Week 1 class 3 - Chaos and Fractals

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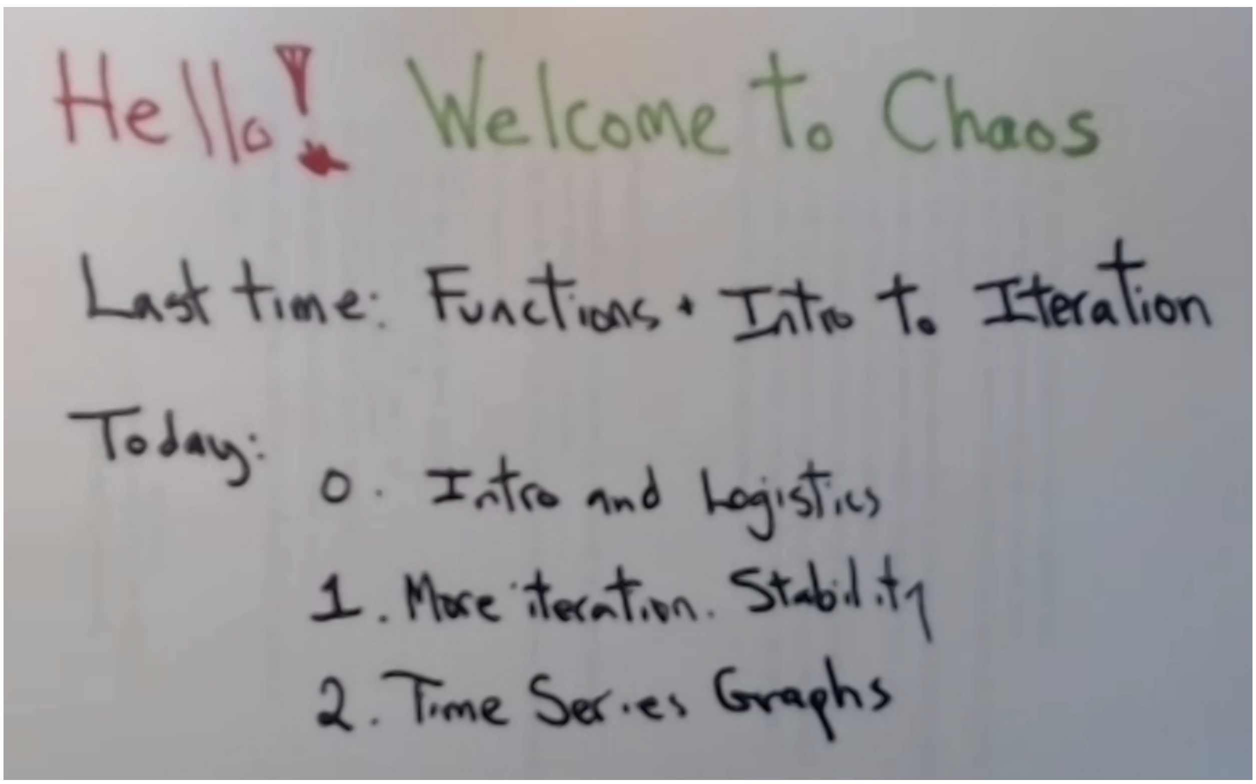
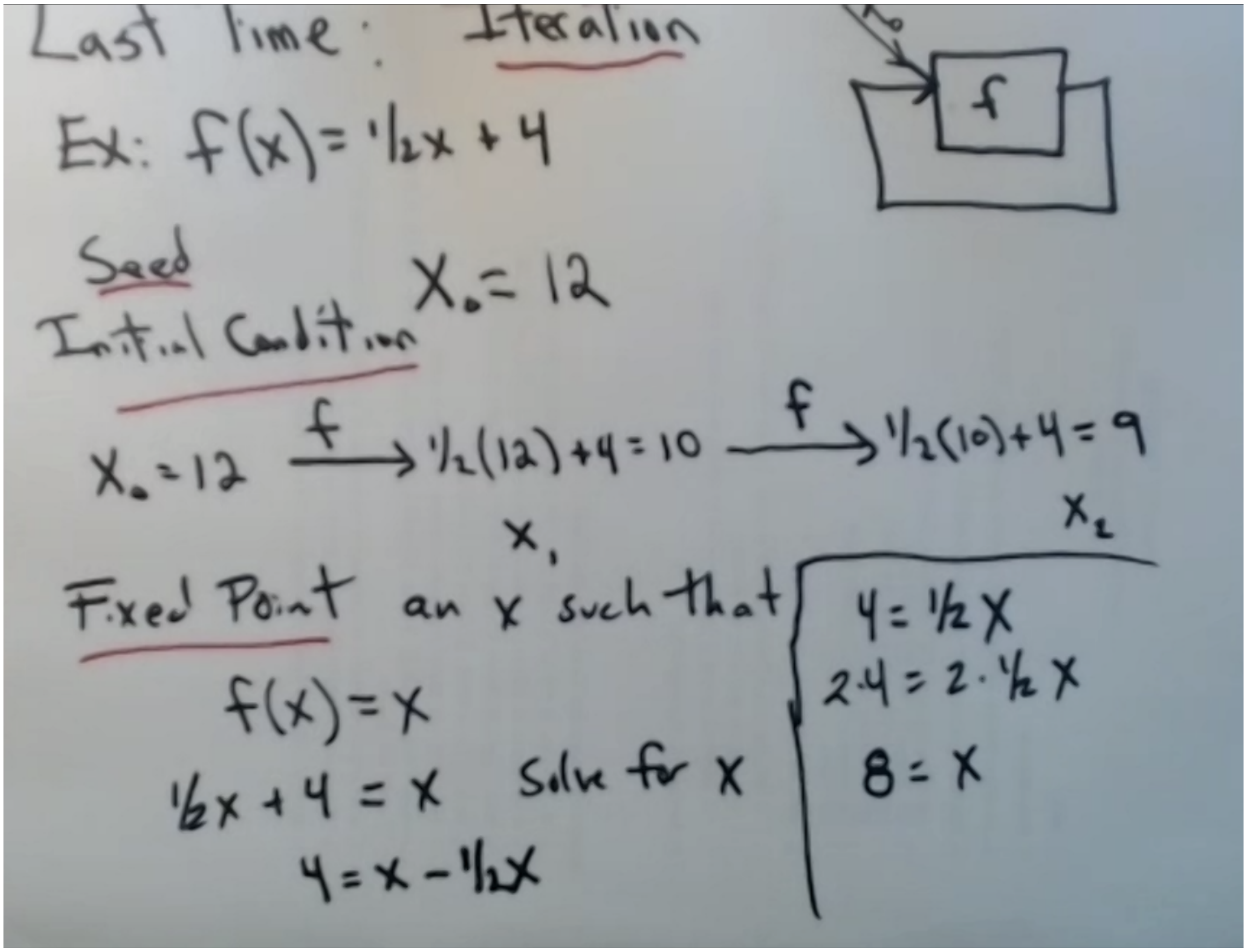
Today's goals

- More iteration, stability
- Time series graphs

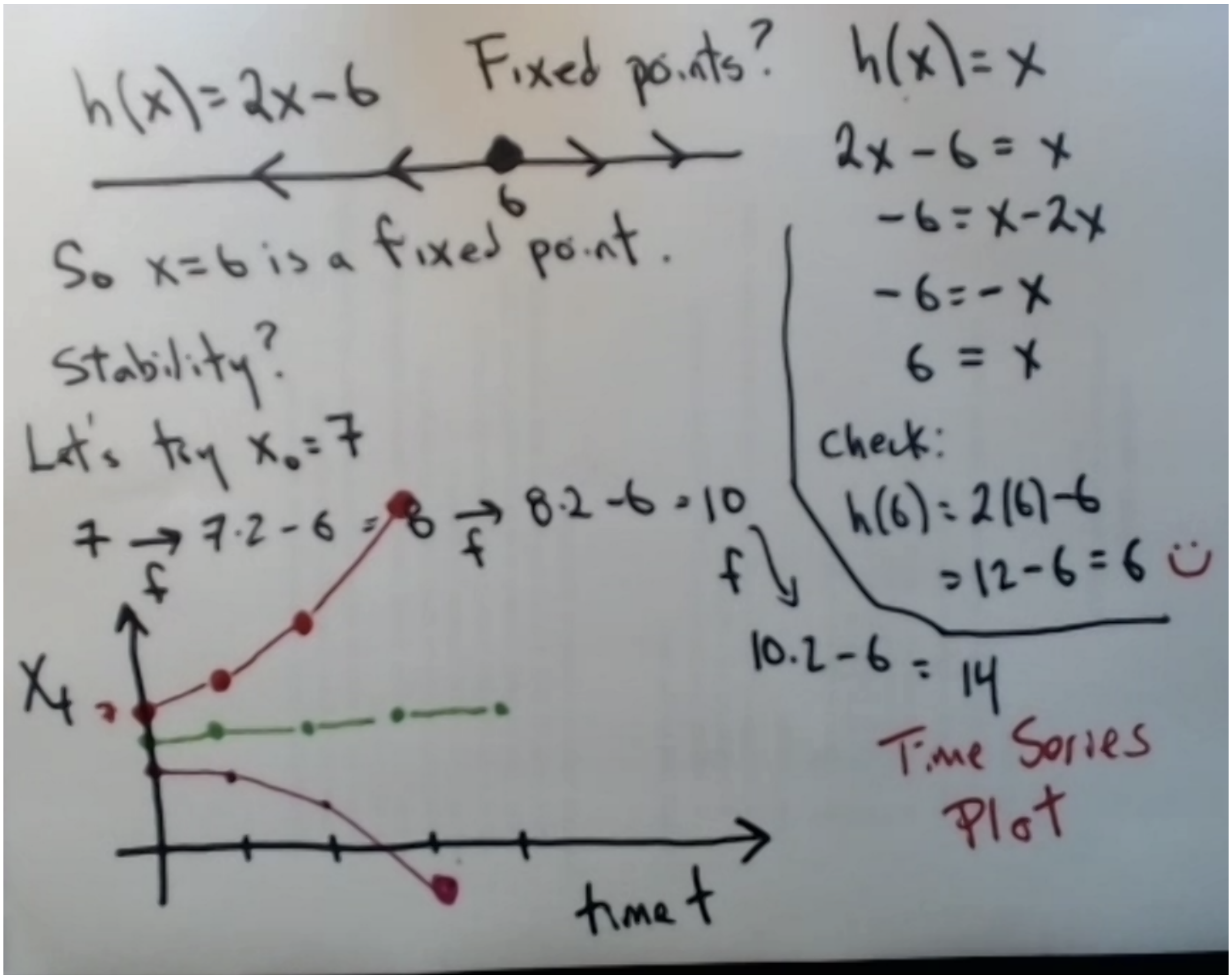


Definitions from last time



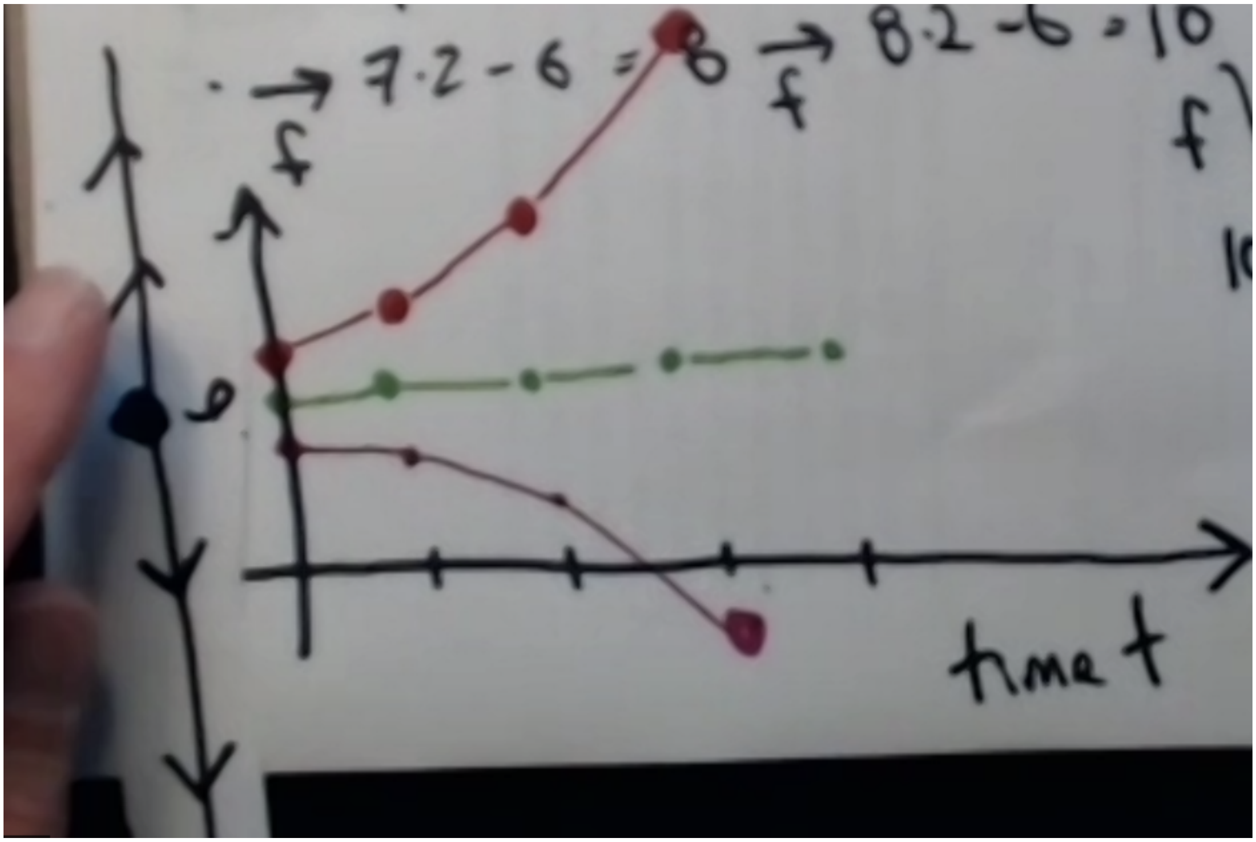
More iteration:

A **phase line** shows the behaviour of a function around fixed points. See the top of the picture below:

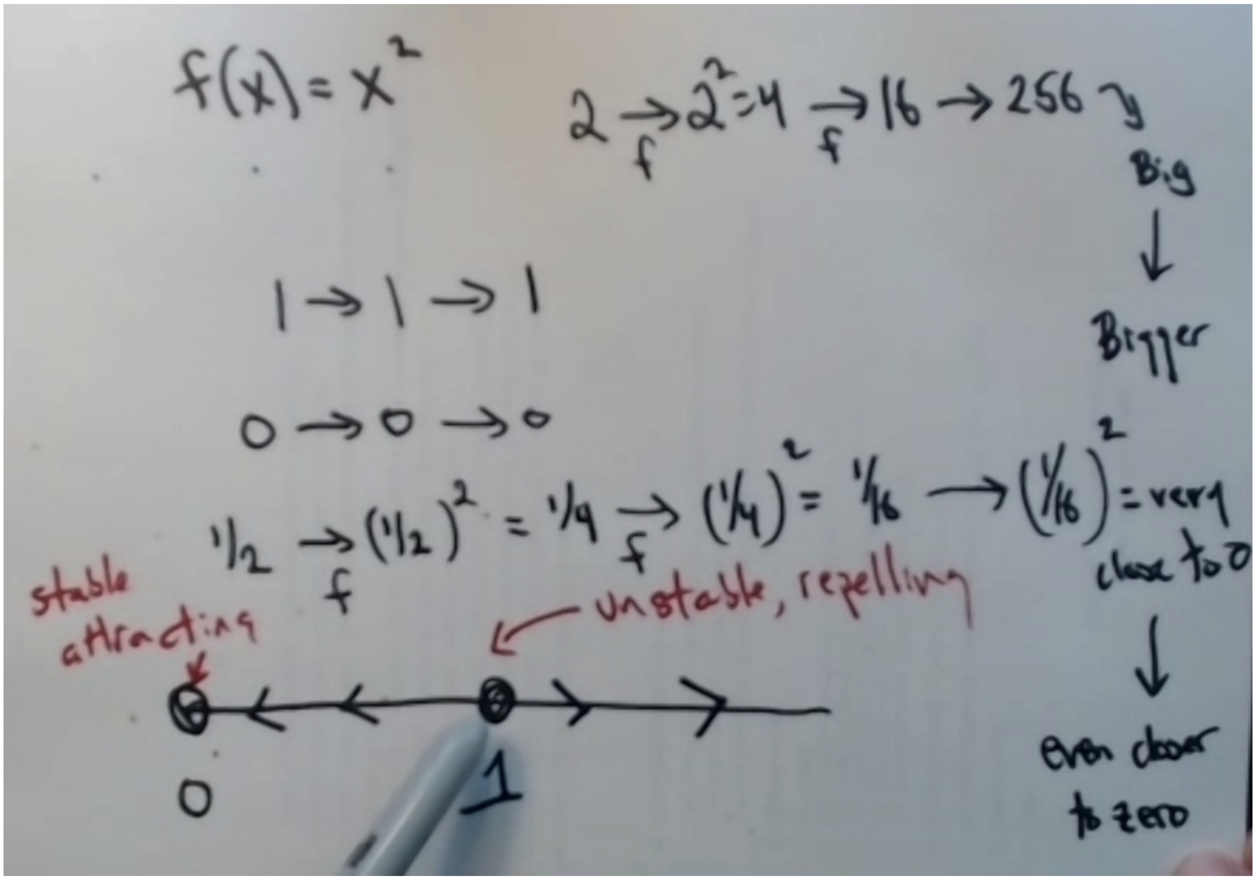


At the bottom of the last picture, we plot the outputs of the function for each time step, such that the y axis is the value returned by the function, and the x axis is the time step. This is called a **time series plot**.

Notice that if you take the phase line and put it along the y axis of the time series plot, it summarises the bahaviour of the function around its fixed points:



The behaviour of a function arount a fixed point can vary. Typically, a fixed point can either be attractive (**stable**) or repelling (**unstable**).



Questions for next time:

- Can a fixed point be a mix of attractive and repelling?
- Can two adjacent fixed points both be repelling at the same time? What happens in the space between them then?

Let's play around with a few functions.

Function 1:

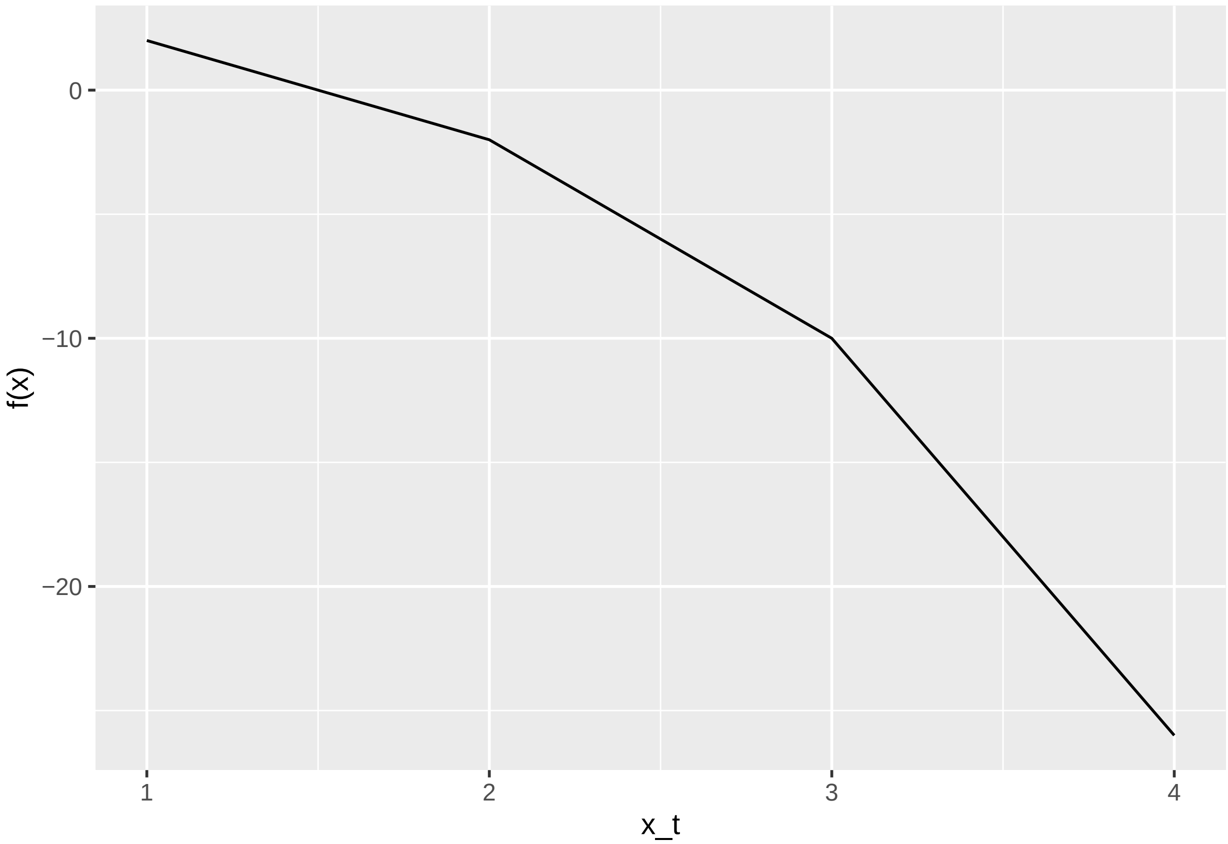
```
# in code, you can iterate a function manually, or in a loop, or you can have the loop
# be part of the function, causing it to run every time the function is called: this
# type of function is called an iterative function

iterative_function_example <- function(x_0, N){
  x_n <- x_0
  itinerary <- c()
  for(i in 1:N){
    itinerary <- c(itinerary, 2 * x_n - 6)
    x_n <- 2 * x_n - 6
  }
  return(data.frame(itinerary = itinerary))
}
```

```r
x_0 <- 4
N <- 4
iterative_function_example(x_0, N) # requests an itinerary of N iterations of a function for a
```

```
##   itinerary
## 1         2
## 2        -2
## 3       -10
## 4       -26
```

```r
iterative_function_example(x_0, N) %>%
  ggplot(aes(x = 1:N, y = itinerary)) +
  geom_line() +
  labs(x = "x_t", y = "f(x)")
```



Phase line: ← 6 →

Function 2:

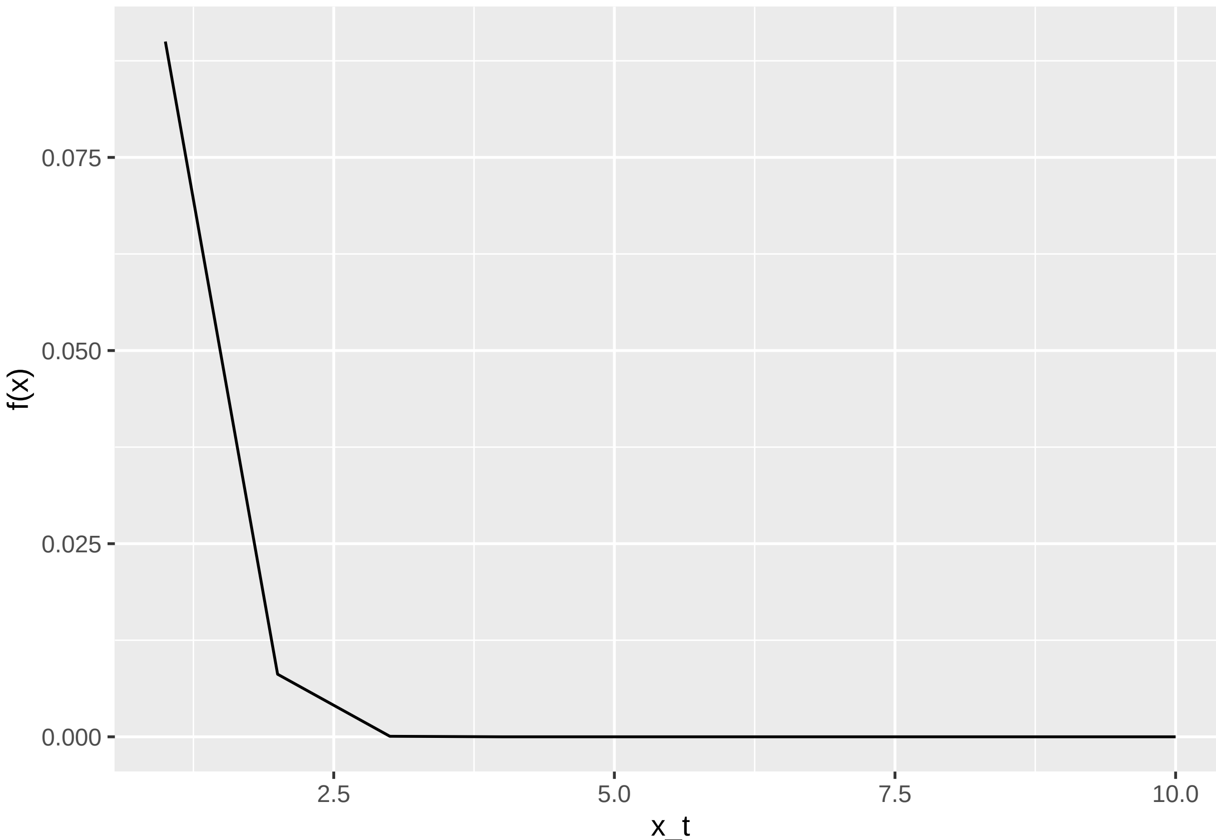
```r
iterative_function_example <- function(x_0, N){
  x_n <- x_0
  itinerary <- c()
  for(i in 1:N){
```

```
    itinerary <- c(itinerary, x_n ^2)
    x_n <- x_n ^2
  }
  return(data.frame(itinerary = itinerary))
}

x_0 <- 0.3
N <- 10
iterative_function_example(x_0, N)
```

```
##        itinerary
## 1   9.000000e-02
## 2   8.100000e-03
## 3   6.561000e-05
## 4   4.304672e-09
## 5   1.853020e-17
## 6   3.433684e-34
## 7   1.179018e-67
## 8  1.390085e-134
## 9  1.932335e-268
## 10  0.000000e+00
```

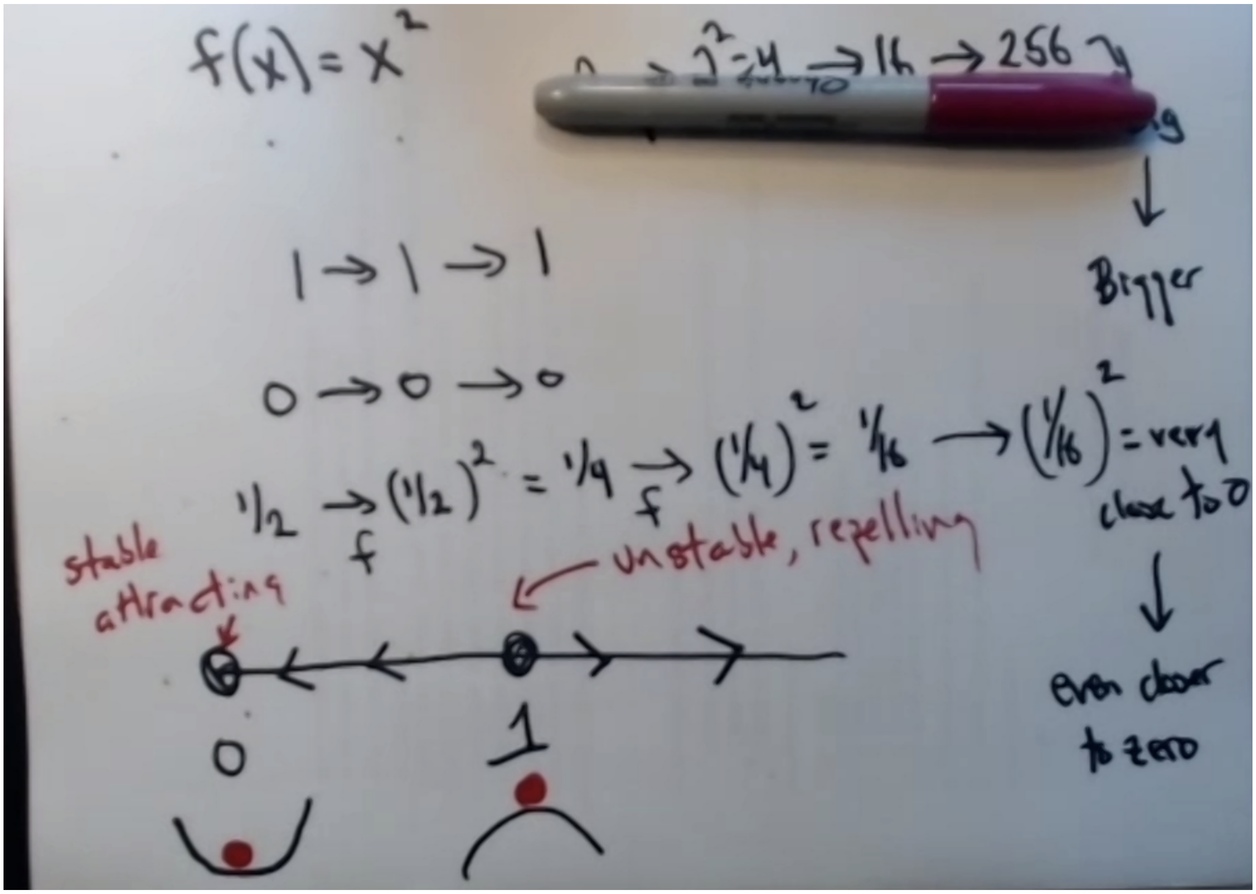
```
iterative_function_example(x_0, N) %>%
  ggplot(aes(x = 1:N, y = itinerary)) +
  geom_line() +
  labs(x = "x_t", y = "f(x)")
```



$\rightarrow 0 \leftarrow 1 \rightarrow$

This function contains two fixed points. 1 **stable** (attracting), and one **unstable** (repelling).

Stable: rock in the bottom of a valley Unstable: rock at the top of a hill



f(x) = x^2
2^2 = 4 → 16 → 256
Bigger
1 → 1 → 1
0 → 0 → 0
1/2 → (1/2)^2 = 1/4 → (1/4)^2 = 1/16 → (1/16)^2 = very close to 0
f
f
even closer to zero
stable attracting
unstable, repelling
0
1