Where Have You Gone, Joe DiMaggio?

Probability and Hitting Streaks

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1941 Baseball Season

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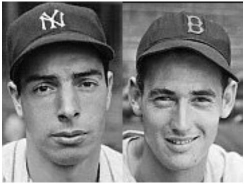

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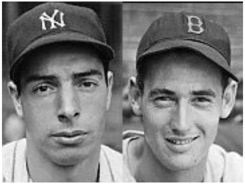

Joe DiMaggio (Player A – won the AL MVP) and Ted Williams (Player B)

Longest Hitting Streaks in MLB History

Year	Name	Team	Games
1941	Joe DiMaggio	New York Yankees	56
1896-97	Willie Keeler	Baltimore Orioles	45
1978	Pete Rose	Cincinnati Reds	44
1894	Bill Dahlen	Chicago Colts (Cubs)	42
1922	George Sisler	St. Louis Browns	41
1911	Ty Cobb	Detroit Tigers	40
1987	Paul Molitor	Milwaukee Brewers	39
2005-06	Jimmy Rollins	Philadelphia Phillies	38
1945	Tommy Holmes	Boston Braves	37
1896-97	Gene DeMontreville	Washington Senators	36

Source: MLB.com

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- the streak is ended when a player has at least 1 plate appearance and no hits
- the streak is not terminated if all official plate appearances result in a base on balls (a walk), hit by pitch, defensive interference or a sacrifice bunt
- the streak shall terminate if the player has a sacrifice fly and no hit

Source: Official Rules, Major League Baseball, 10.23 Guidelines For Cumulative Performance Records

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All possible outcomes in an experiment. Denoted by S

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Complement A^c: A player gets zero hits in four at bats in a single game

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Probability and Hitting Streaks

Basic concepts of probability needed to analyze hitting streaks:

Rules of Probability

Complement Rule:

$$P\left(A^c\right) = 1 - P(A)$$

Multiplication Rule for Independent Events:

Let A and B be two independent events. Then
$P(A \text{ and } B) = P(A) \times P(B)$.

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Fifty-six consecutive games? $0.7599^{56} = \mathbf{0.0000002101}$

Joe DiMaggio: Probability of 56 Game Hitting Streak

In 1941, Joe DiMaggio had 541 at bats in 139 games.

3.892 at bats per game — let's round to 4 at bats (can't have a fraction of an at bat)

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1	7	$(1-0.643^7)\times(1-0.643^1) = \mathbf{0.3408}$

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Solution? Vary the at bats for each game.

During the 56 game hitting streak, DiMaggio had:

- 3 games with 2 at bats
- 11 games with 3 at bats
- 26 games with 4 at bats
- 16 games with 5 at bats

Source: Cliff Blau, a member of SABR (Society for American Baseball Research)

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Problem? This probability is specific to these 56 games and not necessarily **any** 56 consecutive games over the course of an **entire** baseball season.

Simulation: Another Way To Estimate Probability

Let's say we have a player that has a 0.333 batting average and has 4 at bats each game. The baseball season has 162 games in a season (154 when DiMaggio played). How can we estimate the probability of this player having a hitting streak as long as Joe DiMaggio's 56 game hitting streak?

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Problem? This is just one simulated "season." To estimate the probability we would need to repeat this process thousands of times!

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How do we do this? Using a computer program (statisticians love `R`!)

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- We will look at the longest streak in each simulated baseball season. This streak is the number of consecutive "games" with a hit. We will have 10,000 of these longest streaks!
- Estimate the probability of having a hitting streak of 56 games by counting the number of simulated seasons with a streak of 56 consecutive games or longer and divide by the number of simulated seasons – here 10,000.

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- At bats are independent of one another. Why might this not be a good assumption to make? Is it a reasonable assumption to make?