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Stage Rotation Position Correction

Overview

In order to perform the calibration of the relationship between the beam deflection angle (2× the specimen deflection angle) and the beam position on the detector, it necessary to be able to rotate the optical assembly about a point which is offset from the stage rotation centre (the pin).

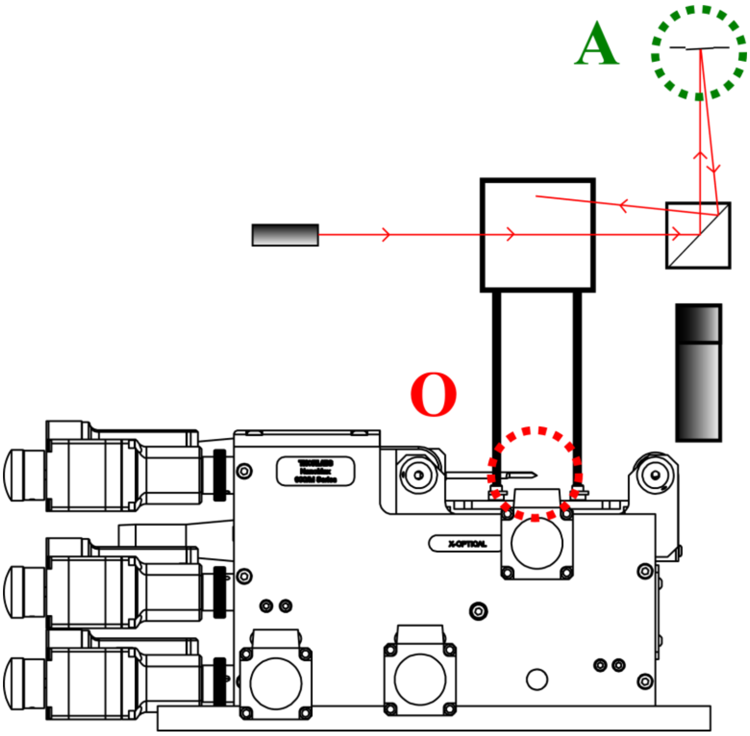

Figure 1. Schematic of stage and Thorlabs optical assembly. The stage rotation centre is at the tip of the pin (highlighted in red).

The optics of the UFR are mounted on a Thorlabs 6-axis stage (model number XXX) (**Error! Reference source not found.**). Translation of the stage is possible along the three orthogonal directions (x, y, and z), as well as rotation about these axes; θ_x (roll), θ_y (pitch), and θ_z (yaw). The rotations are centred on a common point, **O**, located at the tip of the pin attached to the stage body. The desired rotation point, **A**, has a fixed offset in x, y, and z from the stage rotation centre, **O**.

Theory

Co-ordinate system

The x, y, and z axes are defined by the motor axes, and translate the rotation centre **O**. These axes of movement are unaffected by the tilt angle of the stage. – VERIFY THIS

Desired object rotation

The calibration of the laser deflection requires the stage (and attached optics) to rotate about a point coincident with the lower face of the specimen, **A**, as shown in figure 2(a).

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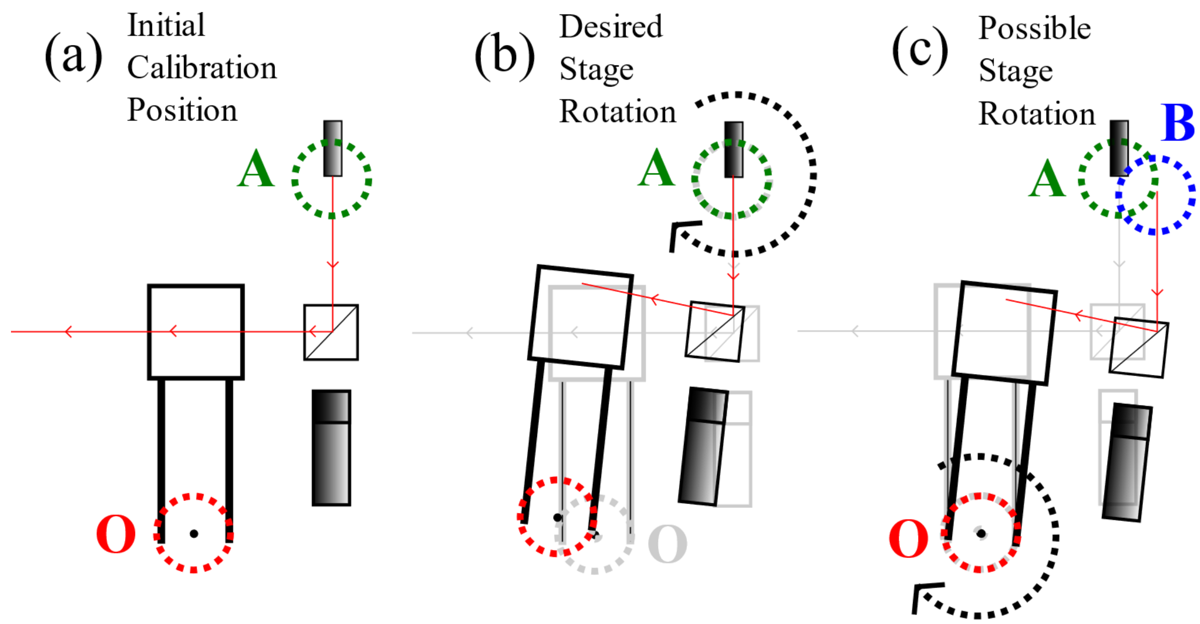

Figure 2. Desired stage rotation about specimen position

Issue with offset stage rotation centre

To apply the desired rotation angle at point **A**, stage rotations θ_x, θ_y, and θ_z are applied. This has the effect of translating point **A** by an amount which depends on the rotation angles and the offset $\overrightarrow{\mathbf{OA}}$. Let the position of the sample after movement be point **B** (figure 2(c)). To correct the rotation centre back to point **A**, a stage translation equal to the vector $\overrightarrow{\mathbf{BA}}$ must be applied.

Vector Representation

It is therefore necessary to calculate vector **B** in terms of vector **A** and the rotation matrix $\boldsymbol{R}_{\mathrm{tot}}$. Vector **B** can be calculated by applying the rotation about the origin (**O**) to point **A**:

$$\boldsymbol{B} = \boldsymbol{R}\boldsymbol{A}$$

Where

$$\boldsymbol{A} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix}$$

The rotation matrix depends on the axis of rotation, as discussed below.

Rotation matrices

The general 3D rotation matrix for a rotation of angle θ about the unit vector passing through the origin (l, m, n) is as follows[1]:

$$R(l,m,n,\theta) = \begin{bmatrix} ll\ 1-\cos\theta\ +\cos\theta & ml\ 1-\cos\theta\ -n\sin\theta & nl(1-\cos\theta)+m\sin\theta \\ lm\ 1-\cos\theta\ +n\sin\theta & mm\ 1-\cos\theta\ +\cos\theta & nm\ 1-\cos\theta\ -l\sin\theta \\ ln\ 1-\cos\theta\ -m\sin\theta & mn\ 1-\cos\theta\ +l\sin\theta & nn\ 1-\cos\theta\ +\cos\theta \end{bmatrix}$$

[1] Szymanski, John E. (1989). Basic Mathematics for Electronic Engineers: Models and Applications. Taylor & Francis. p. 154. ISBN 0278000681.

For the cases of rotation about the x, y, and z axes, the equations simplify as follows:

x-axis	**y-axis**	**z-axis**
$l = 1, m = 0, n = 0$	$l = 0, m = 1, n = 0$	$l = 0, m = 0, n = 1$
$\boldsymbol{R}_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$	$\boldsymbol{R}_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$	$\boldsymbol{R}_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Calculation of translation

The desired stage translation can be found as:

$$\overrightarrow{\boldsymbol{BA}} = \boldsymbol{B} - \boldsymbol{A} \qquad [1]$$

$$= \boldsymbol{RA} - \boldsymbol{A} \qquad [2]$$

$$\overrightarrow{\boldsymbol{BA}} = \ \boldsymbol{R} - \boldsymbol{I} \ \boldsymbol{A} \qquad [3]$$

Terminology

Phrase	**Description**
UFR	Ultrasonic Fatigue Rig
Specimen deflection angle, α_{sample}	Deflection angle of fatigue cantilever
Beam deflection angle, θ_{beam}	Deflection angle of laser beam after reflection from cantilever surface. $\theta_{\text{beam}} = 2\alpha_{\text{sample}}$
Stage rotation centre, **O**	Defined by the metal pin. Acts as the origin for the cartesian co-ordinate system used.
Desired rotation centre, **A**	Typically the specimen surface.
Translated rotation centre, **B**	Position of **A** after application of the desired rotation angle θ
Desired rotation angle, θ	
Required translation, $\mathbf{T}_{\mathbf{BA}}$	Displacement of **A** from **B**. This translation is required to align the translated rotation centre (at **B**) back to point **A**

Implementation

Python Script

LabVIEW