Week 2 class 3 - Chaos and Fractals

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Today's goals

- Logistic Equation
- Derivation
- Graphical iteration practice
- Bifurcation diagram

Logistic equation

Logistic equation: $f(x)rx(1-x)$ where$r$ is the growth rate, and x is a fraction of the maximum population.

The logistic equation is used a lot in population ecology to model population variation through time.

There is a continuous variation of this equation (look this up).

Logistic Equation Derivation

$$f(P) = rP\left(1 - \frac{P}{A}\right) \qquad A = \text{"annihilation population"}$$

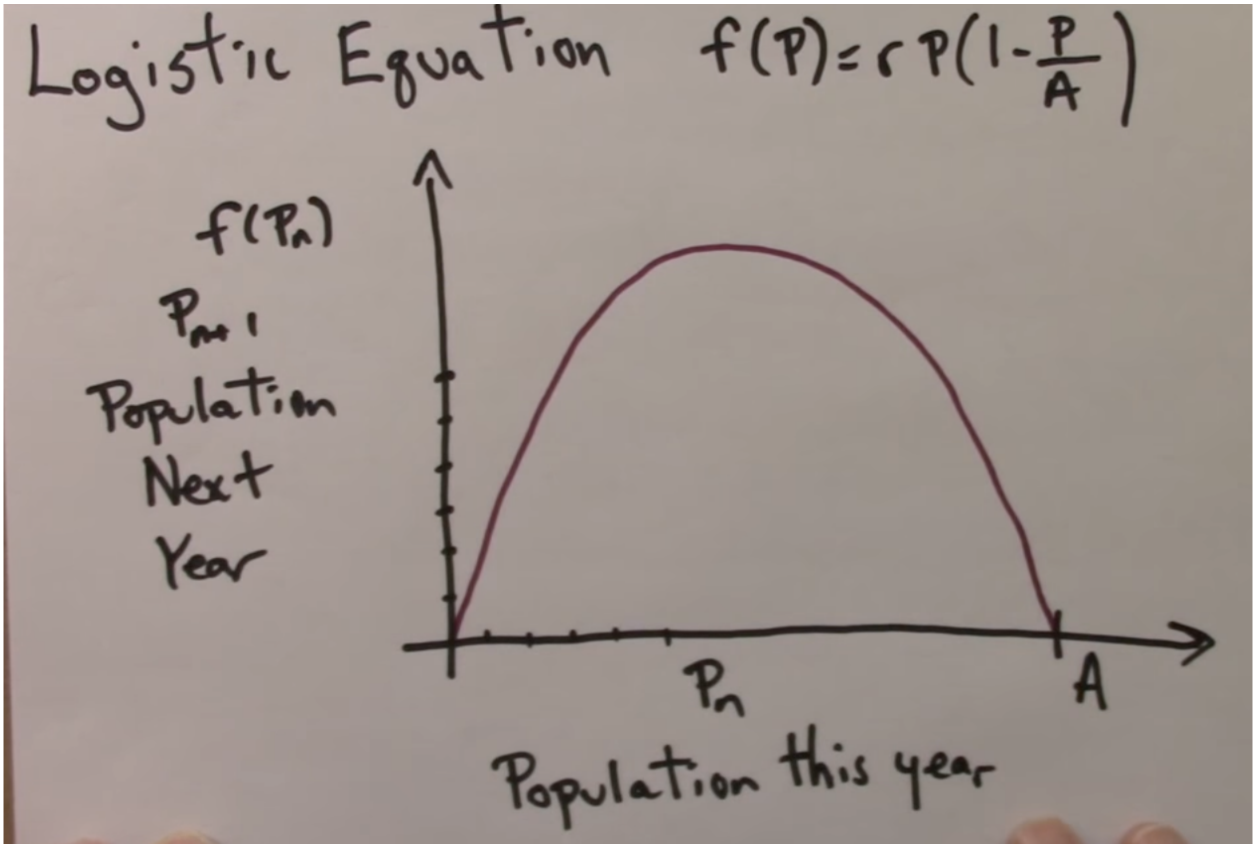
* If population = A, all rabbits die:

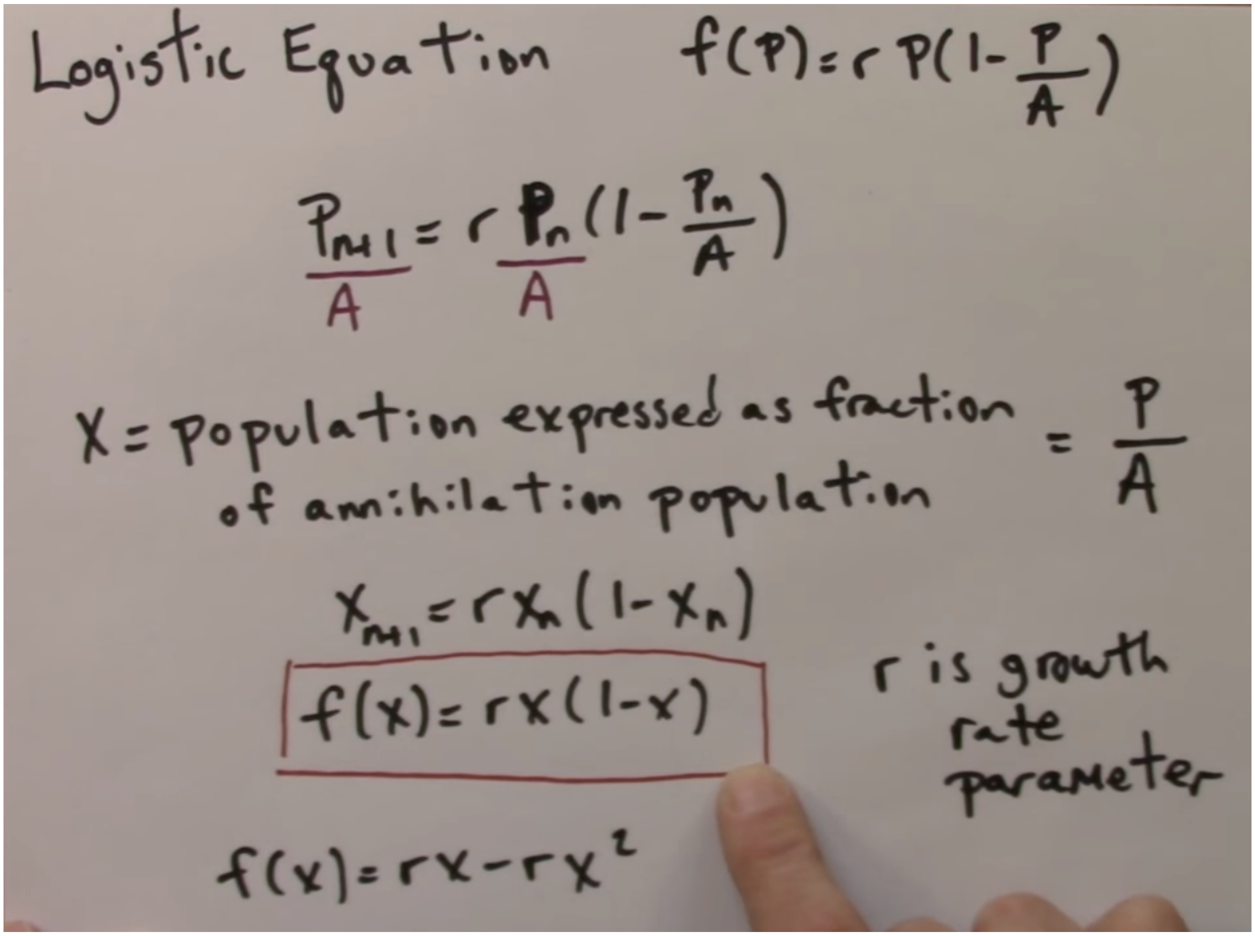
$$f(A) = rA\left(1 - \frac{A}{A}\right) = rA(1-1) = rA(0) = 0$$

* If population is small, $f(P) \simeq rP$

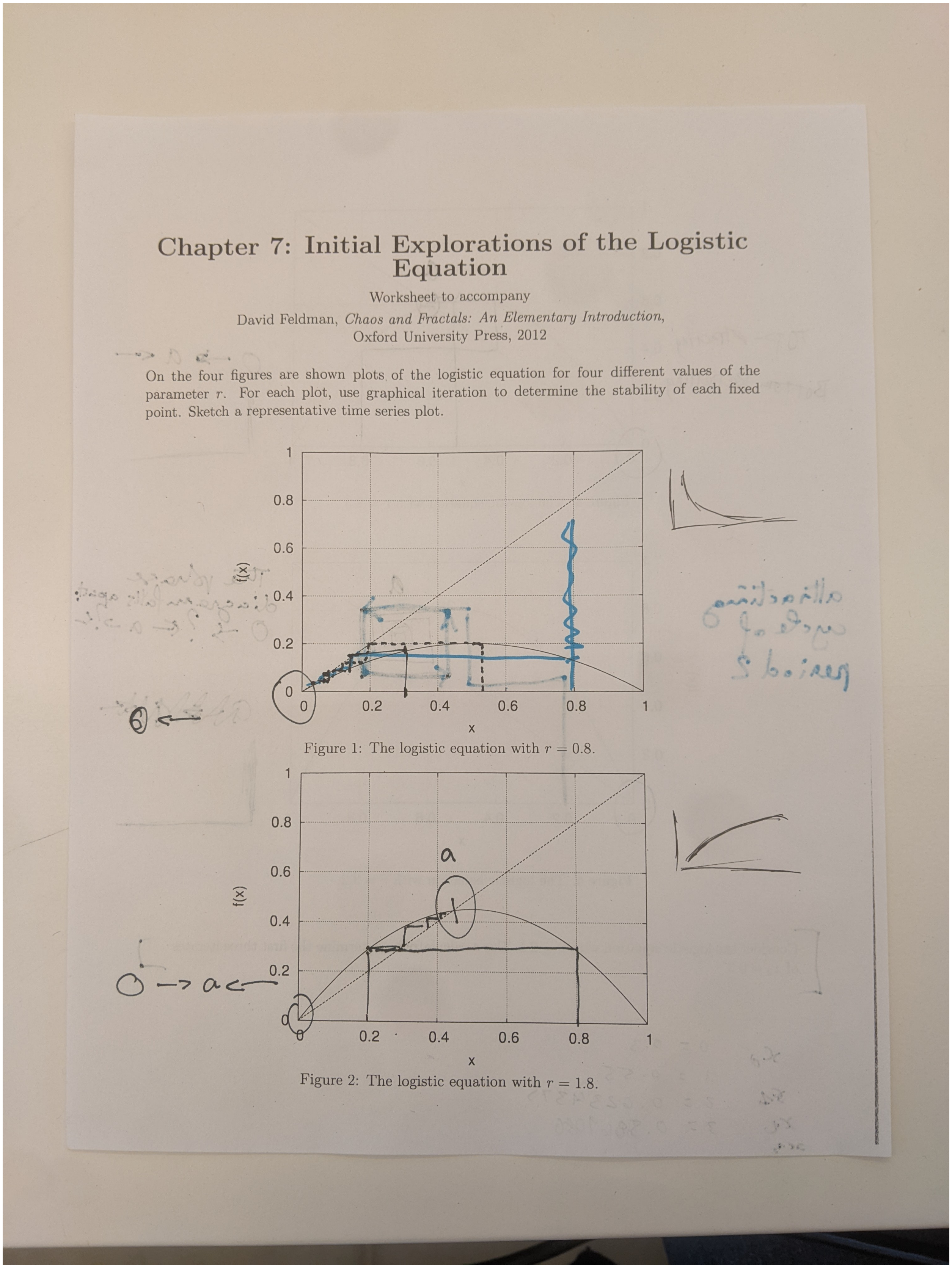
small: P is much less than A $\qquad \frac{P}{A} \simeq 0, \quad \left(1 - \frac{P}{A}\right) \simeq 1$

So $f(P) \simeq rP$


Logistic Equation $f(P) = rP(1 - \frac{P}{A})$
$f(P_n)$
P_{n+1}
Population Next Year
P_n
A
Population this year


Logistic Equation $f(P) = rP(1 - \frac{P}{A})$
$\frac{P_{n+1}}{A} = r\frac{P_n}{A}(1 - \frac{P_n}{A})$
X = population expressed as fraction of annihilation population $= \frac{P}{A}$
$X_{n+1} = rX_n(1 - X_n)$
$\boxed{f(x) = rx(1-x)}$
r is growth rate parameter
$f(x) = rx - rx^2$

Graphical iteration practice



Chapter 7: Initial Explorations of the Logistic Equation

Worksheet to accompany

David Feldman, *Chaos and Fractals: An Elementary Introduction*, Oxford University Press, 2012

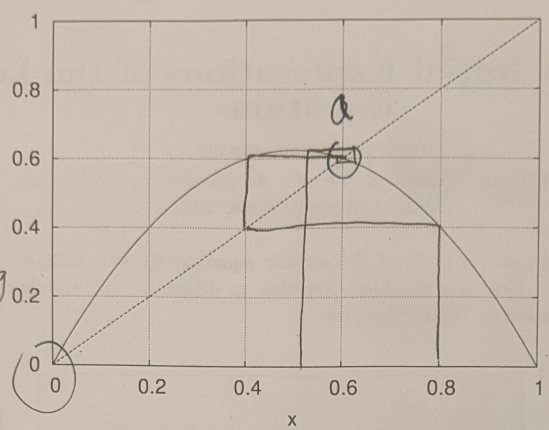
On the four figures are shown plots of the logistic equation for four different values of the parameter r. For each plot, use graphical iteration to determine the stability of each fixed point. Sketch a representative time series plot.

Figure 1: The logistic equation with $r = 0.8$.

Figure 2: The logistic equation with $r = 1.8$.

TOP- attracting

Bottom Repelling



0 → a ←

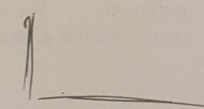
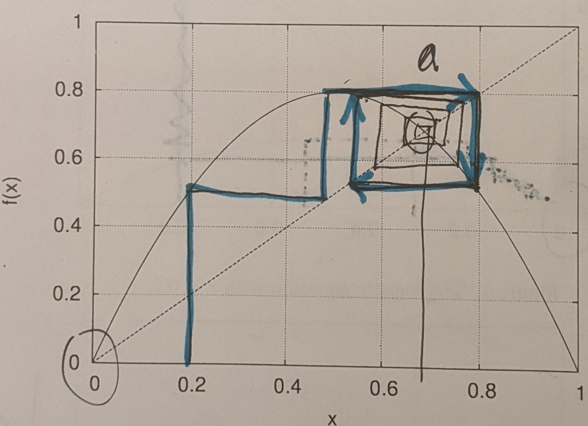


Figure 3: The logistic equation with $r = 2.5$.

attracting cycle of period 2



The phase diagram falls apart.

0 → ? ← a → ? ←

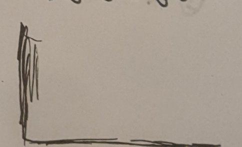


Figure 4: The logistic equation with $r = 3.2$.

[Consider the logistic equation with $r = 2.5$. Use a calculator to determine the first three iterates of $x_0 = 0.3$.]

x_0 0 = 0.3

1 = 0.55

x_1 2 = 0.6234375

x_2 3 = 0.5869080

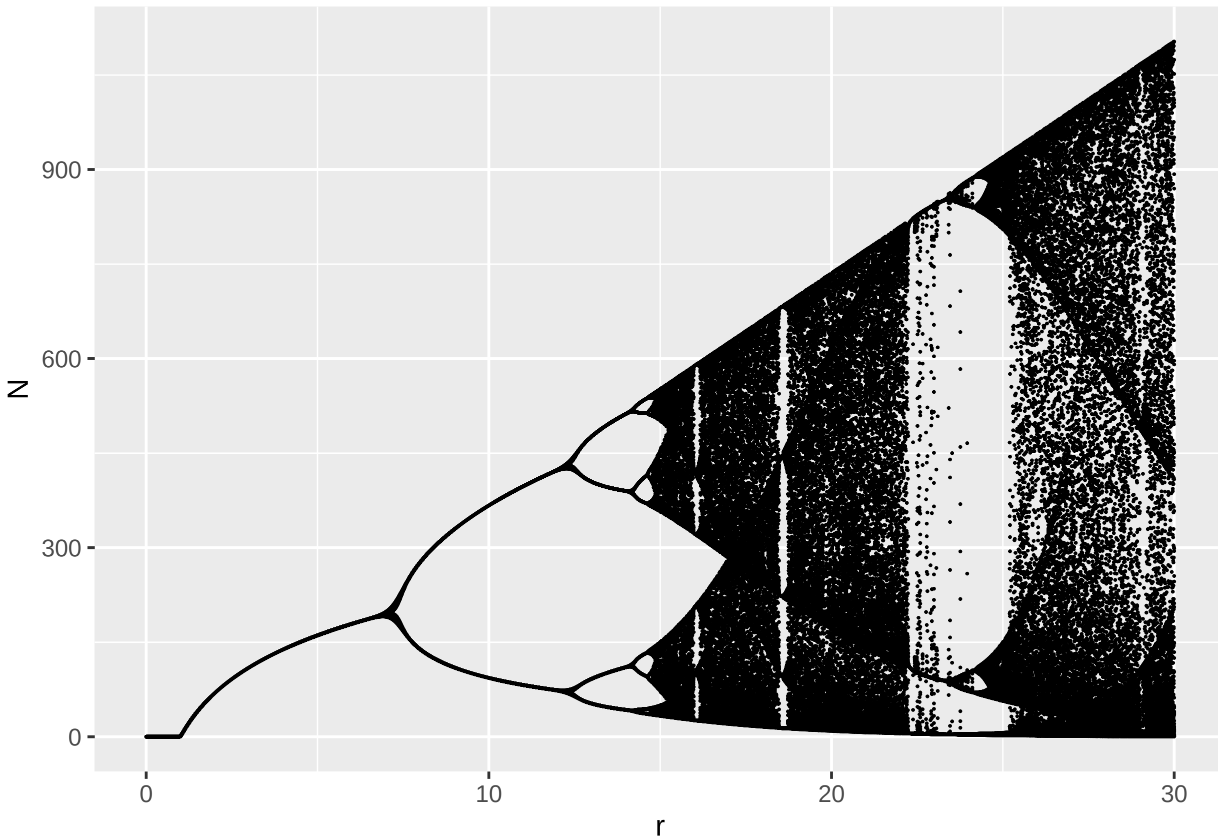
x_3

Bifurcation diagram

How might we represent the long term fate of the itinerary for an initial condition?

Let's imagine a plot with the parameter r as the x axis, and the long term fate of the itinerary of the logistic map as the y axis. What would this look like?

```
# Solution provided by Edmund Hart at https://rpubs.com/DistribEcology/880 based on the work o

library(ggplot2)
rmax <- 30
out.df <- matrix(NA, ncol = 2, nrow = 0)
a <- 0.01
r <- seq(0, rmax, by = 0.01)
n <- 100

for (z in 1:length(r)) {

    xl <- vector()
    xl[1] <- 10
    for (i in 2:n) {

        xl[i] <- xl[i - 1] * r[z] * exp(-a * xl[i - 1])

    }
    uval <- unique(xl[40:n])
    ### Here is where we can save the output for ggplot
    out.df <- rbind(out.df, cbind(rep(r[z], length(uval)), uval))
}
out.df <- as.data.frame(out.df)
colnames(out.df) <- c("r", "N")
ggplot(out.df, aes(x = r, y = N)) + geom_point(size = 0.001)
```



Wowzers!!!! That's so cool!