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MET CS 566 Assignment 2

① Strassen's Algorithm

$[C] = \begin{bmatrix} 1 & 3 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} 6 & 8 \\ 4 & 2 \end{bmatrix}$

$[C] = $ $C_{11} = P_5 + P_4 - P_2 + P_6$

$C_{12} = P_1 + P_2$

$C_{21} = P_3 + P_4$

$C_{22} = P_5 + P_1 - P_3 - P_7$

$S1 = B_{12} - B_{22} = 8 - 2 = \underline{6}$

$S2 = A_{11} + A_{12} = 1 + 3 = 4$

$S3 = A_{21} + A_{22} = 12$

$S4 = B_{21} - B_{11} \quad 4 - 6 = -2$

$S5 = A_{11} + A_{22} \quad 1 + 5 = 6$

$S6 = 6 + 2 = 8$

$S7 = 3 - 5 = -2$

$S8 = 4 + 2 = 6$

$S9 = 1 - 7 = -6$

$S10 = 6 + 8 = 14$

$P_1 = 1 \cdot 6 = \underline{6}$

$P2 = 4 \cdot 2 = \underline{8}$

$P3 = 12 \cdot 6 = \underline{72}$

$P4 = -2(5) = -10$

$P5 = 6(8) = 48$

$P6 = -2 \cdot 6 = -12$

$P7 = -6(14) = -84$

$C_{11} = 48 + (-10) - 8 - 12 = \underline{18}$

$C_{12} = 6 + 8 = \underline{14}$

$C_{21} = 72 - 10 = \underline{62}$

$C_{22} = 48 + 6 - 72 + 84 = \underline{66}$

$[C] = \begin{bmatrix} 18 & 14 \\ 62 & 66 \end{bmatrix}$

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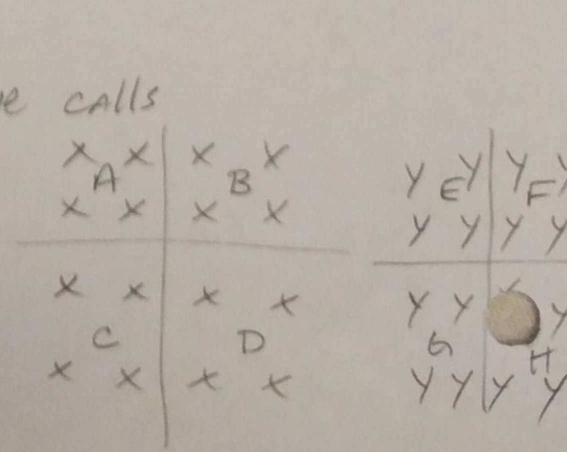
(2) Pseudocode for Strassen Algorithm (SA.)

```
SA (A, B, Dim)
{
  if (Dim ≤ 2) {
    P = (A11 + A12) · (B11 + B22)
    Q = (A21 + A22) · B11
    R = A11 (B12 - B22)
    S = A22 (B21 - B11)
    T = (A11 + A12) · B22
    U = (A21 - A11) · (B11 + B12)
    V = (A12 - A22) · (B21 + B22)

    C11 = P + S - T + V
    C12 = R + T
    C21 = Q + S
    C22 = P + R - Q + U
  }
  ELSE
    NewDim = Dim / 2
    # Split the matrix into smaller sections
    # Matrix must be a multiple of the power of 2.
    # if not you should PAD with zeros to construct
      it.

    SA (AB, EG, NewDim)   # Recursive calls
    SA (AB, EF, NewDim)
    SA (CD, EG, NewDim)
    SA (CD, FH, NewDim)
}
```



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#3. Using Strassen's Algorithm is it possible to compute the following MM?

$$\begin{bmatrix} 1 & 3 & 2 \\ 7 & 5 & 2 \\ 4 & 2 & 1 \end{bmatrix} \begin{bmatrix} 6 & 8 & 6 \\ 4 & 2 & 1 \\ 5 & 3 & 1 \end{bmatrix}$$

(A) YES - I think so.

(B) STEPS - 1.) GET the Dimensions of the matrices.

2.) If the Dimension is NOT a power of 2 I would add A Row of zeros and a column of zeros.

$$\left[\begin{array}{cc|cc} 1 & 3 & 2 & 0 \\ 7 & 5 & 2 & 0 \\ \hline 4 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \left[\begin{array}{cc|cc} 6 & 8 & 6 & 0 \\ 4 & 2 & 1 & 0 \\ \hline 5 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

I would Run this through my Algorithm in #2 And expect the Matrices to be split into smaller sections (Divide & Conquer) until the fundamental Algo. could be computed

(C) I think it could be used for any Matrix Mult. with the caveat of a power of 2 Dimension,

$\Theta(n^{2.8})$

- The Divide and conquer method Results in a Algorith with $O(n^3)$ Because of $T(n) = \begin{cases} 1 \text{ (just computed)} & n \le 2 \\ \underbrace{8}_{\text{8 calls to multiplications}} T(\frac{n}{2}) + \underbrace{n^2}_{\text{count of Adding two matrices } n+n} & n > 2 \end{cases}$

- with STRASSEN's there are only 7 multiplications called between the matrices.

$$T(n) = \begin{cases} 1 & \text{for } n \le 2 \\ \underline{7}\, T(\frac{n}{2}) + n^2 & \text{for } n > 2 \end{cases}$$

$\log_2 7 = 2.81$

$\Theta(n^{2.81})$

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(4) Solve the following using substitution.

(A) $T(n) = T(n-1) + n$ $\therefore T(n-1) = T(n-2) + n - 1$

$T(n-2) = T(n-3) + n - 2$

$T(n) = [T(n-2) + n - 1] + n$

$T(n) = T(n-2) + (n-1) + n$

$T(n) = [T(n-3) + n - 2] + (n-1) + n$

$= T(n-3) + (n-2) + (n-1) + n$

subst. K $\vdots$

$T(n) = T(n-k) + (n-(k-1) + n - (k-2) \ldots + (n-1) + n$

Assume $n = k$.

$= T(n-n) + (n-n+1) + (n-n+2) + \ldots (n-1) + n$

This was not defined in the problem

$= T(\emptyset) + 1 + 2 + \ldots n-1 + n$

$T(n) = T(\emptyset) + \frac{n(n+1)}{2}$

if $T(0) = 1$

$T(n) = 1 + \frac{n(n+1)}{2} \Rightarrow 1 + \frac{n^2+n}{2}$

Drop the constants, and lower n term

$\cancel{1} + \frac{n^2 + \cancel{n}}{2} \Rightarrow \cancel{\tfrac{1}{2}} n^2 = n^2 \Rightarrow O(n^2)$

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2 = 32

(5) Use the Master method.

(A)

$T(n) = 2T(n/4) + 1$

$a = 2$
$b = 4$
$f(n) = \Theta(1)$
$= \Theta(n^0 \log^0 n)$ (2)
$K = 0 \quad P = 0$

$\log_b a$ (1)

$= \log_4 2 > K$ ✓

so $\Theta(n \log_b a)$
$= \Theta(n \log_4 2)$
$= \Theta(n)$

$T(n) = aT(n/b) + f(n)$

$a \geq 1$
$b > 1$ $f(n) = \Theta(n^K \log^P n)$

Find two values

(1) $\log_b a$
(2) K

if (1) > (2) $\Rightarrow \Theta(n^{\log_b a})$
if (1) = (2) $\Rightarrow$
if $p > -1 \Rightarrow \Theta(n^K \log_n P{+}1)$
if $P = -1 \Rightarrow \Theta(n^K \log \log n)$
$P < -1 \Rightarrow \Theta(n^K)$
if $\log_b a < K$
if $P \geq 0$ $\Theta(n^K \log_n P)$
$P < 0 \Rightarrow \Theta(n^K)$

(B) $T(n) = 2T(n/4) + \sqrt{n} \Rightarrow 2T(n/4) + n^{1/2}$

$a = 2$
$b = 4$
$f(n) = \sqrt{n} \Rightarrow \Theta(n^K \log_n P)$
$\log_4 2 = .5$
$K = 1/2$
$P = 0$
$= \Theta(n^K \log_n(P+1))$
$= \Theta(n^{1/2} \log_n(1))$
$= \Theta(\sqrt{n})$

$\left.\right\}$ something wrong here

?

(C) $T(n) = 2T(n/4) + n^2$

$a = 2$
$b = 4$
$f(n) = (n^2 \log_n P)$
$K = 2$
$P = 0$
$\log_4 2 = .5$
K is greater
$= \Theta(n^K \log_n P)$
$= \Theta(n^2 \log 0)$
$= \Theta(n^2)$