Week 2 class 2 - Chaos and Fractals (lab)

Phileas Dazeley-Gaist

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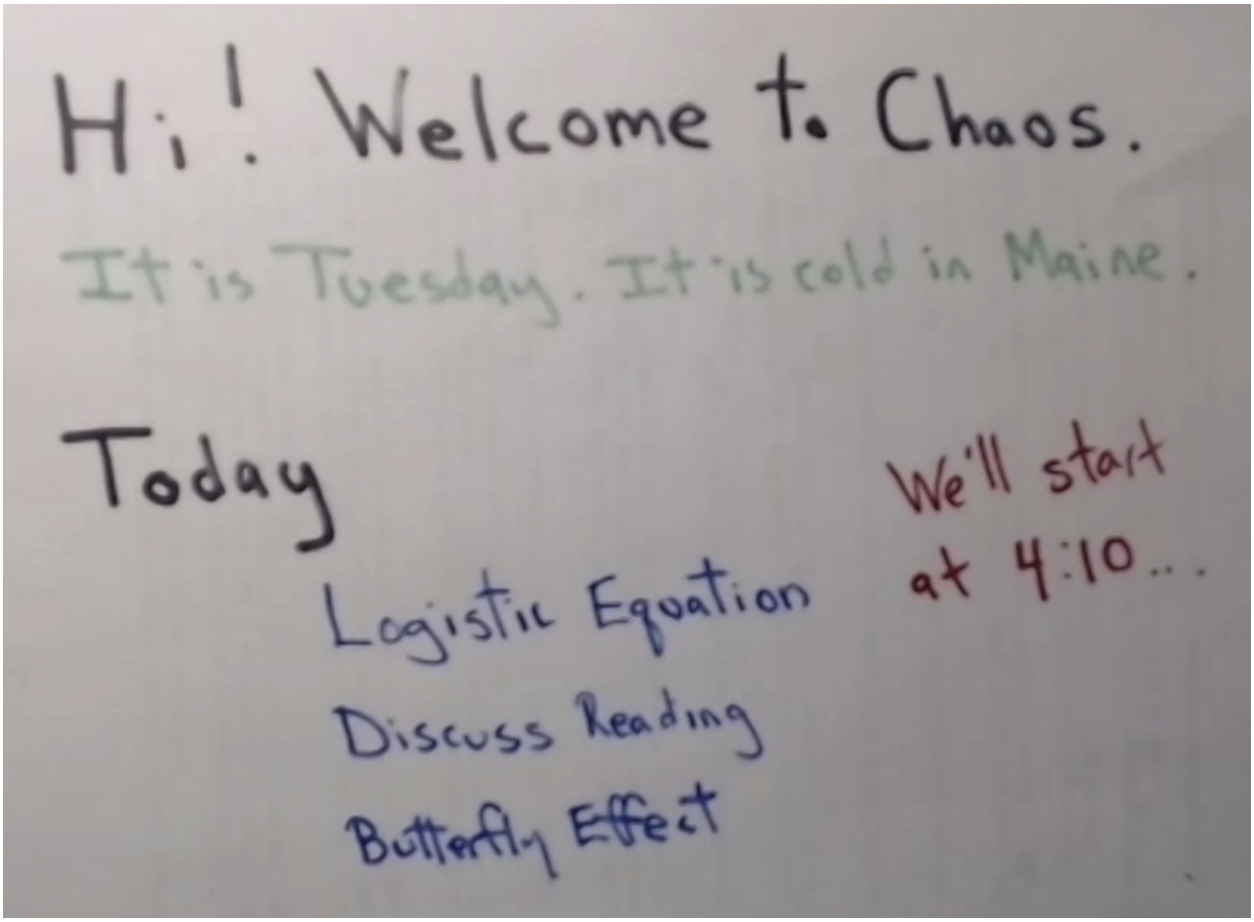
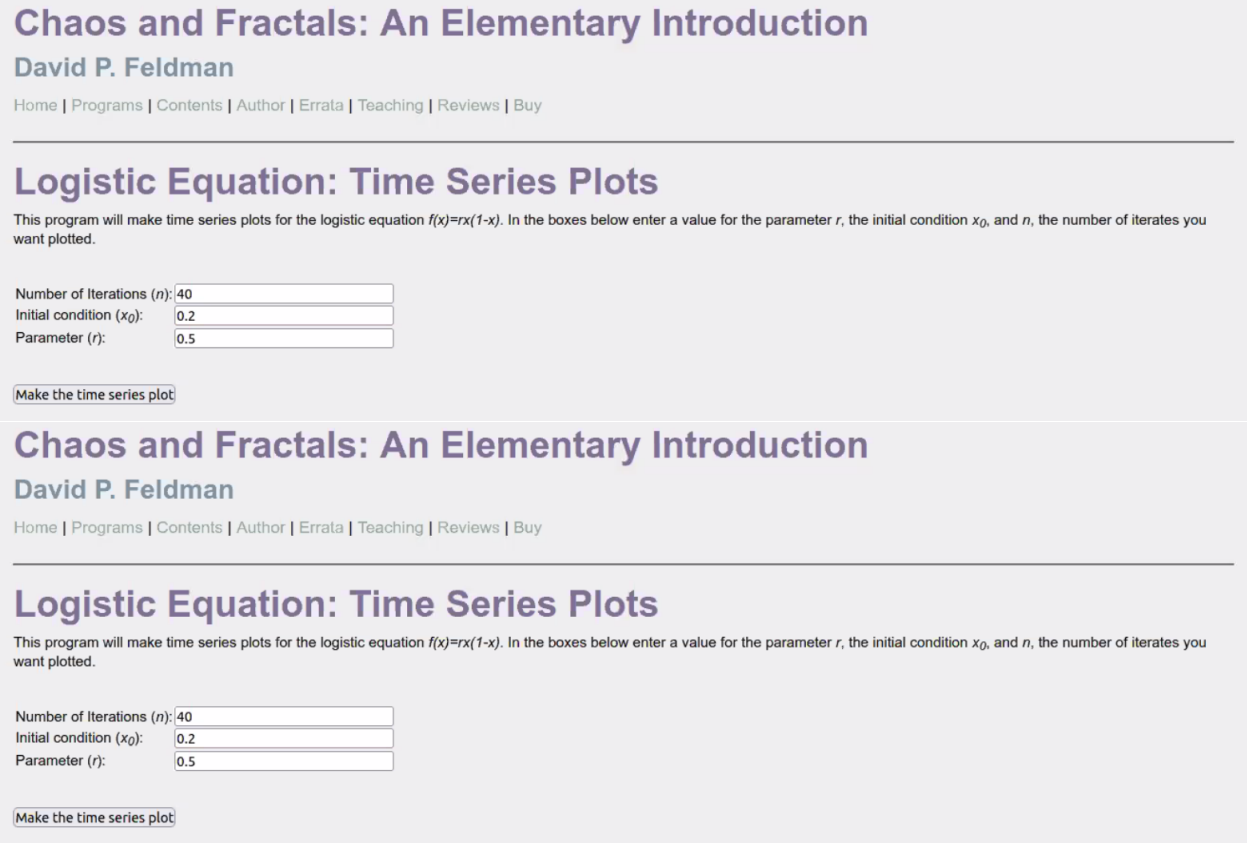
Today's goals



Trajectory types on the logistic map

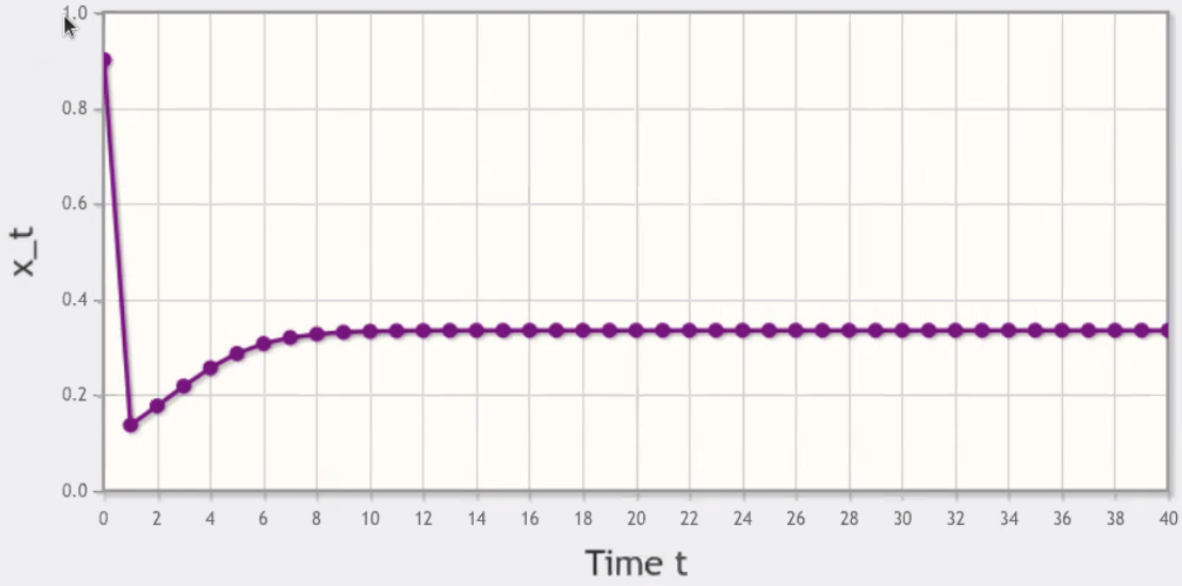
For $0 \leq r \leq 1$: The trajectory approaches 0



For $1 < r < 3$: The trajectory approaches a fixed point.

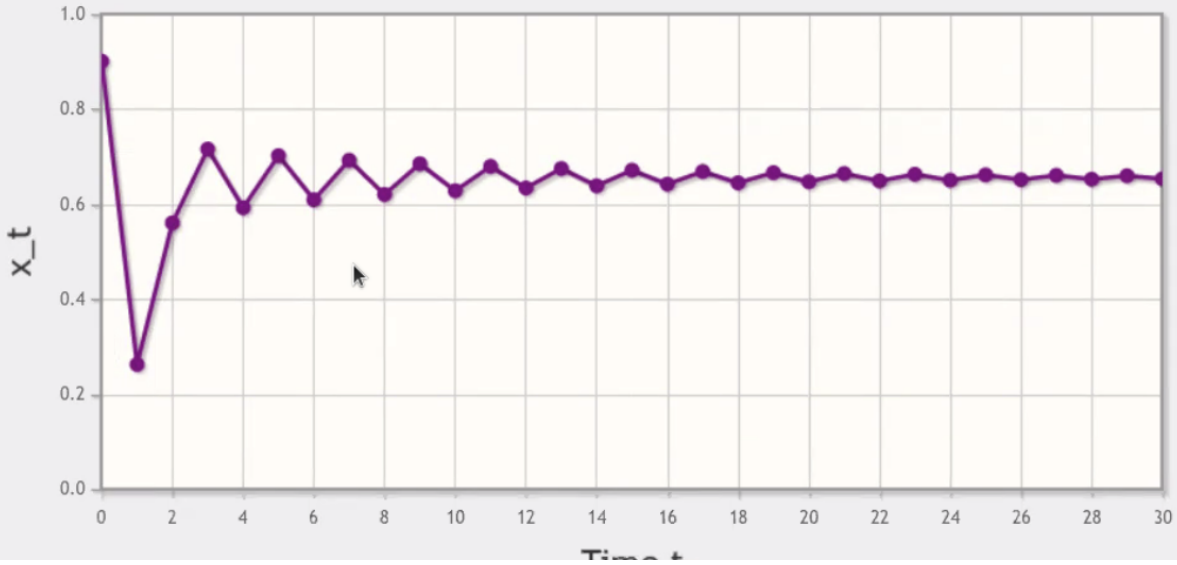
Number of Iterations (n): 40
Initial condition (x_0): 0.9
Parameter (r): 1.5

Make the time series plot



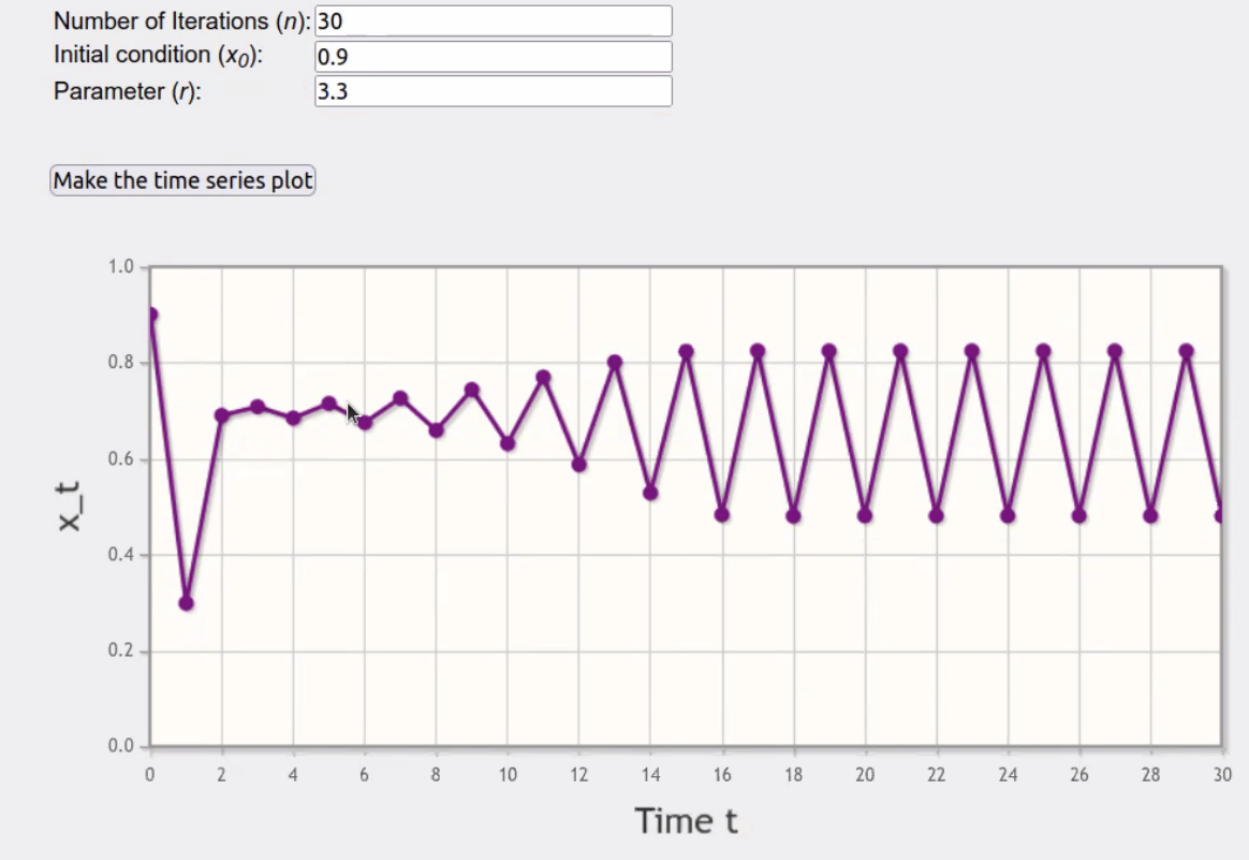
Number of Iterations (n): 30
Initial condition (x_0): 0.9
Parameter (r): 2.9

Make the time series plot



For $3 \leq r < 4$: Things get weird.

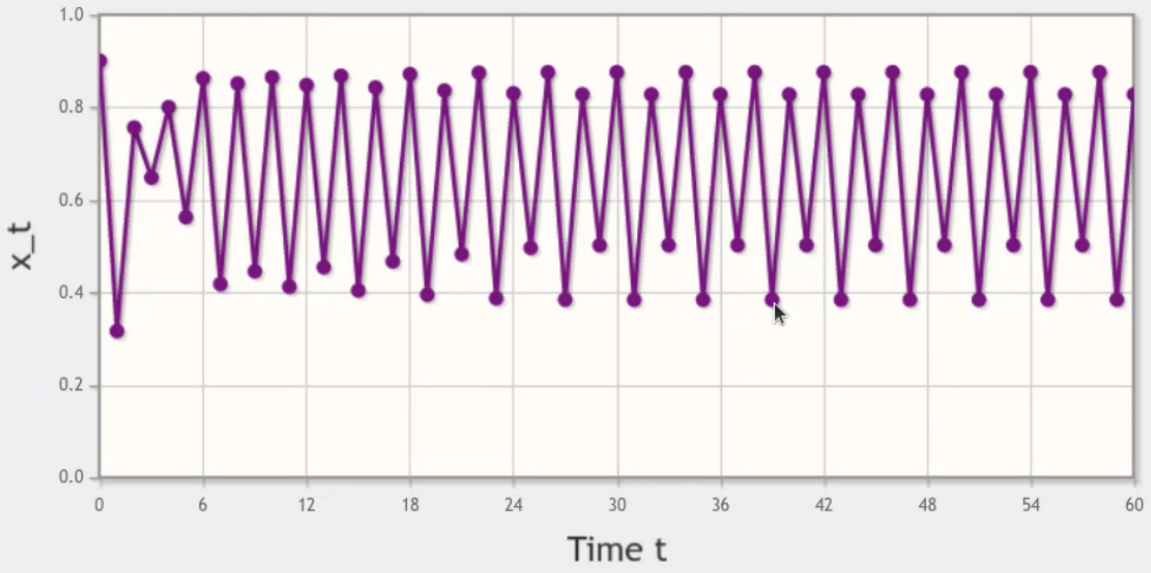
For a range of values of r, the trajectory enters a periodic behaviour of period 2.



For higher values of r the period of the cycle doubles, resulting in period 4. Then for even higher values, it doubles again, and then again for even higher values!

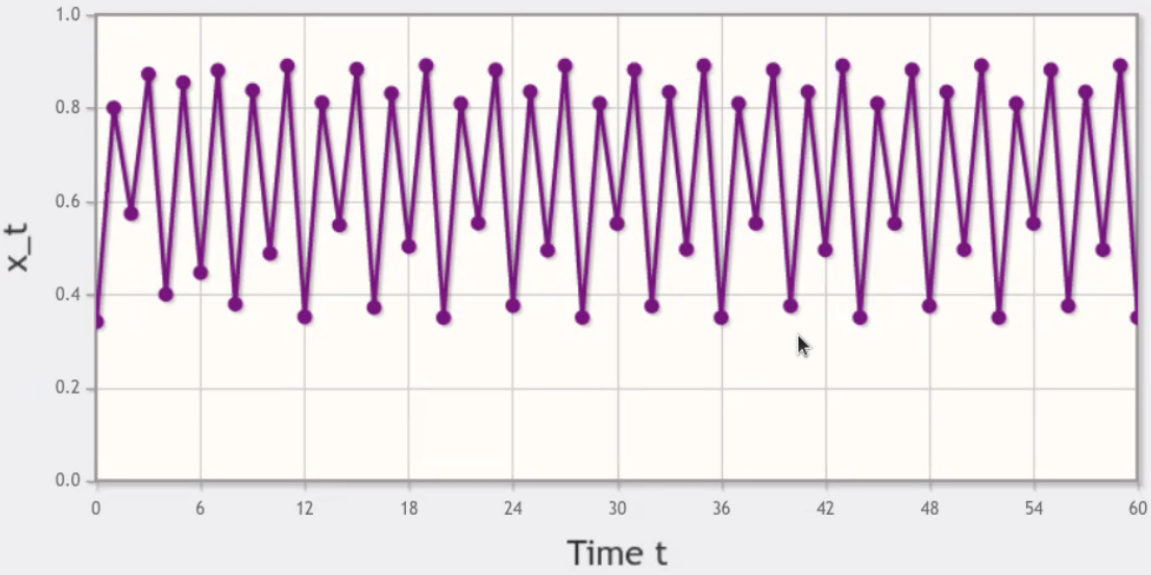
Number of Iterations (n): 60
Initial condition (x_0): 0.9
Parameter (r): 3.5

Make the time series plot

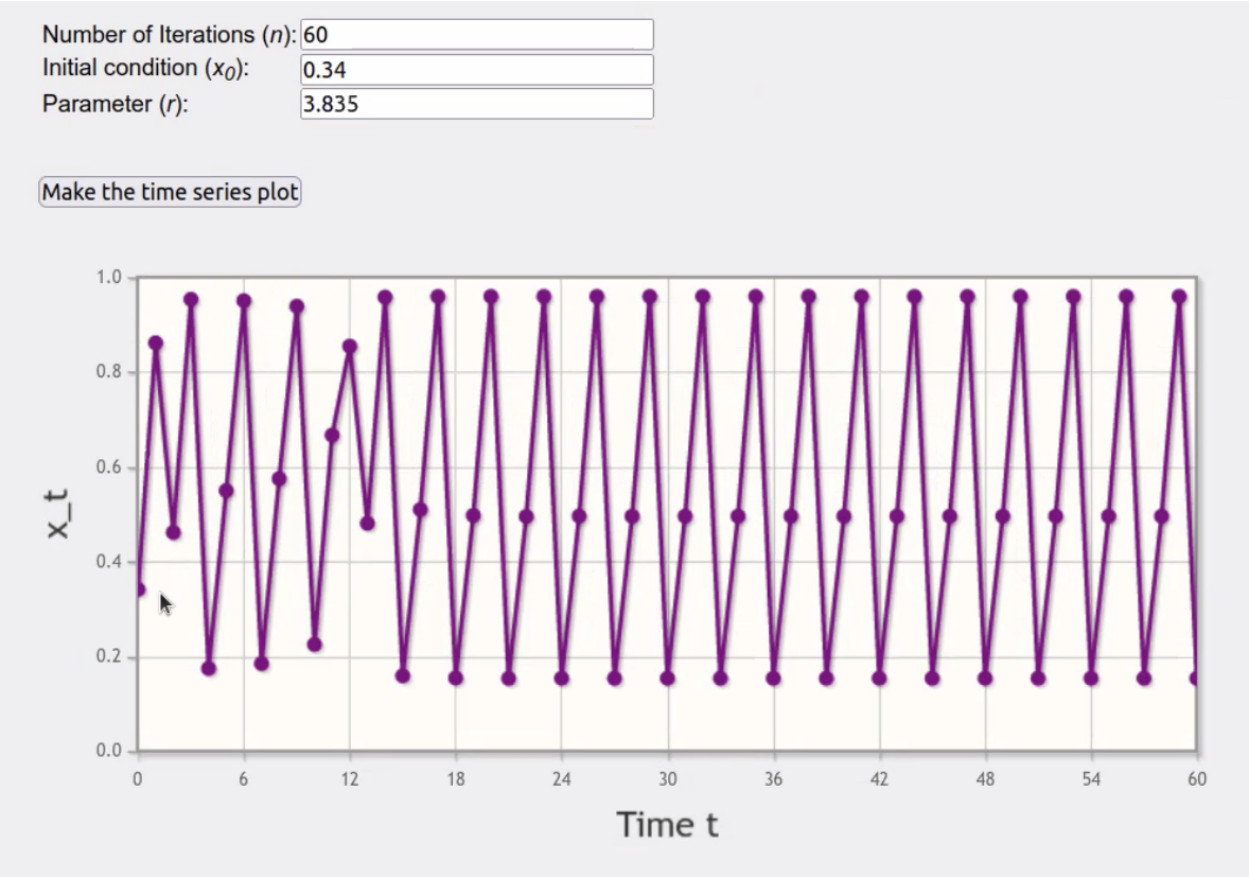


Number of Iterations (n): 60
Initial condition (x_0): 0.34
Parameter (r): 3.56

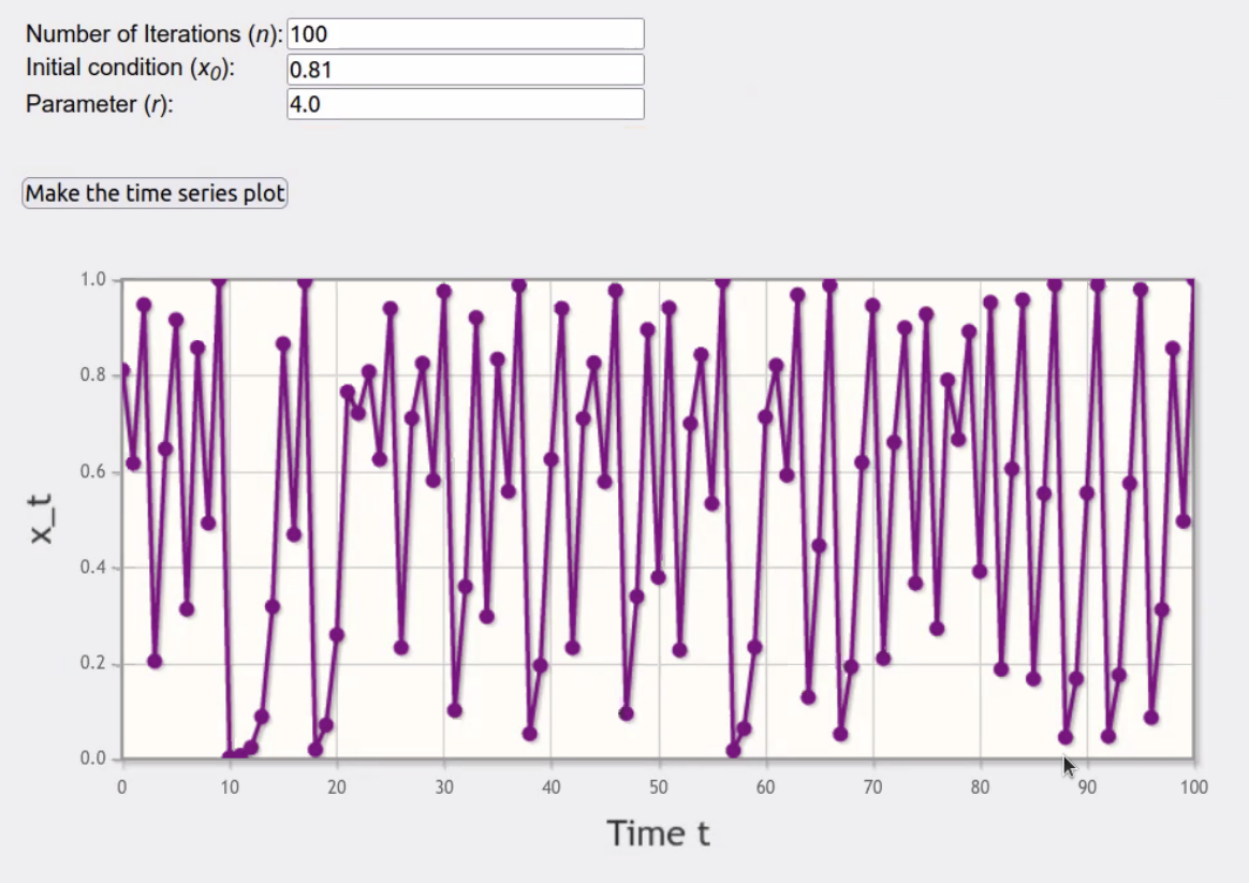
Make the time series plot



Then, for high enough values of r the period stops settling, and we get aperiodic behaviour. But within this aperiodic streak, some values of r will return periodic trajectories.



Finally, for $r = 4$, we get an aperiodic trajectory.



Sensitive dependence on initial conditions

Notice that in aperiodic trajectories, a very slight variation in r will result in a completely different trajectory somewhere down the line. This is known as the butterfly effect, or sensitive dependence on initial conditions.

Number of Iterations (n): 30
First Initial condition (x_0): 0.2
Second Initial condition (y_0): 0.2000000001
Parameter (r): 4

Make the time series plot