

Aerospike Nozzle: Advantages and Complications

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This inquiry-based paper explores why there are efficiency advantages with aerospike nozzle geometry and discusses the complications associated with development and implementation of the aerospike nozzle, with respect to thrust performance, heat exchange, and mass. While the efficiency advantages of the aerospike nozzle are significant and repeatedly shown in computational fluid dynamic simulations, cold flow tests, and hot fire tests, the industry requires more small to medium-scale nozzle integration and flight tests before development of an aerospike configuration for a large-scale launch vehicle is undertaken.

Nomenclature

CBN	= conventional bell nozzle
α	= half-angle
I_{sp}	= specific impulse
F	= thrust
\dot{m}	= mass flow rate
g_o	= acceleration due to gravity
v_e	= exit velocity
A_e or A_2	= exit area
A_t	= throat area
p_e	= exit pressure
p_a	= ambient pressure
C_F	= thrust coefficient
ε	= expansion ratio
p_c	= combustion chamber pressure
ACN	= altitude-compensating nozzle
R_c	= distance between cowl-lip and nozzle axis (see Figure 13)
CFD	= computational fluid dynamics
N_2O	= nitrous oxide

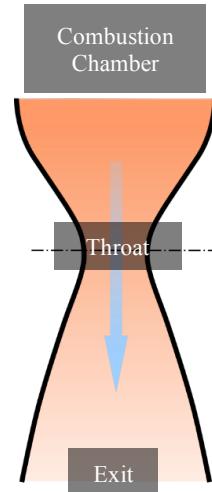


Figure 1. Diagram of a conventional bell nozzle.

I. Introduction

Spaceflight is expensive. During the Apollo program the average cost per flight was \$9.9 billion, after adjusting for inflation [1]. The average cost per flight during the Space Shuttle program was \$450 million [2]. Today, the average cost to launch a satellite on Europe's Ariane 5 is \$137 million [3]. While these examples span multiple decades and programs with different objectives and payloads, the numbers underscore the magnitude of expense that has impeded many governmental, commercial, and academic programs alike. With the rising demand for low-cost satellite and microsatellite launches and the recent growth in the commercial spaceflight industry [4, 5], minimizing the cost per pound to the common satellite destinations of low Earth orbit and geostationary orbit has never been more critical [6]. The efficiency of a launch vehicle, the largest section of a multi-stage rocket, has a significant impact on the overall cost of mission. This is because the greatest amount of fuel is used to lift the entire mass of the rocket off the launch pad, fly under atmospheric drag, and perform a gravity turnⁱⁱ. Because the performance of a launch vehicle's rocket engine and, especially, its nozzle can have a significant impact on cost per pound to orbit, alternative designs and methods have been researched in pursuit of the optimal propulsion system. The scope of this paper will be limited

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ⁱⁱ A flight maneuver executed by rockets during ascent to reduce time traveled through the atmosphere while still utilizing a ballistic trajectory as a smooth transition into a circular orbit, thereby minimizing the propellant required.

to nozzle design. The focus of this paper concerns the advantages and complications associated with the development and implementation of the aerospike nozzle, as an alternative to the conventional bell nozzle (CBN), with regard to thrust performance, heat exchange, and mass.

II. Technical Background

Nozzle geometry largely dictates the performance and efficiency of a launch vehicle. The two supersonic nozzle designs used for launch vehicles are conical and bell. The subtle geometric difference between the two (see Figure 2) reflect the different methods used to design them. Conical nozzles are typically designed using only a series of equations that assume one-dimensional, isentropic flowⁱⁱⁱ. While these assumptions simplify calculations, they do not account for several factors that affect real nozzles, such as jet divergence. Jet divergence is an efficiency loss defined by the angle the fluid flow deviates from the central axis (see Figure 3).

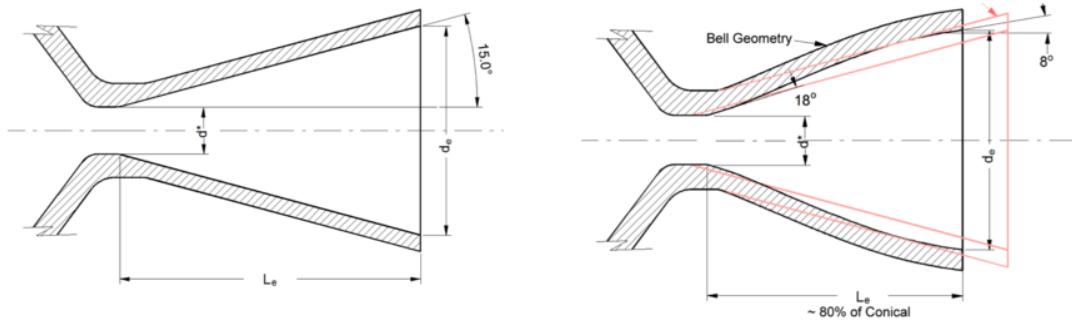


Figure 2. Conical nozzle vs. bell nozzle. Conical nozzles are rarely used in modern rockets today due to their efficiency losses.

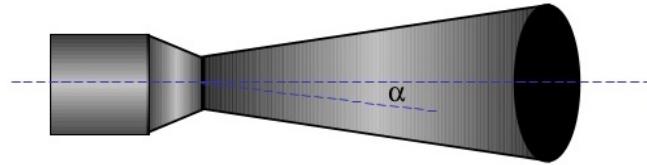


Figure 3. Reducing half-angle, α , reduces divergence losses, but increase length and weight (for a given throat and exit area).

If an optimum nozzle contour is desired, then a bell nozzle will typically be designed by parabolic approximation or by a technique known as method of characteristics^{iv}. To evaluate the efficiency of a nozzle, a value called specific impulse, I_{sp} , is calculated. As defined in Equation 1, specific impulse is a measure of the amount of thrust a nozzle can produce for the rate at which it can expel mass (propellant).

Equation 1

$$I_{sp} = \frac{F}{\dot{m}g_o}$$

Exploring this equation further:

Equation 2

$$F = \dot{m}v_e + A_e(p_e - p_a)$$

ⁱⁱⁱ Fluid flow that can only travel along one axis and does not transfer internal heat or frictional heat to the surrounding materials.

^{iv} An approach to solving hyperbolic partial differential equations.

As shown in Equation 2, I_{sp} is also a function of ambient pressure, p_a . Higher pressure gas particles will travel more readily along the path of least resistance (i.e. toward the nozzle exit) than lower pressure gas particles. Therefore, all nozzles are more efficient at higher altitudes, regardless of their design (see Figure 4). Another important parameter of nozzle design is the area ratio or expansion ratio, A_e/A_t or ϵ . As also shown in Figure 4, for a given pressure ratio there is an optimal area ratio that maximizes the coefficient of thrust, C_F (a representation of efficiency). Figure 5 is a more detailed ϵ vs. C_F graph, including more functions at different pressure ratios. As stated before, at any altitude there is an optimal area ratio for which C_F is maximized. This optimal area ratio allows the fluid to expand, lose

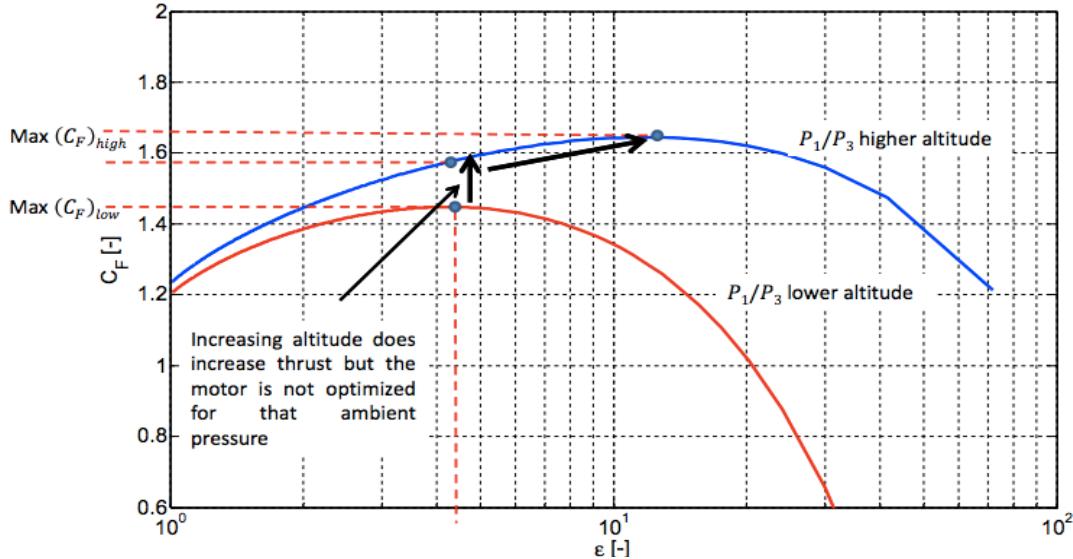


Figure 4. C_F , and I_{sp} represent nozzle efficiency similarly. The same applies for p_c/p_a and p_e/p_a .

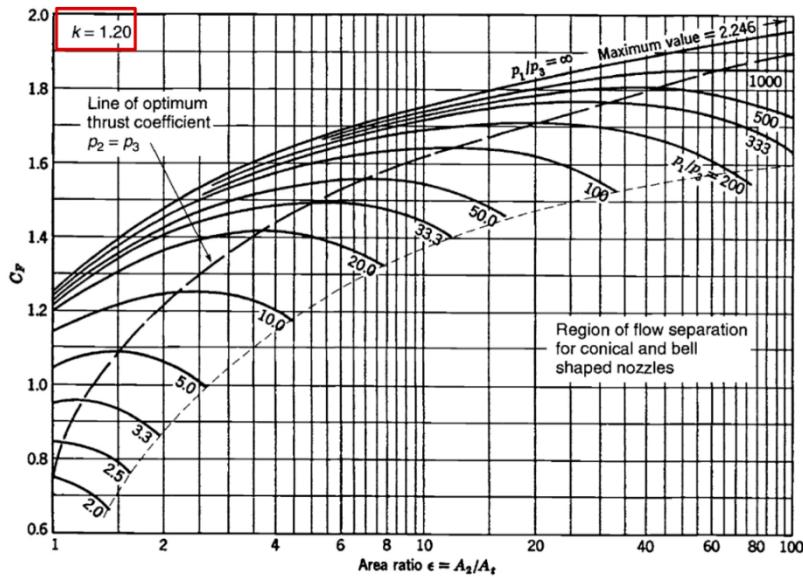


Figure 5. Operating in an environment of decreasing p_a will improve the efficiency of any nozzle configuration. Increasing the area ratio, at a certain rate, as p_a decreases, will yield ideal efficiency at all altitudes, provided all other parameters are fixed.

pressure, and accelerate^v until it reaches the exit plane of the nozzle, at which point, the fluid's pressure is equal to the

^v Behavior of a compressible fluid traveling through a pipe flips when the fluid's velocity exceeds the local speed of sound (i.e. a sub-sonic diffuser acts as a super-sonic nozzle, accelerating the fluid, and vice versa).

ambient pressure. By having $p_e = p_a$, thrust and C_F are maximized. A general, physical explanation for this phenomenon is that if the exit pressure is not equal to the ambient pressure, then the exhaust plume, which exerts a reactionary force on the nozzle, will not be as parallel to the nozzle axis, causing only a component of the reaction forces to contribute to the rocket's acceleration. For a fixed-area ratio nozzle, there is only one altitude where $p_e = p_a$: The "design altitude". At altitudes below the design altitude ($p_e < p_a$) the fluid is allowed to expand too much and is then said to be "overexpanded". The direction of the expanding fluid is turned toward the axis of the nozzle,

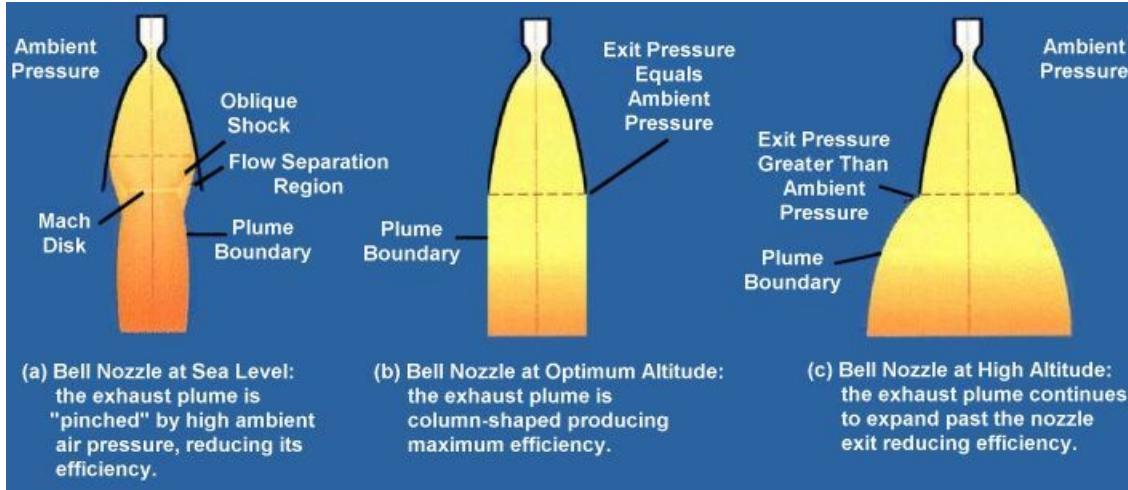


Figure 6. Efficiency losses of overexpanded exhaust and underexpanded exhaust are the result of untapped component exhaust forces that do not act directly along axis of nozzle.

then reflected toward the plume boundary (see (a) in Figure 6). When these expansion waves meet, the local exhaust temperature and luminosity rises (see Figure 7). At altitudes above the design altitude, ($p_e > p_a$) the fluid still has a considerable amount of differential pressure and, therefore, hasn't expanded enough. As the fluid exits the nozzle, its "underexpanded" state will cause a rapid expansion and the formation of a large exhaust plume as seen in Figure 8. The design altitude is primarily chosen as the approximate altitude at which a nozzle is operating for the longest period during ascent, with consideration to other launch vehicle design criterion. For example, during the design of the Space Shuttle Main Engine (SSME), it was determined that the nozzle's design altitude would be relatively high [7]. This is understandable because the Space Shuttle was an orbital vehicle that also used its engines to provide thrust at launch. The SSME couldn't generate enough thrust, at its overexpanded state, during launch. Even if the nozzle geometry was optimized for the ambient pressure at launch, a considerable performance loss would be seen across the wide-range of pressure ratios and long duration of powered, high altitude flight. To combat this issue, affordable and relatively simple Solid Rocket Boosters were designed to provide additional, low altitude-optimized thrust during and shortly after launch. Whether or not an orbital vehicle's engines must operate at launch, a higher area ratio may appear as the best choice because the majority of the vehicle's operation will be at high altitudes and in the vacuum of space. However, as the area ratio is taken to an extreme, the exit pressure of the nozzle reaches a limit, at which its fluid flow has expanded so much that a relatively high ambient pressure can separate the fluid flow from the nozzle wall. This critical exit pressure value is 40% of the ambient pressure and is often referred to as the "Summerfield Criterion" [8]. When a nozzle operates at or below this value, it is said to be "grossly overexpanded". Because separation rarely occurs symmetrically, the nozzle is likely to experience side-loads which could result in the structural failure of the nozzle. Therefore, the ideal nozzle would be one that can change its area ratio so that its exit pressure is equal to that of its ambient surroundings, thereby producing a uniform exhaust plume parallel to the nozzle axis and yielding the best performance at all altitudes. This rate of change is represented by the "Line of optimum thrust coefficient $p_2 = p_2^*$ " in Figure 5.

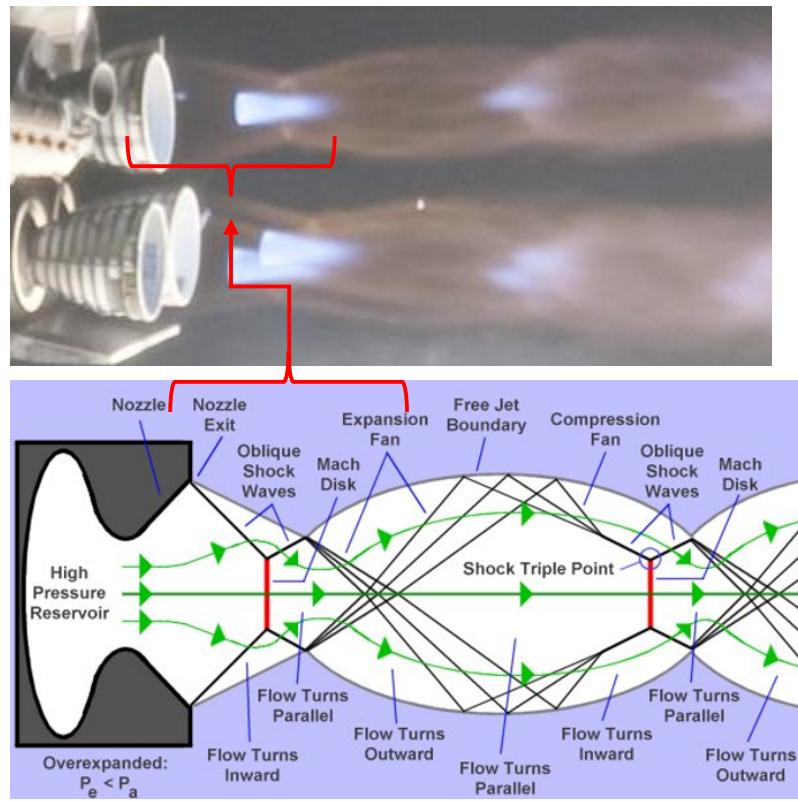


Figure 7. Because the expansion waves of the exhaust are turned inward, toward the nozzle axis, only a component of their reactionary force acts upon the nozzle. The result is a loss in thrust and efficiency.



Figure 8. Five F-1 engines of the Saturn V rocket at ideal conditions during launch compared their underexpanded state during high-altitude flight.

This ideal, variable area ratio nozzle is often referred to as an altitude-compensating nozzle (ACN). There are several ACN designs. Among the most prominent are: aerospike, expansion-deflection, dual-bell, and extended-skirt (see Figure 9). While several types of ACN are worth investigating, the focus of this paper is to investigate the aerospike nozzle.

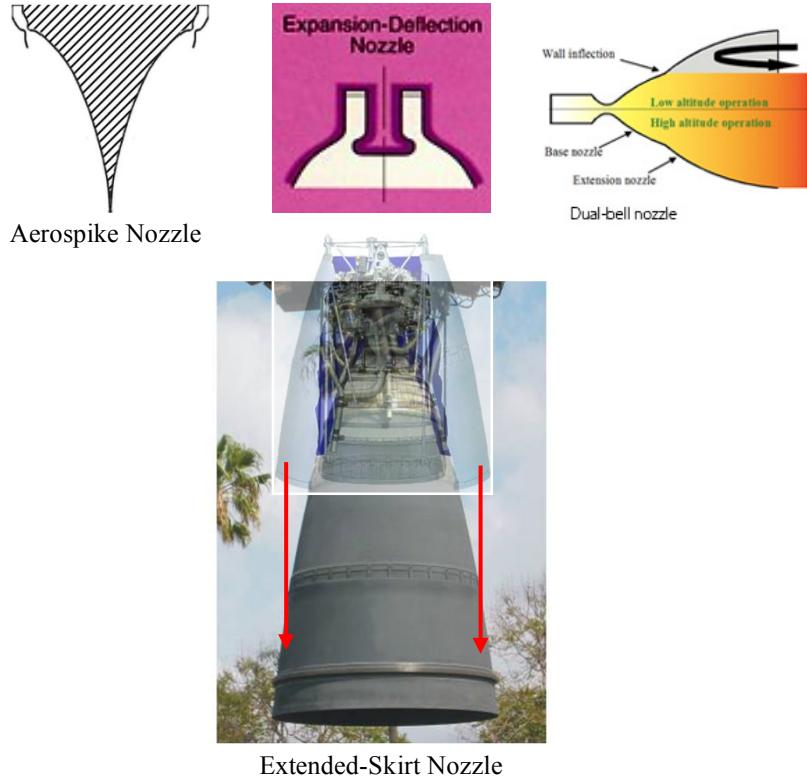


Figure 9. Four examples of altitude-compensating nozzles. The aerospike nozzle and expansion-deflection change their area ratio by exposing the expanding fluid flow to the ambient pressure. The dual-bell nozzle changes its area ratio by using the second bell to constrict the underexpanded plume of the first bell. The extended-skirt nozzle changes its area ratio by lowering a nozzle extension, thereby changing the exit area.

III. Performance & Efficiency of Aerospike Nozzle

The aerospike nozzle is an ACN that changes its area ratio by allowing half of the fluid flow to expand along a rigid spike and the other half to expand along the plume boundary. The plume boundary is where the fluid flow is in contact with the ambient gas (labeled in Figure 6). The exit area of an aerospike nozzle is defined by how offset the exit plume boundary is from the tip of the spike [9]. Because the offset of the exit plume boundary is governed by ambient pressure, the exit area is as well. At lower altitudes, the high ambient pressure keeps the expanding fluid flow against the nozzle wall (i.e. spike wall). In other words, the high ambient pressure prevents the flow from expanding to the point where the ambient pressure is greater than the pressure at any given point along the spike wall. Thus, at these higher ambient pressures the aerospike's exhaust flow is kept more axial, yielding less divergence loss and a better thrust coefficient than the CBN [10] (see Figure 10). Unfortunately, at these higher ambient pressures a series of recompression shock waves form along the walls of the spike. While these recompression shock waves serve to adapt the pressure of the expanding fluid flow to that of the ambient surroundings (see Figure 11) [7], some efficiency losses are incurred on nozzle performance [11], due to the non-isentropic nature of shock waves. These losses are significantly lower than those associated with an overexpanded or

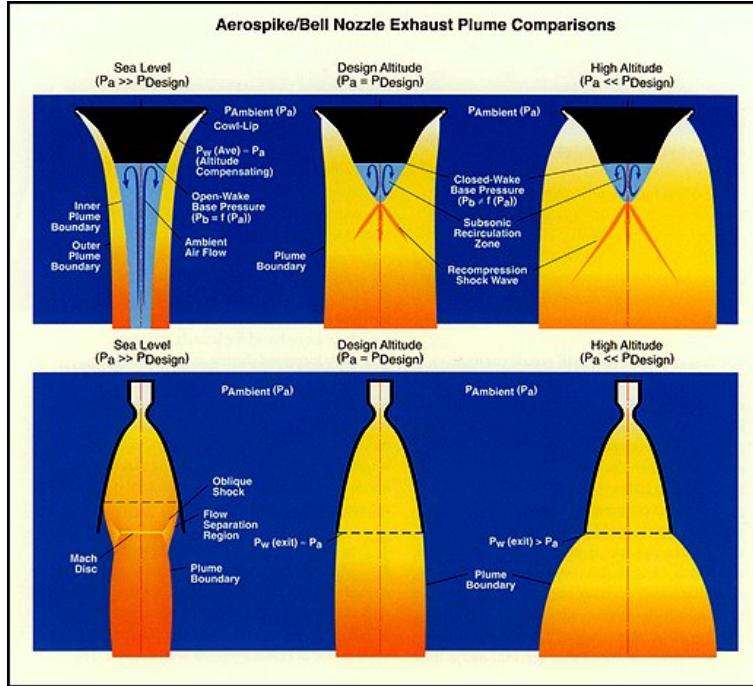


Figure 10. Higher ambient pressures impact the thrust of bell nozzles more than aerospikes because a higher ambient pressure thwarts the overexpansion of fluid flow and directs the exhaust more axially, in comparison to the conventional bell nozzle.

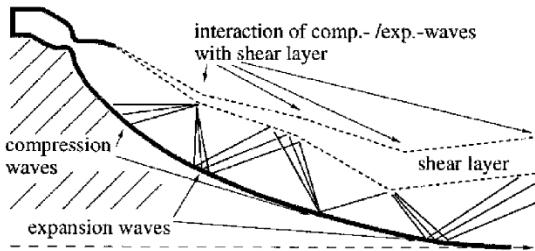


Figure 11. Cross section of half of an aerospike operating below its design altitude where recompression shockwaves adapt the pressure of the expanding fluid flow to the ambient surrounds as it reaches the end of the spike.

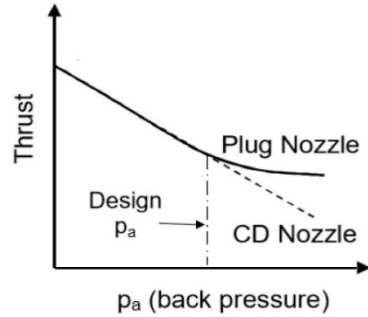


Figure 12. The plug nozzle (truncated aerospike, performs similarly to the aerospike) produces more thrust, at a given area ratio, in an overexpanded state than a CBN (referred to as "CD Nozzle" here).

grossly overexpanded CBN, as depicted by the clear thrust advantage the aerospike has over the CBN during low altitude operation (see Figure 12). As also depicted in Figure 12, the aerospike nozzle has a design altitude. This design altitude corresponds to a fixed area ratio that is expressed in Equation 3 and depicted in Figure 13. Aerospike nozzles follow the same trend as CBNs where a higher fixed area ratio corresponds to a higher design altitude and more efficient high-altitude performance. In fact, when an aerospike reaches its design altitude, where the plume boundary diameter is equal to that of the cowl-lip it is performing at maximum efficiency (see Figure 13). It should also be noted that this is the point at which recompression shockwaves are not present along the spike wall. At and above this design altitude, the aerospike performs the same as the CBN because there is no method in either configuration that restricts the large, underexpanded exhaust plume. Consequently, aerospikes can be designed with sizeable area ratios because they can withstand much higher ambient pressures at launch and low altitude operation

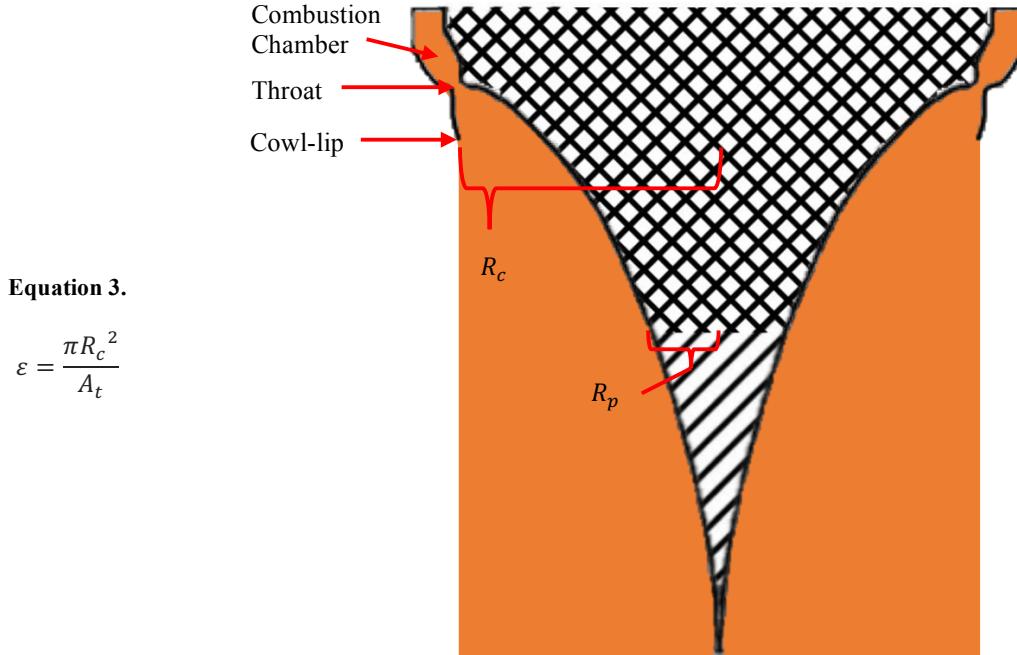


Figure 13. Diagram of aerospike nozzle operating at its design altitude, depicting various components and area dimensions. Excluding the spike segment toward the bottom (filled with a diagonal pattern) would result in a plug nozzle configuration.

without flow separation, as stated previously. This was the principal reason why an aerospike nozzle was a candidate for the SSME [13].

Hagemann et al. [11] numerically simulated the specific impulse of an aerospike nozzle (referred to as a plug nozzle in their paper) and a CBN over a range of altitudes (see Figure 14). This graph clearly depicts the considerable efficiency advantage the aerospike nozzle (see (1) in Figure 14), referred to as a plug nozzle in their study, has over the CBN during operation under its design altitude. Similar diagrams depicting this exceptional increase in efficiency are also available in other published papers (see Figure 15). While these diagrams show a significant increase in efficiency, they do not depict a loss of efficiency that is predicted to be incurred on the nozzle

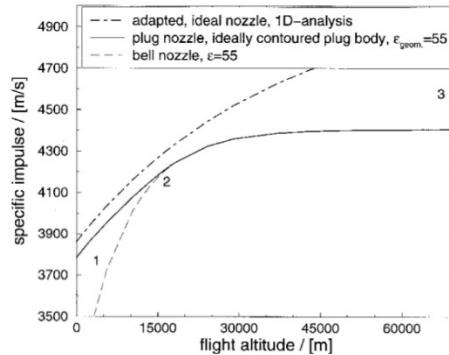


Figure 14. Graph depicting the efficiency of an aerospike nozzle, referred to as a “plug nozzle” here.

performance by external flow around the vehicle during transonic speeds^{vi}, when the vehicle breaks the sound barrier. Many CFD simulations, cold flow tests, and other forms of analysis repeatedly conclude that, for a typical launch vehicle profile, this loss is insignificant compared to the exceptional gains the nozzle attains from its altitude-

^{vi} Speeds slightly below and above the speed of sound.

compensating ability [14-18]. A typical flight profile has the launch vehicle operating in this transonic regime for a relatively short period compared to the total duration of below design-altitude flight. However, there is still a lack of hot fire ground and flight tests that would serve to validate the theory and simulation with empirical data [15, 18]. Toward the end of the 20th century, the Lockheed Martin X-33, a suborbital spaceplane capable of reaching

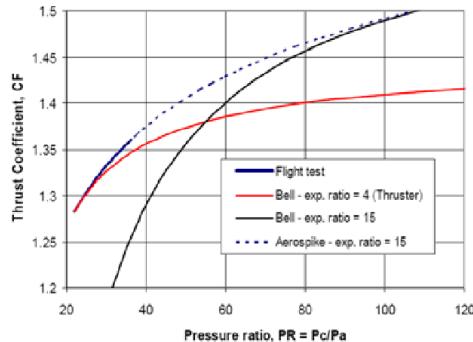
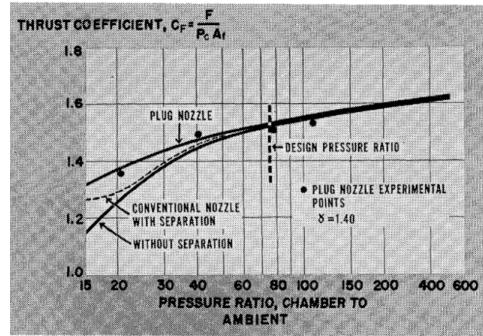
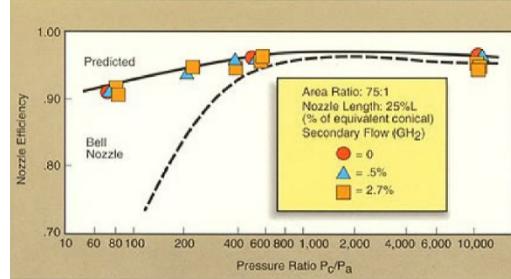


Figure 15. Graphs of nozzle efficiency vs. pressure ratio from several published papers. The bottom graph includes flight test data points which follow simulation data.

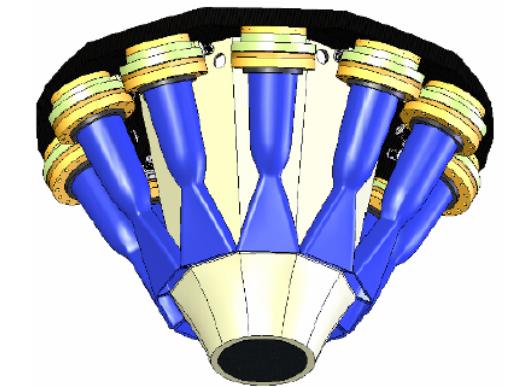


Figure 16. California State University, Long Beach's multi-thruster, aerospike nozzle.

transonic speeds, was planned as the first flight test validation of the aerospike nozzle configuration after many successful engine ground tests. Unfortunately, the program and test flight were canceled due to vehicle issues unrelated to its aerospike nozzle. More recently, in 2009, a university-commercial partnership launched a liquid-propellant rocket sporting a multi-thruster aerospike nozzle configuration (see Figure 16). Unfortunately, this flight failed to test the aerospike's performance under transonic conditions because a hard start^{vii} at launch caused the puncture of two combustion chambers which resulted in the rocket crashing. Dr. Eric Besnard, Project Director for the California Launch Vehicle Education Initiative, the university-commercial partnership mentioned above, provides another reason why the aerospike has not been adopted by industry; "In a business where one failure will ground the vehicle until the failure has been understood and addressed, this can present major operational and business challenges." [19]

^{vii} Start of a liquid rocket engine in which the ignition of an unbalanced fuel and oxidizer mixture causes an uncontrolled combustion and pressure spike that may result in the structural failure of the engine.

IV. Complications with Cooling and Mass

Rocket engine nozzles are cooled by one or more of these methods [20]:

1. Regenerative cooling using fuel or oxidizer
2. Ablative cooling
3. Radiative/convective cooling
4. Heat sink cooling

Regenerative cooling is a widely-used method that pumps fuel or oxidizer around the walls of the nozzle. By adding energy prior to fuel and oxidizer mixture and combustion, the resulting combustion is more energetic and yields an increase in exhaust velocity ranging from 0.1-1.5% [21]; hence the term “regenerative”. While more complex, this cooling method also allows the nozzle walls to be thinner, reducing weight and thermal expansion. Cooling may also be accomplished by using an ablative material to line the entire nozzle or that part of the nozzle that will be subjected to the high heat load (i.e. the throat). The ablative material absorbs heat as it degrades and is expelled out of the nozzle. While this can be a cost-efficient method of cooling, it may change the geometry of the nozzle that it lines which could reduce nozzle efficiency as seen in a NASA Armstrong test flight of the aerospike nozzle configurations. Radiative and convective cooling employs the use of metals or alloys that can rapidly dissipate their heat through radiation and convection with their environment so that their temperature remains sufficiently below their melting point. While this method of cooling simplifies design, it may also introduce large material and manufacturing costs. Heat sink cooling refers to the use of additional features (e.g. fins) along the outside wall of the nozzle to increase the overall heat capacity and heat transfer of the nozzle. This method of cooling may introduce additional manufacturing costs and increase engine mass.

As stated before, the industry needs more hot fire and flight tests before the adoption of the aerospike nozzle. In addition to the empirical validation of fluid flow simulation these tests offer, they also prove that the aerospike can be sufficiently cooled and fully integrated into a launch vehicle. With the aerospike, nearly the entire nozzle is engulfed in hot exhaust gases. This is not the case with CBNs, where the wall of the nozzle is exposed to the surrounding environment. Additionally, aerospikes typically employ relatively small throat areas, which are susceptible to high heat concentrations. While cooling does pose a problem, it appears to be more of a design challenge rather than an unsolvable issue. For example, a student-faculty team at California Polytechnic State University developed a test stand that uses saturated nitrous oxide (N_2O), a widely-used oxidizer in high-powered model rockets and sounding rockets, to cool the throat of a CBN as a proof-of-concept [22]. The following year, the same team developed an N_2O -cooled aerospike nozzle [23]. Their cooling method proved to sufficiently reduce throat temperatures, in computational simulations, enough that the aerospike nozzle’s throat region did not deform significantly and could be reused multiple times even after performing at the highest expected heat loads. Similarly, a thesis study conducted at the Air Force Institute of Technology showed that sufficient cooling for reusability could be achieved with the widely-used liquid hydrogen and liquid oxygen combination. During the X-33 program, the aerospike engines that were ground tested featured fuel-cooling. However, they also used NARloy-Z, a heavy copper alloy with high strength and a high melting point, which serves as an example of how the aerospike engine can quickly become heavy. It should be noted that these engines were serving as a proof-of-concept and their successor would employ the use of a more advanced ceramic matrix composite material, which is considerably lighter [18].

V. Conclusion

While aerospike nozzles have limitations associated with them, their altitude-compensating ability is undeniable. The result is a significant net gain in efficiency, which translates into a significant reduction in the cost per pound to orbit. These savings may even double when applied to powered-descent, reusable launch vehicles such as SpaceX’s Falcon 9R, whose engines operate nearly double the time in higher ambient pressures, such as (see Figure 17). While the

exact value of this price reduction will vary for different launch vehicles, the aerospike nozzle configuration undoubtedly has the potential to expand access to common satellite destinations and beyond.

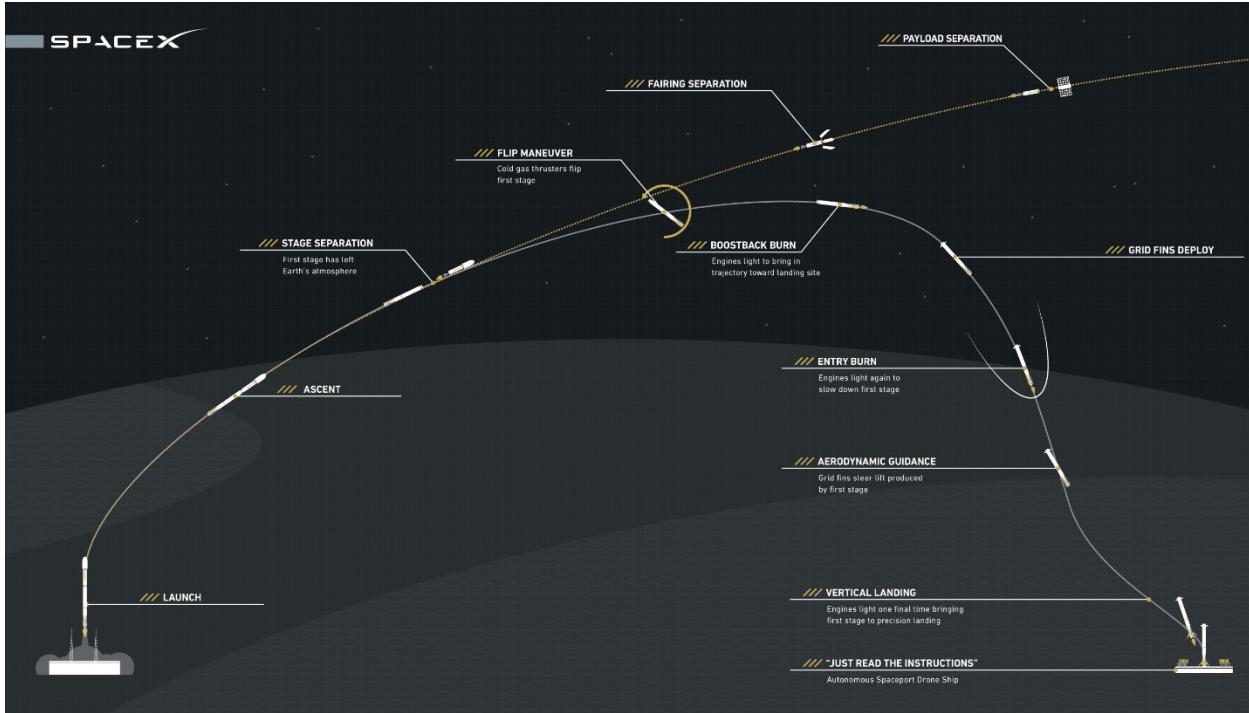


Figure 16. SpaceX's Falcon 9R flight path. If an aerospike nozzle configuration was applied to a reusable launch vehicle, recovered via powered descent and landing, cost savings could nearly double.

Figure References

Figure 2

- Bremer, M., "Combustion Chamber and Nozzle Design," *Ad Astra Per Aspera* blog, Nov. 2014, <https://mrbremer.wordpress.com/2014/11/> [retrieved 25 Nov. 2016]

Figure 3

- See [9]

Figure 4

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Figure 5

- Ibid.

Figure 6

- See [7]

Figure 7

- (TOP) STS-131 launch, NASA/Kenny Allen, https://www.nasa.gov/mission_pages/shuttle/shuttlemissions/sts131/multimedia/photogallery/100405-15.html [retrieved 27 Nov. 2016]
- (BOTTOM) Martinez, I., "Nozzles," *Polytechnic University of Madrid*, 2016, <http://webserver.dmt.upm.es/~isidoro/bk3/c17/Nozzles.pdf> [retrieved 20 Oct. 2016]

Figure 8

- (LEFT) Apollo 11 launch, NASA/Kennedy Space Center, <https://www.youtube.com/watch?v=DKtVpvzUF1Y> [retrieved 23 Oct. 2016]
- (RIGHT) Apollo 11 launch, NASA/Kennedy Space Center, <https://i.stack.imgur.com/Pr8Td.jpg> [retrieved 20 Oct. 2016]

Figure 9

- (TOP LEFT) see [11]
- (TOP CENTER) see [7]
- (TOP RIGHT) "Activities for Advancement of Functionality and High Performance," Japan Aerospace Exploration Agency, <http://www.kspc.jaxa.jp/english/research/functional.html> [retrieved 13 Dec. 2016]
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- (BOTTOM CENTER) RL-10 variant, http://www.b14643.de/Spacerockets/Specials/P&W_RL10_engine/index.htm [retrieved 4 Dec. 2016]

Figure 10

- See [7]

Figure 11

- See [11]

Figure 12

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Figure 13

- See [11]

Figure 14

- See [11]

Figure 15

- (TOP LEFT) see [7]
- (TOP RIGHT) Rao, G.V.R., "Recent Developments in Rocket Nozzle Configurations," *American Rocket Society Journal*. Vol. 31, No. 11, pp. 1488-1494, Nov. 1961
doi: 10.2514/8.5837
- (BOTTOM) see [15]

Figure 16

- “Falcon 9 returns to Vandenberg Launch Pad for Jason-3 Launch on Sunday,” *Spaceflight101*, 2016, <http://spaceflight101.com/jason-3/falcon-9-returns-to-vandenberg-launch-pad-for-jason-3-launch-on-sunday/> [retrieved 2 Dec. 2016]

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