Week 4 class 3 - Chaos and Fractals

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Questions to ask today

- How do I translate the logistic function's discrete form into the continuous form?

Today's goals

- Fractals and fractal dimension

Chapter 16: How Many Things Fit Inside Other Things

Worksheet to accompany
David Feldman, *Chaos and Fractals: An Elementary Introduction*,
Oxford University Press, 2012

1. Draw a short line segment. Now draw a line segment that's three times as long. How many smaller line segments fit inside the big line segment?

2. Draw a small square. Now draw a square that's three times as big. (I.e., three times as tall and deep.) How many smaller squares fit inside the square?

3. Draw a small cube. Now draw a small cube that's three times as long, deep, and tall. How many smaller cubes will fit inside the big cube?

4. Complete the following table:

Shape	Growth Factor	Number of small copies that fit within big copy
Line	3	3
Square	3	9
Cube	3	27

5. What property of a shape determines how many small copies of it fit within a big copy?

6. Come up with a formula that relates the growth factor, the number of small copies.

Magnification factor $\underline{M}$ original shape $\underline{A}$
dimension count $\underline{D}$

$M^D \cdot A$

Making A Snowflake[1]

1. Get some plastic beads. ✓

2. Make a snowflake by starting with a single bead as a seed. Then, at each step, make four copies and place a copy on the corner of the shape you had at the previous step. I'll draw a picture on the board, and Mafe, Nynke, Will, or I can help. Start making the shape in the middle of the table. It gets large quickly and grows in all directions, so you'll need a lot of space.

3. Answer the same questions for the snowflake that you did for the other geometrical shapes.
   (a) How many small copies fit inside the big copy? $3^{1.47\ldots}$
   (b) What is the growth factor? $= 3$
   (c) What is the dimension of the snowflake? $\simeq 1.47$

4. Ponder.

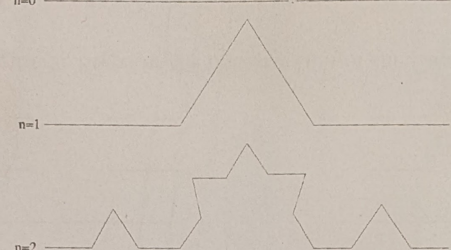
[1] sorta

Chapter 16: Exercises with Fractals and Dimension

Worksheet to accompany

David Feldman, *Chaos and Fractals: An Elementary Introduction*, Oxford University Press, 2012

\# 4^N



1

4

$4 \times 4 = 16$

Figure 1: Steps in the construction of the Koch Curve.

1. **The Koch Curve**

    (a) Complete the following table using the successive steps in the construction of the Koch Curve, as illustrated in Fig. 1:

Step	Number of Segments	Length of Each Segment	Total Length
0	1	1	1
1	4	$1/3$	$4/3$
2	16	$1/9$	$16/9$
3	64	$1/27$	$64/27$
n	4^N	$\left(\frac{1}{3}\right)^N$	$\left(\frac{4}{3}\right)^N$

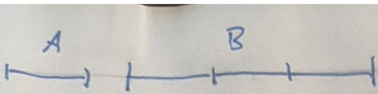
$\dots 0$ $\quad$ $\dots +\infty$

(b) As n goes to infinity, what happens to the total length of the Koch Set? → it also goes up to infinity

(c) What is the dimension of the Koch Set?

$\dots +\infty$

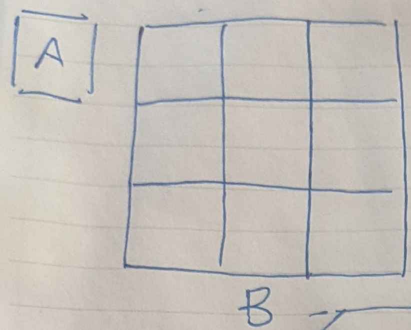
$$\left.\begin{array}{l}\#\text{ copies} = \text{stretch}^D \\ 4 = 3^D\end{array}\right\} \underline{\underline{D \simeq 1.26}}$$



$\to 3A = B$

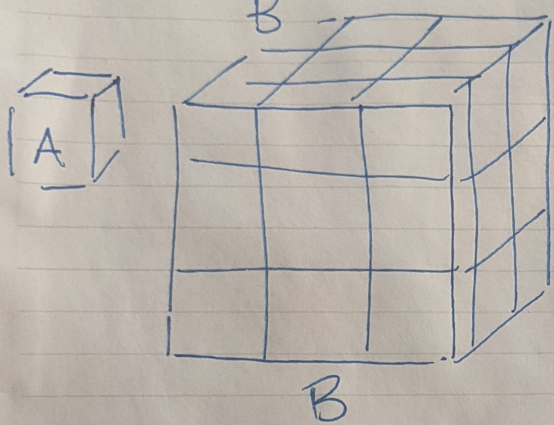
$= M \cdot A = M^1 A$

M { magnification factor = 3



$\to 9A = B$

$= M \cdot M \cdot A = M^2 A$



$27A = A$

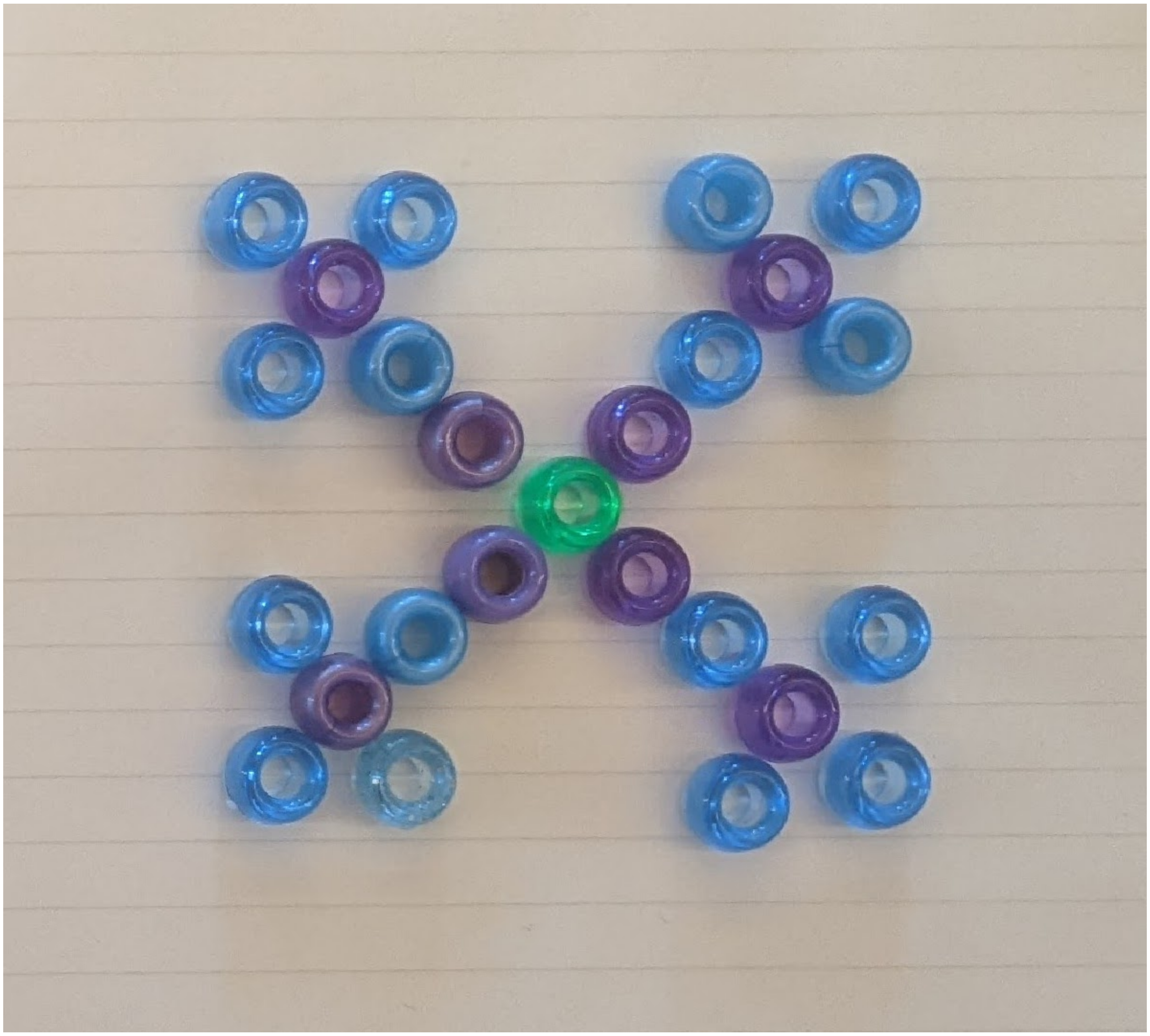
$M \cdot M \cdot M \cdot B = M^3 A$

What property of the shape determines how many small copies of it fit in the big copy?

What matters is:

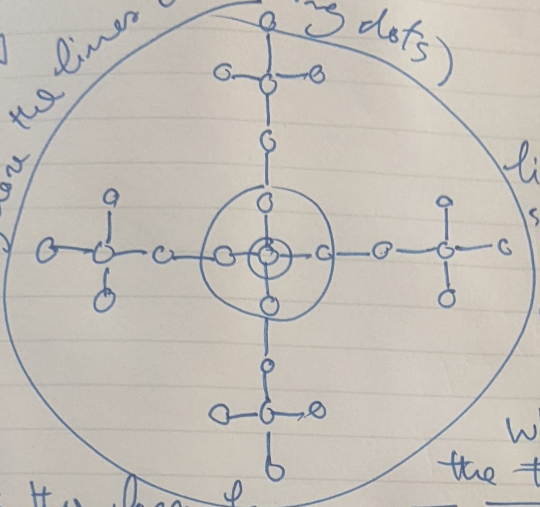
- $\to$ The count of dimensions D
- $\to$ The magnification rate M
- $\to$ The size of the original shape A

formula: $M^D \cdot A$ } Solve for D to find the dimension of an object.



A fractal!

↳ (ignore the lines connecting dots)



	Growth factor	# small copies
line	3	3
snowflake	3	5
square	3	9
cube	3	27

What is the formula describing the # of small copies?

What is the formula describing the number of dimensions D?

$\boxed{3^{\#\text{dimensions}} = 5}$ $\simeq 1.47$

⚠ Fractals typically have non-integer dimensions.