RADIATIVE VIEW FACTORS

VIEW FACTOR DEFINITION

The view factor F_{12} is the fraction of energy exiting an isothermal, opaque, and diffuse surface 1 (by emission or reflection), that directly impinges on surface 2 (and is absorbed or reflected). Some view factors having an analytical expression are compiled below.

View factors only depend on geometry, and can be computed from the general expression below. Consider two infinitesimal surface patches, $\mathrm{d}A_1$ and $\mathrm{d}A_2$ (Fig. 1), in arbitrary position and orientation, defined by their separation distance r_{12}, and their respective tilting relative to the line of centres, β_1 and β_2, with $0\le\beta_1\le\pi/2$ and $0\le\beta_2\le\pi/2$ (i.e. seeing each other). The radiation power intercepted by surface $\mathrm{d}A_2$ coming directly from a diffuse surface $\mathrm{d}A_1$ is the product of its radiance $L_1=M_1/\pi$, times its perpendicular area $\mathrm{d}A_{1\perp}$, times the solid angle subtended by $\mathrm{d}A_2$, $\mathrm{d}\Omega_{12}$; i.e. $\mathrm{d}^2\Phi_{12}=L_1\mathrm{d}A_{1\perp}\mathrm{d}\Omega_{12}=L_1(\mathrm{d}A_1\cos(\beta_1))\mathrm{d}A_2\cos(\beta_2)/r_{12}^2$. Thence:

$$\mathrm{d}F_{12}\equiv\frac{\mathrm{d}^2\Phi_{12}}{M_1\mathrm{d}A_1}=\frac{L_1\mathrm{d}\Omega_{12}\mathrm{d}A_1\cos(\beta_1)}{M_1\mathrm{d}A_1}=\frac{\cos(\beta_1)}{\pi}\mathrm{d}\Omega_{12}=\frac{\cos(\beta_1)}{\pi}\frac{\mathrm{d}A_2\cos(\beta_2)}{r_{12}^2}$$

$$\rightarrow\quad \mathrm{d}F_{12}=\frac{\cos(\beta_1)\cos(\beta_2)}{\pi r_{12}^2}\mathrm{d}A_2\quad\rightarrow\quad F_{12}=\frac{1}{A_1}\int_{A_1}\left(\int_{A_2}\frac{\cos\beta_1\cos\beta_2}{\pi r_{12}^2}\mathrm{d}A_2\right)\mathrm{d}A_1$$

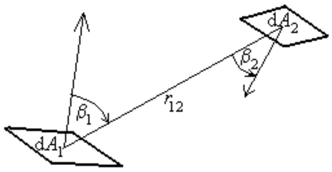


Fig. 1. Geometry for view-factor definition.

When finite surfaces are involved, computing view factors is just a problem of mathematical integration (not a trivial one, except in simple cases). Recall that the emitting surface (exiting, in general) must be isothermal, opaque, and Lambertian (a perfect diffuser), and, to apply view-factor algebra, all surfaces must be isothermal, opaque, and Lambertian.

View factor algebra

When considering all the surfaces under sight from a given one (enclosure theory), several general relations can be established among the N^2 possible view factors, what is known as view factor algebra:

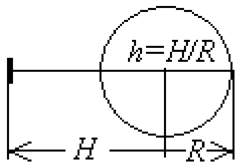
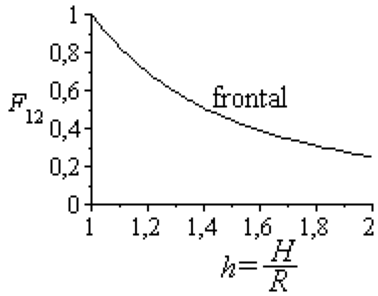
- Bounding. View factors are bounded to $0\le F_{ij}\le 1$ by definition (the view factor F_{ij} is the fraction of energy exiting surface i, that impinges on surface j).
- Closeness. Summing up all view factors from a given surface in an enclosure, including the possible self-view factor for concave surfaces, $\sum_j F_{ij}=1$, because the same amount of radiation emitted by a surface must be absorbed.
- Reciprocity. Noticing from the above equation that $\mathrm{d}A_i\mathrm{d}F_{ij}=\mathrm{d}A_j\mathrm{d}F_{ji}=(\cos\beta_i\cos\beta_j/(\pi r_{ij}^2))\mathrm{d}A_i\mathrm{d}A_j$, it is deduced that $A_iF_{ij}=A_jF_{ji}$.
- Distribution. When two target surfaces are considered at once, $F_{i,j+k}=F_{ij}+F_{ik}$, based on area additivity in the definition.
- Composition. Based on reciprocity and distribution, when two source areas are considered together, $F_{i+j,k}=\left(A_iF_{ik}+A_jF_{jk}\right)/\left(A_i+A_j\right)$.

For an enclosure formed by N surfaces, there are N^2 view factors (each surface with all the others and itself). But only $N(N-1)/2$ of them are independent, since another $N(N-1)/2$ can be deduced from reciprocity relations, and N more by closeness relations. For instance, for a 3-surface enclosure, we can define 9 possible view factors, 3 of which must be found independently, another 3 can be obtained from $A_iF_{ij}=A_jF_{ji}$, and the remaining 3 by $\sum_j F_{ij}=1$.

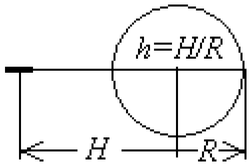
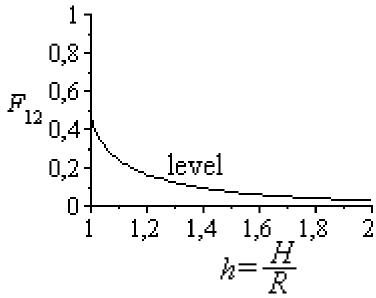
WITH SPHERES

Patch to a sphere

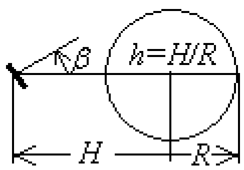
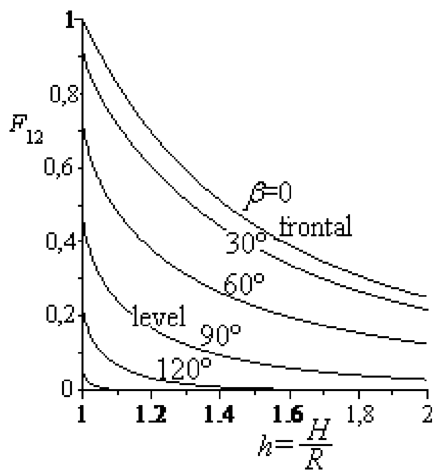
Frontal

Case	View factor	Plot
From a small planar plate facing a sphere of radius *R*, at a distance *H* from centres, with $h\equiv H/R$. 	$F_{12}=\frac{1}{h^2}$ (e.g. for h=2, F_{12}=1/4)	

Level

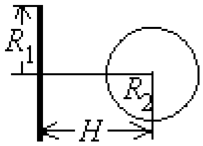
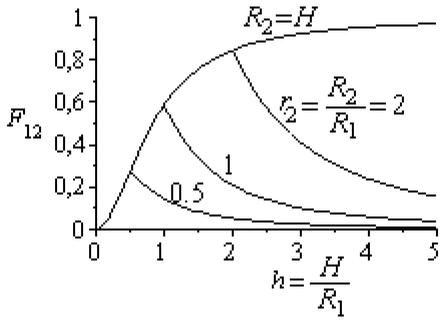
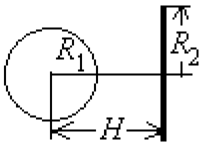
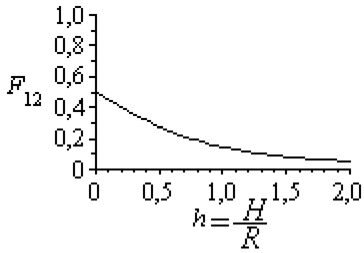
Case	View factor	Plot
From a small planar plate level to a sphere of radius *R*, at a distance *H* from centres, with $h\equiv H/R$. 	$F_{12}=\frac{1}{\pi}\left(\arctan\frac{1}{x}-\frac{x}{h^2}\right)$ with $x\equiv\sqrt{h^2-1}$ ($F_{12}\big\vert_{h\to1}\to\frac{1}{2}-\frac{2\sqrt{2}}{\pi}\sqrt{h-1}$) (e.g. for h=2, F_{12}=0.029)	

Tilted

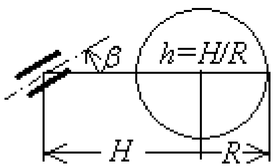
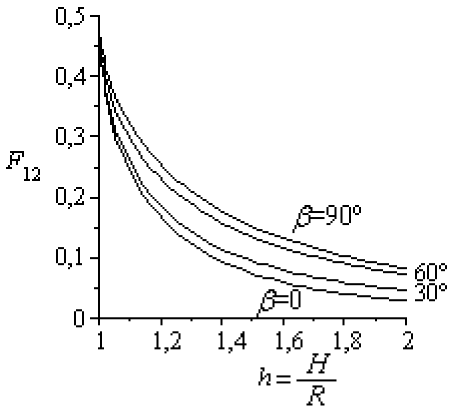
Case	View factor	Plot
From a small planar plate tilted to a sphere of radius *R*, at a distance *H* from centres, with $h\equiv H/R$; the tilting angle β is between the normal and the line of centres. 	-if $\vert\beta\vert<\pi/2-\arcsin(1/h)$ (i.e. $h\cos\beta>1$), $F_{12}=\frac{\cos\beta}{h^2}$ -if not, $F_{12}=\frac{1}{\pi h^2}\left(\cos\beta\arccos y-x\sin\beta\sqrt{1-y^2}\right)+\frac{1}{\pi}\arctan\left(\frac{\sin\beta\sqrt{1-y^2}}{x}\right)$ with $x\equiv\sqrt{h^2-1},\quad y\equiv -x\cot(\beta)$ (e.g. for h=2 and $\beta=\pi/4$ (45º), F_{12}=0.177)	

Disc to frontal sphere

Case	View factor	Plot

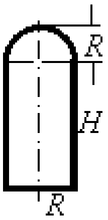
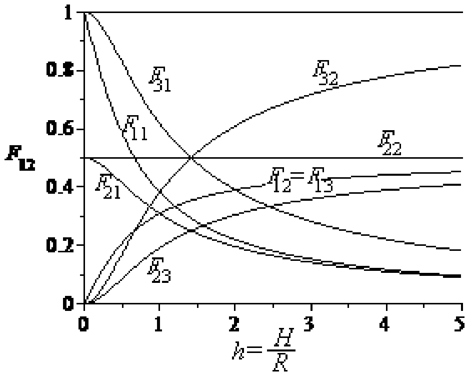
From a disc of radius R_1 to a frontal sphere of radius R_2 at a distance H between centres (it must be $H>R_1$), with $h\equiv H/R_1$ and $r_2\equiv R_2/R_1$.  	$F_{12}=2r_2^2\left(1-\frac{1}{\sqrt{1+\frac{1}{h^2}}}\right)$ (e.g. for $h=r_2=1$, $F_{12}=0.586$)	
From a sphere of radius R_1 to a frontal disc of radius R_2 at a distance H between centres (it must be $H>R_1$, but does not depend on R_1), with $h\equiv H/R_2$.  	$F_{12}=\frac{1}{2}\left(1-\frac{1}{\sqrt{1+\frac{1}{h^2}}}\right)$ (e.g. for $R_2=H$ and $R_1\leq H$, $F_{12}=0.146$)	

Cylinder to large sphere

Case	View factor	Plot
From a small cylinder (external lateral area only), at an altitude $H=hR$ and tilted an angle β, to a large sphere of radius R, β is between the cylinder axis and the line of centres).  	Coaxial (β=0): $F_{12}=\frac{1}{2}-\frac{s}{\pi(1+h)}-\frac{\arcsin(s)}{\pi}$ with $s\equiv\frac{\sqrt{2h+h^2}}{1+h}$ Perpendicular ($\beta=\pi/2$): $F_{12}=\frac{4}{\pi^2}\int_0^{\frac{1}{1+h}}\frac{x\mathrm{E}(x)\mathrm{d}x}{\sqrt{1-x^2}}$ with elliptic integrals E(x). Tilted cylinder: $F_{12}=\int_{\theta=0}^{\arcsin\left(\frac{1}{1+h}\right)}\int_{\phi=0}^{2\pi}\frac{\sin(\theta)\sqrt{1-z^2}\mathrm{d}\theta\mathrm{d}\phi}{\pi^2}$ with $z\equiv\cos(\theta)\cos(\beta)+$ $+\sin(\theta)\sin(\beta)\cos(\phi)$ (e.g. for h=1 and any β, F_{12}=1/2)	

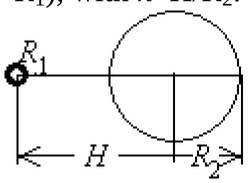
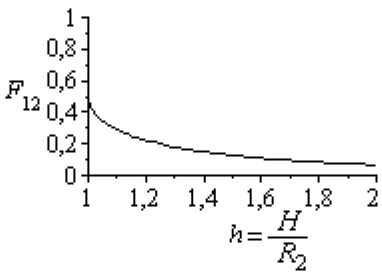
Cylinder to its hemispherical closing cap

Case	View factor	Plot

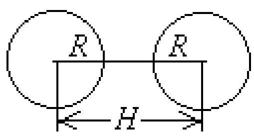
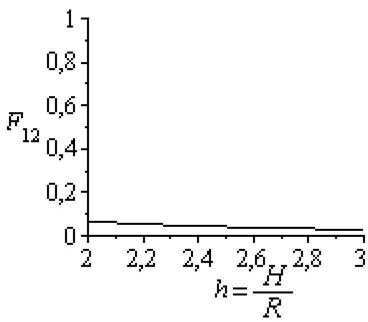
From a finite cylinder (surface 1) of radius R and height H, to its hemispherical closing cap (surface 2), with r=R/H. Let surface 3 be the base, and surface 4 the virtual base of the hemisphere. 	$F_{11}=1-\frac{\rho}{2},\ F_{12}=F_{13}=F_{14}=\frac{\rho}{4}$ $F_{21}=\frac{\rho}{4r},\ F_{22}=\frac{1}{2},\ F_{23}=\frac{1}{2}-\frac{\rho}{4r},$ $F_{31}=\frac{\rho}{2r},\ F_{32}=1-\frac{\rho}{2r},\ F_{34}=1-\frac{\rho}{2r}$ with $\rho=\frac{\sqrt{4r^2+1}-1}{r}$ (e.g. for R=H, F_{11}=0.38, F_{12}=0.31, F_{21}=0.31, F_{22}=0.50, F_{23}=0.19, F_{31}=0.62, F_{32}=0.38, F_{34}=0.38)	

Sphere to sphere

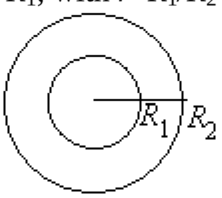
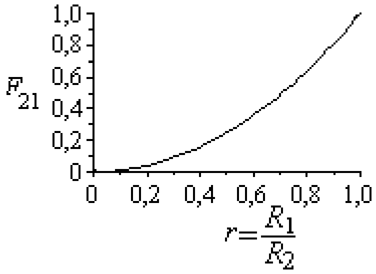
Small to very large

Case	View factor	Plot
From a small sphere of radius R_1 to a much larger sphere of radius R_2 at a distance H between centres (it must be H>R_2, but does not depend on R_1), with $h\equiv H/R_2$. 	$F_{12}=\frac{1}{2}\left(1-\sqrt{1-\frac{1}{h^2}}\right)$ (e.g. for H=R_2, F_{12}=1/2)	

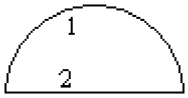
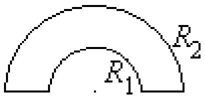
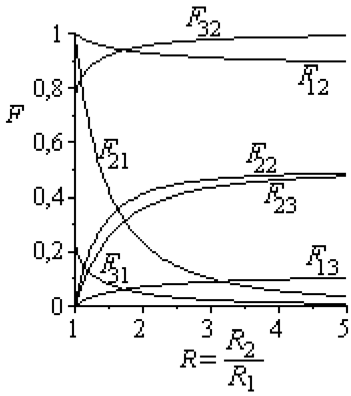
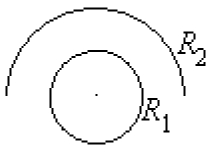
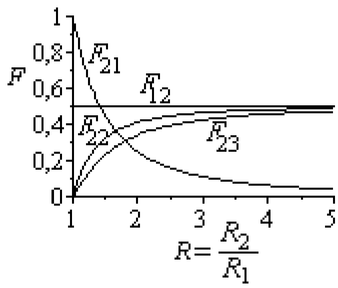
Equal spheres

Case	View factor	Plot
From a sphere of radius R to an equal sphere at a distance H between centres (it must be H>2R), with $h\equiv H/R$. 	$F_{12}=\frac{1}{2}\left(1-\sqrt{1-\frac{1}{h^2}}\right)$ (e.g. for H=2R, F_{12}=0.067)	

Concentric spheres

Case	View factor	Plot
Between concentric spheres of radii R_1 and R_2>R_1, with $r\equiv R_1/R_2$<1. 	F_{12}=1 F_{21}=r^2 F_{22}=1−r^2 (e.g. for r=1/2, F_{12}=1, F_{21}=1/4, F_{22}=3/4)	

Hemispheres

Case	View factor	Plot
From a hemisphere of radius R (surface 1) to its base circle (surface 2). 	$F_{21}=1$ $F_{12}=A_2F_{21}/A_1=1/2$ $F_{11}=1-F_{12}=1/2$	
From a hemisphere of radius R_1 to a larger concentric hemisphere of radius $R_2>R_1$, with $R\equiv R_2/R_1>1$. Let the closing planar annulus be surface 3. 	$F_{12}=1-\frac{\rho}{4},\ F_{13}=\frac{\rho}{4},\ F_{21}=\frac{1}{R^2}\left(1-\frac{\rho}{4}\right),$ $F_{22}=\frac{1}{2}\left(1-\frac{1-\rho}{R^2}\right),$ $F_{23}=\frac{1}{2}\left(1-\frac{1}{R^2}\right)\left(1-\frac{\rho}{2\left(R^2-1\right)}\right),$ $F_{31}=\frac{\rho}{2R^2},\ F_{32}=1-\frac{\rho}{2\left(R^2-1\right)}$ with $\rho=\frac{1}{2}-\frac{1}{\pi}\left[\sqrt{R^2-1}-\left(R^2-2\right)\arcsin\left(\frac{1}{R}\right)\right]$ (e.g. for R=2, F_{12}=0.93, F_{21}=0.23, F_{13}=0.07, F_{31}=0.05, F_{32}=0.95, F_{23}=0.36,F_{22}=0.41)	
From a sphere of radius R_1 to a larger concentric hemisphere of radius $R_2>R_1$, with $R\equiv R_2/R_1>1$. Let the enclosure be '3'. 	F_{12}=1/2, F_{13}=1/2, $F_{21}=1/R^2$, $F_{23}=1-F_{21}-F_{22},\ F_{22}=\frac{1}{2}\left(1-\frac{1-\rho}{R^2}\right)$ with $\rho=\frac{1}{2}-\frac{1}{\pi}\left[\sqrt{R^2-1}-\left(R^2-2\right)\arcsin\left(\frac{1}{R}\right)\right]$ (e.g. for R=2, F_{12}=1/2, F_{21}=1/4, F_{13}=1/2, F_{23}=0.34,F_{22}=0.41)	

WITH CYLINDERS

Cylinder to large sphere

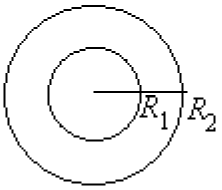
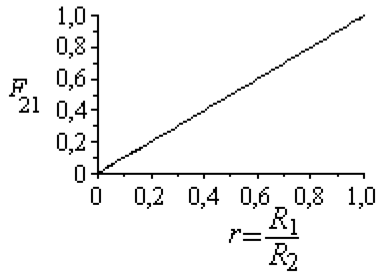
See results under Cases with spheres.

Cylinder to its hemispherical closing cap

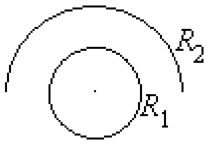
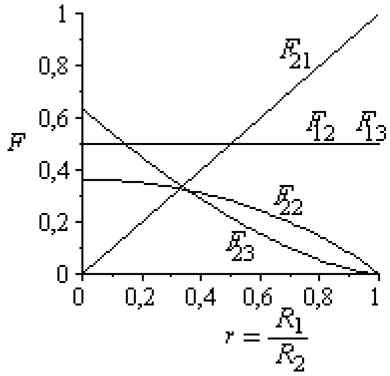
See results under Cases with spheres.

Concentric very-long cylinders

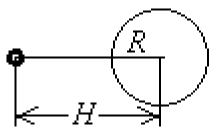
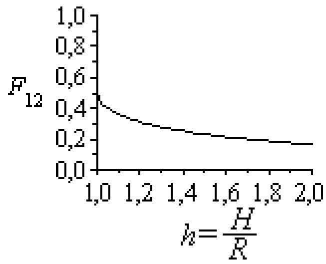
Case	View factor	Plot

Between concentric infinite cylinders of radii R_1 and $R_2>R_1$, with $r\equiv R_1/R_2<1$. 	$F_{12}=1$ $F_{21}=r$ $F_{22}=1-r$ (e.g. for $r=1/2$, $F_{12}=1$, $F_{21}=1/2$, $F_{22}=1/4$)	

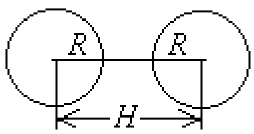
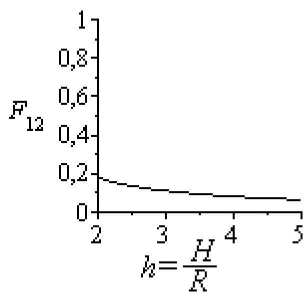
Concentric very-long cylinder to hemi-cylinder

Case	View factor	Plot
Between concentric infinite cylinder of radius R_1 to concentric hemi-cylinder of radius $R_2>R_1$, with $r\equiv R_1/R_2<1$. Let the enclosure be '3'. 	$F_{12}=1/2$, $F_{21}=r$, $F_{13}=1/2$, $F_{23}=1-F_{21}-F_{22}$, $F_{22}=1-\frac{2}{\pi}\left(\sqrt{1-r^2}+r\arcsin r\right)$ (e.g. for $r=1/2$, $F_{12}=1/2$, $F_{21}=1/2$, $F_{13}=1/2$, $F_{23}=0.22$,$F_{22}=0.28$)	

Wire to parallel cylinder, infinite extent

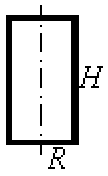
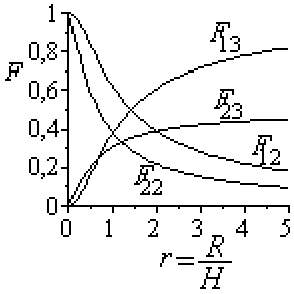
Case	View factor	Plot
From a small infinite long cylinder to an infinite long parallel cylinder of radius R, with a distance H between axes, with $h\equiv H/R$. 	$F_{12}=\frac{\arcsin\frac{1}{h}}{\pi}$ (e.g. for $H=R$, $F_{12}=1/2$)	

Parallel very-long external cylinders

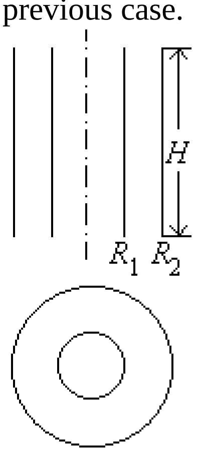
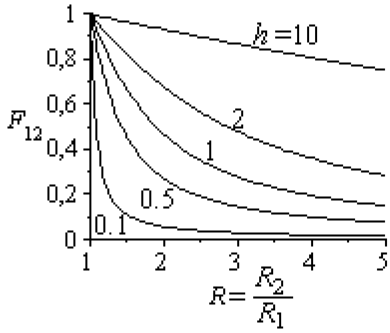
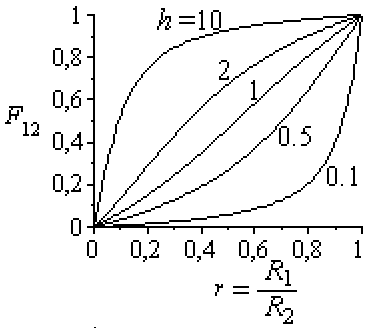
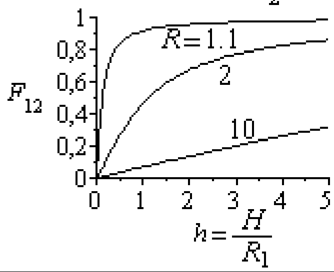
Case	View factor	Plot
From a cylinder of radius R to an equal cylinder at a distance H between centres (it must be $H>2R$), with $h\equiv H/R$. 	$F_{12}=\frac{\sqrt{h^2-4}-h+2\arcsin\frac{2}{h}}{2\pi}$ (e.g. for $H=2R$, $F_{12}=1/2-1/\pi=0.18$)	

Base to finite cylinder

Case	View factor	Plot

From base (1) to lateral surface (2) in a cylinder of radius R and height H, with $r \equiv R/H$. Let (3) be the opposite base. 	$F_{12}=\frac{\rho}{2r}$, $F_{13}=1-\frac{\rho}{2r}$, $F_{21}=\frac{\rho}{4}$, $F_{22}=1-\frac{\rho}{2}$, $F_{23}=\frac{\rho}{4}$ with $\rho=\frac{\sqrt{4r^2+1}-1}{r}$ (e.g. for $R=H$, $F_{12}=0.62$, $F_{21}=0.31$, $F_{13}=0.38$, $F_{22}=0.38$)	

Equal finite concentric cylinders

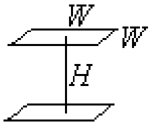
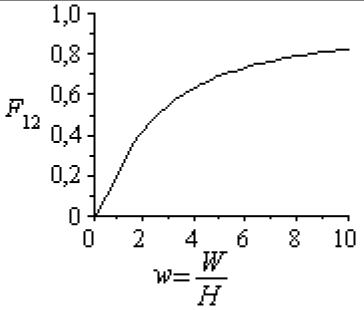
Case	View factor	Plot
Between finite concentric cylinders of radius R_1 and $R_2>R_1$ and height H, with $h=H/R_1$ and $R=R_2/R_1$. Let the enclosure be '3'. For the inside of '1', see previous case. 	$F_{12}=1-\frac{1}{\pi}\left(\arccos\frac{f_2}{f_1}-\frac{f_4}{2h}\right)$, $F_{13}=1-F_{12}$, $F_{22}=1-\frac{1}{R}+\frac{2}{\pi R}\arctan\frac{2\sqrt{R^2-1}}{h}-\frac{hf_7}{2\pi R}$, $F_{23}=1-F_{21}-F_{22}$ with $f_1=h^2+R^2-1$, $f_2=h^2-R^2+1$, $f_3=\sqrt{(A+2)^2-4R^2}$, $f_4=f_3\arccos\frac{f_2}{Rf_1}+f_2\arcsin\frac{1}{R}-\frac{\pi f_1}{2}$, $f_5=\sqrt{\frac{4R^2}{h^2}+1}$, $f_6=1-\frac{2h^2}{R^2\left(h^2+4R^2-4\right)}$, $f_7=f_5\arcsin f_6-\arcsin\left(1-\frac{1}{R^2}\right)+\frac{\pi}{2}(f_5-1)$ (e.g. for $R_2=2R_1$ and $H=2R_1$, $F_{12}=0.64$, $F_{21}=0.34$, $F_{13}=0.33$, $F_{23}=0.43$, $F_{22}=0.23$)	

WITH PLATES AND DISCS

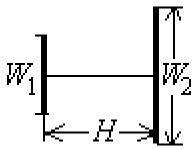
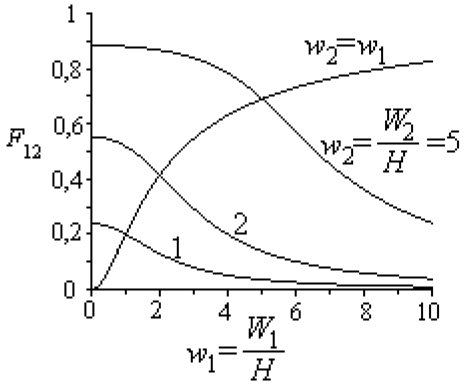
Parallel configurations

Equal square plates

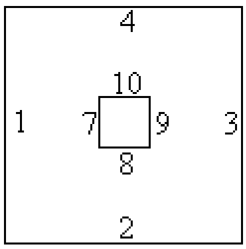
Case	View factor	Plot

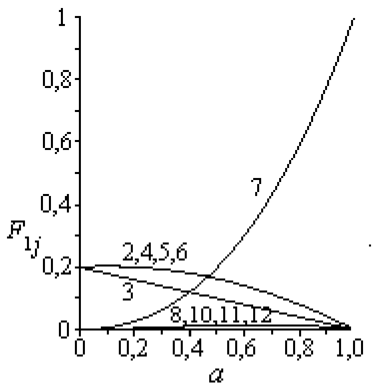
<table>
<tr>
<td>Between two identical parallel square plates of side L and separation H, with w=W/H.

</td>
<td>$$F_{12}=\frac{1}{\pi w^2}\left(\ln\frac{x^4}{1+2w^2}+4wy\right)$$ with $x\equiv\sqrt{1+w^2}$ and $$y\equiv x\arctan\frac{w}{x}-\arctan w$$ (e.g. for W=H, F_{12}=0.1998)</td>
<td>
</td>
</tr>
</table>

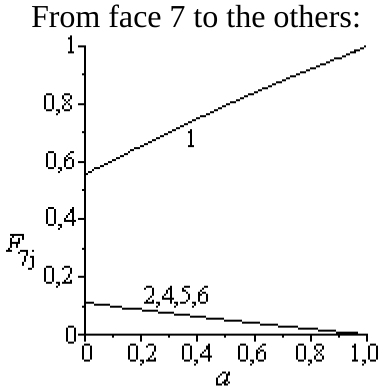
Unequal coaxial square plates

<table>
<tr><th>Case</th><th>View factor</th><th>Plot</th></tr>
<tr>
<td>From a square plate of side W_1 to a coaxial square plate of side W_2 at separation H, with $w_1=W_1/H$ and $w_2=W_2/H$.

</td>
<td>$$F_{12}=\frac{1}{\pi w_1^2}\left(\ln\frac{p}{q}+s-t\right),\ \text{with}$$ $$\begin{cases} p\equiv\left(w_1^2+w_2^2+2\right)^2 \\ q\equiv\left(x^2+2\right)\left(y^2+2\right) \\ x\equiv w_2-w_1,\quad y\equiv w_2+w_1 \\ s\equiv u\left(x\arctan\frac{x}{u}-y\arctan\frac{y}{u}\right) \\ t\equiv v\left(x\arctan\frac{x}{v}-y\arctan\frac{y}{v}\right) \\ u\equiv\sqrt{x^2+4},\quad v\equiv\sqrt{y^2+4} \end{cases}$$ (e.g. for $W_1=W_2=H$, F_{12}=0.1998)</td>
<td>
</td>
</tr>
</table>

Box inside concentric box

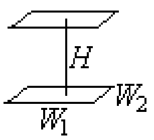
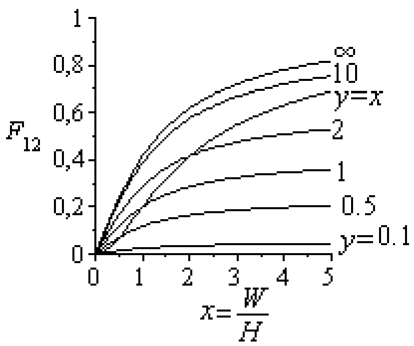
<table>
<tr><th>Case</th><th>View factor</th><th>Plot</th></tr>
<tr>
<td>Between all faces in the enclosure formed by the internal side of a cube box (faces 1-2-3-4-5-6), and the external side of a concentric cubic box (faces (7-8-9-10-11-12) of size ratio a≤1.


(A generic outer-box face #1, and its corresponding face #7 in the inner box, have been chosen.)</td>
<td>From an external-box face: $$\begin{cases} F_{11}=0,\,F_{12}=x,\,F_{13}=y,\,F_{14}=x, \\ F_{15}=x,\,F_{16}=x,\,F_{17}=za^2,\,F_{18}=r, \\ F_{19}=0,\,F_{1,10}=r,\,F_{1,11}=r,\,F_{1,12}=r \end{cases}$$ From an internal-box face: $$\begin{cases} F_{71}=z,\,F_{72}=\left(1-z\right)/4,\,F_{73}=0,\,F_{74}=\left(1-z\right)/4, \\ F_{75}=\left(1-z\right)/4,\,F_{76}=\left(1-z\right)/4,\,F_{77}=0,\,F_{78}=0, \\ F_{79}=0,\,F_{7,10}=0,\,F_{7,11}=0,\,F_{7,12}=0 \end{cases}$$ with z given by:</td>
<td>From face 1 to the others:


From face 7 to the others:

</td>
</tr>
</table>

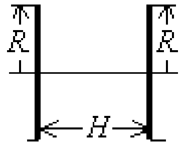
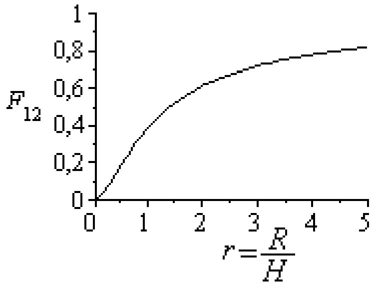
	$\begin{cases} z=F_{71}=\dfrac{(1-a)^2}{4\pi a^2}\left(\ln\dfrac{p}{q}+s+t\right) \\ p\equiv\left(2\dfrac{3-2a+3a^2}{(1-a)^2}\right)^2 \\ q\equiv 2\dfrac{18+12a+18a^2}{(1-a)^2} \\ s\equiv u\left(2\arctan\dfrac{2}{u}-w\arctan\dfrac{w}{u}\right) \\ t\equiv v\left(2\arctan\dfrac{2}{v}-w\arctan\dfrac{w}{v}\right) \\ u\equiv\sqrt{8},\ v\equiv\dfrac{\sqrt{8(1+a^2)}}{1-a},\ w\equiv 2\dfrac{1+a}{1-a} \end{cases}$ and: $\begin{cases} r\equiv a^2(1-z)/4 \\ y\simeq 0.2(1-a) \\ x\equiv(1-y-za^2-4r)/4 \end{cases}$ (e.g. for a=0.5, F_{11}=0, F_{12}=0.16, F_{13}=0.10, F_{14}=0.16, F_{15}=0.16, F_{16}=0.16, F_{17}=0.20, F_{18}=0.01, F_{19}=0, $F_{1,10}$=0.01, $F_{1,11}$=0.01, $F_{1,12}$=0.01), and (F_{71}=0.79, F_{72}=0.05, F_{73}=0, F_{74}=0.05, F_{75}=0.05, F_{76}=0.05, F_{77}=0, F_{78}=0, F_{79}=0, $F_{7,10}$=0, $F_{7,11}$=0, $F_{7,12}$=0). Notice that a simple interpolation is proposed for $y\equiv F_{13}$ because no analytical solution has been found.	

Equal rectangular plates

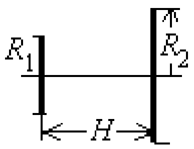
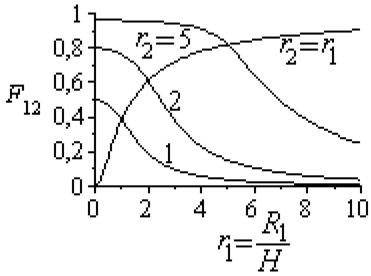
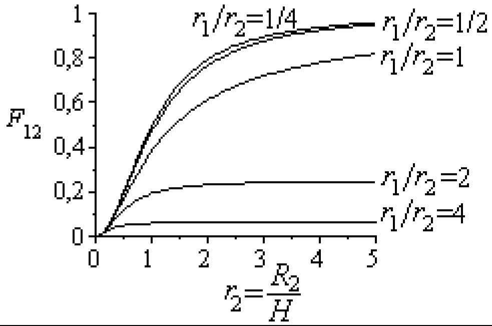
Case	View factor	Plot
Between parallel equal rectangular plates of size $W_1\cdot W_2$ separated a distance H, with $x=W_1/H$ and $y=W_2/H$.  	$F_{12}=\dfrac{1}{\pi xy}\left[\ln\dfrac{x_1^2y_1^2}{x_1^2+y_1^2-1}+2x\left(y_1\arctan\dfrac{x}{y_1}-\arctan x\right)+2y\left(x_1\arctan\dfrac{y}{x_1}-\arctan y\right)\right]$ with $x_1\equiv\sqrt{1+x^2}$ and $y_1\equiv\sqrt{1+y^2}$ (e.g. for x=y=1, F_{12}=0.1998)	

Equal discs

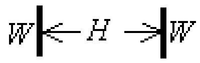
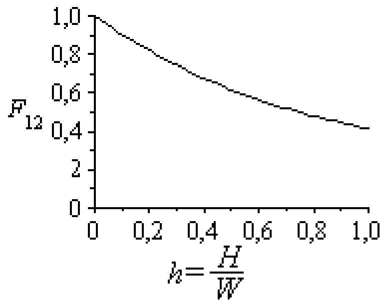
Case	View factor	Plot

Between two identical coaxial discs of radius R and separation H, with $r=R/H$. 	$$F_{12}=1+\frac{1-\sqrt{4r^2+1}}{2r^2}$$ (e.g. for r=1, F_{12}=0.382)	

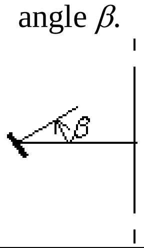
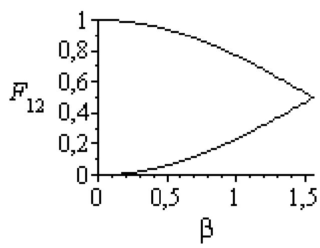
Unequal discs

Case	View factor	Plot
From a disc of radius R_1 to a coaxial parallel disc of radius R_2 at separation H, with $r_1=R_1/H$ and $r_2=R_2/H$. 	$$F_{12}=\frac{x-y}{2}$$ with $x=1+1/r_1^2+r_2^2/r_1^2$ and $y=\sqrt{x^2-4r_2^2/r_1^2}$ (e.g. for r_1=r_2=1, F_{12}=0.382)	 

Strip to strip

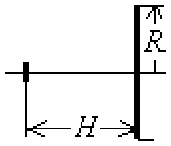
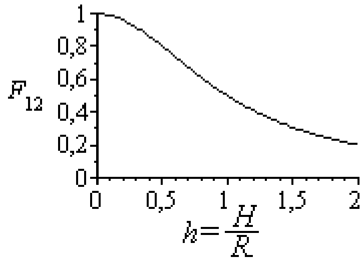
Case	View factor	Plot
Between two identical parallel strips of width W and separation H, with $h=H/W$. 	$$F_{12}=\sqrt{1+h^2}-h$$ (e.g. for h=1, F_{12}=0.414)	

Patch to infinite plate

Case	View factor	Plot
From a finite planar plate at a distance H to an infinite plane, tilted an angle β. 	Front side: $F_{12}=\frac{1+\cos\beta}{2}$ Back side: $F_{12}=\frac{1-\cos\beta}{2}$ (e.g. for $\beta=\pi/4$ (45º), $F_{12,\text{front}}$=0.854, $F_{12,\text{back}}$=0.146)	

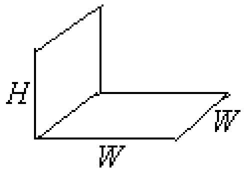
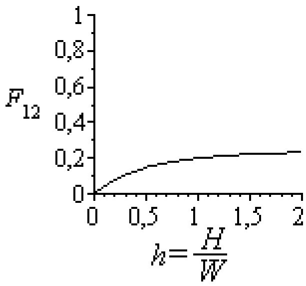
Patch to disc

Case	View factor	Plot

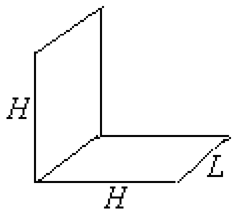
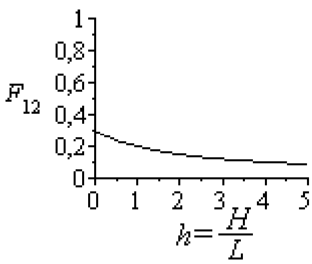
From a patch to a parallel and concentric disc of radius R at distance H, with $h=H/R$.  	$$F_{12}=\frac{1}{1+h^2}$$ (e.g. for h=1, F_{12}=0.5)	

Perpendicular configurations

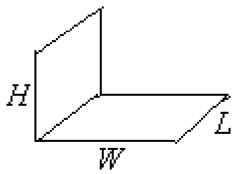
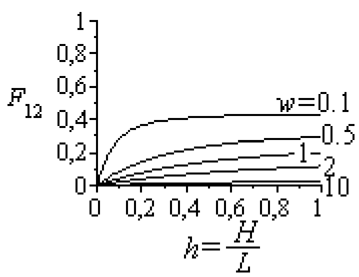
Square plate to rectangular plate

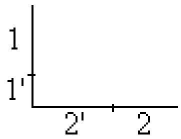
Case	View factor	Plot
From a square plate of with W to an adjacent rectangles at 90º, of height H, with $h=H/W$.  	$$F_{12}=\frac{1}{4}+\frac{1}{\pi}\left[h\arctan\frac{1}{h}-h_1\arctan\frac{1}{h_1}-\frac{h^2}{4}\ln h_2\right]$$ with $h_1=\sqrt{1+h^2}$ and $h_2=\frac{h_1^4}{h^2\left(2+h^2\right)}$ (e.g. for h=→∞, F_{12}=→1/4, for h=1, F_{12}=0.20004, for h=1/2, F_{12}=0.146)	

Rectangular plate to equal rectangular plate

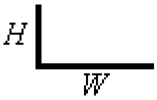
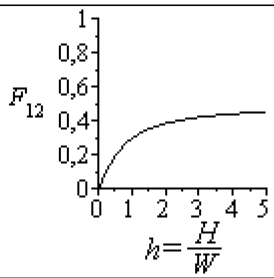
Case	View factor	Plot
Between adjacent equal rectangles at 90º, of height H and width L, with $h=H/L$.  	$$F_{12}=\frac{1}{\pi}\left[2\arctan\left(\frac{1}{h}\right)-\sqrt{2}\arctan\left(\frac{1}{\sqrt{2}h}\right)+\frac{1}{4h}\ln\left(\frac{h_1h_2}{4}\right)\right]$$ with $h_1=2\left(1+h^2\right)$ and $h_2=\left(1-\frac{1}{h_1}\right)^{2h^2-1}$ (e.g. for h=1, F_{12}=0.20004)	

Rectangular plate to unequal rectangular plate

Case	View factor	Plot
From a horizontal rectangle of $W{\cdot}L$ to adjacent vertical rectangle of $H{\cdot}L$, with $h=H/L$ and $w=W/L$.  	$$F_{12}=\frac{1}{\pi w}\left[h\arctan\left(\frac{1}{h}\right)+w\arctan\left(\frac{1}{w}\right)-\sqrt{h^2+w^2}\arctan\left(\frac{1}{\sqrt{h^2+w^2}}\right)+\frac{1}{4}\ln\left(ab^{w^2}c^{h^2}\right)\right]$$ with $a=\frac{\left(1+h^2\right)\left(1+w^2\right)}{1+h^2+w^2}$, $b=\frac{w^2\left(1+h^2+w^2\right)}{\left(1+w^2\right)\left(h^2+w^2\right)}$, $c=\frac{h^2\left(1+h^2+w^2\right)}{\left(1+h^2\right)\left(h^2+w^2\right)}$	

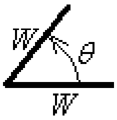
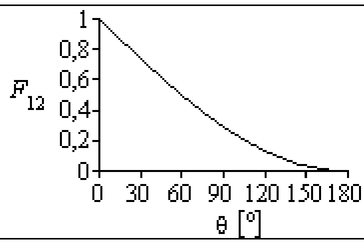
	(e.g. for h=w=1, F_{12}=0.20004)	
From non-adjacent rectangles, the solution can be found with view-factor algebra as shown here 	$$F_{1\to 2}=F_{1\to 2+2'}-F_{1\to 2'}=\frac{A_{2+2'}}{A_1}F_{2+2'\to 1}-\frac{A_{2'}}{A_1}F_{2'\to 1}=$$ $$=\frac{A_{2+2'}}{A_1}\left(F_{2+2'\to 1+1'}-F_{2+2'\to 1'}\right)-\frac{A_{2'}}{A_1}\left(F_{2'\to 1+1'}-F_{2'\to 1'}\right)$$	

Strip to strip

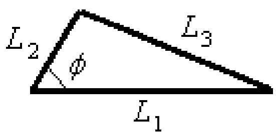
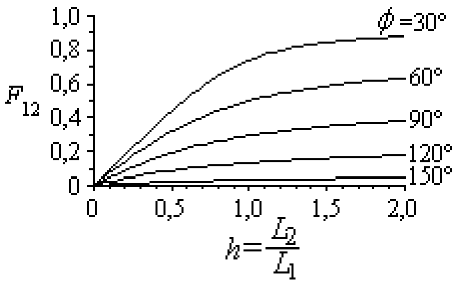
Case	View factor	Plot
Adjacent long strips at 90º, the first (1) of width W and the second (2) of width H, with h=H/W. 	$$F_{12}=\frac{1+h-\sqrt{1+h^2}}{2}$$ (e.g. $F_{12}\big\|_{H=W}=1-\frac{\sqrt{2}}{2}=0.293$)	

Tilted configurations

Equal adjacent strips

Case	View factor	Plot
Adjacent equal long strips at an angle α. 	$$F_{12}=1-\sin\frac{\alpha}{2}$$ (e.g. $F_{12}\big\|_{\frac{\pi}{2}}=1-\frac{\sqrt{2}}{2}=0.293$)	

Triangular prism

Case	View factor	Plot
Between two sides, 1 and 2, of an infinite long triangular prism of sides L_1, L_2 and L_3 , with h=L_2/L_1 and ϕ being the angle between sides 1 and 2. 	$$F_{12}=\frac{L_1+L_2-L_3}{2L_1}=$$ $$=\frac{1+h-\sqrt{1+h^2-2h\cos\phi}}{2}$$ (e.g. for h=1 and ϕ=π/2, F_{12}=0.293)	

References

Howell, J.R., “A catalog of radiation configuration factors”, McGraw-Hill, 1982.

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