Math 156, Sec 4, H.W. 7 *Franchi-Pereira, Philip*

Problem 1 Consider the sequence $\{\frac{3}{7n}+9\}$. Then we have

$$\lim_{n\to\infty}\frac{3}{7n}+9=9$$

Def:

$$\lim_{n\to\infty}\frac{3}{7n}+9 \coloneqq \forall\varepsilon>0,\exists N\in\mathbb{Z}^+,\forall n>N,|(\frac{3}{7n}+9)-(9)|<\varepsilon$$

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Scratch Work: $|\frac{3}{7n}+9-9|<\varepsilon$ so $|\frac{3}{7n}|<\varepsilon$ and since n is positive, we have $\frac{3}{7n}<\varepsilon$. Then $3<7n\varepsilon$ and $\frac{3}{7\varepsilon}\leq n$.

Proof: Given $\varepsilon>0$, take $N=\lceil\frac{3}{7\varepsilon}\rceil+1$. Observe that if $n\geq N$ then $n\geq\lceil\frac{3}{7\varepsilon}\rceil+1$ so $n>\lceil\frac{3}{7\varepsilon}\rceil$ and so $n>\frac{3}{7\varepsilon}$. Then $7\varepsilon n>3$ and $\varepsilon>\frac{3}{7n}$. Thus $|(\frac{3}{7n}+9)-(9)|<\varepsilon$ and $L=9$

Problem 2: Consider the sequence $\{\frac{1}{5n+4}-3\}$. Then we have

$$\lim_{n\to\infty}\frac{1}{5n+4}-3=-3$$

Def:

$$\lim_{n\to\infty}\frac{1}{5n+4}-3=3\coloneqq\forall\varepsilon>0,\exists N\in\mathbb{Z}^+,\forall n>N,|(\frac{1}{5n+4}-3)-(-3)|<\varepsilon$$

Scratch Work: $|\frac{1}{5n+4}-3+3|<\varepsilon$ so $|\frac{1}{5n+4}|<\varepsilon$ and since n is positive, $\frac{1}{5n+4}<\varepsilon$. Then $1<(5n+4)\varepsilon$ and $1<5n\varepsilon+4\varepsilon$ which becomes $\frac{1-4\varepsilon}{5\varepsilon}<n$.

Proof: Given $\varepsilon>0$, take $N=\lceil\frac{1-4\varepsilon}{5\varepsilon}\rceil$. Observe that if $n\geq N$ then $n\geq\lceil\frac{1-4\varepsilon}{5\varepsilon}\rceil$ so then $n\geq\frac{1-4\varepsilon}{5\varepsilon}$. We see that $n\cdot5\varepsilon+4\varepsilon\geq1$ so $\varepsilon(5n+4)\geq1$ and $\varepsilon\geq\frac{1}{5n+4}$. Thus $|(\frac{1}{5n+4}-3)-(-3)|<\varepsilon$ and $L=-3$

Problem 3 Consider the sequence $\{\frac{15}{n^2}+\frac{2}{\sqrt{n}}+3\}$.Then we have

$$\lim_{n\to\infty}\frac{15}{n^2}+\frac{2}{\sqrt{n}}+3=3$$

Def:

$$\lim_{n\to\infty} \frac{15}{n^2} + \frac{2}{\sqrt{n}} + 3 = 3 \coloneqq \forall\varepsilon > 0, \exists N \in \mathbb{Z}^+, \forall n > N, |(\frac{15}{n^2}+\frac{2}{\sqrt{n}}+3)-(3)| < \varepsilon$$

Scratch Work: $|\frac{15}{n^2} + \frac{2}{\sqrt{n}} + 3 - 3| = |\frac{15}{n^2} + \frac{2}{\sqrt{n}}|$. Since $\sqrt{n} \leq n^2$ then $\frac{1}{\sqrt{n}} \geq \frac{1}{n^2}$, so $|\frac{15}{n^2} + \frac{2}{\sqrt{n}}| \leq |\frac{15}{\sqrt{n}} + \frac{2}{\sqrt{n}}|$. So $|\frac{17}{\sqrt{n}}| < \varepsilon$ and we have $\frac{17^2}{\varepsilon^2} \leq n$.

Proof: Given $\varepsilon > 0$, take $N = \lceil \frac{17^2}{\varepsilon^2} \rceil$. Observe that if $n > N$ then $n > \lceil \frac{17^2}{\varepsilon^2} \rceil$ and taking the square root of both sides we have $\sqrt{n} > \frac{17}{\varepsilon}$, then $\varepsilon > \frac{17}{\sqrt{n}}$. Since $\frac{17}{\sqrt{n}} = \frac{15}{\sqrt{n}} + \frac{2}{\sqrt{n}}$ and $\sqrt{n} < n^2$, then we have $\frac{15}{\sqrt{n}} > \frac{15}{n^2}$ and so $\frac{17}{\sqrt{n}} > \frac{15}{n^2} + \frac{2}{\sqrt{n}}$. Therefore $\varepsilon > |\frac{15}{n^2} + \frac{2}{\sqrt{n}}|$, and $\varepsilon > |(\frac{15}{n^2} + \frac{2}{\sqrt{n}} + 3) - (3)|$, so $L = 3$.

Problem 4 Show that the sequence n^7 diverges to infinity.

Def:

$$\lim_{n\to\infty} n^7 = -\infty \forall M \in \mathbb{Z}^+, \exists N \in \mathbb{Z}^+, (n > N \implies n^7 > M)$$

Scratch Work: We need $n^7 > M$. Solve for n by taking the seventh root $n > \sqrt[7]{M}$, so $N = \lceil \sqrt[7]{M} \rceil$.

Proof: Given any number $M \in \mathbb{Z}^+$, choose $N = \lceil \sqrt[7]{M} \rceil$. If $n > N$ then $n > \lceil \sqrt[7]{M} \rceil$, which means $n > \sqrt[7]{M}$ and in turn $n^7 > M$. Therefore the sequence diverges to infinity.