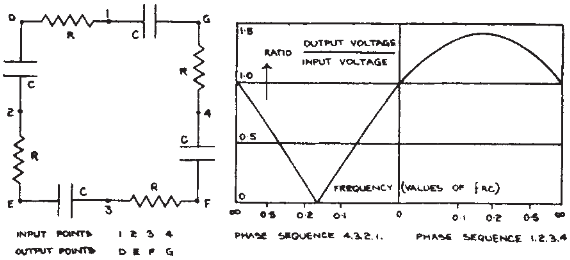

Fig. 2. BRIDGE NETWORK FOR FOUR-PHASE ALTERNATING CURRENT SUPPLY, AND ITS TRANSMISSION CURVE

opposite sequence, and the network transmits one and blocks the other. With this arrangement the impedances between points 2, E and 3 are, of course, omitted, and the arrangement appears as a simple phase-splitting network.

Fig. 2 shows a discriminating bridge for a four-phase supply of frequency f equal to $1/2\pi RC$, together with a calculated transmission curve. If this network is used to generate four-phase supply (or two-phase and neutral) from a single phase, then point 2, the neutral point and point 4 are made common, and voltage applied to points 1, neutral and 3 in the ratio $1:0:-1$. In this case the arrangement becomes a conventional phase-splitting network.

Madella has incorporated his concept of negative multiphase frequency in an improved form of heterodyne wave analyser[2]; he has also shown[3] that the concept gives a full explanation of the action of a modulating filter[4] which has been used in a modified form by D. G. Tucker[5] to give a highly selective transmission measuring equipment. Apart from the greater completeness which appears in the study of multiphase frequency as opposed to single phase frequency the concept seems likely to have considerable technical application and deserves to be more widely known.

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[3] Madella, G. B., *Wireless Eng.*, **24**, 310 (1947).
[4] Barber, N. F., *Wireless Eng.*, **24**, 132 (1947).
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Smoothing Periodograms from Time-Series with Continuous Spectra

IN his review[1] of M. G. Kendall's brochure[2] on oscillatory time-series, David G. Kendall made the pertinent observation that the smoothing of periodograms obtained from autoregressive or other time-series with continuous spectra is equivalent to considering the first few sample autocorrelations. I had arrived at a similar conclusion, though possibly by an alternative route, having noticed that the averaging of periodograms obtained from contiguous lengths of series is approximately equivalent to a truncation of the correlogram at a point represented by the length of the subseries. From preliminary computation already made on M. G. Kendall's artificial series to test out this smoothing device, it appears promising.

In terms of the periodogram intensity $I(\omega)$, standardized to have expectation equal to the variance σ^2 of the series if the latter were entirely random, and related to the correlogram by the identity

$$I(\omega) \equiv \sum_{s=-N+1}^{N-1} C_s \cos \omega s, \tag{1}$$

where C_s is defined in terms of the observations X_r as

$$C_s = \frac{1}{N} \sum_{r=1}^{N-|s|} X_r X_{r+|s|}, \tag{2}$$

the (continuous) spectrum of X_r, apart from a factor $1/(\pi\sigma^2)$, is the limit of $E(I)$ as N increases, where E denotes expectation[3]. For M. G. Kendall's series, with autocorrelations ρ_s satisfying $\rho_{s+2} + a\rho_{s+1} + b\rho_s = 0$, $(s \geqslant -1)$, it may be shown that this limit is

$$\frac{\sigma^2(1-b)\,(1-a^2+b^2+2b)}{(1+b)\{1+a^2+b^2-2b+2a(1+b)\cos\omega+4b\cos^2\omega\}}, \quad (0 < \omega \leqslant \pi). \tag{3}$$

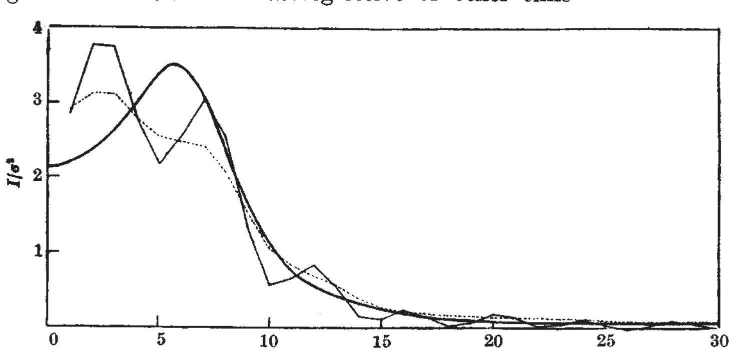
However, in accordance with general spectral theory and as remarked by the late P. J. Daniell, $I(\omega)$ unless smoothed never tends to this limit whatever N, random fluctuations being of the same order of magnitude as the expected value and only the 'resolving power' increasing.

The smoothing procedure first considered was to calculate

$$I_n(\omega) = \sum_{s=-n+1}^{n-1} C_s \cos \omega s, \tag{4}$$

where n is a submultiple of N, chosen in practice to be a compromise between the averaging power required and the avoidance of bias in the expected value of $I_n(\omega)$. The standard deviation of fluctuations should be approximately reduced by the factor $\sqrt{(n/N)}$ and the bias should remain small if n is still large enough for ρ_n to be effectively zero. A variant of (4) which should be less subject to systematic error, though also rather less convenient since it implies fixing n in advance, is to replace C_s by

$$C_s' = C_s\left(1 - \frac{|s|}{n}\right)\bigg/\left(1 - \frac{|s|}{N}\right). \tag{5}$$



SMOOTHED PERIODOGRAM (I_{16} JAGGED LINE, I'_{16} DASHED LINE) COMPARED WITH THEORETICAL SPECTRUM (SMOOTH CURVE).

In the diagram the expected intensity (divided by σ^2) is shown for Series 1 ($N = 480$), together with the values of $I_n(\omega)$ computed for $q = 30\omega/\pi = 1 \ldots 30$ and $n = 16$. There is a tendency to systematic oscillation in $I_n(\omega)$, but this is largely eliminated in $I_n'(\omega)$ (corresponding to C_s' in (5)), the values for which are also shown. Even $I_n(\omega)$, however, is a very great improvement over the unsmoothed periodogram, which includes a peak of 12 at $q = 3$ (see M. G. Kendall's Fig. 4.1, where it should be noted that the intensity contains an extra factor of approximately $2/N$ and is plotted against the trial period p, which is $N/8q$).

The arithmetical results also suggested the increase in the fluctuations about the mean as n was increased, but the computation (for most of which I am indebted to Mr. D. F. Ferguson) was carried out with the aid of the serial correlations recorded by M. G. Kendall, and hence was approximate and limited to n less than 48. It is hoped to test out and compare $I_n(\omega)$ and $I_n'(\omega)$ more precisely in due course. For Series 3 ($N = 240$) M. G. Kendall found the unsmoothed periodogram less confusing, owing to the closer approach of this series to an undamped harmonic, but the results for the modified periodogram still suggested an improvement even for n as low as 16.

Two further comments are: (i) this method of estimating the true spectrum could, of course, be improved if the correct form of the autoregressive scheme were assumed, its unknown constants estimated and the corresponding spectrum calculated, but a direct and more empirical estimation of the spectrum will often be required; (ii) the most rapid methods of obtaining spectra for series defined for continuous time avoid computation (for example, employ optical or electrical methods), and it may be possible to arrange for the smoothing to be carried out automatically without the above machinery of the truncated correlogram.

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Feb. 13.

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[2] "Contributions to the Study of Oscillatory Time-Series" (Cambridge, 1946).
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Race, Pigmentation and Colour Vision

Burt[1] has given evidence of a small correlation between darkness of skin pigmentation and colour-vision weaknesses. Vernon and Straker[2] have shown that red-green blindness is more common in the south and west of the British Isles, where the original dark-skinned inhabitants were pushed by the Nordic invaders. Geddes[3] has shown that red-green blindness is less common among the natives of Fiji than among peoples of Caucasian stock. Clements[4] has shown that red-green blindness is less common among American Indians and American Negroes than among American Whites.

Subjects whose colour vision was studied in Glasgow were classified as 'dark' or 'fair' on the basis of skin and hair pigmentation. There were 571 men and women who had colour vision with normal variations of sensitivity, and 138 colour blind and anomalous subjects (not all found by chance) who had major red-green defects which are sex-linked. There was no significant difference between the proportions of dark and fair among the normal and among the red-green defective subjects. The relative frequencies were:

	Normal subjects	Sex-linked red-green defectives
Dark	248	53
Fair	323	85

Protanopes, in whom the red end of the spectrum is greatly darkened, did not have dark skin pigmentation more frequently than deuteranopes. Sex-linked red-green defectives are on the average about ten times less sensitive to these colours than people with normal variations of colour vision. Any tendency for pigmentation to be coupled with major red-green defects in the population sampled in the south-west of Scotland would have been brought out by this experiment.

Among the normal group as a whole no relationship was found between variations in sensitivity to any colour and skin or hair pigmentation. If, however, twenty members of dark-skinned races were taken separately, they were found to have significantly lower sensitivities to blue and yellow (but not to red and green) than the remainder. Of this group, nine who were Dravidians were significantly less sensitive to yellow and blue than Europeans, while six West Africans were significantly less sensitive to these colours than the Dravidians.

It would appear that pigmentation is not associated with loss of colour sensitivity in Britain, except where there is evidence of racial differences. Negroes and American Indians are less often red-green blind than Europeans; but Dravidians and Negroes are more often weak in yellow and blue.

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[2] Vernon, P. E., and Straker, A., *Nature*, **152**, 690 (1943).
[3] Geddes, W. R., *Brit. J. Psych.*, **37**, 33 (1946).
[4] Clements, E., *Amer. J. Phys. Anthrop.*, **14**, 417 (1930).

Adsorption of Bacterial Polysaccharides to Erythrocytes

We recently reported that saline extracts of smooth strains of *Hæmophilus influenzæ* (Type *b*) contain a substance adsorbable to erythrocytes[1]. Cells treated with such extracts are agglutinated by type-specific antisera. A sample of the type-specific polysaccharide, prepared by the method of Dingle and Fothergill[2], absorbed from immune sera their power of agglutinating erythrocytes so sensitized, but was not itself capable of sensitizing them. We concluded, therefore, that the fraction in saline extracts which was adsorbed to erythrocytes was a combination of the type-specific polysaccharide with some other substance.

Further work has shown that the assumption of another substance in combination is unnecessary. The sample of polysaccharide employed had apparently been degraded during preparation. Influenzal polysaccharide has since been prepared by extraction with 90 per cent phenol[3,4]. The phenol-insoluble, water-soluble fraction of the organisms contains the type-specific polysaccharide in an undegraded state, in which condition it is adsorbed to the red blood corpuscle. If this undegraded polysaccharide is treated with dilute acid or alkali (0·01 N or less), it loses successively, first, the capability of being adsorbed to erythrocytes; secondly, the property of precipitation with antibody; and thirdly, the power of combination with antibody. The second and third of these changes have been shown by Morgan to occur during the hydrolysis of the polysaccharide of *B. Shiga*[5], and we have found that the undegraded Shiga polysaccharide is, like that of *H. influenzæ*, adsorbed to the surface of erythrocytes which then become agglutinable by Shiga antisera.

We have, so far, prepared polysaccharides which are adsorbed to erythrocytes from strains of *H. influenzæ*, *Meningococcus*, *Staphylococcus* and *Salmonella typhi*, *derby*, *typhi murium* and *pullorum*. Similar polysaccharides have been obtained from *Pneumococcus* and *B. proteus* by Dr. J. L. O'Connor, and from *Streptococcus* by Mr. P. L. Bazeley, of these Laboratories. The relationship of these polysaccharides