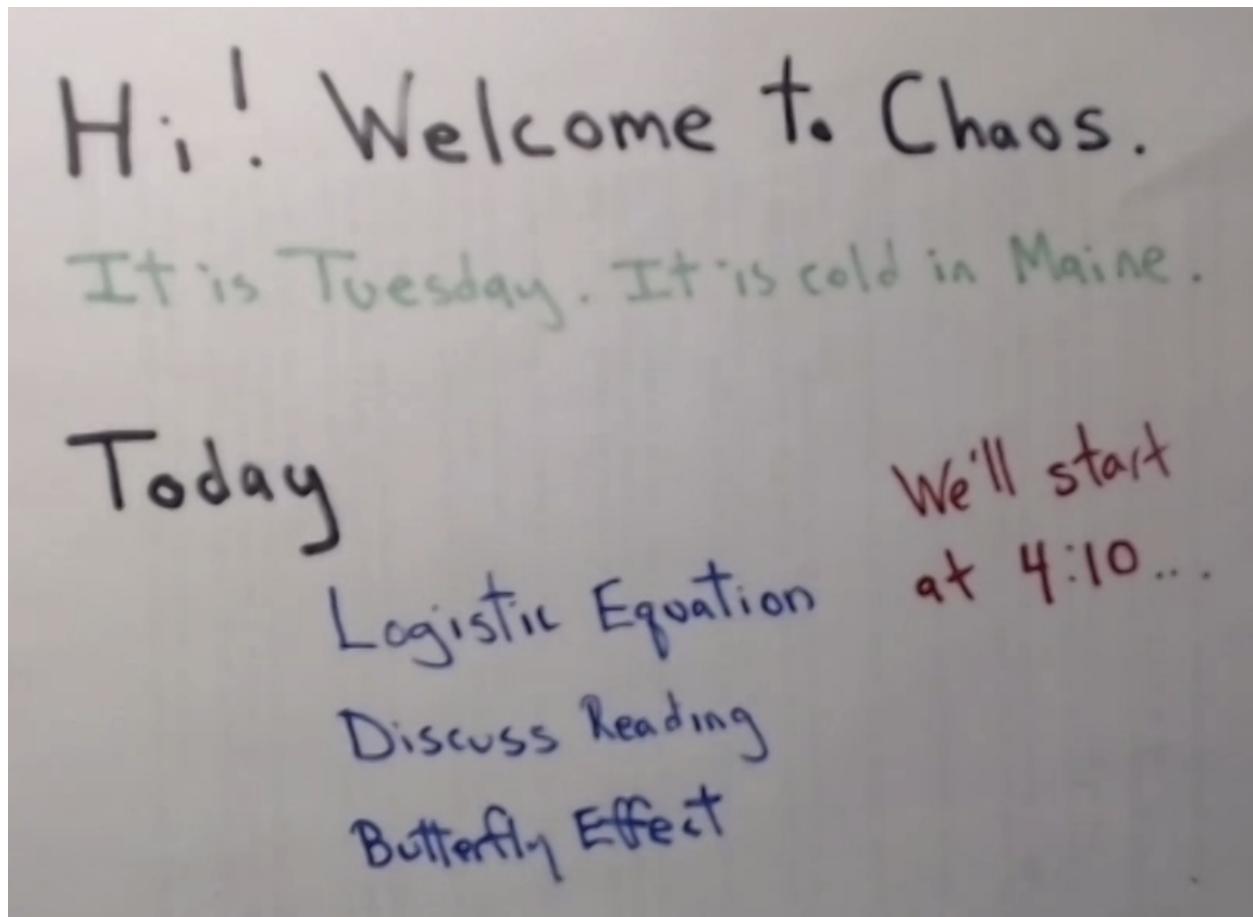


Week 2 class 2 - Chaos and Fractals (lab)

Phileas Dazeley-Gaist

10/01/2022

Today's goals



Trajectory types on the logistic map

For $0 \leq r \leq 1$: The trajectory approaches 0

Chaos and Fractals: An Elementary Introduction

David P. Feldman

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Logistic Equation: Time Series Plots

This program will make time series plots for the logistic equation $f(x)=rx(1-x)$. In the boxes below enter a value for the parameter r , the initial condition x_0 , and n , the number of iterates you want plotted.

Number of Iterations (n):	40
Initial condition (x_0):	0.2
Parameter (r):	0.5

[Make the time series plot](#)

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[Make the time series plot](#)

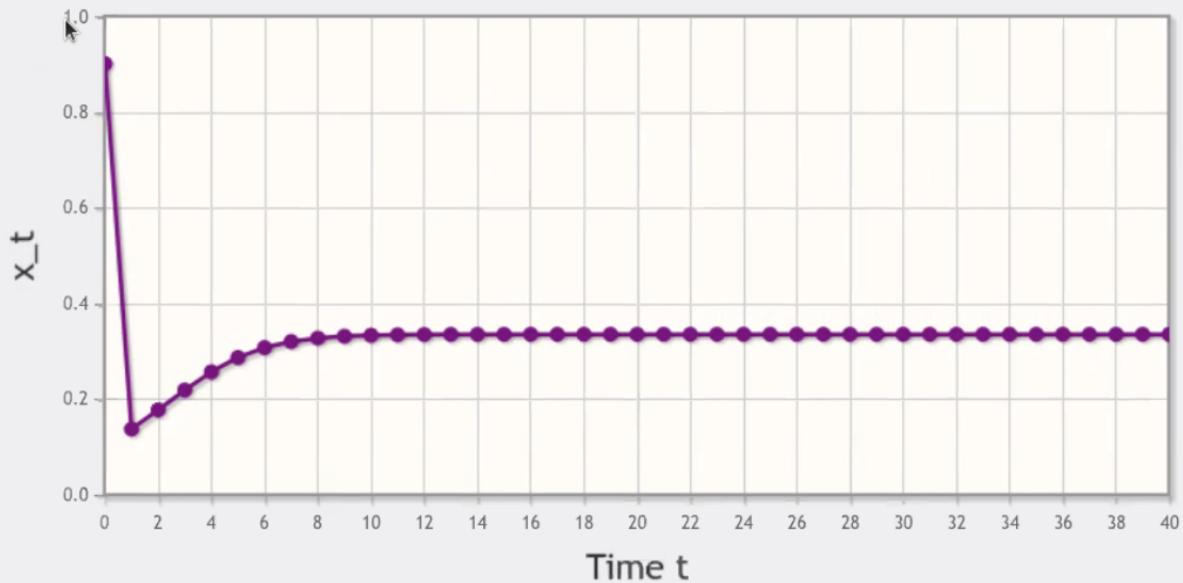
For $1 < r < 3$: The trajectory approaches a fixed point.

Number of Iterations (n):

Initial condition (x_0):

Parameter (r):

[Make the time series plot](#)

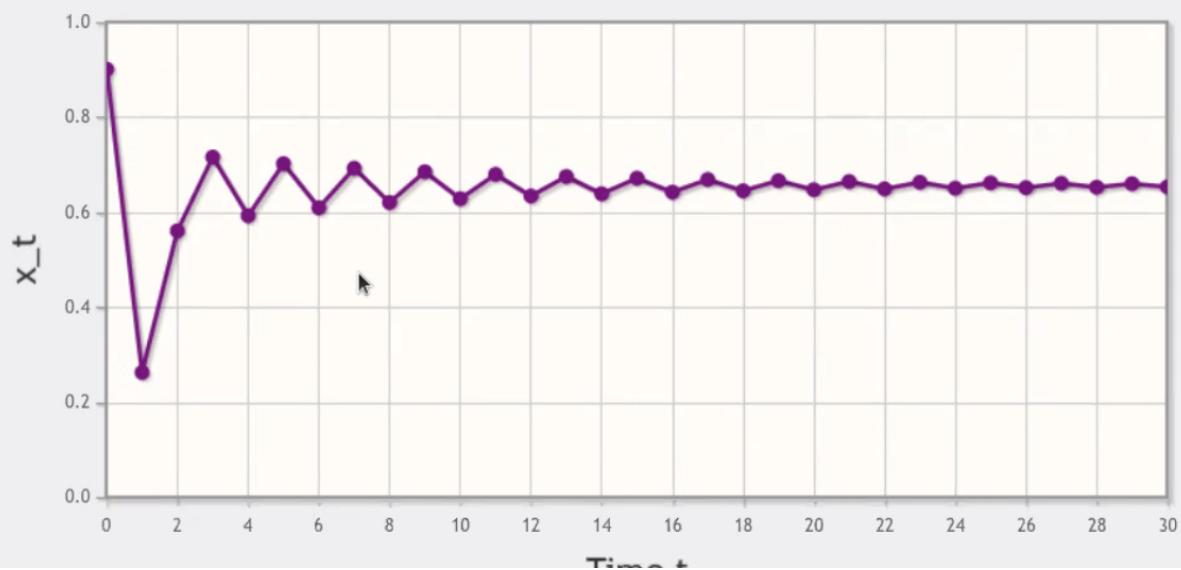


Number of Iterations (n):

Initial condition (x_0):

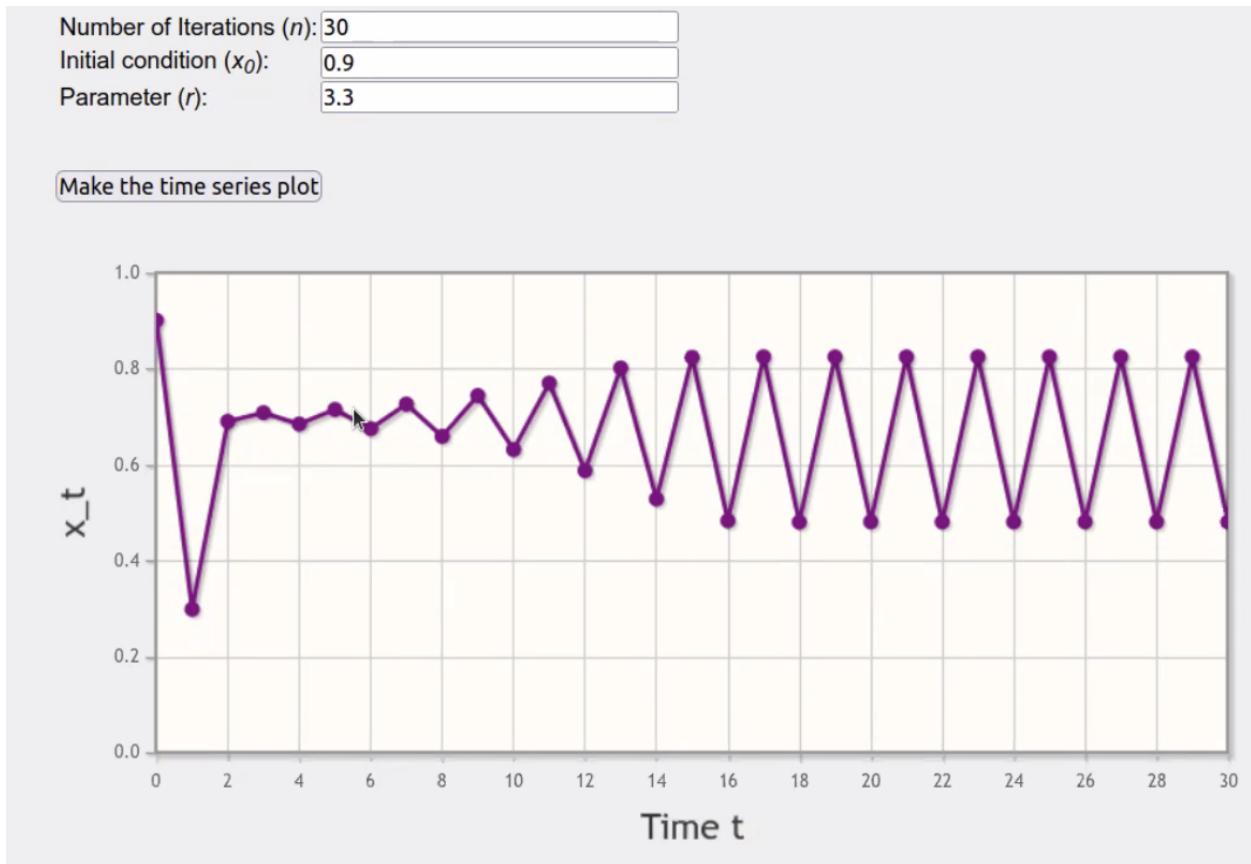
Parameter (r):

[Make the time series plot](#)



For $3 \leq r < 4$: Things get weird.

For a range of values of r , the trajectory enters a periodic behaviour of period 2.

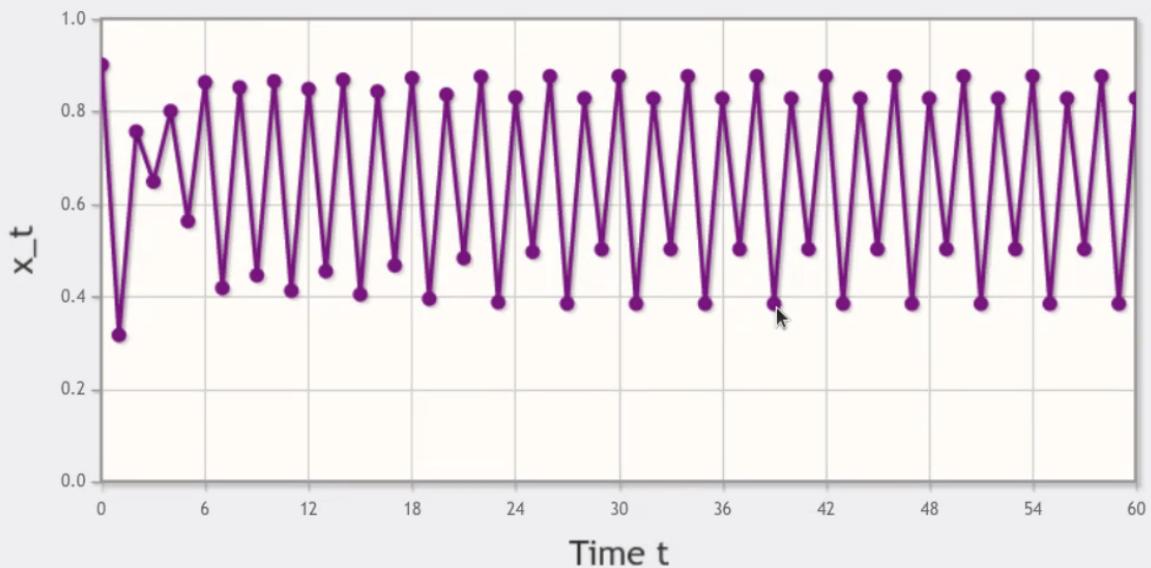


For higher values of r the period of the cycle doubles, resulting in period 4. Then for even higher values, it doubles again, and then again for even higher values!

Number of Iterations (n):

Initial condition (x_0):

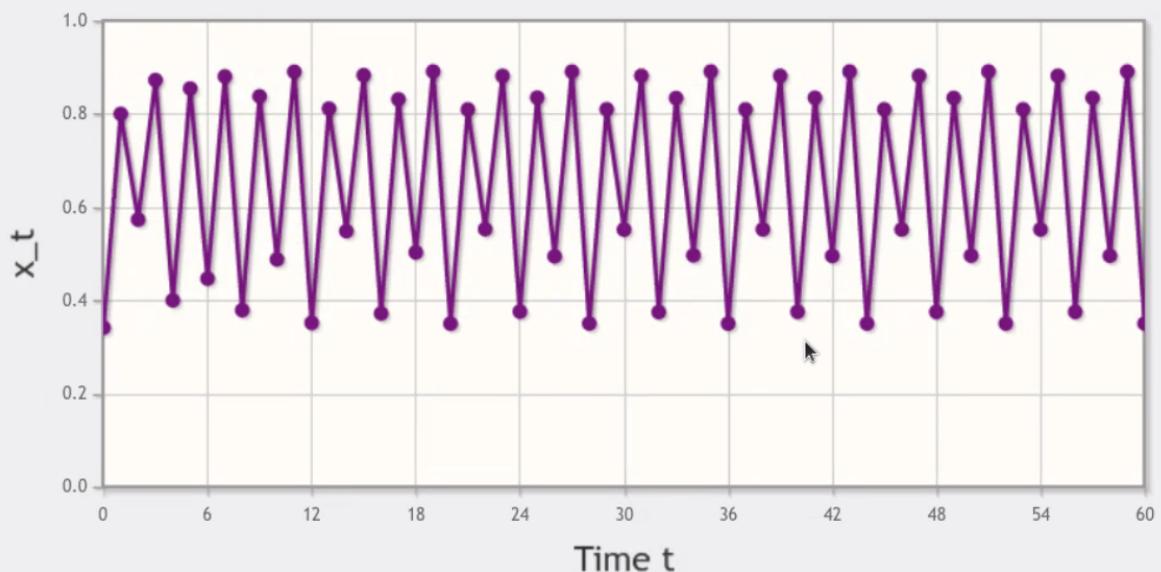
Parameter (r):



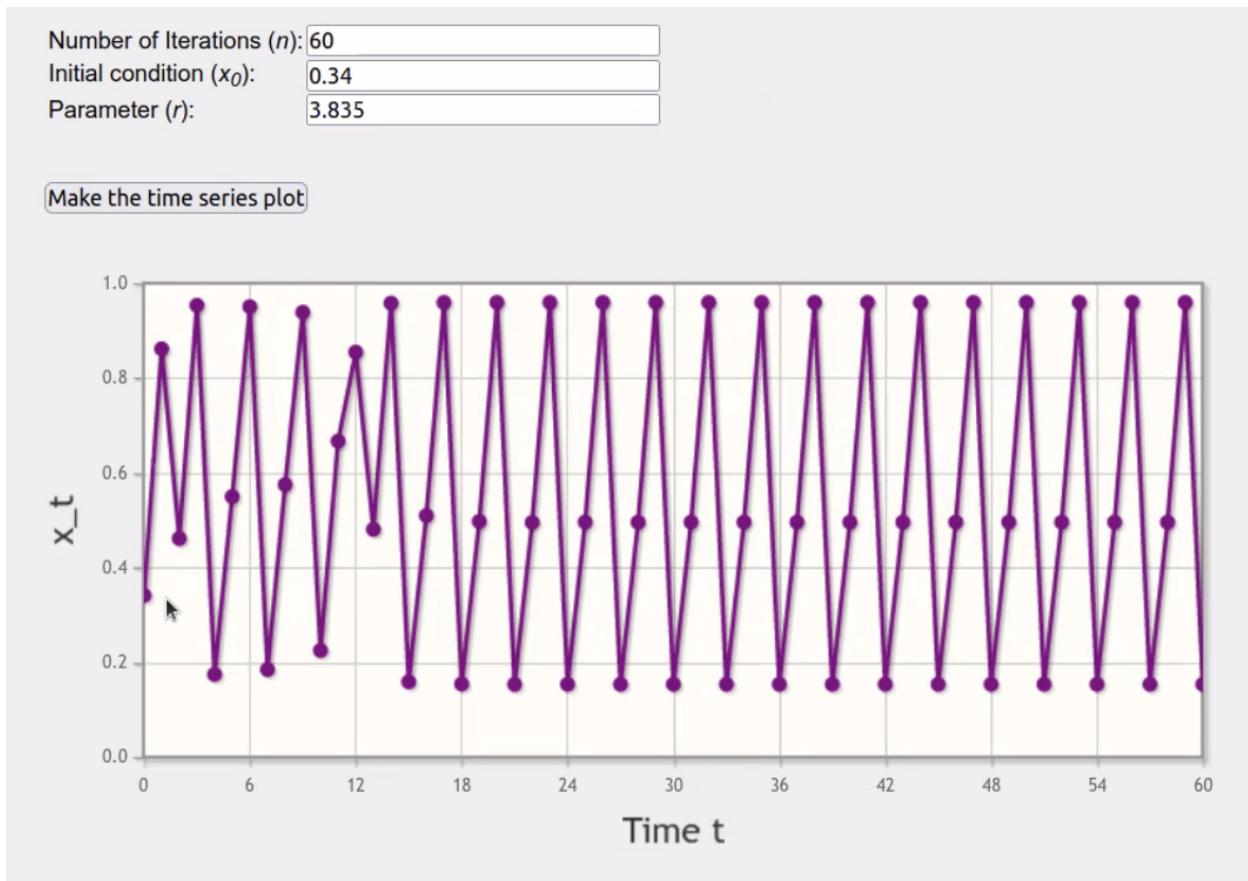
Number of Iterations (n):

Initial condition (x_0):

Parameter (r):



Then, for high enough values of r the period stops settling, and we get aperiodic behaviour. But within this aperiodic streak, some values of r will return periodic trajectories.

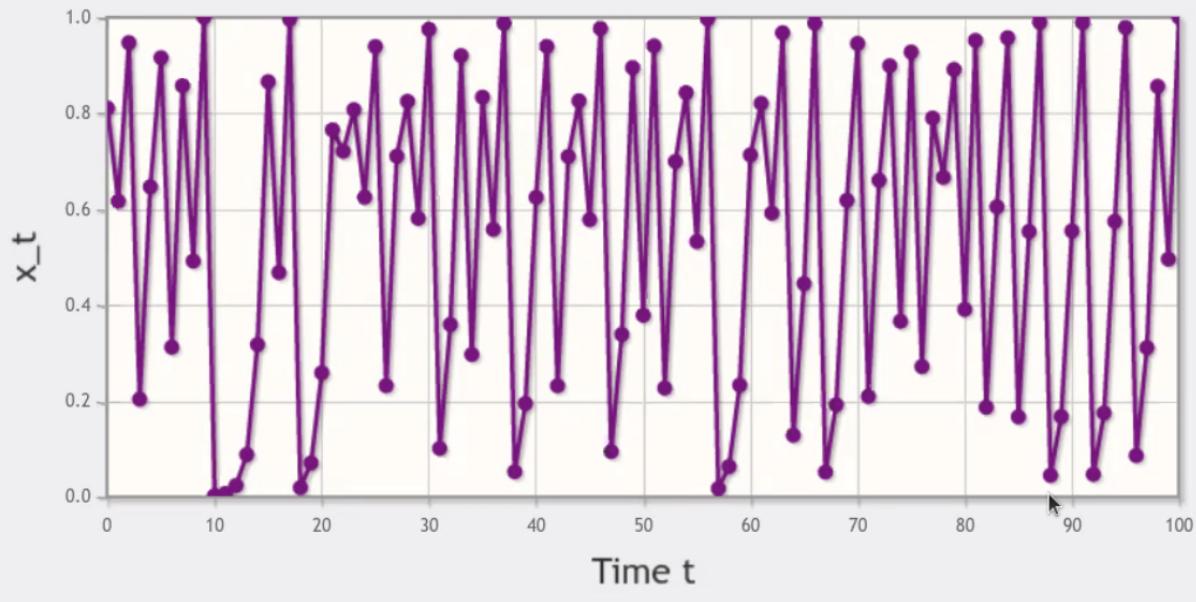


Finally, for $r = 4$, we get an aperiodic trajectory.

Number of Iterations (n): 100

Initial condition (x_0): 0.81

Parameter (r): 4.0



Sensitive dependence on initial conditions

Notice that in aperiodic trajectories, a very slight variation in r will result in a completely different trajectory somewhere down the line. This is known as the butterfly effect, or sensitive dependence on initial conditions.

Number of Iterations (n):
First Initial condition (x_0):
Second Initial condition (y_0):
Parameter (r):

[Make the time series plot](#)

