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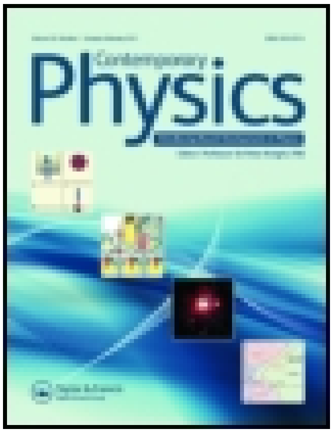

Contemporary Physics
Publication details, including instructions for authors and subscription information:
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Published online: 08 Nov 2010.

To cite this article: Jonathan Leach , Miles Padgett & Johannes Courtial (2003) Fractals in pixellated video feedback, Contemporary Physics, 44:2, 137-143, DOI: 10.1080/0010751021000029598

To link to this article: http://dx.doi.org/10.1080/0010751021000029598

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Contemporary Physics, 2003, volume 44, number 2, pages 137–143

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Fractals in pixellated video feedback

JONATHAN LEACH, MILES J. PADGETT, and JOHANNES COURTIAL*

Using experiments and computer simulations we show that video feedback with a pixel-based monitor can lead to self-similar stationary patterns. We found an astonishing variety of such patterns, which includes many 'classic fractals' such as the Sierpinski gasket and the von Koch snowflake. An important, previously unidentified parameter is the position of the centre of the camera zoom with respect to the individual pixels on the screen. This article is a detailed account of work recently published by the present authors.

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1. Introduction

Very few scientific phenomena have a weekly television show devoted to them. Video feedback is one of them; the half-hour program *Spinning Lights* on a New York cable television station features exclusively 'beautiful and mesmerizing video feedback images with relaxing, meditative music' [1]. Video feedback occurs whenever a video camera is directed at a screen displaying the image currently recorded by the camera. Perhaps the most familiar outcome of this scenario is shown in figure 1; the screen shows a smaller-scale image of the screen, which in turn shows an even smaller image of the screen, etc., a phenomenon sometimes called the monitor-inside-a-monitor effect [2]. Video feedback can be observed in everyday situations, for example at televised sporting events whenever the stadium's display screen comes into view. In the scientific literature it was first mentioned by Abraham [3] and later researched in considerable detail by Crutchfield [4, 5] as a simple model system with complex dynamics. The psychedelic effect of video-feedback sequences has inspired countless enthusiasts to perform their own experiments, many of whom (inclined both scientifically and artistically) display their video-feedback creations on the internet (for a good overview see [6]). Unsurprisingly, video feedback has also found its way into numerous popular science books, for example Douglas Hofstadter's *Gödel, Escher, Bach* [7].

Most of the systematic research on video feedback was carried out in the days when cameras and monitors were based on scan lines. Nowadays, however, cameras and monitors use arrays of pixels. In this paper we present results from experiments and computer simulations which demonstrate that otherwise unmodified pixellated video feedback can lead to stationary patterns in the shape of fractals.

Fractals were introduced by Benoit Mandelbrot as shapes that describe natural objects more adequately than the simple shapes of traditional geometry: 'Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line' [8]. Figure 2 shows two famous examples of fractals: a stylized fern, generated with a few lines of computer program but with an astonishing similarity to real-life ferns, and an abstract fractal called the Sierpinski gasket. A fundamental property of fractals is that small parts of the fractal look similar to the whole fractal. This similarity can be either exact, as in the examples shown in figure 2, or statistical, as in a cloud or a mountain. Soon after their introduction, fractals became the subject of intense mathematical and scientific attention; it has even been suggested that scientists have often been too eager to find the signature of fractals in their data [9], but these most 'organic' of mathematical shapes have also been in vogue with artists (see, for example, [10]) and the general public (countless screen savers, for example, display fractal patterns).

A number of experiments demonstrate that *modified* video feedback can lead to stationary fractal structure. Modified video-feedback set-ups have used added mirrors [11], multiple cameras [12], multiple monitors [13], multiple lenses [14] or multiple imaging arms [15, 16]. These configurations essentially work as what Peitgen *et al.* [17]

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http://www.tandf.co.uk/journals
DOI: 10.1080/0010751021000029598

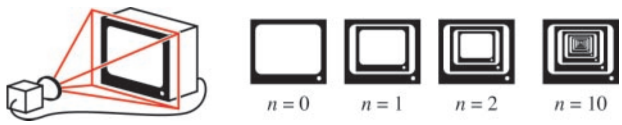

Figure 1. Monitor-inside-a-monitor effect. A camera records an image of a monitor and some of its surroundings (left). This image is then displayed on the monitor's screen. The monitor (complete with the image on the screen) is shown on the right after *n* such video feedback operations, starting with a white screen ($n = 0$). Eventually the screen shows a series of successively smaller images of the monitor, converging towards the *zoom centre*.

refer to in their classic text on fractals as multiple reduction copy machines with feedback.

Some published experimental observations suggest that *unmodified* video feedback can lead to fractal patterns, but all these patterns evolved rapidly in time, that is none of them was stationary. Crutchfield (plate 5 of [4]), for example, observed short 'bursts' of brightness in the shape of a complex spiral pattern†, Peitgen *et al.* [2] saw 'rather wild and almost turbulent motion on the screen', recently Essevaz-Roulet *et al.* [18] observed 'spiral bursts with irregular frequency', and Goodman-Strauss [19] took snapshots of rapidly rotating 'planet-like' fractal structures.

Goodman-Strauss [19] mentioned pixellation as one of the sources of the evolving fractal structure. Pixellation is also recognized as the potential source of fractal structure in video feedback by Peak and Frame [20], who stated that a video-feedback set-up is an example of a cellular automaton (the pixels are the 'cells'), that cellular automata are well known to lead to fractal structure and that video-feedback patterns are (therefore) fractals. Perhaps the most compelling evidence that pixellated video feedback can lead to fractal structure was found by Andersen [21] and Andersen and Petersen [22], who simulated video feedback in a matrix model and found stationary fractal spirals in one particular parameter regime which was previously thought to be the least interesting (namely when the modulus of the magnification associated with one feedback iteration is greater than one, i.e. $|M| > 1$). The matrix elements in the Andersen and Petersen [22] model effectively act like pixels, and indeed these workers mentioned 'pixel truncation' in the context of the creation of structure in video feedback. However, none of the above workers investigated the idea that pixellation gives rise to fractal structure much further.

We discovered the importance of pixellation for the generation of fractal structure in video feedback independently as an analogy to the formation of fractal structure in laser modes [23]. Just like a video-feedback iteration magnifies the pattern on the screen, a round trip through an unstable laser resonator magnifies the intensity cross-section in one plane [24]. In addition to this, in a very general sense the peaks in the diffraction pattern due to apertures inside the resonator play the role of the pixels in video feedback. To contrast this mechanism with the well-known monitor-inside-a-monitor effect, which occurs when the modulus of the magnification is less than one ($|M| < 1$), we christened this new mechanism, which occurs when the modulus of the magnification is greater than one ($|M| > 1$) the *monitor-outside-a-monitor* effect.

This paper reports on the subsequent exploration of the monitor-outside-a-monitor effect. The main results of this study have been summarized in a previous paper [25]; here we present a more detailed account. Using computer modelling we were able to show that unmodified video feedback with pixellated devices can indeed lead to stationary pixel-limited fractal structure. The astonishing variety of fractal patterns that can be created in this way was completely surprising and had not been anticipated in the previous literature. The patterns that can be created include 'classic fractals' such as the Cantor bars, the Sierpinski gasket, the von Koch snowflake, and spiral patterns that resemble parts of Julia sets [26]. Using a video camera that is able to remove flicker-related effects by averaging over a number of frames, we were for the first time able to create many of these patterns in an actual experiment.

2. How pixellation can lead to fractals

Figure 3 gives a step-by-step example of how pixellated video feedback can turn an initially non-fractal intensity distribution into a self-similar fractal pattern. The initially uniform intensity distribution is displayed on the screen. Because of the screen's pixellation, this step 'imprints' the screen's pixel pattern on to the intensity distribution; that is, an observer looking at the screen now sees a combination of the initial intensity distribution and the screen's pixel pattern. (More specifically, the area of each pixel takes on an approximately uniform brightness, while the 'dead' area between the pixels is dark.) In the following step the camera effectively records part of the screen's pixel pattern, which is then displayed again on the screen, magnified by a factor M and freshly pixellated†. The shape of the pattern at this stage depends on the relative alignment of the magnified images of pixels with the actual pixels (figure 4). In figure 3, for example, 7 pixel rosettes appear at positions where the image of a pixel is centred on an actual pixel; this is the situation shown in figure 4 (a).

†Crutchfield did not describe these patterns as fractals. He concentrated on the time evolution and found instead that the chaotic dynamics underlying video feedback can lead to fractal attractors in phase space.

†Note that one feedback step involving magnification and pixellation is well described as a linear process in which the intensity of each pixel is given by the weighted sum of the intensities of one or more pixels after the previous step.

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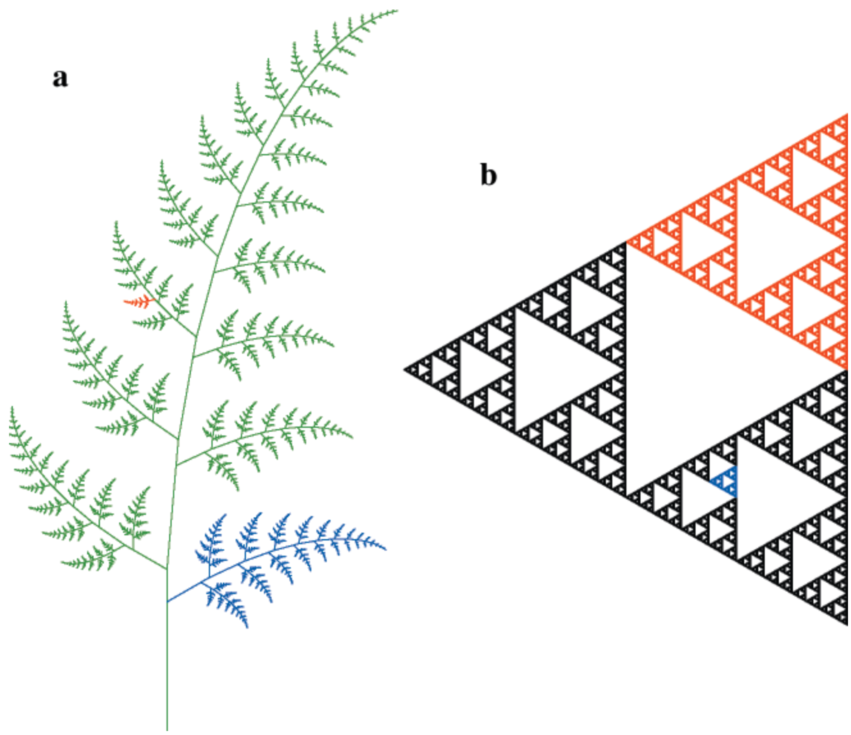

Figure 2. (*a*) Fern and (*b*) Sierpinski gasket, examples of fractal patterns. In each case the pattern consists exclusively of smaller versions of the whole pattern; examples of such smaller ferns and Sierpinski gaskets are shown in red and blue. This presence of similar patterns in many different sizes is a hallmark of fractals.

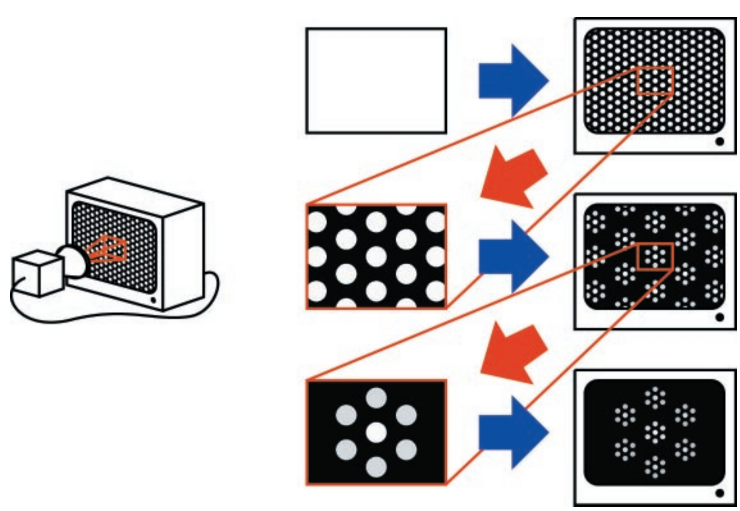

Figure 3. Formation of fractal structure in video feedback with a pixellated monitor. During each feedback iteration the camera records the intensity distribution in a small part of the screen, which is then (magnified and pixellated) displayed again on the monitor. Here the effects of pixellation (blue arrows) and magnification (red arrows) are shown separately for the first three video-feedback iterations, starting with a uniform white pattern. In the example shown here (the magnification factor is $M = 4$), the resulting pattern is a rosette consisting of seven smaller rosettes, each formed by 7 pixels.

Consequently, at positions where the alignment is the same as at the zoom centre the pattern is locally identical with that at the zoom centre. (The zoom centre is the point on the screen the image of which is displayed on the same point again; mathematically this point is the fixed point of the magnifying transformation.) Note that the alignment

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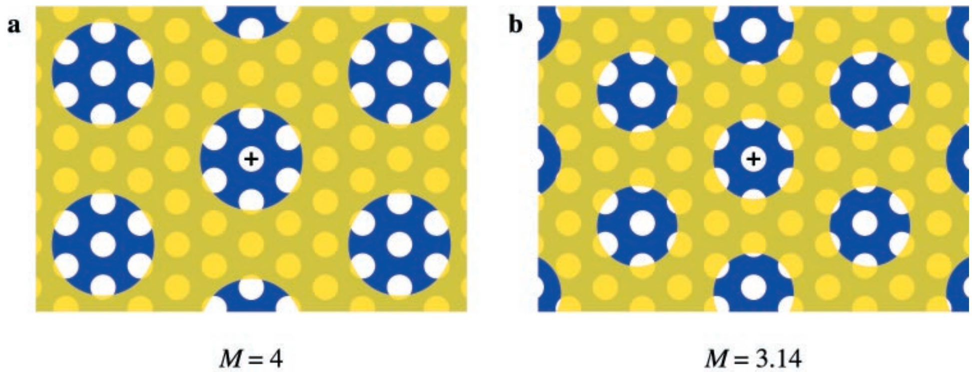

Figure 4. **Alignment between the magnified image of pixels in relation to the actual pixels. The magnified 'dead' area between the pixels is shown as a semitransparent yellow layer on top of the actual pixel mask, shown in blue. (*a*) An example in which the relative alignment between magnified images of pixels and actual pixels is the same in a number of other places as it is at the zoom centre (indicated by a black cross). In the example shown, each image of a pixel is centred on an actual pixel. (*b*) An example where this is not the case. Whereas the alignment shown in (*a*) will result in a self-similar stationary pattern, that shown in (*b*) will not.**

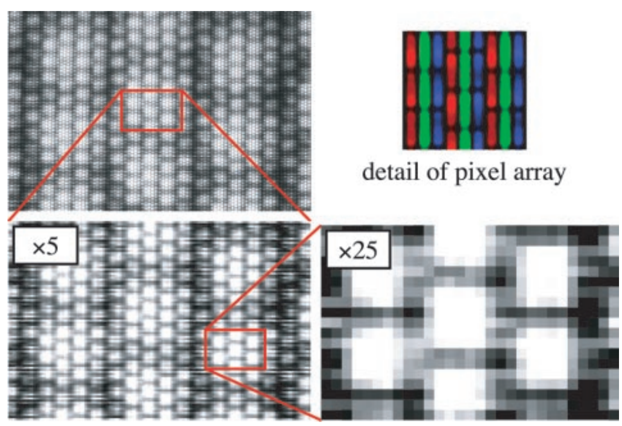

Figure 5. **Self-similarity of a pattern produced experimentally for magnification $M = 5$ with a monitor with a hexagonal pixel array. Note that each individual 'multicolour' pixel is made up of a group of three single-colour pixels (red, green and blue; see photograph of the monitor's pixel pattern). Despite this complication, the resulting stationary pattern clearly exhibits the expected self-similar structure.**

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depends on the position of the zoom centre, magnification, and the shape of the pixel pattern.

Successive magnification steps create images of pixels that are magnified by factors M, M^2, M^3, ... during the previous 1, 2, 3, ... steps. At positions where the relative alignment between the actual pixels and *all* these images of pixels is the same as at the zoom centre the pattern looks similar to that at the zoom centre. The areas surrounding these positions remain similar to the area around the zoom centre, which in turn becomes magnified until it covers the entire screen. In this way, successive video-feedback iterations lead to a self-similar stationary pattern. Figure 5 demonstrates the self-similarity of a pattern of this type that was created with our video-feedback set-up.

Figure 6 shows simulated stationary patterns that correspond to other pixel arrays (defined not only by the positions of the pixel centres but also by the shape and size of the individual pixels), magnifications, and the position of the zoom centre with respect to the pixel array. Note that these patterns depend crucially on every single one of the parameters described above; the two patterns shown in the second row of figure 6, for example, are very different although they differ only in their zoom position, which is shifted by a mere quarter of one vertical pixel period. In particular it is perhaps worth noting that, with suitable pixel arrays, it is possible to create 'classic' fractals such as Sierpinski gaskets, Cantor bars and von Koch snowflakes.

Figure 7 shows some patterns that were created with a video-feedback setup consisting of a semiprofessional video camera (Panasonic MS4) and one of two monitors with different pixel arrays. The camera's motion-blur capability,

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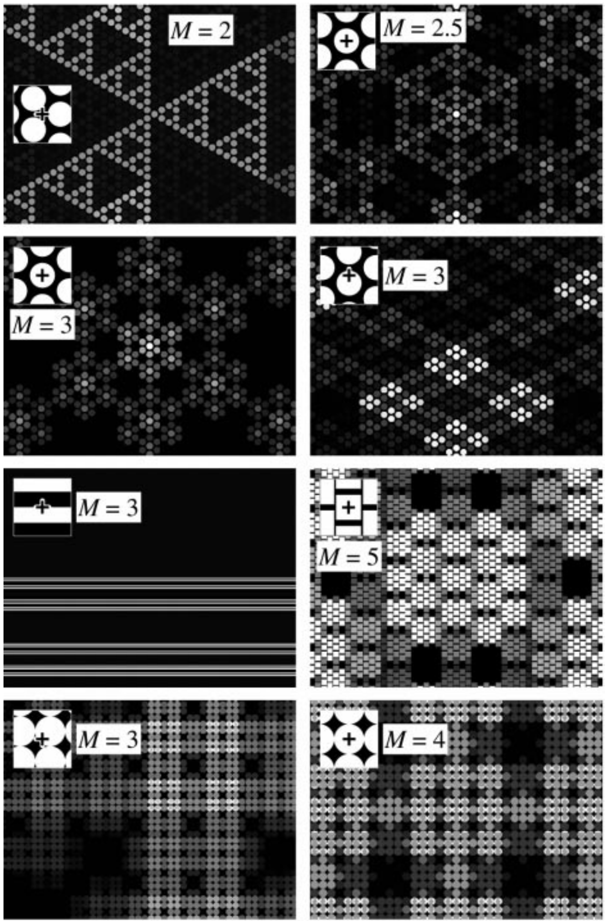

Figure 6. Simulated stationary patterns resulting from pixellated video feedback. The following 'classic' fractals can be found in the patterns in the left-hand column (from top to bottom): Sierpinski gasket, von Koch snowflake, Cantor bars and a tartan. In all cases, the fine detail is limited by the size of the pixels. The insets show the centre of the zoom, indicated by a cross, in relation to a detail of the pixel array. M is the magnification of the set-up.

a mode of operation that effectively averages over the last few frames recorded by the camera, eliminates many effects associated with the monitor's 'flicker', that is the sequential build-up of the picture on the monitor. Note that any patterns which are stationary in this special mode of operation would also be stationary in the 'normal' mode of operation if flicker was absent. This mode of operation therefore allowed us to observe, for the first time, the fully developed stationary fractal patterns.

It was mentioned in the discussion of the computer simulations that the shape of the stationary patterns depends sensitively on the position of the zoom centre. This was very noticable in our experiment, in that even careful changes of the camera angle did not permit sufficiently accurate, that is subpixel width, control over

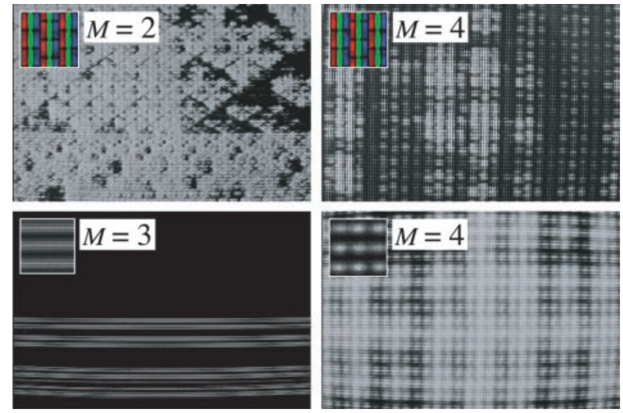

Figure 7. Experimentally created stationary patterns. The insets show close-up photographs of the monitors' pixel arrays. The two patterns in the top row were taken with a colour monitor with a hexagonal pixel array. The bottom patterns were taken with a scan-line-based monochrome monitor. At a low-contrast setting, that monitor's 'pixel' array simply consists of its scan lines (left-hand pictures). At a high-contrast setting the scan lines are modulated (perhaps owing to the signal from the pixel-based camera) such that they effectively form a rectangular 'pixel' array (right-hand pictures).

the important zoom-centre position. The zoom centre was therefore essentially selected at random by disturbing the camera position.

It is worth mentioning that the shape of the stationary pattern in general depends on the initial intensity distribution. When starting with the 'wrong' pattern, it can even be the case that no stationary pattern emerges at all, a fact that became quite clear in our experiments. Note that all the patterns shown in this paper were stationary.

3. Pixellated video feedback with a rotated camera

The patterns discussed so far can be created with a camera that is not rotated with respect to the screen. The patterns that emerge (because of exactly the same mechanism) when the camera is at an angle with respect to the monitor, as shown at the top left of figure 8, are perhaps even more intriguing.

Simulated and experimentally generated examples of such patterns are shown in figures 8 and 9 respectively. Certain combinations of magnification and rotation happen to result in positions at which the alignment of the image of pixels relative to the actual pixels is very similar to that at the zoom centre; the resulting patterns consist of smaller-scale versions of the overall pattern; they are self-similar. As before, it is this alignment that causes the patterns to be self-similar. The additional rotation step merely makes this alignment harder to predict intuitively. The main photograph in figure 9 shows an example of a

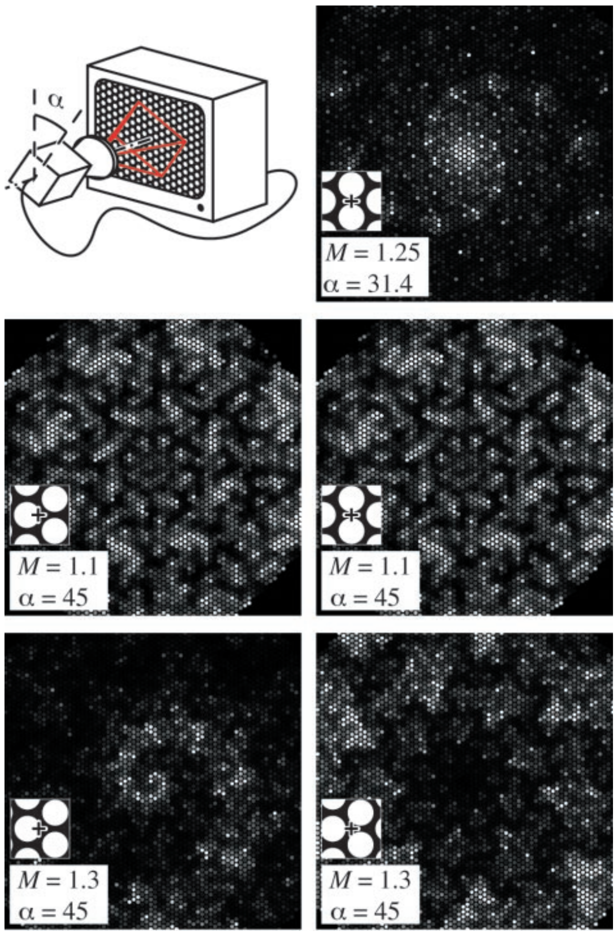

Figure 8. Simulated stationary patterns created by video feedback with a pixel-based monitor. α is the rotation angle of the camera with respect to the monitor (see diagram of the experimental set-up at the top left). The pattern for parameters $M = 1.3$ and $\alpha = 45^\circ$ is a low-resolution simulation of the main photograph in figure 9.

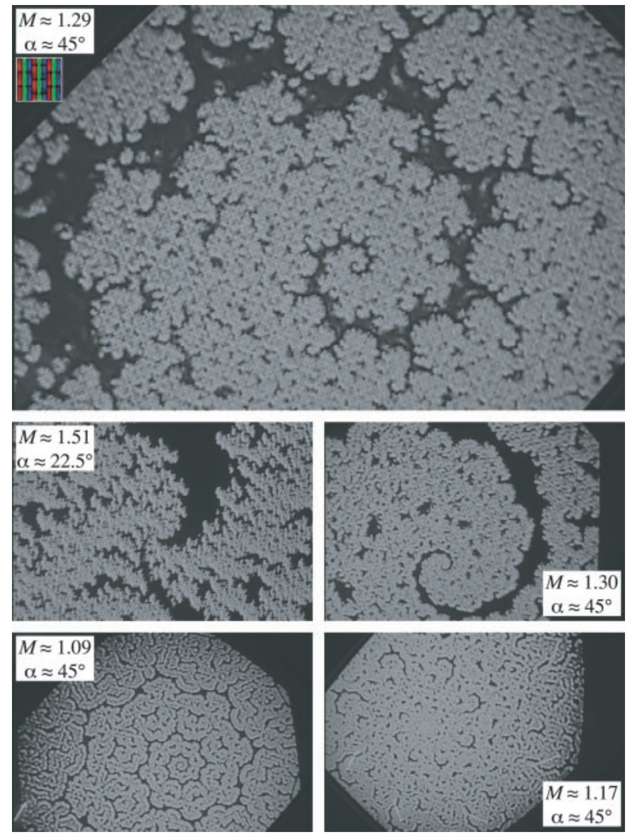

Figure 9. Stationary patterns created with a rotated camera. The same monitor was used to create the patterns shown; the monitor's pixel pattern is shown as an inset in the main photograph.

self-similar pattern which is reminiscent of parts of the Mandelbrot set or Julia sets.

4. Conclusions

This paper represents an update of earlier work on video feedback in the light of the pixellation of modern monitors. Pixellated video feedback is shown to lead to a surprising variety of fractal patterns, notably 'classic fractals,' such as the Cantor bars and the Sierpinski gasket, and fractal spirals reminiscent of the Mandelbrot and Julia sets.

We are currently applying these findings back to laser resonators, which led us to investigate fractal video-feedback patterns in the first place. In particular, we hope to create fractal laser modes that closely resemble 'classic fractals' by manipulating the aperture's diffraction pattern, which plays the role of the pixel pattern. In addition to this scientific value, this work possesses considerable educational potential; it describes a simple effect that leads to highly ordered and complex 'organic' patterns; as such it could perhaps serve as a model for the origin of natural structures such as trees and mountains. The effect can be demonstrated cheaply and easily, using equipment that is available in many households and in most schools and universities, and would consequently be ideally suited to small projects. Most importantly, however, it brings together two scientific fields with great popular appeal: video feedback and fractals.

Acknowledgements

Thanks are due to Ian MacVicar for the loan of his video equipment. Jonathan Leach is supported by the Glasgow–Strathclyde University Synergy Fund. Miles Padgett and Johannes Courtial are Royal Society University Research Fellows.

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Jonathan Leach performed the experiments described in this paper as part of his final-year undergraduate project. He is currently doing a PhD with Miles and Johannes at the University of Glasgow and Steve Barnett at the University of Strathclyde, working on quantum aspects of light beams with orbital angular momentum.

Miles Padgett has worked in many areas related with lasers, including frequency stabilization, laser spectroscopy, laser beams with orbital angular momentum that can cause microscopic particles to rotate, and fractal laser beams. He leads the Optics and Applications Group at the University of Glasgow, where his widespread research interests also include medical imaging and gas detection. Additionally, Miles is actively engaged in numerous public understanding-of-science activities.

Johannes Courtial has been interested in fractals for a number of years. When working with lasers as part of his PhD, he realized that a mechanism for creating fractals can be built into lasers. Together with Miles he developed the analogy between fractal laser beams and video feedback. Johannes is now a Royal Society University Research Fellow in the Optics and Applications Group and is interested in all kinds of exotic light beams and similar states of Bose–Einstein condensates.

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