Introduction to Supersymmetry

Preliminaries to my Master's Thesis

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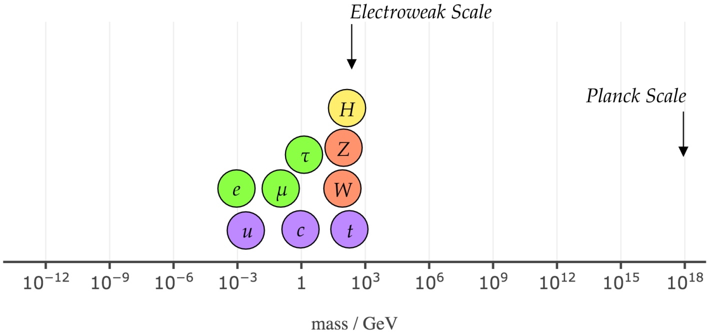
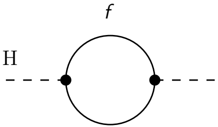
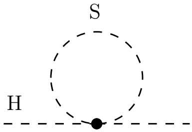

Electroweak Scale
Planck Scale
H
Z
τ
e
μ
W
u
c
t
10^−12
10^−9
10^−6
10^−3
1
10^3
10^6
10^9
10^12
10^15
10^18
mass / GeV

Fine-tuning problem



$$\Delta m_{\mathrm{H}}^2 = -\frac{|\lambda_f|^2}{8\pi^2}\Lambda_{\mathrm{U}V}^2 + \ldots$$



$$\Delta m_{\mathrm{H}}^2 = \frac{\lambda_s}{16\pi^2}\Lambda_{\mathrm{U}V}^2 + \ldots$$

Extension of Poincaré algebra

Goal: Relate bosons and fermions

- Spin is spacetime property; not isolated from Poincaré algebra

$\Rightarrow$ Have to extend Poincaré algebra!

Extension of Poincaré algebra

Coleman-Mandula-Theorem: Full symmetry algebra of S-matrix for consistent four-dimensional QFT (satisfying locality, finiteness of particles, etc.) is

$$\begin{aligned}
[P_\mu, P_\nu] &= 0 \\
[M_{\mu\nu}, P_\lambda] &= i(\eta_{\nu\lambda} P_\mu - \eta_{\mu\lambda} P_\nu) \\
[M_{\mu\nu}, M_{\rho\sigma}] &= -i(\eta_{\mu\rho} M_{\nu\rho} - \eta_{\mu\sigma} M_{\nu\rho} - \eta_{\nu\rho} M_{\mu\sigma} + \eta_{\nu\sigma} M_{\mu\rho})
\end{aligned}$$

combined **trivially** with an internal symmetry algebra.

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\end{aligned}
$$

combined **trivially** with an internal symmetry algebra.
We may not just extend the Poincaré part! But ...

Extension of Poincaré algebra

Haag-Łopuszański-Sohnius-Theorem: If one extends Lie-algebra to a *graded* algebra, the full symmetry may be the internal symmetries and the Poincaré-algebra **extended** by

$$\{Q_a, Q^\dagger_{\dot{a}}\} = 2\sigma^\mu_{a\dot{a}} P_\mu$$
$$[Q_a, P_\mu] = 0$$
$$[Q_a, M_{\mu\nu}] = (\sigma_{\mu\nu})_a^b Q_b$$

where Q_a are anticommuting generators.

Extension of Poincaré algebra

Super-Poincaré-Algebra: or SUSY-Algebra for short:

$$
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[Q_a, P_\mu] &= 0 \\
[Q_a, M_{\mu\nu}] &= (\sigma_{\mu\nu})_a^b Q_b
\end{aligned}
$$

combined trivially with an internal symmetry algebra.[1]

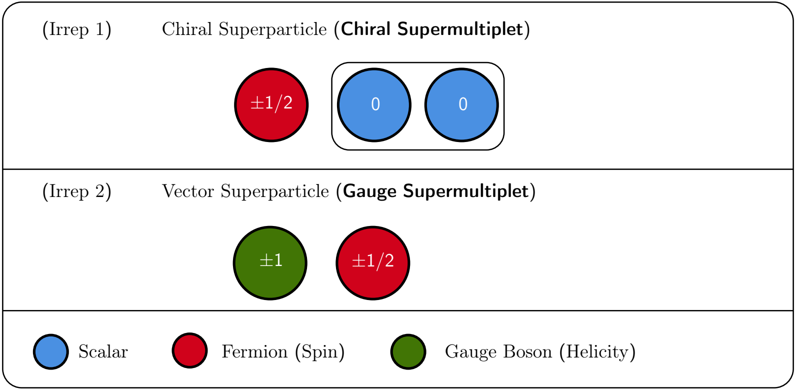
[1]Now: possibility of non-trivially added internal symmetry, c.f. *R-symmetry*

Irreps of the SUSY algebra

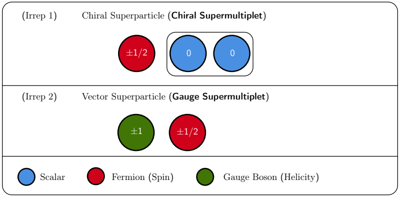
- Irreps of Poincaré algebra: notion of particle of given m and S
- SUSY algebra changes notion of particle!
- Luckily: Poincaré $\subset$ SUSY

$\Rightarrow$ Irreps of SUSY algebra will be (reducible) reps of Poincaré algebra

Irreps of the SUSY algebra



Irreps of the SUSY algebra



Observations:

- Particles within supermultiplet have same m and internal symmetry structure but different S
- Fermionic and bosonic dof match within multiplet

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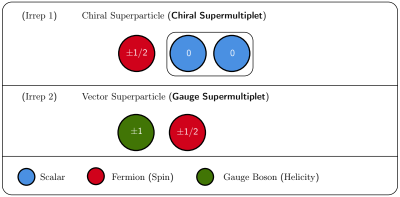

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- Fermions have **2** dof (use Weyl Fermions as basic objects)

Most general Lagrangian

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$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} + i\lambda^{a\dagger}\overline{\sigma}^\mu(D_\mu\lambda)_a + (D_\mu\phi)^\dagger(D^\mu\phi) + i\chi^\dagger\overline{\sigma}^\mu(D_\mu\chi)$$

Most general Lagrangian

$$\mathcal{L} = \underbrace{\boxed{-\frac{1}{4}F^a_{\mu\nu}F_a^{\mu\nu} + i\lambda^{a\dagger}\overline{\sigma}^\mu(D_\mu\lambda)_a}}_{\text{Gauge Multiplet}} + \underbrace{\boxed{(D_\mu\phi)^\dagger(D^\mu\phi) + i\chi^\dagger\overline{\sigma}^\mu(D_\mu\chi)}}_{\text{Chiral Multiplet}}$$

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$$- \tfrac{1}{2}g(\phi^{\dagger}T^{a}\phi)^{2} - \left|\frac{\partial W(\phi)}{\partial\phi}\right|^{2} - \tfrac{1}{2}\left[\left(\frac{\partial^{2}W(\phi)}{\partial\phi\partial\phi}\right)\chi\chi + \mathrm{h.c.}\right] - \sqrt{2}\,g\left[\left(\phi^{\dagger}T^{a}\chi\right)\lambda_{a} + \lambda^{a\dagger}\left(\chi^{\dagger}T_{a}\phi\right)\right]$$

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$$-\underbrace{\boxed{\frac{1}{2}g(\phi^\dagger T^a\phi)^2 - \left|\frac{\partial W(\phi)}{\partial\phi}\right|^2}}_{\text{Scalar Potential}} - \frac{1}{2}\left[\left(\frac{\partial^2 W(\phi)}{\partial\phi\partial\phi}\right)\chi\chi + \text{h.c.}\right] - \sqrt{2}\,g\left[\left(\phi^\dagger T^a\chi\right)\lambda_a + \lambda^{a\dagger}\left(\chi^\dagger T_a\phi\right)\right]$$

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- Multiple gauge and chiral multiplets possible (and mixing)
- $W(\phi) = M^{ij}\phi_i\phi_j + y^{ijk}\phi_i\phi_j\phi_k$ (**no** field adjoints!)
- Independent couplings: $M_{ij},\ y_{ijk},\ g$

What's left to do?

- General Lagrangian restrictive due to SUSY, Renormalizability and Gauge Invariance

$\Rightarrow$ May only choose:
  - Particle content (participating multiplets)
  - Gauge groups (g)
  - Potential W (M_{ij} and y_{ijk})

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Choose them to specify specific model
$\Rightarrow$ **Minimal Supersymmetric Standard Model**

The MSSM: Particle content

	Spin 0	Spin 1/2	Spin 1	Naming convention
Chiral Multiplets				
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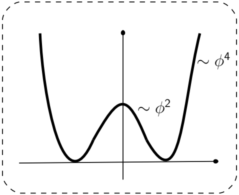
Gauge Multiplets		$\tilde{W}$ $\tilde{Z}$ $\tilde{g}$ $\tilde{\gamma}$	W Z g γ	W $\rightarrow$ wino gluon $\rightarrow$ gluino photon $\rightarrow$ photino

The MSSM: Gauge structure

	Spin 0	Spin 1/2	Spin 1	Gauge Structure
Chiral Multiplets	$\tilde{e}$ $\tilde{u}$ $\tilde{\mu}$ $\tilde{c}$ $\tilde{\tau}$ $\tilde{t}$ H_u H_d	e u μ c τ t $\tilde{H}_u$ $\tilde{H}_d$		**fund. rep.** keep charges from SM and extend to superpartners
Gauge Multiplets		$\tilde{W}$ $\tilde{Z}$ $\tilde{g}$ $\tilde{\gamma}$	W Z g γ	**adj. rep.**

The MSSM: Potential



Standard Model: (right handed, left handed)

$$V(\phi^\dagger\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$$\mathcal{L}_{\mathrm{Yukawa}} = -\bar{u}\mathbf{y}^u Q\phi^{\mathrm{c.c.}} - \bar{d}\mathbf{y}^d Q\phi - \bar{e}\mathbf{y}^e L\phi + \mathrm{h.c.}$$

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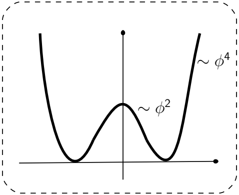

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MSSM:[1]

$$W_{\text{MSSM}} = \mu' H_u H_d + \tilde{\bar{u}}\mathbf{y}^u \tilde{Q} H_u - \tilde{\bar{d}}\mathbf{y}^d \tilde{Q} H_d - \tilde{\bar{e}}\mathbf{y}^e \tilde{L} H_d$$

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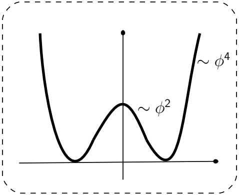

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This is why we need two Higgs in the MSSM!

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- ▸ Soft SUSY breaking terms (couplings with pos. mass dim.)
- × This introduces 105 new parameters

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Extend
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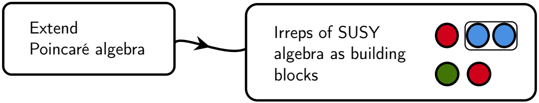
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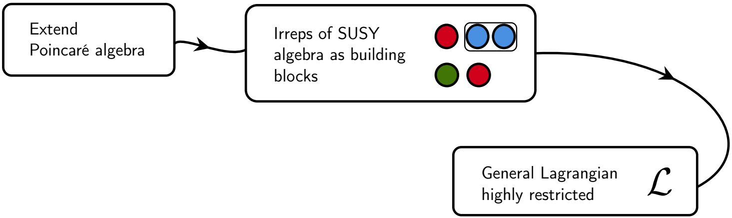
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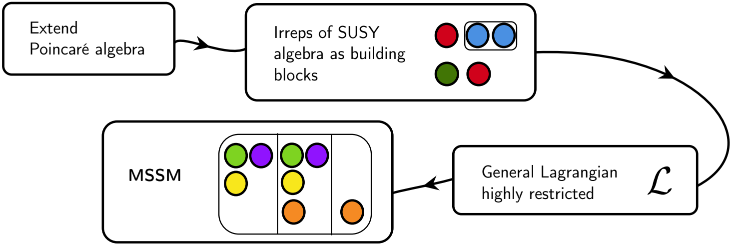
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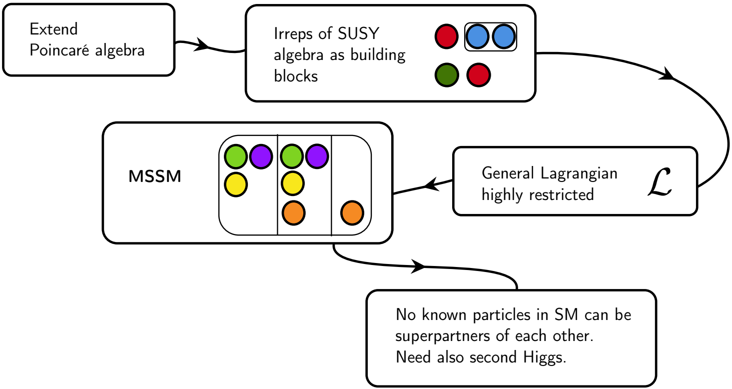
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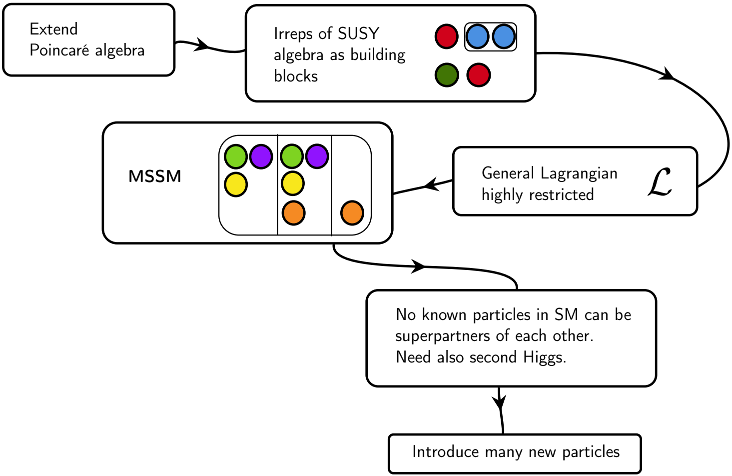
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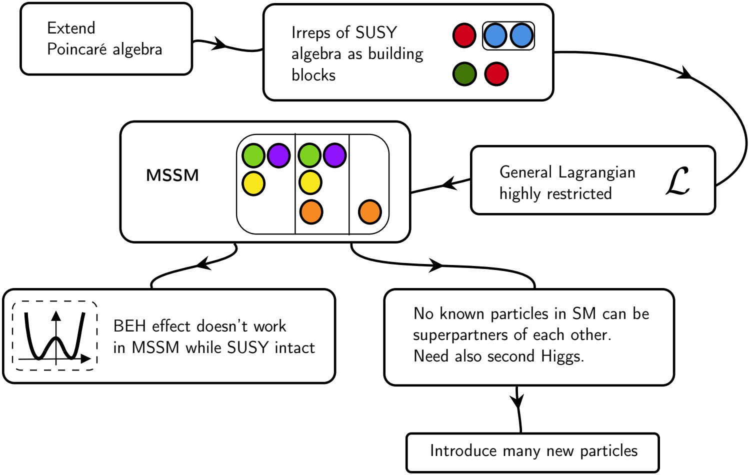
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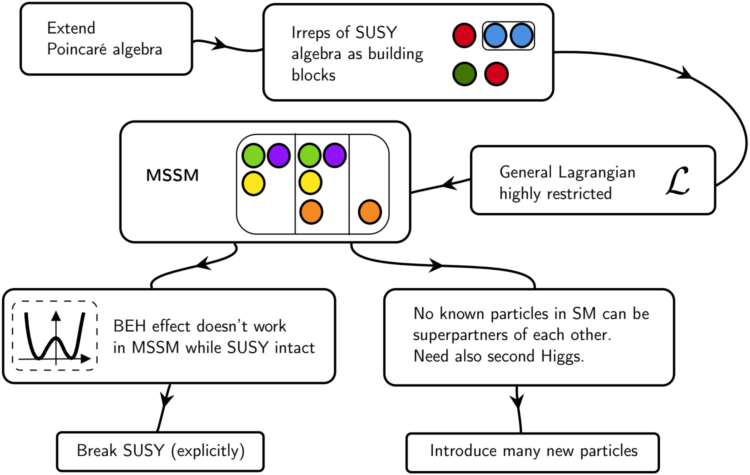
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Outlook

Eventually:

- Apply the FMS-formalism to MSSM-like theories
- Study gauge invariant composite states
- Special interest: Lightest supersymmetric particle (LSP) as DM candidate

Outlook

For now: Consider minimal toy model where BEH works

$$\begin{aligned}\mathcal{L} = &-\frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} + i w^{a\dagger} \bar{\sigma}^\mu (D_\mu w)_a \\ &+ (D_\mu \phi)^\dagger (D^\mu \phi) + i \chi^\dagger \bar{\sigma}^\mu (D_\mu \chi) \\ &- \frac{g}{\sqrt{2}} \left[(\phi^\dagger \tau^a \chi) w_a + w^{a\dagger} (\chi^\dagger \tau_a \phi) \right] \\ &- \left(-\mu^2 \phi^\dagger \phi + \frac{g}{8} (\phi^\dagger \phi)^2 \right)\end{aligned}$$

References

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Stephen P. Martin. "A Supersymmetry primer". In: *Adv. Ser. Direct. High Energy Phys.* 18 (1998). Ed. by Gordon L. Kane, pp. 1–98. DOI: 10.1142/9789812839657_0001. arXiv: hep-ph/9709356.

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