Particle creation in an expanding universe

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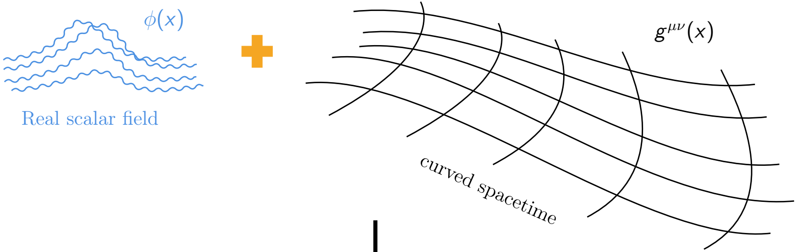
Topic overview

Motivation

$$\left.\begin{array}{c}\text{General Relativity}\\ \text{Quantum Mechanics}\end{array}\right\}\text{particle creation by expansion of universe}$$

L. Parker

Assumptions and Goal



Assumption : ϕ has no effect on $g^{\mu\nu}$

Treat gravity as classical background

Action in Minkowski space

Action: (Minkowski spacetime)

$$S = \int \mathrm{d}^4 x \left(\frac{1}{2} \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right)$$

- Determines motion of free scalar field
- Need to couple to background metric
- Equations of motion follow from $\delta S = 0$

Minimal coupling

Minimally coupled action:

$$S_{\min} = \int \mathrm{d}^4x \sqrt{-|g|} \left(\frac{1}{2} g^{\mu\nu} D_\mu \phi D_\nu \phi - \frac{1}{2} m^2 \phi^2 \right)$$

We performed following modifications of to action:

- Exchange metric: $\eta^{\mu\nu} \to g^{\mu\nu}$
- Adapt derivatives: $\partial_\mu \to D_\mu$
- Covariant volume element: $\mathrm{d}^4x \to \mathrm{d}^4x \sqrt{-|g|}$

Conformal coupling

Conformal coupling action:

$$S_{\rm conf} = \int \mathrm{d}^4 x \sqrt{-|g|} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \left(m^2 + \frac{R}{6} \right) \phi^2 \right]$$

Perks:

- Conformal invariance for m=0: $g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = \Omega^2(x) g_{\mu\nu}$
- energy-momentum tensor of conformally invariant field traceless (classically)
- simplification for conformally flat spacetimes $g_{\mu\nu} = \Omega^2(x) \eta_{\mu\nu}$

Flaws:

- Might violates strong equivalance principle

Equations of Motion

Using the **principle of least action** we obtain

$$\Box\phi + (m^2 + \xi R)\phi = 0$$

$$\text{minimal coupling} : \xi = 0$$
$$\text{conformal coupling} : \xi = \frac{1}{6}$$

$$\Box\phi \equiv \frac{1}{\sqrt{-g}}\partial_\mu \left(g^{\mu\nu}\sqrt{-g}\partial_\nu\phi\right)$$

Notice: for $\eta_{\mu\nu}$ this reduces to the well-known Klein-Gordon-equation

Vacua

1. flat Minkowski space:
   - symmetry in timetranslation
   - positive frequency mode with respect to it
   - define vacuum state
   - define complete Fock space

   ✓ same number of particles for different observers

2. general spacetime
   - no symmetry in timetranslation
   - separation in time-dependent and space-dependent factors impossible
   - no sets of time independent basis modes

   ✗ different number of particles for different observers

Bogolubov Transformations and Coefficients

1. observer *I*
   - complete orthogonal set of mode solutions of field: $\phi(x) = \sum_i [a_i u_i(x) + a_i^\top u_i^*(x)]$
   - vacuum state: $a_i \left|0\right\rangle = 0 \ \forall i$
2. observer *II*
   - different complete orthogonal set of mode solutions of field: $\phi(x) = \sum_j [\bar{a}_j \bar{u}_j(x) + \bar{a}_j^\top \bar{u}_j^*(x)]$
   - vacuum state: $\bar{a}_j \left|\bar{0}\right\rangle = 0 \ \forall j$

Bogolubov transformations:

$$\bar{u}_j = \sum_i [\alpha_{ji} u_i + \beta_{ji} u_i^*]$$

$$u_i = \sum_j [\alpha_{ji}^* \bar{u}_j - \beta_{ji} \bar{u}_j^*]$$

Bogolubov Transformations and Coefficients

Bogolubov coefficients: α_{ji}, β_{ji}

connect creation and annihilation operators

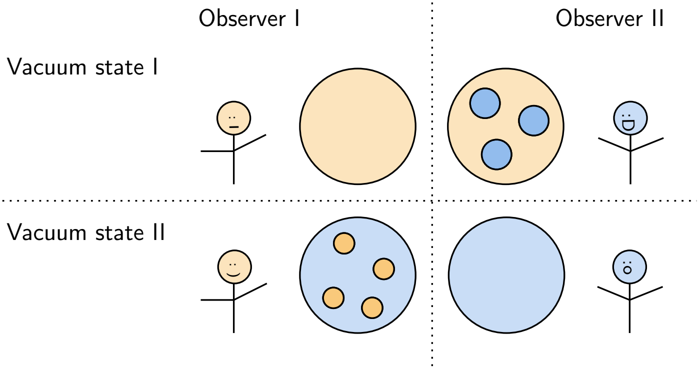
$\Rightarrow$ different Fock spaces for $\beta_{ji} \neq 0$.

$\Rightarrow$ for particle number N_i: $\left\langle \bar{0} \middle| N_i \middle| \bar{0} \right\rangle = \sum_j |\beta_{ji}|^2$

$\Rightarrow$ particle creation ✓

Physical Vacuum

Q: So what is the actual physical vacuum?



Physical Vacuum

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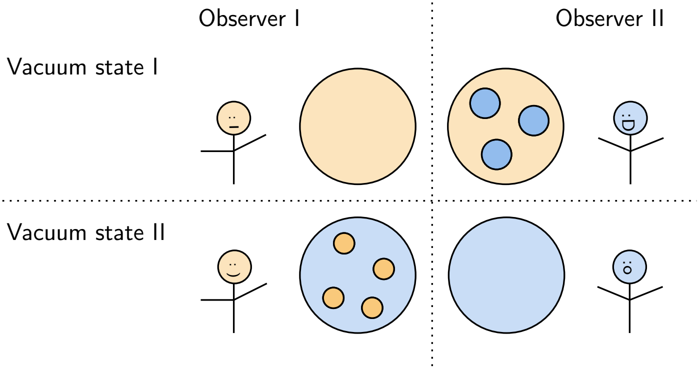

A: A priori, there's no *best* choice.

Minimization of instantaneous energy

Hamiltonian for quantum field:

$$H(t) = \frac{1}{2} \int \mathrm{d}^3x \left[\Pi^2 + (\nabla\phi)^2 + m_{\mathrm{eff}}^2(t)\phi^2\right]$$

$\rightarrow$ explicitly time-dependent $\Rightarrow$ **no time-independent eigenvectors**

Instantaneous vacuum $\left|_{t^*}0\right\rangle$:

lowest-energy state of $H(t^*)$

Ambiguity of vacuum state

- Decomposition of fields into plane waves

$$\exp(i\boldsymbol{kx} - i\omega_k t)$$

- Particle only well defined if

$$\delta k \ll k$$

$$\lambda \gg \frac{1}{k}$$

$$\text{(spatial size of wave packet)}$$

- Geometry!

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example: spatially flat Friedmann modes: $\omega_k^2(\eta) = k^2 + m^2 a^2 - \frac{a''}{a}$

Example: spatially flat Friedmann modes

$$\boxed{\omega_k^2(\eta) = k^2 + m^2 a^2 - \frac{a''}{a}}$$

- curvature might not be large enough
- Hamiltonian bounded below due to gravitational effects

$\Rightarrow$ Definition of instantaneous vacuum ↯

Particle creation: Our model and results

Specifying the metric

Solution to Einstein equations **assuming** a homogeneous, isotropic and spatially flat universe:

$$\boxed{\textbf{FLRW Metric: } \mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j}$$

→ Spatial part: flat but scales with time

→ Spacetime curved!

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After introducing *conformal time*

$$\eta(t) \equiv \int^t \frac{\mathrm{d}t}{a(t)}$$

metric becomes *conformally equivalent* to Minkowski metric:

$$\mathrm{d}s^2 = \underbrace{a^2(\eta)\eta_{\mu\nu}}_{g_{\mu\nu}}\mathrm{d}x^\mu\mathrm{d}x^\nu$$

Equation of motion

For a **conformally coupled** scalar field:

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(g^{\mu\nu}\sqrt{-g}\partial_\nu\phi\right)+(m^2+\frac{R}{6})\phi=0$$

$$\Downarrow_{\ \mathrm{d}s^2=a^2(\eta)\eta_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu \text{ and } \chi(x)\equiv a(\eta)\phi(x)}$$

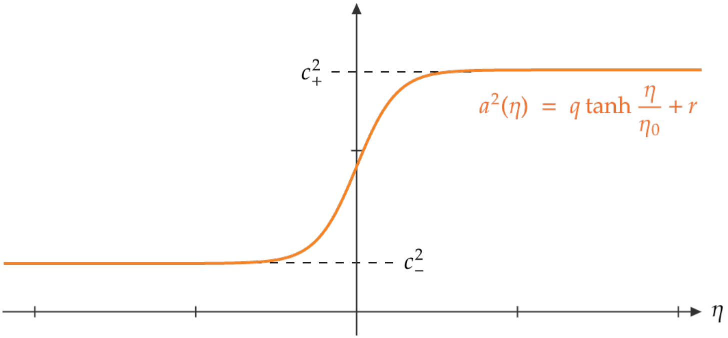
$$\left[\partial_\mu\partial^\mu+a^2(\eta)m^2\right]\chi(x)=0$$

$\rightarrow$ Klein-Gordon-equation with time-dependent mass term

$$\Downarrow_{\ \text{spatial Fourier transform}}$$

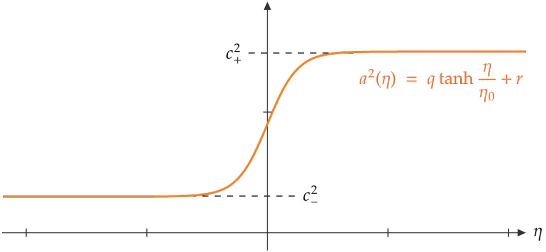
$$\left[\frac{\mathrm{d}^2}{\mathrm{d}\eta^2}+\omega_k^2(\eta)\right]\chi_\mathrm{k}(\eta)=0$$

Expansion model



$$\left[\frac{\mathrm{d}^2}{\mathrm{d}\eta^2} + \omega_k^2(\eta)\right] \chi_k(\eta) = 0 \quad \text{where} \quad \omega_k^2(\eta) = k^2 + m^2 a^2(\eta)$$

Expansion model



Asymptotic behaviour:

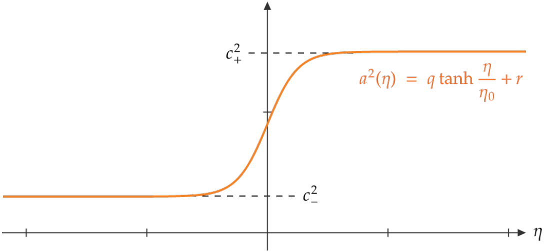
$$\left[\frac{\mathrm{d}^2}{\mathrm{d}\eta^2} + \omega_k^2(\eta)\right] \chi_k(\eta) = 0$$

$$\omega_k^2(\eta) = k^2 + m^2 a^2(\eta)$$

$$\omega_k(\eta \to -\infty) = \sqrt{k^2 + c_-^2 m^2}$$

$$\omega_k(\eta \to +\infty) = \sqrt{k^2 + c_+^2 m^2}$$

Expansion model



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$\Rightarrow$ Plane wave behaviour for $\eta \to \pm\infty$ ✓Notion of particles and vacuum

Solving equation of motion

We find two sets of (equivalently good) solutions

$$v_{\rm in}^{+} \quad v_{\rm in}^{-} \qquad \text{and} \qquad v_{\rm out}^{+} \quad v_{\rm out}^{-}$$

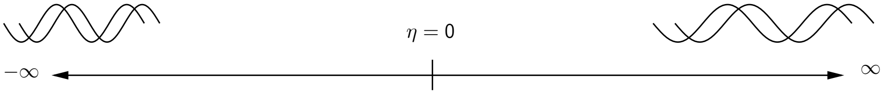
and they are used to define vacua in the past/future, respectively:

$v_{\rm in}^{+} \sim \exp\{-i\omega_{\rm in}\eta\}$

$v_{\rm in}^{-} \sim \exp\{i\omega_{\rm in}\eta\}$

$v_{\rm out}^{+} \sim \exp\{-i\omega_{\rm out}\eta\}$

$v_{\rm out}^{-} \sim \exp\{i\omega_{\rm out}\eta\}$



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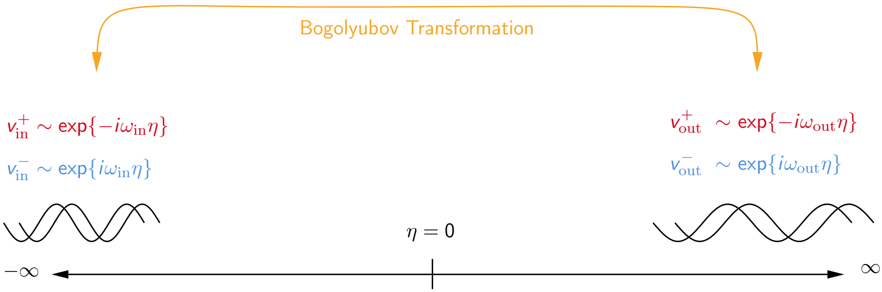
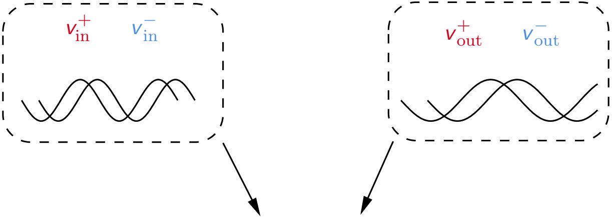


Bogolyubov transformation



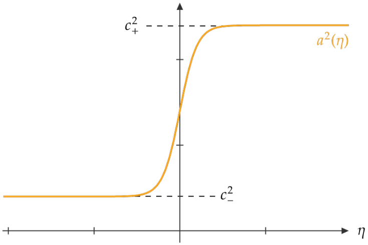
$$\begin{pmatrix} v_{\mathrm{in}}^{+} \\ v_{\mathrm{in}}^{-} \end{pmatrix} = \begin{pmatrix} \alpha_k & \beta_k \\ \beta_k^* & \alpha_k^* \end{pmatrix} \begin{pmatrix} v_{\mathrm{out}}^{+} \\ v_{\mathrm{out}}^{-} \end{pmatrix}$$

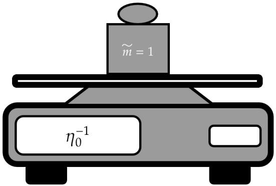
$$\Downarrow$$

$$|\beta_k|^2 = n(k)$$

Natural units and mass reference

1. Want **dimensionless units** $\tilde{m}$ and $\tilde{k}$
   $\rightarrow$ express m and k in units of η_0^{-1}
2. Want to describe produced particles long **after** the expansion ended
   $\rightarrow$ Masses should be measured by scales in the remote future (rest mass)





Particle density

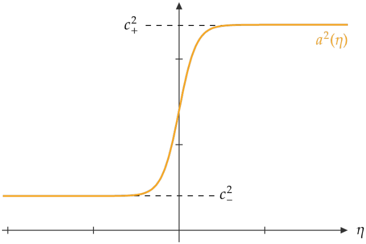
$$n(\tilde{k}) = \frac{\cosh \pi \left(\tilde{\omega}_{\text{out}} - \tilde{\omega}_{\text{in}}\right) - 1}{\cosh \pi \left(\tilde{\omega}_{\text{out}} + \tilde{\omega}_{\text{in}}\right) - \cosh \pi \left(\tilde{\omega}_{\text{out}} - \tilde{\omega}_{\text{in}}\right)}$$

$$\tilde{\omega}_{\text{out}} = \sqrt{\tilde{k}^2 + \tilde{m}^2}$$

$$\tilde{\omega}_{\text{in}} = \sqrt{\tilde{k}^2 + \frac{c_-^2}{c_+^2}\tilde{m}^2}$$

Observations:

- No massless particles are created
- Only depends on relative increase $\frac{c_+}{c_-}$
- No η_0-dependence

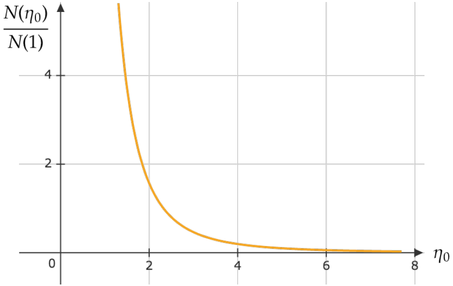


Total number of particles produced

$$N(\eta_0, \tilde{m}) = \frac{4\pi}{\eta_0^3} \int_0^\infty \mathrm{d}\tilde{k}\ \tilde{k}^2\ n(\tilde{k}; \tilde{m})$$

Observations:

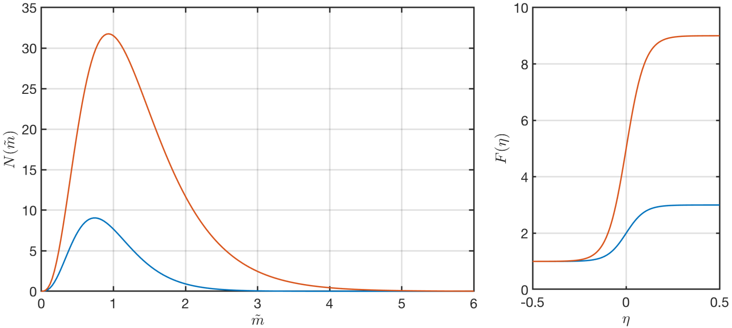
- Things from $n(\tilde{k})$ still apply
- Now also: η_0-dependence
- More rapid expansion produces more particles



... more interested in $N(\tilde{m})$, though.

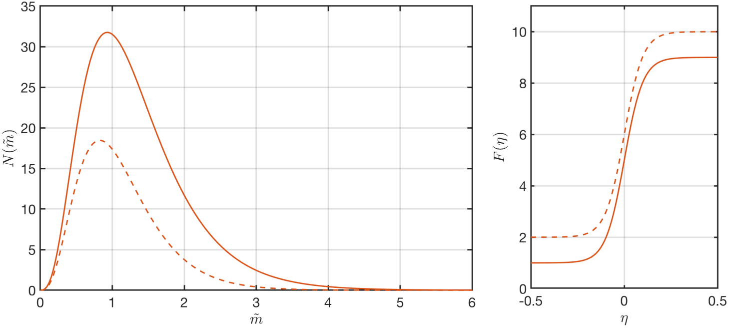
Total number of particles produced

Different absolute scaling and different ratios of $\frac{c_+}{c_-}$:



Total number of particles produced

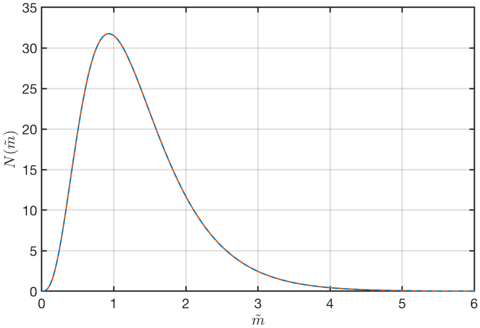
Same absolute scaling but different ratio of $\frac{c_+}{c_-}$:

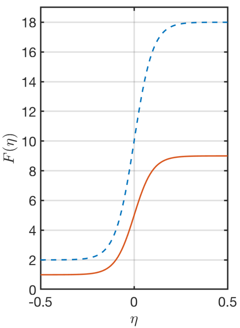


$\rightarrow$ effect of expansion is more pronounced if the universe starts out small

Total number of particles produced

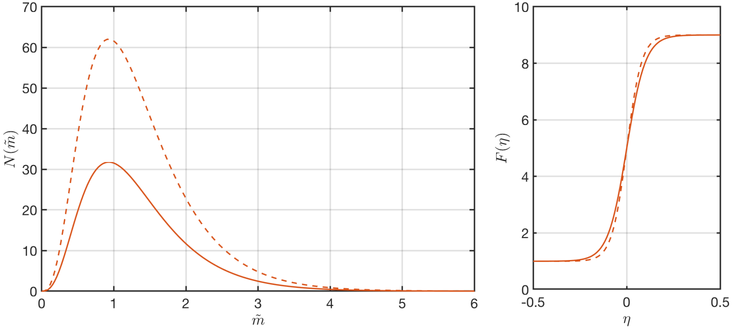
Different absolute scaling but identical ratio of $\frac{c_+}{c_-}$:





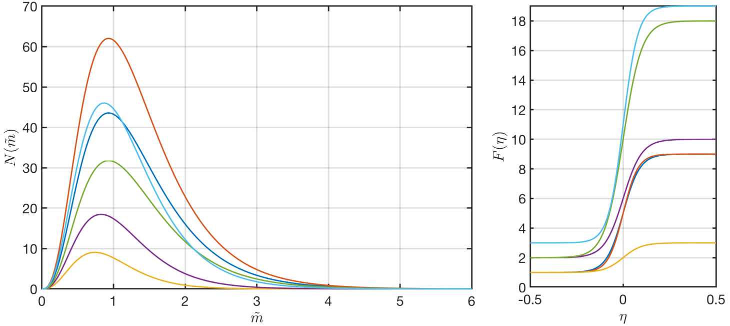
Total number of particles produced

Same scaling but different η_0:



Total number of particles produced

Time- and mass-scales:



$\rightarrow \eta_0$ sets scale of produced particles

Summary

- Notion of vacuum is ambiguous in curved spacetime
- Finding **physical** vacuum is important to start talking about *particles*
- Expansion of universe creates particles

- Only relative increase important
- Stronger relative increase ⇒ more particles
- More rapid increase ⇒ more particles
- Time scale of expansion sets mass scale of produced particles