CV Cheatsheet

Rigid Body Motion

	Mat.	Quat.	Hom. Mat.
T	R t	q t	$ \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix} $
T^{-1}	$R^T - R^T t$	$ q^{-1} - q^{-1}t$	$\begin{bmatrix} R^T & -R^T t \\ 0 & 1 \end{bmatrix}$

 $R \in SO(3)$: 3×3 rotation matrix

 $q: 4 \times 1$ quaternion

 $t: 3 \times 1$ translation vector

Transformations between Coordinate Systems

How to transform a point cloud given in one coordinate system to its coordinates i another coordinate system ?:

$$P_{CCS2} = (T_{VCS}^{CCS2})^{-1}(P_{VCS})$$

$$P_{VCS} = (T_{VCS}^{CCS1})(P_{CCS1})$$

$$P_{CCS2} = (T_{VCS}^{CCS2})^{-1}((T_{VCS}^{CCS1})(P_{CCS1}))$$

Coordinate Transformations

Rotation Matrix, translation vector:

$$X_2 = R_0^2 X_0 + t_0^2$$

$$X_1 = R_0^1 X_0 + t_0^1$$

Quaternion, translation vector:

$$X_2 = q_0^2 X_0 + t_0^2$$

$$X_1 = q_0^1 X_0 + t_0^1$$

Homogeneous coordinates:

$$X_2 = M_0^2 \hat{X}_0$$

$$X_1 = M_0^1 \hat{X}_1$$

Relative transformation:

$$X_2 = T_0^2((T_0^1)^{-1}(X_1))$$

$$T_1^2 = T_0^2 (T_0^1)^{-1}$$

Rotation matrix, translation vector:

$$X_2 = R_0^2((R_0^1)^T X_1 - (R_0^1)^T t_0^1) + t_0^2$$

Quaternion, translation vector:

$$X_2 = q_0^2((q_0^1)^{-1}X_1 - (q_0^1)^{-1}t_0^2) + t_0^2$$

Homogeneous coordinates:

$$M_1^2 = M_0^2 (M_0^1)^{-1}$$

	Mat.	Quat.	Hom. Mat.
T_0^2	$R_0^2 t_0^2$	$q_0^2 t_0^2$	M
T_0^1	$R_0^1 t_0^1$	$q_0^1 t_0^1$	\overline{M}
T_1^2	$R_0^2(R_0^1)^T - R_0^2(R_0^1)^T t_0^1 + t_0^2$	$ q_0^2((q_0^1)^{-1} - (q_0^1)^{-1}t_0^2) + t_0^2$	$M_0^2(M_0^1)^{-1}$

 $T_i^j(X)$: rigid body motion from i to j

 $R \in SO(3)$: 3×3 rotation matrix

 $q: 4 \times 1$ quaternion