Data Structures and Algorithms

Chapter 10

Maps

- *Map* is a data structure to efficiently store and retrieve values based on *search keys*.
- Map stores (*key*, *value*) pairs.
- Each (*key*, *value*) pair is called an *entry*.
- Keys are unique.
- Maps are also known as *associative arrays*.

- Applications:
  - (movie title, movie information)
  - (part number, part information)
  - (reservation number, reservation information)
  - (student id, student information)

Maps
Map ADT

- size( ): Returns the number of entries in *M*.
- isEmpty( ): Returns true if *M* is empty. Returns false, otherwise.
- get(*k*): Returns the value *v* associated with the key *k*, if such entry exists. Returns null, otherwise.
- put(*k*, *v*): If there is no entry in *M* with a key equal to *k*, then adds the entry (*k*, *v*) to *M* and returns null. Otherwise, replaces the existing value associated with the key *k* with *v* and returns the old value.

Maps
Map ADT

- remove(*k*): Removes from *M* the entry with the key *k* and returns its value. If there is not entry in *M* with the key *k*, returns null.
- keySet( ): Returns an iterable collection containing all keys in *M*.
- values( ): Returns an iterable collection containing all values in *M*. If multiple keys map to the same value, then the value appears multiple times in the returned collection.
- entrySet( ): Returns an iterable collection containing all (*key*, *value*) entries in *M*.

Maps
Map ADT

- Map interface

```
public interface Map<K,V> {
  int size();
  boolean isEmpty();
  V get(K key);
  V put(K key, V value);
  V remove(K key);
  Iterable<K> keySet();
  Iterable<V> values();
  Iterable<Entry<K,V>> entrySet();
}
```

- Note: *java.util.Map* interface provides more extensive set of operations than those defined above.

Maps
Map ADT

- Simple application example: Word Frequency

  - Counts frequency of each word in a text.

  - Create an empty map.
  - In the map, an entry is (word, frequency) pair.
  - Read one word at a time.
  - If the word is not in the map, insert it and set frequency = 1
  - If the word is already in the map, increment the frequency of the word.

- *WordCount.java* code.

Maps
Hash Tables

- *Hash table* is an efficient implementation of a map.

- Consider a map that stores *n* entries.
- Assume keys are integers in the range [0, *N* – 1] and values are characters, usually $N \geq n$.
- We can design a lookup table of length *N* as follows, where keys are used as indexes:

0	1	2	3	4	5	6	7	8	9	10
	D		Z			C	Q			

Lookup table's capacity N = 11
Currently there are 4 entries: (1,D), (3,Z), (6,C), and (7,Q)

Maps
Hash Tables

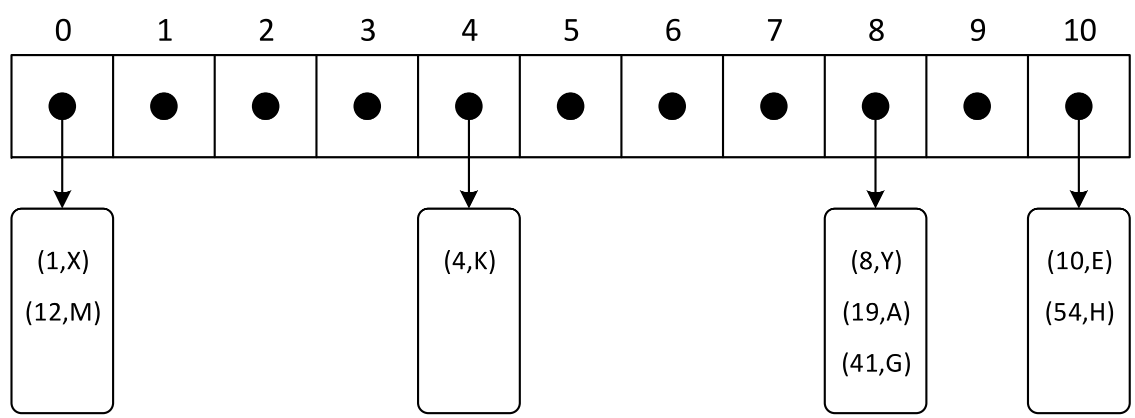
- Issues:
  - The domain of keys may be much larger than the actual number of elements to be stored in the table, i.e., $N >> n$. This is a waste of space.
  - Keys may not be integers. Then, they cannot be used as indexes in the table.

- Solution:
  - Use a *hash function* that maps keys to integers in the range $[0, N – 1]$, distributing keys relatively evenly.
  - N doesn’t have to be very large (could be smaller).

Maps
Hash Tables

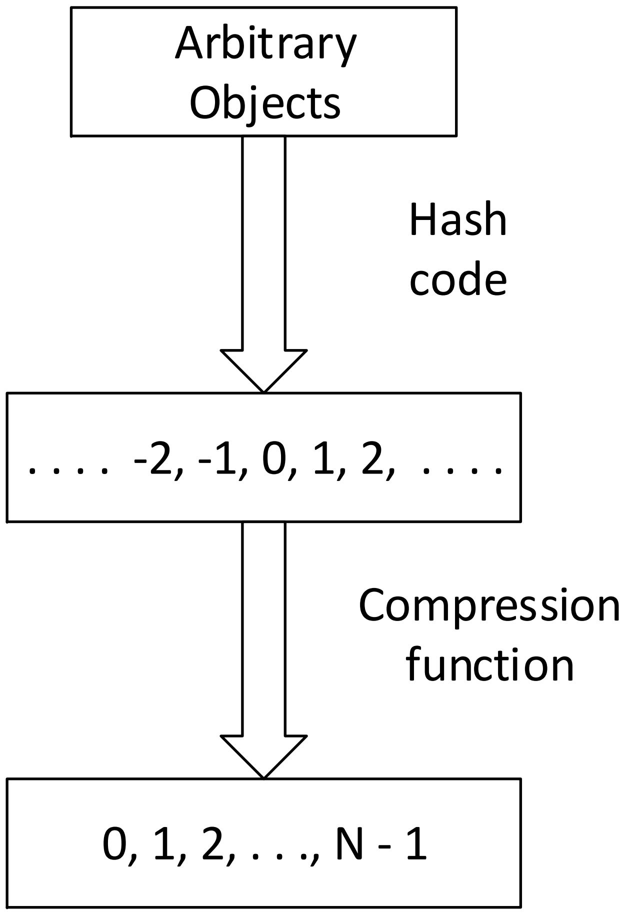
- Ideally a hash function distributes keys evenly across the table.
- In practice, some keys are mapped to the same location.

- One solution: each slot in the table keeps a *bucket* which stores a collection of entries. This table is called *bucket array*.



Maps
Hash Function

- Two step process:
  - *Hash code* maps keys of arbitrary object type to integers. The resulting integer is also called *hash code*.
  - *Compression function* maps the hash code to integers in the range [0, N – 1]



Maps
Hash Code

- Treat bit representation of base types as integers
- Polynomial hash code: used for strings or variable-length objects
- Cyclic-shift hash code: a variant of polynomial hash code

- Java has a default *hashCode*( ) function defined in the *Object* class, which returns a 32-bit integer of ***int*** type.
- When designing a hashCode() for a user-defined class, make sure: If *x*.*equals*(*y*), *x*.*hashCode*( ) = *y*.*hashCode*( )

Maps
Compression Function

- When two keys are mapped to the same hash table index, it is called *collision*.
- A good compression function must distribute hash codes (of keys) relatively uniformly across the hash table to minimize collisions.
- Will discuss two compression functions (compression functions are often called just *hash functions*):
  - *division* method
  - *MAD* (*multiply-add-and-divide*) method

Maps
Compression Function

- Division method: i mod N,

  where i is an integer (such as a hash code) and N is the hash table size.

- *MAD* method: $[(ai + b) \bmod p] \bmod N$,

  where N is hash table size, p is a prime number larger than N, and a and b are integers in $[0, p - 1]$, $a > 0$.

```
private int hashValue(K key) {
    return (int) ((Math.abs(key.hashCode( )*scale + shift)
                                                    % prime) % capacity);
}
```

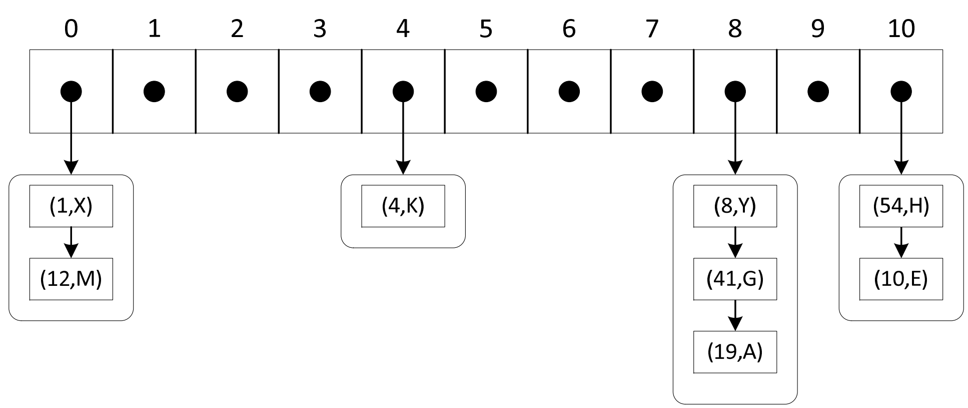
Maps
Compression Function

- *MAD* method is better (in terms of well distributing keys across the has table), but division method is more efficient.

Maps
Collision Handling

- When two keys are mapped to the same slot in the hash table, it is called *collision*.
- Will discuss two collision resolution approaches: *chaining* and *open addressing*.

- Chaining: Each slot in the table keeps an unsorted list and all keys that are mapped to the same slot are kept in the list.



Maps
Chaining Method

- Advantage: Easy to implement
- Drawback:
  - Additional storage
  - In the worst case, all keys are stored in the same list, which increases running time.
- Running time
  - Load factor $\lambda = n / N$, which is expected size of a list.
  - Map operations run in $O(\lceil n / N \rceil)$ or $O(\lambda)$
  - If keys are well distributed, $O(\lambda)$ = O(1) and running time is $O(1)$.
  - In the worst case, $O(n)$.

Maps
Open Addressing

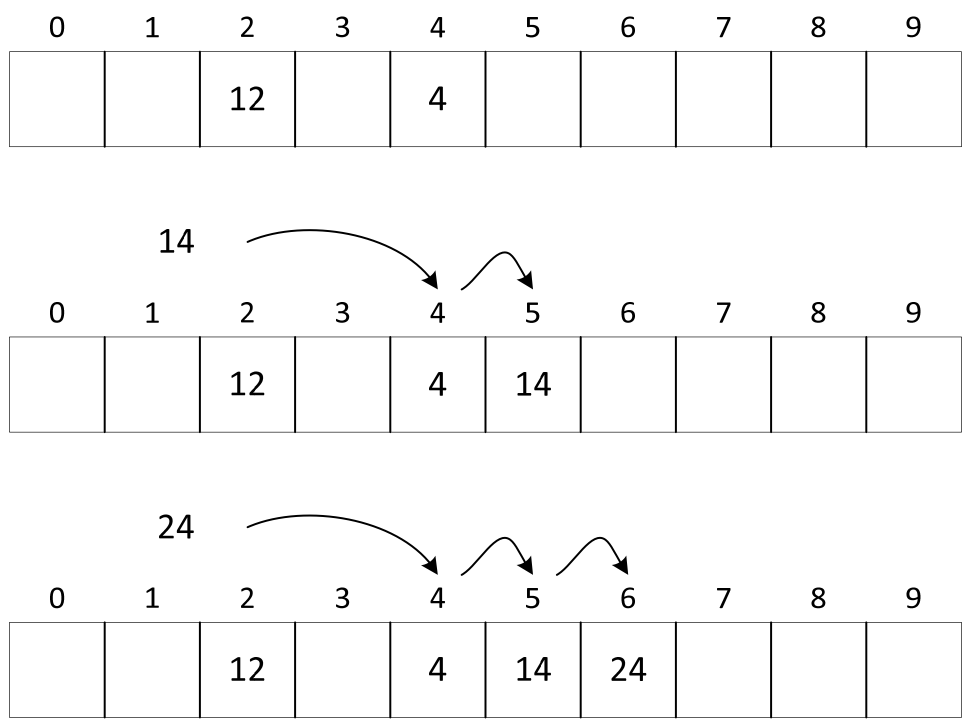
- All entries are stored in a hash table itself.
- No additional data structure and no additional storage space is needed.
- When adding a new key causes a collision, an alternative location in the table is found and the new element is stored in that location.
- Will briefly discuss three open addressing techniques – *linear probing*, *quadratic probing*, and *double hashing*.

Maps
Linear Probing

- Assume A is the array of a hash table.
- Inserting an entry (k, v).
  - Hash function h is applied to key k, i.e., $j \leftarrow h(k)$. We say k is mapped to j.
  - If $A[j]$ is empty, then the entry is stored there, i.e., $A[j] \leftarrow (k, v)$.
  - If that slot is occupied, the next bucket $A[j+1]$ is *probed* to see whether it is available.
  - If it is empty, the entry is stored there. Otherwise, the next bucket, $A[j+2]$, is probed, and so on, until an empty slot is found or all slots have been probed.
  - The sequence of slots probed , called *probe sequence*, is determined by $A[(j+i) \bmod N]$, for $i = 0, 1, 2, . . ., N – 1$.
  - i is called *probe number*.

Maps
Linear Probing

- Illustration: N = 10, h = k mod N, keys are added in the following order: 4, 12, 14, 24.



Maps
Linear Probing

- Searching an entry with key = *k*.
  - A key *k* is mapped to the array index *j,* i.e., $j \leftarrow h(k)$.
  - If $A[j]$ is empty, then conclude the entry is not in the hash table.
  - If that slot is occupied and it has the entry with *k*, then the entry is found.
  - If the slot is occupied and the key of the entry in the slot is not *k,* the next bucket, $A[j+2]$, is probed, and so on, until the entry is found or all slots have been probed.

Maps
Linear Probing

- Deleting an entry:
  – Assume initially all slots are empty.
  – Assume we want to remove an entry in *A*[*j*].
  – We cannot simply remove the entry in *A*[*j*].
  – Assume the current table is:

0	1	2	3	4	5	6	7	8	9
		12		4	14	24			

  – And, we delete an entry with key = 14.

Maps
Linear Probing

- After deleting entry with key = 14

0	1	2	3	4	5	6	7	8	9
		12		4		24			

- Search entry with key = 24

  24 is mapped to A[4]; occupied; A[5] is probed; empty; conclude entry with key = 24 is not in the table => this is wrong.

search 24

0	1	2	3	4	5	6	7	8	9
		12		4		24			

Maps
Linear Probing

- Solution: Put a “special object” or a “*defucnt*” object in the slot from which an entry is deleted.
- For example, place ϕ in the slot when an entry is removed.
- After removing entry with key = 14

0	1	2	3	4	5	6	7	8	9
		12		4	ϕ	24			

- When inserting, the slot with ϕ is considered empty.
- When searching and entry with key = *k*, the slot with ϕ is considered having an entry with a key ≠ *k*.

Maps
Linear Probing

- Linear probing tends to create *primary clustering*.
- A cluster is a contiguous occupied slots.
- Once a cluster is formed, it tends to grow, which is called *primary clustering*.

Maps
Quadratic Probing

- Uses a quadratic function to determine the next slot to probe.
- Example: Probe sequence is determined by

  $A[(h(k) + f(i)) \bmod N]$, for $i = 0, 1, 2, \ldots, N - 1$,

  where $f(i) = i^2$
- Assume that we are inserting a key 24 and it is mapped to $A[4]$, and that it is occupied. Then, the probe sequence is:

  $A[(4 + 1^2) \bmod 10] = A[5]$,

  $A[(4 + 2^2) \bmod 10] = A[8]$,

  $A[(4 + 3^2) \bmod 10] = A[3]$,

  . . . .

Maps
Quadratic Probing

- Quadratic hashing does not have primary clustering.
- But, it still has clustering problem, which is called secondary clustering.

- There are quadratic probing methods that use different quadratic functions.

Maps
Double Hashing

- Does not cause serious clustering problem.
- Uses two hash functions.
- Probe sequence is determined by

  $A[(h(k) + i \cdot h'(k)) \bmod N]$, for $i = 0, 1, 2, \ldots, N - 1$

- One common secondary hash function h' is:

  $h'(k) = q - (k \bmod q)$, for some prime number $q < N$, N is prime
- Another common h' is:

  $h'(k) = 1 + (k \bmod N')$, where N' is slightly smaller than N, N is prime

Maps
Double Hashing

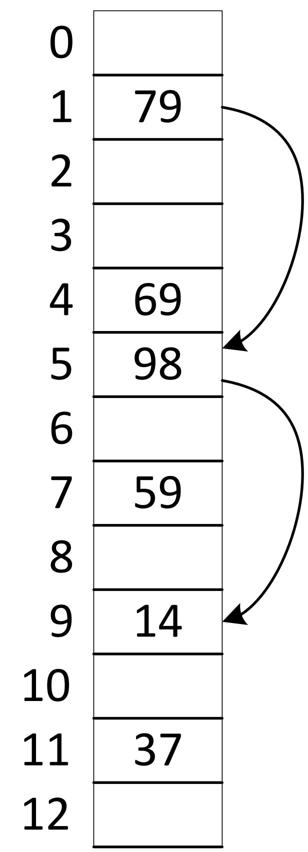
- Example (of the second *h*')

  $h(k) = k \bmod 13$

  $h'(k) = 1 + (k \bmod 11)$

  $h(k, i) = (h(k) + i * h'(k)) \bmod N$

  - Inserting $k = 14$, $h(k) = 1$, $h'(k) = 4$
  - $h(14) = 1$, occupied
  - $i = 1$: $1 + 4 = 5$, occupied
  - $i = 2$: $1 + 8 = 9$, empty, store 14 here



Maps
Load Factor and Efficiency

- Load factor is defined as $\lambda = n / N$
- A larger value of λ means there is higher probability of collisions.
- So, a smaller λ is better.
- With chaining method, λ could be greater than 1.
- With open addressing, $\lambda \leq 1$.

- Performance of chaining method:
  - A theoretical analysis shows that the average number of entries that need to be probed for a successful search is approximately $1+\frac{\lambda}{2}$.

Maps
Load Factor and Efficiency

- Performance of chaining method (continued):
  - Let *C* be the average number of entries that need to be probed for a successful search.

λ	C
0.5	1.25
0.7	1.35
1.0	1.5
2.0	2

  - Java uses chaining method and *λ* is set to 0.75 or less by default.

Maps

Load Factor and Efficiency

- Performance of double hashing:
  - The average number of slots that need to be probed for a successful search is approximately $\frac{1}{\lambda}\ln\frac{1}{1-\lambda}$
  - Let *C* be the average number of slots that need to be probed for a successful search.

λ	C
0.3	1.19
0.5	1.39
0.7	1.72
0.9	2.56

References

- M.T. Goodrich, R. Tamassia, and M.H. Goldwasser, "Data Structures and Algorithms in Java," Sixth Edition, Wiley, 2014.