Week 5 class 1 - Chaos and Fractals

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Today's goals

- More with fractals + dimensions
- Random Koch Curve
- Cantor Set Addresses

For Tuesday:

- Reading
- Bring scissors + ruler

For Thursday

- Fractal show and tell (prepare a fractal of your choice)

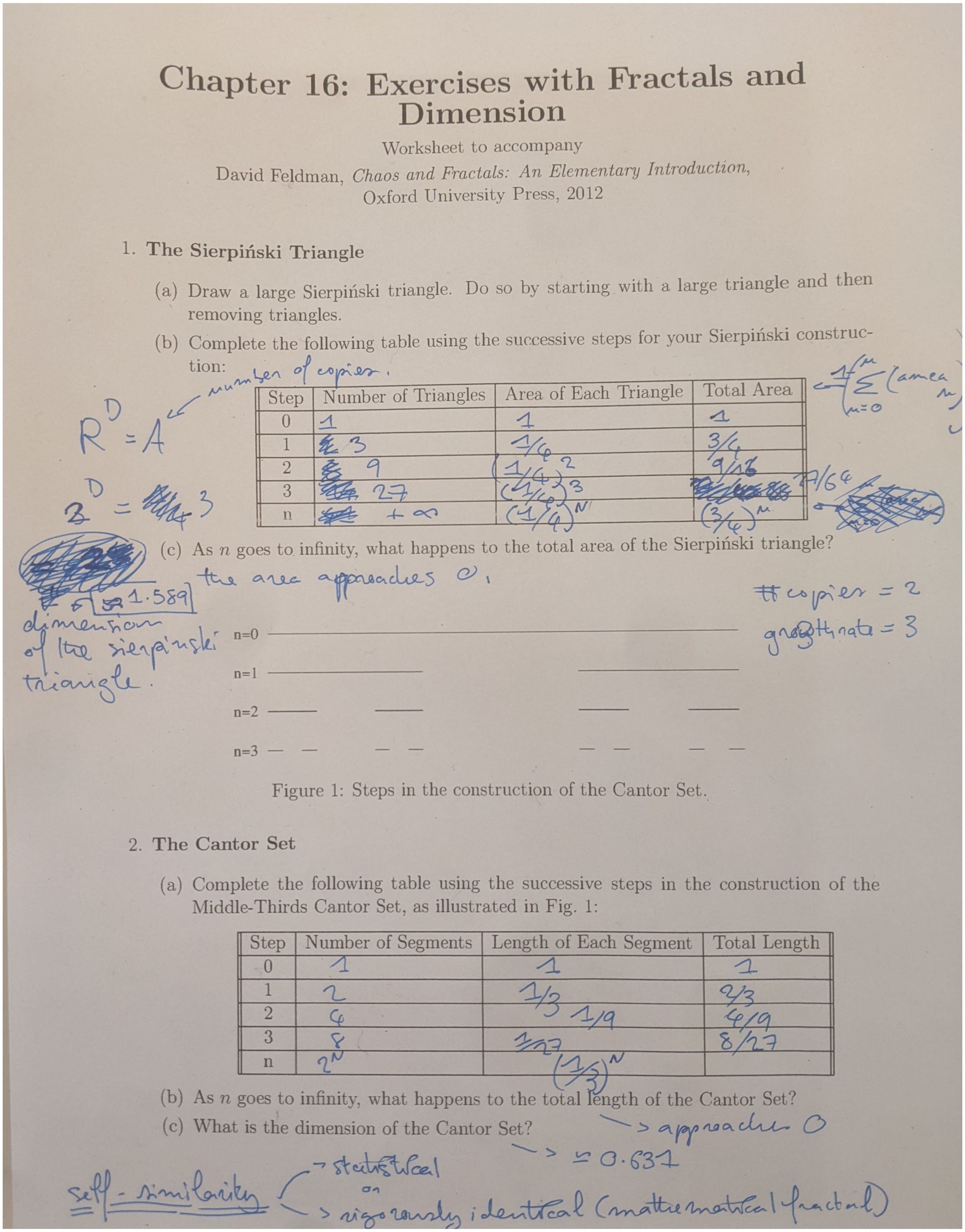
More with fractals + dimensions

From last time:

The number of small copies # is equal to the growth factor R to the power of the dimension D.

$\# = R^D$

More with fractals and fractal dimension



Chapter 16: Exercises with Fractals and Dimension

Worksheet to accompany

David Feldman, *Chaos and Fractals: An Elementary Introduction*, Oxford University Press, 2012

1. **The Sierpiński Triangle**

   (a) Draw a large Sierpiński triangle. Do so by starting with a large triangle and then removing triangles.

   (b) Complete the following table using the successive steps for your Sierpiński construction:

number of copies.

$R^D = A$

Step	Number of Triangles	Area of Each Triangle	Total Area
0	1	1	1
1	3	1/4	3/4
2	9	$(1/4)^2$	9/16
3	27	$(1/4)^3$	27/64
n	$+\infty$	$(1/4)^N$	$(3/4)^n$

$\sum_{n=0}^{n}$ (area n)

$3^D = 3$

   (c) As n goes to infinity, what happens to the total area of the Sierpiński triangle?

the area approaches 0.

$\boxed{\approx 1.589}$ dimension of the sierpinski triangle.

copies = 2

growth rate = 3

n=0

n=1

n=2

n=3

Figure 1: Steps in the construction of the Cantor Set.

2. **The Cantor Set**

   (a) Complete the following table using the successive steps in the construction of the Middle-Thirds Cantor Set, as illustrated in Fig. 1:

Step	Number of Segments	Length of Each Segment	Total Length
0	1	1	1
1	2	1/3	2/3
2	4	1/9	4/9
3	8	1/27	8/27
n	2^N	$(1/3)^N$	

   (b) As n goes to infinity, what happens to the total length of the Cantor Set?

   → approaches 0

   (c) What is the dimension of the Cantor Set?

   → ≈ 0.631

self-similarity → statistical or → rigorously identical (mathematical fractal)

$\text{solve}\left(3=2^{d},d\right)$	$d=\frac{\ln(3)}{\ln(2)}$
$d=\frac{\ln(3)}{\ln(2)}$	$d=1.58496$
$\log_{2}(3)$	1.58496