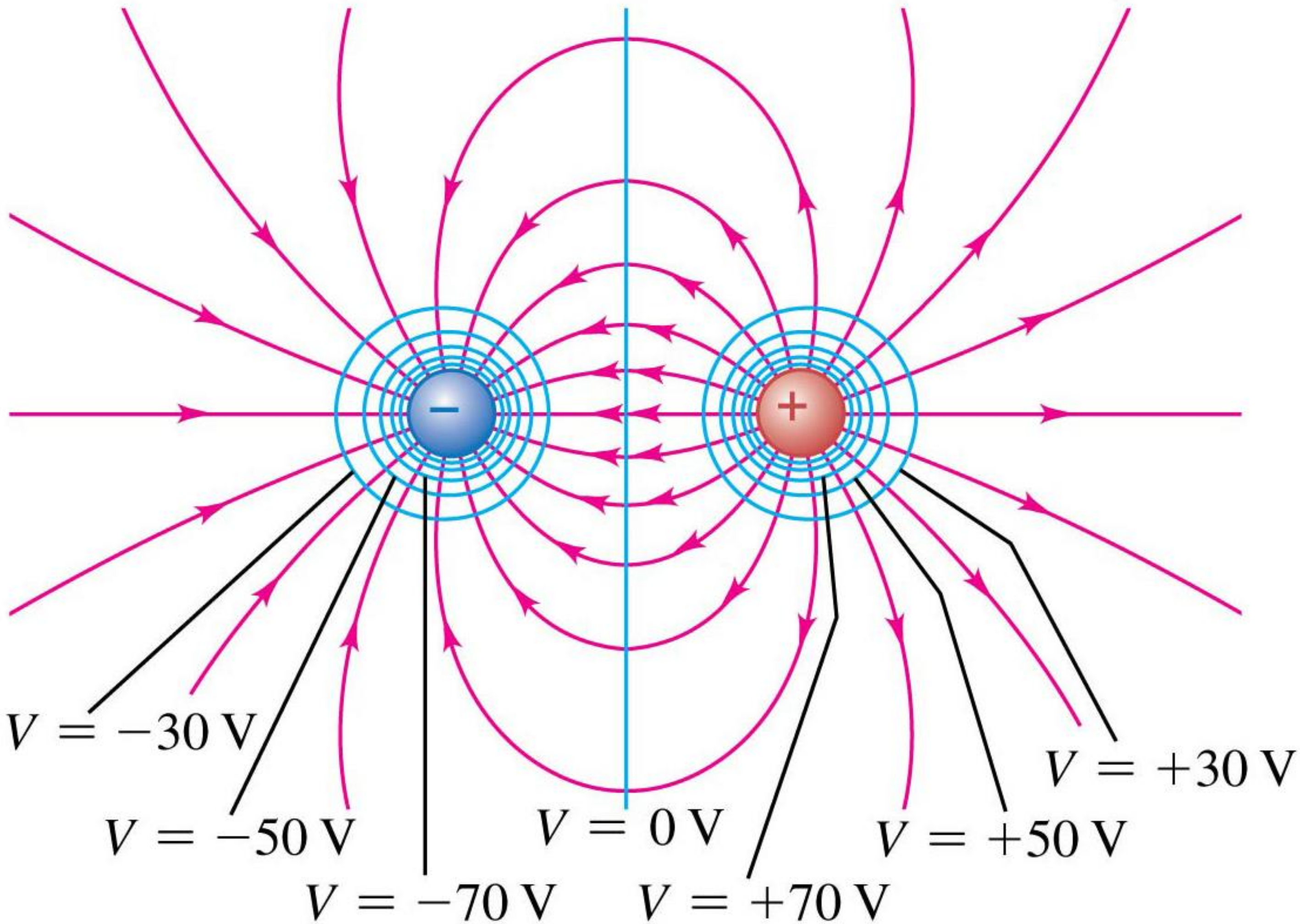


Equipotential surfaces and field lines

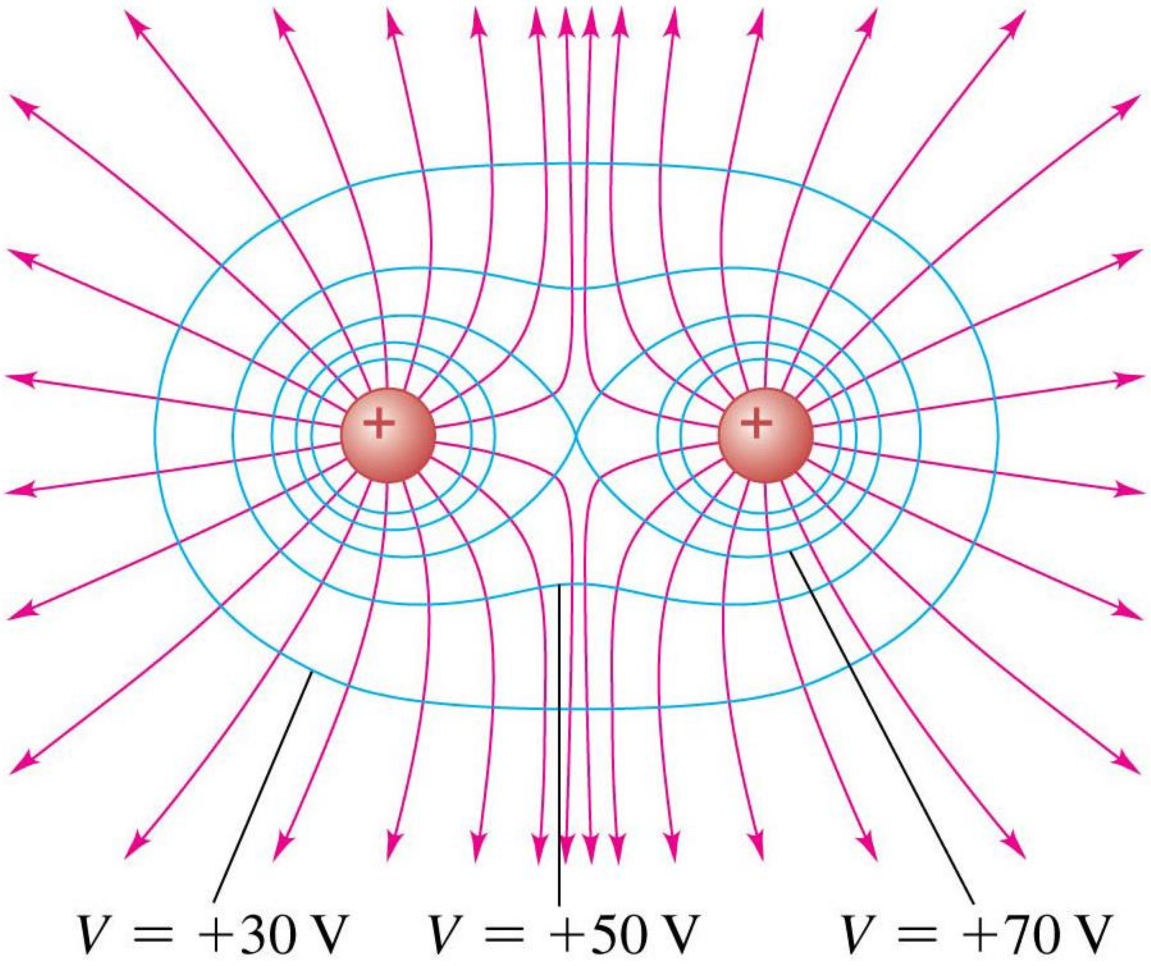
- An **equipotential surface** is a surface on which the electric potential is the same at every point.
- Field lines and equipotential surfaces are always mutually perpendicular.
- Shown are cross sections of equipotential surfaces (blue lines) and electric field lines (red lines) for a single positive charge.



Equipotential surfaces and field lines for a dipole



Field and potential of two equal positive charges



Example 23.11

A point particle that has a charge of +11.1 nC is at the origin. (*a*) What is (are) the shapes of the equipotential surfaces in the region around this charge? (*b*) Assuming the potential to be zero at $r = \infty$, calculate the radii of the five surfaces that have potentials equal to 20.0 V, 40.0 V, 60.0 V, 80.0 V and 100.0 V, and sketch them to scale centered on the charge. (*c*) Are these surfaces equally spaced? Explain your answer.

Example 23.11 (Solution)

- **Picture the Problem** We can integrate the expression for the electric field due to a point charge to find an expression for the electric potential of the point particle.

(*a*) The equipotential surfaces are spheres centered on the charge.

(*b*) From the relationship between the electric potential due to the point charge and the electric field of the point charge we have:

$$\int_a^b dV = -\int_{r_a}^{r_b} \vec{E} \cdot d\vec{r} = -kQ\int_{r_a}^{r_b} r^{-2}\,dr$$

or

$$V_b - V_a = kQ\left(\frac{1}{r_b} - \frac{1}{r_a}\right)$$

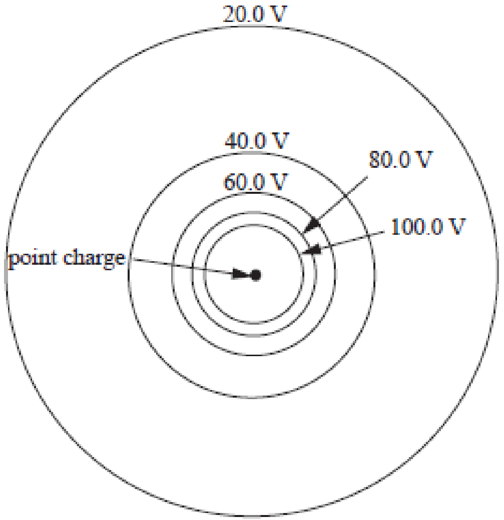
Taking the potential to be zero at $r_a = \infty$ yields:

$$V_b - 0 = kQ\left(\frac{1}{r_b}\right) \Rightarrow V = \frac{kQ}{r} \Rightarrow r = \frac{kQ}{V}$$

Example 23.11 (Solution)

Because $Q = +1.11 \times 10^{-8}$ C:

$$r = \frac{\left(8.988\times10^{9}\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\left(1.11\times10^{-8}\,\mathrm{C}\right)}{V} = \frac{99.77\,\frac{\mathrm{N\cdot m^2}}{\mathrm{C}}}{V} \qquad (1)$$



Use equation (1) to complete the following table:

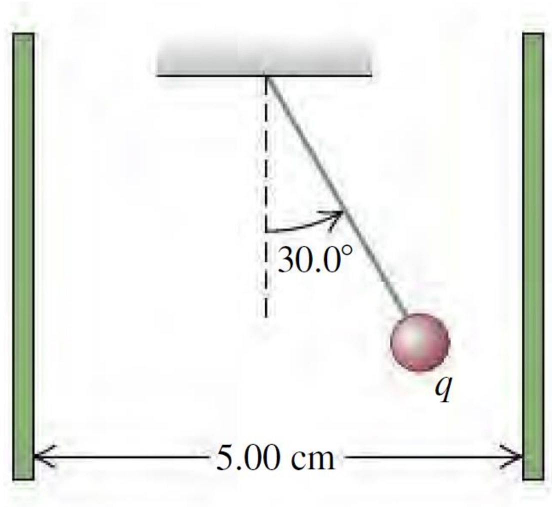
V (V)	20.0	40.0	60.0	80.0	100.0
r (m)	4.99	2.49	1.66	1.25	1.00

The equipotential surfaces are shown in cross-section to the right:

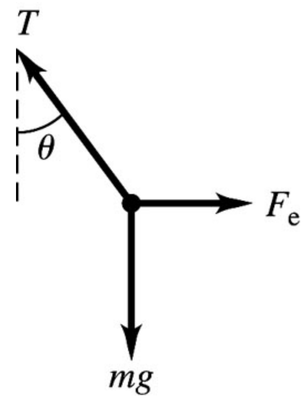
(*c*) No. The equipotential surfaces are closest together where the electric field strength is greatest.

Example 23.12

A small sphere with mass 1.50 g hangs by a thread between two very large parallel vertical plates 5.00 cm apart. The platesare insulating and have uniform surface charge densities $+\sigma$ and $-\sigma$. The charge on the sphere is $q = 8.90 \times 10^{-6}\ C$. What potential difference between the plates will cause the thread to assume an angle of 30.0 with the vertical?



Example 23.12



Identify: Apply $\sum F_x = 0$ and $\sum F_y = 0$ to the sphere. The electric force on the sphere is $F_e = qE$. The potential difference between the plates is $V = Ed$.

Set Up: The free-body diagram for the sphere is given in Figure 23.59.

Execute: $T\cos\theta = mg$ and $T\sin\theta = F_e$ gives

$$F_e = mg\tan\theta = (1.50\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)\tan(30°) = 0.0085\ \text{N}.$$

$$F_e = Eq = \frac{Vq}{d} \text{ and } V = \frac{Fd}{q} = \frac{(0.0085\ \text{N})(0.0500\ \text{m})}{8.90\times10^{-6}\ \text{C}} = 47.8\ \text{V}.$$

Evaluate: $E = V/d = 956\ \text{V/m}$. $E = \sigma/\varepsilon_0$ and $\sigma = E\varepsilon_0 = 8.46\times10^{-9}\ \text{C/m}^2$.

Potential gradient

- The components of the electric field can be found by taking partial derivatives of the electric potential:

Electric field components found from potential:

Each electric field component ...

$$E_x = -\underbrace{\frac{\partial V}{\partial x}} \qquad E_y = -\underbrace{\frac{\partial V}{\partial y}} \qquad E_z = -\underbrace{\frac{\partial V}{\partial z}}$$

... equals the negative of the corresponding partial derivative of electric potential function V.

- The electric field is the negative gradient of the potential:

$$\vec{\boldsymbol{E}} = -\vec{\nabla} V$$

Example 23.13

In a certain region of space, the electric potential is $V(x, y, z) = Axy - Bx^2 + Cy$, where A, B, and C are positive constants. (a) Calculate the x, y, and z components of the electric field. (b) At which points is the electric field equal to zero?

Use $E_x = -\frac{\partial V}{\partial x}$, $E_y = -\frac{\partial V}{\partial y}$, and $E_z = \frac{\partial V}{\partial z}$ to calculate the components of $\vec{\boldsymbol{E}}$.

EXECUTE: $V = Axy - Bx^2 + Cy$.

(a) $E_x = -\frac{\partial V}{\partial x} = -Ay + 2Bx$.

$$E_y = -\frac{\partial V}{\partial y} = -Ax - C.$$

$$E_z = \frac{\partial V}{\partial z} = 0.$$

(b) $E = 0$ requires that $E_x = E_y = E_z = 0$.

$E_z = 0$ everywhere.

$E_y = 0$ at $x = -C/A$.

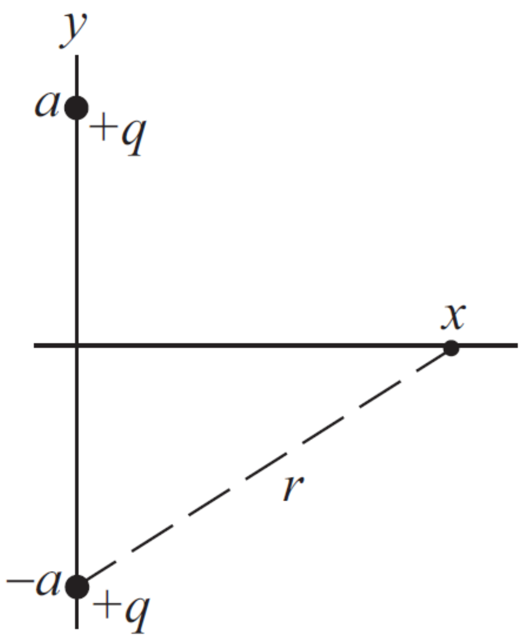
And E_x is also equal to zero for this x, any value of z and $y = 2Bx/A = (2B/A)(-C/A) = -2BC/A^2$.

EVALUATE: V doesn't depend on z so $E_z = 0$ everywhere.

Example 23.14

Two positive point charges, each have a charge of +*q*, and are fixed on the *y*-axis at *y* = +*a* and *y* = –*a*.

- (*a*) Find the electric potential at any point on the *x*-axis.
- (*b*) Use your result in Part (*a*) to find the electric field at any point on the *x*-axis.



The potential *V* at any point on the *x* axis is the sum of the Coulomb potentials due to the two point charges. Once we have found *V*, we can use

$$\vec{E}(x) = -\frac{\partial V_x}{\partial x}\hat{\imath}$$

to find the electric field at any point on the *x* axis.

Example 23.14 (Solution)

(a) Express the potential due to a system of point charges:

$$V = \sum_i \frac{kq_i}{r_i}$$

Substitute to obtain:

$$V(x) = V_{\text{charge at }+a} + V_{\text{charge at }-a}$$

$$= \frac{kq}{\sqrt{x^2 + a^2}} + \frac{kq}{\sqrt{x^2 + a^2}}$$

$$= \boxed{\frac{2kq}{\sqrt{x^2 + a^2}}}$$

(b) The electric field at any point on the x axis is given by:

$$\vec{E}(x) = -\frac{\partial V_x}{\partial x}\hat{\boldsymbol{i}} = -\frac{d}{dx}\left[\frac{2kq}{\sqrt{x^2 + a^2}}\right]\hat{\boldsymbol{i}}$$

$$= \boxed{\frac{2kqx}{\left(x^2 + a^2\right)^{3/2}}\hat{\boldsymbol{i}}}$$

- Extra Solved Problems

Extra Example 1

A point charge q_1 = 4.00 nC is placed at the origin, and a second point charge q_2 = -3.00 nC is placed on the x-axis at x = +20.0 cm. A third point charge q_3 = 2.00 nC is to be placed on the x-axis between q_1 and q_2 . (Take as zero the potential energy of the three charges when they are infinitely far apart.) (a) What is the potential energy of the system of the three charges if q_3 is placed at x = +10.0 cm? (b) Where should q_3 be placed to make the potential energy of the system equal to zero?

Extra Example 1 (Solution)

IDENTIFY: $U = k\left(\dfrac{q_1q_2}{r_{12}} + \dfrac{q_1q_3}{r_{13}} + \dfrac{q_2q_3}{r_{23}}\right)$.

SET UP: In part (a), $r_{12} = 0.200\text{ m}$, $r_{23} = 0.100\text{ m}$ and $r_{13} = 0.100\text{ m}$. In part (b) let particle 3 have coordinate x, so $r_{12} = 0.200\text{ m}$, $r_{13} = x$ and $r_{23} = 0.200\text{ m} - x$.

EXECUTE: **(a)** $U = k\left(\dfrac{(4.00\text{ nC})(-3.00\text{ nC})}{(0.200\text{ m})} + \dfrac{(4.00\text{ nC})(2.00\text{ nC})}{(0.100\text{ m})} + \dfrac{(-3.00\text{ nC})(2.00\text{ nC})}{(0.100\text{ m})}\right) = -3.60\times10^{-7}\text{ J}.$

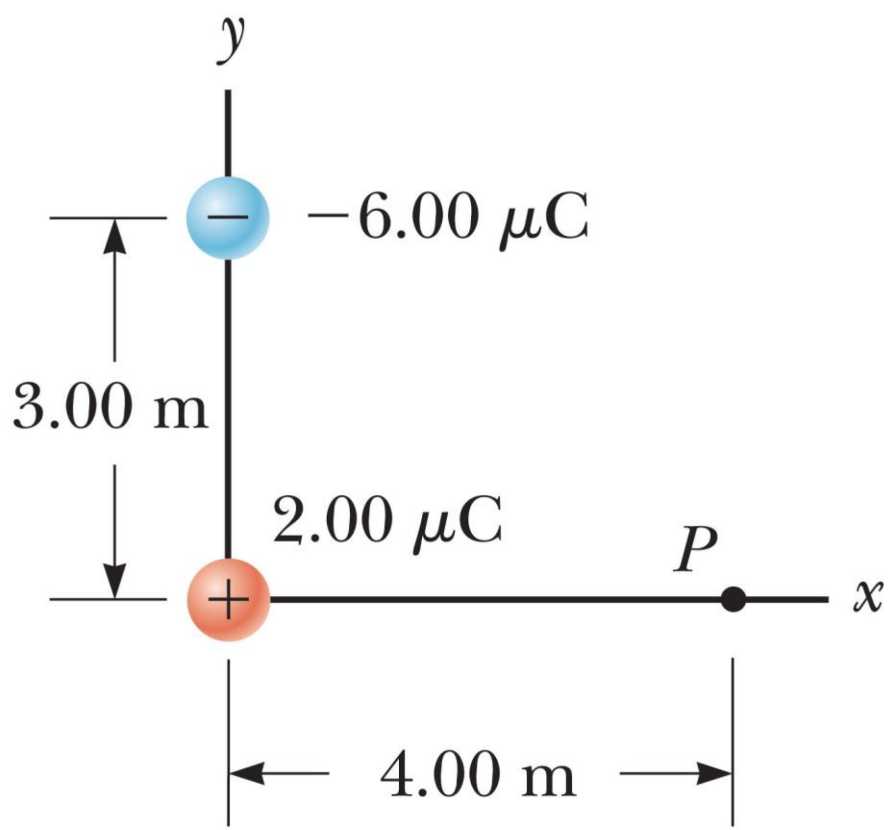
(b) If $U = 0$, then $0 = k\left(\dfrac{q_1q_2}{r_{12}} + \dfrac{q_1q_3}{x} + \dfrac{q_2q_3}{r_{12} - x}\right)$. Solving for x we find:

$0 = -60 + \dfrac{8}{x} - \dfrac{6}{0.2 - x} \Rightarrow 60x^2 - 26x + 1.6 = 0 \Rightarrow x = 0.074\text{ m}, 0.360\text{ m}$. Therefore, $x = 0.074\text{ m}$ since it is the only value between the two charges.

EVALUATE: U_{13} is positive and both U_{23} and U_{12} are negative. If $U = 0$, then $|U_{13}| = |U_{23}| + |U_{12}|$. For $x = 0.074\text{ m}$, $U_{13} = +9.7\times10^{-7}\text{ J}$, $U_{23} = -4.3\times10^{-7}\text{ J}$ and $U_{12} = -5.4\times10^{-7}\text{ J}$. It is true that $U = 0$ at this x.

Extra Example 2: The Electric Potential Due to Two Point Charges

A charge q_1 = 2.00 C is located at the origin and a charge q_2 = 26.00 C is located at (0, 3.00) m.



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Extra Example 2: The Electric Potential Due to Two Point Charges

(A) Find the total electric potential due to these charges at the point *P*, whose coordinates are (4.00, 0) m.

– Find the potential:

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

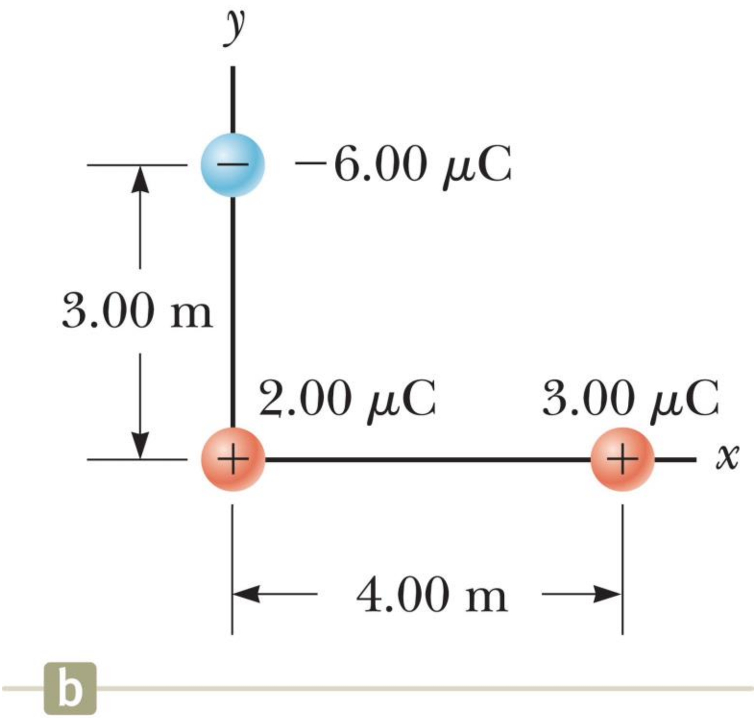
– Substitute numeri

$$V_P = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})\left(\frac{2.00 \times 10^{-6}\ \mathrm{C}}{4.00\ \mathrm{m}} + \frac{-6.00 \times 10^{-6}\ \mathrm{C}}{5.00\ \mathrm{m}}\right)$$

$$= \boxed{-6.29 \times 10^3\ \mathrm{V}}$$

Extra Example 2: The Electric Potential Due to Two Point Charges

(B) Find the change in potential energy of the system of two charges plus a third charge $q_3 = 3.00$ C as the latter charge moves from infinity to point *P*.



Extra Example 2: The Electric Potential Due to Two Point Charges

– Evaluate the potential energy for the configuration in which the charge is at *P*:

$$U_f = q_3 V_P$$

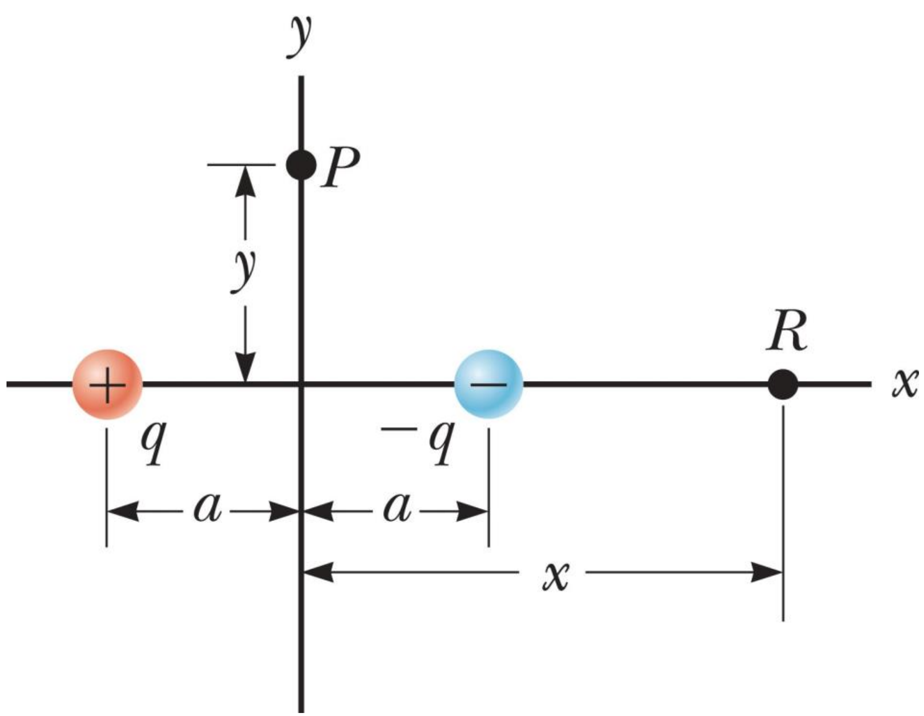
– Substitute numerical values to evaluate ΔU:

$$\Delta U = U_f - U_i = q_3 V_P - 0 = (3.00 \times 10^{-6}\ \mathrm{C})(-6.29 \times 10^{3}\ \mathrm{V})$$

$$= \boxed{-1.89 \times 10^{-2}\ \mathrm{J}}$$

Extra Example 3: The Electric Potential Due to a Dipole

An electric dipole consists of two charges of equal magnitude and opposite sign separated by a distance $2a$. The dipole is along the x axis and is centered at the origin.



Extra Example 3: The Electric Potential Due to a Dipole

(A) Calculate the electric potential at point *P* on the *y* axis

- Find the electric potential at *P* due to the two charges:

$$V_P = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{q}{\sqrt{a^2 + y^2}} + \frac{-q}{\sqrt{a^2 + y^2}}\right) = \boxed{0}$$

Extra Example 3: The Electric Potential Due to a Dipole

(B) Calculate the electric potential at point *R* on the positive *x* axis

- Find the electric potential at *R* due to the two charges:

$$V_R = k_e \sum_i \frac{q_i}{r_i} = k_e\left(\frac{-q}{x-a} + \frac{q}{x+a}\right) = \boxed{-\frac{2k_e q a}{x^2 - a^2}}$$

Extra Example 3: The Electric Potential Due to a Dipole

(C) Calculate V and E_x at a point on the x axis far from the dipole

- For point R far from the dipole such that $x >> a$, neglect a^2 in the denominator of the answer to part (B) and write V in this limit:

$$V_R = \lim_{x >> a} \left(-\frac{2k_e qa}{x^2 - a^2} \right) \approx \boxed{-\frac{2k_e qa}{x^2} \quad (x >> a)}$$

Extra Example 3: The Electric Potential Due to a Dipole

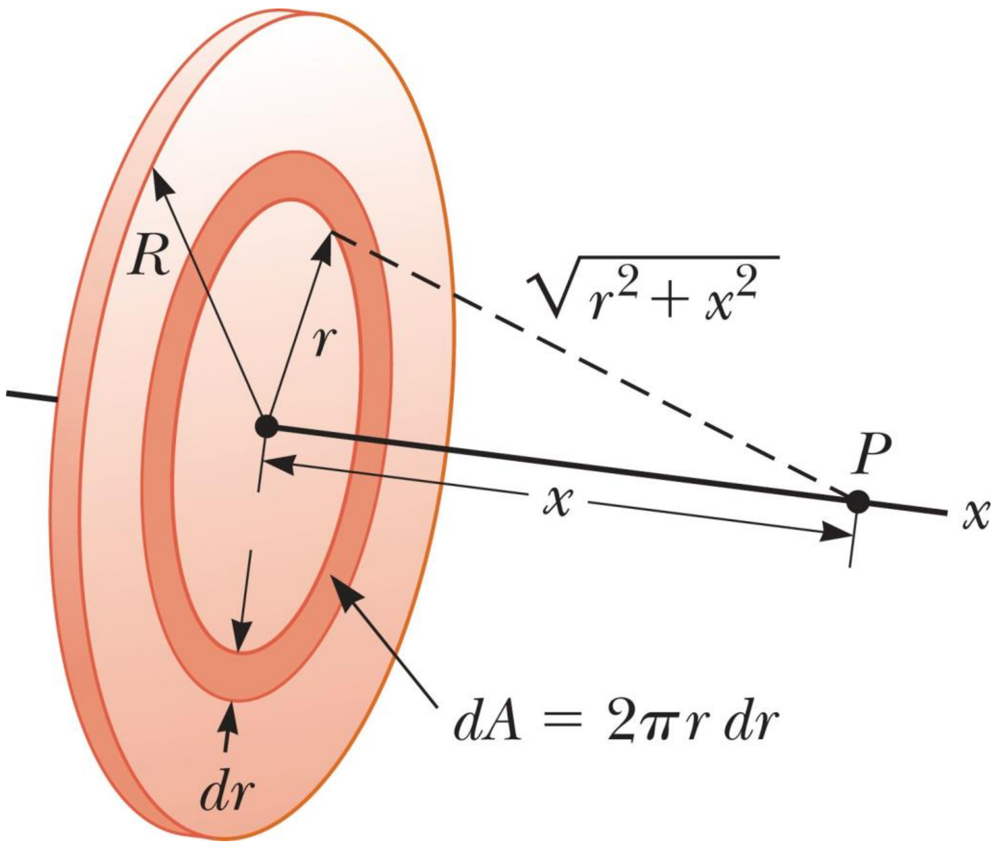
- Calculate the x component of the electric field at a point on the x axis far from the dipole:

$$E_x = -\frac{dV}{dx} = -\frac{d}{dx}\left(-\frac{2k_e qa}{x^2}\right)$$

$$= 2k_e qa \frac{d}{dx}\left(\frac{1}{x^2}\right) = \boxed{-\frac{4k_e qa}{x^3} \quad (x \gg a)}$$

Extra Example 4: Electric Potential Due to a Uniformly Charged Disk

A uniformly charged disk has radius R and surface charge density σ.



(A) Find the electric potential at a point *P* along the perpendicular central axis of the disk

- Find the amount of charge *dq* on a ring of radius *r* and width *dr*:

- Finc

$$dq = \sigma \, dA = \sigma(2\pi r \, dr) = 2\pi\sigma r \, dr$$

$$dV = \frac{k_e \, dq}{\sqrt{r^2 + x^2}} = \frac{k_e 2\pi\sigma r \, dr}{\sqrt{r^2 + x^2}}$$

Extra Example 4: Electric Potential Due to a Uniformly Charged Disk

– Integrate over the limits $r = 0$ to $r = R$:

$$V = \pi k_e \sigma \int_0^R \frac{2r\,dr}{\sqrt{r^2 + x^2}} = \pi k_e \sigma \int_0^R (r^2 + x^2)^{-1/2}\, 2r\,dr$$

– Evaluate the integral:

$$(1) \quad V = \boxed{2\pi k_e \sigma [(R^2 + x^2)^{1/2} - x]}$$

Extra Example 4: Electric Potential Due to a Uniformly Charged Disk

(B) Find the *x* component of the electric field at a point *P* along the perpendicular central axis of the disk

- Find the electric field at any axial point:

$$(2) \quad E_x = -\frac{dV}{dx} = \boxed{2\pi k_e \sigma \left[1 - \frac{x}{(R^2 + x^2)^{1/2}} \right]}$$