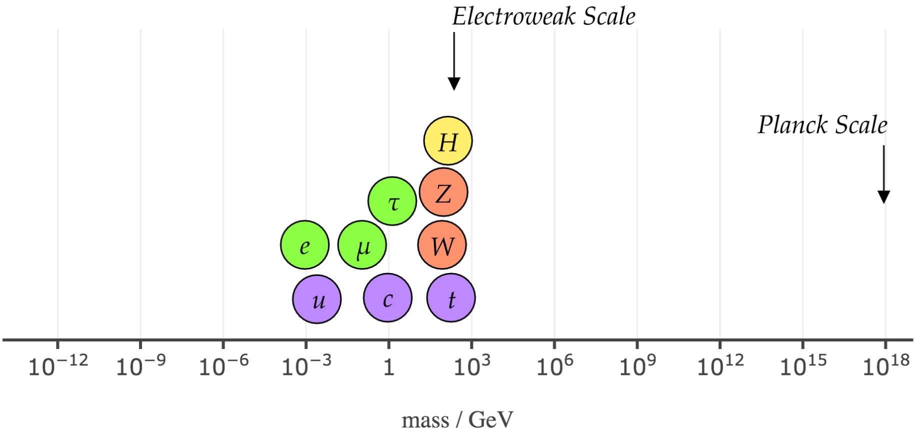
# The Fröhlich-Morchio-Strocchi mechanism in the minimal supersymmetric standard model

Philipp Schreiner

June 14, 2022

# Supersymmetry and the MSSM

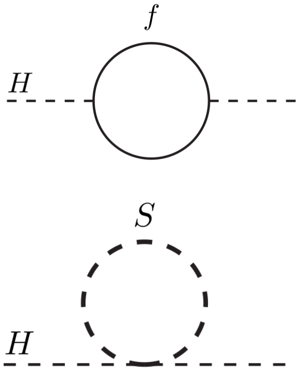
# Motivation


Electroweak Scale
Planck Scale
H
Z
τ
W
e
μ
u
c
t
10^−12 10^−9 10^−6 10^−3 1 10^3 10^6 10^9 10^12 10^15 10^18
mass / GeV

# Motivation

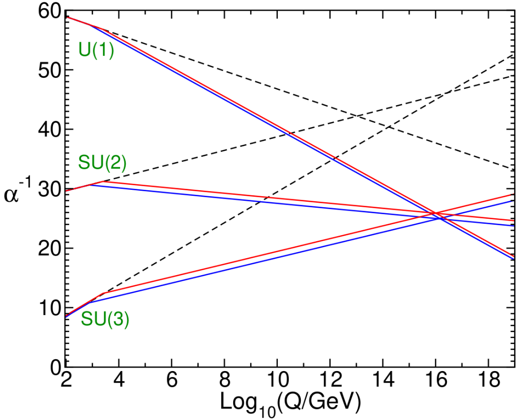
**Cancellation of mass corrections**

[Martin, Adv.Ser.Dir.HEP 18 (1998)]




**Gauge Coupling Unification**

[Martin, Adv.Ser.Dir.HEP 18 (1998)]



# Extension of Poincaré Algebra

[Müller-Kirsten and Wiedemann (2010)]

**Idea: Relate bosons and fermions**

- Spin is spacetime property; not isolated from Poincaré algebra
- **Coleman-Mandula:** Poincaré symmetry is largest possible spacetime symmetry

# Extension of Poincaré Algebra

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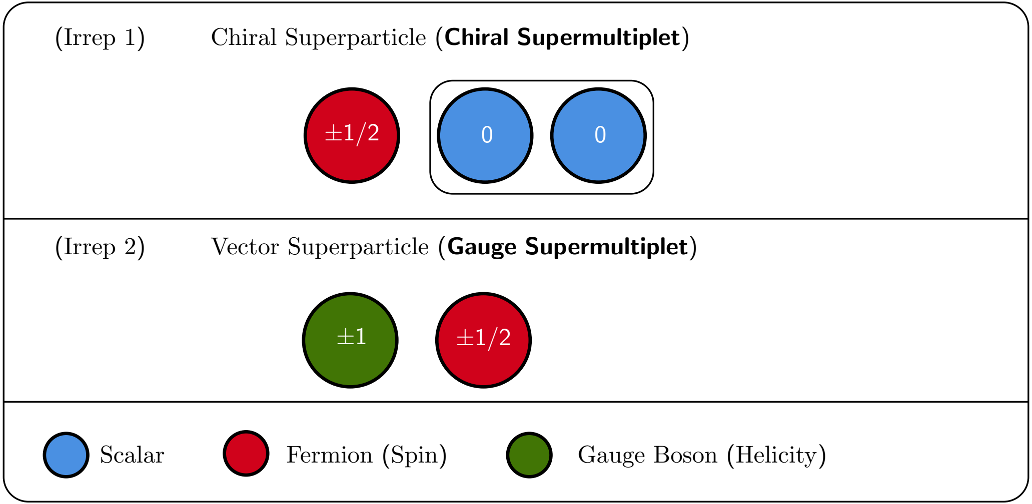
- Spin is spacetime property; not isolated from Poincaré algebra
- **Coleman-Mandula:** Poincaré symmetry is largest possible spacetime symmetry

**Haag-Łopuszański-Sohnius:**

$$\begin{aligned}
[P_\mu, P_\nu] &= 0 \\
[M_{\mu\nu}, P_\lambda] &= i(\eta_{\nu\lambda} P_\mu - \eta_{\mu\lambda} P_\nu) \\
[M_{\mu\nu}, M_{\rho\sigma}] &= -i(\eta_{\mu\rho} M_{\nu\rho} - \eta_{\mu\sigma} M_{\nu\rho} - \eta_{\nu\rho} M_{\mu\sigma} + \eta_{\nu\sigma} M_{\mu\rho}) \\
\{Q_a, Q^\dagger_{\dot a}\} &= 2\sigma^\mu_{a\dot a} P_\mu \\
[Q_a, P_\mu] &= 0 \\
[Q_a, M_{\mu\nu}] &= (\sigma_{\mu\nu})_a^b Q_b
\end{aligned}$$

$\Rightarrow$ **"Super-Poincaré-Algebra" (SUSY Algebra)**

# Irreps of the SUSY algebra



# Building Theories

[Martin, Adv.Ser.Dir.HEP 18 (1998), Aitchison (2007)]

**Most general SUSY Lagrangian**

$$\mathcal{L} = \boxed{-\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + i\lambda^{\dagger a}\bar{\sigma}^\mu(D_\mu\lambda)_a} + \boxed{(D_\mu\phi)^\dagger_i(D^\mu\phi)_i + i\chi^\dagger_i\bar{\sigma}^\mu D_\mu\chi_i}$$

$$\boxed{-\frac{g^2}{2}(\phi^\dagger_i T^a\phi_i)(\phi^\dagger_j T_a\phi_j) - \left|\frac{\partial W(\phi_i)}{\partial\phi_i}\right|^2} - \boxed{\frac{1}{2}\left[\left(\frac{\partial^2 W(\phi_i)}{\partial\phi_i\partial\phi_j}\right)\chi_i\chi_j + \text{h.c.}\right]}$$

$$- \boxed{\sqrt{2}g\left[(\phi^\dagger_i T^a\chi_i)\lambda_a + \lambda^{\dagger a}(\chi^\dagger_i T_a\phi_i)\right]}$$

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$$- \boxed{\sqrt{2}g\left[(\phi^\dagger_i T^a\chi_i)\lambda_a + \lambda^{\dagger a}(\chi^\dagger_i T_a\phi_i)\right]}$$

- Multiple gauge and chiral multiplets possible (and mixing)
- $W(\phi) = M^{ij}\phi_i\phi_j + y^{ijk}\phi_i\phi_j\phi_k$ (**no** field adjoints!)
- Independent couplings: $M_{ij},\ y_{ijk},\ g$

# Building Theories

**Only Freedom:**

- Particle content (participating multiplets)
- Gauge groups ($g$)
- Potential $W$ ($M_{ij}$ and $y_{ijk}$)
- (Breaking Terms)

Particular choice: **Minimal Supersymmetric Standard Model (MSSM)**

# The MSSM: Particle Content

[Martin, Adv.Ser.Dir.HEP 18 (1998), Aitchison (2007)]

| | Spin 0 | Spin 1/2 | Spin 1 | Naming convention |
|---|---|---|---|---|
| Chiral Multiplets | $\tilde{e}$ $\tilde{u}$ $\tilde{\mu}$ $\tilde{c}$ $\tilde{\tau}$ $\tilde{t}$ $H_u$ $H_d$ | $e$ $u$ $\mu$ $c$ $\tau$ $t$ $\tilde{H}_u$ $\tilde{H}_d$ | | lepton $\rightarrow$ slepton<br>quark $\rightarrow$ squark<br>higgs $\rightarrow$ higgsino |
| Gauge Multiplets | | $\tilde{W}$ $\tilde{Z}$ $\tilde{g}$ $\tilde{\gamma}$ | $W$ $Z$ $g$ $\gamma$ | W $\rightarrow$ wino<br>gluon $\rightarrow$ gluino<br>photon $\rightarrow$ photino |

# The MSSM: Gauge Groups

[Martin, Adv.Ser.Dir.HEP 18 (1998), Aitchison (2007)]

| | Spin 0 | Spin 1/2 | Spin 1 | Gauge Structure |
|---|---|---|---|---|
| Chiral Multiplets | $\tilde{e}$ $\tilde{u}$ $\tilde{\mu}$ $\tilde{c}$ $\tilde{\tau}$ $\tilde{t}$ $H_u$ $H_d$ | $e$ $u$ $\mu$ $c$ $\tau$ $t$ $\tilde{H}_u$ $\tilde{H}_d$ | | **fund. rep.** keep charges from SM and extend to superpartners |
| Gauge Multiplets | | $\tilde{W}$ $\tilde{Z}$ $\tilde{g}$ $\tilde{\gamma}$ | $W$ $Z$ $g$ $\gamma$ | **adj. rep.** |

# The MSSM: Potential

[Martin, Adv.Ser.Dir.HEP 18 (1998), Aitchison (2007)]

$$W_{\mathrm{MSSM}} = \mu H_u H_d + \tilde{\bar{u}} \mathbf{y}^u \tilde{Q} H_u - \tilde{\bar{d}} \mathbf{y}^d \tilde{Q} H_d - \tilde{\bar{e}} \mathbf{y}^e \tilde{L} H_d$$

- Differentiation yields Yukawa terms from SM
- Reason for additional Higgs
- No non-trivial Higgs vev $\Rightarrow$ no EWSB

$\Rightarrow$ **Need for (soft) SUSY breaking terms!**

# FMS Mechanism and Augmented PT

# The problem of gauge-invariance

[Berghofer et al. (2021)]

- Gauge Principle is **mathematical Tool** (philosophical depate ongoing)
- Physical quantities must **not** depend on our mathematical description
- In particular: states with open gauge indices cannot be physical states

# The problem of gauge-invariance

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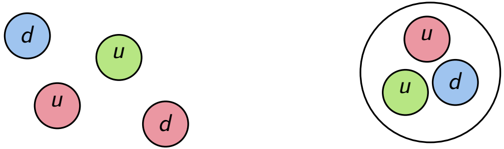
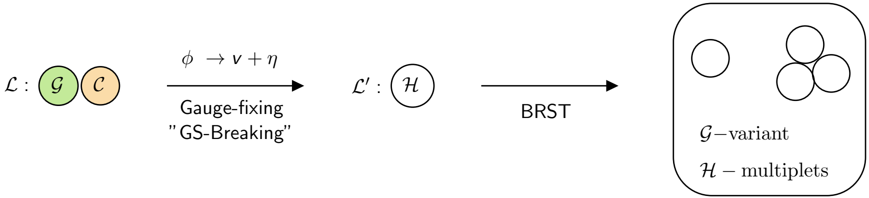



- In particular, explaining mass degeneracy of $W$ is delicate

# Theories with BEH mechanism

[Maas, Prog.Part.Nucl.Phys. 106 (2019)]



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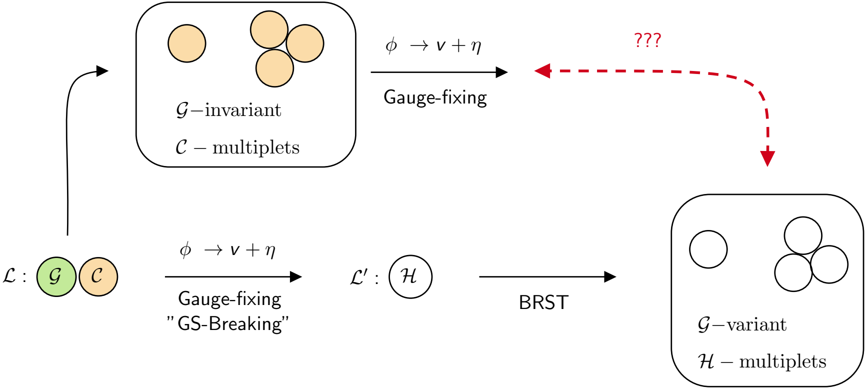

# Example: $W$-Higgs-Sector of SM

[Maas, Prog.Part.Nucl.Phys. 106 (2019)]

$$\mathcal{L} = -\frac{1}{4} W_a^{\mu\nu} W_{\mu\nu}^a + (D^\mu \phi)^\dagger (D_\mu \phi) - \lambda \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^2$$

**Symmetries**

$$\begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \to U_L(x) \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \qquad U_L \in SU(2)$$

$$\begin{pmatrix} \phi \\ i\sigma^2 \phi^* \end{pmatrix} \to U_R \begin{pmatrix} \phi \\ i\sigma^2 \phi^* \end{pmatrix} \qquad U_R \in SU(2)$$

**Making them explicit**

$$\Phi \equiv \begin{pmatrix} \phi_1 & -\phi_2^* \\ \phi_2 & \phi_1^* \end{pmatrix}, \qquad \Phi \to U_L(x) \Phi U_R^\dagger$$

$$\mathcal{L} = -\frac{1}{4} W_a^{\mu\nu} W_{\mu\nu}^a + \frac{1}{2} \mathrm{tr}(D^\mu \Phi)^\dagger (D_\mu \Phi) - \frac{\lambda}{4} \left( \mathrm{tr}\, \Phi^\dagger \Phi - v^2 \right)^2$$

# Example: $W$-Higgs-Sector of SM

[Maas, Prog.Part.Nucl.Phys. 106 (2019)]

Higgs vev: $SU(2) \times SU(2) \to SU(2)$

**Masses for elementary fields**

$$\langle h(x)h(y)\rangle$$

$$\left\langle W_\mu^a(x)W_\nu^b(y)\right\rangle$$

**BUT:** $h$ and $W$ are classified according to $SU(2)$!

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Build gauge-invariant operators first:

$$O = \operatorname{tr} \Phi^\dagger$$

$$O_\mu^A = \operatorname{tr} \tau^A \Phi^\dagger D_\mu \Phi$$

**Masses for composite operators**

$$\left\langle O(x) O^\dagger(y) \right\rangle$$

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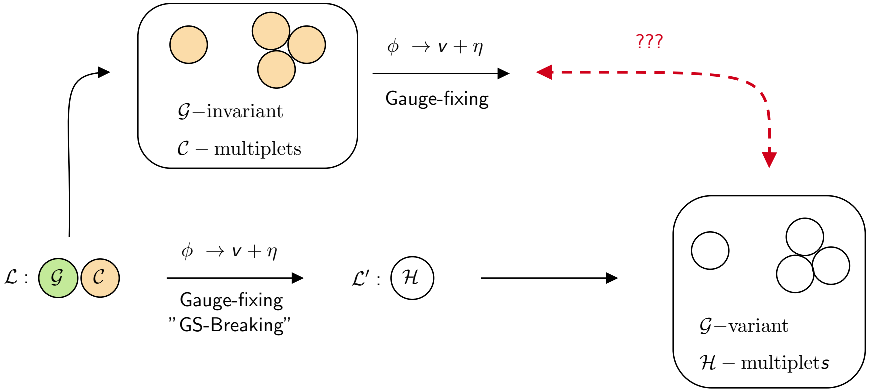
$\Leftarrow$ **Relation?** $\Rightarrow$

# Quick Reminder



# FMS Formalism and Augmented PT

[Fröhlich, Morchio, and Strocchi, Phys.Lett.B 97 (1980)
Maas, Prog.Part.Nucl.Phys. 106 (2019)]

Perform formal split $\phi(x) \to v + \eta(x)$

$$\left\langle O(x)O^\dagger(y)\right\rangle = \text{const.} + 4v^2 \langle h(x)h(y)\rangle + \mathcal{O}(v)$$

$$\left\langle O^A_\mu(x)O^B_\nu(y)\right\rangle = \frac{v^2}{4}\delta^A_a\delta^B_b \left\langle W^a_\mu(x)W^b_\nu(y)\right\rangle + \mathcal{O}(v)$$

**Observations:**

- Propagation of elementary fields $\leftrightarrow$ propagation of gauge-invariant operators with identical quantum numbers
- From here on $\to$ usual PT (hence, "augmented")

# FMS Formalism and Augmented PT

**Higher correlation functions**

$$e^- e^+ \to \mu^- \mu^+$$

$$\left\langle O(x_1)\bar{O}(x_2)O(x_3)\bar{O}(x_4)\right\rangle$$

[See Larissa Egger, Fabian Veider]

# FMS Formalism and Augmented PT

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[See Larissa Egger, Fabian Veider]

**The Catch**

[Török and Maas, PoS LATTICE2016 (2016)]

This only works so well in the SM because of its special structure. Qualitative mismatch e.g. for $SU(3)$ + fund. scalar.

**My thesis:**

Is there a qualitative mismatch for mass spectrum in MSSM?

# Master's Thesis

# Specifying Lagrangian

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu}_a + i\lambda^{\dagger a}\bar{\sigma}^\mu(D_\mu\lambda)_a + (D_\mu\phi)^\dagger_i(D^\mu\phi)_i + i\chi^\dagger_i\bar{\sigma}^\mu D_\mu\chi_i$$
$$-\frac{g^2}{2}(\phi^\dagger_i T^a\phi_i)(\phi^\dagger_j T_a\phi_j) - \left|\frac{\partial W(\phi_i)}{\partial\phi_i}\right|^2 - \frac{1}{2}\left[\left(\frac{\partial^2 W(\phi_i)}{\partial\phi_i\partial\phi_j}\right)\chi_i\chi_j + \text{h.c.}\right]$$
$$-\sqrt{2}g\left[(\phi^\dagger_i T^a\chi_i)\lambda_a + \lambda^{\dagger a}(\chi^\dagger_i T_a\phi_i)\right] + \mathcal{L}_{\text{soft}}$$

- 1 Gauge multiplet ($W$ and $\lambda$), adj. $SU(2)$
- 2 Chiral multiplet ($H_i$ and $\chi_i$), fund. $SU(2)$
- $W = \mu H_1 H_2$
- $\mathcal{L}_{\text{soft}} = -m^2(H^\dagger_1 H_1 + H^\dagger_2 H_2) - m^2_{12}(H_1 H_2 + h.c.)$

# Custodial Symmetry

**Scalar potential:**

$$
\begin{aligned}
V(H_1, H_2) &= \frac{g^2}{8}(H_i^\dagger \sigma^a H_i)(H_j^\dagger \sigma^a H_j) + \left|\frac{\partial W(H_i)}{\partial H_i}\right|^2 - \mathcal{L}_{\text{soft}} \\
&= (|\mu|^2 + m^2)(H_1^\dagger H_1 + H_2^\dagger H_2) + m_{12}^2 (H_1 H_2 + c.c.) \\
&\quad + \frac{g^2}{8}\left[(H_1^\dagger H_1)^2 + (H_2^\dagger H_2)^2\right] - \frac{g^2}{4}(H_1^\dagger H_1)(H_2^\dagger H_2) + \frac{g^2}{2}(H_1^\dagger H_2)(H_2^\dagger H_1)
\end{aligned}
$$

**Naive guess:**

More fields → larger symmetry?

?? $\quad H_1 \to \begin{pmatrix} h_{11} & -h_{12}^* \\ h_{12} & h_{11}^* \end{pmatrix} \qquad H_2 \to \begin{pmatrix} h_{21} & -h_{22}^* \\ h_{22} & h_{21}^* \end{pmatrix} \qquad H_i \to H_i R^\dagger \quad$ ??

Potential structure enforced by SUSY ruins this idea.

# Custodial Symmetry

[Pilaftsis, Phys.Lett.B 706 (2012)]

**Idea:** Translate into 2HDM and use results thereof

$$
\begin{aligned}
V = & -M_1^2(\phi_1^\dagger\phi_1) - M_2^2(\phi_2^\dagger\phi_2) - M_{12}^2(\phi_1^\dagger\phi_2) - M_{12}^{*2}(\phi_2^\dagger\phi_1) + \lambda_1(\phi_1^\dagger\phi_1)^2 + \lambda_2(\phi_2^\dagger\phi_2)^2 \\
& + \lambda_3(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_4(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \frac{\lambda_5}{2}(\phi_1^\dagger\phi_2)^2 + \frac{\lambda_5^*}{2}(\phi_2^\dagger\phi_1)^2 \\
& + \lambda_6(\phi_1^\dagger\phi_1)(\phi_1^\dagger\phi_2) + \lambda_6^*(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_1) + +\lambda_7(\phi_2^\dagger\phi_2)(\phi_1^\dagger\phi_2) + \lambda_7^*(\phi_2^\dagger\phi_2)(\phi_2^\dagger\phi_1)
\end{aligned}
$$

**Our special case**

$$M_1^2 = M_2^2 = -(|\mu|^2 + m^2) \qquad M_{12}^2 = -m_{12}^2 \in \mathbb{R}$$

$$4\lambda_1 = 4\lambda_2 = 2\lambda_3 = -\lambda_4 = \frac{g^2}{2} \qquad \lambda_5 = \lambda_6 = \lambda_7 = 0$$

# Custodial Symmetry

**Find special case in tabulated parameter restrictions:**

[Pilaftsis, Phys.Lett.B 706 (2012)]

| No. | Symmetry | $M_1^2$ | $M_2^2$ | $M_{12}^2$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\mathrm{Re}\,\lambda_5$ | $\lambda_6 = \lambda_7$ |
|---|---|---|---|---|---|---|---|---|---|---|
| 10 | $O(2) \times O(3)$ | - | $M_1^2$ | 0 | - | $\lambda_1$ | $2\lambda_1$ | - | 0 | 0 |
| | | $-(\lvert\mu\rvert^2 + m^2)$ | | $-m_{12}^2$ | $\frac{g^2}{8}$ | | $\frac{g^2}{4}$ | $-\frac{g^2}{2}$ | 0 | |

**Copy group generators:**

[Pilaftsis, Phys.Lett.B 706 (2012)]

$$K^0 = \frac{1}{2}\sigma^3 \otimes \sigma^0 \otimes \sigma^0 \qquad K^8 = \frac{1}{2}\sigma^1 \otimes \sigma^1 \otimes \sigma^0 \qquad K^9 = \frac{1}{2}\sigma^2 \otimes \sigma^1 \otimes \sigma^0$$

# Custodial Symmetry

[Pilaftsis, Phys.Lett.B 706 (2012)]

**Translation back to MSSM**

$$H \equiv \begin{pmatrix} i\sigma^2 H_1^* \\ H_2 \\ -H_1 \\ i\sigma^2 H_2^* \end{pmatrix}$$

$$U_\mathrm{c} = \exp\left\{ i\theta^A K^A \right\}$$

$$\left[ U_\mathrm{c}, \Sigma^{1,2} \right] = 0$$

**Manifestly custodial invariant potential**

$$V = \frac{1}{2}(|\mu|^2 + m^2) H^\dagger H - m_{12}^2 (H^\dagger \Sigma^1 H) + \frac{g^2}{32}(H^\dagger H)^2 - \frac{g^2}{8}[(H^\dagger \Sigma^1 H)^2 + (H^\dagger \Sigma^2 H)^2]$$

**Remarks:** Can be extended to kinetic and Higgsino part; effectively Higgs family transformation

# Gauge-Invariant Operators

| Operator | Spin | $SU(2)_c$ | CP |
|---|---|---|---|
| $H^\dagger H$ | 0 | 0 | even |
| $H^\dagger \Sigma^1 H$ | 0 | 0 | even |
| $H^\dagger i\Sigma^2 H$ | 0 | 0 | odd |
| $H^\dagger K^3 \kappa^A H$ | 0 | 1 | - |
| $H^\dagger \kappa^A D_\mu H$ | 1 | 1 | - |

# Tree-level Spectrum

**Gauge-Fixing:**
(t'Hooft Gauge)

$$C^a = \partial^\mu W_\mu^a + \frac{g\xi}{2} \operatorname{Im}\left\{ V^\dagger \tau^a H \right\}$$

$$\mathcal{L}_{\text{gf}} = -\frac{1}{2\xi} C^a C^a$$

**vev-Expansion:**

$$H_1 \to (v_1, 0)^T + \eta_1$$

$$H_2 \to (0, v_2)^T + \eta_2$$

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu\nu} W^{\mu\nu}_a + \frac{1}{2} m_W^2 W^a_\mu W^\mu_a$$

$$+ \text{ kinetic Terms in } \eta + \text{quadratic Terms in } \eta$$

$$+ \text{ interaction Terms}$$

# Tree-level Spectrum

**Scalar masses:** $$\frac{1}{2}\frac{\partial^2(\text{quadratic Terms in }\eta)}{\partial\eta_i\partial\eta_j}\bigg|_{\eta=0}$$

| **Massive W's** | **Goldstones** | **"neutral" Higgs** | **"charged" Higgs** |
|---|---|---|---|
| $W_1$, $W_2$, $W_3$ | $G^+$, $G^0$, $G^-$ | $H_1^0$, $H_2^0$, $H_3^0$ | $H^+$, $H^-$ |
| $m_W^2 = \frac{g^2}{2}\left(v_1^2 + v_2^2\right)$ | $m_G^2 = \xi m_W^2$ | $m_{H_{1,2}^0}^2 = \mp\frac{g^2}{2}\left(v_1^2 - v_2^2\right)$<br>$m_{H_3^0}^2 = -m_W^2$ | $m_{H^\pm}^2 = 0$ |

# FMS Mechanism

**Expand into vev and fluctuations**

$$H \to V + \eta$$

| Operator | Spin | $SU(2)_c$ | CP |
|---|---|---|---|
| $O_0 = H^\dagger H$ | 0 | 0 | even |
| $O_1 = H^\dagger \Sigma^1 H$ | 0 | 0 | even |
| $O_2 = H^\dagger i\Sigma^2 H$ | 0 | 0 | odd |
| $O^A = H^\dagger K^3 \kappa^A H$ | 0 | 1 | - |
| $O^A_\mu = H^\dagger \kappa^A D_\mu H$ | 1 | 1 | - |

$$O_0 \pm O^3 \sim H_1^0 \quad H_2^0$$

$$O_2 \sim H_3^0$$

$$O^2 \mp iO^1 \sim H^+ \quad H^-$$

$$O^A_\mu \sim c^{Aa} W^a_\mu \quad W_1 \quad W_2 \quad W_3$$

# Summary and Outlook

- Supersymmetric theories have rich phenomenology and some nice properties (even though they might turn out wrong eventually)
- Gauge-invariant formulation absolutely necessary
- GI treatment of MSSM so far: Situation as in SM

**What's next?**

- Extend to higgsinos
- Include at least one fermion generation
- Add hypercharge

# Bonus Content

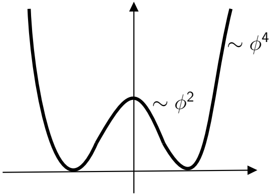
# The MSSM: Potential

**Standard Model**

(right handed, left handed)

$$V(\phi^\dagger\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

$$\mathcal{L}_{\text{Yukawa}} = -\bar{u}\mathbf{y}^u Q\phi^{\text{c.c.}} - \bar{d}\mathbf{y}^d Q\phi - \bar{e}\mathbf{y}^e L\phi + \text{h.c.}$$




**MSSM**

$$W_{\text{MSSM}} = \mu' H_u H_d + \tilde{\bar{u}}\mathbf{y}^u \tilde{Q} H_u - \tilde{\bar{d}}\mathbf{y}^d \tilde{Q} H_d - \tilde{\bar{e}}\mathbf{y}^e \tilde{L} H_d$$

Remember: $W(\phi) = M^{ij}\phi_i\phi_j + y^{ijk}\phi_i\phi_j\phi_k$ interactions from derivatives of $W$

# Hypercharge

**Standard Model**

[Maas, Prog.Part.Nucl.Phys. 106 (2019)]

$$\phi \rightarrow e^{i\alpha(x)}\phi$$

$$\Phi \rightarrow \Phi \; e^{i\alpha(x)\tau^3}$$

$$\begin{pmatrix} \phi_1 & -\phi_2^* \\ \phi_2 & \phi_1^* \end{pmatrix} \begin{pmatrix} e^{i\alpha(x)} & 0 \\ 0 & e^{-i\alpha(x)} \end{pmatrix}$$

This amounts to gauging the $U(1)$ subgroup of $SU(2)_c$:

$$\Phi \rightarrow \Phi U_R^\dagger$$

Mass degeneracy is lifted $\Rightarrow W^\pm, W^3$

**MSSM**

$$H_i \rightarrow e^{\mp i\alpha(x)} H_i$$

$$H \rightarrow e^{i\alpha(x)\kappa^3} H$$

Same situation as in the SM.
Interpretation: Custodial symmetry = Higgs family symmetry (undistinguishable without hypercharge)

# Tree-level mass relation

$$m^2_{H^\pm} = 0$$
$$m^2_{H^0_{1,2}} = \mp\frac{g^2}{2}(v_1^2 - v_2^2)$$
$$m^2_{H^0_3} = -m^2_W$$

**MSSM contraints on tree-level:**

[Rosiek (1995)]

$$m^2_{H^\pm} = m^2_{H^0_3} + m^2_W$$
$$m^2_{H^0_1} + m^2_{H^0_2} = m^2_{H^0_3} + m^2_Z$$