Unrealistic Expectations and Misguided Learning

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people have unrealistically positive views of their traits and prospects.

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We study the beliefs of overconfident individuals about other variables affecting optimal behavior.

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- This adjustment in behavior makes things worse, perpetuating the misdirected learning further.

We characterize the situations in which such misdirected learning occurs, and explore implications.

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Most ignored literature: that studying the positive consequences of unrealistic self-views.

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where f has mean 0 is log-concave.

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- A tractable stand-in for forces to generate overconfidence.
- Equivalent to biased assessment of her contribution to output.

Example:

- Loss Function:

$$Q(e_t, a, \Phi) = a + \Phi - L(e_t - \Phi)\,.$$

where L is a symmetric loss function with $|L'| \leq k < 1$.

- Useful for illustrations.

- Delegation:

$$Q(e_t, a, \Phi) = e_t\,\Phi + (1 - e_t)a - c(1 - e_t)\,.$$

Applications (Beyond Delegation)

Control in organizations.

- $-e_t$: control/punishments/extrinsic incentives.
- More control lowers intrinsic motivation (Benabou and Tirole 2003) or morale (Fang and Moscarini 2005).
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Working.

- $-e_t$: amount of work.
- q_t: life satisfaction.
- $-\Phi$: social norm for how much one should work.

Outline:

- Long-Run actions in a "heuristic" model
  - Reinforcement effect
  - It's the important things the agent get's wrong
  - Lower persistence & outside options
  - Underconfidence
- Convergence
  - Interdependence of actions & beliefs
  - New technique: extremal beliefs
- Extensions
  - Non-Myopic Behavior
  - Different production functions & asymptotic disagreement
  - Learning misspecification

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A kind of self-serving attributional bias.

- Because agent believes she's able, she attributes bad outcome to external factors.

Endogenous Actions

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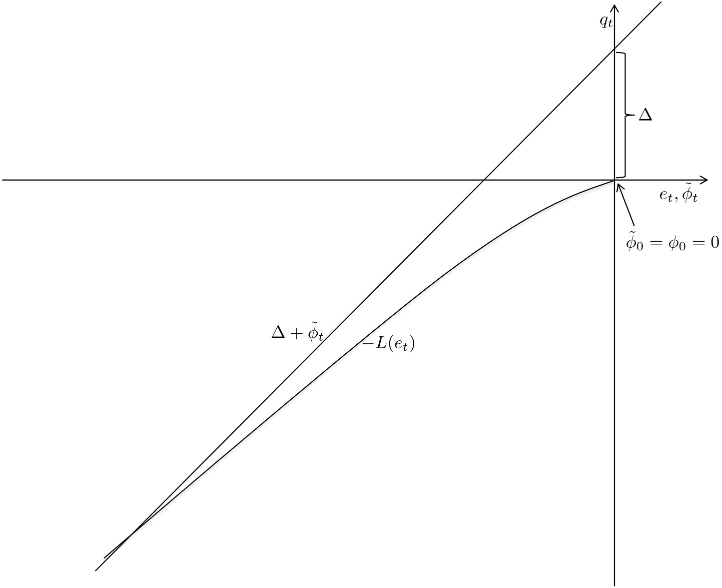
- For illustrations focus on the Loss function case
- Consider the dynamics where
  - the agent can change his action infrequently
  - the frequency at which he can change his action goes to zero quickly
- in this case:
  - noise does not matter
  - the agent updates only based on the output corresponding to his previous action

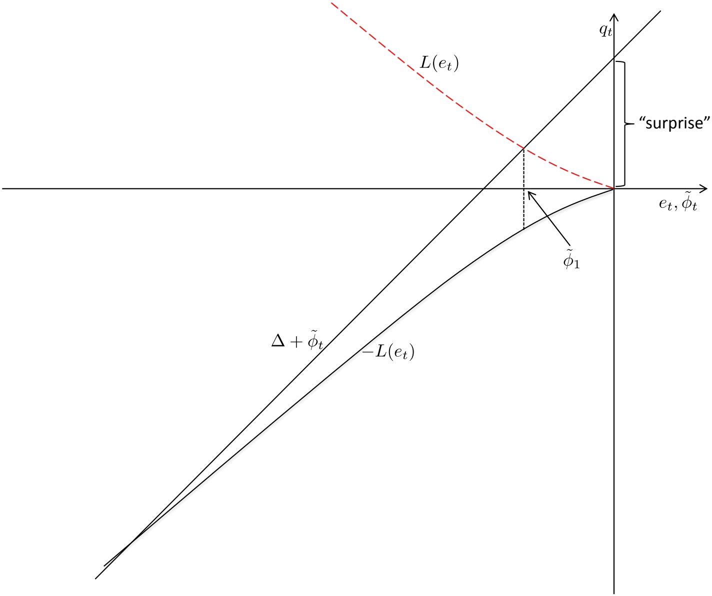
Endogenous Action

Now suppose e_t is chosen by the agent each period.



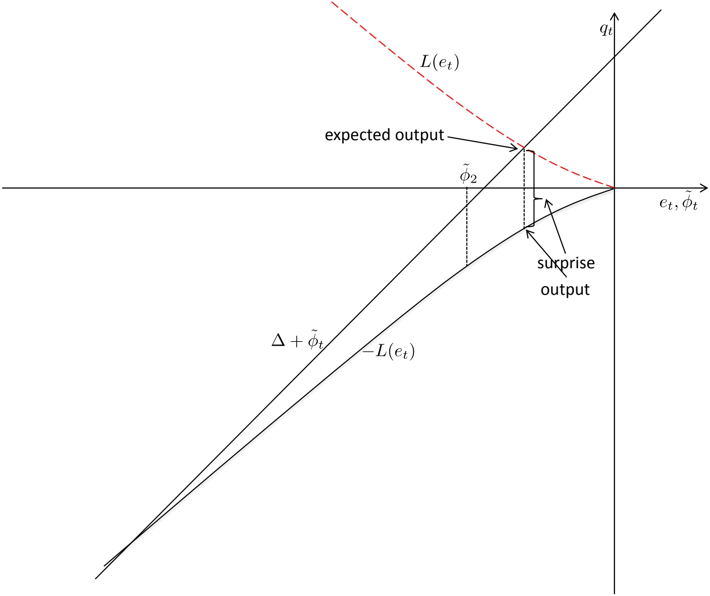
Endogenous Action

Agent concludes that the state of the world is worse.



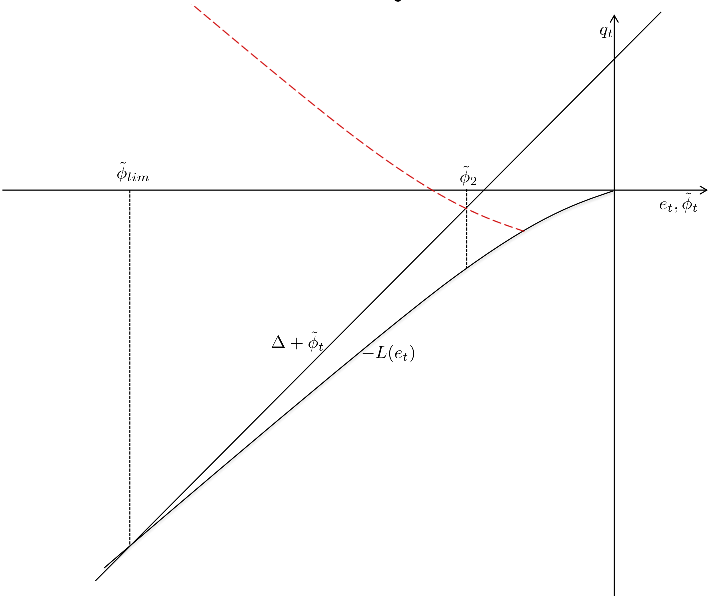
Endogenous Action

Hense she adjusts her action to what she believes to have learned.



Endogenous Action

Conclues the state is even worse, readjusts...



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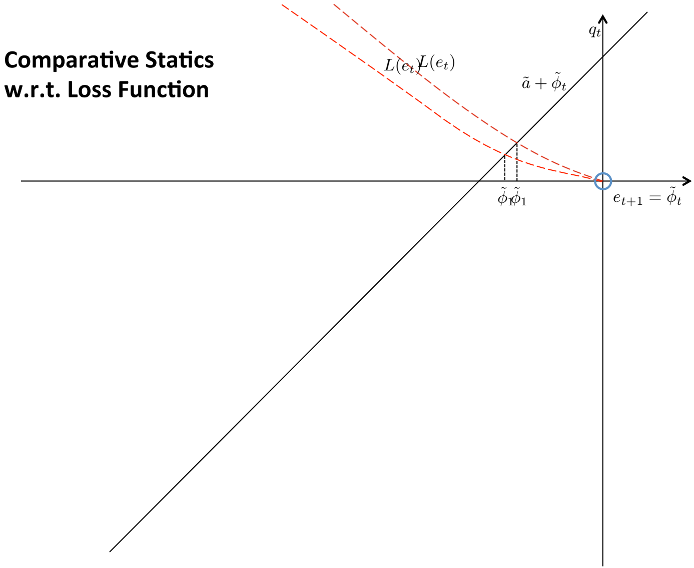
Note: final theory really bad at explaining early observations, but agent endogenously creates enough signals to overwhelm early (less misleading) ones.

Comparative Static: The Importance of Being Right

Short run: update less.



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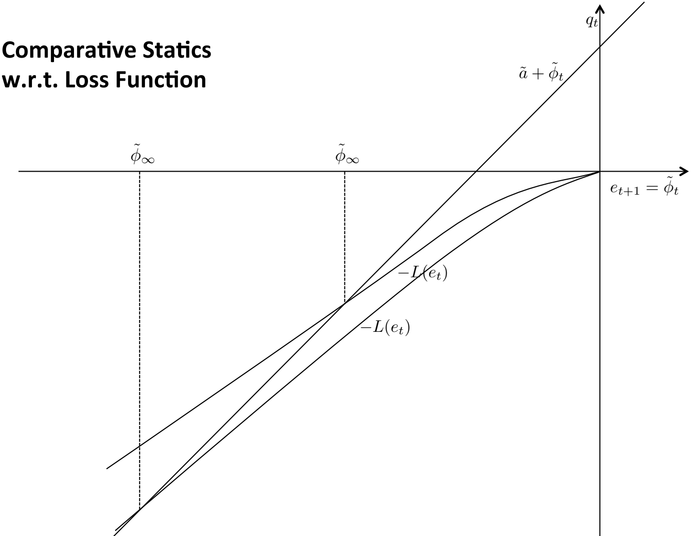

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Intuition:

- Agent is hurting herself more $\Rightarrow$ to explain in a consistent way, she must become more pessimistic.

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- E.g., getting frustrated with coauthor and looking for new one.

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Observe: when the agent exits, her learning doesn't affect future beliefs regarding environments where Φ doesn't apply.

- In fact, she tends to seek out exactly these situations.

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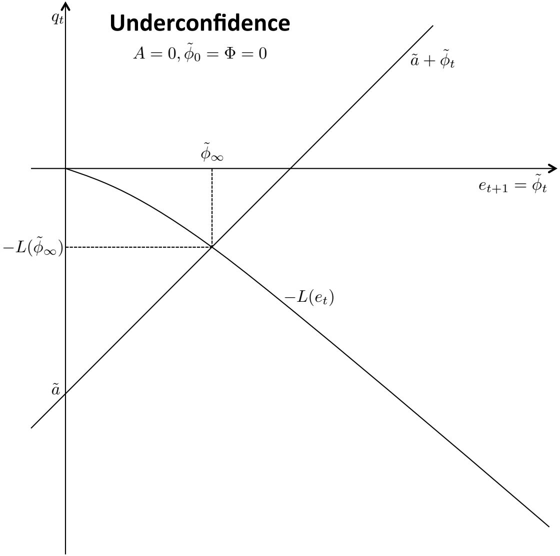

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- So misdirected learning is self-limiting.

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- But, actions depend on beliefs.
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- Beliefs are complicated processes
- We introduce a novel idea of looking at extremal beliefs.

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Theorem

If there is a unique ϕ^* with $\Gamma(\phi^*) = 0$ then a.s.

$$\lim_{t \to \infty} \pi_t \xrightarrow{d} \delta_{\phi^*}$$

Proofidea

- Define

$$\underline{\phi} = \sup\{\phi \colon \lim_{t\to\infty} \tilde{\mathbb{P}}_t[\Phi \le \phi] = 0\}$$
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- $\underline{\phi}$ and $\overline{\phi}$ imply a bound on the long-run actions
- Bound on long-run actions implies a bound on the the average surprise

$$\liminf_{t\to\infty} Q(e_t, A, \Phi) - Q(e_t, \tilde{a}, \underline{\phi}) \ge \Gamma(\underline{\phi}) > 0$$
$$\limsup_{t\to\infty} Q(e_t, A, \Phi) - Q(e_t, \tilde{a}, \overline{\phi}) \le \Gamma(\underline{\phi}) < 0\,.$$

Proofidea

- The log-likelihood is given by (up to a constant)

$$\ell_t(\phi) = \sum_{s=1}^{t} \log f(Q(e_t, A, \Phi) - Q(e_t, \tilde{a}, \phi) + \epsilon_t)$$

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- Bound on the surprise at $\underline{\phi}, \overline{\phi}$ implies a long-run bound on the derivative of ℓ in a neighbourhood of $\underline{\phi}, \overline{\phi}$.
- With uniform convergence this implies that if the agent is surprised at $\underline{\phi}$ and $\overline{\phi}$ and thus assigns probability zero to an environment around $\underline{\phi}$ and $\overline{\phi}$.

- Sufficient conditions for single crossing of Γ

Proposition

The limiting belief ϕ^* satisfies $-\Delta\, \overline{Q}_a / \underline{Q}_b \leq \tilde{\phi}_\infty - \Phi \leq -\Delta\, \underline{Q}_a / \overline{Q}_b$

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- Hence, ϕ^* decreases in L.

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Proposition

The following are equivalent:

I. *For any A, $\tilde{a}$, and Φ the agent behaves optimally in the long run* $\lim_{t\to\infty} e_t = e^*(\Phi)$.

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Conclusion: qualitatively, self-defeating learning occurs if the optimal action depends on the state, and either depends less on ability or does so in the opposite way.

Extensions

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- Idea: Beliefs concentrate almost surely
- The experimentation motive vanishes almost surely

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- Speed of convergence potentially informative.

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Important possibility outside the misspecified model framework: people eventually realize they're on the wrong track, and reconceptualize.

Thank You!