Week 1 class 2 - Chaos and Fractals (lab)

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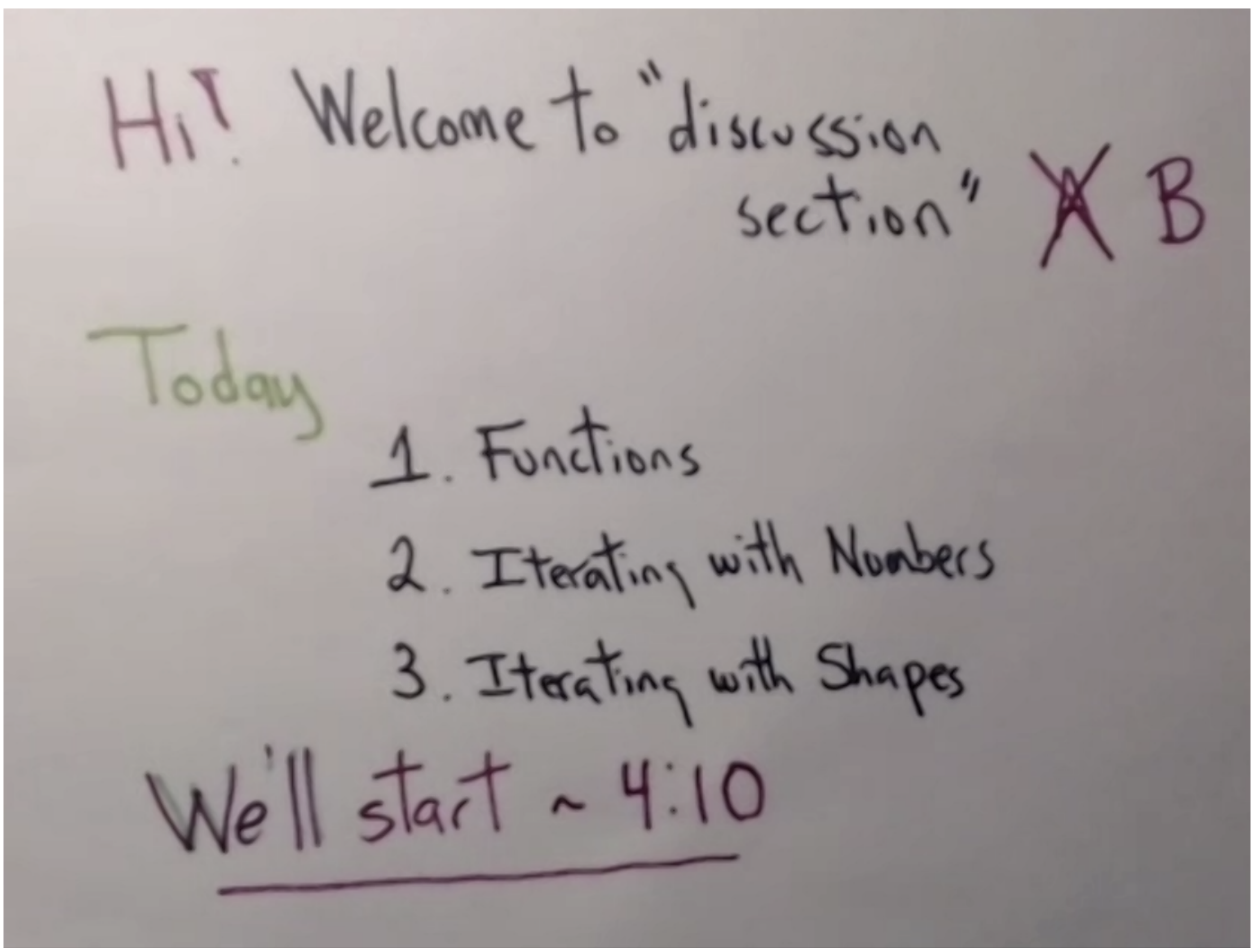
Today's goals

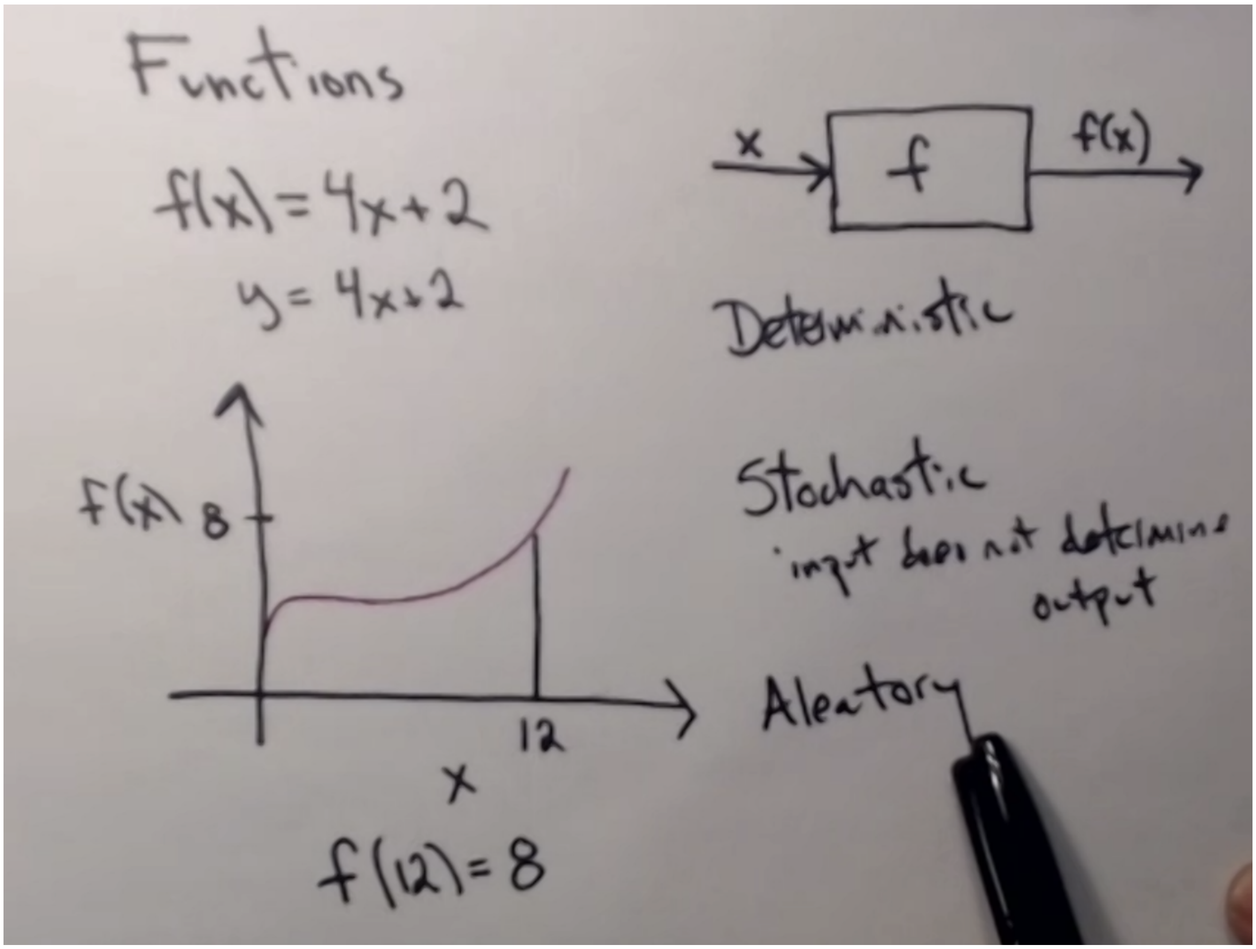
- Functions
- Iterations with numbers
- Iterations with shapes



Functions

General functions reminder: You've likely come across functions a bunch in the past. Functions are sets of instructions, rules expressed as an equation (formula). It is a mathematical expression that takes in an input and returns an output.

There are deterministic functions which always return the same output for a same input. There are stochastic, or aleatory functions, which where the input does not define (or at least does not completely define) the output: for the same input, the function run multiple times may return different results. The nature and distribution of these results will depend on the properties of the function.




```
function_example <- function(x){
  return(3 * x + 6)
}
```

Fixed points are values which, when inputted into a given function remain unchanged.

In algebraic notation, a fixed point equation is when $f(x) = x$

You can find a fixed point for a function by solving the function's fixed point equation. For example, for the function $f(x) = 3x + 6$:

$f(x) = 3x + 6 = x$

$x = 3x + 6$

$3x + 6 - x = 0$

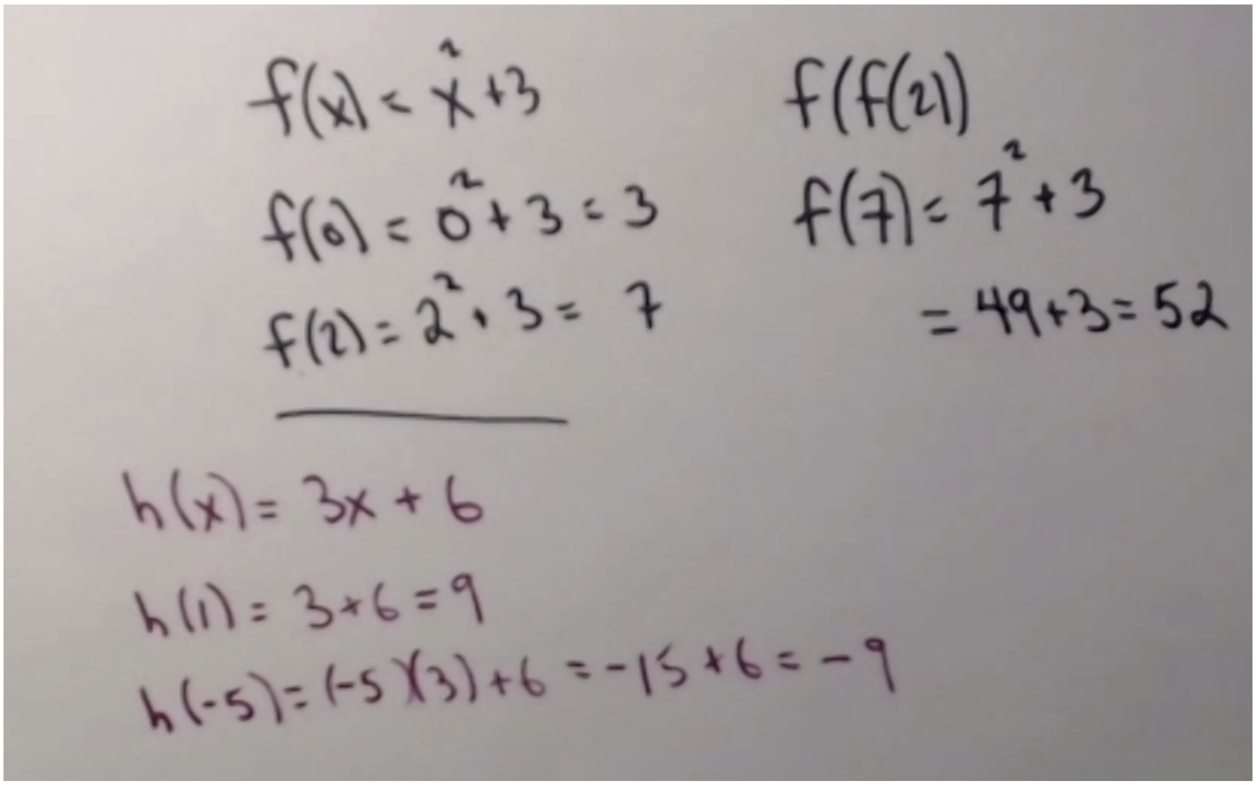
$2x + 6 = 0$

$2x = -6$

$x = -3$

Ta-dah! We just found a fixed point of the function at $x = -3$!

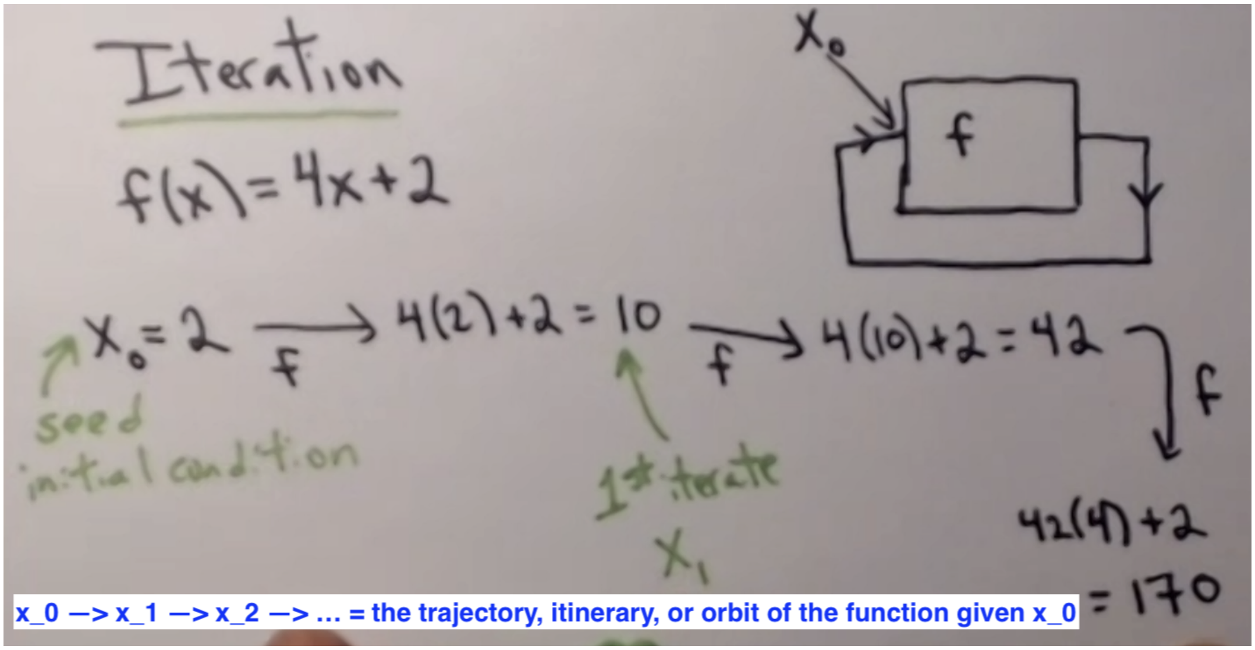
Function examples



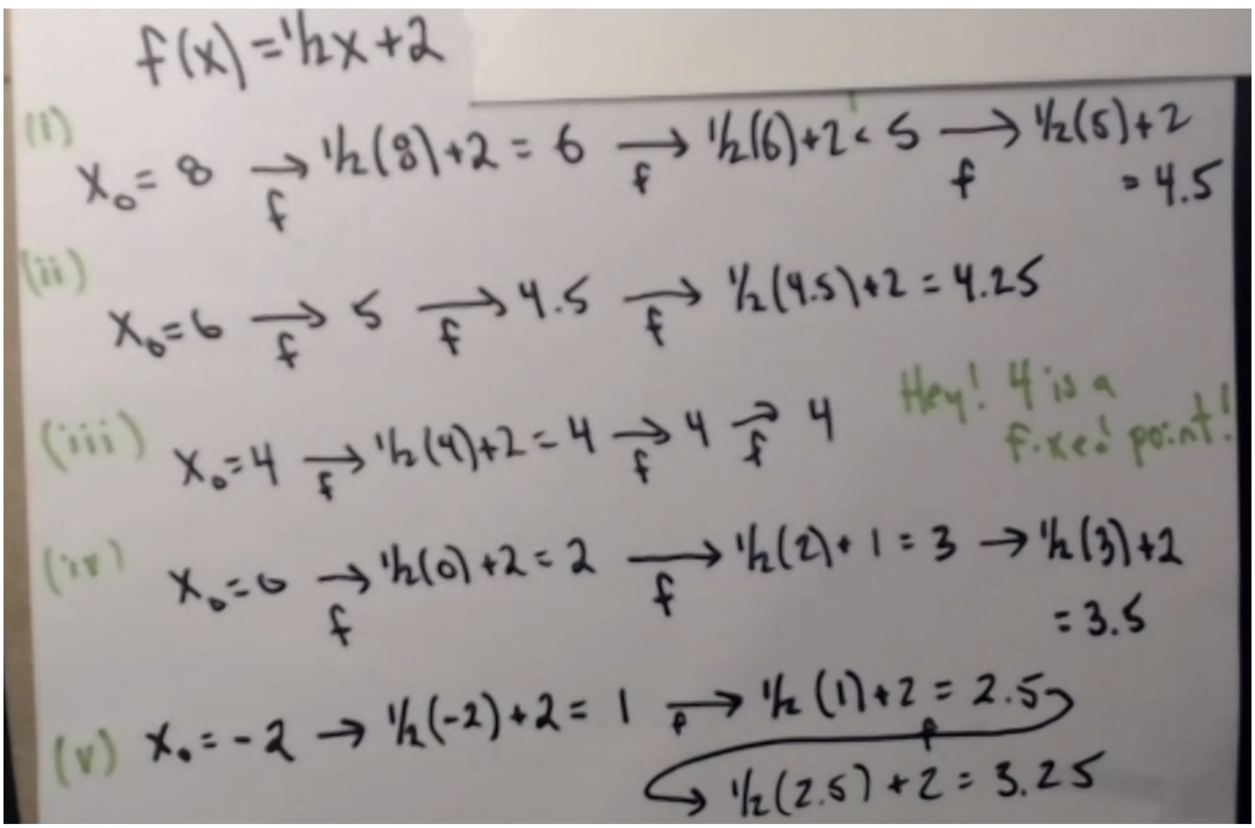
Iteration

Iteration describes a process where you repeatedly apply a function, starting from an initial value known as the **initial condition**, or **seed**.

The sequence of values from $x_0 \to x_1 \to x_2 \to ... \to x_n$ is known as the **trajectory**, **itinerary**, or **orbit** of a function for a given value of the initial condition x_0.



Iteration examples:



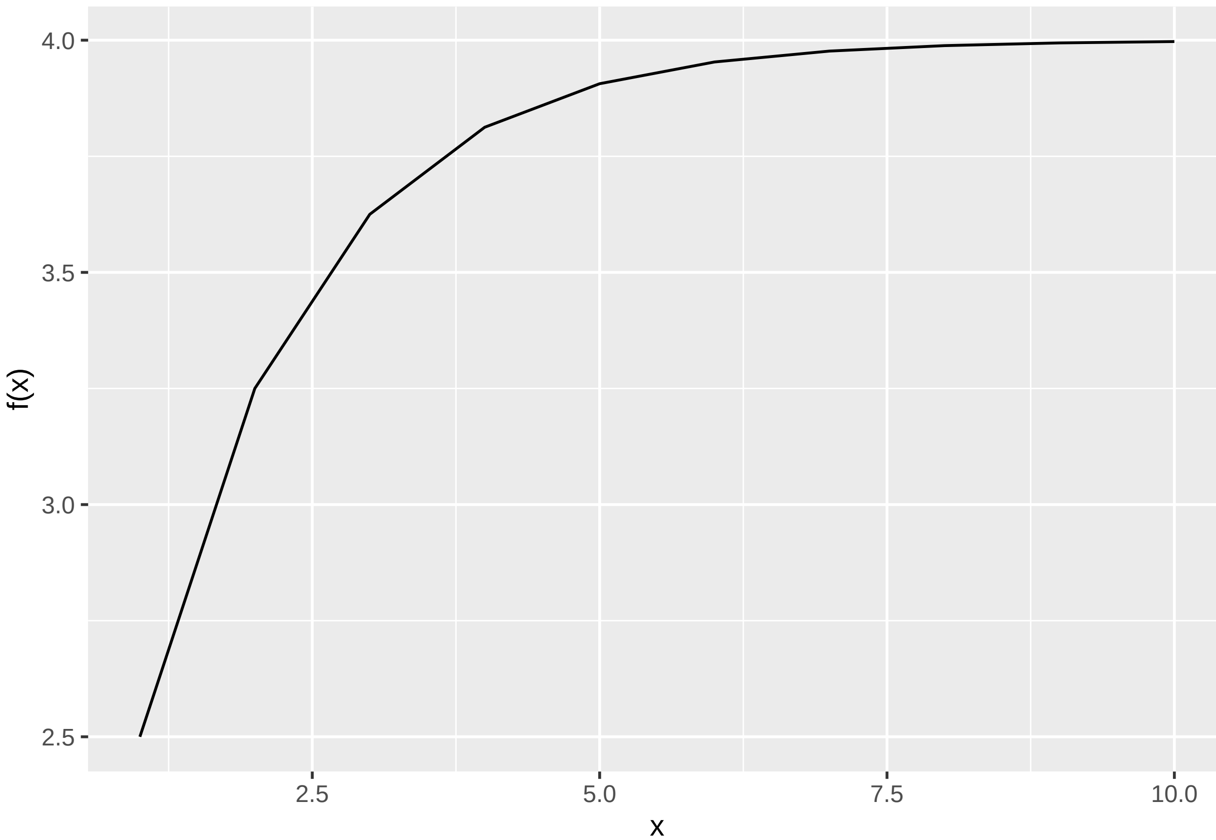

```
# in code, you can iterate a function manually, or in a loop, or you can have the loop
# be part of the function, causing it to run every time the function is called: this
# type of function is called an iterative function
```

```r
iterative_function_example <- function(x_0, N){
  x_n <- x_0
  itinerary <- c()
  for(i in 1:N){
    itinerary <- c(itinerary, x_n / 2 + 2)
    x_n <- x_n / 2 + 2
  }
  return(data.frame(itinerary = itinerary))
}

x_0 <- 1
N <- 10
iterative_function_example(x_0, N) # requests an itinerary of N iterations of a function for a
```

```
##    itinerary
## 1   2.500000
## 2   3.250000
## 3   3.625000
## 4   3.812500
## 5   3.906250
## 6   3.953125
## 7   3.976562
## 8   3.988281
## 9   3.994141
## 10  3.997070
```

```r
# let's plot the resulting trajectory
iterative_function_example(x_0, N) %>%
  ggplot(aes(x = 1:N, y = itinerary)) +
  geom_line() +
  labs(x = "x", y = "f(x)")
```



In the example above, we can see that whatever our initial condition, it seems that over a number of iterations, the values we get converge to 4.

Here, 4 also happens to be a fixed point of the function ($f(x) = \frac{1}{2}x + 2$)

Because the iteration process "attracts values to the fixed point 4 over N iterations", we call 4 a fixed-point attractor of the function.

We will see in this course that there are other types of attractors.

Important questions to keep in mind:

- How can we know if there is only one of multiple point attractors for a function given different initial conditions?
- How can we know how many fixed-point attractors/other types of attractors there are?
- How can we know the nature of these attractors?
- How can we know how attracting/repelling points are in a function? How can we know the range of the attraction of an attractor?

These are all cool questions, but we will have to wait a little bit before we get to them. We will see them in later classes.

Iterations with shapes

Rotation of a square around its centre is a fixed point. 90 degree rotations will always return the same shape in the same configuration.

Rotation of 90° = cycle of period 2 when rotating a rectangle around its centre.