1. Background topics

Matrix multiplication

$$A: \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \qquad B: \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

1. a: $\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix} \circ \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2\cdot 1 - 1\cdot 3 & 2\cdot 2 - 1\cdot 4 \\ 1 + 3\cdot 3 & 2 + 3\cdot 4 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 10 & 14 \end{bmatrix}$

Matrix inversion

1.b: $\left[\begin{array}{cc|cc} -1 & 0 & 1 & 0 \\ 10 & 14 & 0 & 1 \end{array}\right] \cdot -1 \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -1 & 0 \\ 10 & 14 & 0 & 1 \end{array}\right] + (-10 \cdot R_1)$

$$\left[\begin{array}{cc|cc} 1 & 0 & -1 & 0 \\ 0 & 14 & 10 & 1 \end{array}\right] \cdot \frac{1}{14} \rightarrow \left[\begin{array}{cc|cc} 1 & 0 & -1 & 0 \\ 0 & 1 & \frac{5}{7} & \frac{1}{14} \end{array}\right]$$

$$\begin{bmatrix} -1 & 0 \\ \frac{5}{7} & \frac{1}{14} \end{bmatrix}$$

Eigenvectors and eigenvalues

1.c: $\begin{bmatrix} 2-\lambda & -2 \\ 1 & -1-\lambda \end{bmatrix}$

$$\text{ad - cb}$$

$$= (2-\lambda)(-1-\lambda) - 1 \cdot (-2)$$

$$= -2 - 2\lambda + \lambda + \lambda^2 + 2$$

$$= \lambda^2 - \lambda$$

$$\lambda(\underline{\lambda - 1}) = 0$$

$$\lambda = 0 \hookleftarrow \quad \hookrightarrow \lambda - 1 = 0$$

$$\lambda = 1$$

Eigenvalues:

$\lambda_1 = 1$, $\lambda_2 = 0$

For $\lambda = 1$

$$\begin{bmatrix} 2-1 & -2 \\ 1 & -1-1 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 1 & -2 \end{bmatrix}$$

$x - 2y = 0$

$x - 2y = 0$

$x = 2y$

Solution $x = 2y$ with y as free parameter

$$\begin{bmatrix} y \\ 2y \end{bmatrix} = y \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} \text{ or } \begin{bmatrix} 1 \\ 1/2 \end{bmatrix}$$

For $\lambda = 0$

$$\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$$

$2x - 2y = 0$

$x - y = 0$ $\quad x = y$

Solution $x = y$ with y as free parameter

$$y \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Gradient

1.e: $f(x,y) = x^2 + 2y^2 - xy$

$f'x = 2x - y$ $\qquad$ $f'y = 4y - x$

$f''xx = 2$ $\qquad$ $f''yy = 4$

$f''xy = f''yx = -1$

Gradient to $f(x,y)$: $\nabla f(x,y) = \underline{(2x - y,\ 4y - x)}$

Minima / maxima of a function

1.f: $\nabla f(x,y) = 0$

$f'_x = 2x - y = 0$ $\qquad$ $f'_y = 4y - x = 0$ $\qquad$ $x = -4y$

$y = 2x \quad \rightarrow \quad 8x - x = 0$

$2x - 4y = 0 \quad \leftarrow \quad 7x = 0$

$-2y = 0 \qquad x = 0$

$(x, y) = 0$

Lagrange Multipliers

$2x\lambda + y\lambda - 22y$

$$L(x, y, \lambda) = x^2 + 2y^2 - xy - \lambda(2x + y - 22)$$

$$\frac{\partial L(x, y, \lambda)}{\partial x} = 2x - y - 2\lambda = 0$$

$$\frac{\partial L(x, y, \lambda)}{\partial y} = 4y - x - \lambda = 0$$

$$\frac{\partial L(x, y, \lambda)}{\partial \lambda} = 2x + y - 22 = 0$$

$$\left[\begin{array}{ccc|c} 2 & -1 & -2 & 0 \\ -1 & 4 & -1 & 0 \\ 2 & 1 & 0 & 22 \end{array}\right] \cdot -1 \longrightarrow \left[\begin{array}{ccc|c} 1 & -4 & 1 & 0 \\ 2 & -1 & 2 & 0 \\ 2 & 1 & 0 & 22 \end{array}\right] \begin{array}{l} \\ +(R_1 \cdot -2) \\ +(R_1 \cdot -2) \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & -4 & 1 & 0 \\ 0 & 7 & -2 & 0 \\ 0 & 9 & 0 & 22 \end{array}\right] \begin{array}{l} \\ \cdot \frac{1}{7} \\ \\ \end{array} \rightarrow \left[\begin{array}{ccc|c} 1 & -4 & 1 & 0 \\ 0 & 1 & -\frac{4}{7} & 0 \\ 0 & 9 & -2 & 22 \end{array}\right] \begin{array}{l} +(R_2 \cdot 4) \\ \\ +(R_2 \cdot -9) \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{9}{7} & 0 \\ 0 & 1 & -\frac{4}{7} & 0 \\ 0 & 0 & \frac{22}{7} & 7 \end{array}\right] \begin{array}{l} \\ \\ \div \frac{22}{7} \end{array} \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & -\frac{9}{7} & 0 \\ 0 & 1 & -\frac{4}{7} & 0 \\ 0 & 0 & 1 & 7 \end{array}\right] \begin{array}{l} +(R_3 \cdot \frac{9}{9}) \\ +(R_3 \cdot \frac{4}{7}) \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 9 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 7 \end{array}\right] \qquad x = 9 \;,\; y = 4 \;,\; z = 7$$

Statistics

1. h: $P(\overset{K}{\text{card is a king}} \mid \overset{E}{\text{card is not an ace}})$

$\Rightarrow P\left(\frac{1}{13} \mid \frac{12}{13}\right)$

$$P(K) = \frac{1}{13}$$

$$P(E) = \frac{1}{13} \Rightarrow P(E) = 1 - P(E) = \frac{12}{13}$$

$$P(K \mid E) = \frac{P(K \cap E)}{P(E)} = \frac{1/13}{12/13} = \frac{1}{12}$$

1. i: Mean value:

$$\bar{x} = \frac{1+2+3+4+5+6}{6} = 3.5$$

Variance :

$$Var(x) = (1-3.5)^2 + (2-3.5)^2 + (3-3.5)^2 + (4-3.5)^2 + (5-3.5)^2 + (6-3.5)^2$$

$$= \frac{17.5}{6} =$$

discrete

A die has a uniform distribution because the probability of getting a number is 1/6 for each throw regardless of the previous throw

2. Machine learning problems

1. Grocey store problem
2.a:

The task T:	The input for this machine learning may be data/information from another Rema 1000 store that may be near Marineholmen or from another store that may be in the same location. See what customers are buying in the other stores and have that amount of products in the new store. Afterward, collect data from what customer is buying in this store in Marineholmen. See what product is getting bought most. The output should be an increasement in the products that gets sold most. - Increasement in revenue because a product is populare - Increasement in number of a particular product because customer often buy this product
The performance measure P:	If the machine learning works for this situation, an increasement in sales should appear because of the increasement of some popular products in the store.
The experience E:	Data from other stores. Number of products sold in this new store. Data from customer in this new store; what they are buying and see what product is getting bought most.

2. Oil drilling problem

2.b:

The task T:	The task for this machine learning is to determine what drill is most efficient for the type of rock it is about to drill. Input may be information about the rock it is about to drill. How dense and how deep that kind of rock is. The outcome of this should be the an efficient drilling.
The performance measure P:	See if the machine learning is helping the plaforms getting more oil or getting access to oil more easily.
The experience E:	Get information about the rock level and what type of drill is best for this kind of rock.

3. Self-driving car problem

2.c:

The task T:	The input may be data from different sensors to find information about its surroundings. Also data from traffics, e.g. speed limits, where there are many cars, how fast the car in front is driving, etc. Output should be a selfless driving car. The car ride should be as close as a person would normally drive. The car should also be able to stop before a collision occurs.
The performance measure P:	If the passenger is having a comfortable car ride, e.g. car not steering randomly off the road or doing something abnormal. To actually know if the car is avoiding collisions is to test if the sensors are working properly and if the car is actually stopping before a collision occurs, e.g. if a car in front is suddenly braking hard.
The experience E:	The machine learning can use different sensors on the car to collect data to find more about its surrounding. As well as sharing data and information between the cars. For example if one car occurs in an accident, it will share it information/experience with the other cars.

3. K-Nearest-Neighbours

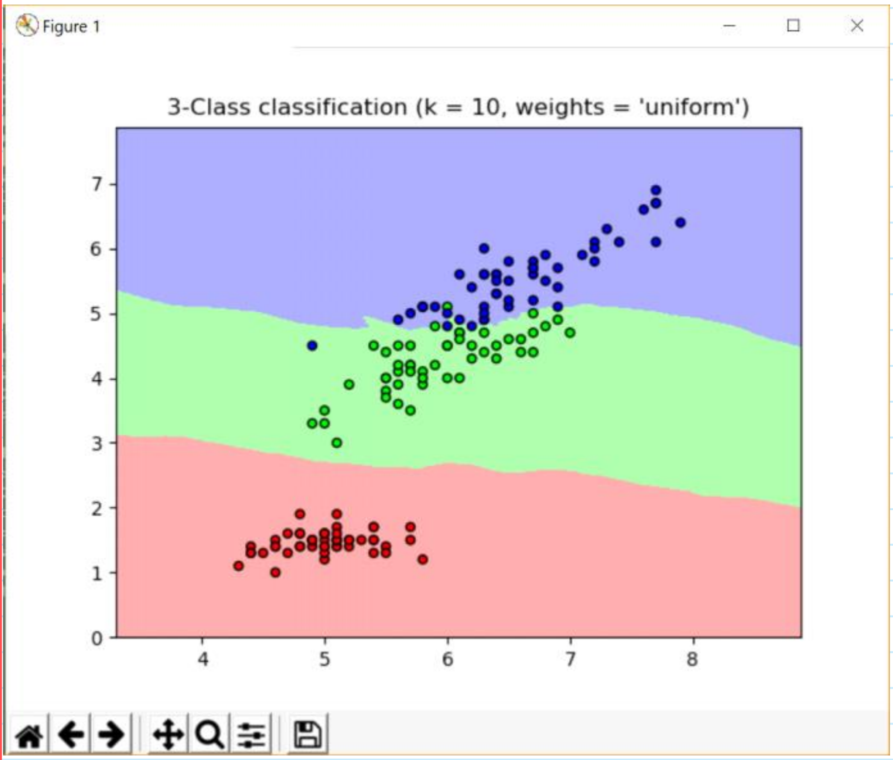
3.a:

K = 3; depends, but can be both. This is depends on the implementation.

K = 9 gives us dogs. The cat at x = 1 is too far away.

KNN using programming

3.b:



3. How many of each group:

0- red: 35
1- green: 47
2- blue: 41

4. Total of each group, with misclassified flowers

0- red: 35
1- green: 47 - 3 + 1 = 45
2- blue: 41 + 3 - 1 = 43

5. The kNN may be run in to some problem:
   - The prediction phase may be slow with big data sets; lot of observations
   - High memory requirement
   - KNN keeps the training data. The training data may have an impact on the decision making. If a certain class is very common in the training data, it might dominate the majority voting in the decision making.