INF283 | Weekly Exercise 06 | Bayesian inference

Exercise 1.1

1.

$P(A) = 1/5$

$P(B) = 1/5 \cdot (0 + 0 + 1/7 + 1/11 + 1/20) \simeq 0{,}05675$

$P(B|A) = 1/11$

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)} = \frac{1/11 \cdot 1/5}{0{,}05675} \approx 0{,}32$$

2.

$P(A) = 1/5$

$P(B) = 0{,}05675$

$P(B|A) = 1/20$

$$P(A|B) = \frac{1/11 \cdot 1/20}{0{,}05675} \approx 0{,}176$$

3. Not possible to get an 18 with an 11-sided die, so the probability of getting this die is 0

4. The 20-sided die is the only die that can give a 18, therefore the probability of getting this die is 1.

$$P(\text{Dic} = 20 \mid x = 18) = \frac{1/20 \cdot 1/5}{1/100} = 1$$

$$P(B) = 1/5 \cdot 1/20 = \frac{1}{100}$$

Exercise 1.2

θ ∈ [0, 1]

P(H) = θ (getting a head)

P(T) = 1 - θ (getting a tail)

1. Probability mass function $P(X=H) = \binom{n}{k} \theta^k (1-\theta)^{n-k}$

Fair coin: $\theta = 0{,}5$

$= \binom{15}{9} 0{,}5^9 (1-0{,}5)^6$

$\theta^{N_h} = k \qquad n = 15$

$= 0{,}15274$

b)

b1. Maximum likelihood

$$P(D \mid \theta) = \theta^{9} \cdot (1-\theta)^{6}$$

Solve $P'(D \mid \theta) = 0$

$$\hat{\theta} = \frac{9}{15} = 0.6 \quad \left\{ \frac{\text{Number of succesful events}}{\text{Total number of sample space}} \right.$$

b2. Likelihood with $\theta = 0.6$

$$P(D \mid \theta) = \theta^{N_h} (1-\theta)^{N_t} = 0.6^{9} (1-0.6)^{6}$$

$$\simeq 0.00004$$

b3. Likelihood when coin is fair, $\theta = 0.5$

$$P(D \mid \theta) = \theta^{N_h} (1-\theta)^{N_t} = 0.5^{9} (1-0.5)^{6}$$

$$\simeq 0.00003$$

Uniform:
alpha = 1
beta = 1

b4. Posterior distribution $P(\theta \mid D)$, uniform prior

$$\theta \sim Beta(\alpha, \beta)$$

Uniform prior : parameters

$$\alpha = \beta = 1, \quad N_h = 9, \quad N_t = 6$$

Posteriori :

$$\theta \sim Beta(N_h + \alpha, N_t + \beta)$$

$$\theta \sim Beta(9+1, 6+1) = Beta(10, 7)$$

b5.

Maximimum likelihood

$P(x=H)$ = Undefined, cannot divide on 0

$P(x=H \mid D=H) = 1$

$P(x=T \mid D=HH) = 0$

$P(x=H \mid D=HHT) = 2/3$

$P(x=H \mid D=HHTH) = 3/4$

$P(x=T \mid D=HHTHH) = 1/5$

Bayesian inference with uniform prior:

$$P(x=H) = \frac{1+0}{1+0+1+0} = 1/2$$

$$P(x=H \mid D=H) = \frac{1+1}{1+1+1+0} = 2/3$$

$$P(x=T \mid D=HH) = \frac{1+0}{1+0+1+2} = 1/4$$

$$P(x=H \mid D=HHT) = \frac{1+2}{1+2+1+1} = 3/5$$

$$P(x=H \mid D=HHTH) = \frac{1+3}{1+3+1+1} = 4/6 = 2/3$$

$$P(x=T \mid D=HHTHH) = \frac{1+1}{1+1+1+4} = 2/7$$

$N_h = 7 \quad N_t = 3$

c)

c1. Maximum likelihood

$P(D \mid \theta) = \theta^7 \cdot (1-\theta)$

Solve $P'(D/\theta) = 0$

$\hat{\theta} \frac{7}{10} = 0,7$

c2. Likelihood with $\theta = 0,7$

$P(D|\theta) = 0,7^7 (1-0,7)^3 \simeq 0,00222$

c3. Likelihood when coin is fair, $\theta = 0,5$

$P(D|\theta) = 0,5^7 (1-0,5)^7 \simeq 0,00097$

c4. Posterior distribution $P(\theta|D)$, uniform prior

$\theta \sim Beta(\alpha, \beta)$

Uniform prior : parameters

$\alpha = \beta = 1, \quad N_h = 7, \quad N_t = 3$

Posteriori :

$\theta \sim Beta(N_h + \alpha, N_t + \beta)$

$\theta \sim Beta(7+1, 3+1) = Beta(8,4)$

c5.

Maximimum likelihood

$P(x=T)$ = Undefined, cannot divide on 0

$P(x=T \mid D=T) = 1$

$P(x=T \mid D=TT) = 1$

$P(x=H \mid D=TTT) = 0$

$P(x=H \mid D=TTTH) = 1/4$

$P(x=H \mid D=TTTHH) = 2/5$

Bayesian inference with uniform prior:

$$P(x=T) = \frac{1+0}{1+0+1+0} = 1/2$$

$$P(x=T \mid D=T) = \frac{1+1}{1+1+1+0} = 2/3$$

$$P(x=T \mid D=TT) = \frac{1+2}{1+2+1+0} = 3/4$$

$$P(x=H \mid D=TTT) = \frac{1+0}{1+0+1+3} = 1/5$$

$$P(x=H \mid D=TTTH) = \frac{1+1}{1+1+1+3} = 2/6 = 1/3$$

$$P(x=H \mid D=TTTHH) = \frac{1+2}{1+2+1+3} = 3/7$$

Exercise 1.3

weather: Sunny
temperatur: Hot
humidity: Normal
wind: false

weather: Sunny

PlayTennis	Rainy	Overcast	Sunny	**Total**
Yes	3	4	2	**9**
No	2	0	3	**5**
Total	**5**	**4**	**5**	**14**

humidity: Normal

PlayTennis	High	Normal	**Total**
Yes	4	5	**9**
No	3	2	**5**
Total	**7**	**7**	**14**

temperatur: Hot

PlayTennis	Hot	Mild	Cool	**Total**
Yes	3	4	2	**9**
No	1	2	2	**5**
Total	**4**	**6**	**4**	**14**

wind: False

PlayTennis	True	False	**Total**
Yes	3	6	**9**
No	3	2	**5**
Total	**6**	**8**	**14**

$$P(\text{PlayTennis} = \text{Yes} \mid \text{weather} = \text{Sunny}, \text{temp} = \text{Hot}, \text{humidity} = \text{Normal}, \text{wind} = \text{False}) \propto$$

$$P(\text{PlayTennis} = \text{Yes}, \text{weather} = \text{Sunny}, \text{temp} = \text{Hot}, \text{humidity} = \text{Normal}, \text{wind} = \text{False})$$

$$= 9/14 \cdot 2/9 \cdot 3/9 \cdot 5/9 \cdot 6/9 \approx \underline{0{,}0176}$$

$$P(\text{PlayTennis} = \text{No}, \text{weather} = \text{Sunny}, \text{temp} = \text{Hot}, \text{humidity} = \text{Normal}, \text{wind} = \text{False})$$

$$= 5/14 \cdot 3/5 \cdot 1/5 \cdot 2/5 \cdot 2/5 \approx 0{,}0069$$

$$= \frac{0{,}0176}{0{,}0176 + 0{,}0069} \approx \underline{\underline{0{,}718}}$$

Exercise 1.4

weather: Sunny
temperatur: Hot
humidity: Unknown
wind: Unknown

weather: Sunny

PlayTennis	Rainy	Overcast	Sunny	**Total**
Yes	3	4	2	**9**
No	2	0	3	**5**
Total	**5**	**4**	**5**	**14**

temperatur: Hot

PlayTennis	Hot	Mild	Cool	**Total**
Yes	3	4	2	**9**
No	1	2	2	**5**
Total	**4**	**6**	**4**	**14**

$P(\text{PlayTennis} = \text{Yes} \mid \text{weather} = \text{Sunny}, \text{temp} = \text{Hot}) \propto$

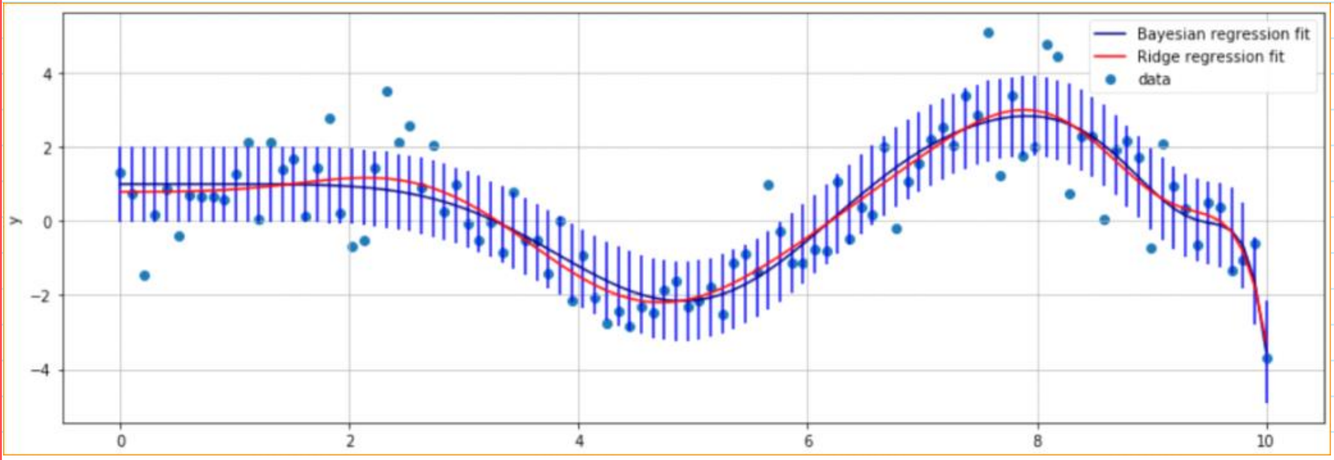
$P(\text{PlayTennis} = \text{Yes}, \text{weather} = \text{Sunny}, \text{temp} = \text{Hot})$

$= 9/14 \cdot 2/9 \cdot 3/9 \simeq 0{,}047$

$P(\text{PlayTennis} = \text{No}, \text{weather} = \text{Sunny}, \text{temp} = \text{Hot})$
$= 5/14 \cdot 3/5 \cdot 1/5 \simeq 0{,}043$

$$= \frac{0{,}047}{0{,}047 + 0{,}043} \simeq \underline{\underline{0{,}52}}$$

Exercise 1.5



Modified code:

```
plt.figure(figsize=(15, 5))
plt.scatter(X, y, label='data')
plt.plot(X, y_predict_bayesian, color='navy', label='Bayesian regression fit')
plt.plot(X, y_predict_ridge, color='r', label='Ridge regression fit')

for i in range(0, len(y_predict_bayesian_std[0])):
    plt.vlines(X[i], y_predict_bayesian_std[0][i] - y_predict_bayesian_std[1][i],

plt.grid(b=True)
plt.xlabel('X')
plt.ylabel('y')
plt.legend(loc='best')
```