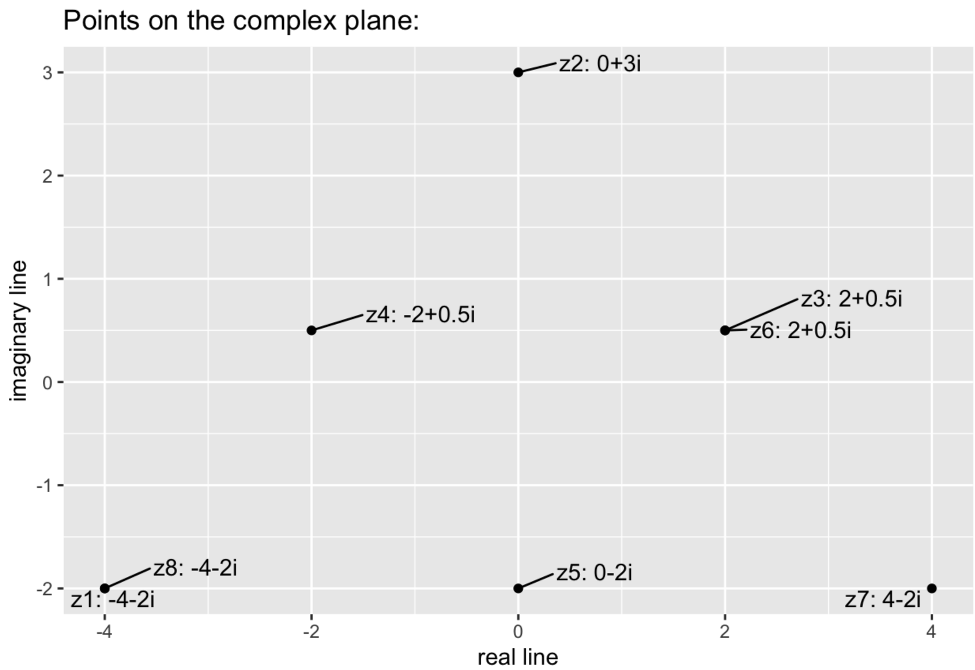
Non-textbook textbook problems

1. Consider the following complex numbers:

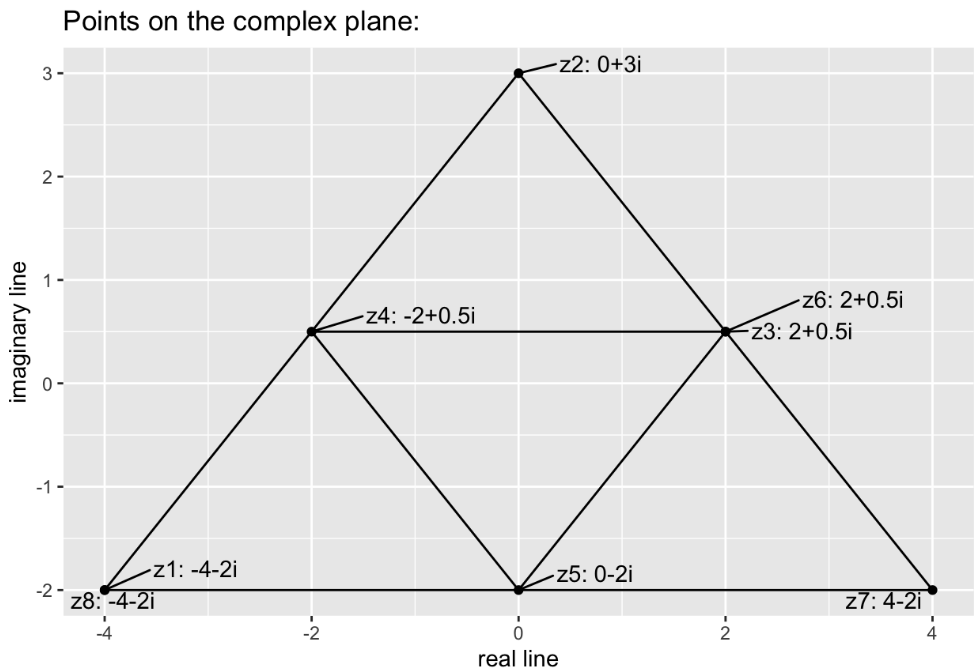
$$z_1 = -4 - 2i \ , \quad z_2 = 3i \ , \quad z_3 = 2 + 0.5i \ , \quad z_4 = -2 + 0.5i \ ,$$

$$z_5 = -2i \ , \quad z_6 = 2 + 0.5i \ , \quad z_7 = 4 - 2i \ , \quad z_8 = -4 - 2i \ .$$

(a) Plot the above numbers on the complex plane.



(b) Connect the dots. The pattern should look familiar.



2. Consider the function f(z) = iz.

```
funky <- function(z){return(z*1i)}
```

(a) Determine first four iterates of z0 = 3.

```
n <- 4
z_0 <- 3
funky <- function(z){return(z*1i)}

x_n <- z_0
for(i in 1:(n-1)){
  x_n <- c(x_n, funky(tail(x_n, 1)))
}
x_n
```

```
3+0i  0+3i -3+0i  0-3i
```
,

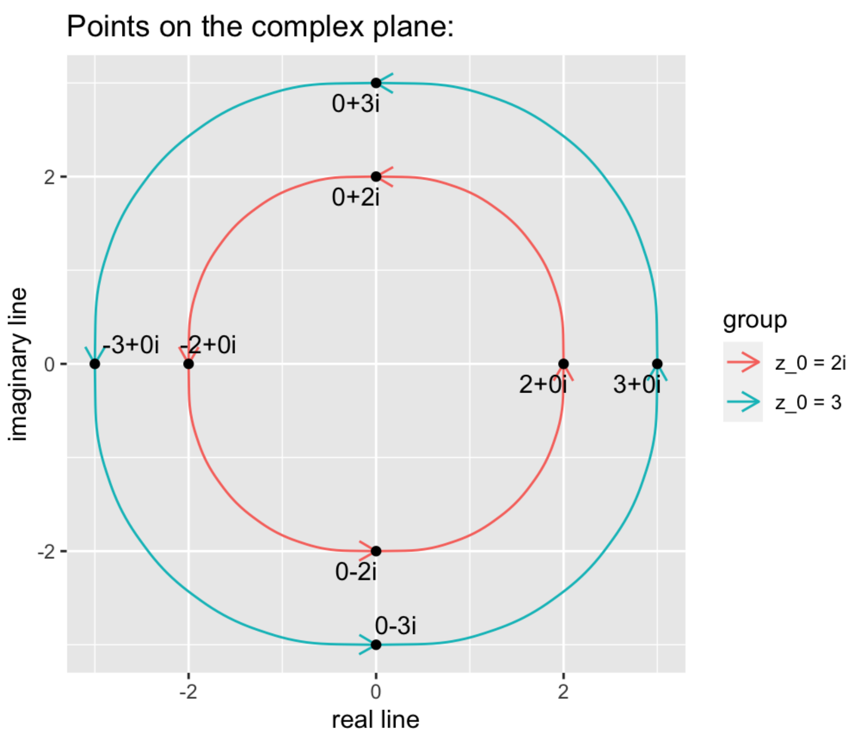
which is to say: $\{3 + i, 3i, \text{-}3, \text{-}3i\}$

(b) Determine first four iterates of z0 = 2i.

```
0+2i -2+0i  0-2i  2+0i
```
,

which is to say: $\{2i, \text{-}2, \text{-}2i, 2 + i\}$

(c) Plot the iterates for each of the seeds in the complex plane.



(d) How would you describe the behavior of the orbits?

I would likely describe the behaviour of these orbits as a rotation.

Chapter 21

(21.1) Consider the set of all perfect squares: {1, 4, 9, 16, 25, …}. What is the cardinality of this set?

The cardinality of this set is $\aleph_0$, it is countably infinite.

(21.2) What is the cardinality of the following infinite set: $\{^1/_2, ^1/_3, ^1/_4, \ldots\}$?

The cardinality of this set is $\aleph_0$, it is countably infinite.

(21.3) What is the cardinality of the following infinite set: {613, 614, 615, 616, 617, …} ?

The cardinality of this set is $\aleph_0$, it is countably infinite.

(21.4) What is the cardinality of the following infinite set: $\{3^1, 3^2, 3^3, 3^4, 3^5, \ldots\}$?

The cardinality of this set is $\aleph_0$, it is countably infinite.

(21.5) What is the cardinality of all numbers contained in the interval between 0 and $^1/_2$?

The cardinality of this set is $\aleph_1$, it is uncountably infinite.

Chapter 21 - Optional problems

(21.8) Convert the following numbers from base-10 to binary:

(a) 8

Decimal value	8	4	2	1
Presence/absence	1	0	0	0

The binary form of base-10 8 is 1000.

(b) 9

Decimal value	8	4	2	1
Presence/absence	1	0	0	1

The binary form of base-10 9 is 1001.

(c) 48

Decimal value	32	16	8	4	2	1
Presence/absence	1	1	0	0	0	0

The binary form of base-10 48 is 110000.

(d) 100

Decimal value	64	32	16	8	4	2	1
Presence/absence	1	1	0	0	1	0	0

The binary form of base-10 100 is 1100100.

(21.9) Convert the following numbers from binary to base-10:

(a) 100

Decimal value	4	2	1
Presence/absence	1	0	0

4 + 0 + 0 = 4

The decimal form of base-2 100 is 4.

(b) 111

Decimal value	4	2	1
Presence/absence	1	1	1

4 + 2 + 1 = 7

The decimal form of base-2 111 is 7.

(c) 1001

Decimal value	8	4	2	1
Presence/absence	1	0	0	1

8 + 0 + 0 + 1 = 9

The decimal form of base-2 1001 is 9.

(d) 10101

Decimal value	16	8	4	2	1
Presence/absence	1	0	1	0	1

16 + 0 + 4 + 0 + 1 = 21

The decimal form of base-2 10101 is 21.

Since there was no question (21.20), I assumed that the correct question might have been (21.10) and answered it instead.

(21.10) Convert the following numbers from binary to base-10:

(a) 0.1

Decimal value <dbl>	Presence/Absence <dbl>
1.0000000000	0
0.5000000000	0
0.2500000000	0
0.1250000000	0
0.0625000000	1
0.0312500000	1
0.0156250000	0
0.0078125000	0
0.0039062500	1
0.0019531250	1

One binary form of base-10 0.1 is approximately 0.0001100111, accurate to 3 decimal places.

(The R table looks fancy, but I only used it to get the decimal values column without having to think too much, I solved the binary bit by trial and error).

(b) 0.01

Decimal value <dbl>	Presence/Absence <dbl>
1.0000000000	0
0.5000000000	0
0.2500000000	0
0.1250000000	0
0.0625000000	0
0.0312500000	0
0.0156250000	0
0.0078125000	1
0.0039062500	0
0.0019531250	1

One binary form of base-10 0.01 is approximately 0.0000001011, accurate to 3 decimal places.

(c) 0.001

Decimal value <dbl>	Presence/Absence <dbl>
1.00000000000	0
0.50000000000	0
0.25000000000	0
0.12500000000	0
0.06250000000	0
0.03125000000	0
0.01562500000	0
0.00781250000	0
0.00390625000	0
0.00195312500	0
0.00097656250	1
0.00048828125	0
0.00024414062	0
0.00012207031	0
0.00006103516	1

One binary form of base-10 0.001 is approximately 0.00000000010001, accurate to 4 decimal places.