Eathan Faust & Philip Pounds

EM204 – Riley – 10am

Project 1: Half-shaft Project

Results

Diameter of long shaft	Outer diameter of short shaft	Inner diameter of short shaft
1.75 inches	2.6875 inches	2.5 inches

Initial Calculations

Before the design process could begin, the amount of torque within each of the half-shafts needed to be calculated. Under the design specifications, we were given the following worst-case scenario of the system being:

$$P_{engine} = 150\ hp, \omega_{engine} = 4000\ rpm$$

The following equation was then done to find the total torque within the drive shaft of the engine:

$$T_{engine} = \frac{P_{engine}}{\omega_{engine}} = \frac{150\ [hp] \cdot \frac{550\ [ft \cdot lbf \cdot s^{-1}]}{1[hp]}}{4000\ [rpm] \cdot \frac{2\pi}{1\ [rev]} \cdot \frac{1\ [min]}{60\ [s]}}$$

The final value of the total torque within the drive shaft was then found to be:

$$T_{engine} = 196.954\ [ft \cdot lb]$$

Within the project statement, we know that the total torque within the engine is evenly split between both gears leading to the half-shafts. Considering the gear ratio between the drive shaft and the half-shafts of 12:1, we get the following equation:

$$T_{shaft} = \frac{1}{2} \cdot T_{engine} \cdot 12$$

The final value of the torque within the shafts was then found to be:

$$T_{shaft} = 1181.72\ [ft \cdot lbf] = 14180.7\ [in \cdot lbf]$$

The final torque value within the shafts was then converted into inch-pound force for easier calculations for torsion in pound-force per square inch.

In the textbook, the normal elastic strength was found, and under the assumption that the shear elastic strength is ½ that of the normal we found that:

$$\tau_{failure} = \frac{1}{2}\sigma_{failure} = 38{,}000\ [psi]$$

With the allowable stress calculated, we then calculated the shafts' minimum cross-sectional area.

Design

We initially started by finding non-optimized axle diameter parameters that do not shear under the torque experienced by the drivetrain. To calculate the diameter for the longer axle, D_{long}, we utilized the equation that describes the normal elastic strength in terms of diameter and driveshaft torque:

$$\tau_{failure} = (FOS) \cdot \frac{T_{shaft} \cdot C_{long}}{J_{long}}$$

Where C_{long} is the distance from the center of the axle to the outer edge of the axle or in this case the radius of the axle or $\frac{D_{long}}{2}$. FOS is the factor of safety given in the project statement to be 2.5. And J_{long} is the polar moment of inertia of the long shaft and is calculated as:

$$J_{long} = \frac{\pi}{2}({C_{long}}^4)$$

Meaning the final equation in terms of diameter is:

$$\tau_{failure} = (FOS) \cdot \frac{T_{shaft} \cdot \frac{D_{long}}{2}}{\frac{\pi}{2}((\frac{D_{long}}{2})^4)}$$

We can then solve for optimal diameters for the long shaft. The diameter of the short shaft is determined by the same equation as the diameter of the long shaft. However, given that the short shaft has a hollow axle we must recalculate the J_{short} where $C_{short\ inner}$ is half of the inner diameter of the short shaft and $C_{short\ outter}$ is half of the outer diameter of the short shaft:

$$J_{short} = \frac{\pi}{2}({C_{short\ outer}}^4 - {C_{short\ inner}}^4)$$

Therefore the failure stress for the short shaft is calculated as:

$$\tau_{failure} = (FOS) \cdot \frac{T_{shaft} \cdot \frac{D_{short\ outter}}{2}}{\frac{\pi}{2}(\left(\frac{D_{short\ outer}}{2}\right)^4 - (\frac{D_{short\ inner}}{2})^4)}$$

Finally, as the project statement recommends, the axle must not twist more than 1.5 degrees from the end of the short shaft to the end of the long shaft, so we incorporated the following constraint equation:

$$\theta_{long} - \theta_{short} \leq 1.5^o$$

Which can be expressed as:

$$(\frac{T_{shaft} \cdot L_{long}}{G \cdot J_{long}} - \frac{T_{shaft} \cdot L_{short}}{G \cdot J_{short}}) \cdot \frac{180}{\pi} \leq 1.5^o$$

We wrote a Python script to increment .0625 inches for these calculations with every possible diameter with a difference of size from .125 inches to 10 inches for all the outer short shaft diameters and the long shaft diameters, and we went from 0 to 10 inches on the short shaft inner diameter with the constraint that the inner diameter must be at least .0625 inches less than the long diameter. The script then saves all values of the diameters, that are correct within the project statement, along with relevant variables including the total volume for the shafts in a Panda's data frame. This data frame is then sorted by total volume from smallest to largest and exported as a .csv file where we take the top rows numbers. With the optimal values from our Python script, we then checked them by hand and with maple to ensure that consumer drive shafts do not shear at max torque.

Long Shaft Dia	Short Shaft Outer	Short Shaft Inne	Long Shaft volume	Short Shaft volu	Both Shafts total	Long Shaft Shear	Short Shaft Shear Str	Theta
1.75	2.6875	2.5	45.70035563	9.930991621	55.63134725	33689.44126	37029.15626	-1.475096024
1.6875	2.0625	1.6875	42.49433579	14.35806017	56.85239596	37573.06379	37289.40576	-1.485806893
1.8125	3.1875	3.0625	49.02295802	7.976700097	56.99965812	30323.12168	37702.00054	-1.2968994
1.8125	3.25	3.125	49.02295802	8.136234099	57.15919212	30323.12168	36224.61602	-1.366970619
1.75	2.375	2.125	45.70035563	11.48644814	57.18680377	33689.44126	37530.61345	-1.267148325
1.8125	3.3125	3.1875	49.02295802	8.295768101	57.31872612	30323.12168	34832.38178	-1.431756703
1.8125	3.375	3.25	49.02295802	8.455302103	57.47826012	30323.12168	33518.87899	-1.491746665
1.75	2.4375	2.1875	45.70035563	11.80551614	57.50587178	33689.44126	35484.62871	-1.395011745
1.6875	2	1.5625	42.49433579	15.91351669	58.40785248	37573.06379	35968.66585	-1.495605866
1.75	2.1875	1.875	45.70035563	12.96213766	58.66249329	33689.44126	37479.48816	-1.130393038
1.8125	2.6875	2.5	49.02295802	9.930991621	58.95394964	30323.12168	37029.15626	-1.096539953
1.75	2.25	1.9375	45.70035563	13.36097266	59.0613283	33689.44126	35212.27111	-1.282812191
1.8125	2.75	2.5625	49.02295802	10.17029262	59.19325064	30323.12168	35280.26914	-1.194048084
1.8125	2.8125	2.625	49.02295802	10.40959363	59.43255165	30323.12168	33652.34274	-1.282803969
1.75	2.3125	2	45.70035563	13.75980767	59.4601633	33689.44126	33144.22226	-1.41814404
1.8125	2.875	2.6875	49.02295802	10.64889463	59.67185265	30323.12168	32134.48033	-1.3637678
1.6875	1.9375	1.4375	42.49433579	17.22967221	59.724008	37573.06379	35617.18408	-1.454641456
1.8125	2.9375	2.75	49.02295802	10.88819563	59.91115365	30323.12168	30716.98448	-1.437776103
1.75	2.0625	1.6875	45.70035563	14.35806017	60.05841581	33689.44126	37289.40576	-1.033179799
1.875	3.1875	3.0625	52.46214295	7.976700097	60.43884304	27390.7635	37702.00054	-0.978333061
1.8125	2.375	2.125	49.02295802	11.48644814	60.50940616	30323.12168	37530.61345	-0.888592254

Screenshot of exported Panda's data frame