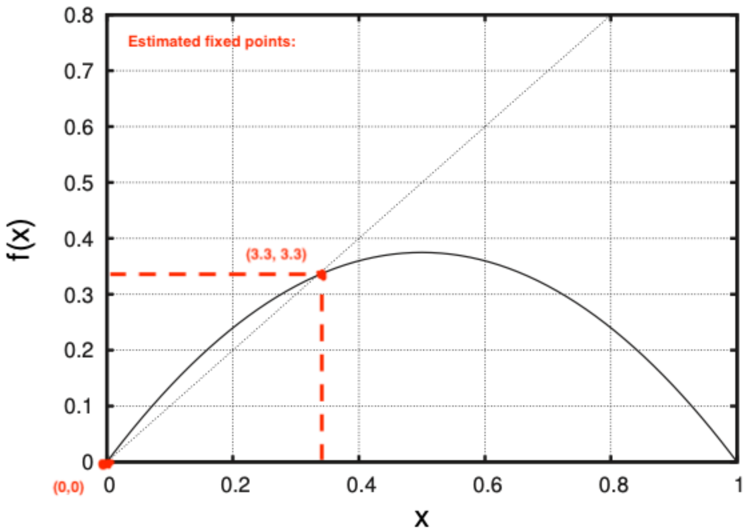
Chapter 5, problems 5.2, 5.6, 5.7

5.2

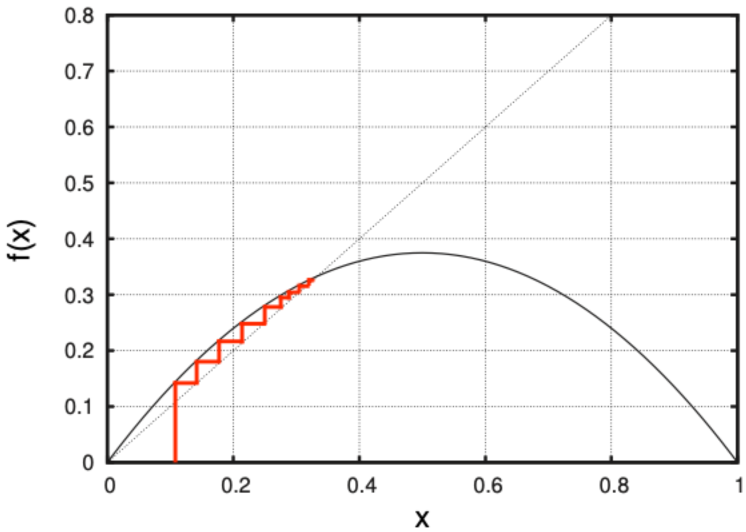
Figure 5.10 shows a plot of the function $f(x) = 1.5x(1 - x)$.

(a) Use the plot to determine approximate values for all fixed points of f(x)



(b) Graphically iterate the seed $x_0 = 0.1$.

- Hand version (software assisted):



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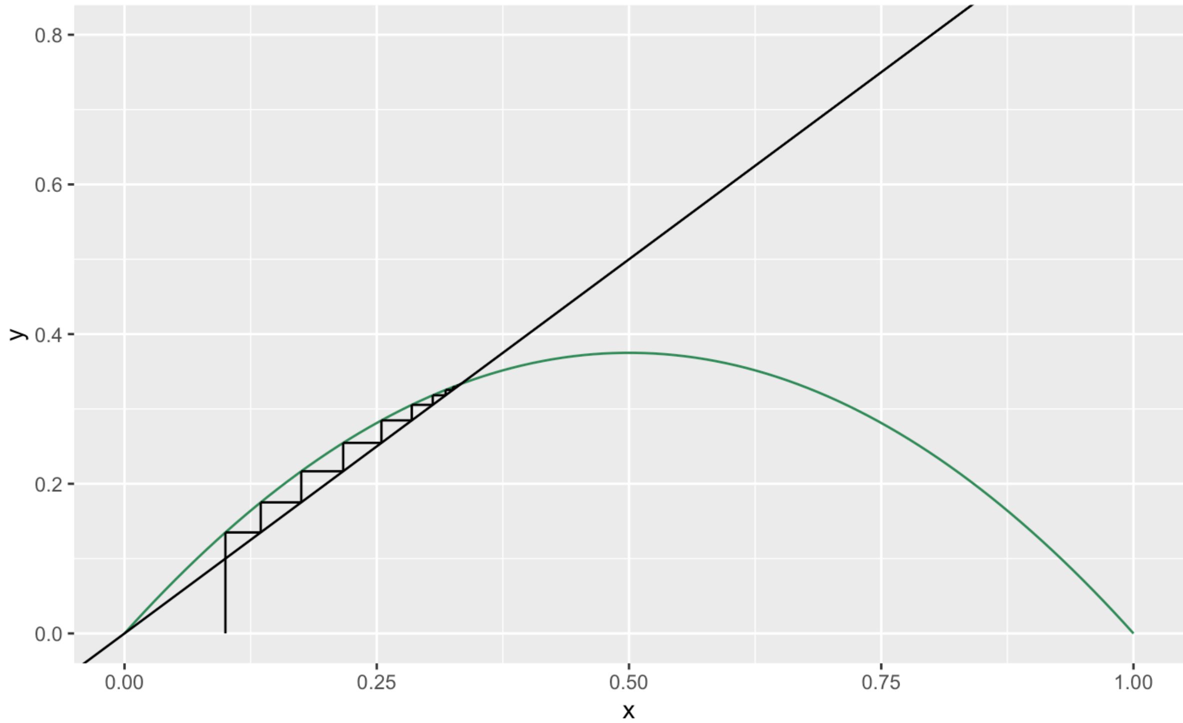
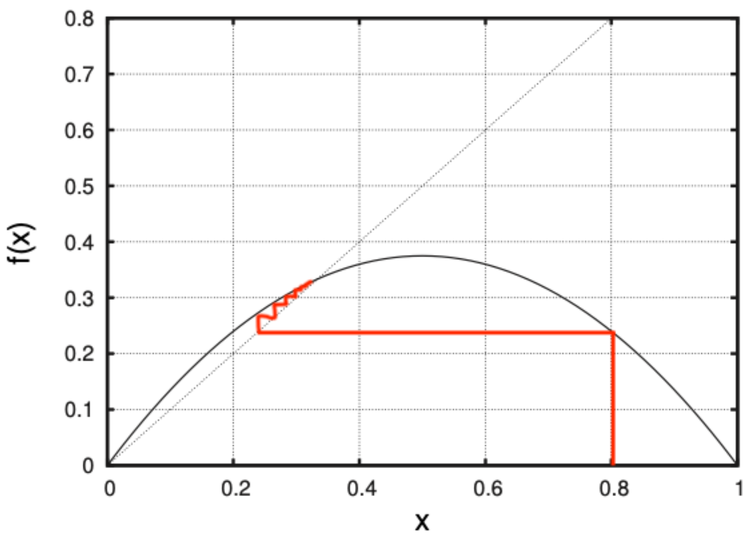
- R program version using ggplot: (I'm tentatively including these because I find them nicer and more interesting to make, but I made sure to also include a couple by-hand versions in case the act of doing the iteration manually was meant to be an important part of the exercise)



(c) Graphically iterate the seed $x_0 = 0.8$.

- Hand version (software assisted)



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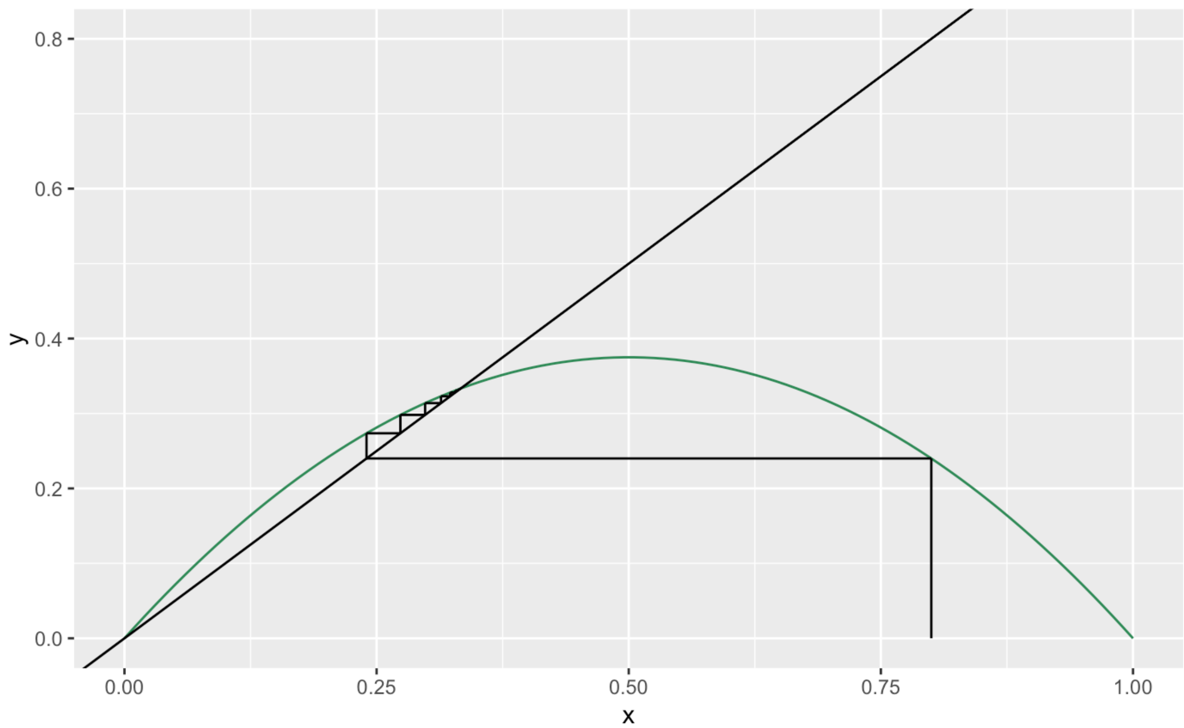
- R program version using ggplot:



(d) What do you conclude about the stability of the fixed point near x = 0.35?

I conclude that the fixed point near x = 0.35 is stable. Over n iterations from x_0 to x_n, the values of the outputs converge to it.

(e) What is the stability of the fixed point at x = 0?

The fixed point at x = 0 appears to be unstable. Over n iterations from x_0 to x_n, the values of the outputs diverge from it.

(f) Use algebra to find the fixed points exactly.

$x = 1.5x(1 - x)$

$x = (1.5x)(-x + 1)$

$x = 1.5x \cdot -x + 1.5x$

$x = 1.5x^2 + 1.5x$

$x + 1.5x^2 - 1.5x = 0$

$x + 1.5x^2 - 1.5x = 0$

$-0.5x + 1.5x^2 = 0$

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$x(-0.5 + 1.5x) = 0$

$$\begin{cases} x = 0 \\ 1.5x - 0.5 = 0 \end{cases}$$

$$\begin{cases} x = 0 \\ 1.5x = 0.5 \end{cases}$$

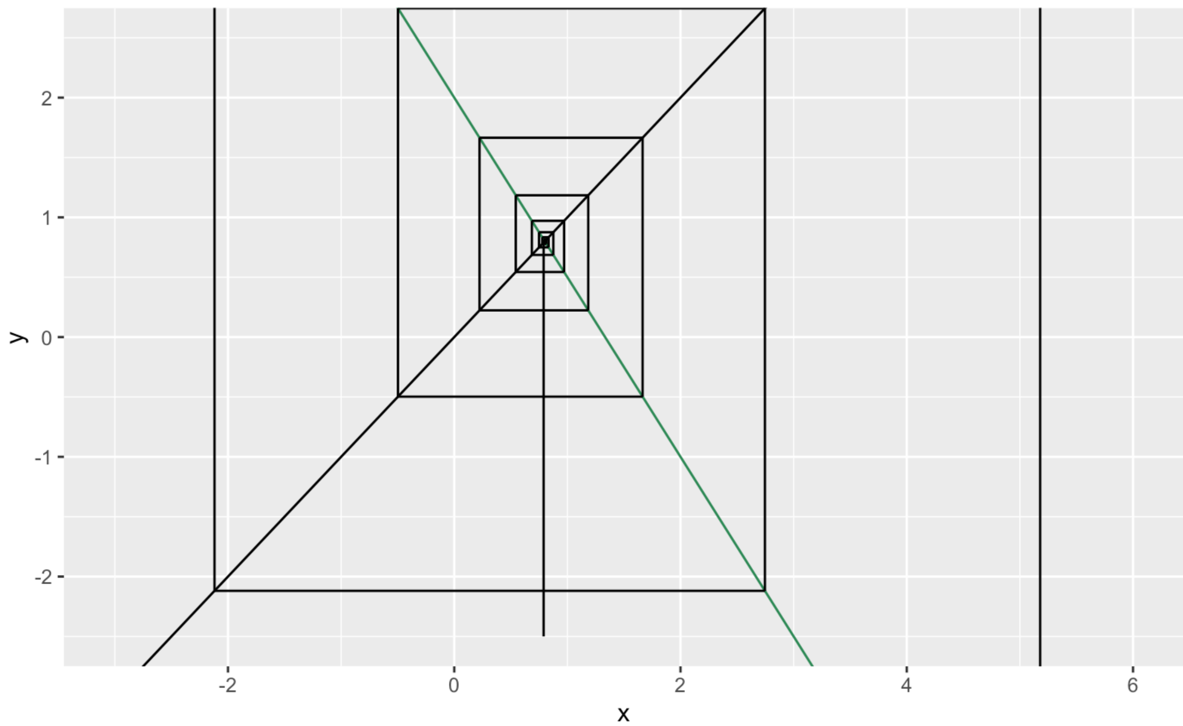
$$\begin{cases} x = 0 \\ x = \dfrac{0.5}{1.5} = \dfrac{1}{3} \end{cases}$$

5.6

Consider the function $f(x) = -\frac{3}{2}x + 2$

(a) Determine the stability of the fixed point by graphically iterating an initial condition very near to, but not exactly at, the fixed point.

- R program iteration using ggplot:



(b) What does your graphical iteration let you conclude about the fixed point's stability?

The fixed point at 0.8 is unstable. Over n iterations from x_0 to x_n, the values of the outputs diverge from it.

(c) Determine the fixed point using algebra.

$$x = -\frac{3}{2}x + 2$$

$$x = \frac{-3x + 2 \cdot 2}{2}$$

$$x = \frac{-3x + 2 \cdot 2}{2}$$

$$2x = -3x + 4$$

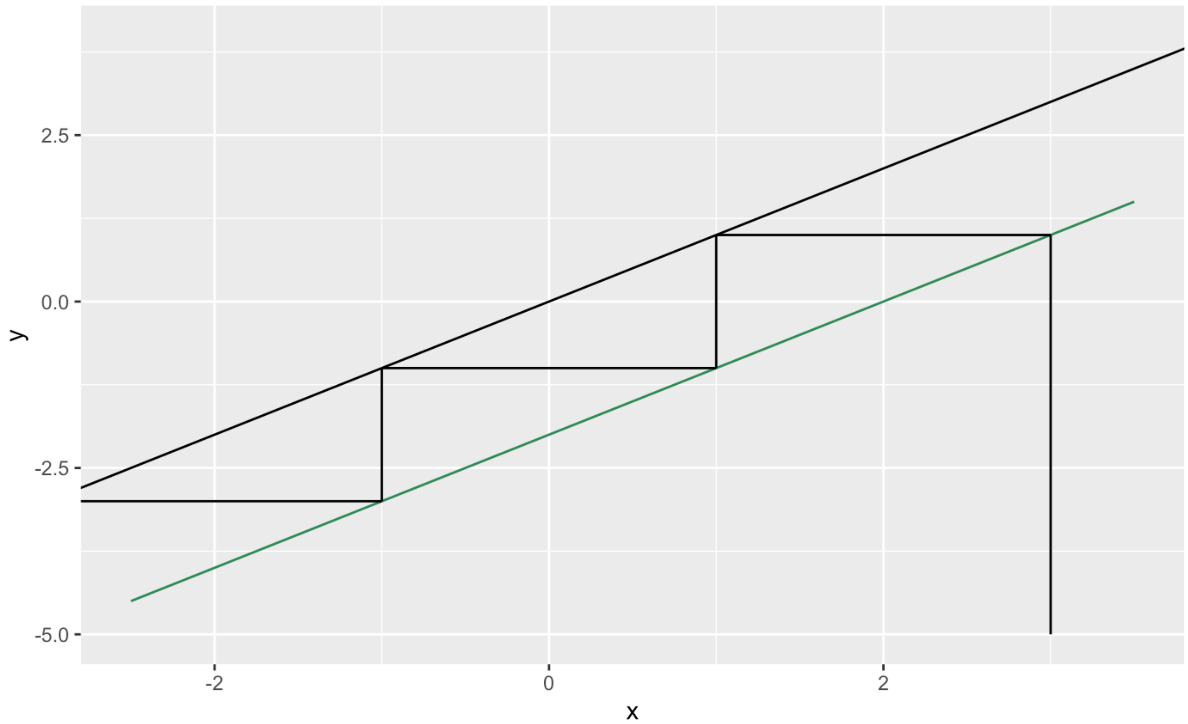
$$5x = 4$$

$$x = \frac{4}{5} = 0.8$$

<u>5.7</u>

Consider the function f(x) = x − 2

(a) Choose an arbitrary initial condition and iterate it graphically. What long-term behavior do you observe?



The trajectory is ever decreasing.

(b) What does your graphical iteration let you conclude about the existence of fixed points and the long-term behavior of the itinerary?

Since the line described by the function f(x) = x-2 runs parallel to y = x, I can be certain that the function has no fixed points. Since the behaviour of the trajectory is linear (as is the function), I can confidently assume it will continue for any initial condition to decrease ad infinitum, tending towards negative infinity.

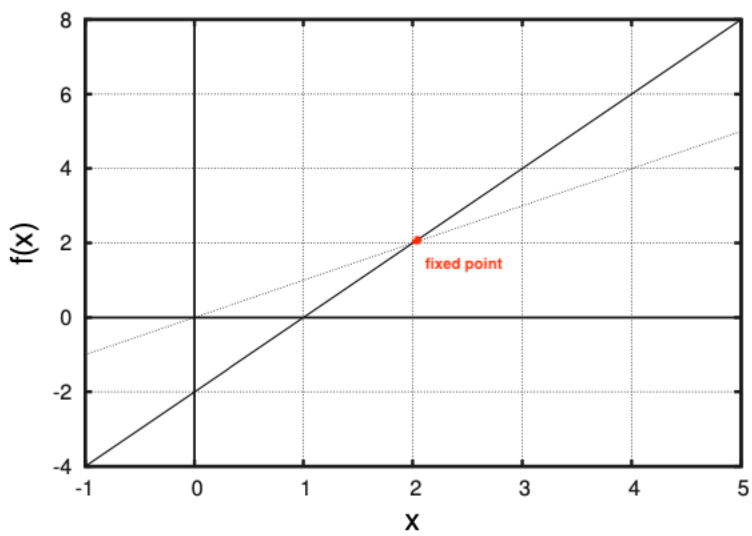
6.8

Consider the function shown in Fig. 6.10.

(a) Determine the equation of the function.

The equation of the function is f(x) = 2x-2

(b) Find the fixed point graphically.

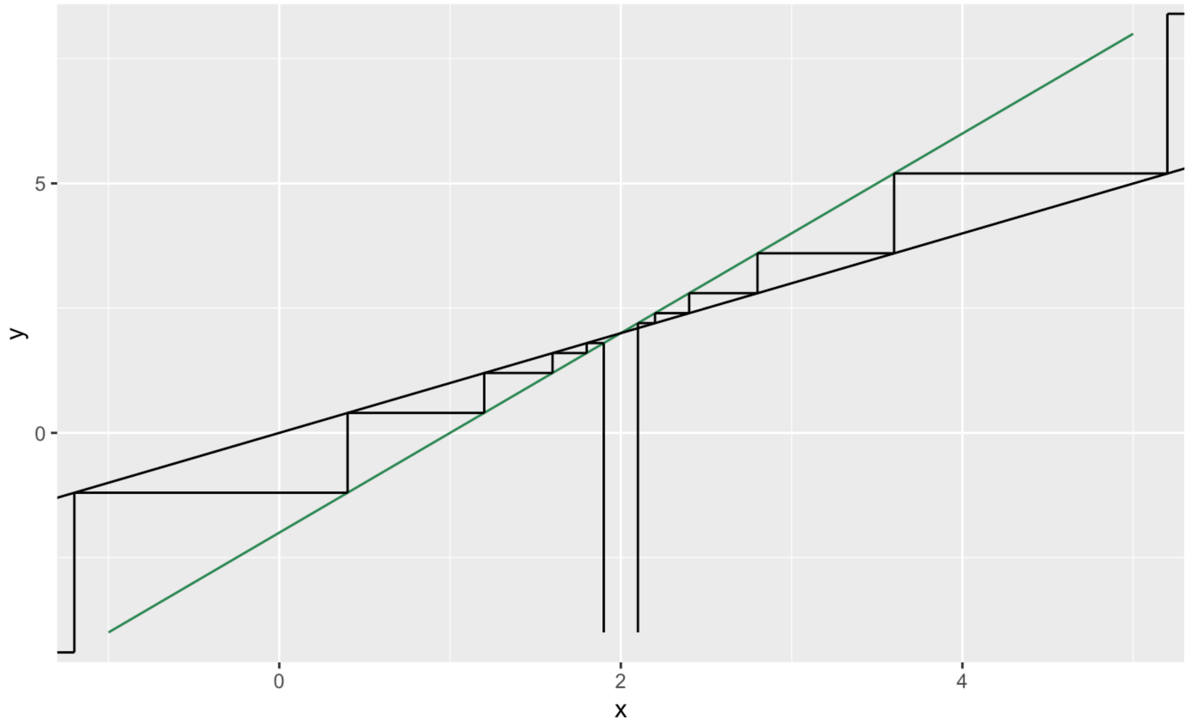


(c) Find the fixed point using algebra.

$$x = 2x - 2$$

$$x = 2$$

(d) Use graphical iteration to determine the stability of the fixed point.



The fixed point is unstable.

(e) Sketch the phase line for this function.

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