

Using Algebraic Geometry in F-Theory

└ Physical Context

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- Type-IIB String Theory: all classical Lie Groups
- F-Theory: all classical Lie Groups as Gauge Group possible

- Type-IIB only A_n and D_n
- F-Theory is non-perturbative and other Lie groups require bound states

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└ F-Theory

Internal space: 6-real-dimensional (B_3) = elliptic fibre
(2-real-dimensional)

Locus	Physical content	dim-
Torus degenerates into one \mathbb{P}^1	7-branes	2
7-branes intersect / degeneration into two \mathbb{P}^1 s	matter curves	1
matter curves intersect / degeneration into three \mathbb{P}^1 s	intersection locus	0

- due to symmetries it is possible to work over \mathbb{C}

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Internal space: 6-real-dimensional (B_6) + elliptic fibre
(2-real-dimensional)

Locus	Physical content	dim-
Torus degenerates into one \mathbb{P}^1	7-branes	2
7-branes intersect / degeneration into two \mathbb{P}^1 s	matter curves	1
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- Internal space: 6 dim + elliptic fibre 2 dim
- Fibration can degenerate in the following ways.
- explain table
- Physical Reasons: B_6 has complex structure

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└ Given Equations

$$\begin{bmatrix} 0 &= & d_1 c_2^2 + b_0^2 c_2 - b_0 b_1 c_2 \\ 0 &= & d_1 b_1 c_2 - b_0^2 b_1 - c_2^2 d_1 \\ 0 &= & d_0 b_2 c_1 - b_0 b_2^2 - c_1^2 d_2 \\ 0 &= & d_1 c_1^2 - b_1 b_2 c_1 + b_2^2 c_2 \\ 0 &= & d_0 c_1^2 c_2^2 + b_0^2 c_1^2 - b_0 b_1 c_1^2 c_2 + c_2^3 (b_1 b_2 c_1 - b_2^2 c_2 - c_1^2 d_1) \\ && d_1 c_1^2 c_2^2 + (b_0 c_1^2 + c_2 (-b_1 c_1 + b_2 c_2)) \times \\ && \times (b_0 b_2 c_1^2 + c_2 (-b_1 b_2 c_1 + b_2^2 c_2 + c_1^2 d_1)) \end{bmatrix}$$

→ Algebraic Geometry

- Variables $\{b_i, c_i, d_i\}$ are polynomial functions on B_6
- Have: 8 variables + 5 scaling relations \rightarrow 3 complex dimensions
- Every two equations describe a possibly complicated 7-brane around an intersecting curve
- Every two equations define matter curve possibly more than one
- We have to analyse the zero set of polynomials. This is predesignated for the use of algebraic geometry.