1. A region of space contains an electric field $\overrightarrow{E} = E_1\widehat{i} + E_2\widehat{j}$ where E_1 and E_2 are positive constants. A frame whose corners are located at (x, y, z) = ($\frac{a}{2}$, 0, $\frac{a}{2}$), ($\frac{-a}{2}$, 0,$\frac{-a}{2}$), ($\frac{a}{2}$, 0,$\frac{-a}{2}$), and ($\frac{-a}{2}$, 0, $\frac{a}{2}$). What is the magnitude of the electric flux through the frame?

$$\overrightarrow{E} = E_1\widehat{i} + E_2\widehat{j}$$

$$A = \ (\frac{a}{2}, 0, \frac{a}{2}),\ B = \ (-\frac{a}{2}, 0, -\frac{a}{2}),\ C = \ (\frac{a}{2}, 0, -\frac{a}{2}),\ D = \ (-\frac{a}{2}, 0, \frac{a}{2})$$

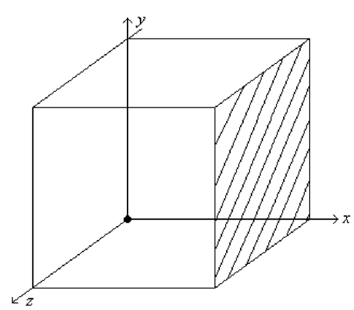
$$\text{Area of rectangle: } (L \cdot W)\widehat{j}$$

$$\ldots$$

$$\begin{aligned}
\phi_E &= \overrightarrow{E} \cdot \overrightarrow{A} \\
&= (E_1\widehat{i} + E_2\widehat{j}) \cdot a^2(\pm\widehat{j}) \\
&= E_1 a^2(\widehat{i} \cdot (\pm\widehat{j})) + E_2 a^2(\widehat{j} \cdot (\pm\widehat{j})) \\
&= 0 + E_2 a^2(\pm 1) \\
&= \ \pm E_2 a^2 \\
\widehat{i}\widehat{j} &= \widehat{i}(-\widehat{j}) = 0 \\
\widehat{j}\widehat{j} &= 1 \cdot \widehat{j}(-\widehat{j}) = -1
\end{aligned}$$

$$|\phi_E| = |\pm E_2 a^2| = \boxed{E_2 a^2}$$

FIGURE 22-2



2. A uniform electric field with a magnitude of $6 \times 10^6 \frac{N}{C}$ is applied to a cube of edge length 0.1 m as shown in Fig. 22-2. If the direction of the E-field is along the +x-axis, what is the electric flux passing through the shaded face of the cube?

$$E = 6 \times 10^6 \frac{N}{C},\ L = 0.1m,$$

$$along\ xaxis,\ flux\ shaded\ side\ of\ cube$$

$$\begin{aligned}
\phi &= E \times A \\
&= (6 \times 10^6 \frac{N}{C}) \times (0.1)^2 \\
&= \boxed{6.0 \times 10^4 \frac{N}{C} \cdot m^2}
\end{aligned}$$

3. A spherical, non-conducting shell of inner radius $r_1 = 10$ cm and outer radius $r_2 = 15$ cm carries a total charge Q = 15 μC distributed uniformly throughout its volume. What is the electric field at a distance r = 12 cm from the center of the shell? (SHOW ALL YOUR WORK, DERIVE THE EXPRESSION N)

$$\begin{aligned} \mathcal{V}_{out} &= \frac{4\pi(15^3 - 10^3)}{3} \\ &= 9948cm^3 \\ \mathcal{V}_{in} &= \frac{4\pi(12^3 - 10^3)}{3} \\ &= 3049cm^3 \end{aligned}$$

$$\begin{aligned} \rho = \frac{Q}{\mathcal{V}} &= \frac{15 \times 10^{-6}}{9.9484 \times 10^3} \\ &= 1.5078 \times 10^{-9} C/cm^3 \\ Q_{in} = \rho \times \mathcal{V}_{in} &= (1.5078 \times 10^{-9} C/cm^3) \times (3049cm^3) \\ &= 4.59 \times 10^{-6} C \end{aligned}$$

$$\begin{aligned} E &= \frac{kQ_{in}}{r_1}^2 \\ &= \frac{(9 \times 10^9 N \cdot m^2/C^2)(4.59 \times 10^{-6} C)}{0.12^2 m} \\ &= \boxed{2.87 \times 10^6 N/C} \end{aligned}$$

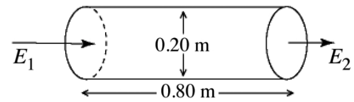
4. A nonconducting spherical shell of inner radius R_1 and outer radius R_2 contains a uniform volume charge density throughout the shell. Use Gauss's law to derive an equation for the magnitude of the electric field at the following radial distances r from the center of the sphere. Your answers should be in terms of ρ, R_1, R_2, r, ϵ_0 and π. (SHOW ALL YOUR WORK, DERIVE THE EXPRESSION N)

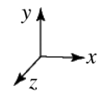
$$\begin{aligned} E &= \frac{q}{4\pi\epsilon_0 \cdot r^2} \\ a.\ \ r < R_1,\ E \cdot 4\pi \cdot r^2 &= \frac{q}{\epsilon_0} \\ q &= 0 \\ E &= \boxed{0 \frac{N}{C}} \end{aligned}$$

$$\begin{aligned} b.\ \ R_1 < r < R_2,\ E \cdot 4\pi \cdot r^2 &= \frac{\rho(\frac{4}{3}\pi r^3 - \frac{4}{3}\pi {R_1}^3)}{\epsilon_0} \\ E &= \frac{\rho(r^3 - {R_1}^3)}{3\epsilon_0 \cdot r} \\ &= \boxed{\frac{\rho}{3\epsilon_0} \cdot (r - \frac{{R_1}^3}{r^2})} \end{aligned}$$

$$\begin{aligned} c.\ \ r > R_2,\ E \cdot 4\pi \cdot r^2 &= \frac{\rho(\frac{4}{3}\pi {R_2}^3 - \frac{4}{3}\pi {R_1}^3)}{\epsilon_0} \\ E &= \boxed{\frac{\rho({R_2}^3 - {R_1}^3)}{3\epsilon_0 \cdot r^2}} \end{aligned}$$

5. A nonuniform electric field is directed along the x-axis at all points in space. This magnitude of the field varies with x, but not with respect to y or z. The axis of a cylindrical surface, 0.80 m long and 0.20 m in diameter, is aligned parallel to the x-axis, as shown in the figure. The electric fields E_1 and E_2, at the ends of the cylindrical surface, have magnitudes of 9000 $\frac{N}{C}$ and 5000 $\frac{N}{C}$ respectively, and are directed as shown. ($\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N} \cdot m^2$) The charge enclosed by the cylindrical surface is closest to





$$
\begin{aligned}
D = 0.2m,\ r = 0.1m,\ &\mathit{cross\ section} = \pi \cdot (0.1)^2 \\
\phi_1 &= \overrightarrow{E_1} \cdot \overrightarrow{A_1},\ \phi_2 = \overrightarrow{E_2} \cdot \overrightarrow{A_2} \\
\phi_1 &= 9000 \times \pi \times 10^{-2} \\
&= 282.74 \frac{N}{C} \cdot m^2 \\
\phi_2 &= 5000 \times \pi \times 10^{-2} \\
&= 157.08 \frac{N}{C} \cdot m^2 \\
\phi_{net} &= \phi_2 - \phi_1 \\
&= 157.08 - 282.75 \\
&= -125.67 \\
&\cong \boxed{-1.1nC}
\end{aligned}
$$