Fitting Dynamical Systems to Neural Data

Project Statement

Motor cortex role in generating voluntary movements has been a very interesting yet challenging problem. Many hypotheses and models were developed to explain the role of motor cortex processing in generating muscles activity. In particular, a new hypothesis stated that neuron populations may serve as a dynamical engine where neurons interact together to generate basis functions (oscillations with different frequencies) that are combined and sent downstream via spinal cord to generate muscles activity (EMG signal). In this model, the firing rate of neuron populations are assumed to follow a linear dynamical system of the following form:

$$\dot{r}(t,c) = M^\top r(t,c) \tag{1}$$

where $r(t) \in I\!R^{N\times 1}$ is the instantaneous firing rate vectors for N neurons. $M \in \mathbb{S}^{N\times N}$ is the oscillatory dynamics matrix ($\mathbb{S}$: skew-symmetric matrix). t is time and $c \in \{1,\ldots,C\}$ are different movement conditions.

To investigate this hypothesis, experiments were performed on different monkeys. In these experiments, the monkeys were instructed to reach different targets, and the firing rates of many neurons were recorded. The firing rates at the movement times and at different reaching conditions were sampled (T samples acquired with sampling rate $\frac{1}{\delta t}$) and grouped into the data matrix $R \in I\!R^{CT\times N}$ where

$R = [r^\top(1\times\delta t,1);\ldots\ldots\ldots;r^\top(T\times\delta t,1);r^\top(1\times\delta t,2);\ldots\ldots\ldots;r^\top(T\times\delta t,2);$
$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots;r^\top(1\times\delta t,C);\ldots\ldots\ldots;r^\top(T\times\delta t,C)]\,.$

This allows us to write the discrete version of equation 1 in matrix form as:

$$\dot{R} = RM \tag{2}$$

To validate the hypothesis, it is desired to fit the neurons' population responses with oscillatory dynamical system. The fitting procedure is equivalent to solving the following optimization problem:

$$\hat{M} = \underset{M \in \Re^{N \times N}}{\operatorname{argmin}} ||\dot{R} - RM||_F^2 \tag{3}$$

Since the data is highly dimensional (N is large), it is desired to solve this problem (equation 3) using an efficient iterative method.

Project Requirements:

- Develop an iterative solver to solve the problem in equation 3.
- Justify your choice of the iterative method implemented.
- Analyze the convergence properties of your solver (i.e. rate of convergence, and whether the algorithm is guaranteed to converge).
- Derive the optimality conditions of the solution.

Hint: *In developing and testing your algorithm, you may assume that R is a random matrix of size greater than or equal to (1000×100). $\dot{R}$ may be constructed from R by calculating the time difference of R between the subsequent samples, assuming $\delta t = 1$.*