

O Frabjous Day!

150th Anniversary of *Alice's Adventures in Wonderland*

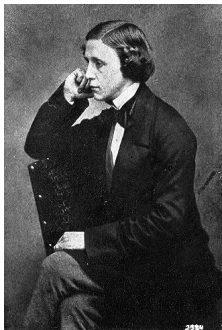
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Charles Dodgson: 1832-1898



Dodgson the Mathematician

Mathematical Lecturer at Christ Church, University of Oxford, England

Areas of research: geometry, linear and matrix algebra, mathematical logic, probability

Big findings: First proof of Kronecker-Capelli Theorem (linear algebra, 1866), Dodgson's method (a voting system for candidates and committees, 1876), probability (*72 Pillow Problems*, 1893)

As a tutor and lecturer, he valued Euclid's *Elements* as the epitome of mathematical thinking – start with axioms and build up complex arguments through logical steps. Propositions are stated and proved (and signed QED)

In Dodgson's time: "new" mathematics started to appear – imaginary numbers ($\sqrt{-1} = i$) and abstract algebra



"Advice from a caterpillar"

- absurdity of symbolic algebra?
- *Trigonometry and Double Algebra* – De Morgan, 1849
- Alice moves from a "rational" world to a land where numbers behave erratically (she forgets her multiplication tables)
- Alice is confused about her height fluctuations – perhaps bound by "conventional" math; The caterpillar's response: "It isn't" and "Keep your temper"

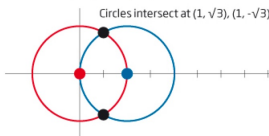
"Pig & Pepper"

Dodgson's target? Projective Geometry

- properties of figures that stay the same even when the figure is projected onto another surface
- Alice is her proper size and shape, shrinks herself down to enter a small house. The Duchess gives the baby to Alice, it turns into a pig

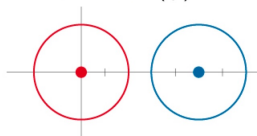
Jean-Victor Poncelet and the Continuity Principle – a mathematical figure should retain some of its original properties even under drastic transformations

CONSIDER TWO CIRCLES
CENTRED ON $(0,0)$ AND $(2,0)$



These circles intersect at two places, and under the principle of continuity you can assume that they will always intersect in two places, even if they move apart and are no longer touching!

SO IF THE BLUE CIRCLE MOVES
SO THAT ITS CENTRE IS NOW $(5,0)$...



...these circles still intersect at two points
 $(\frac{5}{2}, \frac{3}{2}i)$ and $(\frac{5}{2}, \frac{3}{2}i)$, where i is $\sqrt{-1}$

The Tea Party

Dodgson's target? Irish mathematician William Rowan Hamilton and his work on quaternions

- A number system that extends the complex numbers (works with two terms: $a + bi$) to a number system based on four terms – one for each of the three-dimensions and a fourth for time
- The Hatter, the March Hare and the Dormouse are the three guests. Missing? The character Time!
- The guests are constantly moving around the table – this is based on Hamilton's early attempts to calculate motion
- Hamilton: in realm of pure time, cause and effect are no longer linked – nonsensical riddles like "Why is a raven like a writing desk?"
- The multiplication of quaternions are noncommutative ($a \times b \neq b \times a$) – "Say what she means" does not mean "At least what I say" – might as well say "I see what I eat" is the same as "I eat what I see"

Melanie Bayley (2009). "Alice's adventures in algebra: Wonderland solved", *New Scientist*, 16 December 2009, <https://www.newscientist.com/article/mg20427391.600-alices-adventures-in-algebra-wonderland-solved>

Eugene Seneta (1984). "Lewis Carroll as a Probabilist and Mathematician," *Math. Scientist*, 9, 79-94.

Robin Wilson (2008). *Lewis Carroll in Numberland: His Fantastical Mathematical Logical Life*, Allen Lane, London.