MIN-557: Finite Element Methods
Assignment 3

(Due Date: Saturday, 31st October 2020)

Note: Both groups will solve all the problems.

1. Starting from energy develop discrete equations for axially loaded elastic bar.

2. Write a computer program for 1D elastic bar. Your program
   (a) should be capable of taking total number of elements from the user
   (b) should be capable of taking type of element (either linear or quadratic) from the user
   (c) should accommodate geometric and material parameters (i.e., length, area, Young's modulus) of the bar from the user
   (d) should accommodate loading and boundary condition at desired points from the user
   (e) should accommodate proper element assembly
   (f) should be able to compute displacement, strain, and stress
   (g) should be able to interpolate at desired point within an element
   (h) should be self explanatory (i.e., any other person from similar background should be able to easily understand your code and should be able to run it seamlessly for problem of his/her interest). To facilitate above add proper comments throughout the code.

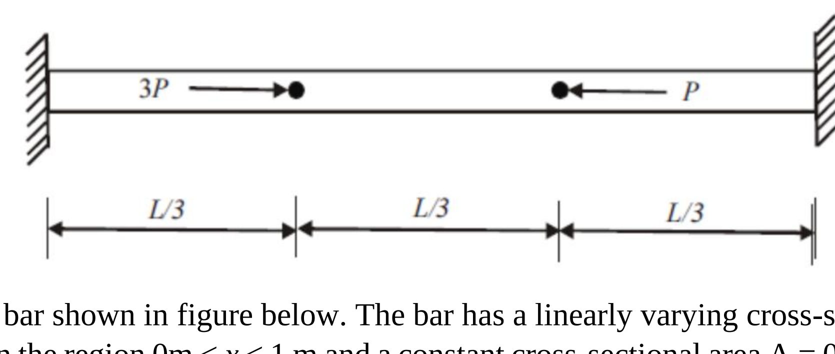
   Please attach a source code (e.g., *.m file) along with a pdf copy of a code during the submission. Also attach concise but complete documentation (in pdf format) of your code so that anyone who wish to use your code should benefit from this documentation.

3. Test your code for linear and quadratically varying body force (i.e., $b(x) = ax \; and \; b(x) = ax^2$. Assume suitable values of A, E, L and a. Use following boundary conditions:

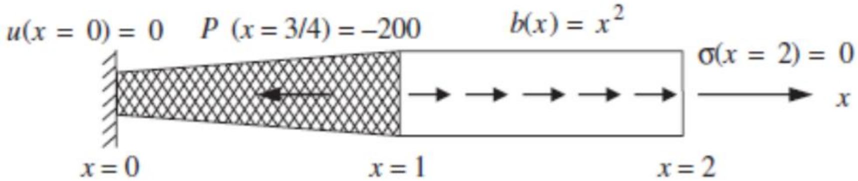
$$u(0) = 0; \; \frac{du}{dx}(l) = 0$$

   Study the sensitivity of results to number of elements and order of element. Compare your results against exact solution. Comment on the obtained results. Also, check for convergence and comment on the rate of convergence.

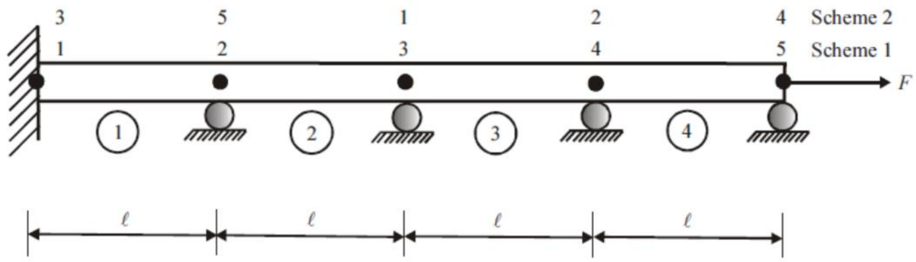
4. Using code developed in # 1 compute the displacements along the length of the bar and reaction forces. Compare the results against exact solution.



5. Consider a bar shown in figure below. The bar has a linearly varying cross-sectional area A= (x+1) m^2 in the region 0m < x < 1 m and a constant cross-sectional area A = 0.2 m^2 in the region 1m < x < 2 m. The Young's modulus is E = 50 MPa. The bar is subjected to the point load P = 200 N at x = 0.75 m and a quadratically varying distributed loading b = x^2 Nm^{-1} in the region 1m < x < 2 m. The bar is constrained at x = 0 m and is traction free at x = 2 m. Find the stresses along the length of the bar using linear and quadric element. Compare the results against exact solution. Study mesh convergence.



6. For the two meshing schemes shown below, obtain an assembled stiffness matrix. Discuss the effect of node numbering on assembled matrix. Assume c/s area is A and Young's modulus is E.

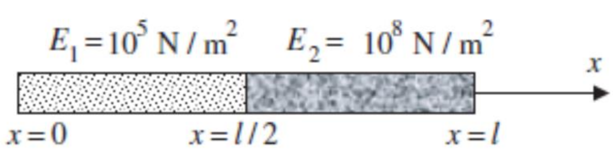


7. For the shape (interpolation) functions pertaining to linear and quadratic bar elements, verify and comment on the implications of the following property:

$$\sum_{e=1}^{n_{el}} N_i = 1$$

Additional practice/testing problems (no need to turn in):

8. Displacement and stress field for the following problem:



Compare the solution against direct stiffness method.

9. Modify code in problem # 2 to accommodate following stress-strain law

$$\sigma(x) = E(x)[\varepsilon(x) - \alpha(x)T(x)]$$

10. Discuss the continuity and completeness of 1D FEM