Chapter 7

(7.2) Suppose a population of rabbits starts at 50 in 2010 and doubles every year. What is the rabbit population in 2020? In what year would there be more rabbits than there are people?

- In 2020, there are 51200 rabbits.

```
> iterator <- function(x_0, n){
+   x <- x_0
+   for(i in 1:n){
+     x <- x*2
+   }
+   print(x)
+ }

> iterator(50, 2020-2010)
[1] 51200
```

- The rabbit population will overtake the human population in numbers by 2038.

```
> iterator <- function(x_0, stop){
+   x <- x_0
+   year <- 2010
+   while(x<=stop){
+     x <- x*2
+     year <- year + 1
+   }
+   print(year)
+ }

> iterator(50, 7900000000)
[1] 2038
```

Chapter 7 Optional Problems:

(7.9) Consider the logistic equation, $f(x) = rx(1 - x)$.

(a) Determine an algebraic expression for the non-zero fixed point. (Your answer will depend on r.)

$$rx(1-x) = x$$

$$rx - rx^2 = x$$

$$rx - rx^2 - x = 0$$

$$x(r - rx - 1) = 0$$

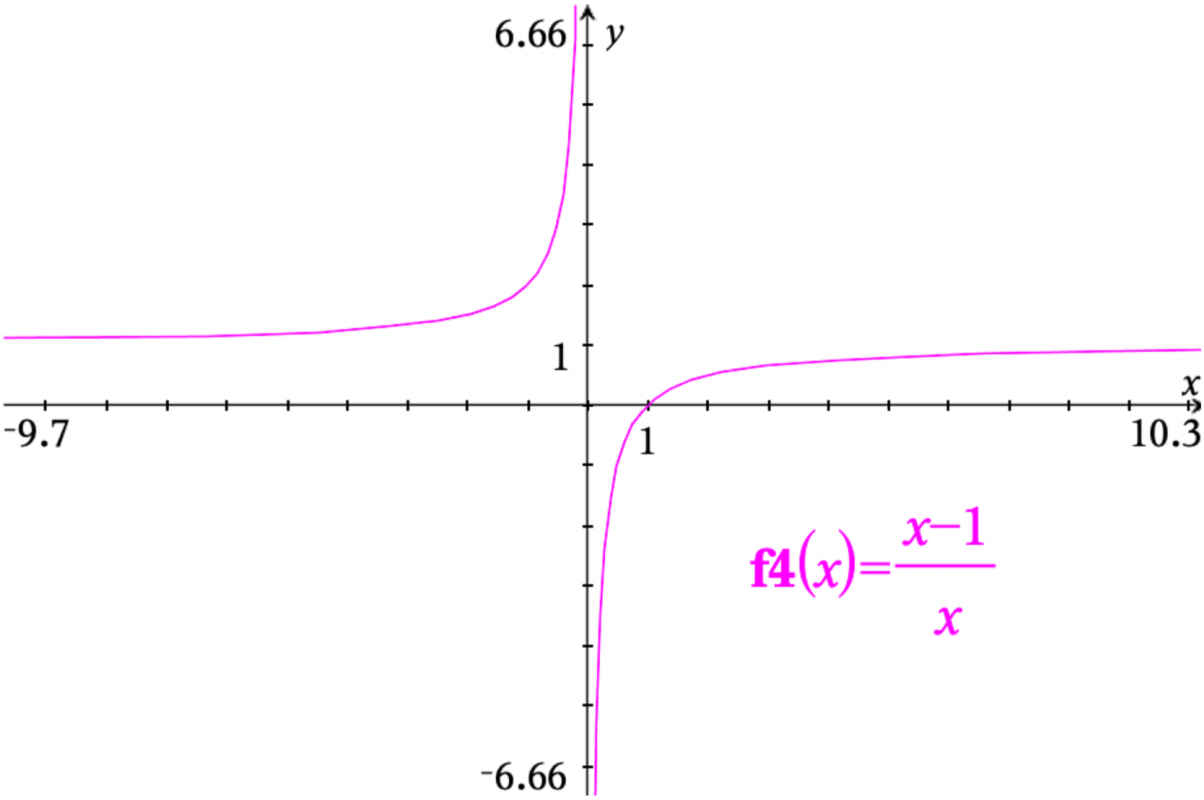
$$x = 1 - \frac{1}{r} =$$

$$x = \frac{r-1}{r}$$

The fixed point of the logistic map is given by the equation $\frac{r-1}{r} = rx(1-x)$.

(b) For what r values is this fixed point positive?

This fixed point is positive for r>1 (or r<0)



(Above, r is in its best x Mardi Gras disguise.)

(c) For what r values is this fixed point less than 1?

This fixed point is less than 1 for 0<r<1

(7.10) ♯ Consider the logistic equation with r = 3.2, as shown in Fig. 7.10.

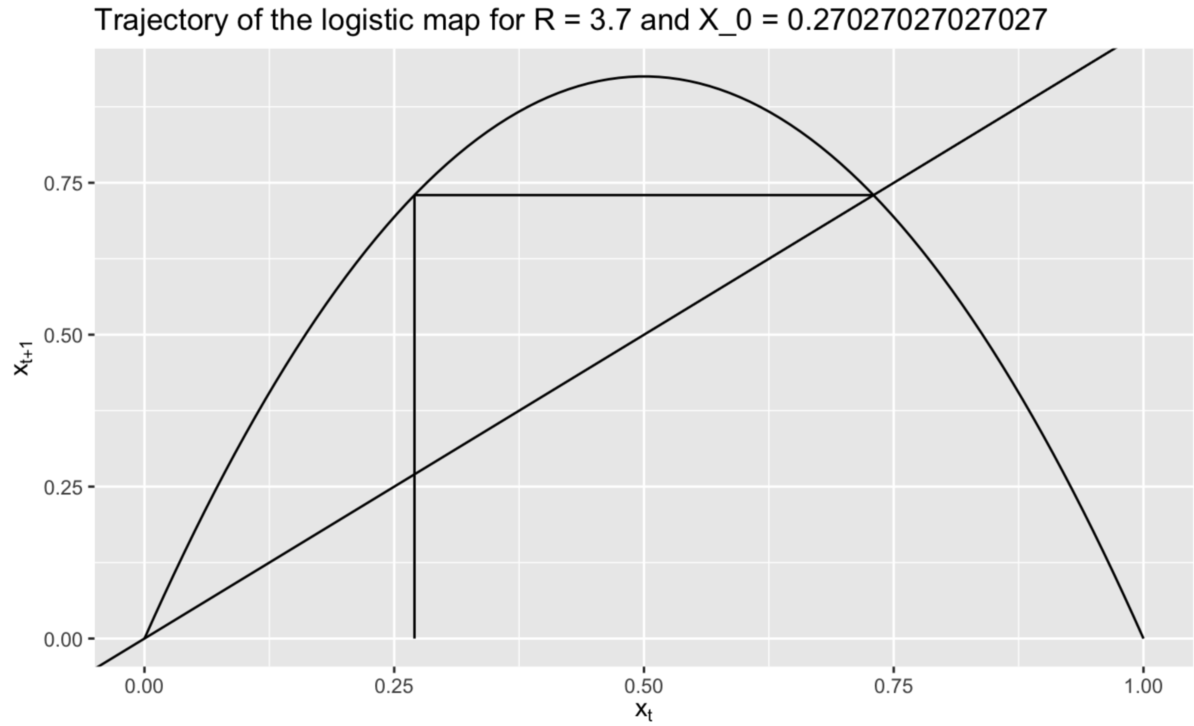
(a) Find the exact value for the fixed-point x∗ near x = 0.7.

(Personal note: x is just notation for a fixed point.)*

$$x^* = \frac{r-1}{r} = \frac{3.7-1}{3.7} = 0.\overline{729}$$

(b) There are two values which, when acted upon by f(x), yield the fixed point. One, of course, is the fixed point itself. But there is another point, denoted x∗−1 that lands on the fixed point after just one iteration.

(i) Sketch the function and illustrate the point x∗−1 by graphically iterating and finding an input value that lands on the fixed point.



(ii) Use algebra to find x∗−1 and check that it is consistent with the point you found graphically.

$$\frac{r-1}{r} = rx(1-x)$$

$$\frac{3.7-1}{3.7} = 3.7x(1-x)$$

Solve for x:

I'm going to skip doing this by hand because this is an optional assignment:

$$\text{solve}\left(\frac{3.7-1}{3.7}=3.7\cdot x\cdot(1-x),x\right) \qquad x=0.27027 \text{ or } x=0.72973$$

The fixed-point x*-1 is at $x = 0.\overline{270}$.

(7.11) ♯ Consider the logistic equation f(x) = rx(1 − x) with r = 3.2. We have seen that for this parameter value the logistic equation has a cycle of period two. We will use algebra to determine the x values that make up this cycle. To do so, note that if a point x∗ is periodic with period two, then it satisfies the following equation: f(2)(x∗) = x∗. In other words, if we start with x∗, apply f to it twice, we return to x∗.

(a) Determine an algebraic expression for f (2) (x). It might be easier to not plug in for r until the very end.

$$f\big(f(x^*)\big) = x^*$$

$$f\big(f(x^*)\big) = rrx(1-x)(1-rx(1-x)$$

(2)∗ ∗ ∗ (b) Solve the equation f (x) = x for x.

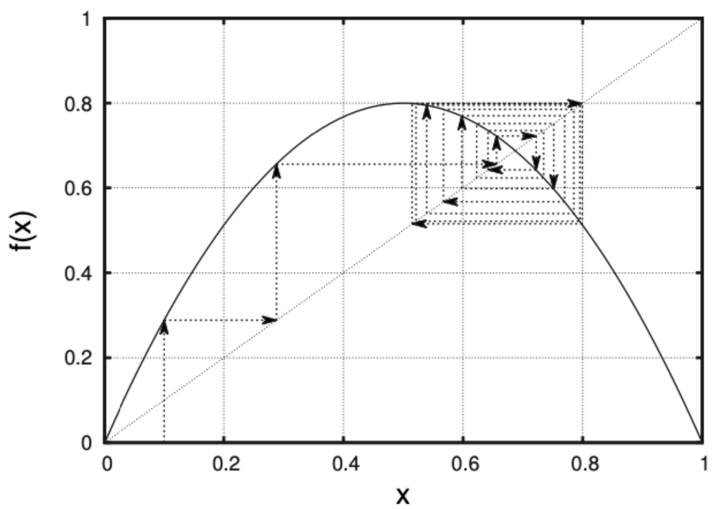
$r{:=}3.2$	3.2
$f(x){:=}r\cdot x\cdot(1-x)$	*Done*
$f(f(x))=r\cdot r\cdot x\cdot(1-x)\cdot(1-r\cdot x\cdot(1-x))$	true
solve$(f(f(x))=x,x)$	$x=0.$ or $x=0.513045$ or $x=0.6875$ or $x=0.799455$

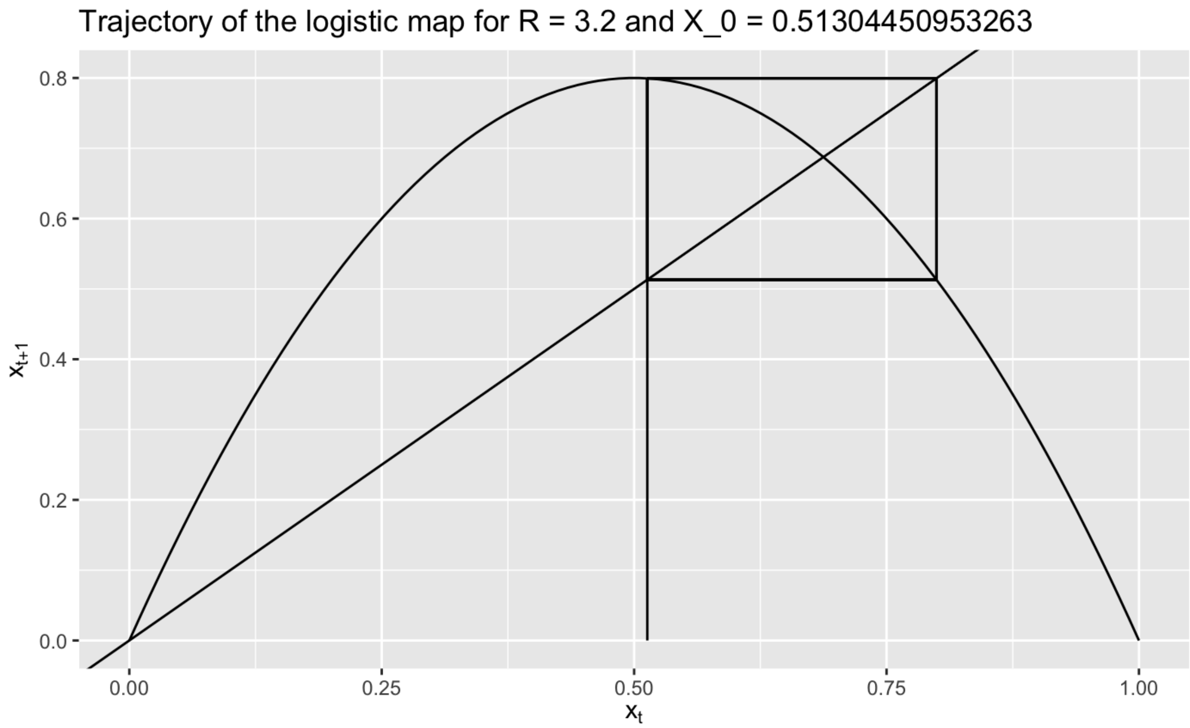
Which of your solutions have period two?

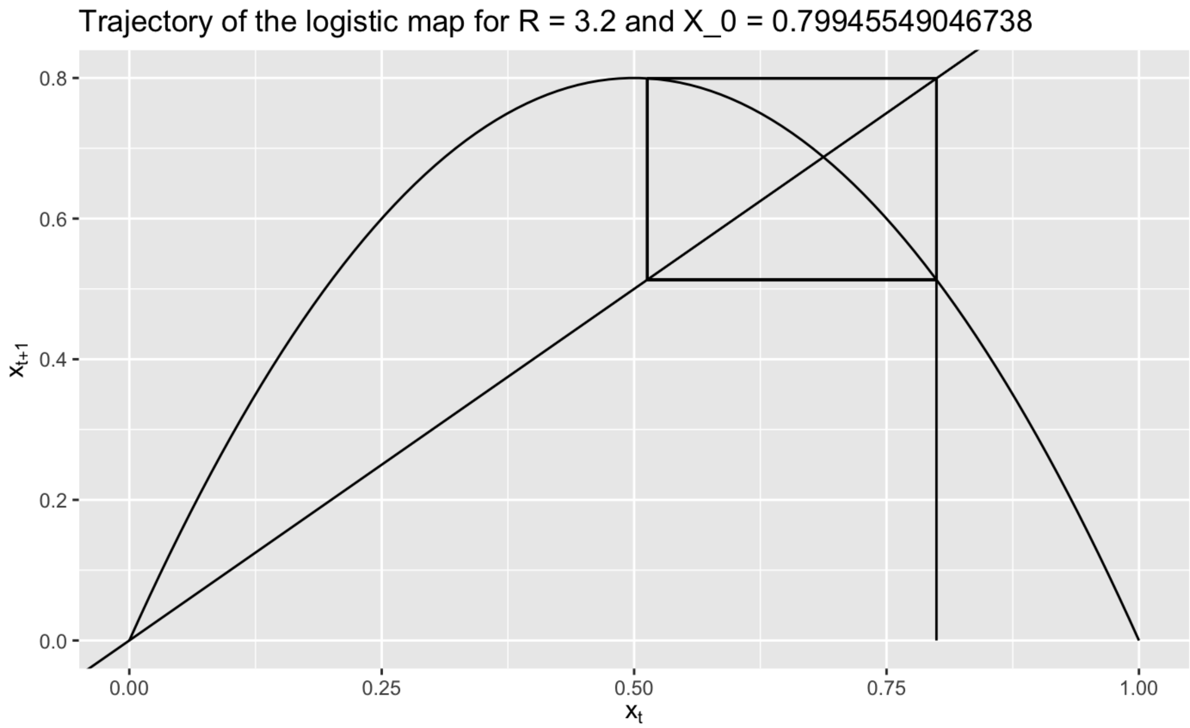
x* = 0.51304450953263 and x* = 0.79945549046738 have period = 2.

(c) How do the two periodic points you found compare with those shown in Fig. 7.10?

The trajectories for $x^* \sim 0.513$ and $x^* \sim 0.799$ immediately oscillate between the fixed points of the periodic a tractor for r = 3.2. The orbits do not spiral out, as the orbit in figure 7.10 does.




Trajectory of the logistic map for R = 3.2 and X_0 = 0.51304450953263
0.8
0.6
0.4
0.2
0.0
x_{t+1}
0.00
0.25
0.50
0.75
1.00
x_t

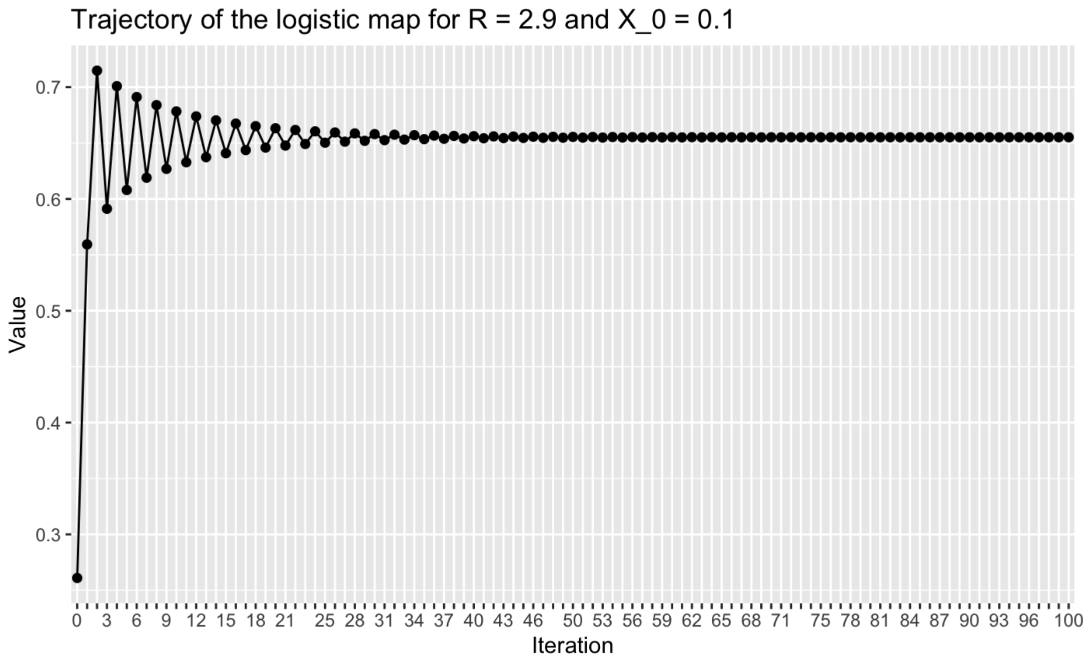

Trajectory of the logistic map for R = 3.2 and X_0 = 0.79945549046738
0.8
0.6
0.4
0.2
0.0
x_{t+1}
0.00
0.25
0.50
0.75
1.00
x_t

Chapter 9

(9.5) Use the logistic orbits program to determine the long-term behavior of the following r values. For each r value determine as best you can if the orbits are periodic or aperiodic. If they are periodic, state the period:

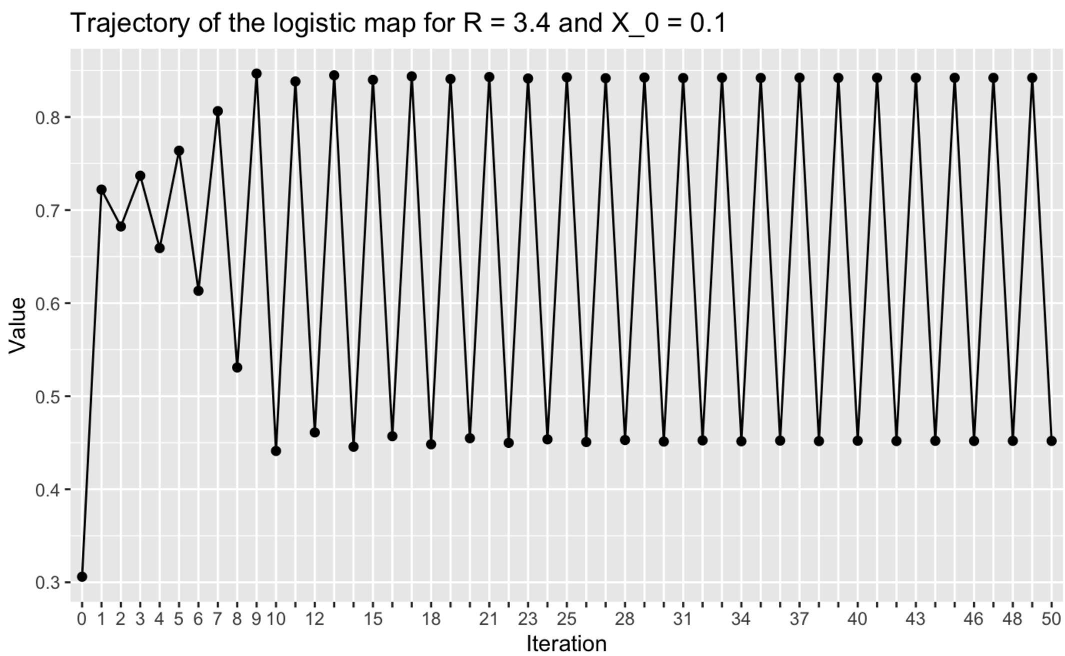
(a) r = 2.9

The orbit converges to a fixed point: ~ 0.655.



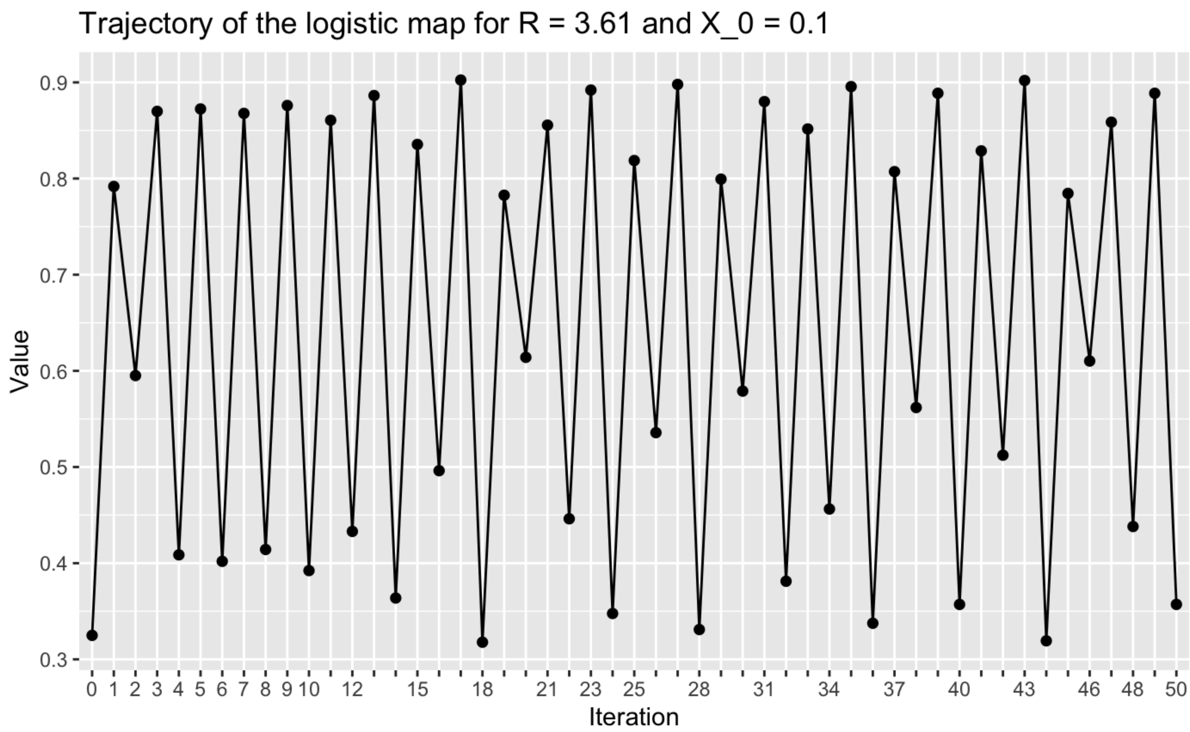
(b) r = 3.4

The orbit enters a cycle of period 2, oscillating between the approximate values 0.84 and 0.45.



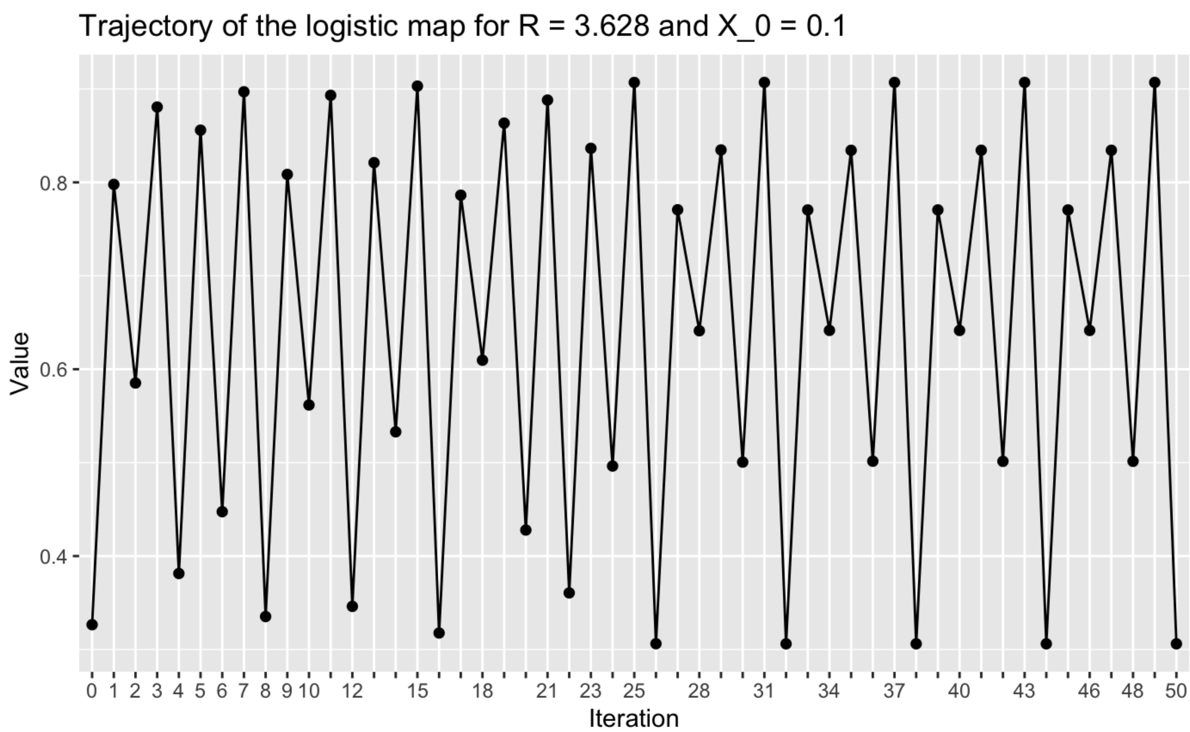
(c) r = 3.61

The orbit enters a cycle of period 4, oscillating between the approximate values 0.610, 0.859, 0.438, and 0.889.



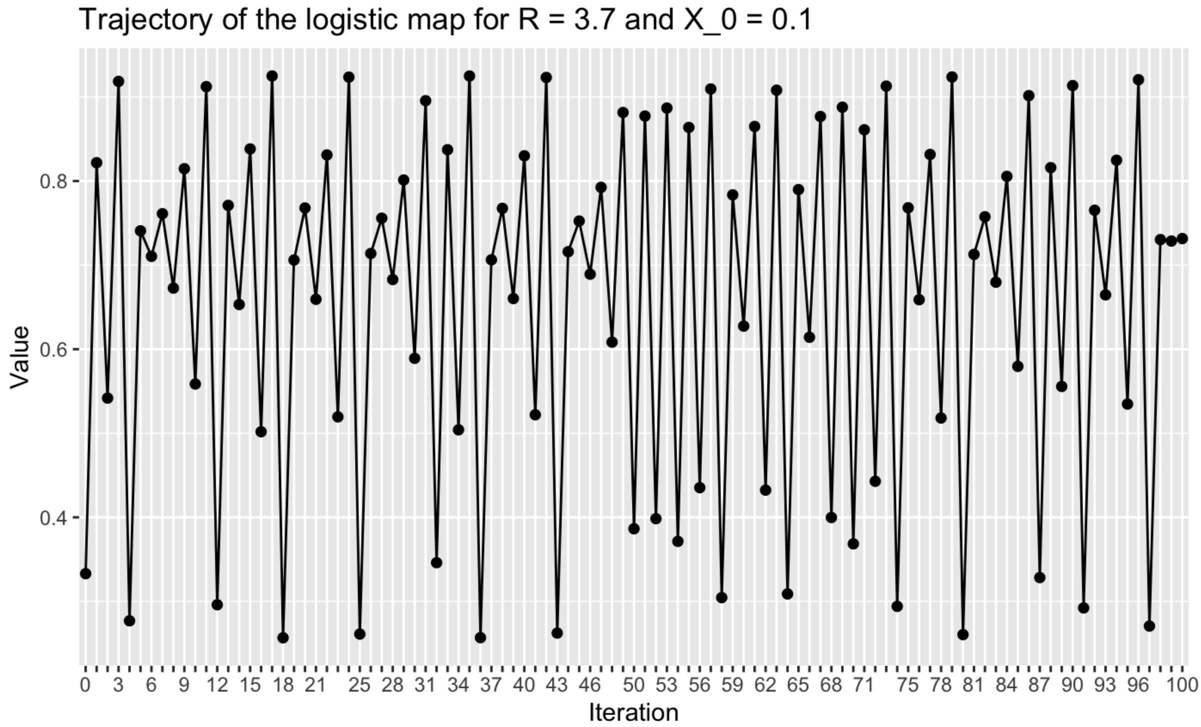
(d) r = 3.628

The orbit enters a cycle of period 6, oscillating between the approximate values 0.501, 0.907, 0.306, 0.770, 0.641, and 0.834.



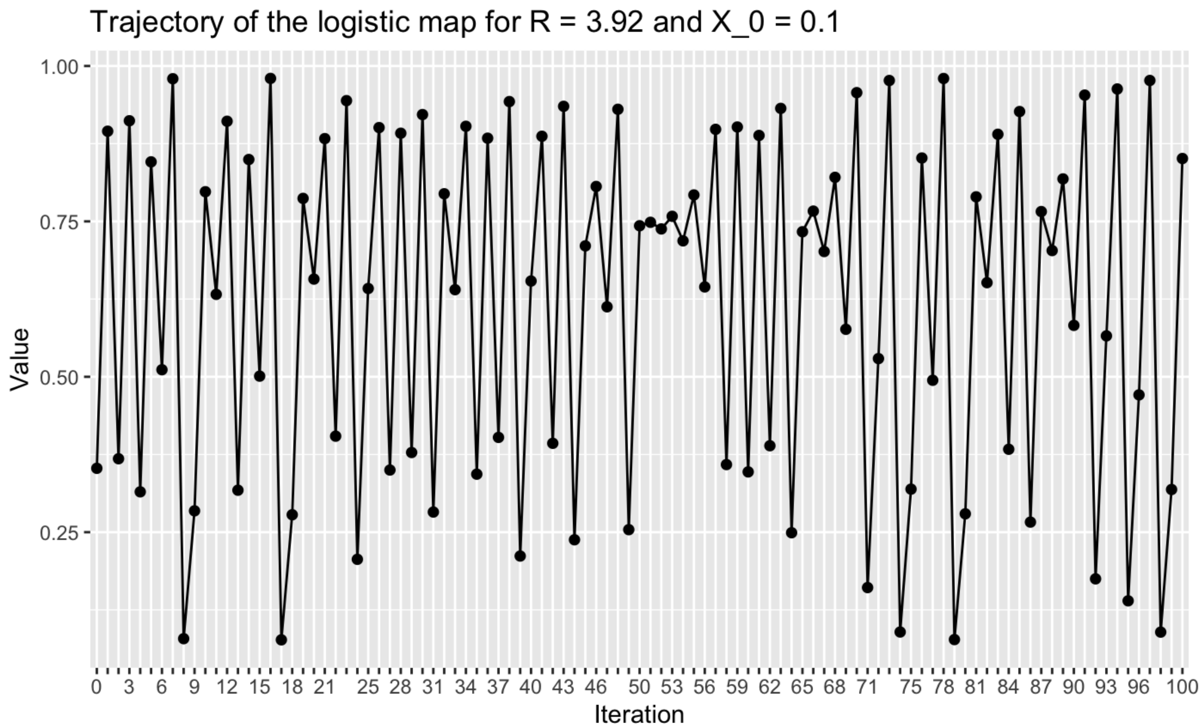
(e) r = 3.7

The orbit is aperiodic.



(f) r=3.92

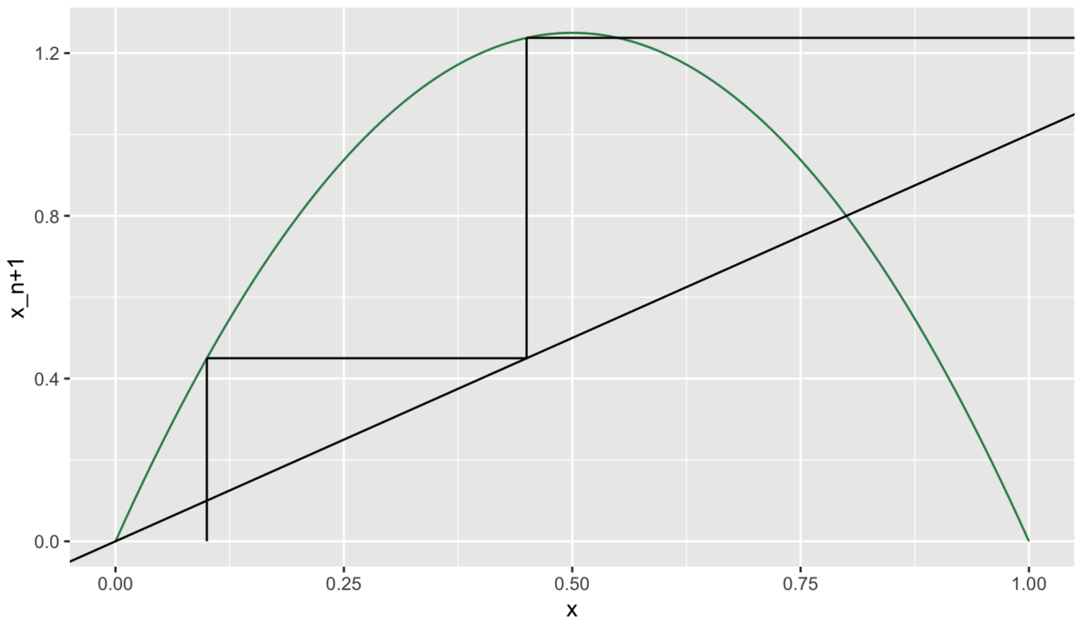
The orbit is aperiodic.



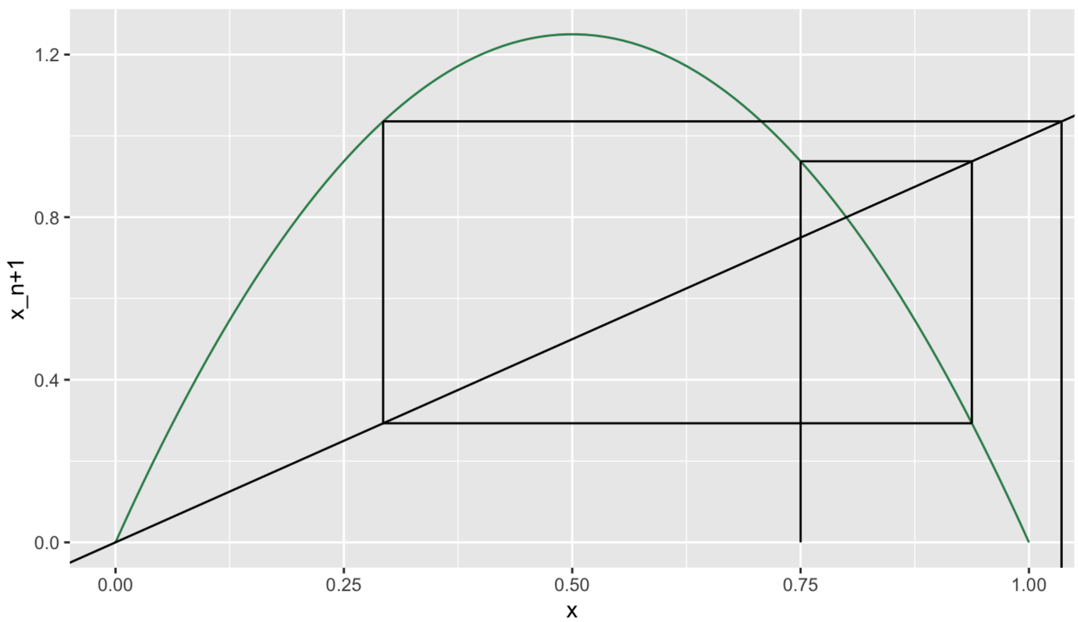
(9.7) The logistic equation is not used for r values above 4.0. If r is above 4.0 the model of population growth no longer makes sense. To see why, we will consider the logistic equation with r = 5.0. This function is plotted in Fig. 9.24.

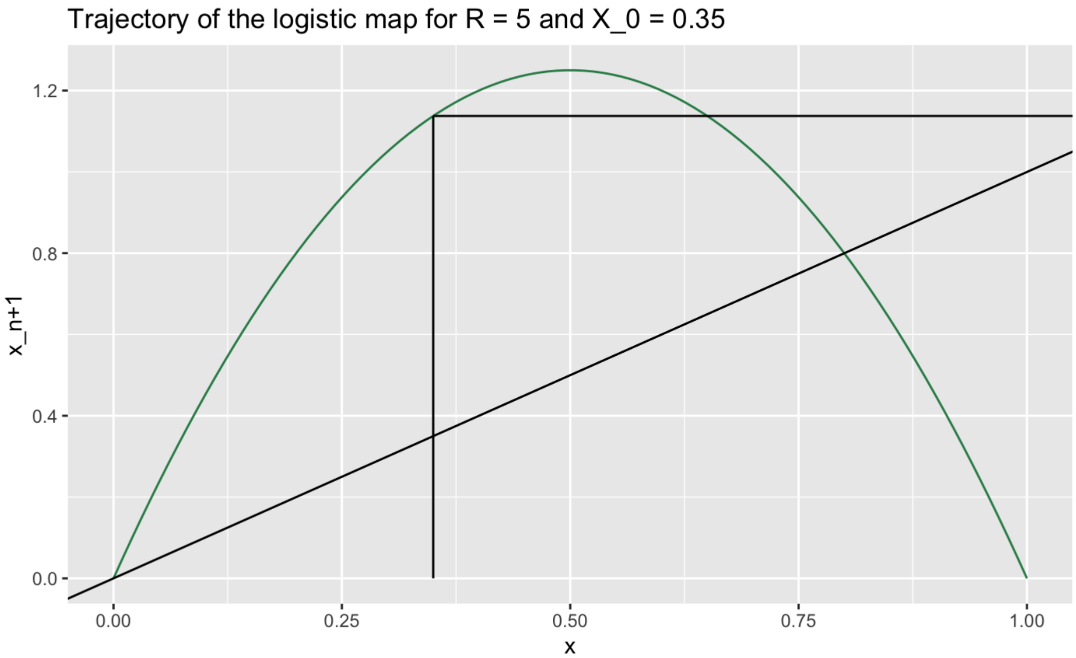
(a) Choose a few initial conditions and iterate them graphically.

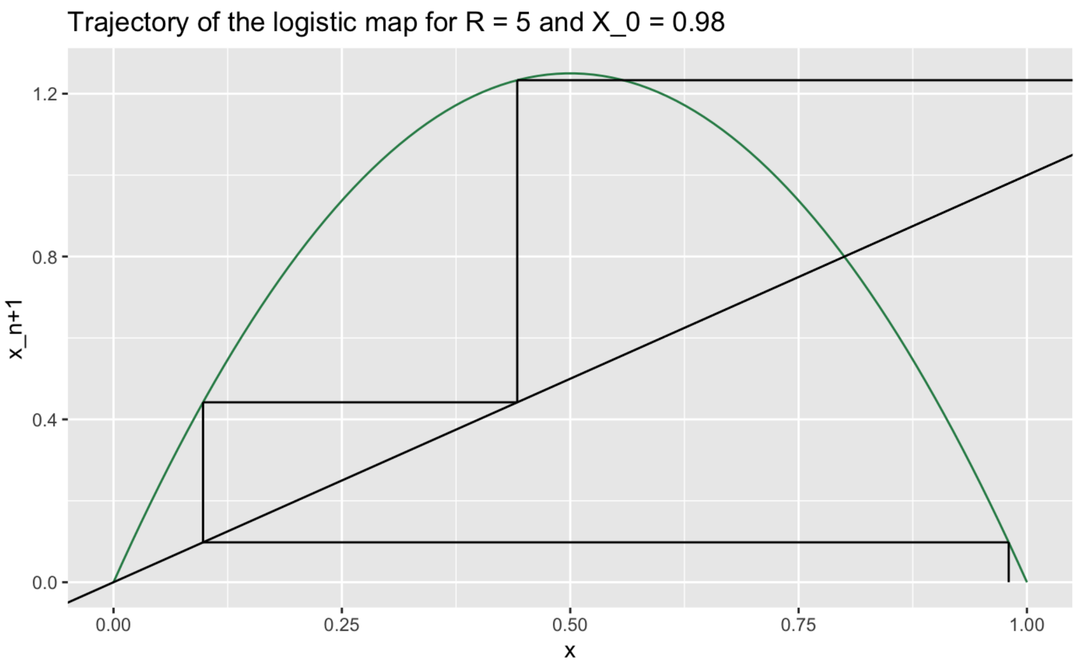
Trajectory of the logistic map for R = 5 and X_0 = 0.1



Trajectory of the logistic map for R = 5 and X_0 = 0.75







(b) What is the long-term fate of these orbits?

The orbits rapidly go negative. The values returned by the orbit tend rapidly towards negative infinity.

(c) What does this let you conclude about the model for r = 5.0? Why does this model not make sense if used to describe the growth of a population?

If used to describe the growth of a population, using r = 5.0 in the logistic equation presupposes that we accept the existence of negative population sizes. This does not make sense.

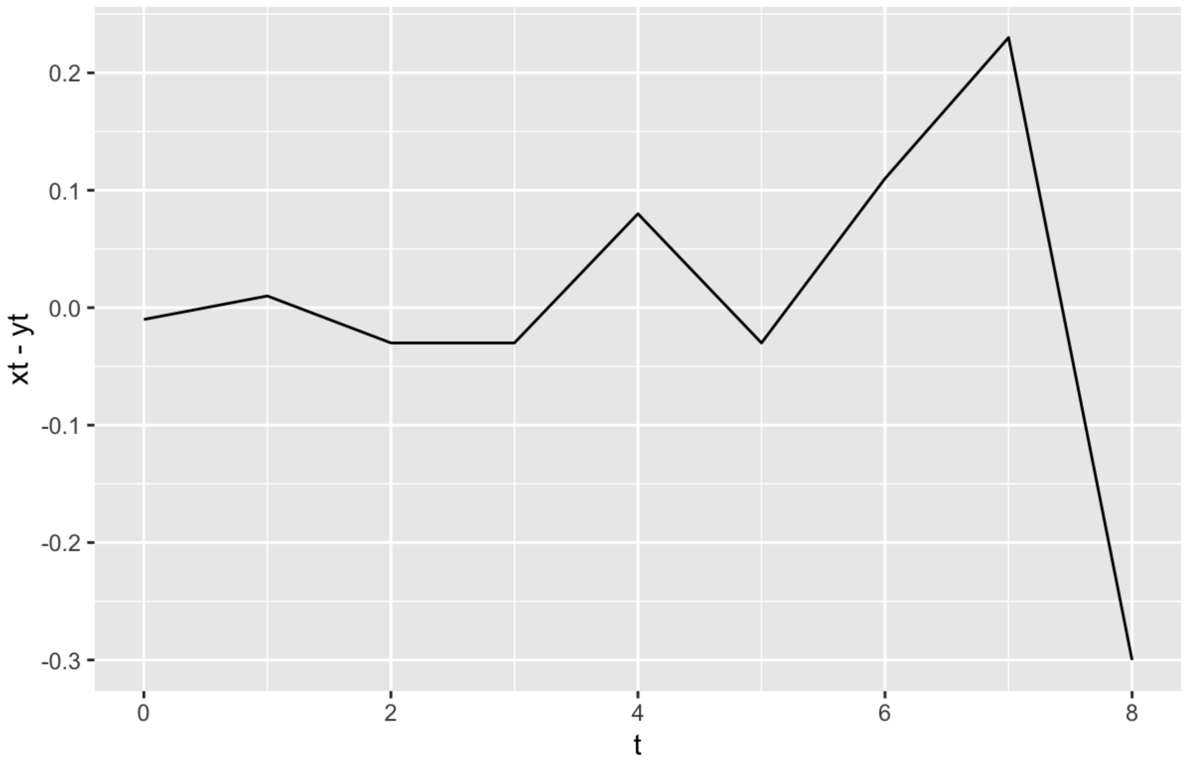
Chapter 10

(10.1) In Table 10.2 are shown the first eight iterates for two different initial conditions. The function is the logistic equation with r = 3.8. Make a plot of the difference between x_t and y_t, as was done in Fig. 10.4.

```
library(tidyverse)

t <- 0:8
xt <- c(0.60, 0.91, 0.30, 0.81, 0.59, 0.92, 0.29, 0.79, 0.63)
yt <- c(0.61, 0.90, 0.33, 0.84, 0.51, 0.95, 0.18, 0.56, 0.93)

data <- tibble(t, xt, yt)
data %>% ggplot(aes(t, xt-yt)) + geom_line()
```



<u>Chapter 10 Optional Problems:</u>

(10.3) In this exercise and the next we will examine the function f(x) = 2x. Iterating this function yields a dynamical system that is not chaotic, since the orbits are not bounded. However, the system does possess SDIC. In this exercise you will consider a few particular cases and use the definition in the second paragraph of Section 10.3.

(a) Let x0 = 2.0. Compute the first ten iterates of x0.

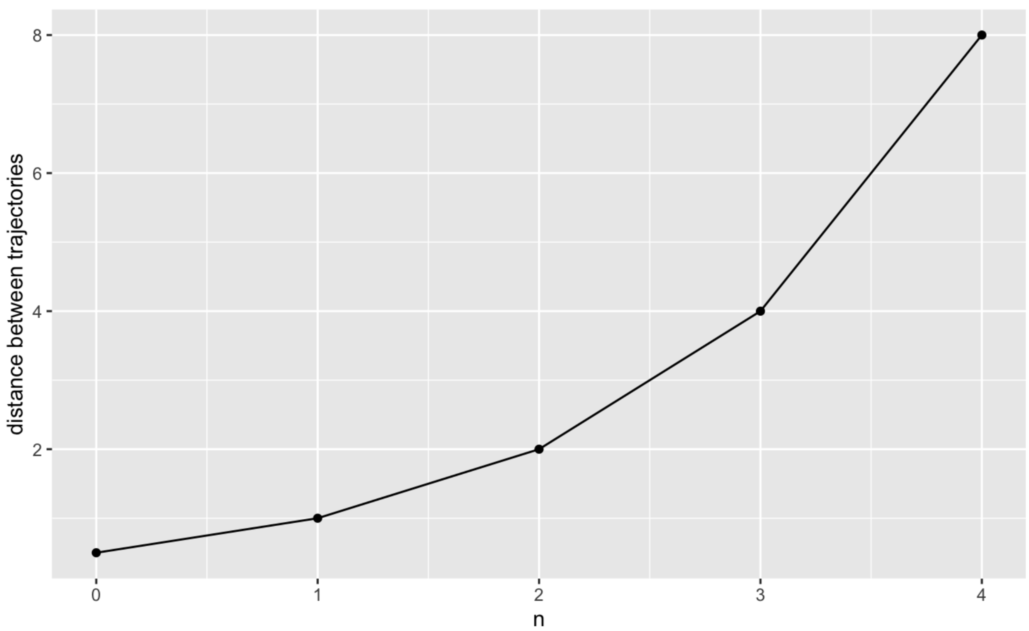
iterate n	value
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024
11	2048
12	4096

(b) Let x0 = 2.0, $\varepsilon = 1.0$, and $\delta = 4.0$. Find an initial condition y0 that is within ε of x0 and which has the property that eventually its orbit is a distance δ away from the orbit of x0.

Starting at y0 = 2.5, by iteration 4, the orbit of y0 is a distance $> \delta$ away from the orbit of x0 = 2.0.

Personal note: p.112 for definitions of epsilon and delta (how big the difference between different tested x_0s is epsilon, delta is what happens after a long time).

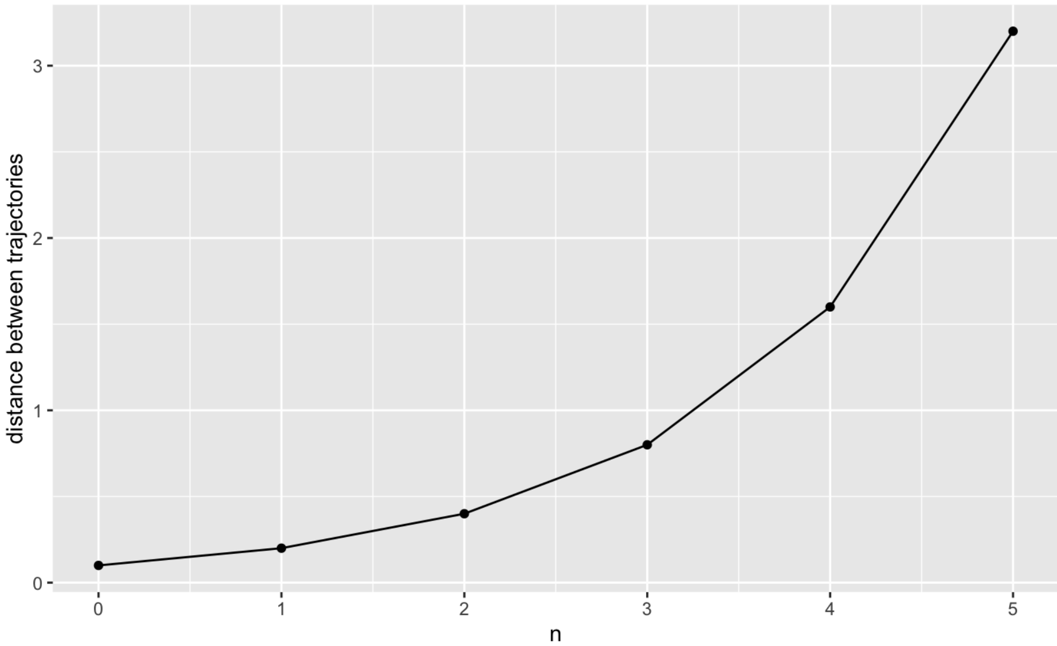
ns <int>	x_0_2 <dbl>	x_0_2.5 <dbl>	distance <dbl>
0	2	2.5	0.5
1	4	5.0	1.0
2	8	10.0	2.0
3	16	20.0	4.0
4	32	40.0	8.0



(c) Let x0 = 2.0, ε = 0.50, and δ = 2.0. Find an initial condition y0 that is within ε of x0 and which has the property that eventually its orbit is a distance δ away from the orbit of x0.

Starting at y0 = 2.1, by iteration 5, the orbit of y0 is a distance > δ away from the orbit of x0 = 2.0.

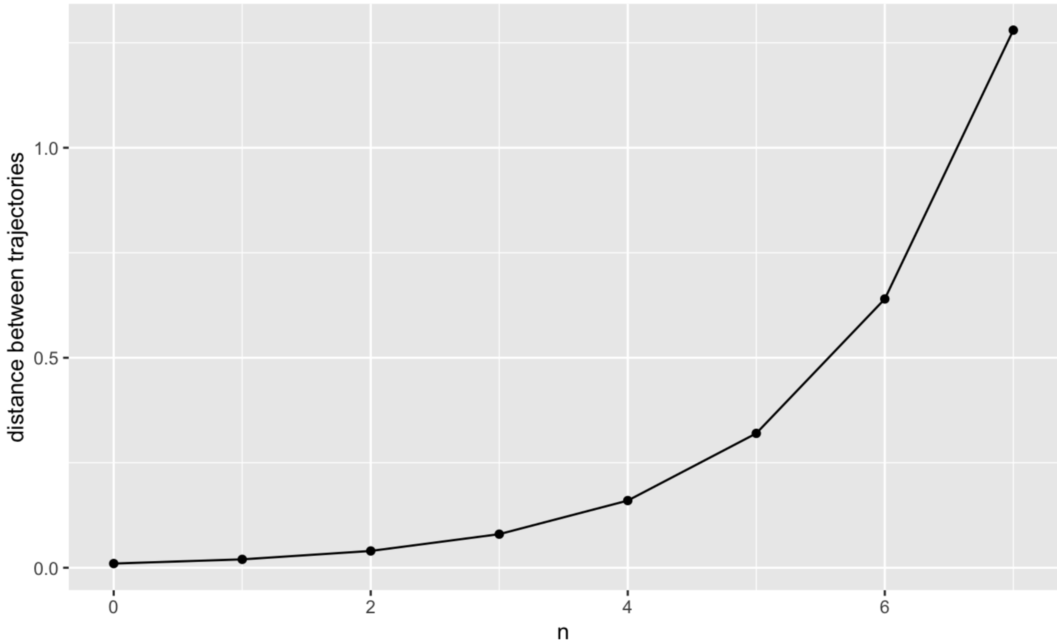
ns <int>	x_0_2 <dbl>	x_0_2.1 <dbl>	distance <dbl>
0	2	2.1	0.1
1	4	4.2	0.2
2	8	8.4	0.4
3	16	16.8	0.8
4	32	33.6	1.6
5	64	67.2	3.2



(d) Let x0 = 2.0, ε = .1, and δ = 1.0. Find an initial condition y0 that is within ε of x0 and which has the property that eventually its orbit is a distance δ away from the orbit of x0.

Starting at y0 = 2.01, by iteration 7, the orbit of y0 is a distance > δ away from the orbit of x0 = 2.0.

ns <int>	x_0_2 <dbl>	x_0_2.01 <dbl>	distance <dbl>
0	2	2.01	0.01
1	4	4.02	0.02
2	8	8.04	0.04
3	16	16.08	0.08
4	32	32.16	0.16
5	64	64.32	0.32
6	128	128.64	0.64
7	256	257.28	1.28



(10.4) ♯ This is a continuation of Exercise 10.3. Again consider the function f(x) = 2x.

(a) Suppose we have two different initial conditions, x0 and y0. Show that after one iteration, the difference between these two initial conditions has doubled. In other words, show that: x1 −y1 = 2(x0 −y0). (10.8)

$f(x):=2\cdot x$ Done

$f(y)-f(x)=2\cdot(y-x)$ true

(b) Use this result to argue that the function has SDIC.

The results above, which show that the distance between the orbits of x0 and y0 double at each iteration signify that for any values of x0, ε, and δ, and y0 within ε of x0, the orbit of y0 will differ by an absolute value > δ from the orbit of x0 after some number of iterations.