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ME6984
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Midterm Project Write-Up

Background and Motivation

The purpose of this project is to present the results of a motion planning optimization for the planar 5 link biped, RABBIT. This robot utilizes feedback linearization control techniques to asymptotically drive virtual constraint output equations to zero. In the stance phase, this robot has five degrees of freedom and four degrees of actuation, resulting in one degree of under-actuation. In the flight phase, there are seven degrees of freedom and four degrees of actuation. Since there are four actuated joints, there are four virtual constraint output equations: one corresponding to the angle of each actuated joint.

The hybrid system describing the robot dynamics is given by:

$$\sum = \begin{cases} \dot{x} = f(x) + g(x)u, & x^- \notin S \\ x^+ = \Delta(x^-), & x^- \in S \end{cases}$$

Where

$$f(x) = -D^{-1}(q)(C(q,\dot{q})\dot{q} + G(q)$$

$$g(x) = D^{-1}(q)B$$

Where $D(q)$ is the NxN mass-inertia matrix, $C(q,\dot{q})$ is the NxN matrix of Coriolis and centrifugal terms, $G(q)$ is the Nx1 vector of gravity terms, and B is the NxM input matrix. The function Δ is the impact map for the discrete time dynamics. Note that x^+ and q^+ indicate the state vectors immediately after the impact, and x^- and q^- indicate the state vectors immediately before the impact. The switching manifold, S, is defined as

$$S := \{(x,y) \in R^2 \mid x > 0, y = 0\}$$

The impact dynamics are given by the following system of equations:

$$\begin{bmatrix} D_e(q_e) & -J_e^T(q_e) \\ J_e(q_e) & 0_{[2x2]} \end{bmatrix} \begin{bmatrix} \dot{q}_e^+ \\ \delta F \end{bmatrix} = \begin{bmatrix} D_e(q_e)\dot{q}_e^- \\ 0_{[2x1]} \end{bmatrix}$$

Where $D_e(q_e)$, $J_e(q_e)$ are the extended mass-inertia matrix and extended swing foot Jacobian matrix, and δF is the magnitude of the impulsive force at impact. These equations are used to solve the impact map Δ for the discrete time dynamics and subsequent ODE reinitialization at x^+ each time the state vector x reaches the switching manifold.

During the gait, a strictly increasing phasing variable $\theta(q)$ is defined as the angle between the virtual leg (a line connecting the stance leg-end and the hip) and the negative x axis. In the code, the phasing variable is calculated as follows:

$$\theta = c_1 q + c_0$$

Where

$$c_1 = \begin{bmatrix} -1 & 0 & \frac{1}{2} & 0 & -1 \end{bmatrix}$$

$$c_0 = \frac{3\pi}{2}$$

Using the robot model described above, an ideal gait is determined by optimizing the initial conditions x^-, and α, the matrix of Bezier coefficients describing the trajectory of the desired joint angles for the virtual constraint controller.

Optimization Problem and Decision Variables:

The objective of the optimization problem is to find the desired trajectory of the controlled variables on the gait, $h_d(\theta)$, that minimizes a cost function subject to periodicity (equality) and feasibility (inequality) constraints. The decision variables for this optimization are x_0, which is the full robot state vector just before impact, and the two middle coefficients of α for each of the *m* number of M^{th}-order Bezier curves that describe the shape of the trajectory. All other Bezier coefficients are determined using the final and initial positions and velocities of the joints; therefore, the final trajectory $h_d(\theta)$ is dependent on the initial conditions.

Cost Function:

The cost function *J* is the norm of the input torque vector squared and integrated over the time duration of a single step, divided by the total time of the step. The value of this function is essentially a measure of the energy required to travel forward by one step.

$$J(x_0, \alpha) \coloneqq \frac{1}{T}\int_0^T \|u(t)\|^2 dt$$

Equality Constraints:

For periodicity, we assume that x_0 is on the switching manifold *S*. The equality constraint is defined such that the solution to the swing leg ODE at the instant before impact (at time *T*, or time of one step) must be equal to the initial condition, x_0. In other words,

$$\varphi[T(\Delta(x_0), \alpha), \Delta(x_0), \alpha] - x_0 = 0$$

Inequality Constraints:

For inequality constraints, we need to make sure the following conditions are met:

-The phasing variable θ is strictly increasing during the gait.

-Input torques are within acceptable bounds (u_min < u < u_max)

-Joint angles are within acceptable bounds (q_min < q < q_max)

-Joint velocities within acceptable bounds (dq_min < dq < dq_max)

-The step length must be greater than 0.2m

-The average horizontal velocity of the robot must be at least 0.8m/s during the gait

<u>Ground reaction force feasibility:</u>

-The ground reaction force must be upwards

-The horizontal ground reaction force cannot exceed the product of the friction coefficient and vertical reaction force:

$$F_x < \mu F_y$$

-The impulsive ground reaction force must meet these same requirements.

Swing Leg-end Clearance:

-The swing leg-end must start at or behind x = -0.1m

-The swing leg-end must finish at or beyond x = 0.1m

-The y-coordinate of the swing leg-end must reach 0m

-The swing leg-end cannot go below y=0m during the gait

-The swing leg-end must be moving downwards at the end of the gait

-The swing leg-end must come up to at least 0.02m above the ground during the gait

-Time constraint: The step time must be at least 0.2s

Optimal Value of Cost Function and Decision Variables (from FMINCON):

Cost Function:

$J = 4793.99$

Decision Variables:

$q^- = [3.3075;\ 3.7886;\ 0.3504;\ 0.1015;\ -0.2709]$

$dq^- = [-0.4286;\ 0.0589;\ 0.0014;\ 0.7109;\ -0.7630]$

$\alpha_{mid} = [3.5765;\ 3.4834;\ 0.0477;\ 0.3367;$

$3.4044;\ 3.7020;\ 0.1331;\ 0.1796]$

Simulation Results

The following sections will present results from two closed-loop simulations of the RABBIT robot. First, we will look at simulation results for five steps with optimal boundary conditions, then we will look at the results from 20 steps starting from an initial condition off the optimal boundary.

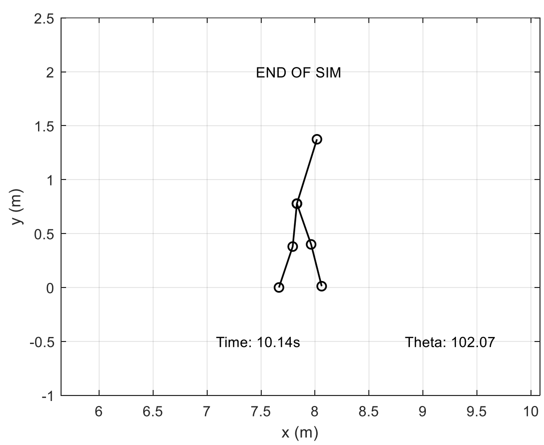

Figure 1. Final frame of the simulation of 20 steps with an initial condition slightly off the optimal boundary. Note the total time is 10.14s, and the total distance is about 8.1m (average velocity of about 0.8 m/s). This demonstrates the stability of the optimized gait.

Closed Loop Simulation with Optimal Boundary Conditions

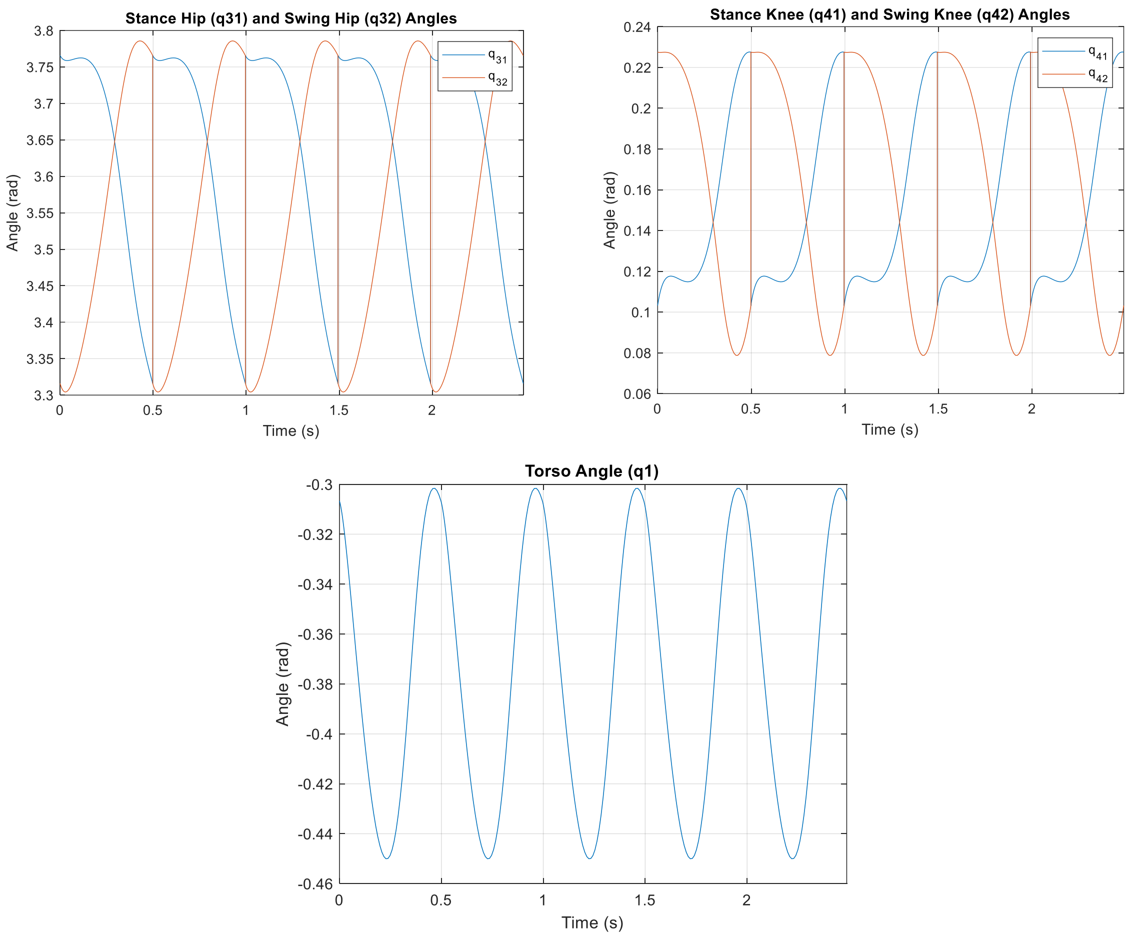

Figure 2. Joint angles (rad) from the RABBIT robot taking five steps in the optimal gait.

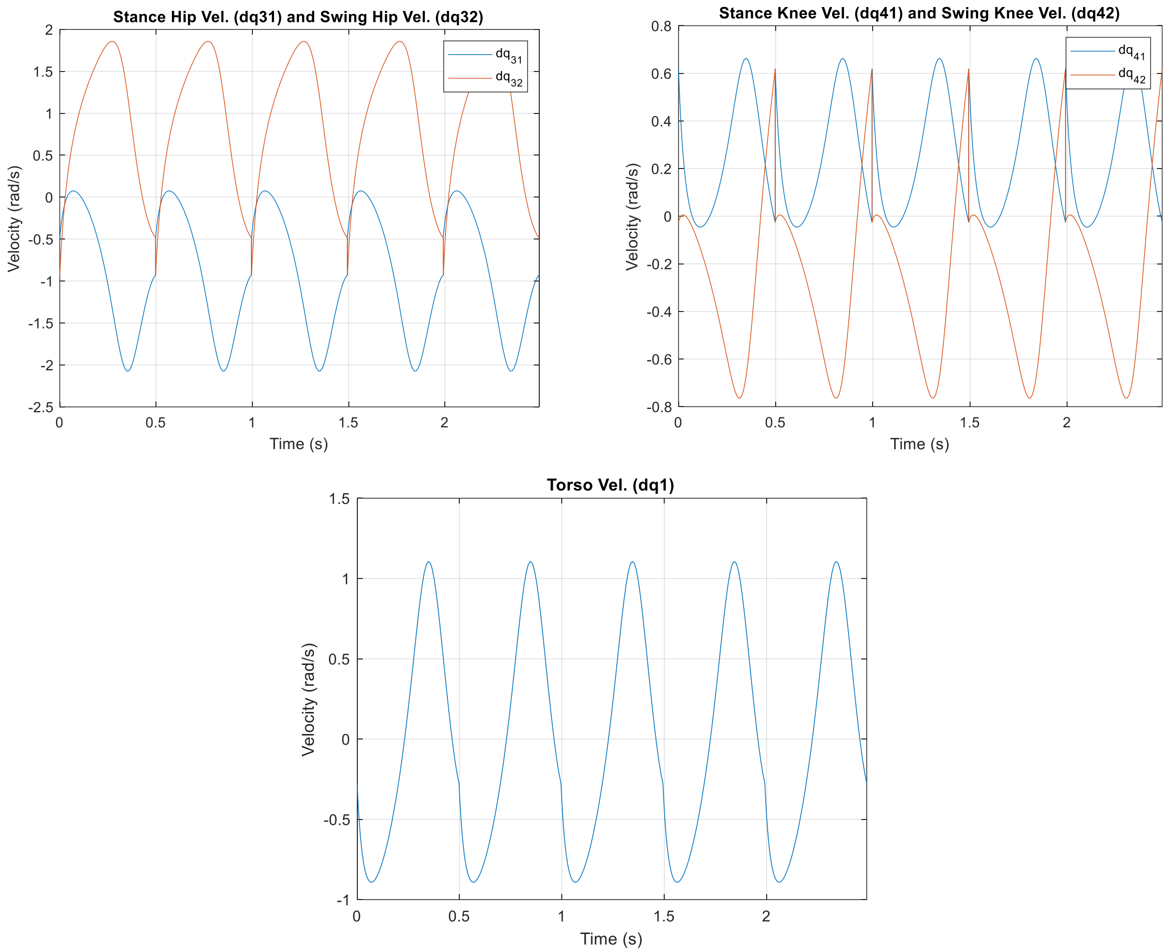

Figure 3. Joint velocities (rad/s) from five steps of the optimal gait.

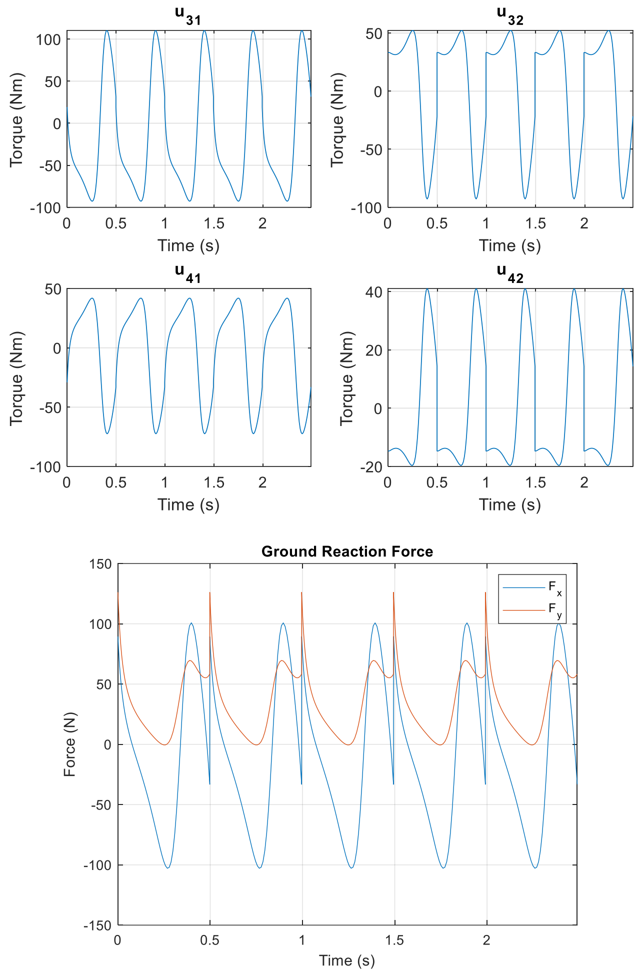

Figure 4. Actuator torques (Nm) and ground reaction forces (N) at the stance leg-end for five steps of the optimal gait.

Closed Loop Simulation with Gait Disturbance

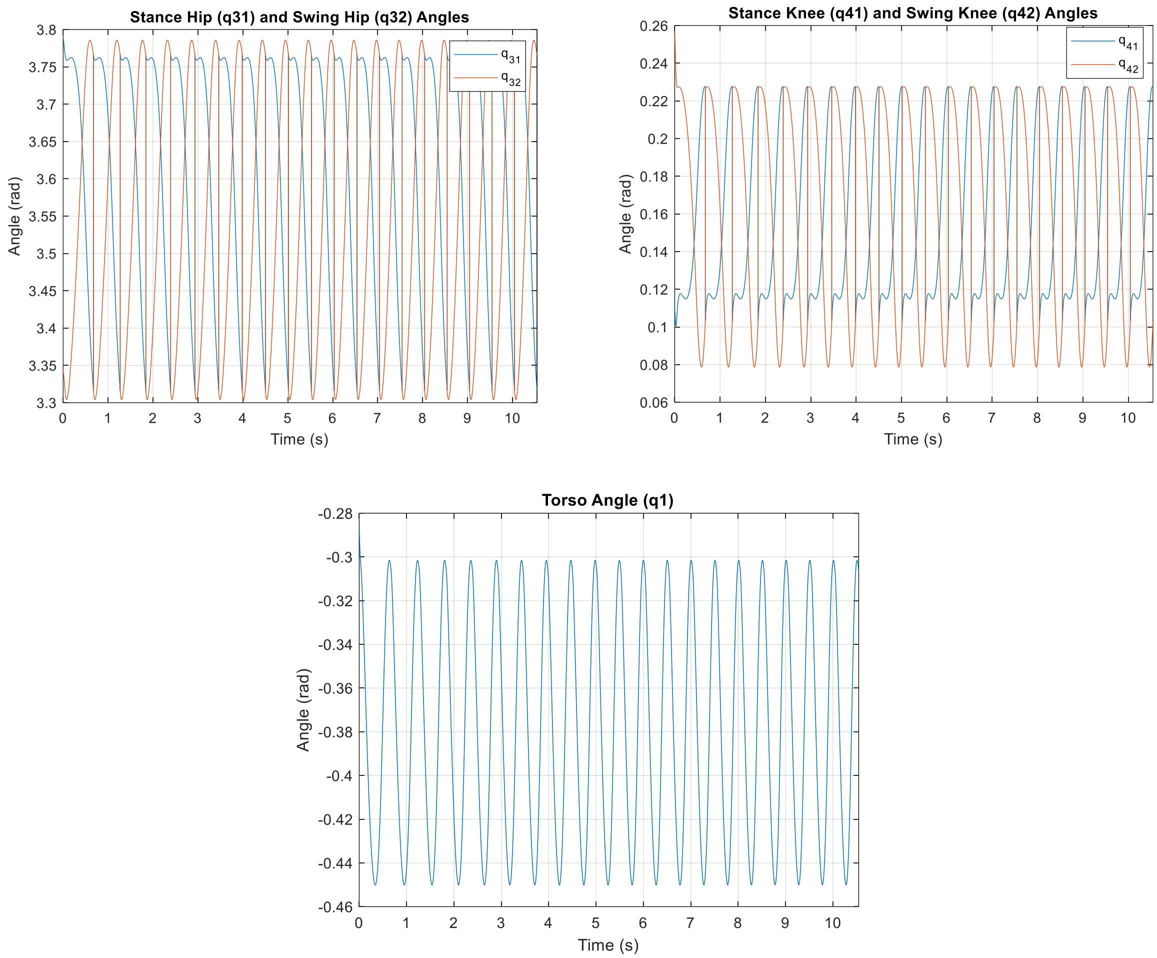

Figure 5. Joint angles for 20 steps with initial conditions off the ideal boundary.

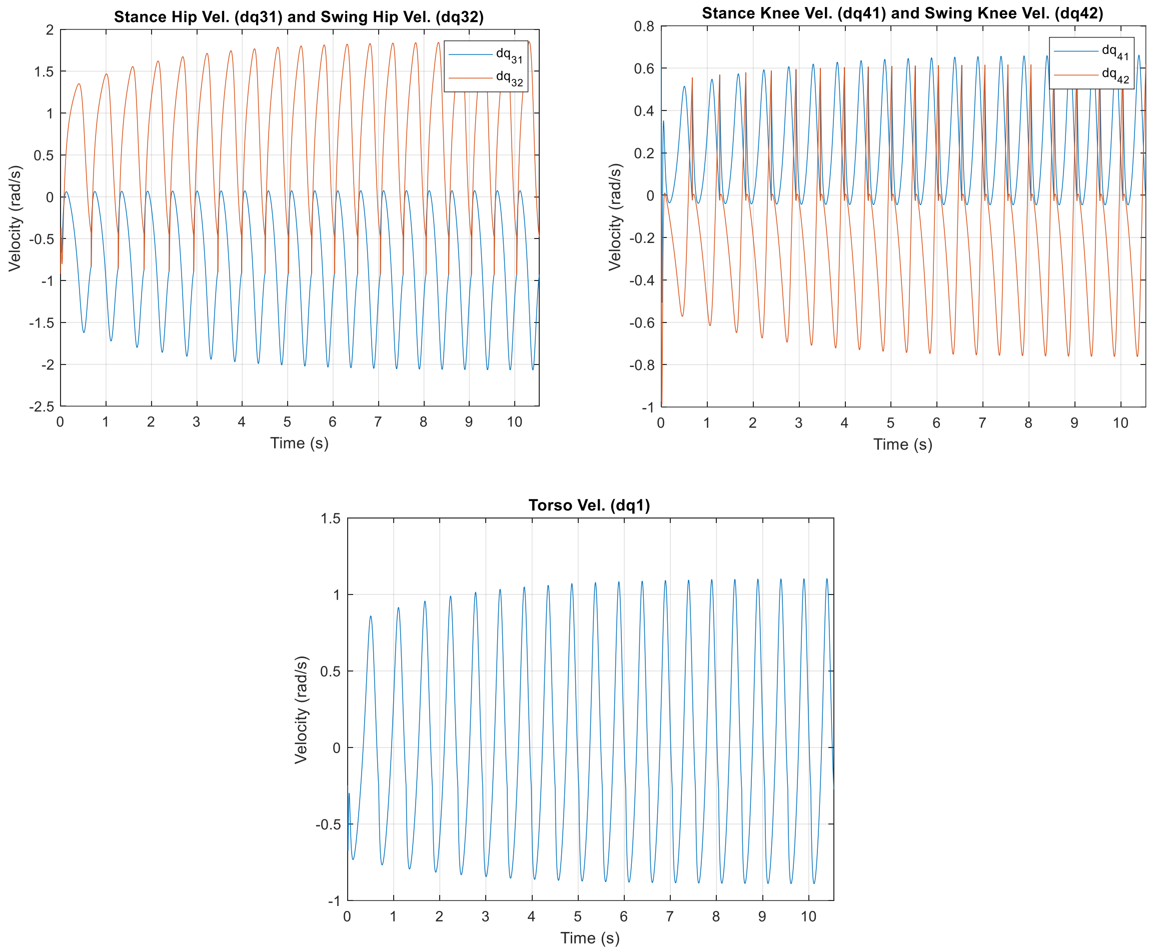

Figure 6. Joint velocities for 20 steps from the same simulation. Note that the velocities converge back to the optimal gait after about seven steps.

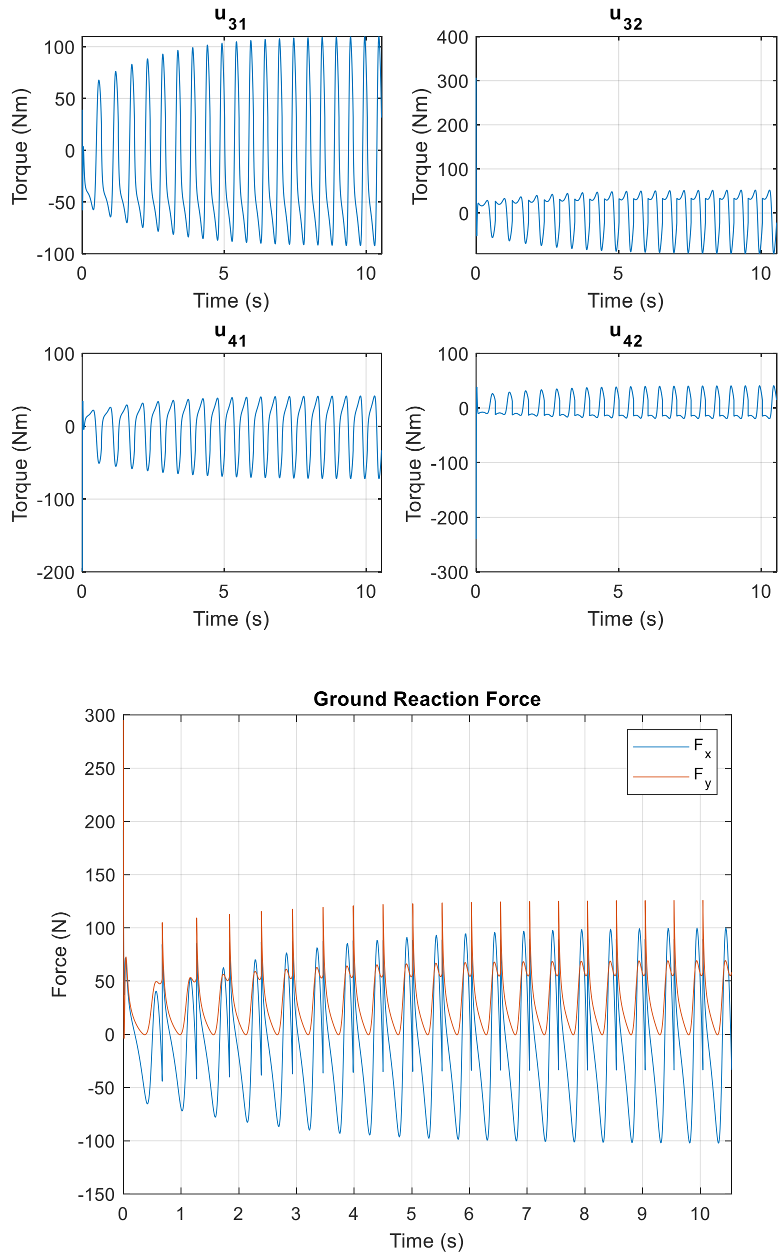

Figure 7. Actuator torques and ground reaction forces for 20 steps.

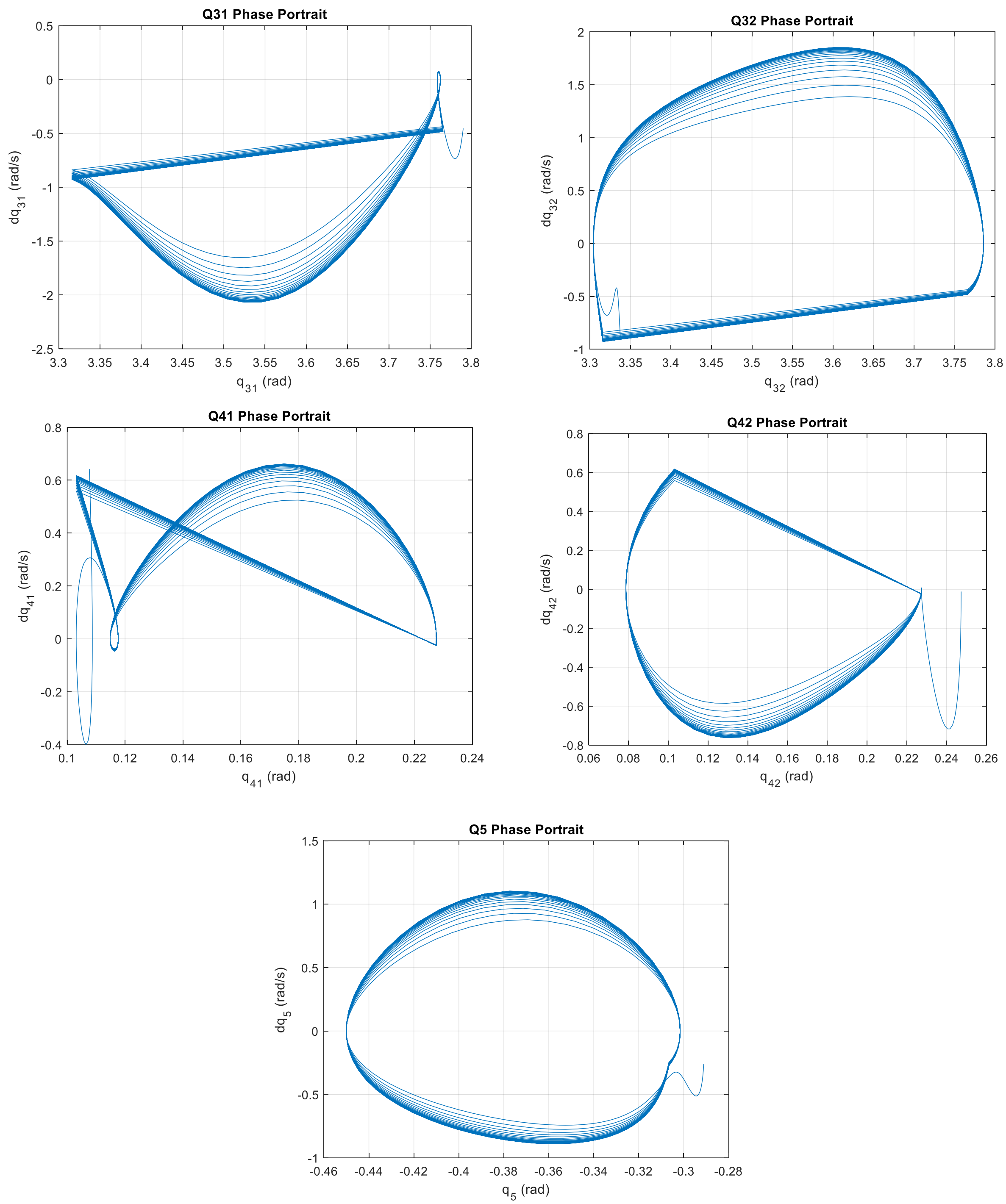

Figure 8. Phase portraits for each joint from a simulation of 20 steps with initial conditions slightly off the optimal boundary condition.