Modeling With Networks

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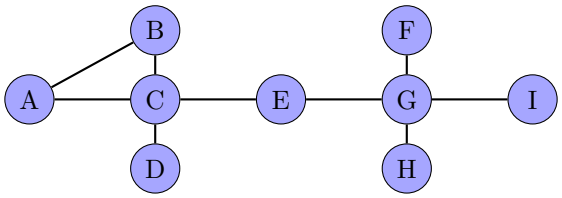
1. Introduction

Graphs are some of the most versatile and interesting structures in mathematics. A graph G is characterized as an ordered pair of vertices and edges, $G = (V(G), E(G))$. The edges in a graph imply the existence of a relationship between two vertices or objects. They can be directed (implying a relationship based on ordering) or undirected where the edges have no orientation. Edges can also be weighted, having several real world applications like distance, cost, etc. These structures evidently can represent countless applications which have discrete and related objects. A simple graph which will operate as our running example is shown below.



Influence is the capacity to produce an effect in indirect or intangible ways. If one wants to be successful in their field, they must share their ideas and persuade their peers to accept them. This idea is present in all types of industries and specializations. A significant portion of this project focuses on the widespread influence of a famous mathematician, Paul Erdos, and his large co-author network.

In the following project report, we analyze how influence can be determined in a graph and describe our exploration into mathematical modeling with networks. There exist several known measures to explore the importance of a vertex in a graph, including degree centrality, betweenness centrality, etc. We will model various networks, analyze them, find who/what has the most influence in the network, and the reasons behind this result. We seek to combine real world scenarios with some of these mathematical ideas to accurately describe the concept of influence in graphs. Furthermore we will argue how these methods can show steps to rapidly boost ones own influence.

The prompt asked us to consider a number of specific tasks, briefly outlined below:

- Build and analyze the Erdos1 co-author network using the data from https://files.oakland.edu/users/grossman/enp/Erdos1.html. The Erdos1 co-author network is characterized by the authors who have collaborated with the famous mathematician, Paul Erdos. Rather than including every entry in this data file, we were asked to consider a network where each author has collaborated directly with Erdos (omitting Erdos himself). After building this network, we are asked to explore the graph and find some interesting features and properties.
- Define and study two critical measurements by which to determine the influence of authors in the network we created in part a.
- Gather data, build, and analyze a network showing the relationship between some foundational papers in the emerging field of network science. Apply the influence measures used for parts a,b and discuss their effectiveness. Also, discuss methodology and other factors surrounding this network.
- Gather data, build, and analyze a real life scenario which can be modeled as a network. Again apply influence measures and discuss the results, shortcomings, external factors, etc..
- Discuss how influence and impact can be used in real life situations. Consider business decisions, improving influence, selecting a graduate school etc.

To address the above tasks, we utilized the R programming language and our knowledge from various math and computer science courses.

2. Statement and Analysis of the Problem

While our project does have a number of specific requirements discussed in the introduction, essentially the problem we are trying to approach is how to measure and interpret the influence of a vertex within a network. We define two influence measures to address this problem, discussed in depth in the latter sections. Once these are defined, we are collect and clean multiple data sets, create multiple networks, and analyze these networks under our two metrics.

While the field of graph theory is certainly not young, we noticed a wealth of more recent works geared towards our topic and general social network analysis. We began by finding a modern survey of various graph centrality measures, notably the work by Bloch, et al. titled *Centrality Measures in Networks* [NUMBER]. Rather than focusing on a concrete example, this work provides a much needed overview of several fundamental graph measurements and their properties. This publication helped us determine what different types of graph metrics exist and how they are calculated.

Another work which we used to help shape our understanding of the problem was the comment released by Vishesh Karwa and Sonja Petrovic titled *Coauthorship and citation networks for statisticians* [NUMBER]. This paper looks directly at how co-author networks and citation networks can be analyzed which is very relevant to our project. Regrettably, the networks and paper which Karwa and Petrovic are commenting on are more advanced than what we are able to make. Consequently, their analysis is also more advanced. As we will discuss, our Erdos1 co-author network is slightly oversimplified. An edge in our network simply implies the presence of these two authors collaborating at least once, but the exact number is unknown. We are able to give no weight to these edges since our data set simply shows some form of collaboration. Regardless, we were able to take a number of pointers from this paper. For a complex network, the paper suggests that a metric like counting the degree of each node is far too simple. For our simple network, such a measure may be more appropriate.

A number of publications also exist which detail analyses of specific co-author networks. One such work is *Co-authorship and citation networks in Spanish history of science research* by Osca-Lluch, et al [NUMBER]. The authors data allows them to look at papers published by various journals, number of papers published by Spanish scientists over time, types of publications, etc. While we did not have this type of data readily accessible, the paper also looks at co-author collaboration and the impact of these publications. The paper interestingly notes that a very high percentage of authors have no collaboration whatsoever. They create a visualization of their graph where the edge thicknesses represent number of collaborations. To analyze the publication impact, the authors weigh in on using a quantative measurement vs a qualitative one. They argue that a qualitative approach can be effective due to its ability to take into consideration the expertise, reach, and prestige apart from the publishing journal.

Despite that our networks were based on much more simple data sets, it was interesting and useful to see how others have approached similar analyses of co-author and citation networks. This problem is not totally novel, but it's apparently getting increasing amount of attention in recent years, as we were able to find several publications referencing co-author and citation networks in the last 10 to 15 years.

3. Description of the Model

We selected two primary metrics by which to determine influence in a graph. While these metrics are not entirely novel, we explain their formulation mathematically and conceptually.

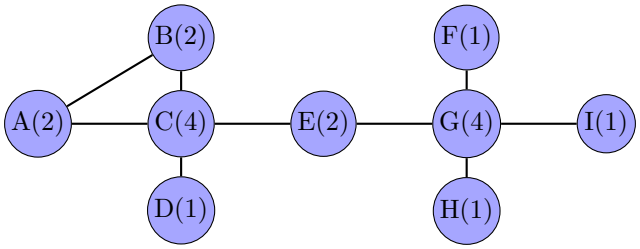
3.1 Total Degree

The degree of a vertex v, commonly denoted $deg(v)$, represents the number of edges incident to the vertex. This calculation can vary slightly depending on if the graph is directed or undirected. For the purpose of our application, we choose to define our measurement to be the total degree of a vertex, where total degree is defined as follows:

For a directed graph $G = (V(G), (E(V))$, Total Degree of a vertex v = $\sum edges\ in\ (v) + \sum edges\ out\ (v)$

where edges in (v) represents the incident edges directed into a vertex v and edges out (v) represents the incident edges directed out of the vertex v. For an undirected graph, the Total Degree of a vertex v is simply $\sum edges\ incident\ to\ (v)$.

We show an example calculation of the total degree of each node in a simple graph below. Note that the degree of each vertex is written inside the vertex next to the label.



By the total degree metric, we would say that the most important/influential vertices in this graph would be C and G, each having a degree of 4. These vertices have the most direct connections to the other vertices in the network.

We selected this metric due to its simplicity and relevance in the example networks which we will consider. More reasoning will be present in the sections which follow, but a subset is mentioned here. The first network which we will analyze, the Erdos1 co-author network, is a simple undirected network which shows authors who have collaborated with the mathematician Paul Erdos. We can apply the total degree metric to help us determine influence of authors in this network because more collaborators will imply that an author has more influence. If a vertex (representing an author) has a higher degree, we could assume that their works are more diverse and their influence is greater. The second network which we will analyze, the citation network, is a directed network which shows academic papers which have cited each other. Our metric will also be relevant in this scenario, as a paper which has a higher degree will mean that more papers have cited it, making it more influential. The third network is a movie/actors network. Total degree has a solid meaning here too, as more connections (edges) will imply that the actor has worked with a larger number of other actors, meaning that they have more of an influence in their industry.

3.2 Eigenvector Centrality

The eigenvector centrality of a vertex in network is a more advanced measurement which can help determine influence of a vertex. While this measurement also suggests that the importance of a vertex depends on the vertices it is directly connected to, it adds a layer of complexity. Rather than just considering the number of incident edges to a vertex, eigenvector centrality suggests that the importance of a vertex is relative to the importance of the vertices it is connected with [NUMBER].

Given a graph $G = (V(G), (E(V))$ the adjacency matrix A for this graph is a V by V matrix. Every entry in this matrix is zero unless there exists an edge between the two corresponding vertices. In that case, the value of the entry is equal to the edge weight between the two vertices (commonly 1). Given these brief definitions, the eigenvector centrality is then defined as follows [NUMBER]:

$$\sum_{n=1}^{\infty} 2^{-n} = 1$$

4. Analysis and Testing of the Model

5. Results and Quality of Model

6. References

To be completed at a later time: (APA)

Paper about calculating eigenvector centrality, why this measurement is stable, etc: https://arxiv.org/pdf/1410.5119.pdf Paper with a whole bunch of graph metrics and explanation about them: https://arxiv.org/pdf/1608.05845.pdf Paper about analyzing a co-author network of spanish scientists: https://link.springer.com/article/10.1007/s11192-008-2089-5 Paper with IIT author regarding co-author and citation networks: https://arxiv.org/pdf/1608.06667.pdf

7. Appendix