The manifestly gauge-invariant spectrum of the Minimal Supersymmetric Standard Model

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Supersymmetry – Basics

[Martin, Adv.Ser.Dir.HEP 18 (1998); Aitchison (2007)]

Motivations

- supergravity
- dark matter
- bosons $\leftrightarrow$ fermions
- largest spacetime symmetry
- explain small Higgs mass
- gauge coupling unification

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How to

Extend Poincaré algebra by

[Müller-Kirsten and Wiedemann (2010)]

$$\{Q_a, Q_b\} = \{Q^\dagger_{\dot a}, Q^\dagger_{\dot b}\} = 0$$
$$\{Q_a, Q^\dagger_{\dot a}\} = 2(\sigma^\mu)_{a\dot a} P_\mu$$
$$[Q_a, P_\mu] = [Q^\dagger_{\dot a}, P_\mu] = 0$$
$$[Q_a, M^{\mu\nu}] = i(\sigma^{\mu\nu})_a{}^b Q_b$$

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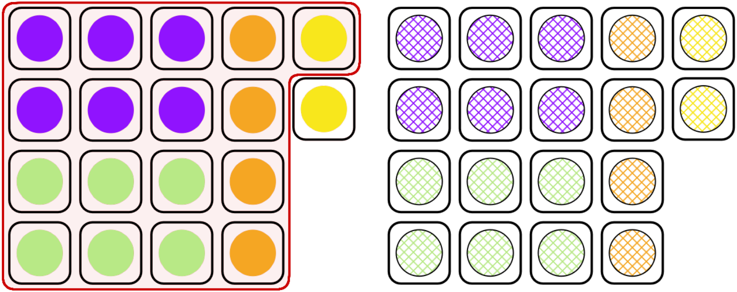
Reality

- necessarily broken
- no new particles found

Supersymmetry – the MSSM

[Martin, Adv.Ser.Dir.HEP 18 (1998); Aitchison (2007)]



- naming convention: *sfermions*, *gauginos*, *higgsinos*
- SUSY breaking parametrization; $>$ 100 parameters
- altered Higgs sector
- gauge theory like SM

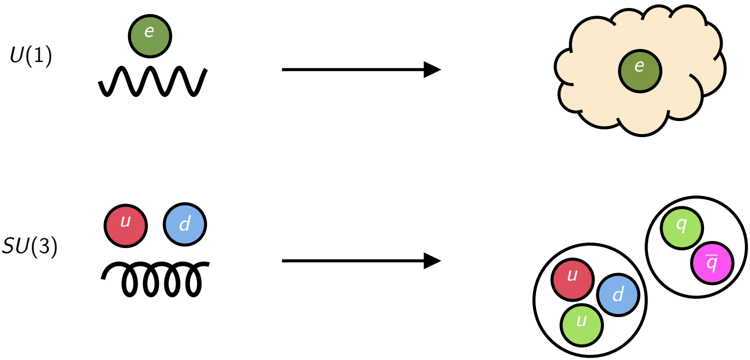
The MSSM – A Gauge Theory

- Well established construction principle for fundamental theories in particle physics
- Elegant mathematical framework
- Introduction of redundant degrees of freedom

What are the **physical** degrees of freedom?

Gauge-invariant states!

The MSSM (SM) – Ensuring Gauge-Invariance



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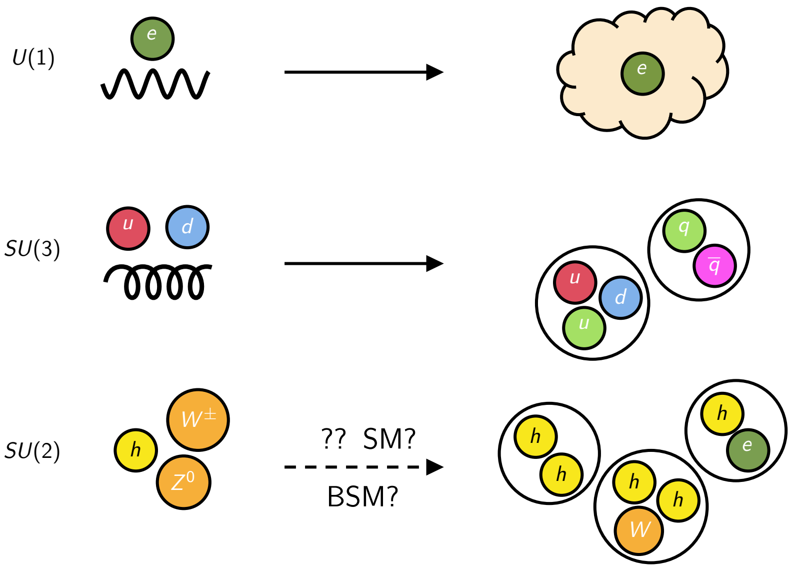
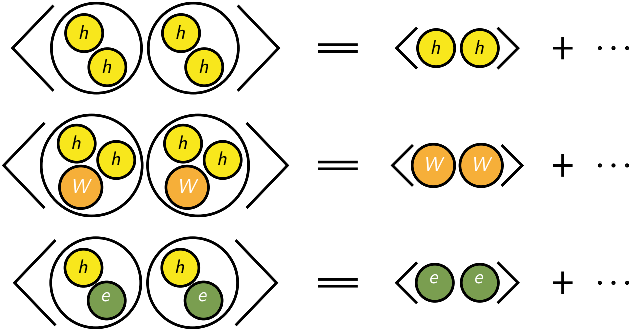


The FMS Mechanism

[Fröhlich, Morchio, and Strocchi, Phys.Lett.B 97 (1980)
Fröhlich, Morchio, and Strocchi, Nucl. Phys. B 190 (1981)]

- In theories with Brout-Englert-Higgs effect
- Expand Higgs field; $h(x) = v + \varphi(x)$



- Propagation of elementary fields $\leftrightarrow$ propagation of gauge-invariant operators with identical quantum numbers

The FMS Mechanism – SM vs. BSM

Standard Model

Duality between spectra; no qualitative differences

Review: [Maas, Prog. Part. Nucl. Phys. 106 (2019)]

Beyond Standard Model

Countless GUT scenarios with qualitative mismatches

[Maas and Török, Phys. Rev. D 95 (2017)
Maas, Sondenheimer, and Török, Annals Phys. 402 (2019)]

Conclusion

Especially for BSM theories one should investigate whether the spectrum predicted by conventional treatment is physical!

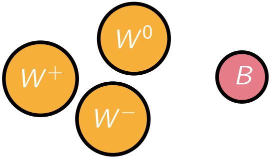
Gauge-invariant spectrum of the MSSM

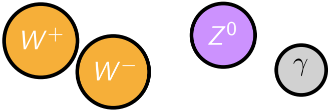
Proceeds in three steps

1. Find elementary mass eigenstates
2. Build gauge-invariant bound states
3. Apply FMS mechanism

The MSSM – Custodial Symmetry

SM: $h \overset{SU(2)}{\longleftrightarrow} i\sigma^2 h^{\dagger T}$





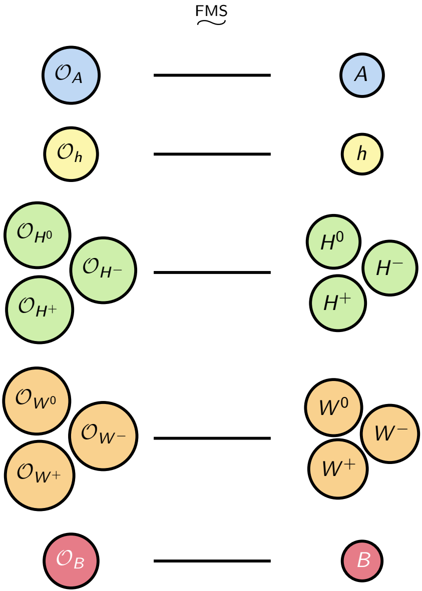
MSSM: $h_u \overset{SU(2)}{\longleftrightarrow} h_d$

Same effect

Violated by g', Yukawas, soft breaking and $v_u \neq v_d$

Needless to say: working with symmetry intact ⇒ ☺

The MSSM – Weak-Higgs Spectrum and Mixing



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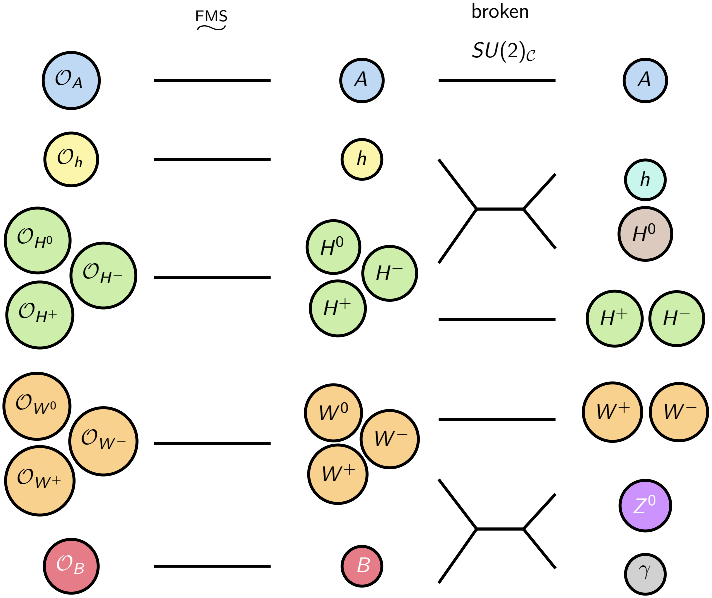

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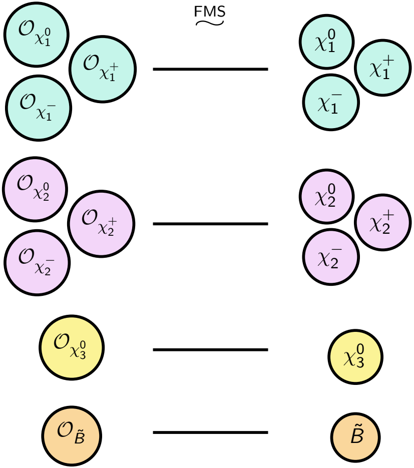

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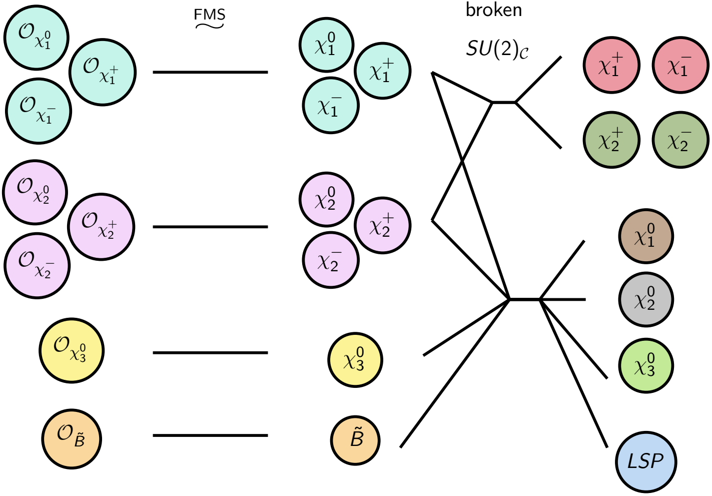


Additional Results and Conclusion

Additional Results

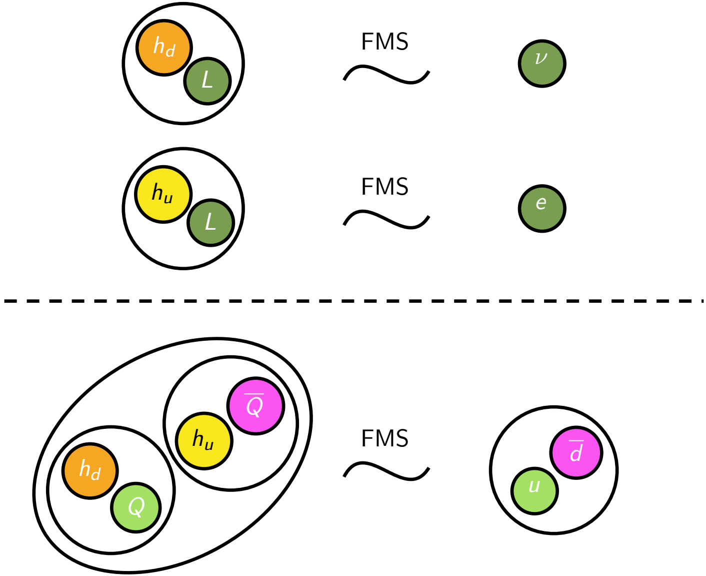
- Extends to leptons and quarks
- Extends to superpartners

Conclusion

- Duality between elementary and bound state spectrum
- Experiments are searching the correct channels
- Sub-leading FMS contributions could nevertheless be relevant

Bonus Content

The MSSM – Leptons and Quarks



Name	Operator(s)	$SU(2)_{\mathcal{C}}$	$\underset{\sim}{\text{FMS}}$
light Higgs	$\mathrm{tr}\, H^\dagger H$ $\mathrm{tr}\, H^\dagger H \sigma^3$	$\mathbf{1}$	h
pseudoscalar Higgs	$\mathrm{Im} \det H$	$\mathbf{1}$	A
heavy Higgs	$\mathrm{tr}\, H^\dagger H \sigma^3$ $\mathrm{tr}\, H^\dagger H$	$\mathbf{3}$, 0	H^0
charged Higgs	$\varphi\ \ \mathrm{tr}\, H^\dagger H \sigma^\mp$	$\mathbf{3}$, $\pm$	$H^\pm$
Z boson	$\mathrm{tr}\, H^\dagger D_\mu H \sigma^3$ DB_μ	$\mathbf{3}$, 0	Z_μ
charged W	$\varphi\ \ \mathrm{tr}\, H^\dagger D_\mu H \sigma^\pm$	$\mathbf{3}$, $\pm$	$W_\mu^\pm$
photon	DB_μ $\mathrm{tr}\, H^\dagger D_\mu H \sigma^3$	$\mathbf{1}$	Γ_μ

neutralino (LSP)	$\mathrm{tr}\, H^\dagger \widetilde{H}$ $\mathrm{tr}\, H^\dagger \widetilde{H} \sigma^3$, $\mathrm{tr}\big(H^\dagger \sigma^a H \sigma^3\big)\, \widetilde{W}_a$ $\widetilde{B}$	**1**	$\widetilde{\chi}^0_3$
other neutralinos	$\mathrm{tr}\, H^\dagger \widetilde{H} \sigma^3$, $\mathrm{tr}\big(H^\dagger \sigma^a H \sigma^3\big)\, \widetilde{W}_a$ $\mathrm{tr}\, H^\dagger \widetilde{H}$ $\widetilde{B}$	**3**, 0	$\widetilde{\chi}^0_{1,2}$
charginos	$\varphi\ \mathrm{tr}\, H^\dagger \widetilde{H} \sigma^\mp$, $\varphi\ \mathrm{tr}\big(H^\dagger \sigma^a H \sigma^\mp\big)\, \widetilde{W}_a$	**3**, $\pm$	$\widetilde{\chi}^\pm_{1,2}$
l.h. leptons	$\varphi(H^\dagger L)_1$, $(H^\dagger L)_2$	**2**	e, ν
‘l.h.’ sleptons	$\varphi(H^\dagger \widetilde{L})_1$, $(H^\dagger \widetilde{L})_2$	**2**	$\widetilde{e}$, $\widetilde{\nu}$
r.h. electron	$\varphi \bar{e}$	**1**	$\bar{e}$
‘r.h.’ selectron	$\varphi \widetilde{\bar{e}}$	**1**	$\widetilde{\bar{e}}$

Mixing Example

$$
\begin{aligned}
\mathcal{L} \supset \frac{1}{2} \begin{pmatrix} h & H^0 \end{pmatrix} \begin{pmatrix} a_1/2 & b_1/2 \\ b_1/2 & c_1/2 \end{pmatrix} \begin{pmatrix} h \\ H^0 \end{pmatrix} \\
+ \frac{1}{2} \begin{pmatrix} A & G^0 \end{pmatrix} \begin{pmatrix} a_2/2 & b_2/2 \\ b_2/2 & c_2/2 \end{pmatrix} \begin{pmatrix} A \\ G^0 \end{pmatrix} \\
+ \begin{pmatrix} H^+ & G^+ \end{pmatrix} \begin{pmatrix} a_3/2 & -ib_3/2 \\ ib_3/2 & c_3/2 \end{pmatrix} \begin{pmatrix} H^- \\ G^- \end{pmatrix}
\end{aligned}
$$

Electric Charge

$$H \to H' = H \ \exp\left(i\alpha\frac{\sigma^3}{2}\right), \quad L \to L' = e^{-i\alpha/2}L, \quad \bar{e} \to \bar{e}' = e^{2\alpha/2}\bar{e}$$

$$\mathrm{tr}\left(H^\dagger H\sigma^A\right) \to \mathrm{tr}\left(H^\dagger H e^{i\alpha\sigma^3/2}\sigma^A e^{-i\alpha\sigma^3/2}\right) = R^{AB} \ \mathrm{tr}\left(H^\dagger H\sigma^B\right)$$

$$\begin{pmatrix} \mathcal{O}_{H^+} \\ \mathcal{O}_{H^-} \\ \mathcal{O}_{H^0} \end{pmatrix} = \begin{pmatrix} \mathrm{tr}\, H^\dagger H(\sigma^2 - i\sigma^1) \\ \mathrm{tr}\, H^\dagger H(\sigma^2 + i\sigma^1) \\ \mathrm{tr}\, H^\dagger H\sigma^3 \end{pmatrix} \to \begin{pmatrix} e^{i\alpha} & & \\ & e^{-i\alpha} & \\ & & 1 \end{pmatrix} \begin{pmatrix} \mathcal{O}_{H^+} \\ \mathcal{O}_{H^-} \\ \mathcal{O}_{H^0} \end{pmatrix}$$

Higgs bidoublet

$$H \equiv (H_u, -H_d) = \begin{pmatrix} H_u^{(1)} & -H_d^{(1)} \\ H_u^{(2)} & -H_d^{(2)} \end{pmatrix}$$

$$H \to H' = L(x) H R^\dagger \quad L(x) \in SU(2)_L,\ R \in SU(2)_{\mathcal{C}}$$

Higgs potential

$$V(H) = \mathrm{tr}\, H^\dagger H M - 2m_{ud}^2\ \mathrm{Re}\det H^\dagger - \frac{g^2}{2}\det H^\dagger H + \frac{g^2}{8}(\mathrm{tr}\, H^\dagger H)^2 + \frac{g'^2}{8}(\mathrm{tr}\, H^\dagger H \sigma^3)^2$$

Higgs vev

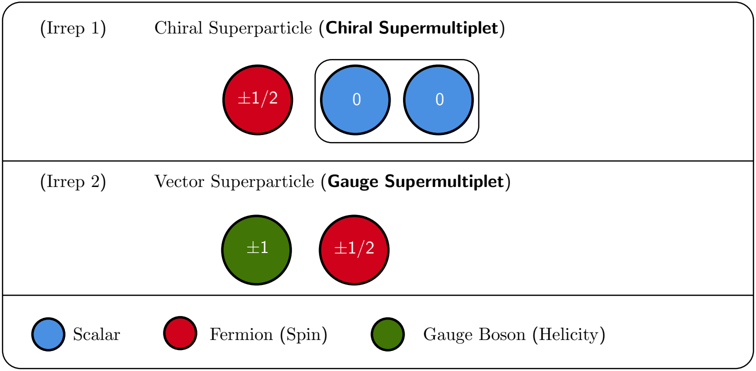
$$H = \begin{pmatrix} 0 & -v_d \\ v_u & 0 \end{pmatrix} + \eta = \frac{v_u + v_d}{2}(-i\sigma^2) + \frac{v_u - v_d}{2}\sigma^1 + \eta$$

MSSM Superpotential

$$W_{\mathrm{MSSM}} = \widetilde{\bar{u}}\boldsymbol{y}_u\widetilde{Q}\cdot H_u - \widetilde{\bar{d}}\boldsymbol{y}_d\widetilde{Q}\cdot H_d - \widetilde{\bar{e}}\boldsymbol{y}_e\widetilde{L}\cdot H_d + \mu H_u\cdot H_d$$

MSSM soft breaking terms

$$\begin{aligned}\mathcal{L}^{\mathrm{soft}}_{\mathrm{MSSM}} = &-\frac{1}{2}\left[M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B} + h.c.\right]\\ &-\left[\widetilde{\bar{u}}\boldsymbol{a}_u\widetilde{Q}\cdot H_u - \widetilde{\bar{d}}\boldsymbol{a}_d\widetilde{Q}\cdot H_d - \widetilde{\bar{e}}\boldsymbol{a}_e\widetilde{L}\cdot H_d + h.c.\right]\\ &-\widetilde{Q}^\dagger\boldsymbol{m}^2_Q\widetilde{Q} - \widetilde{L}^\dagger\boldsymbol{m}^2_L\widetilde{L} - \widetilde{\bar{u}}\boldsymbol{m}^2_{\bar{u}}\widetilde{\bar{u}}^\dagger - \widetilde{\bar{d}}\boldsymbol{m}^2_{\bar{d}}\widetilde{\bar{d}}^\dagger - \widetilde{\bar{e}}\boldsymbol{m}^2_{\bar{e}}\widetilde{\bar{e}}^\dagger\\ &- m^2_u H^\dagger_u H_u - m^2_d H^\dagger_d H_d - \left[m^2_{ud}H_u\cdot H_d + h.c.\right]\end{aligned}$$



$$\mathcal{L} = \overbrace{\boxed{-\tfrac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a \; + \; i\lambda^{a\dagger}\overline{\sigma}^\mu(D_\mu\lambda)_a}}^{\text{Gauge Multiplet}} + \overbrace{\boxed{(D_\mu\phi)^\dagger\,(D^\mu\phi) + i\chi^\dagger\overline{\sigma}^\mu(D_\mu\chi)}}^{\text{Chiral Multiplet}}$$

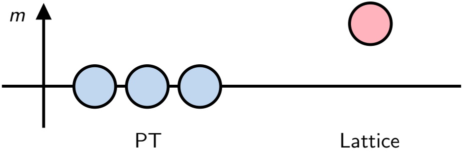
$$- \underbrace{\boxed{\tfrac{1}{2}g^2(\phi^\dagger T^a\phi)^2 - \left|\frac{\partial W(\phi)}{\partial\phi}\right|^2}}_{\text{Scalar Potential}} - \underbrace{\tfrac{1}{2}\left[\left(\frac{\partial^2 W(\phi)}{\partial\phi\partial\phi}\right)\chi\chi + \text{h.c.}\right]}_{\text{Yukawa Couplings}} + \underbrace{\sqrt{2}\,g\left[\left(\phi^\dagger T^a\chi\right)\lambda_a \; + \lambda^{a\dagger}\left(\chi^\dagger T_a\phi\right)\right]}_{\text{Gaugino Couplings}}$$

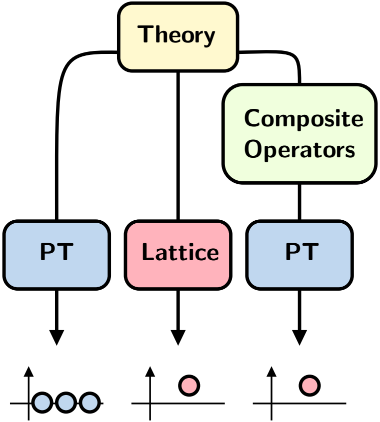
Beyond Perturbation Theory

- Non-perturbative methods for large coupling, bound states
- Examples: Lattice Field Theory, Functional Methods

Subtleties and Problems

- Fixing gauge uniquely in general not possible (locally)
  [Gribov, Nucl. Phys. B 139 (1978); Singer, Commun. Math. Phys. 60 (1978)]
- Perturbative BRST construction fails in general
  [Neuberger, Phys. Lett. B 183 (1987)]
- Non-perturbative **physical** states might differ from PT
  [Maas and Törek, Phys. Rev. D 95 (2017)]




Theory
Composite Operators
PT
Lattice
PT