Data Structures and Algorithms in Java™

Sixth Edition

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Instructor's Solutions Manual

WILEY

Chapter

5 Recursion

Hints and Solutions

Reinforcement

R-5.1) Hint Don't forget about the space used by the method stack.

R-5.1) Solution If the array has 1 element, that is the maximum. Otherwise, consider the bigger of the first element or the maximum of the other $n-1$ elements. The running time and space usages is $O(n)$.

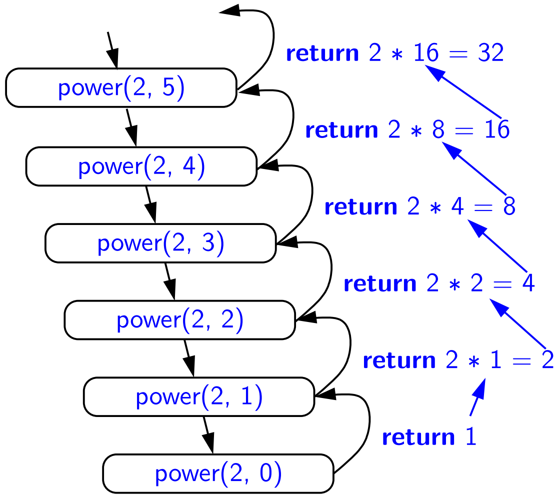
R-5.2) Hint When the algorithm finds a match, does it know where?

R-5.2) Solution

```
public static int binarySearch(int[ ] data, int target, int low, int high) {
  if (low > high)
    return −1;                              // failed search
  else {
    int mid = (low + high) / 2;
    if (target == data[mid])
      return mid;                           // index of found match
    else if (target < data[mid])
      return binarySearch(data, target, low, mid − 1);
    else
      return binarySearch(data, target, mid + 1, high);
  }
}
```

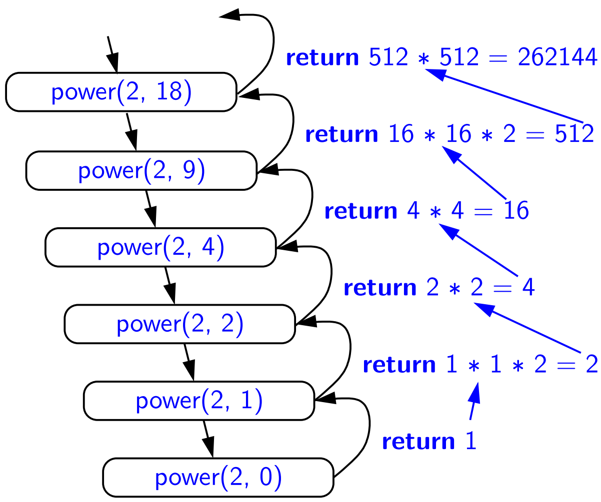
R-5.3) Hint This is probably the first power algorithm you were taught.

R-5.3) Solution



R-5.4) Hint Be sure to get the integer division right.

R-5.4) Solution



R-5.5) Hint You can model your figure after Figure 5.11.

R-5.5) Solution

0	1	2	3	4
4	3	6	2	5

5	3	6	2	4

5	2	6	3	4

R-5.6) Hint You should draw small boxes or use a big paper, as there are a lot of recursive calls.

R-5.7) Hint Start with the last term.

R-5.7) Solution The general case is $H_n = H_{n-1} + \frac{1}{n}$.

R-5.8) Hint Process the string from right to left.

R-5.8) Solution Use a single-digit as the base case. For a multiple-digit string, let $s' = sd$ for digit d. We have that $value(s') = d + 10 * value(s)$.

R-5.9) Hint You can rely on bitwise operations to interpret n in binary.

R-5.9) Solution

```
public static double power(double x, int n) {
  int k = 0;
  while ((1 << k) <= n)
    k++;

  double answer = 1;
  for (int j=k−1; j >= 0; j−−) {
    answer *= answer;
    if (((1 << j) & n) > 0)
      answer *= x;
  }
  return answer;
}
```

R-5.10) Hint You can use two recursive methods that look like BinarySum.

Creativity

C-5.11) Hint The integer part of the base-two logarithm of n is the number of times you can divide by two before you get a number less than 2.

C-5.12) Hint Consider reducing the task of telling if the elements of an array are unique to the problem of determining if the last $n-1$ elements are all unique and different than the first element.

C-5.13) Hint You need subtraction to count down from m or n and addition to do the arithmetic needed to get the right answer.

C-5.13) Solution The recursive algorithm, ***product(n, m)***, for computing product using only addition and subtraction, is as follows: If $m = 1$ return n. Otherwise, return n plus the result of a recursive call to the method ***product*** with parameters n and $m-1$.

C-5.14) Hint Define a recurrence equation.

C-5.14) Solution let $R(c)$ denote the number of dashes drawn by drawInterval(c). We prove by induction that $R(c) = 2^{c+1} - c - 2$. As a base case, we note that drawInterval(0) does not produce any output, and that $R(0) =$

$2^{0+1} - 0 - 2 = 0$. For $c > 0$, we note that drawInterval(c) invokes two recursive calls of drawInterval($c-1$), and a call to drawLine that produces c dashes. Therefore, $R(c) = c + 2R(c-1)$, and by the inductive hypothesis, $R(c) = c + 2\left(2^{(c-1)+1} - (c-1) - 2\right) = c + 2\left(2^c - c - 1\right) = c + 2^{c+1} - 2c - 2 = 2^{c+1} - c - 2$.

C-5.15) Hint Start by removing the first element x and computing all the subsets that don't contain x.

C-5.16) Hint Consider first the subproblem of moving all but the n^{th} disk from peg a to another peg using the third as "temporary storage."

C-5.17) Hint Output to System.out one character at a time.

C-5.17) Solution

```
void printReverse(String s, int n) {
  if (n >= 0) {
    System.out.print(s.charAt(n))
    printReverse(s, n−1);
  }
}

void printReverse(String s) {
  printReverse(s, s.length( ) − 1);
}
```

C-5.18) Hint Check the equality of the first and last characters and recur (but be careful to return the correct value for both odd- and even-length strings).

C-5.19) Hint Write your recursive method to first count vowels and consonants.

C-5.20) Hint Consider whether the last element is odd or even and then put it at the appropriate location based on this and recur.

C-5.20) Solution

```
void organize(int[ ] data, int low, int high) {
  if (low < high) {
    if (data[high] & 1 == 0) {            // even
      int temp = data[high];
      data[high] = data[low];
      data[low] = temp;
      organize(data, low+1, high);        // data[low] is known to be even
    } else {
      organize(data, low, high−1);        // data[high] is known to be odd
    }
  }
}

void organize(int[ ] data) {
  organize(data, 0, data.length − 1);
}
```

C-5.21) Hint Begin by comparing the first and last elements in a range of indices in A.

C-5.21) Solution This problem can effectively be solved using the same technique as Exercise C-5.20.

C-5.22) Hint The beginning and the end of a range of indices in A can be used as arguments to your recursive method.

C-5.22) Solution The solution makes use of the method FindPair(A,i,j,k) below, which given the sorted subarray $A[i..j]$ determines whether there is any pair of elements that sums to k. First it tests whether $A[i]+A[j]<k$. Because A is sorted, for any $j' \leq j$, we have $A[i]+A[j']<k$. Thus, there is no pair involving $A[i]$ that sums to k, and we can eliminate $A[i]$ and recursively check the remaining subarray $A[i+1..j]$. Similarly, if $A[i]+A[j]>k$, we can eliminate $A[j]$ and recursively check the subarray $A[i..j-1]$. Otherwise, $A[i]+A[j]=k$ and we return true. If no such pair is ever found, eventually all but one element is eliminated ($i=j$), and we return false.

Algorithm FindPair(A,i,j,k):

Input: An integer subarray $A[i..j]$ and integer k

Output: Returns true if there are two elements of $A[i..j]$ that sum to k

if i == j **then**
  return false
else
  if $A[i]+A[j]<k$ **then**
    return FindPair$(A,i+1,j,k)$

else
  if $A[i] + A[j] > k$ **then**
    return FindPair$(A, i, j-1, k)$
  else
    return true

C-5.23) Hint Check the last element and then recur on the rest of A.

C-5.24) Hint Look for a geometric series.

C-5.24) Solution The running time is $O(n)$, as it is $O(n + n/2 + n/4 + n/8 + \cdots)$.

C-5.25) Hint Recur on the first $n-1$ positions.

C-5.25) Solution Let us define a method reverse(L, n), which reverses the first $n \leq L$.size() nodes in L, and returns a pointer *end* to the node just after the nth node in L (*end* = **null** if $n = L$.size()). If L.size() ≤ 1, we are done, so let us assume L has at least 2 nodes. If $n = 1$, then we return L.first().next(). Otherwise, we recursively call reverse$(L, n-1)$, and let *end* denote the returned pointer to the nth node in L. We then set *ret* to *end*.next() if $n < L$.size(), and to **null** otherwise. We then insert the node pointed to by *end* at the front of L and we return *ret*. The total running time is $O(n)$.

C-5.26) Hint View the chain of nodes following the head node as forming themselves another list.

Projects

P-5.27) Hint Review use of the java.io.File class.

P-5.28) Hint Use recursion in your main solution engine.

P-5.29) Hint Consider a small example to see why the binary representation of the counter is relevant.

P-5.30) Hint Note the recursive nature of the problem.